Social costs and incentives for optimal control of soil nutrient depletion in the central highlands of Ethiopia

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Abstract

This study analysed trade-offs between short- and long-term objectives of soil use by smallholder teff farmers in Ethiopia. Compared to socially optimal solutions it was found that smallholder farmers discount the future at higher private rates leading to overexploitation of soil nutrients. Current soil conservation efforts, however, are well above static optimization levels suggesting smallholder farmers consider the long-term (dynamic) costs of soil degradation. There is evidence of high social gains from better utilization of soil resources through appropriate policy such as tenure security, to improve incentives for smallholder farmers to adjust input use towards socially desirable dynamic optimization levels.

1. Introduction

Soil degradation is considered one of the most important natural resource management problems in Ethiopia. Among the various types of soil degradation, soil nutrient mining due to net nutrient extraction by crops and nutrient lost along with eroded soil is of utmost concern to smallholder farmers in the highlands of Ethiopia (areas higher than 1500 m above sea level). These lands constitute 44% of the total area and support 88% of the human and 75% of the livestock population in the country. Stoorvogel and Smaling (1990) estimated soil nutrient losses from the highlands of Ethiopia to be in excess of 80 kg of N, P2O5, and K2O per cultivated hectare whereas addition of nutrient through fertilizer applications amounts to only 10 kg/ha (i.e. 12% of the loss) (Ofori, 1995). Soil erosion imposes on-site costs to individual farmers in terms of reduced yield and off-site costs to society as a result of several negative externalities such as sedimentation (Sutcliffe, 1993; Bojo and Cassells, 1995; Shiferaw and Holden, 1999; Sonneveld and Keyzer, 2003).

Depletion of soil fertility leads to declining crop yields and a rise in the number of food insecure people (Bojo and Cassells, 1995; Sinebo and Yirga, 2002). Unlike soil erosion, however, soil nutrient mining can be relatively easily reversed through the addition of organic and inorganic fertilizers. Nonetheless, in Ethiopia, where 50–80% of animal manure and 70–90% of crop residues are removed from the farm and used for fuel, feed and construction purposes (Sinebo and Yirga, 2002), this seriously limits the potential for replacement of soil nutrients through these fertility management options.

The notion that soil is a natural capital that could provide sustained flows of productive and supporting environmental services if managed properly received some recognition among various agricultural development researchers in Ethiopia (Hurni, 1988; Sutcliffe, 1993; Bojo and Cassells, 1995; Sonneveld and Keyzer, 2003). Despite the seriousness of the soil nutrient mining problem and its negative consequences on food security and income to individual households and the nation at large, empirical studies estimating impact of soil degradation are rare in Ethiopia. Consequently, the magnitude of the threat that soil nutrient mining has posed, particularly on future income and how best to address the long-term impacts of the problem, is not well known. Few studies estimated the national economic loss to be substantial for Ethiopia ranging from 2% to 6.7% of agricultural gross domestic product – GDP (Sutcliffe, 1993; Bojo and Cassells, 1995; Kappel, 1996). These studies, however, employed static cost-benefit analysis (CBA) approaches and hence failed to consider the dynamic consequences and long-term impacts of soil degradation. Various authors stressed that soil nutrient mining has become the overriding cause of soil degradation in sub-Saharan Africa (SSA) in general and in Ethiopia in particular (Bojo and Cassells, 1995; Brekke et al., 1999; Yirga et al., 1998; Sinebo and Yirga, 2002). Available studies focused on erosion and neglected soil degradation due to nutrient mining and its dynamic costs. Two exceptions that explicitly considered nutrient mining in SSA are the studies by Brekke et al., 1999) for Tanzania and Nakhumwa, 2004) for Malawi. Brekke
et al. (1999) considered both soil nutrient mining and soil depth depletion but did not include conservation efforts in their analysis. The only study known to us that estimated social costs of nutrient mining explicitly accounting for soil conservation efforts is that of Nakhumwa (2004) for Malawi.

Admittedly, an agricultural country such as Ethiopia needs to adopt a long-term and dynamic perspective to the soil degradation problem if the country is to conserve its fragile soil resources. Policy prescriptions based on short-term assessment of costs and benefits cannot provide optimal management recommendations for long-term sustainability of soil quality. This study, therefore, adopted a dynamic optimization framework in order to assess the inter-temporal trade-offs (the true social costs of soil nutrient depletion relative to the value of output expected) that farmers face in their production decisions. We developed an optimal control model and applied it to analyze the nutrient mining problem smallholder farmers face in the highlands of Ethiopia. The empirical model was then solved to derive the dynamic (socially desirable steady state) and static profit maximization solutions and compare the dynamic and static solutions with average current farmers’ practices.

Section 2 presents the analytical model and discusses the empirical specifications. Section 3 describes the data and information used to estimate model parameters. Model results are presented and discussed in Section 4. The last section summarizes the results and their implications for development policy and research.

2. Methodology

2.1. The analytical model

Agricultural land use removes nutrients from the land lowering soil quality and reducing productivity over time. Soil quality is therefore continuously subject to both natural and human induced factors. Optimal soil management thus entails careful weighing of current costs and benefits from actions taken today with consequent costs and benefits expected in future. Dynamic control formulations are considered appropriate direct determination of the optimal time path of control variables being management/policy instruments that influence change in soil quality as the state variable. Building on the works of McConnel (1983), Brekke et al. (1999) and Nakhumwa (2004), this study used a farm level optimal control model that links changes in soil nutrient stock to crop production practices and soil conservation efforts. Assuming smallholder farmers maximize the sum of discounted streams of future net returns by choosing levels of fertilizer (F), production and conservation labor (L_F and L_s, respectively), and capital (K_s) inputs for production and soil conservation activities, the optimal control problem for a given area of land with an infinite time is given by (time subscripts suppressed):

Max \Pi_{F, L_F, K_s, L_s}

= \int_0^{\infty} e^{-\delta t} [Pf(L_F, K_s, L_s, N) - (W_F F + W_L L_F + W_s L_s + W_k K_s)] dt

Subject to equations of motion measuring initial conditions and changes in soil nutrient stocks:

\dot{N} = G(F) - D(Y) + M(Z, L_s, Y)

N(0) = N_0

where P is the price of output, f is the production function, \delta is the social discount rate, W_F is the input unit costs of fertilizer (F), W_L, W_s and W_k denote unit costs of labor in production (L_F), labor in conservation (L_s) and capital (K_s) in production, respectively. The production function \( f \) at time \( t \) is defined as a function of stock of soil nutrients \( N_t \), two productive inputs labor \( L_{Ft} \) and capital \( K_{st} \). Capital for production in this study refers to the services of a pair of oxen, which could be owned by a household, solicited from fellow farmers through cash rentals, exchange for labor services, livestock feed or other social arrangements. The production function \( f \) is assumed to have all the properties of a well-behaved production function. Eq. (2) denotes the inter-temporal evolution of the stock of soil nutrients \( N_t \), while Eq. (3) gives the initial level of the resource stock. The time rate of change of the resource stock is governed by three processes: fertilizer inputs \( G(F) \) in the form of organic and inorganic nutrients, nutrient removal through crop harvest \( D(Y) \) and nutrient build up and decay \( M(Z) \) due to natural soil formation processes \( Z \), conservation effort through \( L_s \) and cultivation/harvesting of output \( Y \). The canopy of output \( Y \), by reducing the kinetic energy of raindrops hitting the soil surface, deters (lowers) erosion, which consequently reduces nutrient loss. Similarly, soil conservation efforts through labor input \( L_s \) by reducing soil loss contribute to minimizing nutrient decay.\footnote{No significant use of inputs other than labor in soil conservation has been observed in the study area.} The function \( M(Z) \) above, therefore, implies that smallholder farmers can manipulate nutrient loss by varying conservation effort and/or by influencing yields (canopy) via the control variables in the optimization problem. The Hamiltonian (dynamic profit) for this maximization problem is given by:

\[ \Pi(F, L_F, K_s, L_s, N, \mu) = e^{-\delta t}[P f(L_F, K_s, N) - (W_F F + W_L L_F + W_s L_s + W_k K_s)] + \mu[G(F) - D(Y) + M(Z, L_s, Y)] \]

(4)

The first-order conditions for optimal fertilizer, labor and capital use based on the maximum principle are derived establishing rules for optimal use of inputs by balancing short-term costs against long-term benefits. Optimal levels of fertilizer for instance are achieved when the discounted unit price of fertilizer \( e^{-\delta t}W_F \) equals the marginal contribution of an extra unit of fertilizer to the stock of soil nutrients \( G(F) \) evaluated at dynamic price of nutrient stock \( \mu \).

Labor in cultivation should be used up to the point where the discounted net marginal value \( e^{-\delta t}[P f_{L_F} - W_{L_F}] \) equals the net marginal contribution to soil quality or equivalently to the net dynamic benefit \( \mu(D_{L_s} - M_{L_s}) \) from the use of soil quality for production. The first term of the dynamic benefit represents the marginal nutrient loss due to higher output achieved (the marginal increase in nutrient damage function) while the second term denotes the marginal reduction in nutrient decay (nutrient saved) due to better canopy. Similarly, capital use in cultivation is optimized at the point where the discounted net marginal value \( e^{-\delta t}[P f_{L_k} - W_{L_k}] \) equals the net marginal contribution to soil quality.

At the optimum, the discounted unit cost of labor used in soil conservation is equated to the long-term marginal benefit expected from the marginal reduction in soil decay. The shadow value of soil nutrient stock \( \mu \) declines at the rate at which soil nutrients contribute to current profits \( e^{-\delta t}P f_{L_k} \) plus the sum of the marginal contribution of soil nutrient stock to nutrient decay through crop harvest and build up through canopy \( \mu(D_{L_s} - M_{L_s}) \).

2.2. The empirical control model and optimal solutions

The components of the control model that required empirical specification include the production function \( f \), the nutrient decay and regeneration function \( H \), prices and production costs. Among the functional forms widely used in empirical studies of production relationships are the Cobb–Douglas (C–D) and translog.
The C–D functional form is often preferred in empirical studies due to its convenience in estimation and interpretation of parameter estimates. Therefore, for our purpose, a C–D functional form relating crop yield to labor, tillage, and nitrogen is adopted,

\[
f(L_Y, K_Y, N) = Y = AL_Y^aK_Y^cN^d \quad (5)
\]

where \( Y \) is annual yield in tons/ha; \( A \) is a scale parameter; \( L_Y \) measures labor input for production in person-days/ha; \( K_Y \) is capital for production (oxen hours for plowing); \( N \) is nitrogen in tons/ha in the top 10 cm soil depth while \( a \), \( c \), and \( d \) are the technology parameters.

The nutrient regeneration and depletion function \((H)\) has three components: the nutrient augmentation function \(G(F)\), nutrient depletion due to crop harvest \(D(Y)\), and nutrient regeneration and decay due to natural processes, conservation effort and cultivation \(M(Z, L_Y, Y)\).

Smallholder farmers in the highlands of Ethiopia use inorganic fertilizers, farmyard manure, seasonal fallowing and legume rotation. For tractability purposes and following Nakhumwa (2004), the nutrient augmentation function is specified as an aggregate linear function:

\[
G(F) = \beta_1F \quad (6)
\]

where \( F \) is the amount of nutrient inputs in kg/ha and \( \beta_1 \) is a parameter that links nutrient inputs to soil nutrients. Similarly, the second component (nutrient depletion due to crop harvest) is specified as a linear function of the amount of grain and other biomass (crop residues) leaving the plot:

\[
D(Y) = \beta_2Y \quad (7)
\]

where \( Y \) is total biomass (grain and crop residues) harvested in tons/ha while \( \beta_2 \) is a parameter representing the proportion of nutrients per unit of harvested grain and residue.

The other component of the nutrient regeneration and depletion function that requires empirical specification is the soil decay or erosion damage due to natural soil processes, soil conservation effort and cultivation. The nutrient damage function due to natural processes and soil erosion damage is specified as follows:

\[
M(Z, L_Y, Y) = \beta_3[Z - \gamma e^{-\theta L_Y} + \phi(1 - e^{-\nu Y})] \quad (8)
\]

where \( \beta_3 \) is a coefficient that converts soil depth reductions into nutrient loss per unit of eroded soil. Soil decay is a function of soil characteristics such as natural susceptibility of soil to erosion (soil erodibility), plot slope, rainfall intensity (erosivity of rainfall), land cover and land management factors such as presence or absence of soil conservation structures. Households in the highlands could manipulate the rate of erosion either by constructing physical soil conservation structures (conservation effort) and/or intensifying production thus altering the crop cover factor. In this study, the soil decay function relating soil loss to conservation effort (labor inputs for conservation) is specified as an exponential function:

\[
E(L_Y) = \gamma e^{-\theta L_Y} \quad (9)
\]

where \( E(L_Y) \) is the soil loss in tons/ha with conservation effort \( L_Y \) in person-days/ha; \( \gamma \) is a calibrating parameter representing the average rate of soil loss on the ith plot in the absence of soil conservation structures (depends on rainfall, slope, crop cover and other plot specific characteristics); and \( \theta \) is a positive constant denoting the elasticity of conservation effort. Eq. (9) implies that the higher the conservation effort in the form of labor time used for construction of physical structures, the lower the soil loss as conservation effort reduces soil decay. The second component of the soil damage function relates canopy (crop cover) to soil decay. Brekke et al. (1999) indicated that soil erosion decreases with crop cover (increased production). Building on the specifications of Brekke et al. (1999) and Nakhumwa (2004) the relationship of canopy to soil damage is specified as:

\[
J = \phi(1 - e^{-\nu Y}) \quad (10)
\]

where \( \phi \) is a calibrating parameter denoting soil loss on the ith plot of known crop cover in the presence of soil conservation structures; \( \nu \) is the elasticity of canopy effects and \( Y \) is canopy (output). The third component of the soil regeneration and decay function is the natural soil regeneration function, \( Z \), assumed constant. Accordingly, combining Eqs. (9) and (10) gives the additive function:

\[
h = Z - (E - J) = Z - \gamma e^{-\theta L_Y} + \phi(1 - e^{-\nu Y}) \quad (11)
\]

where \( h \) is the net soil loss in tons/ha while other variables are as described above.

Given Eqs. (6)–(11), the aggregate soil nutrient regeneration and depletion function is specified as follows:

\[
\hat{N}(F, Z, L_Y, Y) = \beta_1F - \beta_2 Y + \beta_3[Z - \gamma e^{-x L_Y} + \phi(1 - e^{-\nu Y})] \quad (12)
\]

Substituting the specified functions discussed above in the analytical control model, the empirical nutrient mining control model (time subscripts suppressed) is given by:

\[
\max \Pi F L_Y K F = \int_0^\infty e^{-\xi} [PAL_Y^aK_Y^cN^d] + (WF + WL_Y + W_L Y + W_L K) \mathrm{d}t \quad (13)
\]

Subject to the equation of motion and initial condition:

\[
\dot{N} = \beta_1F - \beta_2 Y + \beta_3[Z - \gamma e^{-\theta L_Y} + \phi(1 - e^{-\nu Y})] \quad (14)
\]

\[
N_0 = \hat{N} \quad (15)
\]

where \( \delta, \phi, W_F, W_L, W_K \) and \( \hat{N} \) are as defined earlier, and \( \hat{N} \) denotes the initial soil nitrogen stock giving the following Hamiltonian (dynamic profit function) for the nutrient mining scenario:

\[
\Pi(F, L_Y, K_Y, N, \mu) = e^{-\xi} [PAL_Y^aK_Y^cN^d] - (WF + WL_Y + W_L Y + W_L K) + (W_L K) \mu + \mu(\beta_1F - \beta_2Y + \beta_3[Z - \gamma e^{-\theta L_Y} + \phi(1 - e^{-\nu Y})]) \quad (16)
\]

In a steady state the rate of change of the resource stock and its implicit price are necessarily zero \( (\hat{N} = \mu = 0) \). The reduced form steady state optimal solutions of the four choice variables and the resource stock denoted by \( L_Y, K_Y, L_F, F \), and \( N \) for optimal values of labor and capital for production, labor for conservation, fertilizer, and the optimal nutrient stock, respectively, are derived using the condition on \( F \) and \( \mu \) given above (see Appendix A).3

### 2.3 Data and parameter estimation

In an ideal situation all relevant economic and environmental data required for numerical analysis need to be obtained from a single unified source for a common reference year. For this study, however, no such data existed. The study, therefore, draws heavily on several secondary sources and some primary data for estimating parameter values. The components of the empirical model that need to be estimated or calibrated include the production technology, erosion damage, nutrient decay and regeneration function and prices of inputs and outputs.

In Ethiopia, smallholder subsistence farmers cultivate 95% of the cropped area and produce more than 90% of the agricultural

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2 Similar formulations have been proposed for modeling interactions between agroforestry systems and soil fertility (Babu et al., 1995).

3 Detailed derivation of this system of equations can be obtained from the authors on request.
output dominating the agricultural sector. Average farm sizes vary across the country depending on population density but are generally very small and declining over time. For instance in 1995, of those households that have access to some type of farmland, 62.9% owned less than 0.5 ha of land against 69.1% in the year 2000 (CSA, 2004).

Smallholder crop production in the mixed crop-livestock farming systems of the highlands involves intensive use of family labor with very little external inputs (Yirga et al., 1998; Yirga and Hassena, 2001). Land preparation is mainly done by oxen drawn local plough. Most farmers use local crop varieties and seeds from own harvest. The major agricultural operations such as land preparation, weeding and harvesting are accomplished mainly by family labor. Indeed, ownership of a team of oxen, adequate seed reserves from own harvest and availability of family labor constitute the major farming inputs of smallholder farming in the highlands. Soil fertility management practices used in the highlands include moderate levels of inorganic fertilizers, farmyard manure as well as fallow and legume rotations (Yirga et al., 1998; Yirga and Hassena, 2001) whereas soil conservation practices used are traditional ditches (boyi), cut-off drains (golenta) and stone and soil bunds. Like elsewhere in SSA, labor input with very little capital constitutes the soil conservation effort in the highlands of Ethiopia.

The control model discussed above assumes smallholder farmers cultivate a single crop (teff). Furthermore, cultivated area is assumed to be fixed. In reality, however, crop rotations are the norm rather than the exception and hence crop choice itself could be considered rampant, the natural rate of soil regeneration is believed to vary between 4.5 ton/ha and 12 ton/ha (McConnell, 1983; Goetz, 1997). In the highlands of Ethiopia, while erosion is generally considered rampant, the natural rate of soil regeneration is believed to be low. In this study, the natural rate of soil regeneration (Z) is fixed at a conservative rate of 5 ton/ha.

As stated earlier, the nutrient augmentation function is assumed to be an aggregate function represented by \( \beta_1F \). Assuming that inorganic N is a perfect substitute to natural soil N and following Nakhumwa (2004), the parameter \( \beta_1 \) is set at one implying a unit external source of nutrient contributes exactly the same unit of nutrients to the soil nutrient pool. A sensitivity analysis using an augmentation coefficient of 0.75 is conducted to assess the sensitivity of model results to changes in the augmentation coefficient.

Likewise the parameter for the crop damage function, \( \beta_2 \), is assumed to be constant representing the proportion of nitrogen present in the removed biomass (grain and residue). Various studies in Ethiopia reported N content ranging from 2.09% to 2.20% from 0.74% to 0.80% in the grain and straw of the teff crop, respectively (Kidanu et al., 1999). Using average values, the crop damage parameter (\( \beta_2 \)) is set at 29.15 kg/ton of harvested product. The parameter \( \beta_2 \) representing soil nitrogen lost along with eroded soil is a constant proportion of soil nitrogen available in the soil. Soil analysis based on composite soil samples taken from 15 smallholder farmers' fields in the Central highlands conducted as part of a soil fertility management on-farm trial gave a total N content ranging from 0.17% to 0.31% with a mean of 0.22%. Considering a soil bulk density of 1 g/cc3, which translates to 100 tons of soil per cm of soil depth (Shiferaw and Holden, 1999) and an average total nitrogen content of 0.22%, the total N content would be 220 kg/cm. Therefore, the coefficient of the nutrient depletion and regeneration function (\( \beta_2 \)) is set at 2.2 kg/ton. Sensitivity analysis using the lowest and highest reported soil N content are also conducted. All parameters are presented in Table 2.

The price of teff grain is set at Birr 1825/ton (one US $ was equivalent to Ethiopian Birr 8.6 in 2002) based on 2001/2002 weighted annual average producer prices of white, mixed and red seeded grain collected at Holetta local market, some 45 km west of Addis Ababa. Similarly, the price of nitrogen is calculated

### Table 1

<table>
<thead>
<tr>
<th>Parameter estimate (Cobb–Douglas production function for smallholder teff production in the Central highlands of Ethiopia, 2003)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable name</td>
</tr>
<tr>
<td>ln N</td>
</tr>
<tr>
<td>ln ( K_T )</td>
</tr>
<tr>
<td>ln ( L_T )</td>
</tr>
<tr>
<td>ln Constanta</td>
</tr>
<tr>
<td>No. of observations</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
</tr>
<tr>
<td>F-value</td>
</tr>
</tbody>
</table>

The anti-log value is 0.092648.

from the widely used commercial fertilizer DAP which contains 18/46 N/P₂O₅. Based on the 2001/2 price, which was Birr 141.7 per a 50 kg bag of DAP fertilizer in the Holetta area, the price of a kg of N is calculated to be Birr 15.74.

In the study area, oxen rental market is highly imperfect due to the skewed distribution of oxen and the seasonality of demand for traction. Nonetheless, farmers reported a rental rate of Birr 15–20/ day for the services of a pair of oxen for plowing which have been used to calculate the cost of tillage inputs. Assuming a pair of oxen is used for 5 h in a normal working day and a daily wage rate of Birr 10 for the oxen handler (cultivator) the hourly oxen rental rate is calculated to be Birr 5.

Payments to hired labor in the highlands of Ethiopia often involve a combination of cash payment of Birr 5–8/day as well as lunch and refreshments. Taking into account the in kind payments, the wage rate, is thus set at Birr 6 and Birr 10/day for the slack and peak periods of agricultural activities, respectively. Given that most soil conservation public projects are implemented during the off-season, the appropriate wage rate to use for labor in conservation would be the off-season rate, which is Birr 6/day.

Labor requirements for constructing soil conservation structures on croplands are based on the soil conservation requirement parameters (SCRP) work norms cited in Shiferaw and Holden (2001). Conservation labor requirements depend on the type of structure (soil or stone bunds) as well as the slope of the plot. In general, labor requirements are higher for stone bunds than soil bunds. Also, labor requirement tend to increase, as the plot gets steeper. For our purpose, the initial labor requirement for constructing soil bunds is fixed at 56 person-days/ha. This figure is used to calculate the elasticity of conservation effort (x) in Eq. (8).

### Table 2
Summary of model parameters of the soil nutrient dynamics of the control model.

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial soil depth (cm)</td>
<td>SD₀</td>
<td>70</td>
</tr>
<tr>
<td>Initial soil N level (%)</td>
<td>NO</td>
<td>0.22</td>
</tr>
<tr>
<td>Initial N stock in the upper 10 cm (kg)</td>
<td>N(10)</td>
<td>2200</td>
</tr>
<tr>
<td>Natural rate of soil regeneration (ton/ha)</td>
<td>Z</td>
<td>5</td>
</tr>
<tr>
<td>Estimated soil loss with conservation (ton/ha)</td>
<td>ϕ</td>
<td>6.97</td>
</tr>
<tr>
<td>Estimated soil loss without soil conservation (ton/ha)</td>
<td>γ</td>
<td>16.94</td>
</tr>
<tr>
<td>Elasticity of conservation effort</td>
<td>x</td>
<td>0.01911</td>
</tr>
<tr>
<td>Elasticity of canopy</td>
<td>ν</td>
<td>0.12</td>
</tr>
<tr>
<td>Coefficient of nutrient augmentation function</td>
<td>β₁</td>
<td>1</td>
</tr>
<tr>
<td>Coefficient of depletion function (N kg/ton of grain)</td>
<td>β₂</td>
<td>29.15</td>
</tr>
<tr>
<td>Coefficient of net nutrient depletion and regeneration</td>
<td>β₃</td>
<td>2.2</td>
</tr>
<tr>
<td>due to erosion (kg/ton)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Ethiopia, long-term institutional credit to smallholders is unavailable. Nonetheless, short-term institutional credit for the purchase of inorganic fertilizers and related inputs are provided at an annual interest rate of 12%. For other needs, most smallholders rely on the informal credit market that charges an exorbitant interest rate reaching 120% per annum (10% per month). Considering the institutional interest rate, the discount rate for the base scenario is set at 9%. A lower discount rate of 6% and higher rate of 12% and 24% are also used to test the sensitivity of model results to changes in the discount rate.

### 3. Results and discussion

Optimal values of the choice variables (LY, KV, LS and F), output (Y) and the resource stock, N under the dynamic (steady state equilibrium) and static decision rules along with average current resource use pattern for smallholder teff production in the Central highlands of Ethiopia are presented in Table 3. Model results of the base run are based on the parameters given in Table 2.

Results of the base run revealed that optimal output and input levels under the dynamic decision rule are much higher than solution levels of the static decision rule (Table 3). For instance, steady state optimal output under the dynamic decision rule is 1.53 ton/ha compared to 0.42 ton/ha of the static decision rule. The optimal inorganic N input necessary to achieve and sustain the optimal production level indefinitely under the dynamic decision rule, albeit other things constant, stands at 55 kg/ha compared to the requirements of the static decision rule, which averages 14 kg/ha. A comparison of the net benefits also clearly shows the superiority of the dynamic steady state optimal solution over the pure profit maximizing static solutions. These results suggest that static profit maximization decision rules lead to sub-optimal choices compared to the socially desirable steady state optimal input and output levels.

On the other hand, the static decision rule concerned with the maximization of short-term benefits ignores the effect of current actions (level of erosion and nutrient application rates) on future levels of the resource stock and output and thus provides insufficient erosion control and N fertilizer application. Static optimizers, ignoring long-term costs and enjoying considerable savings in current annual costs, pay a higher long-term price in terms of reduced soil quality and hence lower yields. The result that the dynamic decision rule provides a sustainable use of the soil resources (higher inorganic N inputs, soil conservation effort and lower soil loss

### Table 3
A comparative analysis of resource use pattern among dynamic and static decision rules and current farmer practice for the nutrient mining scenario.

<table>
<thead>
<tr>
<th>Item</th>
<th>Variable</th>
<th>Decision rule Dynamic (steady state)</th>
<th>Static</th>
<th>Current practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Labor for production (person-days/ha)</td>
<td>32</td>
<td>11</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>Labor for conservation (person-days/ha)</td>
<td>28</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>Capital for production (oxen hours/ha)</td>
<td>51</td>
<td>18</td>
<td>94</td>
</tr>
<tr>
<td>4</td>
<td>Inorganic fertilizer (N kg/ha)</td>
<td>54.83</td>
<td>14.38</td>
<td>24.12</td>
</tr>
<tr>
<td>5</td>
<td>Output (teff grain ton/ha)</td>
<td>1.53</td>
<td>0.42</td>
<td>0.71</td>
</tr>
<tr>
<td>6</td>
<td>Net soil loss (ton/ha)</td>
<td>0.00</td>
<td>11.60</td>
<td>8.90</td>
</tr>
<tr>
<td>7</td>
<td>Net N extraction (kg/ha)</td>
<td>0.00</td>
<td>23.27</td>
<td>16.22</td>
</tr>
<tr>
<td>8</td>
<td>Resource stock (N kg/ha)</td>
<td>448.25</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>9</td>
<td>Marginal user costs of N (Birr/ha)</td>
<td>15.74</td>
<td>15.74</td>
<td>15.74</td>
</tr>
<tr>
<td>10</td>
<td>Total user costs of N (Birr/ha)</td>
<td>366.32</td>
<td>255.33</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Net private benefit (Birr/ha)</td>
<td>1189.24</td>
<td>329.33</td>
<td>157.92</td>
</tr>
<tr>
<td>12</td>
<td>Net social benefit (Birr/ha)</td>
<td>1189.24</td>
<td>−36.99</td>
<td>−97.41</td>
</tr>
</tbody>
</table>

N.A. = Not applicable.

a Calculated based on Eq. (16).
b Calculated based on Eq. (17).
c Calculated based on Eq. (22).
d Toal user costs of N are calculated by multiplying marginal user costs of N (item 9) by the net N extraction (item 7).
e Gross benefit minus total costs.
f Net private benefit (item 11) minus total user costs (item 10).
rates and hence higher soil quality and consequently higher output level) is because the dynamic decision rule considers the effects of current erosion and N extraction rates on levels of the resource stock and output in subsequent years. The dynamic decision rule, therefore, requires that smallholder farmers increase their investment levels not only on yield increasing non-soil inputs (labor and capital for production) but also raise the level of use of soil inputs (labor for conservation and inorganic N) that have long-term desirable effect on soil quality and soil productivity.

Comparisons of current average farmer practice with the dynamic and static decision rules suggest that current farmer practice follows neither the dynamic nor the static decision rule. Output under current practice is higher than the static solution (0.71 ton/ha against 0.42 ton/ha) but much lower than the dynamic (steady state) optimum. Levels of use of soil and non-soil inputs also diverged considerably. Of particular significance is the current level of capital input, which averages 94 oxen hours/ha compared to only 18 oxen hours/ha under static optimization. Moreover, current inorganic N application rate is well above optimal levels under static optimization by about 67% (24.1 kg/ha against 14.4 kg/ha) but much lower than the long-term dynamic (steady state) and socially desirable level of 54.8 kg/ha, which entails a net nutrient extraction of 16.2 kg/ha. Consequently, current resource use pattern implies a total user cost of Ethiopian Birr 253.3/ha. User costs are the annual loss in soil productivity due to changes in the nutrient stock. Hence, total user costs are calculated by multiplying the dynamic price of N with the net change in the nutrients stock. Current soil fertility management and conservation practices are thus far from optimal to offset the soil nutrient depletion problem characterizing tef production in the highlands of Ethiopia. The above results confirm the widely claimed hypothesis that the private optimal path of soil use diverges from the socially optimal path. Among the reasons for the existence of this divergence is the high rate of time preference that smallholder farmers display in their production and consumption decision-making processes. It is worth noting that the steady state socially desirable optimum inorganic N (55 kg/ha) is close to the agronomically-recommended N fertilizer rate of 60 kg/ha currently promoted by the extension package program for the cultivation of small cereals including teff in the highlands of Ethiopia.

Nonetheless, despite the fact that current smallholder tef production practices are sub-optimal compared to the socially desirable dynamic optimum, the fact that current inorganic N application rate is higher by 67% than the static optimal level (24.1 kg/ha against 14.4 kg/ha) suggests that smallholder farmers seem somehow to consider the long-term consequences of the nutrient mining externality.

The optimal values of the choice variables, output and resource stock derived above were based on mean soil and non-soil parameter values. Average values, however, hide valuable information as rates of soil erosion and other soil characteristics are plot- and location-specific and change considerably over time due to climatic variations, slope, topography, etc. Model results are also sensitive to assumed discount rates and other input and output prices. Sensitivity analysis is thus conducted to assess the robustness of the dynamic optimal solutions to changes in parameter values and key assumptions.

The initial soil N stock is the most important variable in the nutrient mining control model and varies considerably across farms and plots managed by the same household. Changing the N content of soil from the average level (0.22%) used in the base run to the lowest observed level (0.17%) and still maintaining the assumption that inorganic fertilizers are perfect substitutes for natural N, appears to have little impact on the dynamic equilibrium solution levels of the N stock, output and non-soil inputs (labor and capital for production) (Table 4). On the other hand, the assumption of above-average soil N content of 0.34% resulted in increased level of conservation effort (from 28 to 55 person-days/ha, up by 96%) but reduced level of inorganic N input from 55 to 48 kg/ha (lower by 13%). The optimal level of inorganic N under the assumption of below-average soil N content of 0.17% increased by 6% to 58 kg/ha, whereas needed conservation efforts have been reduced to 12 person-days/ha (i.e. lower by 57% than the base run). These results suggest that conservation labor and inorganic N are substitutes and hence the optimal soil fertility management strategy depend on the soil fertility status (actual or perceived) of the plot in question. On plots with above-average soil fertility where the marginal reduction in soil quality due to the use of one additional unit of conservation labor is higher than the marginal contribution of inorganic N to soil quality, the optimal soil management strategy would be to increase conservation effort (more conservation labor input) but reduce inorganic N. On plots with lower-than-average soil fertility where the marginal reduction of soil quality due to the use of one additional unit of conservation labor is less than the marginal contribution of inorganic N to soil quality, the optimal strategy would be to use less soil conservation but more inorganic N. Therefore, initial N content of soil affects optimal levels of conservation effort and inorganic N inputs but not the optimal steady state N stock level. It should be noted that the above results depend on the strong assumption of perfect

<table>
<thead>
<tr>
<th>Variable</th>
<th>Base run Soil N (0.22%)</th>
<th>Soil N (0.17%)</th>
<th>Soil N (0.34%)</th>
<th>Nutrient augmentation coefficient (β1 = 0.75)</th>
<th>Output elasticity of N (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β1 = 1</td>
<td>β1 = 1</td>
<td>β1 = 1</td>
<td>Soil = 0.22%</td>
<td>Soil = 0.17%</td>
</tr>
<tr>
<td></td>
<td>β2 = 29.15</td>
<td>β2 = 29.15</td>
<td>β2 = 29.15</td>
<td>Soil = β1 = 1</td>
<td>Soil = β1 = 1</td>
</tr>
<tr>
<td></td>
<td>β3 = 1.7</td>
<td>β3 = 1.7</td>
<td>β3 = 1.7</td>
<td>Soil = β1 = 2.2</td>
<td>Soil = β1 = 2.2</td>
</tr>
<tr>
<td></td>
<td>β4 = 3.4</td>
<td>β4 = 3.4</td>
<td>β4 = 3.4</td>
<td>soil = β1 = 3.4</td>
<td>soil = β1 = 3.4</td>
</tr>
<tr>
<td>Labor for productiona</td>
<td>32</td>
<td>31</td>
<td>33</td>
<td>32</td>
<td>31</td>
</tr>
<tr>
<td>Labor for conservationb</td>
<td>28</td>
<td>12</td>
<td>55</td>
<td>28</td>
<td>12</td>
</tr>
<tr>
<td>Capital for productionc</td>
<td>51</td>
<td>50</td>
<td>52</td>
<td>51</td>
<td>50</td>
</tr>
<tr>
<td>Inorganic fertilizer (kg/ha)</td>
<td>55</td>
<td>58</td>
<td>48</td>
<td>73</td>
<td>77</td>
</tr>
<tr>
<td>Yield of tef (ton/ha)</td>
<td>1.53</td>
<td>1.52</td>
<td>1.55</td>
<td>1.53</td>
<td>1.52</td>
</tr>
<tr>
<td>Net soil loss (ton/ha)</td>
<td>4.65</td>
<td>7.83</td>
<td>0.82</td>
<td>4.65</td>
<td>7.83</td>
</tr>
<tr>
<td>MUC (Birr/kg of N)</td>
<td>15.74</td>
<td>15.74</td>
<td>15.74</td>
<td>20.99</td>
<td>20.99</td>
</tr>
<tr>
<td>Resource stock (N kg/ha)</td>
<td>448</td>
<td>444</td>
<td>459</td>
<td>448</td>
<td>444</td>
</tr>
<tr>
<td>Net benefit (Birr/ha)</td>
<td>1189</td>
<td>1231</td>
<td>1158</td>
<td>902</td>
<td>928</td>
</tr>
</tbody>
</table>

Please refer to Table 2 for what the β symbols measure and mean.

a Person-days/ha.
b Oxen hours/ha.
c Marginal user cost.

substitutability of inorganic N for natural soil N. These results, therefore, suggest that improving smallholder farmers’ skills in soil fertility assessment techniques through extension education and other appropriate medium is likely to contribute to a more efficient use of household resources including the soil wealth.

Changing the coefficient of the augmentation function ($\beta_1$) from 1 to 0.75, which in effect implies that inorganic fertilizers are less than perfect substitutes of natural soil N, but still maintaining other parameter values at base run levels, has very little effect on the dynamic optimal levels of labor and capital for production, labor for conservation and the level of the nutrient stock. It, however, increased the optimal level of inorganic N fertilizer required for maintaining optimal output levels indefinitely by about 33%. The inorganic N input requirements were increased from 55 kg/ha to 73 kg/ha, from 48 kg/ha to 64 kg/ha and 58 kg/ha to 77 kg/ha on plots with average, above-average and below-average N content of soil, respectively, clearly indicating that increased inorganic N levels are needed to compensate for the less than perfect substitutability of inorganic N for natural soil N. The simulation results thus suggest that improved agronomic practices that enhance nutrient use efficiency (e.g. practices that reduce N leaching such as N fertilizer placement techniques and split N fertilizer application) would have a positive contribution to soil quality and hence to a more sustainable use of soil resources.

Sensitivity analysis was also conducted with respect to changes in the output elasticity of N and capital (oxen hours). A 10% improvement in the output elasticity of N fertilizer from 0.2980 to 0.3278, all other parameter values kept at base run levels, raises optimal levels of non-soil inputs (labor and capital for production), inorganic N, output and the resource stock by more than 50%. On the other hand a 10% decrease in the output elasticity of N lowers levels of non-soil inputs, inorganic N, output and the resource stock. Similarly a rise (fall) in the output elasticity of capital would have a similar effect as in the elasticity of output with respect to inorganic N fertilizer. These results, therefore, suggest that technical innovations such as improved agronomic practices and improved crop varieties that improve nutrient use efficiency would play a key role in raising productivity and conservation of soil nutrient stocks.

Dynamically optimal values are found to be highly sensitive to the assumed discount rate (Table 5). For instance raising the discount rate from 9% (base run) to 12% and further to 24%, all other parameter values kept at base run levels, reduced optimal levels of non-soil inputs, the resource stock and output considerably but raised the net rate of soil loss, suggesting households over-exploit the resource stock as the resource is considered worth more now than in the future. It is worth noting that the optimal steady state production labor and inorganic N input and output levels tend to converge to current average practice levels as the discount rate increases beyond 24%, suggesting smallholder farmers discount the future heavily. Lowering the discount rate from 9% (base run) to 6% would have the opposite effect: raising the dynamic optimal levels of labor and capital use for production, inorganic N fertilizer, the resource stock and output with a concomitant fall in the rate of soil loss. The above simulation results agree with the widely held view that smallholder farmers discount the future heavily (have high rate of time preference) and that private optimal paths of soil use diverge considerably from the desirable social optimum (long-term sustainability) (Clark, 1992; Bishop, 1995; Holden et al., 1998).

In many developing countries including Ethiopia the high rate of time preference displayed by smallholder farmers is believed to be associated with poverty, risk aversion behavior and land tenure insecurity. Therefore, measures that reduce smallholder farmers’ rate of time preference such as improved land tenure security, access to credit and actions targeted at reducing poverty would raise the future worth of soil resources, thus providing incentives for the adoption of soil and water conservation (SWC) measures which in turn contribute to a more sustainable use of soil resources.

In most developing countries input and output pricing policies have remained the most important policy tools employed to attain various development objectives deemed desirable by government. It is therefore important to assess the effect of input and output price changes on the desirable dynamic optimal values. Simulation results of a 25% increase in the price of inorganic N lowered the optimal input levels of labor and capital for production, inorganic N, the resource stock and output. It, however, induced a rise in conservation effort and hence reduced the net soil loss. It should be noted that although the level of conservation effort is lower than before the fall in price, the optimal nutrient stock increased. This might be due to the fact that the increase in the level of fertilizer use triggered by the fall in fertilizer price more than compensated the nutrient lost along with eroded soil. This negative relationship of an increase (decrease) in fertilizer price and a rise (fall) in conservation effort could be explained by the relative price changes of fertilizer and conservation labor, inducing a substitution effect. As the price of fertilizer increases, the opportunity cost or shadow price of the nutrient stock rises relative to the price of conservation labor, providing the resource manager with incentives to substitute conservation effort for inorganic N. On the other hand, a fall in the price of fertilizer lowers the shadow price of the nutrient stock, which consequently raises the relative price of conservation effort thereby reducing the manager’s incentive for conservation. Therefore, policies targeted at improving market access (improvement in road networks) and improving the efficiency of existing input and output markets (reduce transaction costs) that ensure the delivery of inorganic fertilizers at the right time, product mix and reasonable price is likely to raise the use of inorganic fertilizers, which ultimately contributes to a more sustainable use of soil resources.

Table 5
Sensitivity analysis with respect to changes in the discount rate, inorganic fertilizer and output price.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Base run</th>
<th>Change in discount rate (δ)</th>
<th>Change in price of N fertilizer ($W_F$)</th>
<th>Change in output price ($P_Y$)</th>
<th>25% rise</th>
<th>25% fall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>δ = 9%</td>
<td>δ = 6%</td>
<td>δ = 12%</td>
<td>δ = 24%</td>
<td>25% rise</td>
</tr>
<tr>
<td>Labor for production&lt;sup&gt;a&lt;/sup&gt;</td>
<td>32</td>
<td>42</td>
<td>26</td>
<td>16</td>
<td>23</td>
<td>46</td>
</tr>
<tr>
<td>Labor for conservation&lt;sup&gt;a&lt;/sup&gt;</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>42</td>
<td>10</td>
</tr>
<tr>
<td>Capital for production&lt;sup&gt;a&lt;/sup&gt;</td>
<td>51</td>
<td>67</td>
<td>42</td>
<td>26</td>
<td>36</td>
<td>74</td>
</tr>
<tr>
<td>Inorganic fertilizer (N/kg/ha)</td>
<td>55</td>
<td>68</td>
<td>47</td>
<td>34</td>
<td>40</td>
<td>77</td>
</tr>
<tr>
<td>Yield of teff (ton/ha)</td>
<td>1.53</td>
<td>2.02</td>
<td>1.26</td>
<td>0.78</td>
<td>1.18</td>
<td>2.06</td>
</tr>
<tr>
<td>Soil loss (ton/ha)</td>
<td>4.65</td>
<td>4.32</td>
<td>4.84</td>
<td>5.19</td>
<td>2.73</td>
<td>7.90</td>
</tr>
<tr>
<td>Resource stock (N/kg/ha)</td>
<td>448</td>
<td>889</td>
<td>276</td>
<td>86</td>
<td>255</td>
<td>866</td>
</tr>
<tr>
<td>Net benefit (Birr/ha)</td>
<td>1189</td>
<td>1689</td>
<td>910</td>
<td>426</td>
<td>701</td>
<td>1951</td>
</tr>
</tbody>
</table>


<sup>a</sup> Person-days/ha.

<sup>b</sup> Oxen hours/ha, other parameters are set at the baseline scenario level: soil N of 0.22%, $\beta_1 = 1, \beta_2 = 29.19$ and $\beta_3 = 2.2$. 

Please see Table 2 for what the $\beta$ symbols measure and mean.
Appendix A. First-order conditions for soil nutrients’ mining in Ethiopian Highlands

\[
\frac{\partial \Pi}{\partial F} = 0 = -e^{-a}W_0 + \mu \beta_1 \\
\frac{\partial \Pi}{\partial L_Y} = 0 = e^{-a} \left( \frac{p_h Y}{L_Y} - W_0 \right) + \mu \frac{b Y}{L_Y} z \\
\frac{\partial \Pi}{\partial K_Y} = 0 = e^{-a} \left( \frac{p_Y K_Y}{K_Y} - W_0 \right) + \mu \frac{b Y}{K_Y} z \\
\frac{\partial \Pi}{\partial L_N} = 0 = -e^{-a}W_N + \mu \beta_2 z e^{-a} \\
\mu = -\frac{\partial \Pi}{\partial N} = -e^{-a}p_g Y N + \mu \frac{b Y \xi}{N}
\]

(22)

where

\[
Y = ALYK_N^v \\
\beta = \beta_1 \phi e^{-a}
\]

References


