

Spillovers in Higher-Order Moments of Crude Oil, Gold, and Bitcoin

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Highlights

- We analyze spillovers in jumps and realized second, third, and fourth moments among crude oil, gold, and Bitcoin markets
- We use Granger causality and generalized impulse response analyses
- Results suggest evidence of predictability and emphasize, among others, the need of jointly modeling linkages across those three markets with higher-order moments

Abstract

We extend existing studies by considering the higher-order moments relationships among crude oil, gold, and Bitcoin markets. Using high-frequency data from December 2, 2014 to June 10, 2018, we analyze spillovers in volatility jumps and realized second, third, and fourth moments among crude oil, gold, and Bitcoin markets via Granger causality and generalized impulse response analyses. Results suggest evidence of predictability and emphasize, among others, the need of jointly modeling linkages across those three markets with higher-order moments; otherwise, inaccurate risk assessment and investment inferences may arise. In fact, the responses of realized volatility shocks and volatility jump are generally positive. Further analyses indicate evidence of a weaker relationship between gold – crude oil, and Bitcoin – crude oil compared to the case of Bitcoin - gold. Practical implications are discussed.

Keywords: crude oil; gold; Bitcoin; realized moments; spillover effect

JEL code: C46; G10

1. Introduction

Spillovers across markets represent an important aspect of international finance, given that they are related to financial modelling and forecasting as well as asset pricing and risk management. Especially spillovers in (realized) higher-order moments computed from intraday returns, such as second, third, and fourth moments of the daily return distribution help market actors to map, and thereby, make inferences not only on cross-markets volatility risk but also the spillover of asymmetry risk and the fat tail risk (i.e., the occurrence of extreme deviations) (see Amaya et al., 2015).¹ Furthermore, recent empirical evidence from intraday data underlines the important role of volatility discontinuities called jumps in improving realized moments forecasts.²

Being the second moment of the price process of an asset, volatility is extensively examined in the empirical finance literature. Furthermore, most of the related studies on volatility jump consider the case of conventional assets, such as stocks. As argued above and given that assets' return distributions are generally non-normal, skewed, and have fat-tails, the studies on spillovers which often neglect higher moments, such as skewness and kurtosis, are in fact incomplete in terms of empirical inferences on cross-assets linkages.

In addition to crude oil and gold markets, those issues are relevant to Bitcoin and the dynamics of its relationship with other assets. Bitcoin market is known to be one of the most volatile markets. It presents non-Gaussian behavior, such as asymmetry and heavy tails (Osterrieder and Lorenz, 2017; Gkillas and Katsiampa, 2018; Gangwal and Longin, 2018), is prone to speculative bubbles (Cheah and Fry, 2015; Corbet et al., 2018b), and violates the hypothesis of efficient markets (Urquhart, 2016; Nadarajah and Chu, 2017). Recently, heated debate has mounted on the relationship between Bitcoin and other assets (e.g., Brière et al., 2015; Dyhrberg, 2016a; Baur et al., 2018a; Bouri et al., 2017, 2018a; Guesmi et al., 2018; Klein et al., 2018; Selmi et al., 2018; Symitsi and Chalvatzis, 2018). In particular, the relationship between Bitcoin and strategic commodities such as gold and crude oil is examined (Bouri et al., 2018b;

¹ Ruan and Zhang (2018) also mentioned that the analysis of moments is also useful for pricing and prediction of options.

² Lai and Sheu (2010) argued that the use of realized volatility incorporating jumps helps to improve the intraday management of portfolio and the hedging performance.

Symitsi and Chalvatzis, 2019), based on some rational that Bitcoin is a hybrid commodity and is affected by crude oil prices.³ However, volatility is latent, thus non-parametric estimates from high frequency data estimates quadratic variation which is the best estimator of integrated (latent) volatility.⁴ More importantly, volatility can be jumpy and prior studies stress the significance of jumps in modeling and forecasting realized volatility (see, among others, Vortelinos and Thomakos, 2013; Duong and Swanson, 2015). Furthermore, the literature only focuses on first and second moments, although higher moments are crucial to characterize the whole distribution.⁵

Accordingly, in this paper, we extend the related literature by examining the spillover effect of volatility jumps and realized second, third, and fourth moments of the price process across crude oil, gold, and Bitcoin markets. More specifically, our focus is on the spillover effect on realized estimators for higher distribution moments, namely realized volatility, realized skewness, and realized kurtosis via Granger causality and generalized impulse response analyses.⁶ We also focus on realized volatility jumps given prior empirical evidence that the volatility of markets can be jumpy.

This study differs from previous research which uses daily returns and GARCH-type models (see Dyhrberg, 2016; Baur et al., 2018a; Bouri et al., 2017; 2018a, b; Guesmi et al. 2018; Klein et al., 2018; and Symitsi and Chalvatzis, 2019). However, by using intraday data we are able to capture important information not easily seen at daily closing prices. The high-frequency data contain much more information about the market, such as the intraday changes and the market microstructures. In this vein, in a bid to capture more information about the market, we construct daily realized volatility, which was proven to be a better proxy for volatility in the high-frequency analysis. Such extension is useful to traders and investors in order to avoid wrong conclusions regarding the casual relation among crude oil, gold, and Bitcoin.

³ As argued by Selgin (2015), Bitcoin is a synthetic commodity money given it combines features from both gold and fiat currencies. Bitcoin is regulated as a commodity in the US (Bouri et al., 2018b).

⁴ Merton (1980) introduced non-parametric volatility estimates, whereas Andersen and Bollerslev (1998) argued via the theory of quadratic variation that the realized volatility estimation is a consistent estimate of the actual volatility.

⁵ As argued by Del Brio et al. (2017), the stylized facts of financial time series, including that of Bitcoin, require researching different moments of the price process, as each moment reveals different properties for the underlying asset.

⁶ See Rafiq et al. (2009), Borovkova and Mahakena (2015), among others for applications of Granger causality in realized moments and volatility jumps, respectively.

Our analyses show that crude oil, gold, and Bitcoin are indeed are not only linked through realized volatility channel but also through the volatility jump as well as the asymmetry of the daily return's distribution (realized skewness) and the occurrence of extreme deviations (realized kurtosis).

The rest of the paper is structured in four sections. The next section reviews the related literature involving Bitcoin and strategic commodities (crude oil and gold). Section 3 describes the data and methodology, where we explain the construction of realized volatility, realized volatility jumps, realized skewness and realized kurtosis which are the methods used to study spillover effect across crude oil, gold and Bitcoin markets. Section 4 presents and discusses the empirical results. Finally, section 5 provides concluding remarks.

2. Related studies

Given many evidences on the relationship between crude oil and gold markets, the focus of this short review of literature will involve Bitcoin and its relationships with those strategic commodities.

Being the first and largest cryptocurrency,⁷ Bitcoin remains at the center of debate related to its potential role within the global financial system. Bitcoin allows its users to exchange value digitally without intermediation and almost anonymously (. Its independence from central banks and government agencies makes it free of sovereign risk. Several studies admire the genuinity of the protocol and the mass collaboration framework that characterize Bitcoin (Ober et al., 2013). More analytically, Bitcoin's underlying technology, called blockchain, has shown boundless promise among financial institutions⁸. However, some other studies question the future of Bitcoin and its prospects for disintermediation. In some countries, financial regulators have been trying to regulate or even ban the use of Bitcoin in their domestic economies⁹, making the financial inclusion of Bitcoin more difficult.

⁷ Bitcoin continues to dominate the cryptocurrency market. At the end of August 2018, Bitcoin's market share accounted for more than 55% of the total market capitalization of all cryptocurrencies (<https://coinmarketcap.com>).

⁸ <https://www.forbes.com/sites/rogeraitken/2018/04/01/despise-bitcoins-sell-off-the-cryptocurrency-space-continues-to-attract-investors/#11d230652ced>

⁹ <https://www.theguardian.com/business/2018/mar/02/bitcoin-faces-regulatory-crackdown-bank-england-warns>

Regardless of regulators' scrutiny, the total market value of Bitcoin increased exponentially, reaching more than \$140 billion at the end of July 2018 (<https://coinmarketcap.com/>). Yet, Bitcoin is almost unrelated to economic and financial developments (see, among others, Polasik et al., 2015; Corbet et al., 2018a). This is probably due to the unique factors that determine Bitcoin price, such as social sentiment (Kristoufek, 2015; Bouoiyour and Selmi, 2015), ratio of exchange-trade volume and hash rate (Bouoiyour and Selmi, 2015), trading volume (Blacilar et al., 2017), electricity prices (Hayes, 2017), user anonymity (Ober et al., 2013), computer programming enthusiasts (Yelowitz and Wilson, 2015), technology (Li and Wang, 2017), cyber-attacks (Böhme et al., 2015) and economic policy uncertainty (Demir et al., 2018).

The relationship between Bitcoin and conventional assets (e.g., equities, bonds) and commodities (e.g., gold, crude oil) has recently attracted an appealing stream of research given its important implications for investors, scholars and policy-makers (e.g., Ji et al., 2018; Symitsi and Chalvatzis, 2019). Despite Bitcoin's extreme price volatility (Cheah and Fry, 2015; Corbet et al., 2018b), market manipulation (Gandal et al., 2018)¹⁰ and exchange security flaws, the interest in Bitcoin investment is still growing. Many scholars provide evidence that Bitcoin is a medium of exchange and a digital investment (see, among others, Polasik et al., 2015). Generally, Bitcoin is weakly correlated with conventional assets and commodities, offering diversifying and/or hedging capabilities. Brière et al. (2015) found that Bitcoin offers significant diversification benefits by improving the risk-return trade-off of well-diversified portfolios from different assets. Using regression models augmented by dummy quantile variables, Baur et al. (2018b) indicated that Bitcoin can diversify equity indices in both normal and stress periods. Quite similar results were reported by Ji et al. (2018) and Corbet et al. (2018a), although the former applied a directed acyclic graph approach, whereas the latter employed connectedness measures in the time and frequency domains.

Focusing on the market dynamics between Bitcoin and commodities, some methods have been used, but overlooked the inter-market dynamics via higher-order moments of the distribution. Bouri et al. (2017) applied a regression analysis based on

¹⁰ <https://hackernoon.com/breaking-news-bitcoin-market-manipulation-detected-by-artificial-intelligence-a4534b7be369>

GARCH dynamic conditional correlations and found that Bitcoin was a strong hedge and a safe-haven against movements in the commodity market before its pre-crash period in December 2013. Guesmi et al. (2018) applied a multivariate GARCH model and reported evidence that Bitcoin can hedge financial assets, such as gold, crude oil and equities. Other studies have considered the relationship between Bitcoin and strategic commodities, such as gold and crude oil by taking into account return and volatility linkages. Bouri et al. (2018a) studied return and volatility spillovers between the market of Bitcoin and the markets of equities, currencies, bonds, and commodities, including energy and gold, under different market states. Using a smooth transition VAR GARCH-in-mean model, the authors showed that Bitcoin returns are related to most of those four markets, particularly commodities, providing additional evidence that the largest digital currency is not completely isolated. More importantly, the authors argued that Bitcoin relates to other assets mostly via return and via volatility. Still, Bitcoin is found to be more a volatility receiver than a volatility transmitter, especially vis-à-vis gold. Symitsi and Chalvatzis (2018) applied a multivariate VAR-GARCH approach and showed evidence of significant spillovers between Bitcoin and the stock indices of energy and technology companies. Using a different approach based on nonlinear and quantile ARDL models, Bouri et al. (2018b) provided evidence that Bitcoin price formation is non-linearly and asymmetrically affected by the aggregate commodity index and gold prices in some low and high quantiles, contradicting earlier evidence from Kristoufek (2015) and Bouoiyour and Selmi (2015) that Bitcoin prices and gold prices are unrelated. With regard to the comparison between Bitcoin and gold, three studies are worthy of mentioning. Dyhrberg (2016) applied a univariate GARCH model and argued that Bitcoin shows some similarities to Gold, given its ability as a hedge¹¹. Klein et al. (2018) used different approaches and argued, among others, that the conditional variance properties of Bitcoin are different from those of Gold. The authors also indicated that Bitcoin moves in tandem with downward stock markets. Selmi et al. (2018) applied a quantile-on-quantile regression and found that both Bitcoin and gold are diversifiers, hedges and safe havens against oil price movements. They also provided implications that adding them to an oil portfolio leads to a reduction in the downside risk.

¹¹ Extending the work of Dyhrberg (2016), Baur et al. (2018a) demonstrated that exact replication is not possible and that alternative statistical methodologies provide very different results.

At least two research gaps emerge from the above review of the related literature. First, there is a lack of understanding of the use of jumps in volatility spillovers between Bitcoin and the two strategic commodities, gold and crude oil. Second, prior studies make inferences on the relationship between Bitcoin and other commodities, mostly by considering the first moment and/or the second moment of return distribution, thus overlooking spillovers in third and fourth moments.

Consequently, in this paper, we examine for the first time realized moments spillovers among Bitcoin, gold and crude oil markets. More specifically, we study the spillover effect on realized estimators for higher distribution moments, namely realized volatility, realized volatility jumps, realized skewness and realized kurtosis. Our study forms a natural extension to existing papers, such as Dyhrberg (2016), Baur et al. (2018a), Bouri et al. (2017, 2018a, b), Guesmi et al. (2018), and Klein et al. (2018), albeit in more sophisticated and different ways that involve higher frequency data and higher (realized) moments as well as volatility jumps. Such extension is important given ample evidence that modeling the volatility jumps help to improve the overall fit of volatility models (see Duffie et al., 2000; Eraker et al., 2003; Todorov and Tauchen, 2011; Gkillas et al., 2018; among others). As for the examination of spillovers in higher moments, it generally leads to a better assessment of the risk inherent in higher realized moments that represent different types of risk, such as volatility risk, asymmetry risk and the fat tail risk (Amaya et al., 2017).

The choice of the gold and crude oil is driven not only by their importance to both investors and policy-makers but by other motivations, as well. In fact, Bitcoin is compared to gold and often called digital gold (e.g., Dyhrberg, 2016), while crude oil is often considered as proxy of the energy market that is related to the mining (i.e. production) process of Bitcoin (Bouri et al., 2017). Crucially, Bitcoin is often classified as a commodity (Bouri et al., 2018b), and thus, the examination of its relationship with gold and crude oil arises as a natural extension of the related literature.

3. Data and methodology

In this paper, we study the spillover effect on realized estimators for distribution moments, namely realized volatility, realized volatility jumps, realized skewness and realized kurtosis across Bitcoin, gold and crude oil markets. First, we discuss the intraday data, data adjustments and the framework for the computation of the daily

realized moment estimators. Then, we apply Granger causality/block exogeneity Wald tests in a tri-variate vector autoregressive (VAR) framework to test the causal relationship among the three markets under consideration. Finally, we proceed to generalized impulse response analyses (GIRF).

2.1. Data

We employ high-frequency (intraday) data for Bitcoin, gold and crude oil for the period starting from December 2, 2014 to June 10, 2018. We select prices every fifteen minutes (15-min) and construct 15-min log-returns. The starting day and this frequency were selected in order to avoid lack liquidity issues from no-activity periods in very small-time windows. Furthermore, we define a trading day from Monday to Friday from 00:00 EST to 23:55 EST, which also allows us to have enough observations to avoid extreme high-frequency noise (see Anderson, 2000). We have collected intraday prices for Bitcoin from Bitcoincharts (<https://www.bitcoincharts.com>) which offers data on a number of liquid Bitcoin markets.¹² Data for gold and crude oil are drawn from the “*Pi-Trading*” database.

The entire dataset has been cleaned, similar to Barndorff-Nielsen et al. (2009). Furthermore, we apply the estimators to the series of mid-quotes after filtering out spread outliers (less than 0.1 percent of each series). We omit days with recorded prices for less than 70 percent of the expected observations on operating time, short trading days around major holidays and any non-common trading day among the series. Lastly, we obtain a final sample of 742 trading days.

For each price series under consideration, we construct intraday returns on day t for the i -th intraday observation as the logarithmic difference between two consecutive observed prices within a day, following the works of Andersen et al. (2001) and Diebold et al. (1999), as follows:

$$r_{t,i} = \log(p_{t,i}) - \log(p_{t,i-1}) \quad (1)$$

where $r_{t,i}$ refers to intraday returns and $p_{t,i}$ stands for the intraday price i , $i = (1, \dots, T)$, for the day t .

¹² We select the Bitcoin prices from the BTC-e exchange which constitutes one of most liquid Bitcoin exchanges (see Bouri et al., 2017).

For each day t , we retrieve a daily point estimate to construct the daily realized volatility RV_t by employing all intraday returns from the final sample. RV_t is referred to as the estimator of the second realized moment and represents the dispersion risk of a univariate price process. RV_t constructs the average value of how dispersed the observed returns are from the mean return. For each day t , we construct RV_t as follows:

$$RV_t \equiv \sum_{i=1}^T r_{t,i}^2 \quad (2)$$

Aligning with the theory of quadratic variation, RV_t is in convergence uniformly in terms of probability with implied volatility as $T \rightarrow 0$. This way, it enables us to provide more accurate measurements of integrated volatility in a non-parametric framework. Ait-Sahalia et al. (2005) and Zhang et al. (2005) argue that RV_t constitutes a consistent and asymptotically normal estimator when we carry out suitable scaling.

Then, we detect realized volatility jumps (henceforth jumps) from realized volatility. To this end, we apply Duong and Swanson's (2015) detection scheme. Detection schemes depend on the selection of the jump-robust realized volatility estimator. We employ the threshold bi-power variation of Corsi et al. (2010) as a jump-robust realized volatility estimator.¹³ The jump statistic $\left(ZJ_t^{(TBPV)}\right)$ is defined as follows:

$$ZJ_t^{(TBPV)} = \sqrt{T} \frac{(RV_t - TBPV_t)RV_t^{-1}}{\left[(\xi_1^{-4} + 2\xi_1^{-2} - 5) \max\{1, TQ_t TBPV_t^{-2}\}\right]^{1/2}} \quad (3)$$

where TQ_t stands for the realized tri-power quarticity. TQ_t is equal to $T\xi_{4/3}^{-3} \sum_{i=1}^T |r_{t,i}|^{4/3} |r_{t,i+1}|^{4/3} |r_{t,i+2}|^{4/3}$ and converges in probability to integrated quarticity. We construct the threshold bi-power variation ($TBPV_t$) as a jump-free volatility estimator as follows:

$$TBPV_t = \sum_{i=2}^T |r_{t,i-1}|, |r_{t,i}| I_{\{|r_{t,i-1}|^2 \leq \theta_{i-1}\}} I_{\{|r_{t,i}|^2 \leq \theta_i\}} \quad (4)$$

¹³ Multipower variation measures are sensitive to the presence of very small (or very close to zero) returns ascending from stale quotes and rounding to a discrete price grid. The prevalence of zero returns is limited in our dataset.

where $I_{\{\cdot\}}$ represents an indicator function and the threshold function, $r_{t,i}$ is the daily return series and t is the time in daily frequency. A jump is statistically significantly different from zero if $ZJ_t^{(TBPV)}$ exceeds the appropriate critical value of the standard gaussian distribution. Hence, we define the jump component of volatility in daily frequency as follows:

$$J_t = |RV_t - TBPV_t| I_{\{ZJ_t^{(TBPV)} > \Phi_\alpha\}} \quad (5)$$

where $I_{\{\cdot\}}$ is an indicator function of $ZJ_t^{(TBPV)}$ exceeds of a given critical value of a Gaussian distribution denoted by Φ_α , at an α significant level.

While the second moment (RV_t) contains both jump and continuous parameters; the third and fourth moments only depend on jump parameters. Moreover, the second moment converges to quadratic variation; yet, the third and fourth moment do not converge to the cubic and quartic variations, respectively.

The third realized moment computes the conditional skewness of a univariate price process. The realized skewness RS_t estimates the asymmetry risk, as a proxy for crash risk (Barndorff-Nielsen et al., 2010). RS_t is a measure of the asymmetry of the daily return distribution. RS_t can be positive or negative, or undefined. A zero value means that the tails on both sides of the mean balance out overall (symmetric distribution). A negative value is drawn out when the left tail is longer or fatter than the tail on the right side (left-skewed distribution). A positive value is drawn out when the right tail is longer or fatter than the tail on the left side (right-skewed distribution). The daily realized skewness is constructd as follows:

$$RS_t = \frac{\sqrt{T} \sum_{i=1}^T r_{t,i}^3}{RV_t^{3/2}} \quad (6)$$

RS_t separates the jump contribution from the continuous to cubic variation and captures just the jump part. Also, RS_t shows the sign of the average jump size (Amaya et al., 2015).

The fourth realized moment estimates the kurtosis risk of a univariate price process with tailedness around the mean (Barndorff-Nielsen and Shephard, 2004). The

realized kurtosis RK_t is related to the tails of the daily distribution.¹⁴ Following Amaya et al. (2015), the daily realized kurtosis is constructed as follows:

$$RK_t = \frac{T \sum_{i=1}^T r_{t,i}^4}{RV_t^2} \quad (7)$$

RK_t captures the jump yet not the continuous component of quadratic variation.

2.2. Econometric framework

To study the spillover effect among the realized estimators of moments for Bitcoin, Gold and Oil, we employ Granger causality/block exogeneity Wald tests via a VAR model. By employing this test, we identify whether a realized moment estimator, of Bitcoin for instance, Granger causes the corresponding realized moment estimator for gold and crude oil in a tri-variate system.

In a general form, a k -dimensional VAR model can be expressed as follows:

$$Y_t = v + A_1 Y_{t-1} + \dots + A_p Y_{t-p} + \varepsilon_t \quad (8)$$

where Y_t corresponds to the $K \times 1$ vector of variables on day t , v is the $K \times 1$ vector for the intercept, A stands for the $K \times K$ matrix of coefficients, and ε_t is the $K \times 1$ vector of the error term. We estimate the parameters of the VAR model consistently using the ordinary least squares (OLS) method based on Newey and West (1987) heteroskedasticity and autocorrelation consistent (HAC) standard errors.

Then, we apply the Granger causality/block exogeneity Wald tests. In simpler terms, causality in Granger (1969) is construed as the case when the values of a variable play an explanatory role in a regression of another variable on lagged values of the two variables. In this paper, for each system, we use Wald χ^2 statistics to identify whether one variable Granger causes another variable in the system.

Finally, following the study of Pesaran and Shin (1998), we proceed to a GIRF. We apply this procedure due to the lack of economic guidance on the direction of the instantaneous causality among Bitcoin, gold and crude oil. Furthermore, the GIRF does

¹⁴ The interpretation of kurtosis is straightforward: constructs whether and how much there are extreme deviations (outliers) from the normal distribution. In case the extreme deviations (outliers) are similar to normal distribution (kurtosis equals to three and is called mesokurtic distribution), there are fewer and less extreme deviations (outliers) than normal distribution (kurtosis is less than three and is called platykurtic distribution) or there are more extreme deviations (outliers) than normal distribution (kurtosis is higher than three and is called leptokurtic distribution).

not require orthogonalization of shocks, and is invariant to the ordering of the variables in the VAR. The generalized responses of the system at time $t + h$ to one-standard-deviation exogenous shock to the j th variable at time t is given as follows:

$$\hat{\psi}_j(h) = \sigma_{jj}^{-1/2} \Pi_h \sum_{\varepsilon} e_j, \quad h = 0, 1, 2, \dots \quad (9)$$

where $\Sigma_{\varepsilon} = \{\sigma_{ij}\}$ corresponds to the $K \times K$ variance-covariance matrix of error term ε_t , while e_j corresponds to a $K \times 1$ selection vector with unity as its j th element and zeros elsewhere for $i, j = 1, 2, \dots, K$. Π_i is the $K \times K$ matrix of coefficients generated from an infinite moving average representation from the previous equation, while the Π_i matrix can be constructed recursively via the following equation:

$$\Pi_i = \begin{cases} \sum_{j=1}^i \Pi_{i-j} A_j, & i = 1, 2, \dots, p \\ \sum_{j=1}^p \Pi_{i-j} A_j, & i > p \end{cases} \quad (10)$$

for Π_0 equal to I_K , which corresponds to a K -dimensional identity matrix.

4. Empirical results

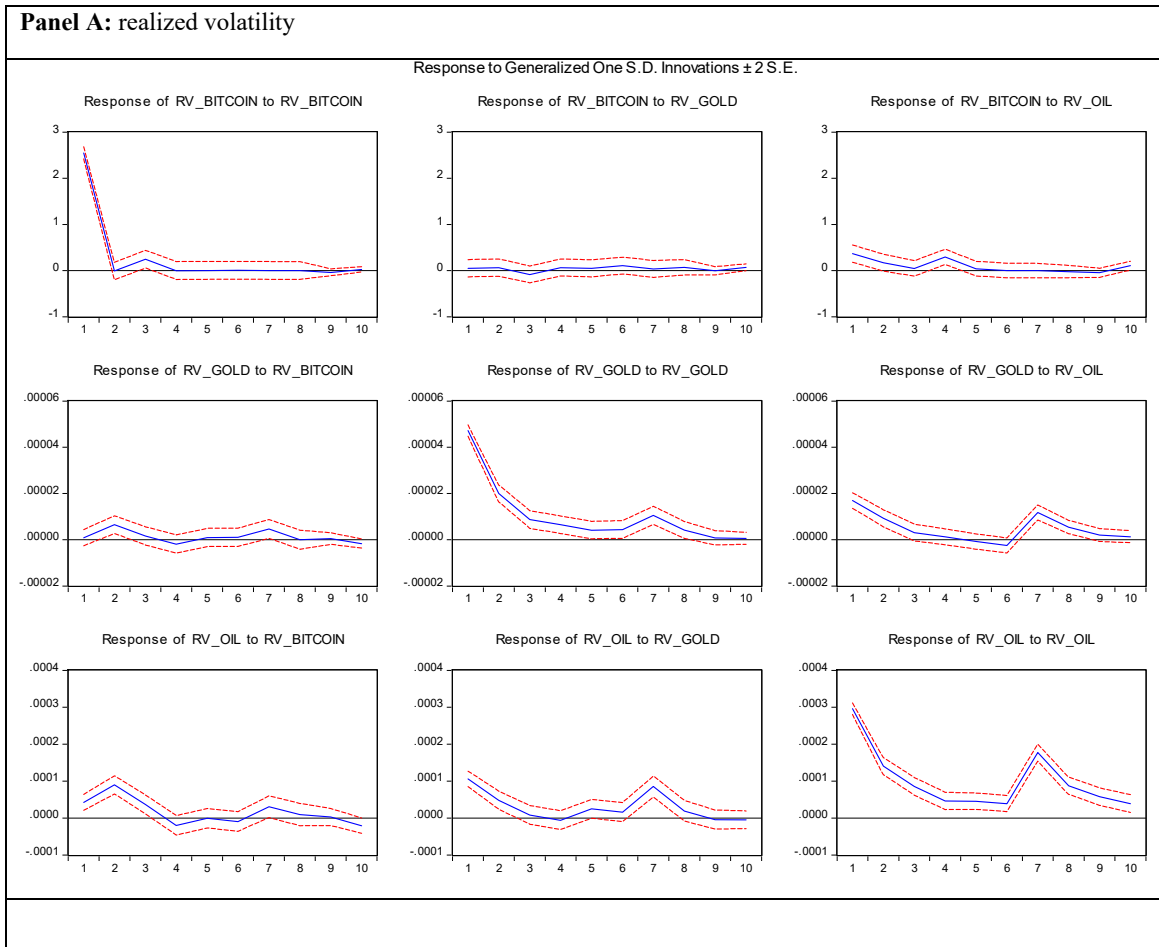
In this section, we present in Table 1 results of the Granger causality/block exogeneity Wald test among the realized estimators of moments for Bitcoin, gold and crude oil in a tri-variate VAR system. The p -values of the test are given in brackets. Panels A, B, C, and D cover respectively the realized second moment (RV), the jump (J), the realized third moment (RS), and the realized fourth moment (RK). The number of lags is selected via the Akaike information criterion (AIC). In each panel, the vertical notation of the variables corresponds to dependent variables of the system, while the horizontal one corresponds to the explanatory variables. Furthermore, we illustrate in Figure 1 the results of the generalized impulse responses. As in Table 1, each Panel of Figure 1 covers the results for the corresponding realized estimator of moments. Since our VAR system has three variables (Bitcoin, gold and crude oil), a total of nine impulses are generated in each Panel. Following Efron and Tibshirani (1993), 95 percent bootstrap confidence intervals are estimated via a bias-corrected procedure.

Table 1. VAR Granger causality tests among realized estimators of the moments of Bitcoin, gold and crude oil

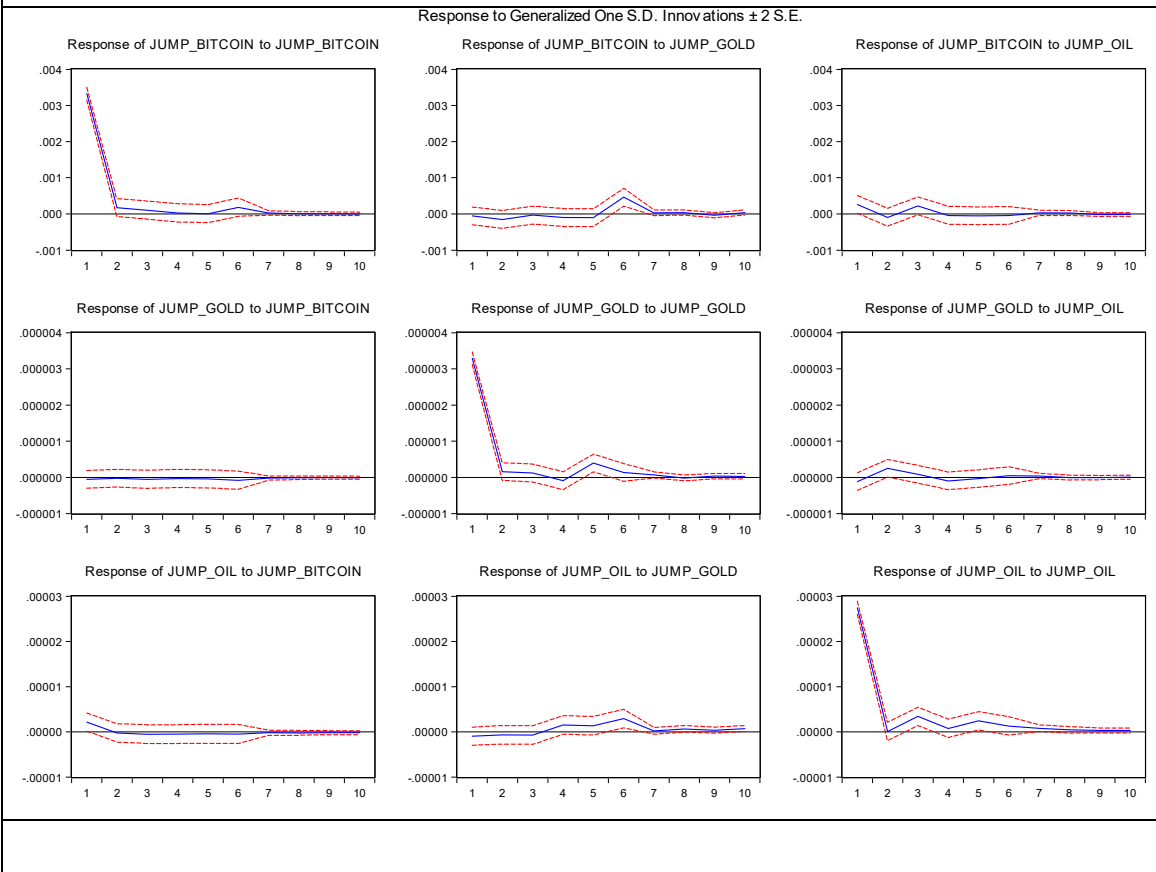
Panel A: realized volatility				
Variables	Bitcoin	Gold	Oil	All
Bitcoin	-	3.923	14.836**	18.783
	-	[0.788]	[0.038]	[0.173]
Gold	28.336***	-	54.705***	74.840***
	[0.000]	-	[0.000]	[0.000]
Oil	71.310***	18.424**	-	90.956***
	[0.000]	[0.010]	-	[0.000]
Panel B: realized volatility jumps				
Variables	Bitcoin	Gold	Oil	All
Bitcoin	-	16.932**	4.797	22.378**
	-	[0.004]	[0.441]	[0.013]
Gold	0.701	-	5.824	6.491
	[0.982]	-	[0.323]	[0.772]
Oil	1.266	12.758**	-	14.507
	[0.938]	[0.025]	-	[0.151]
Panel C: realized skewness				
Variables	Bitcoin	Gold	Oil	All
Bitcoin	-	17.427**	1.620	19.559**
	-	[0.001]	[0.805]	[0.012]
Gold	3.531	-	2.293	5.883
	[0.473]	-	[0.682]	[0.660]
Oil	4.626	4.978	-	8.977
	[0.327]	[0.289]	-	[0.344]
Panel D: realized kurtosis				
Variables	Bitcoin	Gold	Oil	All
Bitcoin	-	14.457*	7.281	21.842*
	-	[0.043]	[0.400]	[0.081]
Gold	3.487	-	7.695	11.860
	[0.836]	-	[0.360]	[0.617]
Oil	4.731	5.311	-	9.975
	[0.692]	[0.622]	-	[0.764]

Note: This Table reports the Wald χ^2 statistics to identify whether any of the realized estimators of moments causes the corresponding estimators of the other variables in a tri-variate system. The p-values of the tests are given below in brackets. Panel A refers to the second moment and realized volatility. Panel B refers to jumps. Panel C refers to the third moment and realized skewness and Panel D refers to the fourth moment and realized kurtosis. The number of lags is selected via the Akaike information criterion. In each panel, the vertical notation of the variables corresponds to dependents variables of the system, while the horizontal one corresponds to the explanatory variables. ***, **, * indicates significance at 1%, 5% and 10% respectively.

Figure 1. GIRF's for a shock to Bitcoin, gold and crude oil



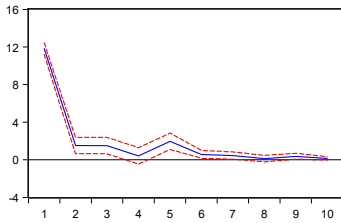
Panel B: realized volatility jumps



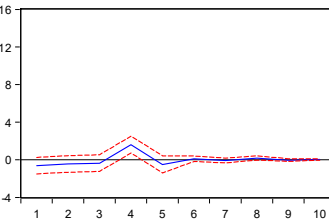
Panel C: realized skewness

Response to Generalized One S.D. Innovations ± 2 S.E.

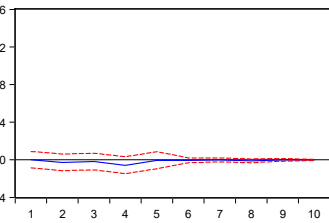
Response of RSKEW_BITCOIN to RSKEW_BITCOIN



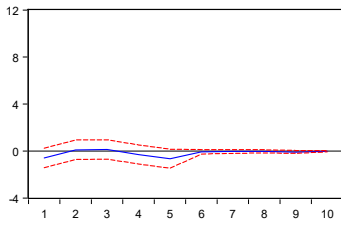
Response of RSKEW_BITCOIN to RSKEW_GOLD



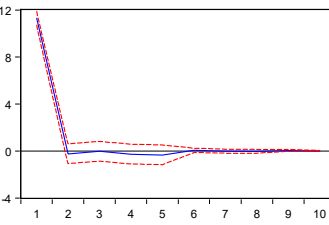
Response of RSKEW_BITCOIN to RSKEW_OIL



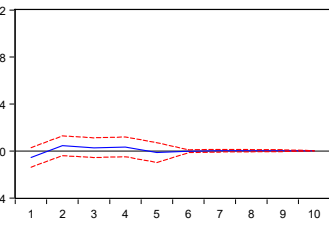
Response of RSKEW_GOLD to RSKEW_BITCOIN



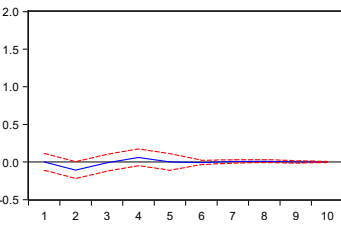
Response of RSKEW_GOLD to RSKEW_GOLD



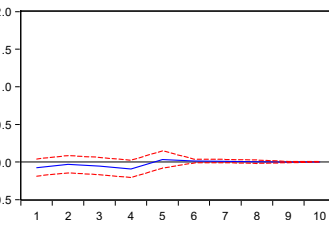
Response of RSKEW_GOLD to RSKEW_OIL



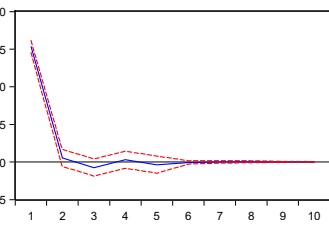
Response of RSKEW_OIL to RSKEW_BITCOIN

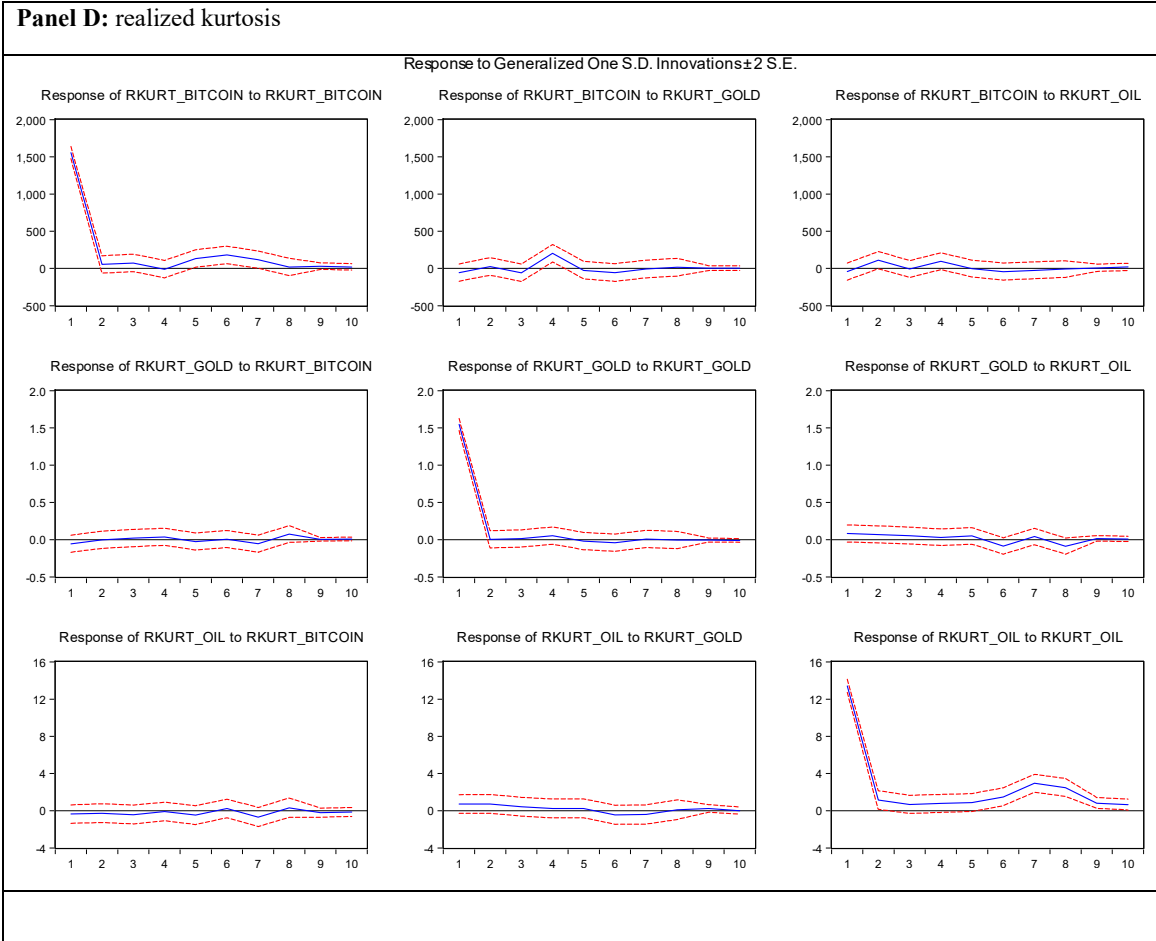


Response of RSKEW_OIL to RSKEW_GOLD



Response of RSKEW_OIL to RSKEW_OIL





4.1. Presentation of results

Panel A of Table 1 shows evidence of a statistically significant bidirectional causality between RV^{Oil} and $RV^{Bitcoin}$. A similar bidirectional causality exists between RV^{Oil} and RV^{Gold} . Additionally, a one-way causality runs from $RV^{Bitcoin}$ to RV^{Gold} . Moving to the GIRF results in Panel A of Figure 1, we observe that RV^{Gold} reacts positively to $RV^{Bitcoin}$ shock, except on days 4 and 10. The initial response of RV^{Oil} to a shock to $RV^{Bitcoin}$ is positive, but it fluctuates around the zero line. It becomes negative in days 4-6, positive in days 7-9, and negative afterwards. RV^{Oil} responds positively to a shock to RV^{Gold} . $RV^{Bitcoin}$ responds positively to a shock to RV^{Gold} , except for day 3. Also, $RV^{Bitcoin}$ responds positively to a RV^{Oil} shock, but the response decays after day 5. RV^{Gold} reacts positively to RV^{Oil} shock, except for days 4 and 6 when the reaction is negative. The effects are statistically significant in all cases. We

also notice that both RV^{Gold} and RV^{Oil} respond positively to their own shocks. The same is true for $RV^{Bitcoin}$, yet its shock decays from the fourth day.

In Panel B of Table 1, we observe a one-way Granger causality from J^{Gold} to $J^{Bitcoin}$ and J^{Oil} at 1 percent and 5 percent significance levels, respectively. For the rest of the cases, there is no significant Granger causality. As for the GIRF results, Panel B of Figure 1 shows that the response of J^{Oil} to a $J^{Bitcoin}$ shock is initially positive, but it becomes negative after the second day. J^{Gold} does not respond to a $J^{Bitcoin}$ shock. When the shock is to J^{Gold} , $J^{Bitcoin}$ responds negatively for the first five days, after which the response becomes positive. Quite similarly, J^{Oil} responds negatively to a shock to J^{Gold} for the first three days, after which the response becomes positive. $J^{Bitcoin}$ initially responds positively to a J^{Oil} shock, however the response becomes negative after day 3. The response of J^{Gold} to a J^{Oil} shock is initially negative, but it fluctuates around the zero line. Interestingly, it becomes positive in days 2-3, negative in days 4-5, and then positive again after day 5. The effects are statistically significant in all cases. The response of both $J^{Bitcoin}$ and J^{Gold} to their own shock is positive.

Turning to Panel C of Table 1, we observe a one-way Granger causality from RS^{Gold} to $RS^{Bitcoin}$, which is statistically significant at 1 percent. For the rest of the cases, there is no causality. Moving to the GIRF results, Panel C of Figure 1 shows that the response of RS^{Gold} and RS^{Oil} to a shock to $RS^{Bitcoin}$ fluctuates around the zero line, but it ends after day 5. Except for day 4 for the case of $RS^{Bitcoin}$, both $RS^{Bitcoin}$ and RS^{Oil} respond negatively to a shock to RS^{Gold} . However, the effect decays after day 5. As for $RS^{Bitcoin}$, it responds negatively to a shock to RS^{Oil} but the effect ends after day 5. The response of RS^{Gold} to RS^{Oil} is initially negative, but after day 1, it becomes positive and decays after day 5. The effects are statistically significant in all cases. Moving to the response to own shocks, $RS^{Bitcoin}$ responds positively to its own shock. RS^{Gold} initially responds positively to its own shock, but negatively after day 2 and it dies off after day 5. RS^{Oil} initially responds positively to its own shock, after which the response fluctuates around the zero line and dies off after day 5.

Finally, Panel D of Table 1 shows evidence of a statistically significant unidirectional Granger causality running from RK^{Gold} to $RK^{Bitcoin}$. For the rest of the cases, there is no causality. As for the GIRF results, Panel D of Figure 1 indicates that the response of RK^{Gold} to a $RK^{Bitcoin}$ shock fluctuates closely around the zero line,

although it was initially negative. RK^{Oil} responds negatively to a shock to $RK^{Bitcoin}$. The response of $RK^{Bitcoin}$ to a RK^{Gold} shock is near the zero line, even though it was initially negative. It decays after day 6. RK^{Oil} responds positively to RK^{Gold} shock, but the response becomes negative after day 5 and ends after day 7. The response of $RK^{Bitcoin}$ to a RK^{Oil} shock fluctuates around the zero line, but it perishes after day 7. RK^{Gold} responds positively to $RK^{Bitcoin}$ shock except for days 6 and 8. In all cases, the effects are statistically significant. Regarding the response to own shocks, both RK^{Oil} and $RK^{Bitcoin}$ exhibit a positive response. The response of RK^{Gold} is initially positive, then it turns negative after day 4 and ends after day 6.

4.2. Discussion of the results

Our analyses showed that the realized volatility of crude oil Granger causes that of Bitcoin with evidence of a feedback effect. This bi-directional causality probably emanates from the close link between Bitcoin and the energy market through the mining process of Bitcoin. However, the realized volatility of Bitcoin is found to Granger cause that of gold, suggesting that investors in the yellow metal should have a close eye on the risk of Bitcoin when considering investment decisions. Furthermore, a bi-directional Granger causality exists between the realized volatilities of crude oil and gold markets, suggesting an interaction between the two markets through the second moment channel.

Gold jump Granger causes Bitcoin jumps, implying that gold risk affects the risk of Bitcoin through the channel of jumps, i.e., through bad volatility. Given jumps represent a source of systematic risk, increased risk in the gold market would make investors in the Bitcoin market demand large risk premia. Such evidence is also useful to policy-makers, who are generally concerned with market stability, and thus make decisions to avoid market chaos potentially induced by jump spillovers and/or fat tail risk or crash risk.

Furthermore, gold realized skewness Granger causes Bitcoin realized skewness. Such evidence implies that gold and Bitcoin markets are linked through the asymmetry of the return distribution. A bi-directional Granger causality exists between the realized kurtosis of gold and that of Bitcoin. This suggests that the market of gold interacts with the Bitcoin market through the channel of fat tail risk. Consequently, market

participants in both markets should not assume that the occurrence of extreme deviations in the gold (Bitcoin) market is independent from that in the other market.

Overall, we indicate that gold is a transmitter of realized skewness and kurtosis to the Bitcoin market, while Bitcoin and crude oil are receivers of jumps from the gold market. Regarding the relationship between gold and crude oil, gold is a transmitter of volatility jump to crude oil, while the two assets have a two-way relationship in realized volatility.

Our analyses above point to a close relationship between Bitcoin and gold in terms of realized volatility, jumps, realized skewness and realized kurtosis. However, for the case of Bitcoin and crude oil, the relationship is weaker. Overall, our empirical evidence contradicts earlier findings that generally imply the isolation of Bitcoin from the global financial system, including gold (Bouoiyour and Selmi, 2015; Kristoufek, 2015) and crude oil (Bouri et al., 2017; Ji et al., 2018). However, our findings are partially in line with and complement earlier studies that found evidence on the ability of commodities, such as gold, to predict Bitcoin prices (Bouri et al., 2018b) and evidence that Bitcoin is sensitive to some economic and financial variables (Li and Wang, 2017). Some of our findings are also partially consistent with prior studies claiming that energy is the main cost in the production of Bitcoin. Yet, they add to the volatility effect from Bitcoin to the stocks of energy companies, as documented by Symitsi and Chalvatzis (2018). Unlike Klein et al. (2018), our results show that Bitcoin and gold have close relationship and interaction. Overall, our findings extend the empirical literature (e.g., Selmi et al., 2018) to higher realized moments, such as skewness and kurtosis, using more realistic and robust estimators of realized volatility and jumps in realized volatility.

5. Conclusions and policy implications

In this paper, we uncovered the different higher moments through which crude oil, gold, and Bitcoin markets can be related. In particular, we attempted to extend the nexus among those markets by considering spillovers in realized higher moments and in the jump component of volatility. This is based on empirical foundations that jumps in volatility are useful to improve the overall fit of volatility models, and that spillovers in realized second, third and fourth moments represent an important aspect of cross-

asset linkages and an input in the pricing and prediction of options (Ruan and Zhang, 2018).

Using high frequency data, we noticed closer relationships between gold and Bitcoin markets compared to the markets of crude oil and Bitcoin as well as the markets of crude oil and Bitcoin. In fact, our analyses showed that crude oil, gold, and Bitcoin are not only linked through realized volatility channel but through the realized skewness (asymmetry risk) and realized kurtosis (fat tail risk). Further analysis based on the generalized impulse response function showed that the spillover effect is generally positive, but in some cases, it switches to negative and lasts for several days. Accordingly, if a shock to a market raises the level of volatility (second moment), the level of asymmetry (third moment) and fat tail (fourth moment), then a spillover effect should be expected to lead to an increase in those three moments in the other markets. Furthermore, a jump in one market (e.g. gold) leads to a jump in the volatility of other markets (e.g. Bitcoin and crude oil).

These findings revealed cross-market linkages which represent a major concern to traders and risk managers. They imply the necessity to consider linkages among crude oil, gold, and Bitcoin via higher moments; otherwise, the ignorance of the significance of symmetry and fat tail risks will have important implications for VaR models. In such cases, subsequently they lead to mistaken financial and risk management decisions and false pricing of those assets. For example, Jurczenko and Maillet (2006) indicated that the significance of the third and fourth moments can be incorporated through the four-moments CAPM model. Moreover, Ruan and Zhang (2018) stressed the importance of analyzing different moments for the pricing and prediction of options.

Regarding Bitcoin and its relationship with gold and crude oil, our findings imply the need for a continuous monitoring of Bitcoin market (European Central Bank 2012), given its ability to transmit volatility risk, jump in volatility risk and fat tail risk as well as asymmetry risk to strategic commodities that are usually considered as hedges (crude oil) and even safe havens (i.e., gold). Therefore, our findings do not rule out the possibility of Bitcoin being a source of instability to those strategic commodities and through them, to the global financial system. Such a potential source of instability might be intensified with the growth of Bitcoin market to its prior levels observed at the end of 2017. Furthermore, considering that policymakers have to make decisions during periods of turbulence in financial (commodity) markets, it is economically vital

to progress an econometric understanding of the behavior of jumps in accordance with their real generating mechanism (see Todorov and Tauchen, 2011).

A future path of research can consider the macroeconomic and financial drivers behind jumps. Another future path of research can examine the diversification benefits arising from the inclusion of the three assets in a single portfolio while considering the spillovers effects of jump, skewness and kurtosis (Liu and Tu, 2012; Hall and Satchell, 2013).

References

- Aït-Sahalia, Y., Mykland, P.A., Zhang, L., 2005. How often to sample a continuous-time process in the presence of market microstructure noise. *Rev. Financ. Stud.* 18, 351–416.
- Andersen, T., Bollerslev, T., 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *Int. Econ. Rev.*, 39, 885-905.
- Andersen, T.G., Bollerslev, T., Diebold, F.X., Ebens, H., 2001. The distribution of realized stock return volatility. *J. financ. econ.* 61, 43–76.
- Balcilar, M., Bouri, E., Gupta, R. and Roubaud, D., 2017. Can volume predict Bitcoin returns and volatility? A quantiles-based approach. *Econ. Model.*, 64, pp.74-81.
- Barndorff-Nielsen, O.E., Shephard, N., 2004. Power and Bipower Variation with Stochastic Volatility and Jumps. *J. Financ. Econom.* 2, 1–37. doi:10.1093/jjfinec/nbh001.
- Barndorff-Nielsen, O.E., Hansen, P.R., Lunde, A., Shephard, N., 2009. Realized kernels in practice: Trades and quotes. *Econom. J.* 12, C1–C32.
- Barndorff-Nielsen, O., Kinnebrock, S., Shephard, N., 2010. Volatility and time series econometrics: Essays in honor of robert f. engle, chapter measuring downside risk-realised semivariance.
- Baur, D.G., Dimpfl, T., Kuck, K., 2018a. Bitcoin, gold and the US dollar – A replication and extension. *Financ. Res. Lett.* 25, 103–110.
- Baur, D.G., Hong, K., Lee, A.D., 2018b. Bitcoin: Medium of Exchange or Speculative Assets? *Journal of International Financial Markets, Institutions and Money*, 54, 177-189.
- Böhme, R., Christin, N., Edelman, B., Moore, T., 2015. Bitcoin: Economics, technology, and governance. *Journal of Economic Perspectives*, 29(2), 213-38.
- Borovkova, S., Mahakena, D., 2015. News, volatility and jumps: the case of natural gas futures. *Quant. Financ.* 15, 1217–1242.
- Bouoiyour, J., Selmi, R., 2015. What does Bitcoin look like? *Annals of Economics and Finance*, 16(2), 449-492.

- Bouri, E., Jalkh, N., Molnár, P., Roubaud, D., 2017. Bitcoin for energy commodities before and after the December 2013 crash: diversifier, hedge or safe haven? *Appl. Econ.* 49, 5063–5073.
- Bouri, E., Das, M., Gupta, R., Roubaud, D., 2018a. Spillovers between Bitcoin and other Assets during Bear and Bull Markets. *Applied Economics*, 50(55), 5935-5949.
- Bouri E., Gupta R., Lahiani A., Shahbaz M., 2018b. Testing for asymmetric nonlinear short- and long-run relationships between Bitcoin, aggregate commodity and gold prices. *Resources Policy*, 57, 224-235.
- Brière, M., Oosterlinck, K., Szafarz, A., 2015. Virtual currency, tangible return: Portfolio diversification with bitcoin. *J. Asset Manag.* 16(6), 365-373.
- Cheah, E.T., Fry, J., 2015. Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Econ. Lett.* 130, 32–36.
- Corbet, S., Meegan, A., Larkin, C., Lucey, B., Yarovaya, L., 2018a. Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Econ. Lett.* 165, 28–34.
- Corbet, S., Lucey, B., Yarovaya, L., 2018b. Datestamping the Bitcoin and Ethereum bubbles. *Finance Research Letters*, 26, 81-88.
- Corsi, F., Pirino, D., Renò, R., 2010. Threshold bipower variation and the impact of jumps on volatility forecasting. *J. Econom.* 159, 276–288.
- Demir, E., Gozgor, G., Lau, C.K.M., Vigne, S.A., 2018. Does economic policy uncertainty predict the Bitcoin returns? An empirical investigation. *Finance Research Letters*, 6, 145-149.
- Diebold, F.X., Hahn, J., Tay, A.S., 1999. Multivariate density forecast evaluation and calibration in financial risk management: High-frequency returns on foreign exchange. *Rev. Econ. Stat.* 81, 661–673.
- Duong, D., Swanson, N. R., 2015. Empirical evidence on the importance of aggregation, asymmetry, and jumps for volatility prediction. *Journal of Econometrics*, 187(2), 606–621.
- Dyhrberg, A.H., 2016. Bitcoin, gold and the dollar - A GARCH volatility analysis. *Financ. Res. Lett.* 16, 85–92.
- Efron, B., Tibshirani, R.J., 1993. *An Introduction to the Bootstrap*. CHAPMAN & HALL/CRC, Boca Raton, London, New York, Washington, D.C.
- Eraker, B., 2003. The impact of jumps in returns and volatility. *J. Finance* 53, 1269–1330.
- European Central Bank, 2012. *Virtual Currency Schemes*. October, 1–55.
- Gandal, N., Hamrick, J.T., Moore, T., Oberman, T., 2018. Price manipulation in the Bitcoin ecosystem. *Journal of Monetary Economics*, 95, 86-96.
- Gangwal, S., Longin, F., 2018. Extreme movements in Bitcoin prices : A study based on extreme value theory. *Work. Pap. Ser.* 1–17.
- Gkillas, K., Katsiampa, P., 2018. An application of extreme value theory to cryptocurrencies. *Econ. Lett.* 164, 109–111.

- Gkillas, K., Gupta, R., Wohar, M.E., 2018. Volatility jumps: The role of geopolitical risks. *Finance Research Letters*, 27, 247-258.
- Granger, C.W.J., 1969. Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica* 37, 424.
- Guesmi, K., Saadi, S., Abid, I., Ftiti, Z., 2018. Portfolio diversification with virtual currency: Evidence from bitcoin. *International Review of Financial Analysis*. <https://doi.org/10.1016/j.irfa.2018.03.004>
- Hall, A. D., Satchell, S. E., 2013. The anatomy of portfolio skewness and kurtosis. *Journal of Asset Management*, 14(4), 228-235.
- Hayes, A.S., 2017. Cryptocurrency value formation: An empirical study leading to a cost of production model for valuing bitcoin. *Telematics and Informatics*, 34(7), 1308-1321.
- Jurczenko E., Maillet, B., 2006. The Four-moment Capital Asset Pricing Model: Between Asset Allocation and Asset Pricing. In: *Multi-moment Capital Asset Pricing and Related Topics*, Adoock-Jurczenko-Maillet Eds, John Wiley & Sons, 113-163.
- Klein, T., Pham Thu, H., Walther, T., 2018. Bitcoin is not the New Gold - A Comparison of Volatility, Correlation, and Portfolio Performance. *International Review of Financial Analysis*, 59, 105-116.
- Kristoufek, L., 2015. What are the main drivers of the Bitcoin price? Evidence from wavelet coherence analysis. *PloS one*, 10(4), art. e0123923.
- Lai, Y.S., Sheu, H.J., 2010. The incremental value of a futures hedge using realized volatility. *Journal of Futures Markets*, 30(9), 874-896.
- Li, X., Wang, C.A., 2017. The technology and economic determinants of cryptocurrency exchange rates: The case of Bitcoin. *Decision Support Systems*, 95, 49-60.
- Liu, Q., Tu, A. H., 2012. Jump spillovers in energy futures markets: Implications for diversification benefits. *Energy Economics*, 34(5), 1447-1464.
- Merton, R., 1980. On estimating the expected return on the market: An explanatory investigation. *Journal of Financial Economics*, 8, 323-361.
- Nadarajah, S., Chu, J., 2017. On the inefficiency of Bitcoin. *Econ. Lett.* 150, 6–9.
- Newey, W.K., West, K.D., 1987. A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55, 703.
- Ober, M., Katzenbeisser, S., Hamacher, K. 2013. Structure and anonymity of the bitcoin transaction graph. *Future internet*, 5(2), 237-250.
- Osterrieder, J., Lorenz, J., 2017. A statistical risk assessment of Bitcoin and its extreme tail behavior. *Ann. Financ. Econ.* 12, 1750003.
- Pesaran, H.H., Shin, Y., 1998. Generalized impulse response analysis in linear multivariate models. *Econ. Lett.* 58, 17–29.
- Polasik, M., Piotrowska, A. Wisniewski, T. P., Kotkowski, R., Lightfoot, G., 2015. Price fluctuations and the use of Bitcoin: An empirical inquiry. *International Journal of Electronic Commerce*, 20(1), 9–49.

- Rafiq, S., Salim, R., Bloch, H., 2009. Impact of crude oil price volatility on economic activities: An empirical investigation in the Thai economy. *Resour. Policy* 34, 121–132.
- Ruan, X., Zhang, J. E., 2018. Risk-neutral moments in the crude oil market. *Energy Economics*, 72, 583-600.
- Selgin, G., 2015. Synthetic commodity money. *Journal of Financial Stability*, 17, 92-99.
- Selmi, R., Mensi, W., Hammoudeh, S., Bouoiyour, J. (2018). Is Bitcoin a hedge, a safe haven or a diversifier for oil price movements? A comparison with gold. *Energy Economics*, 74, 787-801.
- Symitsi, E., Chalvatzis, K.J., 2018. Return, volatility and shock spillovers of Bitcoin with energy and technology companies. *Economics Letters*, 170, 127-130.
- Symitsi, E., Chalvatzis, K. J., 2019. The economic value of Bitcoin: A portfolio analysis of currencies, gold, oil and stocks. *Research in International Business and Finance*, 48, 97-110.
- Todorov, V., Tauchen, G., 2011. Volatility jumps. *Journal of Business and Economic Statistics*, 29(3), 356–371.
- Urquhart, A., 2016. The Inefficiency of Bitcoin. *Econ. Lett.* 150, 1–7.
- Vortelinos, D.I. and Thomakos, D.D., 2013. Nonparametric realized volatility estimation in the international equity markets. *International Review of Financial Analysis*, 28, 34-45.
- Yelowitz, A., Wilson, M. 2015. Characteristics of Bitcoin users: an analysis of Google search data. *Applied Economics Letters*, 22(13), 1030-1036.
- Zhang, L., Mykland, P.A., Aït-Sahalia, Y., 2005. A tale of two time scales: Determining integrated volatility with noisy high-frequency data. *J. Am. Stat. Assoc.* 100, 1394–1411.