## Response Model for flooding events on the Cape Flats

by

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## **Executive summary**

Disasters are known to have no boundaries. Over the last thirty years the annual number of disasters world-wide has tripled which leads to humanitarian crises. Disasters usually occur unexpectedly resulting in incredible suffering and damage within a short period of time. No matter how advanced a nation may be it is never immune to the negative effects of disasters. Disasters have been in the world since its existence. Civilisations have been decimated in an instant. Populations as much as 50% have been reduced due to pandemics and epidemics. Non-profit organisations are actively involved in trying to minimise the amount of damage that can occur and provide aid to the community affected. South Africa is at risk of natural disasters due to the country being located within a region that has a semi-arid to arid climate and the population is living in fragile and vulnerable conditions. Climate change will also increase the occurrence of disasters world-wide and the risks involved. The need for assistance in how to prevent, predict, prepare and manage disasters therefore exists. South Africa can gain from having more control in dealing with these critical issues, by creating a more effective response system, improving information flow in, out and within these disasters and trying to find the root causes of the crises.

By implementing a valuable response model for a critical disaster in South Africa, the disaster's boundaries can be limited and negative impact on the community will be minimised. This is a formal document for a project aimed at developing an optimum response model for flooding events on the Cape Flats which has been identified as a vulnerable area for flooding crises in South Africa. To address this problem, solutions to similar problems existing in the literature were studied in order to find a viable approach to use. It was found that the problem could be best handled through the use of mathematical modeling. This mathematical approach was divided into three stages. LINGO software was used to solve the mathematical models of each stage that was developed. The solutions of the models provided a total system approach to respond optimally to a flooding event on the Cape Flats.

## **Table of contents**

1	Intr	oductio	1					
	1.1	Problem statement						
	1.2	Project	4					
	1.3	Project	4					
	1.4	Deliver	4					
2	Lite	erature r	eview			6		
	2.1	Natural	hazards i	in South A	frica	6		
		2.1.1	Floods -	- general i	nformation	10		
	2.2	Disaste	r manage	ment syste	m	11		
		2.2.1	Key con	nponents		11		
			2.2.1.1	Mitigatio	on	11		
			2.2.1.2	Prepared	ness	12		
			2.2.1.3	Response	2	12		
			2.2.1.4	Recovery	y	13		
		2.2.2	Informa	tion techn	ology in South Africa	13		
		2.2.3	Stakeho	lders		14		
	2.3	Flood forecasting, warning and response systems (FFWRS)						
		2.3.1 Components of FFWRS						
			2.3.1.1	Detection	n and forecasting	17		
			2.3.1.2	Warning		17		
				2.3.1.3.1	Flood warning codes and messages	17		
				2.3.1.3.2	Warning communications			
				2.3.1.3.3	Warning response	19		
			2.3.1.3	Response	2	21		
			2.3.1.4	Learning		21		
		2.3.2	A Socia	l perspecti	ve to improve FFWRS	21		
	2.4	Human	itarian log	gistics		23		
		2.4.1	Quantita	ative methe	ods	23		
			2.4.1.1	A quick re	esponse model	24		

2.5	FFWRS - application in South Africa	.30
2.6	Concluding the literature review	.31

#### 

#### 4. Results

**48** 

#### 5. Conclusion and further research

A Mathematical models in LINGO	60
A.1 Model and solution for stage 2	60
A.2 Model and solution for stage 3 – routing	63
A.3 Model and solution for stage 3 – supply	76

#### 

## List of figures

Figure 2.1: Number of occurrences for each type of hazard (1800-1995)7
Figure 2.2: Fatalities per event type
Figure 2.3: Average annual rainfall in South Africa9
Figure 2.4: Cape Flats10
Figure 2.5: Components and characteristics of FFWRS16
Figure 2.6: Key elements of a flood warning system20
Figure 2.7: Fuzzy membership functions for the measures of relief demands
and supplies26
Figure 3.1: Plotted relief centres and disaster areas on the axis system
Figure 4.1: Grouping of disaster areas into disaster groups
Figure 4.2: Assignment of relief centres to disaster groups
Figure B.1: Plotted disaster areas and relief centres

## List of tables

Table 2.1: Flood warning codes for England and Wales (September 2000) $\dots$	18
Table 2.2: Technologies for communicating flood warnings	19
Table 2.3: Methodology for deriving economic benefits of flood warnings:	
definition and explanation of expressions	22
Table 3.1: Informal settlements from worst to least affected by floods	40
Table 3.2: Disaster relief centres	41
Table 3.3: Supply of relief centres.	42
Table 3.4: Demand of disaster areas	43
Table 3.5: Demand of disaster groups	44
Table 3.6: Fuzzy membership and cost values of disaster groups and relief ce	ntres46
Table 4.1: Disaster areas assigned to disaster groups	48
Table 4.2: Relief centres assigned to disaster groups	49
Table 4.3: Routing for disaster group 3	51
Table 4.4: Routing for disaster group 4	51
Table 4.5: Routing for disaster group 5	51
Table 4.6: Routing for disaster group 6	
Table 4.7: Supply to disaster group 1	52
Table 4.8: Supply to disaster group 2	52
Table 4.9: Supply to disaster group 3	53
Table 4.10: Supply to disaster group 4	53
Table 4.11: Supply to disaster group 5	53
Table 4.12: Supply to disaster group 6	54

## List of acronyms

WHO	World Health Orginisation
DMISA	Disaster Management Institute for Southern Africa
NDMC	National Disaster Management Centre
PDMC	Provincial Disaster Management Committee
ISDR	International Strategy for Disaster Reduction
SADC	Southern African Development Committee
AU	African Union
UNDAC	United Nations Disaster Assessment and Co-ordination
UNEP	United Nations Environmental Program
SAWS	South African Weather Service
GIS	Geographic Information System
NDMIS	National Disaster Management Information System
SRS	Situation Reporting System
SAC	Satellite Application Centre
SANDF	South African National Defense Force
SAQA	South African Qualifications Authority
FFWRS	Flood Forecasting, Warning, and Response System
BANCID	Bangladesh National Committee of the International Commission on
	Irrigation and Drainage
SPRC	Source Pathway Receptor Consequences

## **Chapter 1**

## Introduction and problem background

According to a report by the Fritz-Institute [1], over the last thirty years the annual number of disasters world-wide has tripled. A disaster is an event which disrupts the daily life of the population of a community or country. Disasters can cause a negative impact on humanity such as loss of human lives, damage to infrastructure, suffering, destruction of ecosystems and even economic loss, inflicting a heavy cost on human, material and physical resources, damage to the environment and leading to many persons becoming homeless, helpless and hungry. Disasters represent a potential significant obstacle to economic growth and development. Disasters occur when hazards impact on a community to the extent that available resources cannot cope with the problem effectively. The community itself needs support and assistance to prevent and cope with disasters and their effects. Adequate procedures to deal with disaster situations and relief measures must be planned prior to the event, with strong legislation to empower those responsible to carry out the task. Regular training must be conducted covering all aspects of disaster management. Careful planning must be in place to coordinate the effective use of resources, both human and physical, for the saving of lives and property, limiting damage to the environment, and the return to a normal lifestyle as soon as possible.

Disasters can be classified in short-term, quick moving disasters, such as floods, and long-term, slow moving disasters, such as the HIV/AIDS pandemic. Certain situations world-wide can be classified as disasters such as hurricanes, cyclones, earthquakes, volcanic eruptions, population movement, man-made disasters and technological disasters. Disasters can also be defined as man-made or natural. Man-made disasters include war, civil war, violent conflicts, population mass movement, pollution or technological disasters. Natural disasters include hurricanes, cyclones and typhoons, epidemics, floods and droughts.

In South Africa the most common natural disasters are floods, hailstorms, gale-force winds, bush fires and heavy snowfalls. Poor preparations such as contravening regulations (e.g. people deliberately erecting shacks below the flood line), authorities

pretending there is not a disaster looming (e.g. HIV/AIDS), safety systems being switched off (e.g. Chernobyl), maintenance not being done, and early warning systems being ignored (e.g. the Indian Ocean tsunami) are some of the biggest problems in South Africa when dealing with disaster. There is also a drastic increase of immigration to South Africa from neighbouring countries, where resources and infrastructure are limited and not able to support their needs. This leads to another humanitarian crisis.

Due to global warming and climate change in South Africa it is important to cope with extreme weather conditions that can lead to critical disasters.

Critical disasters can be classified as crises affecting human life and ecosystems where recovery is critically needed and humanitarianism required. If no intervention takes place, it will lead to the destruction of human life and resources.

Humanitarianism can be described as "an active belief in the value of human life, whereby humans practice benevolent treatment and provide assistance to other humans, in order to better humanity for both moral and logical reasons" according to de Torrent [2]. An active partnership between national, provincial and local governments, statutory and voluntary organisations and communities is needed in order to develop and implement effective disaster management strategies. The Fritz Institute, INSEAD, World Health Organization (WHO), World Food Program and the UN Joint Logistics Centre are a few of the institutions and humanitarian agencies involved in disaster management. For these organisations to perform excellent humanitarian operations, an effective support-system and more effective use of technology need to be in place. The support-system consists of objective performance metrics, research and institutionalised learning.

According to Altay *et al* [3] the Disaster Management Institute for Southern Africa (DMISA) is an important role-player involved in humanitarian security which includes normalisation of a country after violent conflict, environmental problems with related risks involved and security problems which arise in everyday life. In South Africa the role of the national government is to provide guidance and support to the provincial and local governments in developing their capacity for dealing with disasters, and provide assistance if requested according to the National Disaster Management Centre (NDMC) [5]. In South Africa, participation of certain forums takes place on a regular basis and international assistance is also provided when required.

According to the analysis of disaster processes, logistics form a critical part when dealing with disasters. Humanitarian logistics specialises in delivery and warehousing of supplies during crises (natural disasters and complex emergencies) to the affected area and people. In order to improve humanitarian logistics an information infrastructure is required. The correct data must be provided so that preparation and management of disaster situations can be performed effectively.

This is needed so that the supply processes in the humanitarian relief chain (component of humanitarian logistics) can be executed as quickly as possible when required. This information must be translated into understandable language and must be accessible to the communities at risk.

Quantitative methods are used to analyse the disasters in humanitarian logistics so that the performance of the relief of chain can be improved. These quantitative methods consist of mathematical programming, heuristic methods, probability theory, queuing theory, etc.

Quantitative measures can be used for conflict scenarios, food crises, inventory modelling, and casualty treatment after earthquake disasters. In a natural disaster case, pre-positioning of facilities, quick response models, routing and scheduling problems can be used.

### **1.1 Problem statement**

South Africa's natural resources and infrastructure are limited and currently struggling to support the community when disasters occur. This leads to incredible suffering and economic loss within a short period of time. There exists a gap between the authorities and the community affected by the disaster where there is an ineffective flow of information. This information entails general precautions, procedures, aid being provided and regulations that are critical when preparing and managing disaster situations. This indicates the dire need for disaster risk reduction with the purpose of minimising vulnerabilities and disaster risks throughout society in order to avoid (prevention) or to limit (mitigation and preparedness) adverse impacts of hazards and facilitate sustainable development. The Cape Flats is a flood prone area, consisting of informal settlements that developed as a result of limited housing available in the surrounding formal areas. Therefore, relocation is not a viable option

at present. The only assistance that can be offered during a disaster situation is to respond effectively and efficiently.

## 1.2 Project aim

The aim of this project is to develop and optimise a disaster response model for potential flooding on the Cape Flats. This model should provide for quick response resulting in minimum damage. The solution that's presented should be realistic and applicable to the humanitarian environment in general including different types of disaster crisis. Disaster management teams must be able to use the mathematical model presented, by applying this model to a specific disaster case leading to optimum decision making.

### **1.3 Project scope**

A review of the natural disaster types occurring in South Africa, established that flooding is the most critical hazard type. The informal settlements on the Cape Flats have been identified as one of the most vulnerable communities to flooding. The main focus of this project is to develop a quick response model for a flooding disaster on the Cape Flats. This quick response model is divided into three stages to form an optimum total response system. The output of the different stages will be analysed to form one response approach for flooding events on the Cape Flats. This response model will be viewed as a general approach that can be taken by all stakeholders involved in this disaster crisis.

## **1.4 Deliverables**

Mathematical models and methods are to be used to obtain a response model for flooding in the Cape Flats. In the first stage the most vulnerable areas in the Cape Flats need to be identified and prioritised. These disaster areas must be divided into disaster groups using an axis system approach. The output that needs to be obtained from the second stage is the identification of relief centres and the assigning of relief centres to disaster groups. The mathematical model must ensure that a relief centre can respond to many disaster groups, while a disaster group can only be served by one relief centre. In stage 3, an optimum route strategy needs to be developed and the supply to the disaster areas needs to be calculated. By integrating these three stages an optimum response approach should be delivered.

## **Chapter 2**

## Literature review

## 2.1 Natural hazards in South Africa

Natural disasters that occur and affect South Africa the most needs to be identified and the most critical disaster will be focused on for this project. Disaster trends in South Africa need to be analysed to identify the most critical disaster. According to the NDMC [5] a record of important weather events of the past is kept by the South African Weather Service (SAWS). These records include date of occurrence, degree of damage, affected area and the frequency of the type of disaster in a specific area. The frequency of the different hazard types in South Africa (1800-1995) are given in Figure 2.1 (Cealum Statistics [5]). The number of hazardous events recorded in South Africa was 946.



Figure 2.1: Number of occurrences for each type of hazard (1800-1995)

The consequences as a result of loss of life and injuries are regarded as the most critical feature of hazardous events. The number of fatalities for each of these hazard types (1800-1995) is depicted in Figure 2.2. The total number of fatalities amounted to 2085 people.

Figure 2.2: Fatalities per event type



This statistical analysis (Figure 2.1 and Figure 2.2) shows that flooding are the hazard type that occur the most frequently and can be seen as the most single "killer event" and therefore has been identified as the most critical disaster in South Africa. The average annual rainfall in South Africa is depicted in Figure 2.3.

Figure 2.3: Average annual rainfall in South Africa



South Africa has been classified by the NDMC [5] as a "wet and windy" country. Approximately 59% of the disaster cases are due to "wet" events and 33% due to "windy" events. In Figure 2.3 the coastal provinces are the areas in South Africa with the highest average annual rainfall. Floods can cause infrastructure damage, loss of lives, increase in epidemics and diseases, contamination of water and water supplies and shortage of food. Entire harvests can be lost and communities affected can be forced to evacuate their property. Floods can also have a tremendous impact in South Africa's economy. According to the NDMC [6] in the year 2000 a total loss of R 361 million was caused by floods.

The Cape Flats is situated south-east of Cape Town. This is a vast area which is lowlying and flat. According to the Department of Environmental Affairs and Tourism [26] people living at the coast, informal settlements and rural areas are regarded as being the most vulnerable to impacts of climate change in the Western Cape. During late summer, increased heavy rainfall often occurs. This leads to extensive flooding that could be a threat to these vulnerable areas. The Department of Environmental Affairs and Tourism stated [26] that the informal settlements of Cape Town are especially vulnerable, since they are living on the Cape Flats which has a high water table with an inadequate infrastructure. According to Powell [32] there are two hundred and twenty five informal settlements across Cape Town where most of them are situated on the Cape Flats. These Informal Settlements are located in a flood prone area. The Cape Flats area can be viewed in Figure 2.4.

#### Figure 2.4: Cape Flats



### **2.1.1 Floods – general Information**

According to Damon [4] flooding is by far the most common natural hazard that occurs throughout the world. Annually more people are killed throughout the world by flooding than by any other hazard, with an average of 20 000 deaths and 75 million people affected each year. Floods can either be slow or fast rising, generally developing over days or weeks. Floods are a hazard resulting from other meteorological processes. These processes can be defined as prolonged rainfall, localised and intense thunderstorms or onshore winds. According to Damon [4] the five most commonly flooded geographic land types are river floodplains, basins and

valleys affected by flash flooding, land below water-retention structures (dams), low lying coastal and inland shorelines and alluvial fans.

Jonkman [21] classified different types of floods such as coastal, flash, river, tsunamis, tidal wave and drainage problems due to high precipitation levels that cannot be handled by regular drainage systems. Becker *et al* [14] identified some technical measures to control extreme floods. These technical measures that can be implemented are reservoir systems, controllable polders and constructions.

Secondary effects of flooding include coastal erosion and soil erosion. Erosion effects can increase the chance of future flooding, resulting in a vicious cycle of repeat flooding and further erosion.

### 2.2 Disaster management system

The general goals of a disaster management system are to reduce the risk likelihood, consequences, avoidance, acceptance and transfer of natural hazards.

According to Damon [4] disaster management can be divided into four distinct components – mitigation, preparedness, response and recovery. Each of these components needs to be fully utilised to form an effective disaster management system.

### 2.2.1 Key components

#### 2.2.1.1 Mitigation

According to Damon [4] mitigation is defined as "any sustained effort undertaken to reduce a hazard risk through the reduction of the likelihood and/or the consequences component of that hazard risk". Mitigation reduces the negative effects of a hazard if it were to occur and seeks to make a hazard less likely to occur. Two types of mitigation are identified as structural mitigation and non-structural mitigation. Structural mitigation examples are relocation resistant construction, building codes and regulatory measures and structural modification. Non-structural mitigation examples are regulatory measures, community awareness and education programs, environmental control and behavioural modification. According to Damon [4] mitigation measures cannot eliminate all disaster risks, but are very effective at reducing them.

#### 2.2.1.2 Preparedness

Disaster preparedness can be defined as actions taken in advance of a disaster to ensure satisfactory response to its impacts, and the relief and recovery from its consequences is performed to eliminate the need for any last-minute actions. The range of activities that constitute to the preparedness component of the disaster management cycle is expansive, and these actions are often the primary factors that determine whether actual response actions are successful. One must know how to respond to the specific disaster and be equipped with the right tools to respond effectively. Being prepared minimizes the negative and harmful effects of hazards. A timely, efficient and appropriate response and relief is ensured by effective measures. According to Damon [4] preparedness must occur at both the government level and the individual level to reduce risk and vulnerability.

#### 2.2.1.3 Response

The response function of disaster management includes actions performed before, during and immediately after a disastrous occurrence which are aimed at minimising injuries, deaths and damage to property. Response is conducted during high stressful period, in an environment with minimal time available and with hardly any information available. Response includes also immediate needs such as first aid, search and rescue, and shelter. Response to a disaster must occur as soon as the hazard is recognized by officials with the authority to commence the response effort. The response actions that may take place during the pre-disaster period are warning and evacuation, pre-positioning of resources and supplies, last-mitigation and preparedness measures. According to Damon [4] once disaster response begins, the first priority is saving lives. As response resources are mobilized, additional functions will be added to the list in increasing priority, to include shelter, sanitation, security, donations management and fatality management. The response actions that may take place during below the list in increasing priority is saving lives. As response resources are mobilized, additional functions will be added to the list in increasing priority, to include shelter, sanitation, security, donations management and fatality management. The response actions that may take place during post-disaster period are provision of relief for the immediate needs of the victims of the disaster, repair of damaged buildings and infrastructure, recovery and

regeneration of the environment and the economic activities in the affected area and review of the activities that was performed in order to improve the process and planning for similar future disaster in the area.

#### 2.2.1.4 Recovery

According to Damon [4] even with the best mitigation, preparedness and response, there will almost always be some level of environmental damage, destruction of property and infrastructure, disruption of social and economic systems, and other physical and psychological health consequences. The process by which all these are rebuilt, reconstructed, repaired, and returned to a functional condition is called recovery. Some actions and activities commonly performed in the recovery period are ongoing communication with the public, provision of temporary housing or long-term shelter, assessment of damages and needs, demolition of damaged structures, rehabilitation of infrastructure, new construction, social rehabilitation programs, creation of employment opportunities, rehabilitation of the injured and reassessment of hazard risk

#### 2.2.2 Information technology in South Africa

For a disaster management system to function effectively a supportive information technology system needs to be available. According to the NDMC [5] effective timely warning systems need to consider hazards and community vulnerabilities such as poverty growth, environmental degradation, populations located in high risk areas, civil strife, a lack of access to information and low preparedness. For early warning systems to work effectively the use of technology as enabler is important, the information must be in an understandable language and accessible for the communities at risk. Weather forecasters observe the start of hurricanes and tropical storms. Drought experts estimate the chances of rain and oceanographers plumb the depths of the Pacific Ocean for signs of the El Nino and plot the travel of ocean-crossing tsunamis caused by earthquakes. According to the NDMC [5] South Africa implemented the Geographic Information System (GIS) for hazard mitigation activities and early warning. GIS plays a crucial role in the development of the NDMC's enhanced Nation Disaster Management Information System (NDMIS). This

system relates to various aspects of Hazard Analysis, Vulnerability Assessment, Risk Reduction and Contingency Planning, Incident Reporting Systems as well as Early Warning Systems. The Situation Reporting System (SRS) is a web-based system used by disaster management officials to capture data relating to disaster events occurring in South Africa according to the NDCM [5]. The NDMC has entered into an agreement with the Satellite Application Centre (SAC) to procure their sensor portfolio to assist and enhance the NDMIS. This space technology will be used in disaster management by interacting satellite imagery, remote sensing techniques and GIS information. This will help to identify the communities and assets at risk. The quantification of the disasters that occur is currently being measured by the extent of live fires, floods and storms. According to the NDMC [5] a substantial amount of archived satellite images are available, that enables analysts to view changes in the environment. Combining this information with historical events, reasons why certain events occurred in the past can be discovered and assist with modelling in future events.

#### 2.2.3 Stakeholders

Another important criterion for a disaster management system to function effectively is for stakeholders to be actively involved to minimise the amount of damage that can occur and provide aid to the community affected. According to the NDMC [5] the following departments are involved in disaster risk reduction in South Africa, The Department of Agriculture – focusing on mitigation and prevention, The Department of Health – significant role in respect of resources and response to disasters, The National Department of Public Works - part of risk management strategies to all assets and infrastructure, The Department of Science and Technology - initiate Research Learnership Programs focusing on Disaster Management and Early Warning Systems, The Department of Water Affairs and Forestry – is the custodian of the country's forest and water resources. Dealing mostly with forest fires and is responsible for the equitable, sustainable and efficient management of South Africa's water resources, The South African National Defence Force (SANDF) – is mandated to provide humanitarian aid and support, nationally and internationally, in disaster situations, The South African Police Service – ensures legal compliances through the development of a disaster management strategy and policy, The South African

Qualifications Authority (SAQA) – development and recognition of qualified Disaster Managers, and the South African Weather Service (SAWS) – maintains a climatology database of weather data over South Africa that is used regularly in disaster risk reduction and mitigation activities by various role-players. In South Africa participation of certain forums takes place on a regular basis such as:

- Provincial Disaster Management Committee (PDMC)
- International Strategy for Disaster Reduction (ISDR)
- Southern African Development Committee (SADC)
- African Union (AU)
- United Nations Disaster Assessment and Co-ordination Committee (UNDAC)
- United Nations Environmental Programme (UNEP)

For the Cape Flats floods the following stakeholders are actively involved:

- Disaster Risk Management Centre
- Department of Transport
- Department of Roads and Storm water
- Department of Heath

# 2.3 Flood forecasting, warning and response systems (FFWRS)

Flood Forecasting and Response System (FFWRS) can be seen as a subcomponent of disaster management. FFWRS is a flood control option that can be installed to reduce tangible flood losses. Bangladesh National Committee of the International Commission on Irrigation and Drainage (BANCID) [7] expressed that "Flood forecasting and warning has been identified as a key component that could exert major benefits on numerous aspects of national life, with considerable potential for improving the national economy. As such it is recognised as a highly cost effective, non structural measure". According to Parker *et al* [8] FFWRS has been implemented in Europe to reduce material, human and cultural losses. According to Davis *et al* [12]

The potential damage that can be caused when not responding effectively to a flood exceeds the installation cost of an FFWRS by far. The cost associated with flood warning is less than the cost required for flood control actions. In figure 2.5 FFWRS is portrayed as a five-stage process for reducing the impacts of floods. It starts with detection of the environmental conditions leading to flooding.



Figure 2.5: Components and characteristics of FFWRS

An important feature of FFWRS is the span of technocratic discipline of meteorology, hydrology and hydraulic engineering and the human behavioural, sociological and organisational disciplines. In A in Figure 2.5, these disciplines are not successfully brought together and integrated as they need to be (as in B). According to du Plessis [6] this is one of the causes of ineffective or partially effective FFWRS. The four main customer groups that can be identified in a FFWRS are the people who occupy, live and work in floodplains, emergency planning and response agencies, broadcast and other media who inform the public of events and who have a role in both disseminating flood warnings and providing information about flooding, and flood defence agencies. According to du Plessis [6] an effective FFWRS must serve the needs of all these customer groups. Members of the affected community also vary according to personal communication networks they possess, their flood risk experience and awareness and their knowledge of appropriate response to flood warnings.

### **2.3.1 Components of FFWRS**

#### 2.3.1.1 Detection and forecasting

Risk analyses can form part of the detection component. According to Mai *et al* [20] risk is defined as the probability of a disaster related to the consequences. One formal approach to risk analysis in Europe [16] is the Source-Pathway-Receptor-Consequences (SPRC) model. The Source and Pathway determine the probability and the nature of the hazard and the Receptor and Consequences determine the impact of flooding on whatever human society values. The detection component consists of data acquisition, data transmission and assembly, while the forecasting component consists of modelling and interpretation. There are different forecasting models available for different types of floods. Charles *et al* [29] used an ingredients-based methodology to develop a forecasting method for flash floods. Campolo *et al* [27] developed a neural model to analyse and forecast the behaviour of the river Tagliamento in Italy. This model makes use of distributed rainfall information coming from several rain gauges in the mountain district and predicts the water level of the river at the section closing the mountain district. According to Chen et al [28] China uses a reservoir flood forecasting and control system as a forecasting method. Although different forecasting models are available, the correct interpretation and understanding of the chosen model is critical in order to facilitate the flow of information to the people involved.

#### 2.3.1.2 Warning

#### 2.3.1.3.1 Flood warning codes and messages

In most hazard warning systems, warning scales are used to provide an appropriate means of categorizing risk and communicating to those who need to know about the risk. The advantage that warning scales can provide is that they save time when a flood occurs, especially in flash flooding. According to Parker [18] in September 2000, England and Wales employed a four stage flood warning code system [18]. This was designed by the Environment Agency to categorise the severity and urgency

of unfolding events so that this can be communicated to the public and other costumers. The flood warning code is presented in Table 2.1.

Flood Watch	Flood Warning	Severe Flood Warning	All Clear	
* Flooding is possible Be aware ! Be prepared ! Watch out !	* Flooding of homes, businesses and main roads is expected Act now !	* Severe flooding is expected Imminent danger to life and property Act now !	* There are no floods watches or warnings currently in force in the area	

Table 2.1: Flood warning codes for England and Wales (September 2000)

Problems that may occur when using these warning codes to often be that the public can become de-sensitised by this over-exposure. Situations do exist where active warning system seem unpractical and physically impossible, especially in smaller communities. The only feasible solution for these smaller communities is to provide facilities which enable communities to handle their own river watch system. Warning messages should also suggest or indicate appropriate response as well as giving information about when the flood will occur, how long the flood will last, where the floodwater will come from, and the depth and velocity of the floodwaters.

According to du Plessis [6] where a short warning time and inaccurate forecasts are evident, flood warnings are found to be ineffective in practice. Therefore flood warnings should reach the communities affected at the right time and will be of no value when this information is not understood by these communities for them to respond effectively.

#### 2.3.1.3.2 Warning communications

Efficient communication and dissemination of warnings will create a more effective FFWRS. A variety of information and communication technology are available. Table 2.2 shows the technology available for communicating flood warnings categorised into well-tried, new methods and near future methods in terms of their application to flood warnings.

Table 2.2: Technologies for communicating flood warnings

Well Tried	New	Near Future
* Standard telephone	<sup>4</sup> Press-button digital	* Electronic mail
* Radio telephone	elephone	* Internet and website
* Radio	<sup>5</sup> Mobile telephone	with real-time warnings
* Facsimile	<sup>6</sup> Pagers	* WAP telephones
* Flood siren	<sup>6</sup> Automatic voice	* Activation of local
* Door to door	messaging	radio/electronic signal
* Loudspeaker	<sup>6</sup> Flashing signs	alerts
* Written letters	<sup>6</sup> Recorded messages	* Integrated AVM/
* Leaflets	<sup>6</sup> Floodline	floodline service
* Flood wardens	<sup>6</sup> Television	* Home computer links
* Automatic water level	<sup>6</sup> Electronic file transfer	* Real time data on web
alert linked to telephone	<i>i</i> a AVM	* MMWDS

#### 2.3.1.3.3 Warning response

Alexander [11] divided flood warning systems into passive and active flood warning systems. Passive warning systems (marking flood lines, beacons and telephone standards) make people aware of flood hazards. Communities could therefore take own responsibility by observing flood waters and take necessary actions. The desired results, however, are seldom achieved since they often lose interest if disasters do not occur regularly. Three conditions should be compiled with when using the more formal active warning system: immediate response to an issued warning; timely and executable information transfer about flood hazards to all people at risk; adequate relief centres. This is crucial for the information to be received, processed and interpreted without which appropriate action is impossible. Situations do occur where active warning system seem unpractical and physically impossible, especially in smaller communities. The provision of facilities enabling these smaller communities to operate their own river watch system is the only viable solution. According to Smith and Handmer [10] false warnings, which can lead to a lowering in response of the inhabitant of flood plains, needs to focus on three important factors: involvement of the relevant institutions; appropriate flood forecasting and issuing technology; adequate education level and response of the communities being warned. The key elements of a flood warning system as discussed by Downing [9], is represented diagrammatically in Figure 2.6.

Figure 2.6: Key elements of a flood warning system



The key elements in Figure 2.6 consists of short-term warning actions which starts with weather forecasting procedures, followed by dissemination of flood warnings by catchment boards, and ending with the response to the flood warning. Furthermore it includes the actions to be taken to enable long-term prediction, aimed at avoidance of

loss of lives and damage to property. Mapping procedures are used to delimitate these possible hazards.

#### 2.3.1.3 Response

Both individuals and organisations will need to respond to a disaster. Humanitarian logistics is a critical component of response methods. In the humanitarian relief chain, supply processes are an important domain for managing the humanitarian logistics. These response processes' ultimate objective is to deliver the correct number of people, goods and monetary resources to the affected locations.

#### 2.3.1.4 Learning

After a flood incident has occurred, FFWRS has to be evaluated, so that the effectiveness of the process can be improved by making the needed adjustments. This will ensure a continual process for improvement. By improving FFWRS can lead to economical advantages. According to du Plessis [6] the evaluation can be done by distinguishing between the forecasting and warning system and the response system. The forecasting and warning system can be regarded as the information system, by determining the potential and actual quality of the information provided. The response system which, according to Smith and Handmer [10], can be regarded as a decision-making system can be evaluated on the basis of an optimal and actual response strategy.

#### 2.3.2 A social perspective to improve FFWRS

In Europe [18] effectiveness of Environment Agency's investment model is measured as actual flood damage avoided (FDA). This investment model is based on the methodology aiming at maximizing the economic benefits of flood warnings. It is measured by:

#### FDA = (AAD x DR x C) x (R x PRA x PHR x PHE)

(Flood damage component) x (Warning response component)

In Table 2.3 expressions are defined and explained.

Expression	Definition	Explanation				
FDA	Actual flood damage avoided	The economic value of the losses avoided through the provision of a flood warning service				
AAD	Annual average flood damages	The anticipated annual average damages to property at the existing levels of protection				
С	Coverage of the flood warning system	The percentage of the properties defined at risk which receive a full, targeted flood warning service				
DR	Damage reduction factor	The potential flood damage which could be avoided from the amount of warning received by at risk properties where flooding occurs				
R	Reliability of the flood warning service	The proportion of at risk properties which are warned with sufficient lead-time to take action				
PRA	Proportion of residents available to respond to a warning	The proportion of properties in which at least one adult is awake/at home/at work and can also receive a flood warning				
PHR	Proportion of residents able to respond to a warning	Proportion of at risk residential properties where the occupants are able to respond (this recognises the numbers who are elderly,disable, ill, pregnant or otherwise unable to respond to reduce flood damage)				
PHE	Proportion of residents who respond effectively	The proportion of residents who are willing to respond effectively and who take appropriate action to minimise flood damage and personal risk				

Table	2.3:	Methodology	for	deriving	economic	benefits	of	flood	warnings:	definition	and
explanation of expressions											

According to Parker [18] well developed methods by which potential damages to different property types as a result of floods are estimated, exists in England and Wales. If the property types, property heights and flood frequency-magnitude-level relationship in any flood plain are known, the annual average flood damages (AAD) can by estimated. This is done on a property-by-property basis. According to Parker [18], some of these expressions can be improved. Development of the reliability of methods used to forecast floods and extension of rainfall and runoff instrumentation and telemetry will improve C. Improvement of the processes used for flood forecasting, which includes modelling, and enhancement of the dissemination system used for flood warning will improve R. Improving PRA is done by the introduction of dissemination systems with improved penetration and reach for warning against floods. An increase in the public awareness of potential flooding events and the associated warning processes, as well as their knowledge of the correct response procedures to be followed in the event of a flood warning, will improve PHE.

### **2.4 Humanitarian logistics**

The Fritz Institute [1] defined humanitarian logistics as "the process and systems involved in mobilising people, resources, skills and knowledge to help vulnerable people affected by natural disasters and complex emergencies. It encompasses a range of activities, including procurement, transport, tracking, customs clearance, local transportation, warehousing and last mile delivery". Humanitarian logistics can be seen as a bridge between being prepared for the disaster and the response to the disaster. Humanitarian logistics can be seen as one of the response activities identified in a FFWRS. To analyse certain disasters in humanitarian logistics, certain quantitative methods can be used such as mathematic programming, heuristic models, probability theory, queuing theory etc. This will result in an improvement in the performance of the relief of the chain.

#### 2.4.1 Quantitative methods

Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise [31]. In classical logic, something is true or its not; there is no in between. An element either belongs to set or to its components. However fuzzy thinking on the other hand introduces degrees of greyness, or degrees belonging to a set. Fuzzy analysis builds on fuzzy logic. According to Kevin [30] fuzzy logic can be defined as a branch of logic that uses degrees of membership in sets rather than a strict true/false membership. "Fuzzy logic is useful just because of its low specificity; it allows a more flexible response to a given input" according to Kevin [30]. The output of a fuzzy system is smooth and continuous. Fuzzy systems base their decisions on inputs in a form of linguistic variables. Linguistic variables are non-numeric variables often used to facilitate the expressions of rules and facts [31]. The variables are tested with a small number of ifthen rules, which produce one or more responses depending on which rules were asserted. The response of each link is weighed according to the confidence or degree of membership of its inputs, and the centroid of the responses is calculated to generate the appropriate output. According to Kevin [30] fuzzy logic are best applied to nonlinear, time-variant, ill-defined systems. If a system to be controlled is very

complex, or cannot be easily represented by if-then rules, fuzzy logic is the appropriate method to be used. According to Kevin [30] fuzzy logic deals with observed, rather than measured variables of a system. Membership functions characterise the fuzziness in a fuzzy set, whether the elements in the set are discrete or continuous in a graphical form for the eventual use in the mathematical formalism of fuzzy set theory. The membership can also have binary values which indicate a belonging to a set or not. Fuzzy logic is powerful in fields such as expert systems, decision-making and information processing.

#### 2.4.1.1 A quick response model

J B Sheu *et al* [23] developed a quick response model consisting of three stages to solve large scale disaster relief distribution problems. To solve the problem of relief distribution they presented a fuzzy-optimisation methodology. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition where an element either belongs or does not belong in the set. The term "Fuzzy" refers to a membership in vaguely defined sets; however it does not refer to randomness [24]. According to Nasseri [25] Fuzzy linear programming refers to linear programming with fuzzy variables. In the developed quick response model the "fuzzy" variables is the distance between the disaster area and relief distribution area, where the "fuzzy" clustering refers to the grouping of disaster areas on the basis of the "fuzzy" distance. The methodology is as follows:

• Stage one – Consists of the grouping of the disaster areas by a fuzzy clustering method. The relative distance R is determined between the disaster area and the relief distribution area, assigning a value between 0 and 1. The distance is then "fuzzified" with a formula, thus making crisp quantity fuzzy. The fuzzy distance represents the similarity of each disaster area. The fuzzy relationship is as follows:

$$R(D_i, D_j) = 1 - \delta [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$$
(2.1)

#### Where:

- $D_i$ ,  $D_j$  = elements of set D, representing the locations associated with a given relief centre *i* and disaster area *j*.
- $\delta$  = represents the maximal reciprocal of distance between elements in a set D. As a result, R(D<sub>i</sub>, D<sub>i</sub>) is between 0 and 1.

 $x_i$  and  $x_j$  represent the x – coordinates of the relief centre *i* and disaster area *j*, while  $y_i$  and  $y_j$  represents the y – coordinates.

The clustering method is one of the algorithms available for developing a fuzzy model. The disaster areas are then clustered according to a method where different categories are determined for the disaster areas and relief areas, and these were divided by "cuts". The disaster areas are clustered by  $\propto$  - cut<sup> $\infty$ </sup> R, where  $\propto \epsilon$  [0, 1]. The value  $\propto$  can be defined by the rescue commander to represent the appropriate similarity of disaster area group. The relationship between the disaster- and relief areas obtained by the formula above, determines in which category the specific disaster area i and relief area j will fall.

• Stage two – After clustering each group is assigned a centre-of-gravity position. The location of each group is determined by its centre of gravity and the total demand of all the disaster areas in the same group. The location is determined by using the (X, Y) coordinates of the centre of gravity.

$$\sum X_{i} \cdot d_{i}$$

$$X = \frac{i}{\sum d_{i}}$$
and
$$Y = \frac{i}{\sum d_{i}}$$
(2.2) and (2.3)
$$\sum d_{i}$$

$$\sum d_{i}$$

$$\sum d_{i}$$

#### Where:

\_

X = the coordinate of the centre of gravity at the X axis

 $X_i$  = the X coordinates of the centre of gravity of the disaster group i

 $\overline{Y}$  = the coordinate of the centre of the centre of gravity at Y axis Y<sub>i</sub> = the Y coordinates of the centre of the gravity of the disaster group i d<sub>i</sub> =calculated demand for disaster group i

Once the disaster areas have been identified using the aforementioned measures, relief distribution centres will be assigned to each of the disaster area groups.

Three assumptions are made in order to facilitate the formulation of the model:

(1) Relief demands are assumed to follow fuzzy properties. The demands and supplies in terms of rescuing goods are generally uncertain at the onset of a disaster. Therefore, J B Sheu *et al* [23] postulate the representation of the fuzziness of demand and supply to be represented by right- and left- skewed triangles membership functions as shown in Figure 2.7. Where  $\mu(Si)$  is the degree of membership of element Si in the fuzzy set and  $\mu(Dj)$  is the degree of membership of element Dj. Thus  $\mu$  is the value on the unit interval that measures the degree to which an element belongs to a fuzzy set.





(2) Rescuing goods are assumed to be homogenous.

(3)Transportation cost is assumed to be linear in relation to the transportation distance.

In addition, the costs with respect to rescuing equipment, human resource, and services are neglected.

The fuzzy-based assignment model is therefore given by:

**Objective function:** 
$$Min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$
 (2.4)

Subject to:

$$\sum_{j=1}^{n} d_j x_{ij} \le s_i \qquad i = 1, 2, ..., m$$
(2.5)

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1, 2, ..., n$$
 (2.6)

$$x_{ij} = 0 \text{ or } 1$$
  $i = 1, 2, ..., m$   $j = 1, 2, ..., n$  (2.7)

#### Where:

i = the index of relief centres, i = 1, 2, ..., mj = the index of disaster groups, j = 1, 2, ..., nc<sub>ij</sub> = distance cost between node *i* and *j* d<sub>j</sub> = demand for disaster group *j* s<sub>i</sub> = supply for relief centre *i* 

 $\mathbf{x}_{ij} = \begin{cases} 1 \text{ if relief centre } i \text{ provides supply for disaster group } j \\ 0 \text{ if relief centre } i \text{ does not provide for disaster group } j \end{cases}$ 

The model is formulated as a 0-1 integer program and fuzzy linear programming model.

Equation (2.4) represents the objective function to minimise the total distance cost. Equation (2.5) is the constraint specifying the maximum supply capability associated with each given relief distribution centre. Equation (2.6) is specified to ensure that each disaster area group is served by only one relief centre. Accordingly, each disaster area group is served by one relief distribution centre; however one relief centre may serve more than one disaster area groups at this stage.

• Stage 3 – A fuzzy-based vehicle routing model is proposed to determine the routing order and distribution relief quantity associated with each damaged area in the same group under the objective of minimum travel time.

To facilitate model formulation, the following assumptions are made:

(1) The supply amount and location associated with each relief distribution centre are given.

(2) The relief demands are allowed to be satisfied partially.

(3) Each disaster group is served by only one vehicle.

(4) The time window limitation associated with each disaggregate disaster area is ignored.

The proposed fuzzy-based vehicle routing model consists of the following two sequential steps.

#### Step 1: Deciding the distribution routes

Min 
$$\sum_{i=1}^{n} \sum_{i=1}^{n} c_{ij} x_{ij}$$
 (2.8)

Subject to:

$$\sum_{i=0}^{n} x_{ij} = 1 \qquad \forall j = 1, 2, ..., n$$
 (2.9)

$$\sum_{j=0}^{n} x_{ij} = 1 \qquad \forall i = 1, 2, ..., m$$
 (2.10)

$$y_i - y_j + (n+1)x_{ij} \le n$$
  $\forall 1 < i \ne j < n$  (2.11)
Step 2: Deciding the shipment for each disaster area group

Min 
$$\sum_{j=1}^{n} c_{j}^{*} d_{j}$$
 (2.12)

Subject to:

$$\sum_{j=1}^{n} d_j \le S$$
(2.13)

$$d_j \ge d_j^*$$
  $\forall j = 1, 2, ..., n$  (2.14)

$$x_{ij} = 0 \text{ or } 1$$
  $d_j \ge 0$   $\forall i = 1, 2, ..., n$   $\forall j = 1, 2, ..., n$  (2.15)

#### Where:

n = Summation of relief centre and disaster area groups, where i = 0 represents the relief centre and  $i = 1 \sim n$  represents the disaster area groups.

 $c_{ij}$  = Distance cost between node i and node j.

- $c_{j}^{*}$  = Distances cost between relief centre and node j.
- $d_j$  = Shipment for disaster area group j.
- S = Total supply of relief centre.

 $d_{j}^{*}$  = Demand of disaster area group j.

 $x_{ij} = 0$ -1 variable, where  $x_{ij} = 1$  represents the link between node i and node j is utilised and  $x_{ij} = 0$  represents the link between node i and node j is not utilised.  $y_i =$ Arbitrary real number.

Equation (2.8) and (2.12) represents the corresponding objective functions used for optimising decisions for vehicle routing and relief distribution. Equations (2.9) and (2.10) represent the constraints which ensure that each disaster area group is served only once by a single relief distribution centre. Equation (2.11) is specified to avoid

the occurrence of sub tours. Equation (2.13) is the maximal limitation of demand and equation (2.14) ensures that demand is met.

This proposed mathematical model can be summarised as follow. In the first stage the fuzzy clustering method is used to group the disaster areas. The relative distance between each disaster area and relief centre, valued from 0 to 1, is "fuzzified" using the relation formula. The  $\propto$  value in using of  $\propto$ -cut method can be defined by the decision maker to match real conditions and get suitable clustering results. In the second stage, the centre-of-gravity position of each group is found as the representation of the group. With the locations of damaged area groups, the fuzzy dispatching model assigns one appropriate relief centre to each group under the objective of minimal rescuing time. In the last stage, the fuzzy vehicle routing programming is utilised to determine distributed order and quantity of each damaged area in the same group under the objective of minimal travelling time.

### 2.5 FFWRS - application in South Africa

According to du Plessis [6] in South Africa the current aim after floods have occurred, is the coverage of losses incurred during a flood and the re-instatement of the existence level of the victims. The use and development of scientific hazard and risk assessments for the determination of a sustainable flood standard for different river reaches have received minimal attention. This will make a valuable contribution to appropriate mitigation and prevention strategies. According to Klopper [15] the forecasting of flood producing rainfall and climate scenarios is primarily the responsibility of the South African Weather Bureau, while the government and public sectors have a role to play in advising the public, and farmers in particular. According to du Plessis [6] there are three systems that are used for forecasting purposes at the South African Weather Bureau, namely a mathematical weather model, geostationary satellite images and a radar observation station. They have successfully forecast major flood incidents using the mathematical weather models. According to Alexander [11] although floods had been forecast by the Weather Bureau five days ahead of occurrence, loss of life and damage to infrastructure still occurred due to the absence of an effective response system.

In Cape Town according to Powell [32] there are weekly meetings between all the stakeholders, especially in winter months when flooding are more likely to occur. At these meetings current situations are reviewed and problem solving strategies are discussed.

### 2.6 Concluding the literature review

The total number of fatalities experienced during a disaster situation is determined by the efficiency of rescue actions, including rescue resource allocation and relief distribution, in the initial phase of emergency rescue. The quick response model is developed to minimise the number of fatalities in a disaster situation. This model utilises fuzzy clustering and linear programming for finding a solution to the disasterrelief resource allocation problem. The insufficient supply situation is considered to make the problem more realistic. According to J B Sheu *et al* [23] the complex problem could be decomposed into smaller scale to accelerate the searching of optimal solutions by using the fuzzy clustering method.

# **Chapter 3**

## **Mathematical Model**

The main objective of the mathematical model is to determine an optimal response system to minimize the total number of fatalities, and with the objective to minimise the time spent in emergency rehabilitation and relief distribution to disaster areas. This response system includes the following:

- Grouping of disaster areas or locations into disaster groups.
- Allocating the identified relief centres to the disaster groups.
- Routing strategy from the relief centres to the disaster areas in each of the disaster groups.
- Determining the actual supply amount from the relief centre to the disaster areas to provide effective and efficient aid to the community affected.

This mathematical model is divided into three stages where the following assumptions were made:

- Rescuing goods are homogenous.
- Transportation cost is to be linear in relation to the transportation distance.
- Costs with respect to rescuing equipment, human resources, and services are neglected.
- Relief demands are allowed to be satisfied partially.
- Each relief centre has one vehicle to serve the disaster areas in each of the disaster groups.

The three stages in the mathematical model entail the following:

 Stage 1 – disaster grouping, where the identified disaster areas on the Cape Flats are divided into disaster groups and each disaster area is assigned to a specific disaster group.

- Stage 2 Assigning, where each of the identified relief centres is assigned to one or more disaster groups that were formed in stage 1.
- Stage 3 Routing and supply, the routing of the transportation vehicle and the supply to the disaster areas in each disaster group is calculated in this stage.

The output gained in stage 1 is used as input for stage 2, as well as the output gained from stage 2 provides an input for stage 3. Stage 3 can be divided into two stages, where each of these two stages has to be executed for every disaster group that was formed in stage1. The three consecutive stages provide a total response system for the flood crisis on the Cape Flats. Certain constraints were incorporated in the mathematical model to develop an optimum response system. Crucial constraints were identified at the following stages:

- Stage 2 The relief centre's supply will be more or equal to the demand of the disaster area which it has been assigned to.
- Stage 3 Sub touring needs to be prevented when determining the route of the transportation vehicle.

### **3.1 Formulation of the Model**

#### • Stage 1:

The model that was developed by J B Sheu *et al* [23] used a fuzzy clustering method to group the disaster areas in disaster groups. The relative distance between the disaster areas and relief centres followed fuzzy properties, where the sets were defined vaguely. Data was gathered on the floods on the Cape Flats and distances could be calculated. This fuzzy property no longer existed and no clustering method was used as method to group the disaster areas into disaster groups. Twenty four informal settlements and six relief centres were identified. Their locations were plotted on a geographical map. On the geographical map a zero point was chosen arbitrary to create an axis system where all the disaster areas and relief centres were islustrated. This can be seen in Appendix B. Coordinates were assigned to each of these locations relative to the chosen zero point. Using these x and y coordinates this axis system was divided into nine blocks. The nine blocks formed nine disaster groups

that consist of the following properties regarding the x and y coordinates in centimetres:

- Block 1:  $0 \le x \le 12$ , and  $0 \le y \le 12$
- Block 2:  $12 < x \le 24$ , and  $0 \le y \le 12$
- Block 3:  $24 < x \le 36$ , and  $0 \le y \le 12$
- Block 4:  $24 < x \le 36$ , and  $12 < y \le 24$
- Block 5:  $12 < x \le 24$ , and  $12 < y \le 24$
- Block 6:  $0 \le x \le 12$ , and  $12 < y \le 24$
- Block 7:  $0 \le x \le 12$ , and  $24 < y \le 36$
- Block 8:  $12 < x \le 24$ , and  $24 < y \le 36$
- Block 9:  $24 < x \le 36$ , and  $24 < y \le 36$

These nine blocks can be seen in Figure 3.1 where the blue stars indicate the twenty four informal settlements and the red stars represent the six relief centres. This figure clearly indicates which disaster areas fall into each disaster group.

Figure 3.1: Plotted relief centres and disaster areas on the axis system



Only six blocks has disaster areas plotted thus only six disaster groups were identified and used for the consecutive stages. The center x and y coordinates of the six disaster groups were calculated using the following formulation:

$$\sum_{i} \sum_{i} X_{i} \cdot d_{i}$$

$$X = \frac{i}{\sum_{i} d_{i}}$$

$$\sum_{i} d_{i}$$

### Where:

X<sub>i</sub> - represent the x coordinate of disaster area 'i'

Y<sub>i</sub> - represent the y coordinate of disaster area 'i'

d<sub>i</sub> - represent the calculated demand of disaster area 'i'

Equation (3.1) and (3.2) calculates the centre of gravity position of the disaster group and are used in the next stages as x and y coordinates.

#### • Stage 2:

The six disaster groups identified in stage 1 need to be assigned to relief centres. The objective for the second stage in the mathematical model is to minimise the total sum of transportation cost when assigning a relief centre to the disaster groups. Hence, the objective function of the model is formulated as follows in (3.3):

min 
$$z = \sum_{i=1}^{6} \sum_{j=1}^{6} c_{ij} x_{ij}$$
 (3.3)

Subject to:

$$\sum_{j=1}^{6} d_j x_{ij} \le s_i \qquad \forall i = 1...6$$
 (3.4)

$$\sum_{i=1}^{6} x_{ij} = 1 \qquad \forall j = 1...6$$
 (3.5)

$$x_{ij} \in \{0, 1\}$$
  $\forall i = 1...6, j = 1...6$  (3.6)

### Where:

 $c_{ij}$  – represent the relative cost between the relief centre 'i' and disaster group 'j'

 $d_i$  – represent the demand the disaster areas in disaster group 'j'

s<sub>i</sub> – represent the supply of relief centre 'i'

Constraint (3.4) ensures that the relief centre that is assigned to the disaster group, where the supply will be more than the demand of the disaster group. Constraint (3.5) ensures that each disaster group is served by only one relief centre. Constraint (3.6) ensures that the relevant variable is binary.

#### • Stage 3:

The third stage is divided into two stages, by first determining the optimal route that needs to be followed by the transportation vehicle, and secondly the actual supply is determined for each disaster area. These two stages will be executed for every disaster group with the assigned relief centre to develop an optimum respond strategy for each of the disaster groups. For the routing stage, the objective is to minimise the total sum of transportation cost of each disaster group. Hence, the objective function is formulated as follows in (3.7):

min 
$$z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$
 (3.7)

Subject to:

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \forall j = 1...m$$
 (3.8)

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad \forall i = 1...n$$
 (3.9)

$$y_i - y_j + n(x_{ij}) \le (n-1)$$
  $\forall 1 < i < n, 1 < j < m, i \neq j$  (3.10)

$$y_1 = 1$$
 (3.11)

$$x_{ij} \in \{0, 1\}$$
  $\forall i = 1...n, j = 1...m$  (3.12)

### Where:

 $c_{ij}$  – represent the cost value between node 'i' and node 'j'

- y<sub>i</sub> represent the position of the node 'i' in the route
- $y_j$  represent the position of the node 'j' in the route
- n represent the number of nodes

Constraints (3.8) and (3.9) ensure that each disaster area will only be visited once by the transportation vehicle of the relief centre. Constraint (3.10) is specified to avoid the occurrence of sub tours. Constraint (3.11) ensures that the transportation vehicle starts its departure at the relief centre and ends at relief centre once all the disaster areas have been reached. Constraint (3.12) ensures that the relevant variable is binary. For the second stage the objective is to minimise the total sum of cost associated with the supply and demand relationship between the disaster areas and the relief centre in each disaster group. Hence, the objective function is formulated as follows in (3.13):

min 
$$z = \sum_{j=1}^{m} c_j h_j$$
 (3.13)

Subject to:

$$\sum_{j=1}^{m} h_j \leq S_j \qquad \forall j = 1...m$$
 (3.14)

$$h_j \ge e_j \qquad \qquad \forall \ j = 1...m \qquad (3.15)$$

$$h_j \ge 0$$
  $\forall j = 1...m, i = 1...n$  (3.16)

$$x_{ij} \in \{0, 1\}$$
  $\forall j = 1...m, i = 1...n$  (3.17)

#### Where:

 $c_j$  – represent the distance between the disaster areas 'j' in the disaster group and relief centre

S<sub>i</sub> – represent the calculated supply of the relief centre

 $e_j$  – represent the calculated demand of each of the disaster areas 'j' in the disaster group

Constraint (3.14) is the maximal limitation of the demand. Constraint (3.15) is specified so that the supply to the disaster area must be more or equal to the demand of the disaster area. Constraint (3.16) ensures that the variable is non-negative. Constraint (3.17) ensures that the relative variable is binary.

### **3.2 Input Data**

The biggest challenge was to obtain correct and valuable information for this humanitarian crisis. This data is transformed into essential input values that are used in the mathematical model. According to Powell [32] there are twenty four informal settlements in the Cape Flats area that are the most vulnerable. There are six relief centres available to provide aid to this community affected by floods. These informal settlements are identified as the disaster areas to be considered in this mathematical model. These twenty four informal settlements are listed from worst to least affected by floods in Table 3.1 with the corresponding suburb, latitudinal and longitudinal values.

Priority	Settlement	Suburb	Latitude	Longitude
1	Kosovo	Phillippi/Weltevreden Valley	33°00'50''S	18°35'30''E
2	Sweet Home	Phillippi	33°00'20''S	18°33'40''E
3	Green Park	Driftsands	33°59'20"S	18°38'30''E
4	Wallacedene	Wallacedene	33°51'30"S	18°43'45"E
5	Bongani	Khayelitsha	34°01'00"S	18 <i>°</i> 39'10"E
6	Never Never	Phillippi	34°00'30"S	18 <i>°</i> 35'50''E
7	Solomon Mahlangu	Khayelitsha	34°01'40"S	18°40'20''E
8	Ekuphumleni	Mfuleni	34°00'20"S	18°40'20''E
9	Barnet Molokwana Corner	Khayelitsha	34°01'15"S	18°40'15"E
10	Masiphumelele Wetlands	Noordhoek	34°07'35"S	18°22'30"E
11	Kanana	Gugulethu	33°58'00"S	18°34'10''E
12	Cross Roads Infills	Crossroads	33°59'20"S	18 <i>°</i> 35'45"E
13	Ezimbacwini	Lwandle	34°07'20"S	18°51'45"E
14	Dunoon	Table View	33°49'10"S	18°32'30''E
15	New Rest	Gugulethu	33°57'40"S	18 <i>°</i> 33'40''E
16	Lotus Park	Gugulethu	34°01'50"S	18°32'10''E
17	Silver Town	Khayelitsha	34°02'35"S	18°41'10''E
18	Monwabisi Park	Khayelitsha	34°03'45"S	18°40'00''E
19	Doornbach	Table View	33°48'30"S	18°32'15"E
20	Gqobasi	Nyanga	33 <i>°</i> 59'15"S	18 <i>°</i> 34'30''E
21	Barcelona	Gugulethu	33°58'20"S	18 <i>°</i> 34'30''E
22	Freedom Farm Airport	Belhar	33°57'00"S	18 <i>°</i> 36'00''E
23	KTC	Gugulethu	33°58'45"S	18°35'05''E
24	Phola Park Phillipi	Phillippi	33°00'10"S	18 <i>°</i> 35'45"E

Table 3.1: Informal settlements from worst to least affected by floods

The relief centres are listed in Table 3.2 also with corresponding suburb, latitudinal and longitudinal values.

Table 3.2: Disaster relief centres

No.	Relief Centres	Location	Latitude	Longitude
1	HDI Support	Paarden Eiland	33°55'00"S	18 <i>°</i> 28'30''E
2	Mustadafin Foundation	Athlone	33°58'00"S	18°31'00''E
3	The Salvation Army	Wynberg	34°00'15"S	18°28'10"E
4	SARCS	Wynberg	34°00'15"S	18°28'30''E
5	SANZAF	Bridgetown	33°57'20''S	18°31'30"E
6	Masibambani	Khayelitsha	34°02'15"S	18°40'20''E

By examining the locations of these twenty four informal settlements and six relief centres, the latitudinal and longitudinal values are obtained from a geographical map. These values are used to plot the locations of the twenty four informal settlements and six relief centres on an axis system. These location values are used in the first stage of the response model. The axis system is divided into nine blocks using certain constraints to create these blocks. Each block in the axis system represents a disaster group to which the disaster areas are assigned. All of the twenty four disaster areas are located in only six blocks. Therefore only six disaster groups are formed. These six disaster groups are used in the second and third stages for the optimum response system. The necessary input data for the mathematical models at different stages are as follows:

- The relative distance between the each of the disaster areas in the disaster group.
- The relative distance between the relief centre in the disaster group and the disaster areas.
- Supply of the relief centres
- Demand of the disaster groups

The supply of the disaster relief centres and the demand of the disaster areas will have fuzzy properties and still need to be calculated. The supply of the relief centres is determined by using the following equation:

$$S_{i} = \underbrace{\sum_{j=1}^{24} \underline{d}_{j}}_{b} (1 \text{ x r})$$
(3.18)

### Where:

b - represents the total number of disaster areas

r - represents a random number between 0 and 1

The supply of the relief centres can be seen in Table 3.3.

### Table 3.3: Supply of relief centres

No.	Relief Centres	Supply
1	HDI Support	35.08
2	Mustadafin Foundation	28.06
3	The Salvation Army	42.09
4	SARCS	25.72
5	SANZAF	37.41
6	Masibambani	30.4

The demand of the disaster areas is determined by combining the priority number and considering factors such as size of the informal settlements and the vulnerability of the community. The demand of the disaster areas can be seen in Table 3.4.

Table 3.4: Demand of disaster areas

Disaster Area	Informal Settlement	Demand
1	Kosovo	3
2	Sweet Home	3
3	Green Park	3.7
4	Wallacedene	8.7
5	Bongani	4
6	Never Never	6
7	Solomon Mahlangu	7.9
8	Ekuphumleni	6
9	Barnet Molokwana Corner	3.5
10	Masiphumelele Wetlands	6.8
11	Kanana	5.2
12	Cross Roads Infills	3
13	Ezimbacwini	5.9
14	Dunoon	3.3
15	New Rest	2.3
16	Lotus Park	4.6
17	Silver Town	4.2
18	Monwabisi Park	3.9
19	Doornbach	3.6
20	Gqobasi	1.5
21	Barcelona	1.2
22	Freedom Farm Airport	1.1
23	KTC	0.8
24	Phola Park Phillipi	0.4

The demand of the disaster groups was also determined by calculating the average demand of the disaster areas allocated to the disaster group. The demand of the disaster groups can be seen in Table 3.5.

#### Table 3.5: Demand of disaster groups

Disaster Group	Demand
1	6.8
2	4.6
3	21.9
4	25
5	23
6	12.3

The relative distance values are used as the transportation cost value in the objective function of the mathematical model when assigning relief centres to disaster area groups. The relative distance measured in kilometers between two nodes (from node 'i' to node 'j') can be calculated by using the x and y coordinated in equation (3.19) as follows:

$$T = ((x_i - x_j)^2 + (y_i - y_j)^2))^{1/2}$$
(3.19)

### Where:

- x<sub>i</sub> represent the x coordinate of node 'i'
- $x_i$  represent the x coordinate of node 'j'
- y<sub>i</sub> represent the y coordinate of node 'i'
- y<sub>j</sub> represent the y coordinate of node 'j'

In Stage 2 however, the cost value that was used between the disaster groups and relief centres was determined by first calculating the fuzzy membership R ( $D_i$ ,  $D_j$ ). This fuzzy membership value is used to indicate the degree of which a disaster group belongs to the relief centre. The fuzzy membership is calculated by using equation (3.20) as follows:

$$R (D_i, D_j) = 1 - T_{ij} / T_{max}$$
(3.20)

### Where:

 $D_i$  – represent the disaster group 'i'  $D_j$  – represent the relief centre 'j'

 $T_{ij}\xspace$  – represents the distance between disaster group 'i' and relief centre 'j'

 $T_{max}\,-\,maximum$  distance between disaster group 'i' and relief centre 'j'

The cost between disaster group 'i' and relief centre 'j' is calculated by using equation (3.21) as follows:

$$C_{ij} = 1 - R (D_i, D_j)$$
 (3.21)

The fuzzy membership and associated cost values between the disaster groups and relief centres can be seen in Table 3.6.



Table 3.6: Fuzzy membership and cost values of disaster groups and relief centres

The values in Table 3.6 are used in stage 2 to allocate relief centres to disaster groups. In stage 3 when the routing strategy in the disaster groups is calculated, the relative distances between the nodes are used.

### **3.3 Execution of the model**

The mathematical model is written in the correct format and executed in LINGO. The output that is obtained from LINGO has to be analysed so that the solutions can be understood and correctly interpreted. Verification and validation of the model is performed throughout the modelling, coding and solving phases. If an error is discovered at any stage, the error is corrected and model resolved. By also inspecting the answers to see if they seem logical and possible the model is again checked.

The mathematical model for stage 2 and stage 3 are formulated into computer algorithms and can be seen in Appendix A. The solution found in stage 2 is used as data for the mathematical models in stage 3.

A zero value was used as the cost value of the relief centre as input data for the mathematical model used in stage 3. This value ensures that the transportation vehicle departs from the relief centre. A value of 200 was used as a big "M" number to ensure that the vehicle that's currently departing that disaster area doesn't return to the same location, but moves to the next disaster area and therefore, follows a circular route (visiting each disaster area only once) back to the relief centre.

# **Chapter 4**

# **Results**

## 4.1 Model results

For stage 1 the disaster areas that are assigned to disaster groups can be seen in Table 4.1. The six disaster groups can be viewed with all the disaster areas that belong in the disaster group.

Block	Disaster Group	Disaster Area
1	1	10
2	2	16
3	3	7, 13, 17, 18
4	4	1, 3, 5, 8, 9, 14, 22, 24
5	5	2, 6, 11, 12, 15, 20, 21, 23
6	0	0
7	0	0
8	0	0
9	6	4, 19

Table 4.1: Disaster areas assigned to disaster groups

In figure 4.2 the disaster areas that are associated with the disaster groups can be seen. The disaster areas are the blue stars in the figure where the red stars represent the location of the relief centres. Figure 4.1: Grouping of disaster areas into disaster groups



The results of stage 2 can be seen in Table 4.2. Here the relief centres are assigned to disaster groups to provide aid to the community affected. This is also graphically presented in Figure 4.2.

Table 4.2 Relief centres	assigned to	o disaster	groups
--------------------------	-------------	------------	--------

<b>Relief</b> Centre	Disaster Group
1	6
2	5
3	1
4	2
5	4
6	3

#### Figure 4.2: Assignment of relief centres to disaster groups



In stage 3, disaster group 1 and disaster group 2 each consists only of one disaster area and one relief centre. Thus there is only one route that the transportation vehicle can take from the relief centre towards the disaster area and back to the relief centre. The supply of the relief centres that are assigned to disaster group 1 and disaster group 2 also exceeds the demand of these disaster groups, meaning that the demand will be met of the individual disaster areas. For the disaster groups 3, 4, 5 and 6 the mathematical models were executed and the results found can be seen in the following tables:

#### • Routing Strategy

In the following Tables, node 1 represents the relief centre where the transportation vehicle departs to all the disaster areas in the disaster group. It can also be seen that a circular route from and back to the relief centre is followed.

Table 4.3: Routing for disaster group 3

Node Number	Location	From Node	Towards Node
1	Relief Centre 6	1	>5
2	Solomon Mahlangu	5 <	≥3
3	Ezimbacwini	3 <	
4	Silver Town	4	2
5	Monwabisi Park	2 <	·····>1

### Table 4.4: Routing for disaster group 4

Node Number	Location	From Node	Towards Node
1	Relief Centre 5	1	8
2	Kosovo	8	7
3	Green Park	7	5
4	Bongani	5	6
5	Ekuphumleni	6	4
6	Barnet Molokwana Corner	4	3
7	Dunoon	3<	9
8	Freedom Farm Airport	9<	2
9	Phola Park Phillipi	2	·> 1

Table 4.5: Routing for disaster group 5

Node Number	Location	From Node	Towards Node
1	Relief Centre 2	1	6
2	Sweet Home	6	4
3	Never Never	4	8
4	Kanana	8	9
5	Cross Roads Infills	9<	7
6	New Rest	7	5
7	Gqobasi	5	3
8	Barcelona	3<	2
9	KTC	2	·····> 1

Table 4.6: Routing for disaster group 6

Node Number	Location	From Node	Towards Node
1	Relief Centre 1	1	
2	Wallacedene	3 <	2
3	Doornbach	2	> 1

### • Supply

The supply mathematical model for disaster group 3, 4, 5 and 6 can be seen in Appendix A. The supply of each relief centre is more than the average demand of the disaster groups, thus the actual supply to each of the disaster areas is equal to the actual demand. The supply to each disaster group and associated disaster areas can be seen in the following tables:

Table 4.7: Supply to disaster group 1

Disaster Area	Location	Supply
1	Masiphumelele Wetlands	6.8

Table 4.8: Supply to disaster group 2

Disaster Area	Location	Supply
1	Lotus Park	4.6

Table 4.9: Supply to disaster group 3

Disaster Area	Location	Supply
1	Solomon Mahlangu	7.9
2	Ezimbacwini	5.9
3	Silver Town	4.2
4	Monwabisi Park	3.9

### Table 4.10: Supply to disaster group 4

Disaster Area	Location	Supply
1	Kosovo	3
2	Green Park	3.7
3	Bongani	4
4	Ekuphumleni	6
5	Barnet Molokwana Corner	3.5
6	Dunoon	3.3
7	Freedom Farm Airport	1.1
8	Phola Park Phillipi	0.4

### Table 4.11: Supply to disaster group 5

Disaster Area	Location	Supply
1	Sweet Home	3
2	Never Never	6
3	Kanana	5.2
4	Cross Roads Infills	3
5	New Rest	2.3
6	Gqobasi	1.5
7	Barcelona	1.2
8	KTC	0.8

 Table 4.12: Supply to disaster group 6

Disaster Area	Location	Supply
1	Wallacedene	8.7
2	Doornbach	3.6

By analysing the output of the mathematical models, all the results were integrated to form a total response system.

## **Chapter 5**

## **Conclusion and further research**

The main aim of this project was to develop an optimum response system for the flooding events on the Cape Flats. Fuzzy logic approach was used to describe the degree to which the disaster areas and disaster groups belong to a relief centre. The mathematical models for the different stages were coded and executed in LINGO. By using the output of one stage as the input for the next stage a total response approach could be developed. This developed response system can be viewed as an optimal total system approach. The solutions of the mathematical models presented suggested as to who should respond to which area, how to respond, and how much is being supplied to each of the disaster areas.

There will always be an area of opportunity for research on this project. Especially applying fuzzy logic in humanitarian crises where complex systems are created and need to be solved as quickly as possible. New mathematical methods should be explored to obtain new approaches in how to respond to humanitarian crises or even prevent these disasters. Humanitarian crises are experienced daily across the world where effective and efficient response is needed. By performing further research on topics like a response model for floods on the Cape Flats, awareness is increased drastically and new solving methods or approaches can be explored or created resulting in a better world for all.

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# Appendix A

## Mathematical models in LINGO

### A.1 Model and solution for stage 2

```
Stage 2 – model MODEL:
```

sets:

```
centre/1..6/:s;
group/1..6/:d;
assigne(centre,group):c,x;
```

#### endsets

#### data:

```
s=35.08 28.06 42.09 25.72 37.41 30.40;
d=6.8 4.6 21.9 25 23 12.3;
c= 0.77 0.64 0.46 0.48 0.73 0.89
0.47 0.24 0.27 0.2 0.3 0.42
0.79 0.59 0.75 0.68 0.56 0.1
0.57 0.4 0.6 0.53 0.35 0.16
0.43 0.23 0.42 0.35 0.2 0.26
0.77 0.79 1 0.96 0.7 0.73;
```

```
Enddata
```

```
min=@sum(assigne(i,j):
c(i,j)*x(i,j)
);
@for(centre(i):
    @sum(group(j):
         d(j)*x(i,j))<=s(i)
);
@for(group(j):
    @sum(centre(i):
         x(i,j)) = 1
);
@for(centre(i):
   @sum(group(j):
        x(i, j)) = 1
);
@for(assigne(i,j):
    @bin(x(i,j))
```

### Stage 2 - solution

Global optimal solution f	Eound.
Objective value:	
Objective bound:	
Infeasibilities:	
Extended solver steps:	
Total solver iterations	s:

2	1	1	0	0	0	0
2	1	1	0	0	0	0
0	0	0	0	0	0	0
						0
						0

Variable	Value	Reduced Cost
S( 1)	35.08000	0.00000
S(2)	28.06000	0.00000
S(3)	42.09000	0.00000
S(4)	25.72000	0.00000
S(5)	37.41000	0.00000
S(6)	30.40000	0.00000
D( 1)	6.800000	0.00000
D(2)	4.600000	0.00000
D(3)	21.90000	0.00000
D( 4)	25.00000	0.00000
D( 5)	23.00000	0.00000
D(6)	12.30000	0.00000
C( 1, 1)	0.7700000	0.00000
C( 1, 2)	0.6400000	0.00000
C( 1, 3)	0.4600000	0.00000
C(1,4)	0.4800000	0.00000
C( 1, 5)	0.7300000	0.00000
C( 1, 6)	0.8900000	0.00000
C(2,1)	0.4700000	0.00000
C(2,2)	0.2400000	0.00000
C(2,3)	0.2700000	0.00000
C(2,4)	0.200000	0.00000
C(2,5)	0.300000	0.00000
C(2,6)	0.4200000	0.00000
C( 3, 1)	0.7900000	0.00000
C(3,2)	0.5900000	0.00000
C(3,3)	0.7500000	0.00000
C(3,4)	0.6800000	0.00000
C(3,5)	0.5600000	0.00000
C(3,6)	0.1000000	0.00000
C(4,1)	0.5700000	0.00000
C(4,2)	0.400000	0.00000
C(4,3)	0.600000	0.00000
C(4,4)	0.5300000	0.00000
C(4,5)	0.3500000	0.00000
C(4,6)	0.1600000	0.00000
C(5, 1)	0.4300000	0.000000
C(5,2)	0.2300000	0.000000
C(5,3)	0.4200000	0.000000
C(5, 4)	0.3500000	0.000000
C(5, 5)	0.2000000	0.000000
C(5, 6)	0.2600000	0.00000
C(6, 1)	0.//00000	0.000000
C( 6, Z)	0./900000	0.00000

); end

C( 6.	3)	1.000000	0.00000
C ( 6 )	4)	0 960000	0 000000
C ( 6	/ 5)	0 700000	0.000000
	()	0.7000000	0.000000
C( 6,	6)	0.7300000	0.000000
X( 1,	1)	0.000000	0.7700000
X( 1,	2)	0.000000	0.6400000
X( 1,	3)	1.000000	0.4600000
X( 1,	4)	0.00000	0.4800000
X( 1,	5)	0.00000	0.7300000
X( 1,	6)	0.00000	0.8900000
X(2.	1)	0 00000	0 4700000
x(2)	2)	0 00000	0 2400000
x(2)	3)	0 000000	0 2700000
x(2)	<u></u>	1 000000	0.2700000
X( 2)			0.2000000
Λ( <i>Δ</i> ,	5)	0.000000	0.3000000
X( 2,	6)	0.00000	0.4200000
Х(З,	1)	0.000000	0.7900000
Х(З,	2)	0.000000	0.5900000
Х(З,	3)	0.000000	0.7500000
Х( З,	4)	0.00000	0.6800000
Х(З,	5)	0.00000	0.5600000
X(3,	6)	1.000000	0.1000000
X(4.	1)	0.00000	0.5700000
X(4.	2)	0 00000	0 4000000
X ( 4	3)	0 000000	0 6000000
X( 1, V( /	4)	0.000000	0.5300000
A( 4, V/ A	4) 5)	1 000000	0.3500000
A(4,	5)	1.000000	0.3300000
X(4,	6)	0.000000	0.1600000
X(5,	1)	0.000000	0.4300000
X(5,	2)	1.000000	0.2300000
X( 5,	3)	0.000000	0.4200000
X( 5,	4)	0.00000	0.3500000
X( 5,	5)	0.00000	0.200000
X( 5,	6)	0.00000	0.2600000
X(6,	1)	1.000000	0.7700000
X(6,	2)	0.00000	0.7900000
X(6,	3)	0.00000	1.000000
X(6.	4)	0 00000	0 960000
X ( 6	5)	0 00000	0 7000000
X( 0, V( 6	6)	0.000000	0.7300000
Δ( 0,	0)	0.000000	0.7300000
	Dou	Slack or Surplus	Dual Drico
	1100 W	2 110000	1 00000
	1 1	2.110000	-1.000000
	2	13.18000	0.000000
	3	3.060000	0.000000
	4	29.79000	0.000000
	5	2.720000	0.00000
	6	32.81000	0.00000
	7	23.60000	0.00000
	8	0.00000	0.00000
	9	0.00000	0.00000
	10	0.00000	0.00000
	11	0.00000	0.00000
	12	0.00000	0.00000
	1 २	0 000000	0 000000
	11	0.000000	
	1 E	0.000000	0.000000
	10	0.000000	0.000000
	10	0.000000	0.000000
	17	0.000000	0.000000
	18	0.000000	0.00000
	19	0.00000	0.00000

## A.2 Model and solution for stage 3: routing

### Group 3 - model:

```
MODEL:
SETS:
      node/1..5/:y;
      link(node, node): c,x;
ENDSETS
DATA:
c=0 0.92 8.45 2.29 1.57
 0.92 200 7.88 1.74 1.57
 8.45 7.88 200 6.18 7.85
 2.29 1.74 6.18 200 2.27
  1.57 1.57 7.85 2.27 200;
ENDDATA
MIN = @SUM(link(i,j):
      c(i,j)*x(i,j)
);
@FOR(node(j):
    @SUM(node(i):
       x(i,j))=1
);
@FOR(node(i):
    @SUM(node(j):
       x(i,j))=1
);
@for(node(i):
      @for(node(j)|i#GT#1#AND#j#GT#1#AND#i#NE#j:
            y(i) - y(j) + 5*x(i,j) <= 4;
));
y(1) = 1;
@FOR(link(i,j):
   @BIN(x(i,j))
);
END
```

#### Group 3 – solution:

Global optimal solution found.	
Objective value:	18.26000
Objective bound:	18.26000
Infeasibilities:	0.00000
Extended solver steps:	13
Total solver iterations:	538

Variable	Value	Reduced Cost
Y( 1)	1.000000	0.00000

	Υ(	2)	3.000000	0.00000
	Υ(	3)	1.000000	0.00000
	Υ(	4)	2.000000	0.00000
	Υ(	5)	0.00000	0.00000
С (	1,	1)	0.00000	0.00000
С (	1,	2)	0.9200000	0.00000
С(	1,	3)	8.450000	0.00000
С(	1,	4)	2.290000	0.00000
С(	1,	5)	1.570000	0.00000
С(	2,	1)	0.9200000	0.00000
С (	2,	2)	200.0000	0.00000
С (	2,	3)	7.880000	0.00000
С(	2,	4)	1.740000	0.00000
С (	2,	5)	1.570000	0.00000
С(	3,	1)	8.450000	0.00000
С (	3,	2)	7.880000	0.00000
С (	3,	3)	200.0000	0.00000
С(	3,	4)	6.180000	0.00000
C (	з,	5)	7.850000	0.00000
С(	4,	1)	2.290000	0.00000
C (	4,	2)	1.740000	0.00000
C (	4,	3)	6.180000	0.00000
C (	4,	4)	200.0000	0.00000
C (	4,	5)	2.270000	0.00000
Ċ	5,	1)	1.570000	0.00000
C (	5,	2)	1.570000	0.00000
C (	5,	3)	7.850000	0.00000
Ċ	5,	4)	2.270000	0.00000
C (	5,	5)	200.0000	0.00000
X (	1.	1)	0.00000	0.00000
X (	1,	2)	0.000000	0.9200000
X (	1,	3)	0.000000	8.450000
X (	1,	4)	0.00000	2.290000
X (	1,	5)	1.000000	1.570000
X (	2,	1)	1.000000	0.9200000
Х (	2,	2)	0.00000	200.0000
Х (	2,	3)	0.00000	7.880000
Х (	2,	4)	0.00000	1.740000
Х (	2,	5)	0.00000	1.570000
Х (	З,	1)	0.00000	8.450000
Х (	З,	2)	0.00000	7.880000
Х (	З,	3)	0.00000	200.0000
Х (	З,	4)	1.000000	6.180000
Х (	З,	5)	0.00000	7.850000
Х (	4,	1)	0.00000	2.290000
Х (	4,	2)	1.000000	1.740000
Х (	4,	3)	0.00000	6.180000
Х (	4,	4)	0.00000	200.0000
Х (	4,	5)	0.00000	2.270000
Х (	5,	1)	0.00000	1.570000
Х (	5,	2)	0.00000	1.570000
Х (	5,	3)	1.000000	7.850000
Х (	5,	4)	0.00000	2.270000
Х (	5,	5)	0.000000	200.0000
	I	Row	Slack or Surplus	Dual Price
		1	18.26000	-1.000000
		2	0.000000	0.000000
		ک ^	0.000000	0.000000
		4 5	0.000000	
		С	0.00000	0.000000
6	0.00000	0.00000		
----	----------	---------		
7	0.00000	0.00000		
8	0.00000	0.00000		
9	0.00000	0.00000		
10	0.00000	0.00000		
11	0.00000	0.00000		
12	2.000000	0.00000		
13	3.000000	0.00000		
14	1.000000	0.00000		
15	6.00000	0.00000		
16	0.00000	0.00000		
17	3.000000	0.00000		
18	0.00000	0.00000		
19	3.000000	0.00000		
20	2.000000	0.00000		
21	7.000000	0.00000		
22	0.00000	0.00000		
23	6.00000	0.00000		
24	0.00000	0.00000		

### Group 4 – model:

#### MODEL:

#### SETS:

node/1..9/:y; link(node,node): c,x;

#### ENDSETS

#### DATA:

```
c= 0 8.58 7.45 13.02 14.77 15.01 20.43 6.92 8.69
8.58 200 6.29 6.65 8.81 8.22 23.40 7.76 3.14
7.45 6.29 200 2.54 3.78 4.41 18.61 5.52 3.30
13.02 6.65 2.54 200 2.16 2 20.60 8.02 4.47
14.77 8.81 3.78 2.16 200 1.62 20 9.14 6.48
15.01 8.22 4.41 2 1.62 200 21.57 9.93 6.4
20.43 23.40 18.61 20.60 20 21.57 200 15.80 20.52
6.92 7.76 5.52 8.02 9.14 9.93 15.8 200 5.35
8.69 3.14 3.3 4.47 6.48 6.4 20.52 5.35 200;
```

#### ENDDATA

```
y(i) - y(j) + 9*x(i,j) <= 8;
));
y(1) = 1;
@FOR(link(i,j):
    @BIN(x(i,j))
);
END</pre>
```

# **Group 4 – solution:**

Global	optimal	solution	found	at	iteration:	818	10
Objec	ctive val	lue:				63.9	0000

Va	rial	ble	Value	Reduced Cost
	Υ(	1)	1.000000	0.00000
	Υ(	2)	7.000000	0.00000
	Υ(	3)	5.000000	0.00000
	Υ(	4)	4.000000	0.00000
	Υ(	5)	2.000000	0.00000
	Υ(	6)	3.000000	0.00000
	Υ(	7)	1.000000	0.00000
	Υ(	8)	0.00000	0.00000
	Υ(	9)	6.00000	0.00000
С(	1,	1)	0.00000	0.00000
С(	1,	2)	8.580000	0.00000
С(	1,	3)	7.450000	0.00000
С(	1,	4)	13.02000	0.00000
С(	1,	5)	14.77000	0.00000
С(	1,	6)	15.01000	0.00000
С(	1,	7)	20.43000	0.00000
С(	1,	8)	6.920000	0.00000
С(	1,	9)	8.690000	0.00000
С(	2,	1)	8.580000	0.00000
С(	2,	2)	200.0000	0.00000
С(	2,	3)	6.290000	0.00000
С(	2,	4)	6.650000	0.00000
С(	2,	5)	8.810000	0.00000
С(	2,	6)	8.220000	0.00000
С(	2,	7)	23.40000	0.00000
С(	2,	8)	7.760000	0.00000
С(	2,	9)	3.140000	0.00000
С(	З,	1)	7.450000	0.00000
С(	З,	2)	6.290000	0.00000
С(	З,	3)	200.0000	0.00000
С(	З,	4)	2.540000	0.00000
С(	З,	5)	3.780000	0.00000
С(	З,	6)	4.410000	0.00000
С(	З,	7)	18.61000	0.00000
С(	З,	8)	5.520000	0.00000
С(	З,	9)	3.300000	0.00000
С(	4,	1)	13.02000	0.00000
С(	4,	2)	6.650000	0.00000
С(	4,	3)	2.540000	0.00000
С (	4,	4)	200.0000	0.00000
С (	4,	5)	2.160000	0.00000
С (	4,	6)	2.000000	0.00000
С (	4,	7)	20.60000	0.00000
С(	4,	8)	8.020000	0.00000

С (	4,	9)	4.470000	0.000000
С(	5,	1)	14.77000	0.000000
С(	5,	2)	8.810000	0.000000
С(	5,	3)	3.780000	0.000000
С (	5,	4)	2.160000	0.000000
С (	5,	5)	200.0000	0.000000
С(	5,	6)	1.620000	0.000000
С(	5,	7)	20.00000	0.000000
С(	5,	8)	9.140000	0.000000
С(	5,	9)	6.480000	0.000000
С(	6,	1)	15.01000	0.000000
С(	6,	2)	8.220000	0.000000
С(	6,	3)	4.410000	0.000000
С(	6,	4)	2.000000	0.000000
С(	6,	5)	1.620000	0.000000
С(	6,	6)	200.0000	0.000000
С(	6,	7)	21.57000	0.000000
С(	6,	8)	9.930000	0.000000
С(	6,	9)	6.400000	0.000000
С(	7,	1)	20.43000	0.000000
С(	7,	2)	23.40000	0.000000
С(	7,	3)	18.61000	0.000000
С(	7,	4)	20.60000	0.000000
С(	7,	5)	20.00000	0.000000
С(	7,	6)	21.57000	0.000000
С(	7,	7)	200.0000	0.000000
С(	7,	8)	15.80000	0.000000
С(	7,	9)	20.52000	0.000000
С(	8,	1)	6.920000	0.000000
С(	8,	2)	7.760000	0.000000
С(	8,	3)	5.520000	0.000000
С(	8,	4)	8.020000	0.000000
С(	8,	5)	9.140000	0.000000
С(	8,	6)	9.930000	0.000000
С(	8,	7)	15.80000	0.000000
С(	8,	8)	200.0000	0.000000
С(	8,	9)	5.350000	0.000000
С(	9,	1)	8.690000	0.000000
С(	9,	2)	3.140000	0.000000
С(	9,	3)	3.300000	0.000000
С (	9,	4)	4.470000	0.000000
С (	9,	5)	6.480000	0.000000
С (	9,	6)	6.400000	0.000000
С (	9,	7)	20.52000	0.000000
С (	9,	8)	5.350000	0.000000
С(	9,	9)	200.0000	0.000000
Х (	1,	1)	0.00000	0.000000
Х (	1,	2)	0.00000	8.580000
Х (	1,	3)	0.00000	7.450000
Х (	1,	4)	0.00000	13.02000
Х (	1,	5)	0.00000	14.77000
Х (	1,	6)	0.00000	15.01000
Х (	1,	7)	0.00000	20.43000
Х (	1,	8)	1.000000	6.920000
Х (	1,	9)	0.000000	8.690000
Х (	2,	1)	1.000000	8.580000
Х (	2,	2)	0.00000	200.0000
Х (	2,	3)	0.00000	6.290000
Х (	2,	4)	0.00000	6.650000
Х (	2,	5)	0.000000	8.810000
Х (	2,	6)	0.00000	8.220000

Χ(	2.	7)	0.00000	23.40000
X (	2.	8)	0.00000	7.760000
X (	2.	9)	0 00000	3 140000
X (	ן ג	1)	0 00000	7 450000
X (	3 7	2)	0.00000	6 290000
X (	3 <b>,</b>	2)	0.000000	200 0000
л ( У (	3,	3) 4)	0.000000	2 540000
	ວ <b>,</b> ວ	4) E)	0.000000	2.340000
A (	ວ <b>,</b> ວ	5)	0.000000	3.780000
A (	ວ <b>,</b>	0)	0.000000	4.410000
Х (	<u>ک</u>	/)	0.000000	18.61000
Х (	<i>з</i> ,	8)	0.00000	5.520000
х (	3,	9)	1.000000	3.300000
Х (	4,	1)	0.000000	13.02000
Х (	4,	2)	0.000000	6.650000
Х (	4,	3)	1.000000	2.540000
Х (	4,	4)	0.000000	200.0000
Х (	4,	5)	0.000000	2.160000
Х (	4,	6)	0.00000	2.000000
Х (	4,	7)	0.00000	20.60000
Х (	4,	8)	0.00000	8.020000
Х (	4,	9)	0.00000	4.470000
Х (	5,	1)	0.00000	14.77000
Х (	5,	2)	0.00000	8.810000
Х (	5,	3)	0.00000	3.780000
Х (	5,	4)	0.00000	2.160000
Х (	5,	5)	0.00000	200.0000
Х (	5,	6)	1.000000	1.620000
Х (	5,	7)	0.00000	20.00000
Х (	5,	8)	0.00000	9.140000
Х(	5,	9)	0.00000	6.480000
Х (	6,	1)	0.00000	15.01000
Х (	6,	2)	0.00000	8.220000
Х(	6,	3)	0.00000	4.410000
X ( X (	6, 6,	3) <b>4)</b>	0.000000 1.000000	4.410000 2.000000
X ( X ( X (	6, 6, 6,	3) <b>4)</b> 5)	0.000000 <b>1.000000</b> 0.000000	4.410000 2.000000 1.620000
X ( X ( X ( X (	6, 6, 6,	3) <b>4)</b> 5) 6)	0.000000 <b>1.000000</b> 0.000000 0.000000	4.410000 2.000000 1.620000 200.0000
X ( X ( X ( X ( X (	6, 6, 6, 6,	3) <b>4)</b> 5) 6) 7)	0.00000 <b>1.000000</b> 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 200.0000 21.57000
X( X( X( X( X( X(	6, 6, 6, 6, 6,	3) <b>4)</b> 5) 6) 7) 8)	0.00000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 200.0000 21.57000 9.930000
X( X( X( X( X( X( X(	6, 6, 6, 6, 6, 6,	3) <b>4)</b> 5) 6) 7) 8) 9)	0.00000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 200.0000 21.57000 9.930000 6.400000
X( X( X( X( X( X( X( X(	6, 6, 6, 6, 6, 7,	3) 4) 5) 6) 7) 8) 9) 1)	0.00000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000
X( X( X( X( X( X( X( X( X( X(	6, 6, 6, 6, 6, 7, 7,	3) 4) 5) 6) 7) 8) 9) 1) 2)	0.00000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000
X( X( X( X( X( X( X( X( X( X( X(	6, 6, 6, 6, 6, 7, 7, 7,	3) 4) 5) 6) 7) 8) 9) 1) 2) 3)	0.00000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000
X( X( X( X( X( X( X( X( X( X( X( X(	6, 6, 6, 6, 6, 7, 7, 7, 7,	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4)	0.00000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000
X( X( X( X( X( X( X( X( X( X( X( X( X( X	6, 6, 6, 6, 6, 7, 7, 7, 7, 7,	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5)</pre>	0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b>	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000
X ( X ( X ( X ( X ( X ( X ( X ( X ( X (	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7,	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6)	0.00000 1.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 1.00000 0.000000 0.000000 0.000000 0.000000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.0000000 0.0000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000 21.57000
X (( X () X () X () X () X () X () X ()	6,, 6,, 6,, 6,, 7,, 7,, 7,, 7,, 7,,	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7)</pre>	0.00000 1.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 1.00000 0.000000 0.0000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000 21.57000 200.0000
X (( X () X () X () X () X () X () X ()	6,, 6,, 6,, 6,, 7,, 7,, 7,, 7,, 7,,	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8)</pre>	0.00000 1.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 1.00000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000 0.00000 0.00000 0.00000 0.00000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000 21.57000 20.0000 15.80000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 6) 7) 8) 9) 9) 1) 2) 3) 9) 9) 9) 9) 9) 9) 9) 9) 9) 9) 9) 9) 9)</pre>	0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000 21.57000 20.0000 15.80000 20.52000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 8,	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 1) 1) 1) 1) 1) 1) 1) 1) 1) 1) 1) 1) 1)</pre>	0.00000 1.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 1.00000 0.000000 0.0000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000 21.57000 20.0000 15.80000 20.52000 6.920000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8.	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 1) 2) 3) 1) 2) 3) 3) 1) 2) 3) 3) 3) 3) 3) 3) 3) 3) 3) 3) 3) 3) 3)</pre>	0.00000 1.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 1.00000 0.000000 0.000000 0.000000 0.000000 0.000000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 21.57000 20.00000 15.80000 20.52000 6.920000 7.760000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8.	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 3) 3) 3) 3) 3) 3)</pre>	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000 21.57000 20.0000 15.80000 20.52000 6.920000 7.760000 5.520000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8.	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 4) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5)</pre>	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 21.57000 20.00000 15.80000 20.52000 6.920000 7.760000 8.020000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8	<pre>3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5)</pre>	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 21.57000 20.00000 15.80000 20.52000 6.920000 7.760000 5.520000 8.020000 9.140000
X (( X (( X () X () X () X () X () X ()	6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 7) 8) 9) 5) 6) 7) 7) 8) 9) 5) 6) 7) 7) 8) 9) 5) 6) 7) 7) 8) 9) 5) 6) 7) 7) 7) 8) 9) 5) 6) 7) 7) 7) 7) 7) 7) 7) 7) 7) 7	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 21.57000 20.00000 15.80000 20.52000 6.920000 7.760000 5.520000 8.020000 9.140000 9.930000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 7) 8) 9) 7) 8) 9) 1) 2) 5) 7) 7) 7) 7) 7) 7) 7) 7) 7) 7	0.00000 1.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 1.00000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 20.0000 9.930000 6.400000 20.43000 23.40000 18.61000 20.60000 20.00000 21.57000 20.0000 15.80000 0.52000 6.920000 7.760000 5.520000 8.020000 9.140000 9.930000 15.80000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8,	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 7) 8) 9) 1) 2) 5) 6) 7) 8) 7) 8) 8) 7) 8) 8) 7) 8) 8) 7) 7) 8) 7) 8) 7) 7) 8) 7) 7) 7) 8) 7) 7) 7) 7) 7) 7) 7) 7) 7) 7	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 20.0000 9.930000 6.400000 20.43000 23.40000 20.60000 20.60000 21.57000 20.00000 15.80000 20.52000 6.920000 7.760000 5.520000 8.020000 9.140000 9.930000 15.80000 20.0000
X (( X (( X (( X (( X (( X (( X (( X ((	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8,	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 9) 1) 2) 5) 6) 7) 8) 9) 9) 1) 2) 5) 6) 7) 8) 9) 9) 1) 2) 5) 6) 7) 8) 9) 9) 1) 2) 8) 9) 9) 9) 9) 9) 9) 9) 9) 9) 9	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 20.0000 9.930000 6.400000 20.43000 23.40000 20.60000 20.60000 20.00000 21.57000 20.0000 15.80000 20.52000 6.920000 7.760000 5.520000 8.020000 9.140000 9.930000 15.80000 20.0000 5.350000
X (( X (( X (( X (( X (( X (( X (( X ((	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8,	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 1) 1) 2) 1) 1) 1) 1) 1) 1) 1) 1) 1) 1	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 20.0000 9.930000 6.400000 20.43000 23.40000 20.60000 20.60000 20.00000 21.57000 20.0000 15.80000 20.52000 6.920000 7.760000 5.520000 8.020000 9.140000 9.930000 15.80000 20.0000 5.350000 8.620000 8.620000 15.80000 20.00000 20.00000 20.00000 20.00000 20.00000 2
X ((XX) X (XX) X	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 8) 9) 1) 2) 3) 4) 5) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 8) 9) 1) 2) 2) 2) 2) 2)	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 0.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 0.000000 1.000000 0.0000000 0.000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 20.0000 9.930000 6.400000 20.43000 23.40000 20.60000 20.60000 20.00000 21.57000 20.00000 15.80000 20.52000 6.920000 5.520000 8.020000 9.140000 9.350000 20.0000 5.350000 8.690000 3.140000
X ((XX) X (XX) X	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 9, 9, 9	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 3) 2) 3) 2) 3) 3) 2) 3) 3) 2) 3) 3) 2) 3) 3) 2) 3) 3) 2) 3) 3) 2) 3) 3) 3) 3) 2) 3) 3) 2) 3) 3) 3) 3) 3) 3) 3) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 1.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 20.60000 20.60000 20.00000 21.57000 20.0000 15.80000 20.52000 6.920000 5.520000 8.020000 9.140000 9.350000 15.80000 3.140000 3.300000
X (( X X () X X	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9	3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4)	0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 1.000000 1.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.00000000	4.410000 2.000000 1.620000 21.57000 9.930000 6.400000 20.43000 23.40000 20.60000 20.60000 20.00000 15.80000 20.52000 6.920000 7.760000 5.520000 8.020000 9.140000 9.930000 15.80000 3.50000 8.690000 3.300000 4.470000

## Group 5 – model:

```
MODEL:
```

```
SETS:
    node/1..9/:y;
    link(node,node): c,x;
ENDSETS
```

#### DATA:

```
c= 0 7.39 9.34 5.79 8.37 4.93 6.54 6.67 7.14
7.39 200 3.30 5.57 3.77 5.79 4.15 5.07 4.79
9.34 3.30 200 5.18 1.62 5.87 3.51 4.18 3.59
5.79 5.57 5.18 200 3.64 0.92 1.69 1.11 1.74
8.37 3.77 1.62 3.64 200 4.40 2.05 2.59 1.99
4.93 5.79 5.87 0.92 4.40 200 2.36 1.99 2.59
6.54 4.15 3.51 1.69 2.05 2.36 200 0.92 0.8
6.67 5.07 4.18 1.11 2.59 1.99 0.92 200 0.63
7.14 4.79 3.59 1.74 1.99 2.59 0.8 0.63 200;
```

```
ENDDATA
```

```
MIN = @SUM(link(i,j):
      c(i,j)*x(i,j)
);
@FOR(node(j):
    @SUM(node(i):
        x(i,j))=1
);
@FOR(node(i):
    @SUM(node(j):
       x(i,j))=1
);
@for(node(i):
      @for(node(j)|i#GT#1#AND#j#GT#1#AND#i#NE#j:
            y(i) - y(j) + 9*x(i,j) \le 8;
));
y(1) = 1;
@FOR(link(i,j):
   @BIN(x(i,j))
);
```

#### END

## **Group 5 – solution:**

Global	optimal	solution	found	at	iteration:	138656	
Objec	ctive val	lue:				22.7500	0

Variable	Value	Reduced Cost
Y( 1)	1.000000	0.00000
Y(2)	7.000000	0.00000
Y(3)	6.00000	0.00000
Y(4)	1.000000	0.00000

	Υ(	5)	5.00000	0.00000
	Υ(	6)	0.00000	0.00000
	Υ(	7)	4.000000	0.00000
	Υ(	8)	2.000000	0.00000
	Υ(	9)	3.000000	0.00000
С(	1,	1)	0.00000	0.00000
C (	1.	2)	7.390000	0.00000
C (	1.	3)	9.340000	0.00000
C	1.	4)	5 790000	0 000000
C	1	5)	8 370000	0 000000
C	1	6)	4 930000	0.000000
C	1	7)	6 540000	0.000000
C	±, 1	8)	6.670000	0.000000
	⊥, 1	0)	7 140000	0.000000
C	⊥ <b>,</b> 2	) 1)	7.390000	0.000000
	2 <b>,</b>	1) 2)	200 0000	0.000000
C	2 <b>,</b>	2)	200.0000	0.000000
	2 <b>,</b>	3)	5.500000	0.000000
	Ζ,	4)	5.570000	0.000000
C (	Ζ,	5)	3.770000	0.000000
С(	2,	6)	5.790000	0.000000
С(	2,	/)	4.150000	0.000000
С (	2,	8)	5.070000	0.000000
С(	2,	9)	4.790000	0.000000
С (	З,	1)	9.340000	0.00000
С(	З,	2)	3.300000	0.00000
С (	З,	3)	200.0000	0.00000
С (	З,	4)	5.180000	0.00000
С(	З,	5)	1.620000	0.00000
С(	З,	6)	5.870000	0.00000
С (	З,	7)	3.510000	0.00000
С (	З,	8)	4.180000	0.00000
С (	З,	9)	3.590000	0.00000
С(	4,	1)	5.790000	0.00000
С(	4,	2)	5.570000	0.00000
С(	4,	3)	5.180000	0.000000
С(	4,	4)	200.0000	0.000000
С(	4,	5)	3.640000	0.000000
C (	4,	6)	0.9200000	0.00000
C (	4,	7)	1.690000	0.00000
Ċ	4.	8)	1.110000	0.00000
C (	4.	9)	1.740000	0.000000
C (	5.	1)	8.370000	0.000000
C (	5.	$2)^{-}$	3.770000	0.00000
C (	5.	3)	1.620000	0.00000
C	5, 5,	4)	3 640000	0 000000
C	5	5)	200 0000	0 000000
C	5	6)	4 400000	0.000000
C	5	7)	2 050000	0.000000
C	5	8)	2.590000	0.000000
C	5	0)	1 990000	0.000000
C	5,	9) 1)	1.990000	0.000000
	ΰ,	1) 2)	4.930000	0.000000
	ΰ,	∠) ⊃`	5.790000	
	ο,	3) ()	000018.0	0.000000
	ΰ,	4)	0.9200000	0.000000
C (	6,	5)	4.40000	0.000000
C (	6,	6) _	200.0000	0.000000
С(	6,	/)	2.360000	0.000000
С(	6,	8)	1.990000	0.00000
С (	6,	9)	2.590000	0.00000
С(	7,	1)	6.540000	0.000000
С (	7,	2)	4.150000	0.00000

С (	7,	3)	3.510000	0.00000
С (	7,	4)	1.690000	0.00000
С(	7,	5)	2.050000	0.00000
С(	7,	6)	2.360000	0.00000
С(	7,	7)	200.0000	0.00000
С (	7,	8)	0.9200000	0.00000
С (	7,	9)	0.800000	0.00000
С (	8,	1)	6.670000	0.00000
С (	8,	2)	5.070000	0.00000
С (	8,	3)	4.180000	0.00000
С (	8,	4)	1.110000	0.00000
С (	8,	5)	2.590000	0.00000
С (	8,	6)	1.990000	0.00000
С (	8,	7)	0.9200000	0.00000
С (	8,	8)	200.0000	0.00000
С (	8,	9)	0.6300000	0.00000
С (	9,	1)	7.140000	0.00000
С (	9,	2)	4.790000	0.00000
С (	9,	3)	3.590000	0.00000
С (	9,	4)	1.740000	0.00000
С (	9,	5)	1.990000	0.00000
С (	9,	6)	2.590000	0.00000
С (	9,	7)	0.800000	0.00000
С(	9,	8)	0.6300000	0.000000
С(	9,	9)	200.0000	0.000000
Х(	1,	1)	0.000000	0.000000
X (	1, 1	2)	0.000000	7.390000
X (	⊥, 1	3)	0.000000	9.340000
A ( V (	⊥, 1	4) 5)	0.000000	9 370000
A (	1'	6)	1 000000	4 930000
X (	1	7)	0.000000	6 540000
X (	1.	8)	0.00000	6.670000
Х (	1,	9)	0.000000	7.140000
X (	2,	1)	1.000000	7.390000
Х (	2,	2)	0.00000	200.0000
Х (	2,	3)	0.00000	3.300000
Х (	2,	4)	0.00000	5.570000
Х (	2,	5)	0.00000	3.770000
Х (	2,	6)	0.00000	5.790000
Х (	2,	7)	0.00000	4.150000
Х (	2,	8)	0.00000	5.070000
Х (	2,	9)	0.00000	4.790000
Х(	З,	1)	0.000000	9.340000
Х (	3,	2)	1.000000	3.300000
X (	<u>ک</u>	3)	0.000000	200.0000
X (	3, 2	4)	0.000000	5.180000
X (	<b>ک</b> ر ک	5)	0.000000	I.620000
X (	ວ <b>,</b> ວ	6) 7)	0.000000	2 510000
Λ ( Υ /	ວ <b>,</b> ຈ	/) 2\		3.510000 A 190000
л ( Х (	י א	9)	0.000000	3 590000
х (	3, 4	ン) 1 \	0 000000	5 790000
X (	4.	21	0.000000	5.570000
Х(	4.	3)	0.000000	5.180000
Х(	4,	4)	0.000000	200.0000
X (	4,	5)	0.00000	3.640000
Х (	4,	6)	0.00000	0.9200000
Х (	4,	7)	0.00000	1.690000
Х (	4,	8)	1.000000	1.110000
Х (	4,	9)	0.00000	1.740000

Х (	5,	1)	0.00000	8.370000
X (	5,	2)	0.00000	3.770000
x (	5.	3)	1.00000	1,620000
хí	5.	4)	0 - 000000	3.640000
X (	5	-, 5)	0 00000	200 0000
X (	5	6)	0 000000	4 400000
X (	5	7)	0.000000	2 050000
	5,	/) 0\	0.000000	2.00000
	э <b>,</b> Б	0)	0.000000	2.590000
A (	5,	9) 1 \	0.000000	1.990000
X (	6,	1)	0.000000	4.930000
X (	6,	2)	0.000000	5.790000
X (	6, C	3)	0.000000	5.870000
х (	ь,	4)	1.000000	0.9200000
Х (	6,	5)	0.000000	4.400000
Х(	6,	6)	0.000000	200.0000
Х(	6,	/)	0.000000	2.360000
Х (	6,	8)	0.000000	1.990000
Х (	6,	9)	0.000000	2.590000
Х (	Ί,	1)	0.000000	6.540000
Х (	7,	2)	0.000000	4.150000
Х (	7,	3)	0.00000	3.510000
Х (	7,	4)	0.00000	1.690000
Х (	7,	5)	1.000000	2.050000
Х (	7,	6)	0.00000	2.360000
Х (	7,	7)	0.00000	200.0000
Х (	7,	8)	0.00000	0.9200000
Х (	7,	9)	0.00000	0.800000
Х (	8,	1)	0.00000	6.670000
Х (	8,	2)	0.00000	5.070000
Х (	8,	3)	0.00000	4.180000
Х (	8,	4)	0.00000	1.110000
Х (	8,	5)	0.00000	2.590000
Х (	8,	6)	0.00000	1.990000
Х (	8,	7)	0.00000	0.9200000
Х(	8,	8)	0.00000	200.0000
Х (	8,	9)	1.000000	0.6300000
Х(	9,	1)	0.00000	7.140000
Х (	9,	2)	0.00000	4.790000
X (	9,	3)	0.00000	3.590000
X (	9.	4)	0.00000	1.740000
X (	9.	5)	0.000000	1.990000
X (	9.	6)	0.000000	2.590000
x (	9,	7)	1.000000	0.800000
хí	9.	8)	0.00000	0.6300000
Х (	9,	9)	0.000000	200.0000
	F	Row	Slack or Surplus	Dual Price
		1	22.75000	-1.000000
		2	0.00000	0.00000
		3	0.00000	0.00000
		4	0.00000	0.00000
		5	0.00000	0.00000
		6	0.00000	0.00000
		7	0.00000	0.00000
		8	0.00000	0.00000
		9	0.00000	0.00000
		10	0.00000	0.00000
		11	0.00000	0.00000
		12	0.00000	0.00000
		13	0.000000	0.00000
		14	0.00000	0.00000

15	0.00000	0.00000
16	0.00000	0.00000
17	0.00000	0.00000
18	0.00000	0.00000
19	0.00000	0.00000
20	7.000000	0.00000
21	2.000000	0.000000
22	6.000000	0.000000
23	1.000000	0.000000
24	5.000000	0.000000
20	3.000000	0.000000
20	4.000000	0.000000
28	3 000000	0.000000
29	7 000000	0.000000
30	2.000000	0.00000
31	6.000000	0.000000
32	4.000000	0.00000
33	5.000000	0.00000
34	14.00000	0.00000
35	13.00000	0.00000
36	12.00000	0.00000
37	7.000000	0.00000
38	11.00000	0.00000
39	0.00000	0.00000
40	10.00000	0.000000
41	10.00000	0.000000
42	0.00000	0.000000
43	4.000000	0.000000
44	7 000000	0.000000
45	5 000000	0.000000
47	6 000000	0.000000
48	15.00000	0.000000
49	14.00000	0.00000
50	0.00000	0.00000
51	13.00000	0.00000
52	12.00000	0.00000
53	10.00000	0.00000
54	11.00000	0.00000
55	11.00000	0.00000
56	10.00000	0.000000
57	5.000000	0.000000
58 F0	0.000000	0.000000
59	4.000000	0.000000
61	7 000000	0.000000
62	13 00000	0.000000
63	12.00000	0.00000
64	7.000000	0.000000
65	11.00000	0.00000
66	6.000000	0.00000
67	10.00000	0.00000
68	0.000000	0.00000
69	12.00000	0.00000
70	11.00000	0.00000
71	6.00000	0.00000
72	10.00000	0.00000
13	5.000000	0.000000
/4 75		
15	1.000000	0.00000

## Group 6 – model:

MODEL:

#### SETS:

node/1..3/:y; link(node,node): c,x;

#### ENDSETS

#### DATA:

c=0 23.54 20.49 23.54 200 7.29 20.49 7.29 200;

#### ENDDATA

```
MIN = @SUM(link(i,j):
    c(i,j)*x(i,j)
);
@FOR(node(j):
    @SUM(node(i):
       x(i,j))=1
);
@FOR(node(i):
    @SUM(node(j):
      x(i,j))=1
);
@for(node(i):
      @for(node(j)|i#GT#1#AND#j#GT#1#AND#i#NE#j:
            y(i) - y(j) + 3*x(i,j) <= 2;
));
y(1) = 1;
@FOR(link(i,j):
   @BIN(x(i,j))
);
```

# END

# **Group 6 – solution:**

Global optimal solution found.	
Objective value:	51.32000
Objective bound:	51.32000
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	14

Variable	Value	Reduced Cost
Y( 1)	1.000000	0.00000
Y(2)	2.00000	0.00000

Y(3)	0.00000	0.00000
C(1, 1)	0.00000	0.00000
C(1,2)	23.54000	0.00000
C(1,3)	20.49000	0.00000
C(2, 1)	23.54000	0.00000
C(2,2)	200.0000	0.00000
C(2,3)	7.290000	0.00000
C(3,1)	20.49000	0.00000
C(3,2)	7.290000	0.00000
C(3,3)	200.0000	0.00000
X(1,1)	0.00000	0.00000
X(1,2)	0.00000	23.54000
X(1,3)	1.000000	20.49000
X(2,1)	1.000000	23.54000
X(2,2)	0.00000	200.0000
X(2,3)	0.00000	7.290000
X(3,1)	0.000000	20.49000
X(3,2)	1.000000	7.290000
X(3,3)	0.000000	200.0000
Row	Slack or Surplus	Dual Price
1	51.32000	-1.000000
2	0.00000	0.00000
3	0.00000	0.00000
4	0.00000	0.00000
5	0.00000	0.00000
6	0.00000	0.00000
7	0.00000	0.00000
8	0.00000	0.00000
9	1.000000	0.00000
10	0.00000	0.00000

# A.3 Model and solution for stage 3: supply

# Group 3 – model:

```
model:
sets:
disasterarea/1..4/:c,d,e;
relief/1/:y;
link(relief,disasterarea):x;
endsets
data:
c= 0.92 8.45 2.29 1.57;
e= 7.9 5.9 4.2 3.9;
enddata
min=@sum(disasterarea(j):
           c(j)*d(j));
@for(disasterarea(j):
    d(j) <= 30.4);
@for(disasterarea(j):
     d(j) >= e(j));
@for(link(i,j):
    @bin(x(i,j)));
@for(link(i,j):
    d(j)>= 0);
```

# end

# **Group 3 – solution:**

Global optimal solution found.	
Objective value:	72.86400
Objective bound:	72.86400
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
C( 1)	0.9200000	0.00000
C(2)	8.450000	0.00000
C(3)	2.290000	0.00000
C(4)	1.570000	0.00000
D( 1)	7.900000	0.00000
D(2)	5.900000	0.00000
D(3)	4.200000	0.00000
D(4)	3.900000	0.00000
E( 1)	7.900000	0.00000
E(2)	5.900000	0.00000

	Ε(	3)	4.200000	0.00000
	Ε(	4)	3.900000	0.00000
	Υ(	1)	0.00000	0.00000
Х (	1,	1)	0.00000	0.00000
Х (	1,	2)	0.00000	0.00000
Х (	1,	3)	0.00000	0.00000
Х (	1,	4)	0.00000	0.00000
	F	Row	Slack or Surplus	Dual Price
		1	72.86400	-1.000000
		2	22.50000	0.00000
		3	24.50000	0.00000
		4	26.20000	0.00000
		5	26.50000	0.00000
		6	0.00000	-0.9200000
		7	0.00000	-8.450000
		8	0.00000	-2.290000
		9	0.00000	-1.570000
		10	7.900000	0.00000
		11	5.900000	0.00000
		12	4.200000	0.00000
		13	3.900000	0.00000

## Group 4 – model:

model:

sets:

disasterarea/1..8/:c,d,e; relief/1/:y; link(relief,disasterarea):x;

endsets

```
data:
```

**Group 4 – solution:** 

Global optimal solution found.	
Objective value:	325.0470
Objective bound:	325.0470
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
C( 1)	8.580000	0.00000
C(2)	7.450000	0.00000
C(3)	13.02000	0.00000
C(4)	14.77000	0.00000
C(5)	15.01000	0.00000
C(6)	20.43000	0.00000
C(7)	6.920000	0.00000
C(8)	8.690000	0.00000
D( 1)	3.000000	0.00000
D(2)	3.700000	0.00000
D(3)	4.000000	0.00000
D(4)	6.000000	0.00000
D(5)	3.500000	0.00000
D(6)	3.300000	0.00000
D(7)	1.100000	0.00000
D(8)	0.400000	0.00000
E(1)	3.000000	0.000000
E(2)	3.700000	0.000000
E(3)	4.000000	0.000000
E(4)	6.000000	0.000000
E(5)	3.500000	0.000000
E(6)	3.300000	0.000000
上( /)	1.100000	0.000000
E(8)	0.4000000	0.000000
Υ( 1) Υ( 1 1)	0.000000	0.000000
X(1, 1)	0.000000	0.000000
$\Lambda(1, 2)$	0.000000	0.000000
X(1, 3)	0.000000	0.000000
X(1, 4) Y(1, 5)	0.000000	0.000000
X(1, 5) X(1, 6)	0.000000	0.000000
X(1, 0) X(1, 7)	0.000000	0.000000
X(1, 7) X(1, 8)	0.000000	0.000000
X( 1 <b>/</b> 0/	0.000000	0.000000
Row	Slack or Surplus	Dual Price
1	325.0470	-1.000000
2	34.41000	0.00000
3	33.71000	0.00000
4	33.41000	0.00000
5	31.41000	0.00000
6	33.91000	0.00000
7	34.11000	0.00000
8	36.31000	0.00000
9	37.01000	0.00000
10	0.000000	-8.580000
11	0.000000	-7.450000
12	0.00000	-13.02000
13	0.000000	-14.77000
14	0.000000	-15.01000
15	0.000000	-20.43000
16	0.00000	-6.920000

17	0.00000	-8.690000
18	3.000000	0.00000
19	3.700000	0.00000
20	4.000000	0.00000
21	6.000000	0.00000
22	3.500000	0.00000
23	3.300000	0.00000
24	1.100000	0.00000
25	0.400000	0.00000

# Group 5 – model:

model:

sets:

```
disasterarea/1..8/:c,d,e;
relief/1/:y;
link(relief,disasterarea):x;
```

endsets

data:

c= 7.39 9.34 5.79 8.37 4.93 6.54 6.67 7.14; e= 3 6 5.2 3 2.3 1.5 1.2 0.8;

enddata

```
min=@sum(disasterarea(j):
    c(j)*d(j));
```

```
@for(disasterarea(j):
    d(j) <= 28.06);</pre>
```

```
@for(disasterarea(j):
    d(j) >= e(j));
```

@for(link(i,j):
 @bin(x(i,j)));

## **Group 5 – solution:**

Global optimal solution found.	
Objective value:	168.2930
Objective bound:	168.2930
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
C( 1)	7.390000	0.00000
C(2)	9.340000	0.00000
C(3)	5.790000	0.00000

C(4) C(5) C(6) C(7) C(8) D(1) D(2) D(3) D(4) D(4) D(5)	8.370000 4.930000 6.540000 7.140000 3.000000 6.000000 5.200000 3.000000 2.300000	0.000000 0.000000 0.000000 0.000000 0.000000
D(6)	1.500000	0.00000
D(7) D(8)	0.800000	0.000000
E(1)	3.000000	0.00000
E(2)	6.000000	0.00000
E(3) E(4)	5.200000	0.000000
E(5)	2.300000	0.000000
E(6)	1.500000	0.00000
E(7)	1.200000	0.00000
Y(1)	0.000000	0.000000
X(1,1)	0.00000	0.00000
X(1,2)	0.00000	0.00000
X(1, 3) X(1, 4)	0.000000	0.000000
X(1,5)	0.00000	0.000000
X(1,6)	0.00000	0.00000
X(1,7) X(1,8)	0.000000	0.000000
Row 1 2 3	Slack or Surplus 168.2930 25.06000 22.06000	Dual Price -1.000000 0.000000 0.000000
4	22.86000	0.000000
5	25.06000	0.00000
6 7	26.56000	0.000000
8	26.86000	0.000000
9	27.26000	0.00000
10	0.000000	-9.340000
12	0.000000	-5.790000
13	0.00000	-8.370000
14		-4.930000 -6.540000
16	0.000000	-6.670000
17	0.000000	-7.140000
18 19	3.000000	0.000000
20	5.200000	0.000000
21	3.000000	0.00000
22	2.300000	0.000000
23	1.200000	0.000000
25	0.8000000	0.000000

# Group 6 – model:

model:

#### sets:

```
disasterarea/1..2/:c,d,e;
relief/1/:y;
link(relief,disasterarea):x;
```

#### endsets

### data:

c= 23.54 20.49; e= 8.7 3.6;

### enddata

```
end
```

# **Group 6 – solution:**

Global optimal solution found.	
Objective value:	278.5620
Objective bound:	278.5620
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable Value		Reduced Cost
C( 1)	23.54000	0.00000
C(2)	20.49000	0.00000
D( 1)	8.700000	0.00000
D(2)	3.600000	0.00000
E( 1)	8.700000	0.00000
E(2)	3.600000	0.00000
Y( 1)	0.00000	0.00000
X(1,1)	0.00000	0.00000
X(1,2)	0.000000	0.000000
Row	Slack or Surplus	Dual Price
1	278.5620	-1.000000
2	26.38000	0.00000
3	31.48000	0.00000
4	0.00000	-23.54000
5	0.00000	-20.49000
6	8.700000	0.00000

7 3.600000 0.000000

Appendix B

# Additional figures



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# Appendix A

Stage 2 – model

MODEL:

# Mathematical models in LINGO

# A.1 Model and solution for stage 2

```
sets:
centre/1..6/:s;
group/1..6/:d;
assigne(centre,group):c,x;
endsets
data:
s=35.08 28.06 42.09 25.72 37.41 30.40;
d=6.8 4.6 21.9 25 23 12.3;
c= 0.77 0.64 0.46 0.48 0.73 0.89
   0.47 0.24 0.27 0.2 0.3 0.42
   0.79 0.59 0.75 0.68 0.56 0.1
   0.57 0.4 0.6 0.53 0.35 0.16
   0.43 0.23 0.42 0.35 0.2 0.26
   0.77 0.79 1 0.96 0.7 0.73;
Enddata
min=@sum(assigne(i,j):
c(i,j)*x(i,j)
);
@for(centre(i):
    @sum(group(j):
         d(j)*x(i,j))<=s(i)
);
@for(group(j):
    @sum(centre(i):
         x(i, j)) = 1
);
@for(centre(i):
   @sum(group(j):
        x(i, j)) = 1
);
@for(assigne(i,j):
    @bin(x(i,j))
);
end
```

# Stage 2 - solution

Global optimal solution found.	
Objective value:	2.110000
Objective bound:	2.110000
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
S( 1)	35.08000	0.00000
S(2)	28.06000	0.00000
S(3)	42.09000	0.00000
S(4)	25.72000	0.00000
S(5)	37.41000	0.00000
S(6)	30 40000	0 00000
D(1)	6 80000	0 00000
D(1)	4 60000	0.000000
D(2)	21 90000	0.000000
D(3)	21.90000	0.000000
D(4)	25.00000	0.000000
D(5)	23.00000	0.000000
D(6)	12.30000	0.000000
C(1, 1)	0.7700000	0.000000
C(1,2)	0.6400000	0.00000
C(1,3)	0.4600000	0.00000
C(1,4)	0.4800000	0.00000
C(1,5)	0.7300000	0.00000
C(1,6)	0.8900000	0.00000
C(2, 1)	0.4700000	0.00000
C(2,2)	0.2400000	0.00000
C(2, 3)	0.2700000	0.00000
C(2, 4)	0.200000	0.00000
C(2, 5)	0.300000	0.00000
C(2, 6)	0.4200000	0.00000
C(3, 1)	0 7900000	0 00000
C(3, 1)	0.5900000	0.00000
C(3, 2)	0.7500000	0.000000
C(3, 3)	0.7500000	0.000000
C(3, 4)	0.8800000	0.000000
C(3, 5)	0.5600000	0.000000
C(3, 6)	0.1000000	0.000000
C(4, 1)	0.5700000	0.000000
C(4,2)	0.4000000	0.000000
C(4,3)	0.6000000	0.000000
C(4,4)	0.5300000	0.00000
C(4,5)	0.3500000	0.00000
C(4,6)	0.1600000	0.00000
C(5,1)	0.4300000	0.00000
C(5,2)	0.2300000	0.00000
C(5,3)	0.4200000	0.00000
C(5,4)	0.3500000	0.00000
C(5, 5)	0.200000	0.00000
C(5, 6)	0.2600000	0.00000
C(6, 1)	0.7700000	0.00000
C(6, 2)	0 7900000	0 00000
C(6, 3)	1 00000	0 00000
$C(6, \Delta)$	0 9600000	
	0.3000000	0.00000
(0, 0)	0.7000000	0.00000

C(6,	6)	0.7300000	0.00000
X(1,	1)	0.000000	0.7700000
$\mathbf{X}(1, \mathbf{X}(1, \mathbf{X}))$	2) 3)	1,00000	0.8400000
X(1,	4)	0.000000	0.4800000
X(1,	5)	0.00000	0.7300000
X( 1,	6)	0.00000	0.8900000
X(2,	1)	0.00000	0.4700000
X(2,	2)	0.000000	0.2400000
X(Z)	3) 1)	1 000000	0.2700000
X(2,	<b></b> / 5)	0.000000	0.3000000
X(2,	6)	0.000000	0.4200000
Х(З,	1)	0.00000	0.7900000
Х(З,	2)	0.00000	0.5900000
X(3,	3)	0.000000	0.7500000
X(3,	4)	0.000000	0.6800000
X(3, X(3)	ວ) 6)	1 000000	0.5600000
X(4,	1)	0.000000	0.5700000
X(4,	2)	0.000000	0.400000
X(4,	3)	0.00000	0.600000
X(4,	4)	0.00000	0.5300000
X(4,	5)	1.000000	0.3500000
X(4,	6) 1)	0.000000	0.1600000
$\mathbf{X}(5, \mathbf{X}(5, 5))$	1) 2)	1 000000	0.4300000
X(5,	3)	0.000000	0.4200000
X(5,	4)	0.000000	0.3500000
X(5,	5)	0.00000	0.200000
X(5,	6)	0.00000	0.260000
X(6,	1)	1.000000	0.7700000
X(6, X(6	∠) 3)	0.000000	1 00000
X( 6, X( 6,	4)	0.000000	0.9600000
X(6,	5)	0.000000	0.700000
X(6,	6)	0.00000	0.7300000
I	Row	Slack or Surplus	Dual Price
	1	2.110000	-1.000000
	2	3 060000	0.000000
	4	29.79000	0.00000
	5	2.720000	0.00000
	6	32.81000	0.00000
	7	23.60000	0.00000
	8	0.000000	0.00000
	10	0.000000	0.000000
	11	0.00000	0.000000
	12	0.000000	0.000000
	13	0.00000	0.00000
	14	0.00000	0.00000
	15	0.00000	0.00000
	16	0.00000	0.00000
	1 / 1 R		
	19	0.000000	0.000000

# A.2 Model and solution for stage 3: routing

## Group 3 - model:

```
MODEL:
SETS:
      node/1..5/:y;
      link(node, node): c, x;
ENDSETS
DATA:
c=0 0.92 8.45 2.29 1.57
 0.92 200 7.88 1.74 1.57
 8.45 7.88 200 6.18 7.85
 2.29 1.74 6.18 200 2.27
  1.57 1.57 7.85 2.27 200;
ENDDATA
MIN = @SUM(link(i,j):
      c(i,j)*x(i,j)
);
@FOR(node(j):
    @SUM(node(i):
        x(i,j))=1
);
@FOR(node(i):
    @SUM(node(j):
       x(i,j))=1
);
@for(node(i):
      @for(node(j)|i#GT#1#AND#j#GT#1#AND#i#NE#j:
            y(i) - y(j) + 5*x(i,j) <= 4;
));
y(1) = 1;
@FOR(link(i,j):
   @BIN(x(i,j))
);
END
```

## **Group 3 – solution:**

Global optimal solution found.Objective value:18.26000Objective bound:18.26000Infeasibilities:0.000000Extended solver steps:13Total solver iterations:538

Variable	Value	Reduced Cost
Y( 1)	1.000000	0.00000

	Υ(	2)	3.000000	0.00000
	Υ(	3)	1.000000	0.00000
	Υ(	4)	2.000000	0.00000
	Υ(	5)	0.00000	0.00000
С(	1,	1)	0.00000	0.00000
С(	1,	2)	0.9200000	0.00000
С(	1,	3)	8.450000	0.00000
С(	1,	4)	2.290000	0.00000
C (	1,	5)	1.570000	0.00000
C (	2.	1)	0.9200000	0.00000
C (	2,	2)	200.0000	0.00000
C (	2,	3)	7.880000	0.00000
Ċ(	2,	4)	1.740000	0.00000
C (	2,	5)	1.570000	0.00000
C (	з,	1)	8.450000	0.00000
C (	з,	2)	7.880000	0.00000
C (	з,	3)	200.0000	0.00000
Ċ(	з,	4)	6.180000	0.00000
C (	з,	5)	7.850000	0.00000
Ċ(	4,	1)	2.290000	0.00000
C (	4,	2)	1.740000	0.00000
C (	4,	3)	6.180000	0.00000
C (	4,	4)	200.0000	0.00000
C (	4,	5)	2.270000	0.00000
C (	5,	1)	1.570000	0.00000
C (	5,	2)	1.570000	0.000000
C (	5,	3)	7.850000	0.000000
C (	5.	4)	2.270000	0.00000
C (	5.	5)	200.0000	0.00000
Х (	1.	1)	0.00000	0.00000
Х(	1,	2)	0.000000	0.9200000
X (	1.	3)	0.00000	8.450000
X (	1.	4)	0.00000	2.290000
X (	1,	5)	1.000000	1.570000
Х(	2,	1)	1.000000	0.9200000
Х(	2,	2)	0.00000	200.0000
Х(	2,	3)	0.00000	7.880000
Х(	2,	4)	0.00000	1.740000
Х (	2,	5)	0.00000	1.570000
Х (	З,	1)	0.00000	8.450000
Х (	З,	2)	0.00000	7.880000
Х (	З,	3)	0.00000	200.0000
Х (	З,	4)	1.000000	6.180000
Х (	З,	5)	0.00000	7.850000
Х (	4,	1)	0.00000	2.290000
Х (	4,	2)	1.000000	1.740000
Х (	4,	3)	0.00000	6.180000
Х (	4,	4)	0.00000	200.0000
Х (	4,	5)	0.00000	2.270000
Х (	5,	1)	0.00000	1.570000
Х (	5,	2)	0.00000	1.570000
Х (	5,	3)	1.000000	7.850000
Х (	5,	4)	0.00000	2.270000
Х (	5,	5)	0.000000	200.0000
	Ι	Row	Slack or Surplus	Dual Price
		1	18.26000	-1.000000
		2	0.000000	0.00000
		3	0.000000	0.000000
		4	0.000000	0.00000
		5	0.00000	0.000000

6	0.00000	0.00000
7	0.00000	0.00000
8	0.00000	0.00000
9	0.00000	0.00000
10	0.00000	0.00000
11	0.00000	0.00000
12	2.000000	0.00000
13	3.000000	0.00000
14	1.000000	0.00000
15	6.00000	0.00000
16	0.00000	0.00000
17	3.000000	0.00000
18	0.00000	0.00000
19	3.000000	0.00000
20	2.000000	0.00000
21	7.000000	0.00000
22	0.00000	0.00000
23	6.00000	0.00000
24	0.00000	0.00000

## Group 4 – model:

#### MODEL:

### SETS:

node/1..9/:y; link(node,node): c,x;

#### ENDSETS

#### DATA:

```
c= 0 8.58 7.45 13.02 14.77 15.01 20.43 6.92 8.69
8.58 200 6.29 6.65 8.81 8.22 23.40 7.76 3.14
7.45 6.29 200 2.54 3.78 4.41 18.61 5.52 3.30
13.02 6.65 2.54 200 2.16 2 20.60 8.02 4.47
14.77 8.81 3.78 2.16 200 1.62 20 9.14 6.48
15.01 8.22 4.41 2 1.62 200 21.57 9.93 6.4
20.43 23.40 18.61 20.60 20 21.57 200 15.80 20.52
6.92 7.76 5.52 8.02 9.14 9.93 15.8 200 5.35
8.69 3.14 3.3 4.47 6.48 6.4 20.52 5.35 200;
```

#### ENDDATA

```
y(i) - y(j) + 9*x(i,j) <= 8;
));
y(1) = 1;
@FOR(link(i,j):
    @BIN(x(i,j))
);
END</pre>
```

## **Group 4 – solution:**

Global optimal solution found at iteration: 81810 Objective value: 63.90000

Variabl	e Value	Reduced Cost
Y( 1	) 1.000000	0.00000
Y(2	) 7.00000	0.00000
Y( 3	) 5.00000	0.00000
Y( 4	) 4.000000	0.00000
Y(5	) 2.000000	0.00000
Y(6	) 3.00000	0.00000
Y(7	) 1.000000	0.00000
Y( 8	) 0.000000	0.00000
Y( 9	) 6.00000	0.00000
C(1, 1	) 0.000000	0.00000
C(1,2	) 8.580000	0.00000
C(1, 3	) 7.450000	0.00000
C(1,4	) 13.02000	0.00000
C(1,5	) 14.77000	0.00000
C(1,6	) 15.01000	0.00000
C(1, 7	) 20.43000	0.00000
C(1,8	) 6.920000	0.00000
C(1, 9	) 8.690000	0.00000
C(2, 1	) 8.580000	0.00000
C(2,2	) 200.0000	0.00000
C(2,3	) 6.290000	0.00000
C(2,4	) 6.650000	0.00000
C(2,5	) 8.810000	0.00000
C(2,6	) 8.220000	0.00000
C(2,7	) 23.40000	0.00000
C(2,8	) 7.760000	0.00000
C(2,9	) 3.140000	0.00000
C(3, 1	) 7.450000	0.00000
C(3,2	) 6.290000	0.00000
C(3,3	) 200.0000	0.00000
C(3,4	) 2.540000	0.00000
C(3,5	) 3.780000	0.00000
C(3,6	) 4.410000	0.00000
C(3,7	) 18.61000	0.00000
C(3,8	) 5.520000	0.00000
C(3,9	) 3.300000	0.000000
C(4,1	) 13.02000	0.000000
C(4,2	) 6.650000	0.000000
C(4,3	) 2.540000	0.00000
C(4,4	) 200.0000	0.00000
C(4,5	) 2.160000	0.000000
C(4,6	) 2.000000	0.000000
C(4,7	) 20.60000	0.000000
C(4,8	) 8.020000	0.00000

C(4,9)	4.470000	0.00000
C(5, 1)	14.77000	0.000000
C(5,2)	8.810000	0.000000
C(5,3)	3.780000	0.000000
C(5,4)	2.160000	0.000000
C(5, 5)	200.0000	0.000000
C(5, 6)	1.620000	0.000000
C(5, 7)	20.0000	0.00000
C(5, 8)	9 140000	0 000000
C(5, 9)	6 480000	0 000000
C(6, 1)	15 01000	0.000000
C(6, 2)	8 220000	0.000000
C(0, 2)	4 410000	0.000000
C(6, 3)	2 000000	0.000000
C(0, 4)	1 620000	0.000000
C(0, 0)	1.020000	0.000000
C(6, 6)	200.0000	0.000000
C(6, 7)	21.57000	0.000000
C(6, 8)	9.930000	0.000000
C(6, 9)	6.400000	0.000000
C( /, 1)	20.43000	0.000000
C(7, 2)	23.40000	0.000000
C(7,3)	18.61000	0.000000
C('/, 4)	20.60000	0.000000
C(7,5)	20.00000	0.000000
C(7,6)	21.57000	0.000000
C(7,7)	200.0000	0.000000
C(7,8)	15.80000	0.000000
C(7,9)	20.52000	0.000000
C( 8, 1)	6.920000	0.000000
C(8,2)	7.760000	0.000000
C(8,3)	5.520000	0.000000
C(8,4)	8.020000	0.00000
C(8,5)	9.140000	0.00000
C(8,6)	9.930000	0.00000
C(8,7)	15.80000	0.00000
C(8,8)	200.0000	0.00000
C(8,9)	5.350000	0.00000
C( 9, 1)	8.690000	0.000000
C(9,2)	3.140000	0.000000
C(9,3)	3.300000	0.00000
C(9,4)	4.470000	0.00000
C(9,5)	6.480000	0.000000
C(9,6)	6.400000	0.000000
C(9,7)	20.52000	0.000000
C(9,8)	5.350000	0.00000
C(9, 9)	200.0000	0.000000
X(1, 1)	0.000000	0.00000
X(1,2)	0.000000	8.580000
X(1,3)	0.000000	7.450000
X(1, 4)	0.00000	13.02000
X(1, 5)	0.00000	14.77000
X(1, 6)	0.000000	15.01000
(1, 7)	0.000000	20.43000
X(1,8)	1.000000	6,920000
X(1, 9)	0 000000	8 690000
x(2, 1)	1 000000	8 580000
X(2, 2)	0 00000	200 0000
		200.0000
X(2) $X(3)$	0 00000	6 290000
X(2, 3)	0.00000	6.290000
X(2, 3) X(2, 4) X(2, 5)	0.000000 0.000000	6.290000 6.650000 8.810000
X(2, 3) X(2, 4) X(2, 5) X(2, 6)	0.000000 0.000000 0.000000	6.290000 6.650000 8.810000

Х(	2,	7)	0.000000	23.40000
Х(	2,	8)	0.000000	7.760000
Х(	2,	9)	0.000000	3.140000
Х(	З,	1)	0.000000	7.450000
Х (	3,	2)	0.00000	6.290000
Х (	3,	3)	0.000000	200.0000
Х(	з,	4)	0.00000	2.540000
Х(	з,	5)	0.000000	3.780000
X (	3.	6)	0.000000	4.410000
X (	3.	7)	0.000000	18,61000
Х (	3.	8)	0.000000	5.520000
X (	3,	9)	1.000000	3.300000
х(	4.	1)	0.000000	13.02000
X (	4,	2)	0.000000	6.650000
X (	4,	3)	1.000000	2.540000
Х(	4,	4)	0.00000	200.0000
Х(	4,	5)	0.00000	2.160000
Х(	4.	6)	0.000000	2.000000
X (	4,	7)	0.00000	20.60000
Х(	4,	8)	0.000000	8.020000
X (	4,	9)	0.00000	4.470000
Х (	, 5,	1)	0.00000	14.77000
Х (	5,	2)	0.00000	8.810000
Х(	5,	3)	0.000000	3.780000
Х(	5,	4)	0.000000	2.160000
Х(	5,	5)	0.000000	200.0000
Х (	5,	6)	1.000000	1.620000
Х (	5,	7)	0.000000	20.00000
Х (	5,	8)	0.000000	9.140000
Х(	5,	9)	0.000000	6.480000
	<u> </u>	1 \	0 00000	15 01000
Х(	6,	⊥)	0.000000	13.01000
X ( X (	6, 6,	1) 2)	0.000000	8.220000
X ( X ( X (	6, 6, 6,	1) 2) 3)	0.000000 0.000000 0.000000	8.220000 4.410000
X ( X ( X ( <b>X</b> (	6, 6, 6,	1) 2) 3) <b>4)</b>	0.000000 0.000000 0.000000 1.000000	8.220000 4.410000 <b>2.000000</b>
X ( X ( X ( X ( X (	6, 6, 6, 6,	1) 2) 3) <b>4)</b> 5)	0.000000 0.000000 1.000000 0.000000	8.220000 4.410000 2.000000 1.620000
X ( X ( X ( X ( X ( X (	6, 6, 6, 6, 6,	1) 2) 3) <b>4)</b> 5) 6)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000	8.220000 4.410000 2.000000 1.620000 200.0000
X ( X ( X ( X ( X ( X ( X (	6,, 6, 6,, 6,, 6,,	1) 2) 3) <b>4)</b> 5) 6) 7)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000	8.220000 4.410000 2.000000 1.620000 200.0000 21.57000
X ( X ( X ( X ( X ( X ( X ( X (	6, 6, 6, 6, 6, 6, 6,	1) 2) 3) <b>4)</b> 5) 6) 7) 8)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000	8.220000 4.410000 2.000000 1.620000 200.0000 21.57000 9.930000
X ( X ( X ( X ( X ( X ( X ( X ( X (	6, 6, 6, 6, 6, 6, 6, 6,	2) 3) <b>4)</b> 5) 6) 7) 8) 9)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000	8.220000 4.410000 2.000000 1.620000 200.0000 21.57000 9.930000 6.400000
X ( X ( X ( X ( X ( X ( X ( X ( X ( X (	6, 6, 6, 6, 6, 6, 7,	1) 2) 3) <b>4)</b> 5) 6) 7) 8) 9) 1)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000
X ( X ( X ( X ( X ( X ( X ( X ( X ( X (	6, 6, 6, 6, 6, 6, 7, 7,	1) 2) 3) <b>4)</b> 5) 6) 7) 8) 9) 1) 2)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000
X ( X ( X ( X ( X ( X ( X ( X ( X ( X (	6,,6,,6,, 6,,6,,7,,7,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000
X (( X () X () X () X () X () X () X ()	6,,6,6,6,7,7,7,7,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000
X (( X () X () X () X () X () X () X ()	6,,66,,66,,77,7,77,77,77,7	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5)	0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 1.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000
X (( X () X () X () X () X () X () X ()	6,,, 6,,, 6,,,,,,,,,,,,,,,,,,,,,,,,,,,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7)	0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.00000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.00000         15.80000
X (( X (( X (( X (( X (( X (( X (( X ((	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> <b>1.000000</b> 0.000000 <b>0.</b> 000000 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.00000         21.57000         200.0000         21.57000         200.0000         15.80000         20.52000
X (( X (( X (( X (( X (( X (( X (( X ((	6, , 6, 6, 6, 7, 7, 7, 7, 7, 7, 8,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.00000         21.57000         20.00000         15.80000         20.52000         6.920000
X (( X () X () X () X () X () X () X ()	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 7, 7, 7, 7, 7, 8, 8, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 6) 7) 8) 9) 1) 2) 3) 6) 7) 8) 9) 1) 2) 3) 2) 3) 2) 5) 6) 7) 8) 9) 1) 2) 3) 2) 2) 2) 2) 2) 2) 2) 2) 2) 2	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         20.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.00000         15.80000         20.52000         7.760000
X (( X () X () X () X () X () X () X ()	6, , , , , , , , , , , , , , , , , , ,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.0000000 0.0000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         20.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.00000         15.80000         20.52000         6.920000         7.760000
X (( X (( X () X () X () X () X () X ()	6, , , , , , , , , , , , , , , , , , ,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 5) 6) 7) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000	13.01000         8.220000         4.410000         2.000000         1.620000         20.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.00000         15.80000         20.52000         6.920000         7.760000         8.020000
X (( X (( X () X () X () X () X () X ()	6,,6,6,6,6,7,7,7,7,7,7,8,8,8,8,8,8,8,8,8	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 5) 6) 5) 6) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.0000000 0.000000 0.0000000 0.000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         20.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.60000         21.57000         20.00000         15.80000         20.52000         6.920000         7.760000         5.520000         8.020000         9.140000
X (( X (( X (( X (( X (( X (( X (( X ((	6,,6,6,6,6,7,7,7,7,7,7,8,8,8,8,8,8,8,8,8	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4)	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.0000000 0.0000000 0.000000 0.000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         20.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.60000         20.60000         21.57000         20.00000         15.80000         20.52000         6.920000         7.760000         5.520000         8.020000         9.140000         9.30000
X (( X (	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 7) 8) 7) 7) 7) 7) 7) 7) 7) 7) 7) 7	0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         20.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         21.57000         20.60000         20.00000         21.57000         20.52000         6.920000         7.760000         5.520000         8.020000         9.140000         9.30000
X (( X (	6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 6) 7) 8) 8) 9) 1) 2) 8) 8) 8) 9) 1) 2) 8) 8) 8) 8) 8) 8) 8) 8) 8) 8	0.000000 0.000000 <b>1.000000</b> 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 <b>1.000000</b> 0.0000000 0.0000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.00.0000         1.620000         201.57000         9.930000         6.400000         20.43000         23.40000         23.40000         20.60000         20.60000         20.00000         21.57000         20.00000         15.80000         9.930000         15.80000         9.930000         15.80000         20.0000
X (( X (	6, , 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 1) 2) 3) 1) 1) 1) 1) 1) 1) 1) 1) 1) 1	0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.000000 0.0000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         18.61000         20.60000         20.00000         21.57000         20.00000         21.57000         20.00000         15.80000         9.140000         9.30000         15.80000         200.0000
X (( X (	6,6,6,6,6,77777777788888888888888888888	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 2) 2) 2) 2) 2) 2) 2) 2) 2) 2	0.000000 0.000000 1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000 0.0000000 0.0000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         201.57000         9.930000         6.400000         20.43000         23.40000         23.40000         23.40000         20.60000         20.60000         20.52000         6.920000         7.760000         5.520000         8.020000         9.140000         9.30000         15.80000         20.0000         5.350000         8.690000
X (( X (	6, , , 6, , 6, , 7, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 9, 9, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 2) 2) 2) 2) 2) 2) 2) 2) 2) 2	0.000000 0.000000 1.000000 0.0000000 0.0000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         200.0000         21.57000         9.930000         6.400000         20.43000         23.40000         23.40000         20.60000         20.60000         20.00000         21.57000         20.00000         21.57000         20.00000         5.80000         20.52000         8.020000         9.140000         9.30000         15.80000         200.0000         5.350000         3.140000         2.00000
X (( X X () X X	6,6,6,6,6,77777777788888888899990	1) 2) 3) 4) 5) 6) 7) 8) 9) 1) 2) 3) 4) 5) 5) 6) 7) 8) 9) 1) 2) 3) 4) 2) 3) 4) 2) 3) 4) 2) 3) 4) 2) 3) 4) 2) 3) 4) 2) 3) 3) 4) 2) 3) 4) 2) 3) 3) 4) 2) 3) 4) 2) 3) 3) 4) 2) 3) 4) 2) 3) 3) 4) 5) 5) 5) 5) 5) 5) 5) 5) 5) 5	0.000000 0.000000 1.000000 0.0000000 0.000000 0.0000000 0.000000 0.000000 0.000000 0.0000000 0.00000000	13.01000         8.220000         4.410000         2.000000         1.620000         20.0000         21.57000         9.930000         6.400000         20.43000         23.40000         23.40000         23.40000         23.40000         20.60000         20.00000         21.57000         20.00000         15.80000         20.52000         6.920000         7.760000         5.520000         8.020000         9.140000         9.30000         15.80000         20.0000         3.140000         3.00000

### Group 5 – model:

MODEL:

```
SETS:
    node/1..9/:y;
    link(node,node): c,x;
ENDSETS
```

DATA:

```
c= 0 7.39 9.34 5.79 8.37 4.93 6.54 6.67 7.14
7.39 200 3.30 5.57 3.77 5.79 4.15 5.07 4.79
9.34 3.30 200 5.18 1.62 5.87 3.51 4.18 3.59
5.79 5.57 5.18 200 3.64 0.92 1.69 1.11 1.74
8.37 3.77 1.62 3.64 200 4.40 2.05 2.59 1.99
4.93 5.79 5.87 0.92 4.40 200 2.36 1.99 2.59
6.54 4.15 3.51 1.69 2.05 2.36 200 0.92 0.8
6.67 5.07 4.18 1.11 2.59 1.99 0.92 200 0.63
7.14 4.79 3.59 1.74 1.99 2.59 0.8 0.63 200;
```

```
ENDDATA
```

```
MIN = @SUM(link(i,j):
      c(i,j)*x(i,j)
);
@FOR(node(j):
    @SUM(node(i):
        x(i,j))=1
);
@FOR(node(i):
    @SUM(node(j):
       x(i,j))=1
);
@for(node(i):
      @for(node(j)|i#GT#1#AND#j#GT#1#AND#i#NE#j:
            y(i) - y(j) + 9*x(i,j) \le 8;
));
v(1) = 1;
@FOR(link(i,j):
   @BIN(x(i,j))
);
```

# END

## **Group 5 – solution:**

Global	optimal	solution	found	at	iteration:	138656	
Objec	ctive val	lue:				22.75000	

Variable	Value	Reduced Cost
Y( 1)	1.000000	0.00000
Y(2)	7.000000	0.00000
Y(3)	6.000000	0.00000

	Υ(	4)	1.000000	0.00000
	Ý(	5)	5.00000	0.00000
	Ϋ́	6)	0.00000	0.00000
	Y (	7)	4 000000	0 000000
	- ( V (	8)	2 00000	0 000000
	1 ( V (	0) Q)	3 000000	0.000000
~ /	1	9) 1)	3.000000	0.000000
C (	1,	1)	0.000000	0.000000
C (	1,	2)	7.390000	0.000000
С (	1,	3)	9.340000	0.000000
С (	1,	4)	5.790000	0.00000
С (	1,	5)	8.370000	0.00000
С (	1,	6)	4.930000	0.00000
С (	1,	7)	6.540000	0.00000
С(	1,	8)	6.670000	0.00000
С(	1,	9)	7.140000	0.00000
С(	2,	1)	7.390000	0.00000
Ċ	2.	$2)^{i}$	200.0000	0.00000
C (	2.	3)	3 300000	0 000000
	2	4)	5 570000	0 000000
	2,	エ) 5)	3 770000	0.000000
	2 <b>,</b>	5)	5.770000	0.000000
	ζ,	6) 7)	5.790000	0.000000
C (	Ζ,	/)	4.150000	0.000000
C (	2,	8)	5.070000	0.000000
С (	2,	9)	4.790000	0.000000
С (	З,	1)	9.340000	0.00000
С (	З,	2)	3.300000	0.00000
С (	З,	3)	200.0000	0.00000
С (	З,	4)	5.180000	0.00000
С(	З,	5)	1.620000	0.00000
С(	3,	6)	5.870000	0.00000
С(	З,	7)	3.510000	0.00000
C (	3.	8)	4.180000	0.00000
ĊĹ	3.	9)	3.590000	0.00000
C (	4.	1)	5.790000	0.00000
	Δ	2)	5 570000	0 000000
	т <b>,</b> Л	2)	5 180000	0.000000
	ч, Л	3) 4)	200 0000	0.000000
	ч, л	4) 5)	200.0000	0.000000
	4,	5)	3.640000	0.000000
C (	4,	6)	0.9200000	0.000000
С (	4,	7)	1.690000	0.000000
С (	4,	8)	1.110000	0.000000
С (	4,	9)	1.740000	0.000000
С (	5,	1)	8.370000	0.00000
С (	5,	2)	3.770000	0.00000
С (	5,	3)	1.620000	0.00000
С (	5,	4)	3.640000	0.00000
С (	5,	5)	200.0000	0.00000
С(	5,	6)	4.400000	0.00000
С(	5,	7)	2.050000	0.00000
С(	5,	8)	2.590000	0.00000
Ċ	5.	9)	1,990000	0.00000
С(	6.	1)	4.930000	0.00000
C (	6	2)	5 790000	0 000000
	6	בי גו	5 870000	
	6	) /\	0 0200000	0.000000
	о <b>,</b>	4) 5)	4 400000	0.000000
	ο,	5)	4.400000	0.000000
	ь,	ю) ,	200.0000	0.000000
C(	6,	/)	2.360000	0.000000
C (	6,	୪)	T.990000	0.000000
С (	6,	9)	2.590000	0.00000
С (	7,	1)	6.540000	0.00000

С(	7,	2)	4.150000	0.00000
С(	7,	3)	3.510000	0.00000
С (	7,	4)	1.690000	0.00000
С (	7,	5)	2.050000	0.00000
С (	7,	6)	2.360000	0.00000
С(	7,	7)	200.0000	0.00000
С(	7,	8)	0.9200000	0.00000
С(	7,	9)	0.800000	0.00000
С(	8,	1)	6.670000	0.00000
С(	8,	2)	5.070000	0.00000
С(	8,	3)	4.180000	0.00000
С (	8,	4)	1.110000	0.00000
С(	8,	5)	2.590000	0.00000
С(	8,	6)	1.990000	0.00000
С(	8,	7)	0.9200000	0.00000
С(	8,	8)	200.0000	0.00000
С(	8,	9)	0.6300000	0.00000
С(	9,	1)	7.140000	0.00000
С(	9,	2)	4.790000	0.00000
С(	9,	3)	3.590000	0.00000
С (	9,	4)	1.740000	0.00000
С(	9,	5)	1.990000	0.00000
С(	9,	6)	2.590000	0.00000
С (	9,	7)	0.800000	0.000000
С (	9,	8)	0.6300000	0.000000
С(	9,	9)	200.0000	0.000000
Х(	1,	1)	0.000000	0.000000
X (	⊥, 1	2)	0.000000	7.390000
X (	⊥, 1	3)	0.000000	9.340000
X (	⊥, 1	4) 5)	0.000000	5.790000
X (	1.	6)	1 000000	4 930000
X (	1.	7)	0 00000	6 540000
Х (	1.	8)	0.000000	6.670000
Х(	1,	9)	0.000000	7.140000
X (	2,	1)	1.000000	7.390000
Х (	2,	2)	0.00000	200.0000
Х (	2,	3)	0.00000	3.300000
Х (	2,	4)	0.00000	5.570000
Х(	2,	5)	0.00000	3.770000
Х (	2,	6)	0.00000	5.790000
Х (	2,	7)	0.00000	4.150000
Х (	2,	8)	0.00000	5.070000
Х (	2,	9)	0.000000	4.790000
Х(	З,	1)	0.000000	9.340000
Х(	3,	2)	1.000000	3.300000
Х(	3,	3)	0.000000	200.0000
X (	з <b>,</b>	4)	0.000000	5.180000
X (	<u>ح</u>	5)	0.000000	1.620000 E 070000
X (	ວ <b>,</b> ວ	6) 7)	0.000000	2 510000
A ( V (	י ג	2) 2)	0.000000	1 180000
х ( х (	י ר א	9) 91	0 000000	3 590000
х (	4.	ン) 1)	0.000000	5.790000
X (	4.	2)	0.000000	5.570000
Х(	4.	3)	0.000000	5.180000
X (	4,	4)	0.00000	200.0000
Х (	4,	5)	0.00000	3.640000
Х (	4,	6)	0.00000	0.9200000
Х (	4,	7)	0.00000	1.690000
Х (	4,	8)	1.000000	1.110000
Х (	4,	9)	0.00000	1.740000
------------	-----------------	-----------	------------------	------------
Х (	5,	1)	0.00000	8.370000
Х (	5,	2)	0.00000	3.770000
Х (	5,	3)	1.000000	1.620000
Х (	5,	4)	0.00000	3.640000
Х (	5,	5)	0.000000	200.0000
Х (	5,	6)	0.000000	4.400000
Х(	5,	7)	0.000000	2.050000
X (	5,	8)	0.000000	2.590000
X ( V (	э <b>,</b> 6	9) 1)	0.000000	1.990000
A ( X (	6	1) 2)	0.000000	4.930000
X (	6, 6,	2)	0.000000	5 870000
X (	ő,	4)	1.000000	0.9200000
Х(	6,	5)	0.00000	4.400000
Х (	6,	6)	0.00000	200.0000
Х (	6,	7)	0.00000	2.360000
Х (	6,	8)	0.00000	1.990000
Х (	6,	9)	0.00000	2.590000
Х(	7,	1)	0.00000	6.540000
Х (	7,	2)	0.00000	4.150000
Х(	7,	3)	0.000000	3.510000
X (	<i>''</i> ,	4) E)	0.000000	1.690000
<b>X</b> (	',	<b>5)</b>	1.000000	2.050000
X (	' <b>'</b>	6) 7)	0.000000	2.360000
л ( Х (	' <b>'</b>	7) 8)	0.000000	0 9200000
X (	7.	9)	0.00000	0.800000
Х(	8,	1)	0.000000	6.670000
Х (	8,	2)	0.00000	5.070000
Х(	8,	3)	0.00000	4.180000
Х(	8,	4)	0.00000	1.110000
Х (	8,	5)	0.00000	2.590000
Х (	8,	6)	0.00000	1.990000
Х (	8,	7)	0.00000	0.9200000
Х(	8,	8)	0.000000	200.0000
X (	8,	9)	1.000000	0.6300000
X(	9,	⊥) 2)	0.000000	7.140000
Λ ( Υ (	9, G	2) 3)	0.000000	4.790000
X (	9.	4)	0 000000	1 740000
X (	9.	5)	0.00000	1.990000
Х(	9,	6)	0.000000	2.590000
Х (	9,	7)	1.000000	0.800000
Х(	9,	8)	0.00000	0.6300000
Х (	9,	9)	0.000000	200.0000
	Ι	Row	Slack or Surplus	Dual Price
		⊥ 2	0 00000	
		2	0.00000	0,00000
		4	0.000000	0.000000
		5	0.00000	0.00000
		6	0.00000	0.00000
		7	0.00000	0.00000
		8	0.00000	0.00000
		9	0.00000	0.000000
		10 11	0.00000	0.000000
		⊥⊥ 1		
		⊥∠ 1 २	0.00000	0.000000
			3.000000	0.000000

14	0.00000	0.00000
15	0.00000	0.00000
16	0.00000	0.00000
17	0.000000	0.000000
18	0.000000	0.000000
19	0.000000	0.000000
20	7.000000	0.000000
21	2.000000	0.000000
22	1 000000	0.000000
23	5 000000	0.000000
25	3 000000	0.000000
26	4.000000	0.000000
27	0.000000	0.000000
28	3.000000	0.00000
29	7.000000	0.00000
30	2.000000	0.00000
31	6.00000	0.00000
32	4.000000	0.00000
33	5.000000	0.00000
34	14.00000	0.00000
35	13.00000	0.000000
36	12.00000	0.000000
3/	1.00000	0.000000
38	11.00000	0.000000
39 40	10 00000	0.000000
40	10.00000	0.000000
42	0.00000	0.000000
43	4.000000	0.000000
44	3.000000	0.000000
45	7.000000	0.00000
46	5.000000	0.00000
47	6.00000	0.00000
48	15.00000	0.00000
49	14.00000	0.00000
50	0.000000	0.000000
51	13.00000	0.000000
52	12.00000	0.000000
53	11 00000	0.000000
55	11 00000	0.000000
56	10.00000	0.00000
57	5.000000	0.000000
58	0.00000	0.00000
59	4.000000	0.00000
60	6.00000	0.00000
61	7.000000	0.00000
62	13.00000	0.00000
63	12.00000	0.00000
64	7.000000	0.00000
65	11.00000	0.000000
66	6.00000	0.000000
68	10.00000	
69	12 00000	
70	11.00000	0.00000
71	6.000000	0.000000
72	10.00000	0.000000
73	5.000000	0.00000
74	0.00000	0.00000

75	7.000000	0.00000
76	0.00000	0.00000

## Group 6 – model:

MODEL:

SETS:

node/1..3/:y; link(node,node): c,x;

ENDSETS

#### DATA:

c=0 23.54 20.49 23.54 200 7.29 20.49 7.29 200;

#### ENDDATA

```
MIN = @SUM(link(i,j):
      c(i,j)*x(i,j)
);
@FOR(node(j):
    @SUM(node(i):
       x(i,j))=1
);
@FOR(node(i):
    @SUM(node(j):
       x(i,j))=1
);
@for(node(i):
      @for(node(j)|i#GT#1#AND#j#GT#1#AND#i#NE#j:
            y(i) - y(j) + 3*x(i,j) \le 2;
));
y(1) = 1;
@FOR(link(i,j):
   @BIN(x(i,j))
);
```

#### END

## **Group 6 – solution:**

Global optimal solution found.	
Objective value:	51.32000
Objective bound:	51.32000
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	14

Variable	Value	Reduced Cost
Y( 1)	1.000000	0.00000

Y(2)	2.000000	0.00000
Y(3)	0.00000	0.00000
C(1, 1)	0.00000	0.00000
C(1,2)	23.54000	0.00000
C(1,3)	20.49000	0.00000
C(2,1)	23.54000	0.00000
C(2,2)	200.0000	0.00000
C(2,3)	7.290000	0.00000
C(3,1)	20.49000	0.00000
C(3,2)	7.290000	0.00000
C(3,3)	200.0000	0.00000
X(1,1)	0.00000	0.00000
X(1,2)	0.00000	23.54000
X(1,3)	1.000000	20.49000
X(2,1)	1.000000	23.54000
X(2,2)	0.00000	200.0000
X(2,3)	0.00000	7.290000
X(3,1)	0.00000	20.49000
X(3,2)	1.000000	7.290000
X(3,3)	0.00000	200.0000
Row	Slack or Surplus	Dual Price
1	51.32000	-1.000000
2	0.00000	0.00000
3	0.00000	0.00000
4	0.00000	0.00000
5	0.00000	0.00000
6	0.00000	0.00000
7	0.00000	0.00000
8	0.00000	0.00000
9	1.000000	0.00000
10	0.00000	0.00000

## A.3 Model and solution for stage 3: supply

## Group 3 – model:

```
model:
sets:
disasterarea/1..4/:c,d,e;
relief/1/:y;
link(relief,disasterarea):x;
endsets
data:
c= 0.92 8.45 2.29 1.57;
e= 7.9 5.9 4.2 3.9;
enddata
min=@sum(disasterarea(j):
            c(j)*d(j));
@for(disasterarea(j):
    d(j) <= 30.4);
@for(disasterarea(j):
      d(j) >= e(j));
@for(link(i,j):
    @bin(x(i,j)));
@for(link(i,j):
    d(j) >= 0);
end
```

## **Group 3 – solution:**

Global optimal solution found.	
Objective value:	72.86400
Objective bound:	72.86400
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
C( 1)	0.9200000	0.00000
C(2)	8.450000	0.00000
C(3)	2.290000	0.00000
C(4)	1.570000	0.00000
D( 1)	7.900000	0.00000
D(2)	5.900000	0.00000
D(3)	4.200000	0.00000
D(4)	3.900000	0.00000
E( 1)	7.900000	0.00000

E(2)	5.900000	0.00000
E(3)	4.200000	0.00000
E(4)	3.900000	0.00000
Y( 1)	0.00000	0.00000
X(1,1)	0.00000	0.00000
X(1,2)	0.00000	0.00000
X(1,3)	0.00000	0.00000
X(1,4)	0.00000	0.00000
Row	Slack or Surplus	Dual Price
1	72.86400	-1.000000
2	22.50000	0.00000
3	24.50000	0.00000
4	26.20000	0.00000
5	26.50000	0.00000
6	0.00000	-0.9200000
7	0.00000	-8.450000
8	0.00000	-2.290000
9	0.00000	-1.570000
10	7.900000	0.00000
11	5.900000	0.00000
12	4.200000	0.00000
13	3.900000	0.00000

## Group 4 – model:

model:

sets:

disasterarea/1..8/:c,d,e; relief/1/:y; link(relief,disasterarea):x;

endsets

#### data:

```
d(j)>= 0);
end
```

## **Group 4 – solution:**

Global optimal solution found.	
Objective value:	325.0470
Objective bound:	325.0470
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
C(1)	8.580000	0.00000
C(2)	7 450000	0 00000
C(3)	13 02000	0 000000
C(4)	14 77000	0 000000
C(5)	15 01000	0.000000
C(5)	20 43000	0.000000
C(0)	6 920000	0.000000
C(7)	0.920000	0.000000
C(8)	8.690000	0.000000
D(1)	3.000000	0.000000
D(2)	3.700000	0.000000
D(3)	4.000000	0.000000
D(4)	6.000000	0.000000
D(5)	3.500000	0.000000
D(6)	3.300000	0.000000
D(7)	1.100000	0.000000
D(8)	0.4000000	0.000000
E( 1)	3.000000	0.000000
E(2)	3.700000	0.00000
E( 3)	4.000000	0.00000
E(4)	6.000000	0.00000
E( 5)	3.500000	0.00000
E(6)	3.300000	0.00000
E(7)	1.100000	0.00000
E( 8)	0.4000000	0.00000
Y( 1)	0.000000	0.00000
X(1,1)	0.000000	0.00000
X(1,2)	0.00000	0.00000
X(1,3)	0.00000	0.00000
X(1,4)	0.000000	0.00000
X(1,5)	0.00000	0.00000
X(1,6)	0.00000	0.00000
X(1,7)	0.00000	0.00000
X(1,8)	0.00000	0.00000
Row	Slack or Surplus	Dual Price
1	325.0470	-1.000000
2	34.41000	0.00000
3	33.71000	0.00000
4	33.41000	0.00000
5	31.41000	0.00000
6	33.91000	0.00000
7	34.11000	0.00000
8	36.31000	0.00000
9	37.01000	0.00000
10	0.00000	-8.580000
11	0.00000	-7.450000
12	0.00000	-13.02000
13	0.00000	-14.77000
14	0.000000	-15.01000

15	0.00000	-20.43000
16	0.00000	-6.920000
17	0.00000	-8.690000
18	3.000000	0.00000
19	3.700000	0.00000
20	4.000000	0.00000
21	6.00000	0.00000
22	3.500000	0.00000
23	3.300000	0.00000
24	1.100000	0.00000
25	0.400000	0.00000

## Group 5 – model:

model:

#### sets:

disasterarea/1..8/:c,d,e; relief/1/:y; link(relief,disasterarea):x;

#### endsets

#### data:

c= 7.39 9.34 5.79 8.37 4.93 6.54 6.67 7.14; e= 3 6 5.2 3 2.3 1.5 1.2 0.8;

#### enddata

```
min=@sum(disasterarea(j):
    c(j)*d(j));
```

```
@for(disasterarea(j):
    d(j) <= 28.06);</pre>
```

@for(link(i,j):
 @bin(x(i,j)));

## **Group 5 – solution:**

Global optimal solution found.	
Objective value:	168.2930
Objective bound:	168.2930
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
C( 1)	7.390000	0.00000

C( 2) C( 3) C( 4) C( 5) C( 6) C( 7) C( 8) D( 1) D( 2) D( 3) D( 4) D( 5) D( 6) D( 7) D( 8) E( 1) E( 2) E( 3) E( 1) E( 2) E( 3) E( 4) E( 2) E( 3) E( 4) E( 5) E( 6) E( 7) E( 8) Y( 1) X( 1, 1) X( 1, 2) X( 1, 3) X( 1, 4) X( 1, 5) X( 1, 6) X( 1, 7)	9.340000 5.790000 8.370000 4.930000 6.540000 6.540000 7.140000 3.000000 5.200000 3.000000 1.500000 1.200000 3.000000 3.000000 3.000000 2.300000 1.500000 3.000000 0.8000000 1.200000 0.800000 0.000000	
X(1,8) Row 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	0.000000 Slack or Surplus 168.2930 25.06000 22.06000 22.86000 25.76000 26.56000 26.56000 27.26000 0.0000000 0.0000000 0.000000 0.0000000 0.00000000	Dual Price -1.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 -7.390000 -9.340000 -9.340000 -9.340000 -5.790000 -8.370000 -4.930000 -6.540000 -6.540000 -7.140000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000000

```
model:
```

```
sets:
```

```
disasterarea/1..2/:c,d,e;
relief/1/:y;
link(relief,disasterarea):x;
```

### endsets

#### data:

c= 23.54 20.49; e= 8.7 3.6;

#### enddata

#### end

## **Group 6 – solution:**

Global optimal solution found.	
Objective value:	278.5620
Objective bound:	278.5620
Infeasibilities:	0.00000
Extended solver steps:	0
Total solver iterations:	0

Variable	Value	Reduced Cost
C( 1)	23.54000	0.00000
C(2)	20.49000	0.00000
D( 1)	8.700000	0.00000
D(2)	3.600000	0.00000
E( 1)	8.700000	0.00000
E(2)	3.600000	0.00000
Y( 1)	0.00000	0.00000
X(1,1)	0.00000	0.00000
X(1,2)	0.000000	0.000000
Dout	Clack on Cumplus	Dual Drigo
ROW	STACK OF SULPIUS	Dual FIICe
1	278.5620	-1.000000
2	26.38000	0.00000
3	31.48000	0.00000
4	0.00000	-23.54000

5	0.00000	-20.49000
6	8.700000	0.00000
7	3.600000	0.00000

# Appendix B

## **B.1 Additional figures**