The application of affine transformations in matching distorted forensic samples with a common origin

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ABSTRACT

The practice of forensic odontology regularly requires the forensic dental expert to establish a degree of concordance between two objects, one or both of which have undergone minimal degrees of warping, shrinkage and distortion collectively described as deformations. These comparisons can be between the suspect’s or victim’s dentition and bite marks on inanimate objects/skin, two radiological images, palatal rugae patterns, dental arches, lip prints and other unique scenarios which are presented in forensic odontological cases.

This paper will define the mathematical concept of affine transformation as it pertains to forensic matters and explains how it can be applied to a case study involving pattern deformations. Although the deformation of the evidence may be minimal in extent, it creates a measure of uncertainty when expert evidence is given in court cases. Any metric discrepancy caused by deformation of the evidence will necessitate the application of pattern association and negate the use of metric analysis. It has been shown that a pattern association analysis of evidence will not be affected by minimal amounts of deformation. The mathematical limits of these deformations which will significantly affect the comparisons have not yet been determined. In the case study presented the deformations were considered minimal and thus explainable by a pattern association analysis.

The mathematical concepts will empower the expert to explain to a court of law how two samples which were not a perfect match, could in fact be deformed equivalents of common origin.

1. Introduction

The practice of forensic odontology regularly requires the expert to establish a degree of concordance between two objects one or both of which have undergone minimal degrees of deformation [1–4]. For the purposes of this paper, deformation will include warping, shrinkage and distortion. The deformation can be the result of a time delay, postural change [4,5], poor photographic technique or physical changes to the evidence [2,6]. These comparisons can be between the dentition of a suspect/victim and bite marks on inanimate objects/skin [1,3], two radiological images [7], palatal rugae patterns [8–10], dental arches, lip prints [8,11] and other unique situations which are presented in forensic odontological cases.

Any attempt at metrically analysing evidence which is known to have undergone deformations will have negative consequences for the expert witness under cross examination in court [1]. It is not possible for any expert witness to quantify the degree of deformation, and thus impossible to justify a metric comparison of any such evidence irrespective of the deformation present [5]. It has been shown that a degree of deformation will not affect a pattern association analysis of evidence [2]. This was demonstrated by comparing a set of facial portraits taken many years apart. The different portraits were clearly identifiable despite the induced digital changes and the deformation that had taken place. The three-dimensional relationship of the individual features present remained the same, irrespective of the deformation which had taken place. This cognitive concept was mathematically tested by Stols and Bernitz [5], who found that deformed bites could be reconstructed by affine transformations. It demonstrated that the deformed bite mark pattern was in fact a transformed image of the dentition. This paper seeks to demonstrate that two samples which differ in appearance as a result of a degree of deformation can be mathematically reconstructed to demonstrate a common origin.

2. Materials and methods

Affine transformations can be used to reconstruct distorted evidence as a result of stretching, shrinkage, enlargement, reduction, rotation, shearing or any combination of the above. This type of transformation preserves co-linearity, midpoints of segments, and segment division ratios [5]. Any affinity is the product

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of a shear, a strain and a similarity. These image distortions preserve both image relationships and key characteristics. The affine transformations do not, however, preserve distances. Applying an affine transformation to a deformed sample of evidence can correct for the enlargement, reduction, stretching and shearing of the mark. In order to reconstruct a transformed sample of evidence, three corresponding points need to be constructed on the evidence to be matched. The open source software GeoGebra (Dynamic Mathematics software) was used in this study to determine the affinity which maps the coordinates of the three pairs of collinear points. GeoGebra combines geometry and algebra and supports matrices which make it an ideal tool for determining the matrix representation of an affine transformation. This case represents the first application of GeoGebra in matching deformed samples.

After determining the matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ using GeoGebra the coordinates of the other identifiable points of the evidence where multiplied by the matrix $T$ in order to determine if it matched similar identifiable points of the evidence.

2.1. Actual case example

A real case study was used to demonstrate the degree of correlation that exists after the affine transformation was applied to a maxillary arch which had undergone alveolar absorption and thus deformed over a period of time. The authors were requested to match two dentures which were manufactured several years apart. Silicone models of the maxillary arches were constructed from the two dentures and compared. Although alveolar absorption is never perfectly uniform, the minimal changes and preservation of distinct features made the analysis possible. The following analytical calculations were carried out to demonstrate that the feature transformations present in the second maxillary arch (newer denture) did not significantly affect the relationships of the individual features present in both maxillary arches.

We need three pairs of points to define an affinity (Fig. 1). We can use the following pairs: $A (4.62, 0.81) \rightarrow A' (-1.13, 0.64)$, $B (4.99, -1.36) \rightarrow B' (-0.78, -1.37)$, and $C (2.93, -0.39) \rightarrow C (-2.8, -0.36)$.

The matrix that represent this affinity is:

$$\begin{bmatrix} 0.98 & 0.00 & -5.67 \\ -0.06 & 0.91 & 0.17 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix $T$ that defines the affine transformation was determined by mapping the point $A$, $B$ and $C$ on the first maxillary arch to the points $A'$, $B'$ and $C'$ on the second maxillary arch (Fig. 1).

To determine if both maxillary arches belong to the same individual we compare the irregularities at $D$, $E$, $F$, $G$ and $H$ on the first maxillary arch with possible irregularities $D'$, $E'$, $F'$, $G'$ and $H'$ on the second maxillary arch.

If the second maxillary arch was from the same individual as the first maxillary arch, the affinity $T$ must map points $D$, $E$, $F$, $G$ and $H$ that indicate characteristic features on the first maxillary arch and similar characteristic features on the second maxillary arch which are indicated by points $D'$, $E'$, $F'$, $G'$ and $H'$. When applying the affinity $T$ to the points $D$, $E$, $F$, $G$ and $H$ the figure shows that the transformed coordinates $D'$, $E'$, $F'$, $G'$ and $H'$ on the second maxillary arch do indeed indicate similar irregularities precisely at $D'$, $E'$, $F'$, $G'$ and $H'$ on the second maxillary arch.

3. Results

The two maxillary arches which appeared macroscopically different as a result of deformation, were shown to be of common origin by applying a geometric affine transformation to the samples. The authors could declare a match with a high degree of certainty.

4. Discussion

The matching of forensic odontological evidence will always involve a certain degree warping, shrinkage and distortion [5]. Defence advocates can use this fact to negate a match advocated by a forensic dental expert. In a court of law these deformations whether microscopic or macroscopic remain deformations and thus represent unexplained discrepancies in terms of a true match, unless they can be satisfactorily explained. Variations in tissue structure, dehydration due to time lapse and photographic technique can induce these minor deformations which will rarely be perfectly uniform in nature.

Stols and Bernitz [5] were the first to show that deformed bite marks could be reconstructed and thus matched to dentitions using affine transformations. The affine transformations in this study were reconstructed using the GeoGebra open source dynamic mathematics software. This software program combines arithmetic, geometry and algebra in determining the transformations. It is an interactive geometry system which makes provision for the exploration of possible relationships. These relationships are between features which can be identified on both samples to be compared irrespective of feature distortions which could have taken place.

This case study involved a Caucasian male who had been murdered. On investigation it was observed that the victim’s denture had been removed. Investigating officers found a set of dentures and other missing goods at the suspect’s house. The dentures were sent for DNA analysis but the results were negative. During further investigation of the case, a denture belonging to the victim was presented to the investigators by a family member. The victim had kept the denture made some years previously as a spare. The investigating officer then requested that the two dentures be compared and an opinion given as to their commonality. Objective comparison showed that very similar characteristic features were present in both maxillary models but that these were distorted as a result of bone resorption which had taken place over a period of time. AFFINE transformation was used to reconstruct the distorted maxillary arch of the second denture and declare a highly probable match. This real case study involving maxillary arches, clearly demonstrated that mathematically calculable affine transformation had taken place in the maxillary arches during the years between the manufacture of the first and second dentures and that the second maxillary arch was in fact a distorted image of the first maxillary arch.

5. Conclusion

This paper demonstrates that affine transformations can be applied to samples of forensic odontological evidence which
possess characteristic features and demonstrate various degrees of transformation, and match them with a high degree of certainty.

References


