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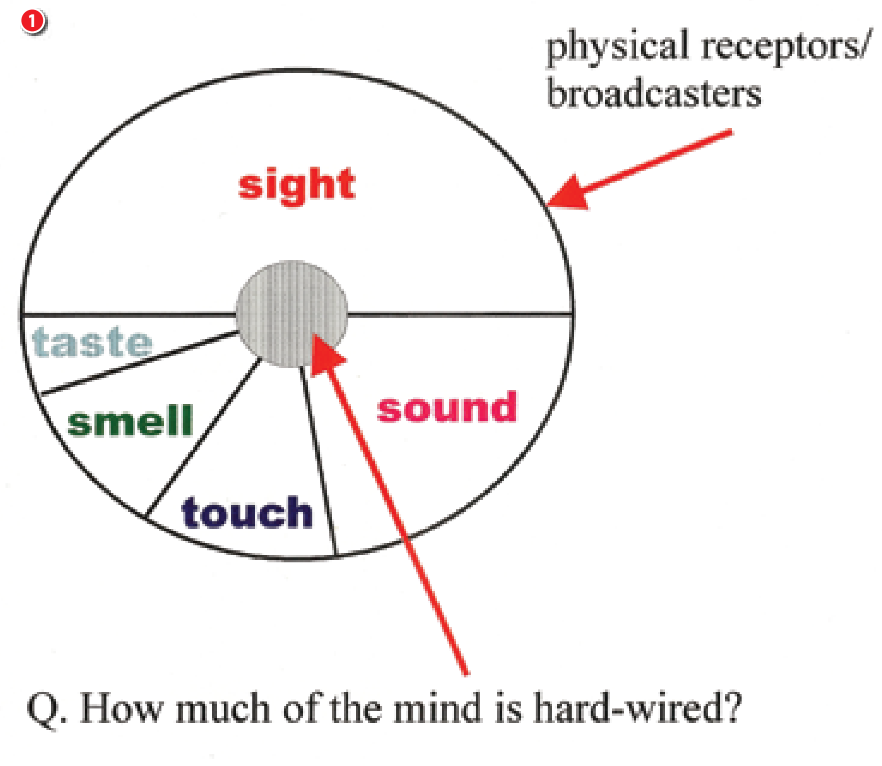
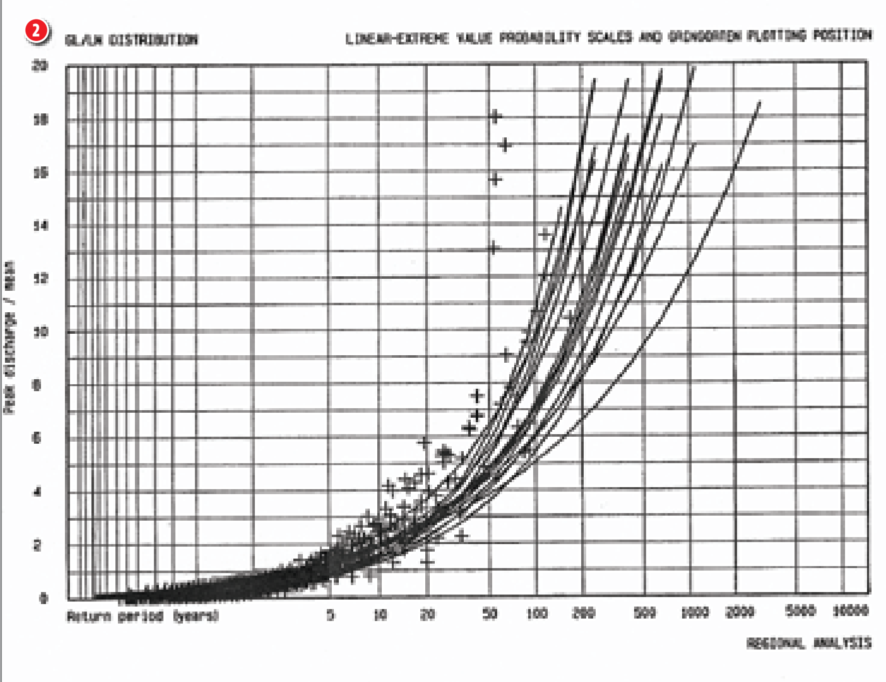
Mathematics vs pattern recognition in water resource studies





PATTERN RECOGNITION IS hard-wired into our brains. Unfortunately, it seems that most civil engineers have forgotten how to apply it to solve complex problems. This article gives some examples of where pattern recognition led to the solution of problems, and the examples date back to biblical times.

My first experience in pattern recognition goes back to my first year at high school. We had to write an essay on the Renaissance. I visited the library and started my essay with a quote: "The Renaissance was a time when the people opened their eyes and saw." I was congratulated in front of the class. I still have strong memories of the occasion and the message. We have to open our eyes and see things before even attempting to describe them mathematically.

The next experience occurred when I was on leave in Rome. I was a non-smoker so I collected my weekly cigarette rations. I bought a 10 cm Nestler slide rule for a few packets of cigarettes. It was my constant companion for many years. Together with graph paper and a ruler, I could solve all the engineering problems that came my way.

After some 20 years in the field, I was promoted to the post of Chief of the Division of Hydrology in the then Department of Water Affairs. My hydrological knowledge was minimal so I was sent on a study tour of the UK and USA. I was also *ex officio* a member of the IAHS (International Association of Hydrological Sciences). In that capacity I attended an international conference at Reading in the UK. Late one afternoon I was enthralled by a vigorous blackboard debate. The opponents were Vujica Yevjevich, a stochastic hydrologist of the Colorado State University in the USA, and James Wallace, an IBM

1 Elements of pattern recognition (*vide* Alexander)
2 The Noah Effect
3 The Joseph Effect

mathematician. The subject was the mathematical description of annual river flow sequences. It was all beyond me. The only words that I recognised were "white noise" and "red noise". It was only years later that I realised that they were debating the nature of the random, and therefore unpredictable, component of annual river flow sequences.

In the years that followed I became increasingly involved in this problem. There were major difficulties in the determination of the flood magnitude/frequency relationships (called the Noah Effect) and the relationships required for water resource studies (called the Joseph Effect).

THE NOAH EFFECT

Figure 2 illustrates the Noah Effect. There are a number of high outliers that have observed return periods in the range from 50 to 100 years but calculated return periods between 200 and 2 000 years. These are serious discrepancies. They cannot be resolved mathematically. (Try drawing a curve that fits the outliers.)

THE JOSEPH EFFECT

The Joseph Effect is a little more difficult to explain as it involves the additional dimension of time. The key diagram is the cumulative departure from the record mean. This is the basis for storage/yield calculations.

Now comes my problem. Thirty years ago and on many occasions since then I demonstrated the presence of the alternating sequences in annual river flow and their linkages with sunspot activity. Why am I having difficulty in persuading others that these relationships exist? Their argument is that these properties cannot be detected mathematically, therefore they cannot be meaningful.

Where should we place our trust – in mathematics or pattern recognition? My response is to point out that a one-year-old child can recognise its feeding bottle without difficulty. None of us can describe it mathematically in such a way that it will be immediately recognised as such by an Internet colleague. In this situation I had to develop a more convincing counterargument.

Remembering my first experience, I decided to challenge my undergraduate students. I introduced them to the Joseph Effect that was clearly present in the cumulative departure plots of the annual flows in the Vaal River. I asked them to resolve the problem. I gave them a hint. The foundation of stochastic hydrology was the assumption of random variations about a constant mean. I asked them to consider an alternative explanation.

One of the students produced the diagram in Figure 3. He was able to remove the alternating pattern in the cumulative departure plot by assuming a variable mean. He then proceeded to demonstrate it graphically. It now became very clear that the Joseph Effect was the consequence of *regular, alternating changes in the mean value*. The residual values in the cumulative departure plot (the bottom panel of the figure) were truly random, i.e. white noise.

We could now replicate this property mathematically by abandoning the Gaussian models in which the random component was independent of the mean. We substituted a Generalised Extreme Value model where the periodicity and randomness were applied directly to the mean itself. It was now possible to replicate any anomalies and trends in the data series mathematically and recover the properties from the synthetic data sequences.

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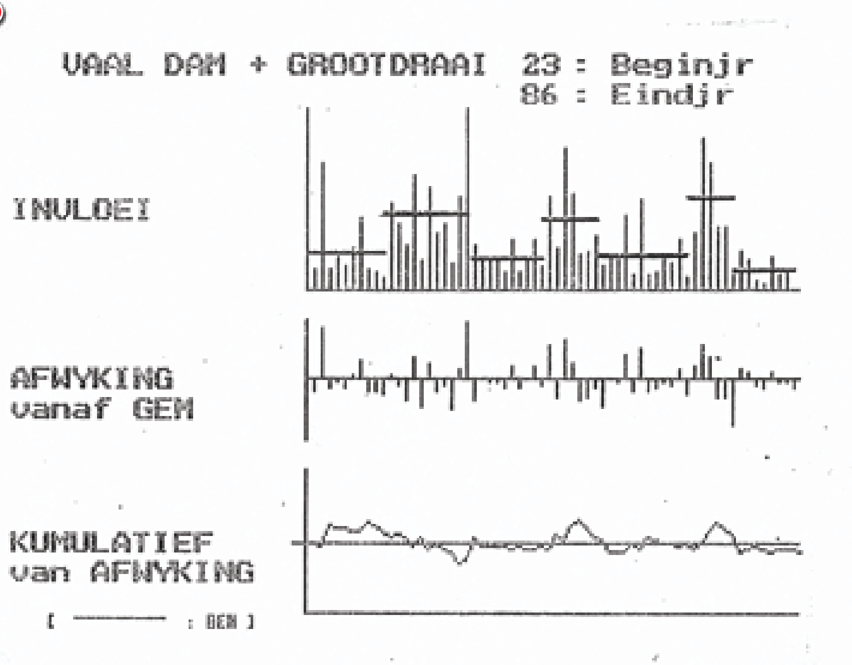

THE HURST PHENOMENON

Continuing with the Joseph Effect, it must be obvious that a long record is likely to contain a more severe drought sequence than a short record. It is also obvious that the greater the variability of the flow in the river, the greater the storage capacity required to meet the specified demand.

In 1950 the civil engineer R E Hurst examined the 1 080-year-long record of the maximum water levels in the Nile River. Not only did he find multiyear anomalies in the data, but he also found the same anomalies in other geophysical data, including deposits in lakes, tree rings, temperatures, rainfall, sunspots and wheat prices. Surely it must be very obvious that all these processes must be related to a single cause. The only conceivable cause is variations in received solar energy, but this was not investigated any further at that time.

Stochastic hydrologists then abandoned their search for the causes of these well-known and well-documented anomalies, the Noah and Joseph Effects and the Hurst Phenomenon. This is what Vit Klemes, a distinguished stochastic hydrologist, wrote in his paper *The Hurst Phenomenon – A Puzzle?*

"We are then, in one of those situations, so salutary to theoreticians, in which empirical discoveries stubbornly refuse to accord with theory. We are forced to the conclusion that either the theorists' interpretation of their own work is inadequate or their theories are falsely based; possibly, both conclusions apply."

The mathematically inclined stochastic hydrologists then departed from the scene. By the end of the 1980s, we were ahead of the pack but we still had much to learn.

PERIODIC FLOW SEQUENCES

They say that fortune favours the brave. I innocently compiled Table 1 by showing the Vaal Dam inflow in columns of 20 values for the sake of convenience. Once I had done this, even to the untrained eye there was a very clear pattern in the data showing approximately 20-year sequences (actually 20,8 years as I later calculated). There is another fundamentally important characteristic. This is the abrupt transitions from low flow to high flow sequences. I was later to discover that these sequences were synchronous with the double sunspot cycle, and that the abrupt changes coincided with the sunspot minima.

Note the clearly evident, abrupt changes from drought sequences to high runoff sequences at the ends of the cycles, shown by the thick horizontal lines.

A number of years went by. As the hydrological records increased in length, another pattern became evident. All the records were chopped up into 21-year sequences, beginning with the clearly evident, abrupt changes from drought to high inflow sequences. They were then plotted together as multiples of the individual record mean values. The product is shown in Figure 4. We are now in period year 13, with period year 14 starting in October this year.

Table 1 Non-random grouping of annual flow sequences in the Vaal River (MAR = 1 942 10^6 m^3)

Year	Inflow	Year	Inflow	Year	Inflow	Year	Inflow
23/24	765	43/44	6 863	63/64	1 136	83/84	1 535
24/25	4 777	44/45	1 696	64/65	2 890	84/85	581
25/26	808	45/46	1 277	65/66	520	85/86	708
26/27	1 283	46/47	1 117	66/67	3 392	86/87	896
27/28	862	47/48	1 100	67/68	597	87/88	4 040
28/29	1 612	48/49	641	68/69	686	88/89	3 209
29/30	2 754	49/50	1 938	69/70	1 172	89/90	1 254
30/31	778	50/51	638	70/71	1 008	90/91	1 138
31/32	698	51/52	1 167	71/72	1 977	91/92	256
32/33	469	52/53	1 951	72/73	440	92/93	501
33/34	3 301	53/54	881	73/74	2 176	93/94	1 780
34/35	2 549	54/55	3 510	74/75	5 727	94/95	331
35/36	1 688	55/56	1 545	75/76	4 803	95/96	9 009
36/37	4 361	56/57	5 379	76/77	2 395	96/97	
37/38	1 145	57/58	3 655	77/78	2 366		
38/39	3 928	58/59	1 344	78/79	602	**Key**	
39/40	2 178	59/60	1 449	79/80	1 231	>2000	
40/41	2 534	60/61	2 039	80/81	1 205	1 500 – 2 000	
41/42	1 039	61/62	961	81/82	364	1 000 – 1 500	
42/43	3 597	62/63	1 315	82/83	227	<1 000	

PERIODIC OSCILLATIONS

One of my colleagues, Alwyn van der Merwe, produced Figure 4. Yet another oscillating pattern is emerging. Another colleague, David Bredenkamp, equates it to a siphon effect where a constant flow into a tank of water can result in alternating outflows.

CHALLENGE

I started this article by describing the challenge that our history teacher gave us a long time ago. Many years later I challenged my students to quantify the Joseph Effect. I now challenge readers of this article to describe the annual river flow sequences shown in Figure 4 mathematically or numerically. It has to be such that it can be used for the determination of likely future inflows in our major rivers for the next ten years, starting in October 2009. I will add a trick question. How will climate change affect your answer? I would be very interested in hearing from you. □

4 Periodic dam inflows (prepared by Alwyn van der Merwe)