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How does traditional option hedging perform in the South African equity market?

ABSTRACT

Derivative securities are frequently priced within the Black-Scholes methodology. Theoretically this entails maintaining a hedge consisting of the underlying asset and cash which needs to be rebalanced continuously. In practice, traders would only rebalance such hedges on a discrete basis. We examine the effects of discrete rebalancing of derivative hedges written on the FTSE/JSE TOP40 index.

1.   INTRODUCTION

The seminal work by Black and Scholes (1973), formalized and extended by Merton (1973), paved the way for modern valuation of contingent liabilities such as options. This valuation framework is computationally simple for plain (or so-called vanilla) options. Furthermore, it requires no knowledge of the individual risk preferences of the option trader (see, for example, Björk (2005) or Musiela and Rutkowski (2005) amongst many other excellent handbooks on this topic).

Within the Black-Scholes valuation methodology options can be seen as "redundant securities", i.e., options can be recreated using only a combination of the underlying asset and cash. It is therefore important to understand that this technique provides both a method to calculate the fair value of a derivative security as well as a "hedging recipe" to replicate the value of the derivative. This hedging approach is frequently referred to as a dynamic trading strategy.

In this paper we examine the performance of the well-known delta-hedging trading strategy within the context of options on the FTSE/JSE TOP40 index. The outline of this paper is as follows: In Section 2 we provide an overview of delta-hedging as it is frequently applied in the financial markets (South Africa and internationally). This is followed by an analysis showing how the delta-hedging strategy would have performed over time for options with different strikes and maturities. In Section 4 we discuss the results and their implications for Black-Scholes modelling within the South African context focusing on the moneyness of an option. We conclude in Section 5.

2.   DYNAMIC OPTION REPLICATION

The central idea behind Black-Scholes is that we can hedge a contingent claim on a stock, for example, by making use of a dynamically adjusted portfolio comprising a certain holding in the underlying stock and cash.

Let C denote the value of a call option on an underlying stock denoted by S. If we hold

$$\Delta = dC / dS,$$

of the stock at any given instant during the life of the option we would in theory perfectly replicate the call option behaviour. This dynamic replication strategy is known as a delta-hedging strategy and a portfolio that is instantaneously hedged in this way (against small moves in the underlying) is called delta-neutral.

It is very important to note that the hedge ratio is not a constant but a function of spot, volatility, interest rates and time. Portfolios being hedged on a delta-neutral basis are therefore only immune to small movements in the underlying asset.

The theoretical conditions needed to ensure that we have perfect replication include (amongst others): continuous portfolio rebalancing, no transaction costs or taxes, constant and known volatility of the underlying asset and unrestricted borrowing and lending of funds at the same constant interest rate. A fundamental assumption is made that the underlying asset returns are log-normally distributed. Based on these assumptions being satisfied, the value of an option should equal the cost of the dynamic replication strategy (Black and Scholes (1973), Björk (2005), Kamal and Derman (1999)).

In practice, these theoretical conditions are not satisfied. Continuous portfolio rebalancing, for example, is totally impractical and impossible. Apart from the obvious transaction cost issue that would arise (Leland (1985), Toft (1996)), it is practically impossible to participate in trades on a continuous basis as would be required by the theoretical approach.

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Option traders therefore typically trade the underlying within the confines of a limit framework where a residual mismatch between the replication portfolio and the derivative security is allowed depending on the risk appetite of the trading institution and liquidity of the underlying. Typically, many traders would only hedge positions at the close of business on a daily basis. The consequence of this is that option traders are exposed to, amongst other things, the rate of change of the delta with respect to changes in the underlying asset, also called the gamma of the derivative (the gamma being highly nonlinear). This means that a trader could incur substantial hedging errors (i.e. increased cost) in the replication of derivative securities (Kamal and Derman (1999)).

In the next section we examine the effect of delta-hedging on the replication of a derivative security, at the close of business on a periodic basis.

3. APPLYING DELTA-HEDGING TO OPTIONS ON THE FTSE/JSE TOP40 INDEX

For the purpose of our analysis we made use of weekly FTSE/JSE TOP40 closing index data, covering the period July 1996 to July 2008. The motivation behind using this index is the fact that options and futures trade on this index on SAFEX, which facilitates a real market comparison of our results.

In our analysis we assume that we have written a call/put option with a specific maturity and strike level on the FTSE/JSE TOP40 index. The option will be written given certain inputs, for example, the current relevant index level. We convert the historical index levels described above to a return series, which we then apply to the current index level to produce a historical simulation framework. The idea is then to estimate the cost of hedging the chosen option over its lifetime by making use of the closing index levels as described. To estimate this cost we use the Black-Scholes based delta-hedging technique described in Section 2. This creates a delta-neutral hedge for the derivative liability at weekly index closing levels, i.e., we assume that we can replicate the derivative based on weekly rebalancing. Note that we choose not to implement this procedure on a daily basis or more frequently such as intra-day, as this may be prohibitively expensive.

In our cost estimation we take careful account of the cost to establish and maintain the delta-hedge. If we denote the total hedge borrowing cost at rebalancing time i with B_i we have that:

$$B_i = B_{i-1}e^{r\delta t} + S_i(\Delta_i - \Delta_{i-1})$$

where the first term describes the previous borrowing plus interest expense (at rate r per period δt) and the last term represents the additional borrowing required to change the hedge position in the underlying from Δ_{i-1} to Δ_i units.

At the expiry of the option we calculate the value of the liability (the option payout) that is then subtracted from the value of the delta-hedge after allowing for borrowing costs (as calculated above). The estimated cost of the option for a given asset path is obtained by discounting this value to the start date of the option. In Figure 1 below we show a histogram detailing the results obtained with the procedure described above for a three-month option written at-the-money on the FTSE/JSE TOP40 index over successive three monthly periods on the dataset. The average of all the different estimates is then used as the estimated cost of replicating an option. Note that we use the average of the three-month realised volatility (which equates to 19.8%) over the historical price series as a volatility input. The histogram was obtained by subtracting the average of all simulations from individual simulations – the idea being to use the average value as the expected cost of the option as stated above. The histogram therefore details profit/loss relative to the estimated option cost. (The process described corresponds directly with the Monte Carlo method discussed in Glasserman (2003).)

The hedging error observed is attributable to three factors. The volatility realised on three-monthly periods will differ from the volatility used to provide the initial option price as well as provide a base for calculation of the delta-hedge. Furthermore hedging error is introduced by virtue of discrete hedging. These results above are in agreement with observations by Kamal and Derman (1999). They note specifically that the hedging error would roughly halve if we hedge four times as frequently. We note that a further hedging error is introduced by the assumption of log-normal asset returns. Equity returns are well known to exhibit fat-tailed behaviour.

We can now use the procedure described above to estimate option costs for any maturity and option strike. We consider options with a three-month tenor over a range of strikes. Given the estimated price we can make use of the Black-Scholes formula to calculate what volatility we would have had to sell the option at to break even. In Figure 2 below we show the results of this calculation compared to market prices at different times.
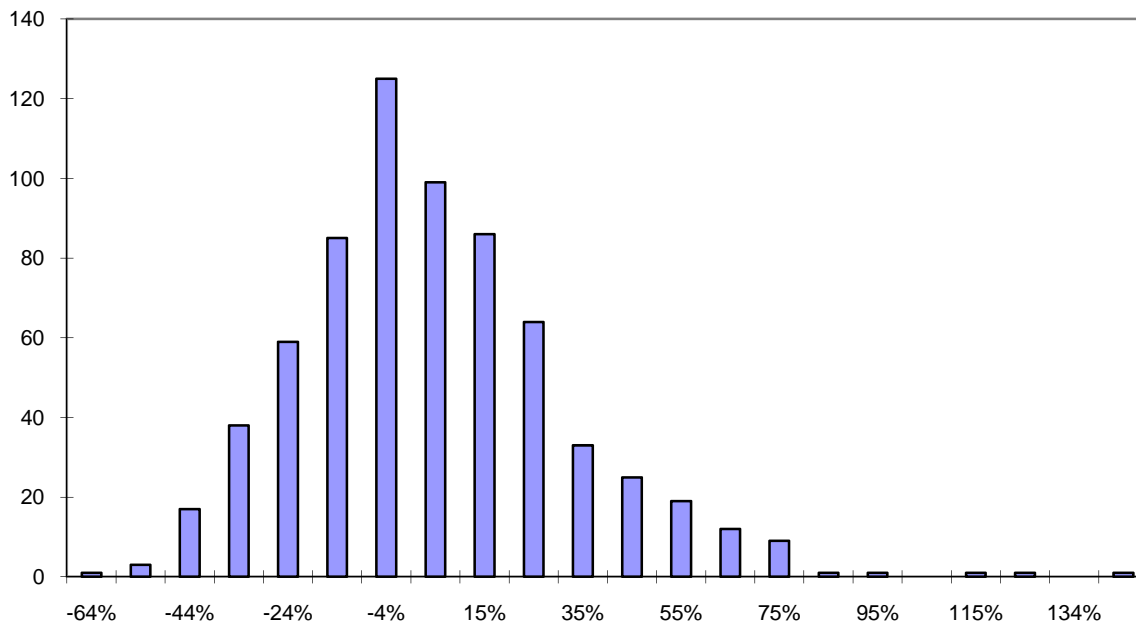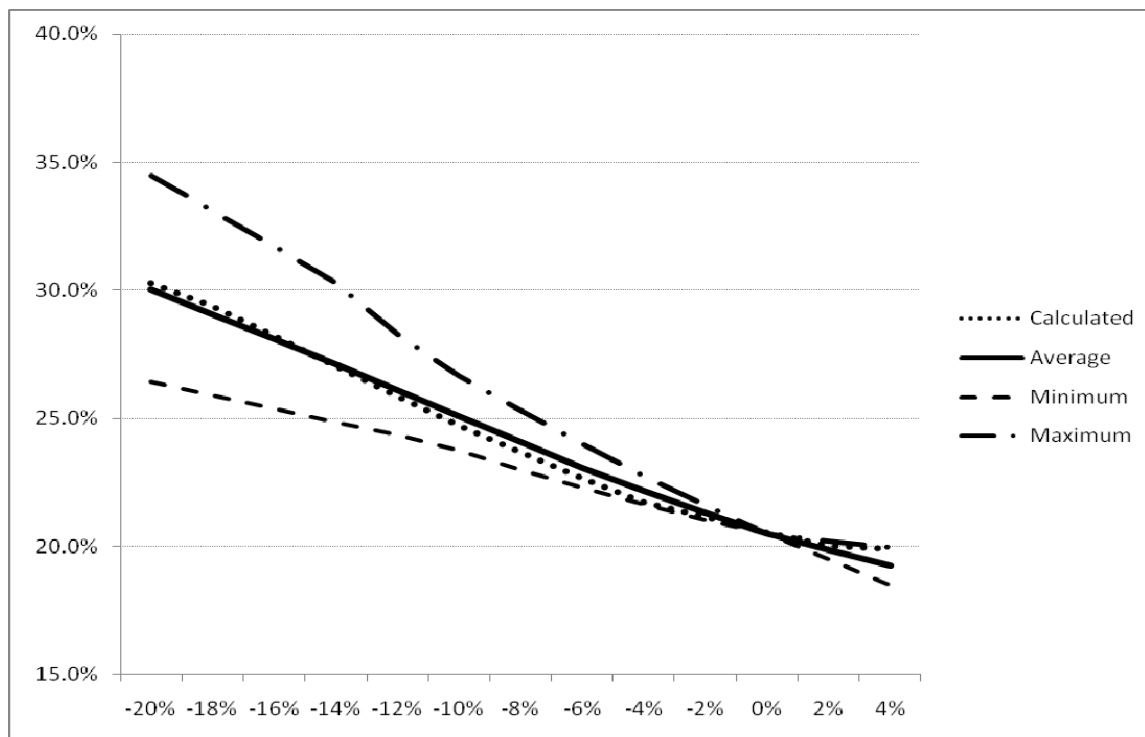
Figure 1: Histogram of delta-hedge Profit/Loss

Figure 2: Implied volatility skews compared to calculated skew for three-month options

The figure shows that we obtain the ubiquitous volatility skew observed in the market from our analysis. (For a description of the volatility skew see, for example, Gatheral (2006) and the many excellent references contained therein.) In the figure, we note that the calculated skew corresponds quite closely with the average implied volatility skew. It is important to notice that our calculation yields a static skew whereas the market skew varies over time as a function of market conditions and liquidity. The calculation methodology yields the average fair value over the period covered by the data whereas the market takes a view of current observed and future events and tries to predict the replication cost. This remark is further explained in the table below.

Table 1 shows the volatility increase per percentage point increase in out-of-the-moneyness (constructed using three-monthly implied volatility skew numbers over the period June 2005 through June 2008). Important to note is the range of the skew between minimum and maximum values over the period. This is indicative of the market uncertainty over the period considered as well as the considerable volatility realised in the market over that time. It is of interest that the calculated skew number mostly falls within the range of the implied skew numbers.

In our analysis above we use implied volatility numbers over the period June 2005 to June 2008. The reader could ask why the historical calculation is based on a much longer dataset. It is important to understand that the methodology we employ effectively acts as a Monte Carlo type analysis – we therefore need as much data as possible to make it work efficiently. The longer dataset also includes a number of significant market events which one would argue would be priced into implied volatility quotes by virtue of market participant's expected view of such events occurring again. Notwithstanding this, the method will work with a shorter dataset as well.

4.   DISCUSSION OF RESULTS

We chose to perform our analysis on options with a three-month expiry for a number of reasons. Firstly, a liquid market exists on SAFEX for options with a short-term to maturity – there is also a three monthly period between option expiries on SAFEX. Secondly, the skew effect is more pronounced in short-term options than in longer-term options. Thirdly, implied volatility data is more readily available for short-term options.

Our results indicate that an estimate of option prices based on historical price movements provide an estimate of the volatility skew observed in the market. In this sense we see that the market prices a volatility skew for out-of-the-money options on the basis of

future estimated replication costs. The historical estimation costs provide a basis for price discovery. The estimation can be used to perform a cheap/dear analysis of implied volatility quotes.

It is of interest that the estimates for replication costs between the market and the process described compare well for options with decreasing strikes from the at-the-money level. For options with increasing strikes the process shows bigger differences. This can partially be attributed to liquidity effects in the market – option traders will frequently be sellers of the option skew, i.e., positioned short of low level struck options and long of higher level struck options. An increase in the underlying market therefore leads to option traders ending with a long volatility position, Moreover, institutions typically implement call overwriting, which leads to an oversupply of high strike options in the market. In summary these differences can be attributed to liquidity in the derivatives market and the effects of the derivatives on the underlying – this is the topic of a follow-on paper.

The South African derivatives market is active but price discovery is difficult i.e. options are executed in the market through the use of structured products (such as 'zero-cost-collars') and not individually, making the implied volatility skew difficult to observe directly. The process we describe provides a means to estimate where the skew should trade on the basis of historical price movements. This technique can be used for derivatives on any share or index where a price series is available taking into account the fact that traded prices would include some bid/offer spread which would probably be biased based on the option market-maker's view of potential moves in the underlying asset. As noted before, implied volatility quotes indicate the market's different perceptions of option replication costs over time.

Table 1: Skew calculation compared with implied volatility skew data (Volatility add-on per percentage increase in moneyness from the at-the-money level)

Moneyness	-20%	-18%	-16%	-14%	-12%	-10%	-8%	-6%	-4%	-2%
Min	0,33	0,36	0,41	0,42	0,43	0,41	0,39	0,37	0,36	0,34
Average	0,49	0,50	0,51	0,52	0,52	0,51	0,49	0,47	0,45	0,44
Max	0,65	0,64	0,64	0,63	0,62	0,61	0,60	0,58	0,57	0,56
Skew Calc	0,49	0,49	0,48	0,46	0,44	0,42	0,39	0,35	0,32	0,30

5.   CONCLUSION

We have examined the replication of derivates on the FTSE/JSE TOP40 based on delta-hedging using the Black-Scholes approach, with weekly hedging. The results obtained are in agreement with the literature.

Option traders can not hedge their derivative liabilities on a continuous basis. They therefore frequently decide to hedge their books on a periodic basis. This approach does lead to a reduction of transaction costs but exposes the trader to a potentially significant variance in profitability.

Furthermore, traders are exposed to their forecast of future volatility which could also lead to big replication mismatches. Associated with the volatility estimation option traders have additional exposure to the kurtosis of the underlying asset return distribution. This effect is usually observed in the market through the so-called volatility skew. We see in particular from our results that the market seems to price the implied volatility skew, on average, based on effective replication cost as can be estimated from a historical analysis. We reproduce the volatility skew observed in the market and produce an explanation for this effect.

We maintain that an option replication analysis can be used for price discovery, in particular, the pricing of illiquid derivative securities.

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