PURCHASING POWER PARITY IN THE LONG RUN: AN EMPIRICAL RE-EVALUATION OF THE SOUTH AFRICAN EVIDENCE

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Abstract

Purchasing Power Parity is a very important equilibrium concept in macroeconomics. Although the concept of purchasing power parity equilibrium is widely used in the academic, public and business sector, the actual existence of purchasing power equilibrium between countries is widely debated. The continuous development of methods to analyse the properties of time series data has contributed to this debate. In this paper, the purchasing power parity equilibrium between South Africa and its major trading partners is tested with some of the recent methods to analyse whether time series data converges towards equilibrium. The conclusion that is reached is that a purchasing power parity equilibrium do exist in the long run, but that this equilibrium breaks down over the short run.

1. Introduction

Numerous studies worldwide have shown that the operation of purchasing power parity (PPP) on the short run is in doubt. According to models of exchange rate determination, different speeds of adjustment in the asset and commodity markets result in short-run negation of PPP. Because prices adjust more slowly than exchange rates, demand shocks cause an immediate change in the nominal as well as real exchange rate. However, economic theory suggests that offsetting movements in commodity prices occur over time to leave the real exchange rate unchanged in the long run (Mark, 1990). While available evidence and tests of the operation of PPP in the long run are distorted by certain problems (e.g. highly volatile floating exchange rates, insufficient long-run floating exchange rate data, inadequate econometric techniques for testing PPP), the past decade has witnessed a great degree of progress in this area and several important conclusions have emerged.

This paper is concerned with testing the PPP relationship between South Africa and its main trade partners during the period 1979 to 1998, inclusive. Even though this

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time span may still be considered relatively short and is characterised by several real shocks to the South African economy, emerging evidence suggests that purchasing power parity does hold between South Africa and its major trade partners in the long run.

The paper is organised as follow: Section 2 briefly explains basic concepts related to do with PPP. Section 3 describes some events which might have affected South Africa's real exchange rate since 1979. In Section 4, the data employed in the study are discussed and some summary statistics reported. Section 5 outlines three approaches to testing PPP, and Section 6 then tests the hypothesis for South Africa by means of the methods described in Section 5. The results suggest that a long-run PPP equilibrium does in fact hold in the South African case. The results derived from applying fractional integration methods to the real exchange rate, however, suggest that the real exchange rate shows long-term memory and that shocks to the real exchange rate exhibit some persistence that only converges over the long-run.

2 Purchasing power parity concepts

Attributed to the Mercantilists and popularised by Gustav Cassell, the concept of purchasing power parity fundamentally rests on the law of one price. Underlying this law is the premise of simple goods market arbitrage, which suggests that in the absence of tariffs and transportation costs, unfettered trade in goods would result in identical prices across countries (Froot, 1995), that is:

\[ p_t(i) = p^*_t(i) + \pi_t \]

where \( p_t(i) \) is the logarithm of the domestic price of good \( i \) at time \( t \), \( p^*_t(i) \) is the logarithm of the foreign currency price of good \( i \) at time \( t \), and \( \pi_t \) is the logarithm of the domestic currency price of foreign currency at time \( t \).

In testing the PPP hypothesis, most studies do not make use of identical goods, but rather compare the prices of baskets of goods between countries. In order to do this comparison, consumer price indices (CPI) or wage price indices (WPI) are usually used to test an absolute version of PPP:

\[ p_t(CPI) = p^*_t(CPI^*) + \pi_t \]

This absolute version of testing the PPP relationship results in some problems because countries do not include the same goods in their consumer price indices, nor do they assign the same weights to the basket of goods used in calculating the CPI. In order to overcome these problems, most of the tests of PPP are based on testing the relative version of PPP:

\[ \Delta p_t(CPI) = \Delta p^*_t(CPI^*) + \pi_t \]

which requires that changes in relative price levels be offset by changes in the exchange rate (Froot, 1995).
Results reported in the empirical literature on the testing of PPP are mixed, although there is broad consensus that a PPP relationship does not hold in the short run but may well exist in the long run. However, in testing the validity of a long-run PPP relationship between the industrialised countries of the world, the null hypothesis that the real exchange rate is a random walk cannot be rejected (see Meese and Rogoff (1988), Mark (1990)). Recent developments in the statistical testing of time-series behaviour have again produced new evidence which rejects the null hypothesis of a random walk real exchange rate (Johnson, 1990; Diebold, Husted and Rush, 1991). In South Africa, available evidence on the PPP relationship indicates that this relationship breaks down in the face of real shocks but could hold over a period of relative stability (Barr and Kahn, 1995).

3. An examination of incidents affecting the real exchange rate during the period 1979-1998

According to economic theory, the real exchange rate is influenced by nominal (monetary) as well as real shocks (Dornbush, 1976). Examining real exchange rate volatility in South Africa, one may start by investigating real shocks to the South African economy that might have a significant influence on the real exchange rate.

The first incident affecting the real exchange rate, during the period under investigation, was the shift of the rand exchange rate policy from a peg to the US dollar, to a more market-orientated managed float where a policy of "variable dollar pegging" was adopted. In August 1983 the South African Reserve Bank abandoned this policy and market forces were then allowed to determine the exchange rate. The effect of this policy change on the real exchange rate is evident in all four cases. The new policy resulted in a significant appreciation of the real exchange rate. Even though a managed floating exchange rate regime was adopted, it must be seen against the background of considerable Reserve Bank intervention in the market.

In 1983, following the De Kock Commission report, the Reserve Bank took the liberalisation of financial markets further. Probably the most significant measure implemented was the abolition of capital controls on non-residents in South Africa. This move, together with the scrapping of the financial rand, which was meant to have protected the capital account against foreign portfolio shifts, was accompanied by a sharp depreciation of the real exchange rate. This was caused by foreign capital outflow for which political unrest, a disinvestment campaign and the debt crises of the mid-1980s were cited as reasons. From late 1984 onwards social and political events hampered the development of the foreign exchange market and moved the authorities to control the foreign exchange market more directly. As a result of financial sanctions against South Africa, the Reserve Bank re-entered the foreign exchange market as an active participant and used direct
controls to regulate the effect of capital flows on the balance of payments and foreign reserves. Exchange controls on non-residents were brought back by the re-establishment of the dual exchange rate system and the restoration of the financial rand. These events contributed to the volatility of the real exchange rate up to mid-1988.

From then onwards, the exchange rate and monetary policy objective shifted towards the maintenance of a current account surplus apart from the existing commitment to lower inflation. The appointment of Dr Chris Stals as Reserve Bank governor in 1989 reaffirmed the Reserve Bank's mission on protecting the internal as well as external value of the rand. The relative stability in monetary policy from 1988 onward resulted in a less volatile exchange rate.

Towards the mid-1990s the monetary authority embarked on a gradual path of liberalising capital controls. The effect of this was an appreciation of the real exchange rate, since the mid-1990s, as can be seen in Figure 1.

4. Data and summary statistics

Due to the problems experienced with low power of statistical tests in the face of short time spans, the behaviour of the South African real exchange rate is examined over the period 1979 to the end of 1998 on a monthly frequency. This period represents a time when South Africa experienced a relatively market-orientated floating exchange rate (Barr and Kahn, 1995).

Instead of using the real effective exchange rate, which consists of a weighted basket of currencies of South Africa's major trading partners, the bilateral real exchange rates between South Africa and each of these trading partners are used separately. The reason for this in the testing of PPP is that it should decrease the problems experienced with different consumption baskets in the different countries as mentioned in Section 2.

The four major trading partners of South Africa during the period 1979 to 1998 were the United States, United Kingdom, Japan and Germany. These four countries were chosen on the basis of the volume of trade between them and South Africa. Another reason was the fact that the South African Reserve Bank used these four countries (up to the end of 1998) to compile the basket of currencies used to calculate South Africa's effective exchange rate. The relationship between South Africa and Japan, the United States, United Kingdom, South Africa and Germany are denoted by SA_JAP, SA_US, SA_UK and SA_GER respectively.
All the variables are expressed in logarithmic form. The logarithm of the nominal exchange rate is denoted by $\pi$ and the logarithm of the real exchange rate is denoted by $q$. The logarithm of the domestic and foreign price levels are denoted by $p$ and $p^*$ respectively, while the logarithm of the relative price levels is defined as $\tau = p - p^*$.

The bilateral nominal exchange rates between South Africa and the four above-mentioned trading partners Germany, Japan, the United Kingdom and the United States were obtained from the South African Reserve Bank's database. These are the middle-of-the-month exchange rates. Consumer price index data were obtained from the IMF *International Financial Statistics*.

![Graphs of real exchange rates](image-url)

**Figure 1: Real exchange rates: Rand-Mark, Rand-Yen, Rand-Pound, Rand-Dollar: 1979-1998**
The approach used in the evaluation of the sample statistics is the same as that used by Mark, 1989. In evaluating some sample statistics, the expected observation over the short term that nominal exchange rate changes are uncorrelated with the relative price levels is confirmed. Table 1 reports the cross-correlation of nominal exchange rate changes and relative inflation rates, estimated from 3 leads to 3 lags for the respective bilateral exchange rates.

It is evident that changes in the nominal exchange rate and relative prices are uncorrelated, with a contemporaneous correlation of nearly zero for each of the paired countries. The contemporaneous correlation ranges from 0.0219 for SA_UK to 0.0557 for SA_Japan. This result confirms the expectation that changes in the nominal exchange rate are not immediately offset by changes in relative prices.

Table 1: Cross-correlation of changes in logarithms of nominal exchange rates and relative price levels

<table>
<thead>
<tr>
<th>lag of prices relative to the exchange rate</th>
<th>Country</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>South African Rand as base currency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>United States</td>
<td>0.1116</td>
<td>0.0315</td>
<td>0.1127</td>
<td>-0.07</td>
<td>-0.1251</td>
<td>-0.0499</td>
<td>0.0352</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>0.0229</td>
<td>-0.0041</td>
<td>-0.0347</td>
<td>0.0219</td>
<td>0.1428</td>
<td>0.0881</td>
<td>0.119</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>0.0288</td>
<td>-0.0198</td>
<td>-0.0078</td>
<td>0.0518</td>
<td>0.1940</td>
<td>0.0287</td>
<td>0.0522</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>0.1158</td>
<td>0.0717</td>
<td>0.2006</td>
<td>0.0557</td>
<td>-0.0009</td>
<td>0.0121</td>
<td>0.0401</td>
</tr>
</tbody>
</table>

Examination of the sample correlation in Table 2 of changes between the real and nominal exchange rates shows that the real and nominal exchange rate are highly correlated with a contemporaneous correlation close to unity in each case.

Table 2: Cross-correlation of changes in logarithms of nominal exchange rates and real exchange rates

<table>
<thead>
<tr>
<th>lag of real exchange rate to nominal exchange rate</th>
<th>Country</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>South African Rand as base currency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>United States</td>
<td>-0.0157</td>
<td>0.1185</td>
<td>0.3510</td>
<td>0.9793</td>
<td>0.3027</td>
<td>0.1020</td>
<td>-0.0311</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
<td>-0.0933</td>
<td>0.0239</td>
<td>0.3234</td>
<td>0.9655</td>
<td>0.2771</td>
<td>-0.0001</td>
<td>-0.1165</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>-0.0691</td>
<td>0.0771</td>
<td>0.2303</td>
<td>0.9793</td>
<td>0.1895</td>
<td>0.0673</td>
<td>-0.0739</td>
</tr>
<tr>
<td></td>
<td>Japan</td>
<td>-0.0256</td>
<td>0.1219</td>
<td>0.3040</td>
<td>0.9684</td>
<td>0.2536</td>
<td>0.1070</td>
<td>-0.0445</td>
</tr>
</tbody>
</table>
While the contemporaneous correlation is close to one in each case, the correlation with non-zero lags is very small.

Table 3 reports sample standard deviations of changes in nominal and real exchange rates as well as the inflation differential.

While the real exchange rate is slightly more variable than the nominal exchange rate, both are nearly four times more variable than the inflation differential in each case. The inflation differential exhibits a very low variability in the short run, while the real and nominal exchange rate exhibits a higher degree of variability.

It may be concluded on examination of these sample statistics that the nominal and real exchange rate are significantly more variable than relative prices in the short run. Furthermore, the real and nominal exchange rates move together, while no such relationship exists in the short run between relative prices and nominal exchange rates. These findings suggest why PPP breaks down in the short run, and why the real exchange rate shows random-walk behaviour in the short run. The low variability of the inflation rate differentials relative to nominal exchange rate movements, implies that nominal exchange rate movements dominate real exchange rate movements.

Table 3: Sample standard deviations of inflation differentials and changes in logarithms of real and nominal exchange rates

<table>
<thead>
<tr>
<th>Country</th>
<th>Inflation differential</th>
<th>Changes in the log of the nominal exchange rate</th>
<th>Change in the log of the real exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>South African Rand as base currency</td>
<td>0.006657</td>
<td>0.031686</td>
<td>0.032832</td>
</tr>
<tr>
<td>United States</td>
<td>0.009083</td>
<td>0.033851</td>
<td>0.034856</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.007020</td>
<td>0.034307</td>
<td>0.034660</td>
</tr>
<tr>
<td>Germany</td>
<td>0.009336</td>
<td>0.036686</td>
<td>0.037348</td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is evident that no PPP relationship between South Africa and her major trading partners does exist in the short run. It remains to be tested whether this relationship may hold in the long run.

5. Testing the hypothesis of purchasing power parity in the long run

The development of tests to test the PPP relationship dissolves into three different approaches. This section describes the three approaches and the available statistical methods of testing each approach as described by Froot and Rogoff.
Although most of the methods set out in this section are employed in section 5, only the more recent methods of testing are explained in detail.

5.1 Stage one tests

Tests of purchasing power parity has evolved from a simple method of testing the null hypothesis of purchasing power parity by running a regression of the form:

$$\pi_t = \alpha + \beta \tau_t + \epsilon_t$$

and testing whether $\beta$ is equal to one. The main problem with this test is that it does not distinguish between long-run and short-run effects, nor does it specifically allow for any dynamics of adjustment to PPP. Another problem is that exchange rates and prices are simultaneously determined and there is no compelling reason to put exchange rates on the left side instead of the relative prices (Froot, 1995).

The most positive results from this test have been applicable to high inflation economies (Frenkel, 1981) where the hypothesis of PPP could not be rejected. For countries with lower inflation rates, however, the null hypothesis of PPP has been mostly rejected.

5.2 Stage two tests

The problems encountered with the testing of PPP by means of the above mentioned method, have resulted in an alternative approach. Using this approach, PPP is tested under a null hypothesis that the real exchange rate $q_t = \pi_t - \tau_t$, is a random walk (contains a unit root) against the alternative hypothesis that the real exchange rate is stationary. (See Mark, 1989; Meese and Rogoff, 1988.)

Three main techniques are used in modern literature to distinguish the real exchange rate from a random walk. The first technique uses the Dickey-Fuller and Augmented Dickey-Fuller tests. The test involves a regression of the real exchange rate $q_t$, on a constant, a time trend $q_{t-1}$ and lagged changes in $q_{t-1}$. Under the null hypothesis that the real exchange rate has a unit root, the coefficient of the one period lagged real exchange rate will be equal to one. Under the alternative hypothesis that PPP holds over the long run, the coefficient will be less than one. The distribution of the ordinary least squares estimates from the estimation are non-standard and the hypothesis must be tested by using the confidence intervals reported by Dickey and Fuller (1979).

The problem with testing the hypothesis of a unit root in the real exchange rate by using the Dickey-Fuller test is low statistical power. Because floating exchange rates are volatile, it is difficult to distinguish between slow mean reversion and a random walk real exchange rate, especially when the time span under investigation is as short as that of post-Bretton Woods data (Froot, 1995).

The second technique uses variance ratios. The null hypothesis of a unit root will be accepted if the variance of the real exchange rate grow linearly over time.
A third technique which has gained more appeal in recent literature (see Dieboldt, Husted and Rush, 1991) is that of fractional integration, which encompasses a broader class of stationary processes. Due to the fact that this technique can be used to test real economic time series, and that it is employed in this paper, a more detailed discussion of the theory behind this method follows below.

Fractionally integrated time-series model (ARFIMA-models) possess long-memory properties that make them useful for modelling real economic series, like the real exchange rate. These models provide generalised approximations to low frequency components in economic time-series, especially where the knife-edged "unit root" phenomenon arises as a potentially restrictive case (Diebold, 1989).

A fractionally integrated process allows the real exchange rate to evolve according to

$$\varphi(L)(1-L)^d q_t = \chi(L)\epsilon_t$$

where $\varphi(L)$ and $\chi(L)$ are polynomial lag operators with roots outside the unit circle and $\epsilon_t$ is white noise. If the parameter $d = 0$, then the real exchange rate is confined to a class of stationary ARMA processes described by $\varphi(L)$ and $\chi(L)$. If $d = 1$ and $\varphi(L) = \chi(L) = 1$, then the real exchange rate follows a random walk. In most tests only integer values of $d$ are considered. The arbitrary nature of the integer, $d$, restriction is overcome by allowing for fractional integration, that is, allowing for a value $0 < d < 1$.

According to Diebold (199*) the intuition of fractional integration emerges from the frequency domain. A series, $q_t$, displays long-term memory if its pseudo-spectrum increases without limit as angular frequency tends to zero:

$$\lim f_q(\lambda) = \infty$$

ARFIMA processes have pseudo-spectra that behave like $\lambda^{-2d}$ as $\lambda \to \infty$. Integration of the order 1 (I(1)) is a special case where $d=1$. The wider range of spectral behaviour near the origin that becomes possible when the "integer $d$" restriction is relaxed, gives the ARFIMA model the potential to provide superior approximations to low frequency dynamics (Diebold, 1989).

Due to the fact that fractionally integrated processes are stationary, but have autocovariance functions that die off slower than ARMA processes, encompassing them under the alternative hypothesis may enhance the chances of rejecting the random walk null hypothesis (see Diebold, Husted and Rush (1991)).

5.3 Stage three tests

The third method of testing PPP is makes use of cointegration techniques. The economic interpretation of cointegration is that if two series are linked to form an equilibrium relationship in the long run, they will move closely together over time.
and the difference between them will be stable, even though the series themselves may contain stochastic trends (Harris, 1995). Most evidence indicates that PPP is a long-run relationship, and for this reason these tests are the logical means for testing whether PPP holds over the long run. In this paper, cointegration techniques tests whether the relationship

\[ \pi_t - \alpha p_t + \alpha^* p^*_t \]

is stationary for any constant \( \alpha \) and \( \alpha^* \). Most cointegration methods to test PPP are based on the method proposed by Engle and Granger (1987), but more recent PPP tests use the test introduced by Johansen (1991).

The Engle and Granger method involves three steps. In the first step the nominal exchange rate and the relative price levels are tested for their order of integration (the existence of a unit root in each of the series) using the augmented Dickey-Fuller test. If it is established that the two series contain a unit root, the second step is to estimate the cointegrating regression, \( \pi_t = \alpha p_t + \alpha^* p^*_t + \epsilon_t \), using the ordinary least squares estimation technique. For relative prices and the nominal exchange rate to be cointegrated, the residuals of this estimation should be stationary. The third step is then to test whether the residuals are in fact stationary. Using the augmented Dickey-Fuller test once again, the stationarity of the residuals are tested.

Johansen proposed a one-step full-information maximum likelihood estimator for estimating the coefficients of a possible cointegrating relationship, while simultaneously testing for the presence of a unit root. Summarising Harris's (1995) explanation of the Johansen technique: if a vector \( Z_t \) of \( n \) potentially endogenous variables exist, then it is possible to specify the following data generation process and model \( z_t \) as an unrestricted vector autoregression (VAR) involving up to \( k \)-lags of \( z_t \)

\[ z_t = A_1 z_{t-1} + \ldots + A_{k-1} z_{t-k} + u_t \]

\[ u_t \sim \text{IN}(0, \Sigma) \]

where \( z_t \) is \((n \times 1)\) and each of the \( A_i \)'s is an \((n \times n)\) matrix of parameters. The system is in reduced form with each variable in \( z_t \) regressed on only lagged values of both itself and all the variables in the system. Thus, ordinary least squares is an efficient way to estimate each equation in the above equation.

This equation can be reformulated into a vector error-correction (VECM) form:

\[ \Delta z_t = \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + u_t \]

where \( \Gamma_i = -(I - A_1 - \ldots - A_i) \) (i=1,..,-A_k) and \( \Pi = -(I - A_1 - \ldots - A_k) \). This way of specifying the system encompasses information on both the short- and long-run adjustment to changes in \( z_t \), via the estimates of \( \Gamma_i \) and \( \Pi \) respectively. \( \Pi = \alpha \times \beta' \), where \( \alpha \) is the speed of adjustment to disequilibrium, and \( \beta \) is a matrix of long-run coefficients such that the term \( \beta' z_{t-k} \) embedded in the VECM represents up to \((n-1)\) cointegration relationships in the multivariate model. The Johansen test establishes
whether $\Pi$ has full rank. If this is the case, then the variables in $z_t$ are stationary ($I(0)$). If the rank of $\Pi$ is zero then there are no cointegrating relationships. Usually, $\Pi$ has reduced rank that indicates that there are $r < n$ cointegrating vectors ($r$ is the rank of the matrix $\Pi$).

In this paper, the null hypothesis of a random walk real exchange rate is empirically tested by conducting stage two tests, employing both the Augmented Dickey Fuller test and fractional integration, as well as by conducting stage three tests, employing both the Engle and Granger and Johansen techniques.

6. Empirical testing of a purchasing power relationship

6.1 Stage two tests

The first method employed to test whether real exchange rates in South Africa exhibit random walk behaviour is the Augmented Dickey Fuller test, which establishes whether the real exchange rate contains a unit root, as described in Section 4. The results of the Augmented Dickey Fuller test on the bilateral real exchange rates of South Africa and its four main trading partners, respectively, are reported in Table 4.

This result confirms that the null-hypothesis of a unit root in the real exchange rate can not be rejected (except in the single case of the South African/Germany exchange rate). This indicates that the real exchange rate is not constant over time and shows random walk behaviour. This result is supported by most studies that make use of Dickey-Fuller tests. Due to the relatively short time span used in the tests, and the problems of low statistical power that could be experienced using the Dickey-Fuller test, the null hypothesis is again tested by making use of fractional integration.

Following the example of Diebold, Husted and Rush (1989), the parameters of the time series model that allow for fractional integration are estimated by the simple linear regression equation

$$\ln[I(\lambda_j)] = \beta_0 + \beta_1 \ln[4 \sin^2(\lambda_j/2)] + \eta_j$$

where $I(\lambda_j)$ is the periodogram of the real exchange rate at the ordinate $j$, and $\lambda_j$ is the harmonic ordinates of the sample, that is, $\lambda_j = 2\pi j/T$ ($j = 0, \ldots, T-1$). The coefficient $\beta_1$ represents the value of $d$, the order of integration of the time series.

Diebold et al. arrive at this result by basing the estimation of $d$ on the order of the spectral density function near $\lambda = 0$. This is done by taking the first difference of the relevant time series, denoted $X_t = (1 - I)dX_t = \phi^{-1}(L)\theta(L)e_t = \mu_t$.

As $d$ of the level series equals $1 + d$, a value of $d$ equal to zero corresponds to a unit root in $Y_t$. 

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Table 4: Augmented Dickey-Fuller tests for the existence of a unit root in the real exchange rate

<table>
<thead>
<tr>
<th>Rand as base currency</th>
<th>Value of t-statistic on time-series in levels</th>
<th>Value of t-statistic on time-series in first differences</th>
<th>Critical value (5%)</th>
<th>Order of integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>-2,553097</td>
<td>-8,574564</td>
<td>-3,4297</td>
<td>I(1)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-2,918559</td>
<td>-9,170280</td>
<td>-3,4297</td>
<td>I(1)</td>
</tr>
<tr>
<td>Japan</td>
<td>-2,867496</td>
<td>-8,411304</td>
<td>-3,4297</td>
<td>I(1)</td>
</tr>
<tr>
<td>Germany</td>
<td>-3,284327</td>
<td>-8,705689</td>
<td>-3,4297</td>
<td>I(0)</td>
</tr>
</tbody>
</table>

The spectral density of $X_t$ is given by

$$f_X(\lambda) = |1 - \exp(-i\lambda)|^{-2d} f_U(\lambda) = [2\sin(\lambda/2)]^{-2d} f_U(\lambda)$$

where $f_U(\lambda)$ is the spectral density of the stationary process $\mu_t$. Taking logarithms of the above equation, adding and subtracting $\ln\{f_U(0)\}$, and evaluating at the harmonic ordinates, we obtain

$$\ln\{f_X(\lambda_j)\} = \ln\{f_U(0)\} - d\ln\{4\sin^2(\lambda_j/2)\} + \ln\{\ln\{f_U(\lambda_j)/f_U(0)\}\}$$

If consideration is restricted to the low-frequency ordinates near zero, say, $\lambda_j$, $j \leq K \leq T$, the last term in the above equation can be dropped as negligible. The final linear equation is obtained by adding $\ln\{I(\lambda_j)\}$ to both sides of the equation and rearranging to obtain:

$$\ln\{I(\lambda_j)\} = \ln\{f_U(0)\} - d\ln\{4\sin^2(\lambda_j/2)\} + \ln\{I(\lambda_j)/f_U(\lambda_j)\}$$

which is the linear regression used to estimate $d$. This result holds true regardless of the orders and parameterisations of the $\phi$ and $\theta$ polynomials underlying the stationary process $\mu_t$. (For further discussion see Diebold and Rudebusch, 1989.)

The result of the above estimation is summarised in Table 5.

The estimates of $d$ are rather striking. For each of the exchange rates tested, the estimated value of $d$ is consistent with the hypothesis of a unit root at a 5 percent level of significance. At a ten percent level of significance, however, both the South African/Germany real exchange rate and the South African/United Kingdom real exchange rate exhibit convergence to a purchasing parity level in the very long run and contain a high degree of long-run memory. The fact that all the estimated $d$'s are bigger than 0 is a promising indication that there could be long-run convergence even though it is not at standard levels of significance.
Table 5: Estimated values for d allowing for fractional integration

<table>
<thead>
<tr>
<th>Rand as base currency</th>
<th>Estimated value for d</th>
<th>t-values (5% level of significance)</th>
<th>t-values (10% level of significance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.03</td>
<td>0.543</td>
<td>-7.92</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.12</td>
<td>1.669</td>
<td>-5.07</td>
</tr>
<tr>
<td>Japan</td>
<td>0.07</td>
<td>1.085</td>
<td>6.25</td>
</tr>
<tr>
<td>Germany</td>
<td>0.12</td>
<td>1.669</td>
<td>-7.42</td>
</tr>
</tbody>
</table>

6.2 Stage three tests

It is clear from the discussion in Section 5 that testing for a cointegration relationship between the nominal exchange rate and the relative price levels will reveal if a PPP relationship exists in the long run. This relationship is tested by means of the Engle and Granger method. The results obtained are confirmed by testing the same relationship using the Johansen technique. The results from the Engle and Granger tests are reported in Table 6.

Table 6 shows that relative prices and nominal exchange rates are uniformly non-stationary in their levels but stationary in first differences. Given the results of the first step, the cointegration equation can be estimated, and the resulting residuals then tested for stationarity. The results of the test on the residuals are reported in the third and fourth columns of Table 6. It seems that the null hypothesis of no cointegration can be rejected in all four cases. The existence of such a cointegrating relationship is evidence that the nominal exchange rate and relative prices are related over the long run.

The three-step method employed in the testing above has been criticised as being inherently inefficient (Froot, 1995). The reason for this is that it requires choosing, arbitrarily, a single right-hand side variable. To overcome this inefficiency the Johansen-test, described in Section 4, is used to test the four cointegrating relationships, and it confirms the results of the Engle and Granger test. Results from the Johansen test are reported in Table 7.

The results of the Johansen test confirm the findings from the Engle and Granger test.

The stage 3 approach confirm that a long-run cointegrating relationship exists between the nominal exchange rate of South Africa and its four major trading partners and the corresponding relative price levels. On the basis of the stage 3 tests, the hypothesis that a purchasing power parity relationship between South Africa and its major trading partners exist over the long run can therefore be accepted.
Table 6: Engle and Granger test results for cointegration of the nominal exchange rate and relative prices.

<table>
<thead>
<tr>
<th>Rand as base currency</th>
<th>Order of integration of $\pi$</th>
<th>Order of integration of $\tau$</th>
<th>ADF-test statistic</th>
<th>Critical Value (5 % level of significance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>I(1)</td>
<td>I(1)</td>
<td>-2.553524</td>
<td>-2.8738</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>I(1)</td>
<td>I(1)</td>
<td>-2.960140</td>
<td>-2.8738</td>
</tr>
<tr>
<td>Germany</td>
<td>I(1)</td>
<td>I(1)</td>
<td>-3.379270</td>
<td>-2.8738</td>
</tr>
<tr>
<td>Japan</td>
<td>I(1)</td>
<td>I(1)</td>
<td>-2.867913</td>
<td>-2.8738</td>
</tr>
</tbody>
</table>

Table 7: Johansen test results for testing the existence of a cointegrating relationship between the nominal exchange rate and relative price levels.

<table>
<thead>
<tr>
<th>Country</th>
<th>1978:01 - 1998:12</th>
<th>Johansen result</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA_US</td>
<td>No intercept or trend in CE or test VAR</td>
<td>1 cointegrating equation</td>
</tr>
<tr>
<td>SA_UK</td>
<td></td>
<td>1 cointegrating equation</td>
</tr>
<tr>
<td>SA_JAP</td>
<td></td>
<td>1 cointegrating equation</td>
</tr>
<tr>
<td>SA_GER</td>
<td></td>
<td>1 cointegrating equation</td>
</tr>
</tbody>
</table>

How can this apparent disparity between the stage two and stage 3 tests be explained? The answer could lie in the low power of the Dickey-Fuller test in establishing whether a real exchange rate contains a unit root. This problem is aggravated by the short time span available for testing. Another factor that could distort the stage two tests is the high level real shocks to the South African economy over the period under investigation. As explained in Section 5, stage two tests are less powerful under such circumstances. It would therefore be difficult to distinguish between slow mean reversion and a random walk real exchange rate, when the real exchange rate exhibits volatility in the short run. Examination of the sample statistics in Section 3 indicates that the real exchange rate is indeed very volatile in the short run. The fractional integration result indicates that there could be a long-term convergence, although not at the conventional levels of significance.

7. Conclusion

The null hypothesis tested in this paper is that the South African real exchange rate follows a random walk over the long run. The conclusion that can be reached after
stage three tests, is that the null hypothesis can be rejected, but only over a very long period.

Both the Engle and Granger and Johansen techniques indicate that a long-run PPP relationship does indeed exist. By allowing for fractional integration in the stage two tests, some evidence exists that the real exchange rate converges to an equilibrium purchasing power parity level, although only over a very long period. Although the fractional integration results are not statistically significant at the usual levels of significance, given the limited time-span available for testing and the stage three results, it would appear that the bilateral exchange rates have a high level of long-run memory components. This indicates that shocks to the real exchange rate are persistent and their effect only disappears over a longer period. This could be the reason why a PPP relationship breaks down in the short run.

The average time period for adjustment of the real exchange rate to its parity level still needs to be tested. With a world that is constantly getting smaller and where a higher degree of international competitiveness exists, it is likely that the time to convergence will shorten. It is highly unlikely, however, that the real exchange rate will converge to this equilibrium in the face of constant real as well as monetary shocks to the South African economy.

REFERENCES


