# Forecasting spare parts demand using condition monitoring information

Nzita A. Lelo, P.Stephan Heyns and Johann Wannenburg.

Centre for Asset Integrity Management, Department of Mechanical and Aeronautical Engineering,

University of Pretoria, Lynnwood Road, Pretoria, South Africa

Email address (corresponding author): lelosud2013@gmail.com

## Abstract

**Purpose:** The control of an inventory where spare parts demand is infrequent has always been difficult to manage because of the randomness of the demand, as well as the existence of a large proportion of zero values in the demand pattern. The purpose of this paper is to propose a just-in-time (JIT) spare parts availability approach by integrating condition monitoring (CM) with spare parts management by means of proportional hazards models (PHM) to eliminate some of the shortcomings of the spare parts demand forecasting methods.

**Design/methodology/approach:** In order to obtain the event data (lifetime) and CM data (first natural frequency) required to build the PHM for the spares demand forecasting, a series of fatigue tests were conducted on a group of turbomachinery blades that were systematically fatigued on an electrodynamic shaker in the laboratory, through base excitation. The process of data generation in the numerical as well as experimental approaches comprised introducing an initial crack in each of the blades and subjecting the blades to base excitation on the shaker and then propagating the crack. The blade fatigue life was estimated from monitoring the first natural frequency of each blade while the crack was propagating. The numerical investigation was performed using the MSC.MARC/2016 software package.

**Findings:** After building the PHM using the data obtained during the fatigue tests, a blending of the PHM with economic considerations allowed determining the optimal risk level, which minimizes the cost. The optimal risk point was then used to estimate the JIT spare parts demand and define a component replacement policy. The outcome from the PHM and economical approach allowed proposing development of an integrated forecasting methodology based not only on failure information, but also on condition information.

**Research limitations/implications:** The research is simplified by not considering all the elements usually forming part of the spare parts management study, such as lead time, stock holding, etc. This is done to focus the attention on component replacement, so that a just-in-time spare parts availability approach can be implemented. Another feature of the work relates to the decision making using PHM. The approach adopted here does not consider the use of the transition probability matrix as addressed by Jardine and Makis (2013). Instead, a simulation method is used to determine the optimal risk point which minimizes the cost.

**Originality/value:** This paper presents a way to address some existing shortcomings of traditional spare parts demand forecasting methods, by introducing the PHM as a tool to forecast spare parts demand, not considering the previous demand as is the case for most of the traditional spare parts forecasting methods, but the condition of the parts in operation. In this paper, the blade bending first mode natural frequency is used as the covariate in the PHM in a laboratory experiment. The choice of natural frequency as covariate is justified by its relationship with structural stiffness (and hence damage), as well as being a global parameter that could be measured anywhere on the blade without affecting the results.

**Keywords -** Proportional Hazards Model, Condition Monitoring, Covariates, Optimal Risk Level.

**Paper type** - Technical paper.

## 1. Introduction

The management of physical assets has become a matter of central interest for the competitiveness of companies. One of the most important life-cycle phases in asset management is the operation and maintenance of the asset. An efficient maintenance program also assumes proper management of the spare parts inventory.

When managing an asset, it is critical to plan and control the spare parts inventory to avoid premature part replacement and overstocking of unnecessary spare parts (Yam, et al., 2001). That is why forecasting the demand of spare parts is important. In fact, forecasting is vital to every business organization and for every spare parts inventory, for it enables estimating the spare parts stock as accurately as possible. A better forecasting technique will most likely allow a more efficient spare parts management policy, as well as cost optimization.

Traditional forecasting methods applied for spare parts management, are inefficient for intermittent demand patterns and cannot accomplish reliable forecasting results under these circumstances. This includes methods such as the time series method, the Croston method and the exponential smoothing method.

Instead of using the classical methods to forecast spare parts demand, recent research proposes an integrated method that combines condition monitoring information with event data associated with the spare parts. The advantage related to this integrated method is the precision estimation of parts failure, and it also avoids downtime of machinery and stock-out. It detects potentially broken parts sufficiently early and allow a just-in-time maintenance and spare parts availability when managing a supply system (Hellingrath & Cordes, 2014).

The following approach is adopted in this research:

- An experimental investigation run in the laboratory consisted of estimating the lifetime and Paris law material constants. The setup was designed in such a way that an initiated crack in an axial fan blade was propagated and measurements were performed using digital image correlation (DIC). The stress intensity factor was calculated analytically, and the measured crack length was used to determine the Paris law constants. Furthermore, a statistical analysis was performed on the material constants and lifetimes that were determined. This study was done as a separate investigation by (Brits, 2016). The experiment served not only for validation of the finite element model (FEM) but also to determine the Paris material constants and lifetimes which served as event data to build the proportional hazards model (PHM).
- A numerical investigation was conducted which consisted of a modal analysis performed with MSC.MARC2015.0 nonlinear finite element software, to determine the coupling between natural frequency and mode shape for a 30-degree axial fan blade. A 2mm crack was initiated in the blade, then propagated to failure. Information such as natural frequencies and mode shapes were recorded as the crack propagated into the axial flow fan blade. For the purposes of this paper only the natural frequency was considered as a covariate to build the PHM.
- Both the natural frequencies and the lifetimes recorded served as covariates and event data respectively in the PHM.
- In this research a simulation procedure was followed to determine the cost function.
- Optimal decision making was performed by means of the cost function built and based on the PHM. An optimal risk point ‘d’ was set up and served as input to define a spare parts demand policy. It is important to emphasize that this paper does not address aspects of spare parts management which deal with the lead time, stock holding etc. The focus here is to provide the inventory manager with information required to correctly estimate the demand of the component at the right time.

When the process described above is properly performed, it results in reduction of the overestimation of spare parts demand, compared to the traditional forecasting methods and a just in time spare parts management and maintenance policy is established.

The core contribution of this paper is therefore to propose an alternative forecasting method for demand, based on condition-based maintenance instead of using traditional methods (time based). The value of this approach lies in the fact that it addresses a fundamental limitation of traditional methods, by employing condition monitoring information that track the progressive advance of failure in the component and using this information to enhance forecasting. This is done by introducing the PHM as a tool to forecast spare parts demand without considering the previous demand as is the case for most of the traditional spare parts forecasting methods, and then inferring the likely time of failure of the component during operation by means of continuous condition monitoring. In this way the paper offers the decision maker a just-in-time replacement tool for spare parts. The paper is organized as follows: Section 2 describes the PHM, its construction and estimation of the parameters needed, and its assessment by means of the Kolmogorov-Smirnov test (K-S test) to establish how well it fits the data. Section 3 is a case study and Section 4 discusses the results obtained by means of the PHM and the associated economical approach. Section 5 concludes the paper while section 6 are recommendation for future research.

## 2. Modelling

### 2.1 Proportional hazards model

The PHM is a statistical procedure used for estimating the risk of failure for a component when it is under condition monitoring (Jardine&Tsang, 2013). PHM is one of the popular statistical models used for survival analysis. Its forms part of a broader class of survival analysis which provides information on the duration of time between the identifiable start and the occurrence of an event (Leclerre, 2005).

The PHM assumes that a component’s hazard rate is the product of two specific functions, $h_0(t)$ the unspecified baseline that describes how the hazard rate changes as function of time, whereas the other function which is the exponentiated set of covariates and coefficient describes how the hazard rate changes as a function of a component covariates (Leclerre, 2005).

The PHM with a Weibull baseline hazard function is presented in the following formula:

$$h[t, Z(t)] = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1} \exp\{\sum_{i=1}^{m} \gamma_i Z_i(t)\} \quad (1)$$

where $h[t, Z(t)]$ is the hazard function, $Z_i(t)$ are the covariates at time t, $\frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}$ is the baseline hazard function with $\beta$ the shape parameter and $\eta$ the scale parameters. The Weibull parameters which allow the construction of the baseline part of the model are determined by maximizing the likelihood function.

## 2.2 Maximum likelihood for parameter estimates

The Weibull PHM parameters are unknown and need to be estimated from the data obtained from previous failures and covariates. The likelihood of the parameters of the PHM given the data, is maximised using optimisation techniques to ensure that the model describes the data well. The process (Kalbfleisch, 2002) is commonly referred to as maximum likelihood estimation. The likelihood function is a mathematical expression which describes the joint probability of obtaining the data observed on the subjects in the study as a function of the unknown parameters (the $\gamma's$) in the model being considered (Kleinbaum, 1999). (Vlok, 1999) addressed the maximization of the likelihood equation to determine the Weibull parameters.

The Weibull parameters $\beta$ and $\gamma$ are estimated by maximizing the likelihood equation given by:

$$L(\beta, \eta, \bar{\gamma}) = \prod_i h(T_i, \overline{Z_i\,(T_i)}) \times \prod_j R(T_j, \overline{Z_j(t))} \quad (2)$$

with the $i$ indices referring to failure times and $j$ indices to failure and suspension times. For this work we consider complete data without suspension.

# 3. Case Study

This paper addresses the numerical and experimental investigation carried out to make the event and condition monitoring (CM) data needed to build the PHM available. The case study focuses on seven 30-degree axial flow fan blades.

The reason for considering the turbomachine blade failure case in this study, was simply to capitalise on the numerical models and experimental results that were already available from a prior study conducted by Brits (2016). In his work Brits worked on estimating the fatigue crack life (FCL) of turbomachine blades by means of a set of fatigue tests in the laboratory. As part of this study Brits conducted extensive numerical investigations and a comprehensive experimental study. Because of the dearth of results of this nature in the open literature, these results were used for the current investigation.

However, unlike the work by Brits where the main goal was to estimate the fatigue crack life of the fan blades, here the same blades were considered with a focus on updating the finite element model to get the natural frequencies corresponding to the FCL. Then both the FCL and natural frequencies obtained were used as inputs to build a PHM prognostic model. The choice of natural frequency as covariate follows from it being relatively easy to measure, compared to the actual crack size, which is difficult to directly measure in practice. A numerical investigation based on models generated by Brits, allowed calculation of the natural frequencies related to the crack propagation.

It is important to note that blade lifetime was not obtained from the finite element model (FEM). Only the stress intensity factors were used as inputs to the Paris Law model and a modal analysis was run by means of MSC.MARC/MENTAT 2016.

The experimental investigation was conducted in the C-AIM Laboratories at the University of Pretoria and entailed the use of a 50 kN Spectral Dynamics electrodynamics shaker to apply base excitation to the axial fan blade specimens. The fatigue lifetime recorded from the experimental approach served as event data required to build the PHM. After having obtained the outcomes from both numerical and experimental investigations, the PHM could be constructed from both types of data, generated through numerical and experimental investigations. (Figures 1 and 2).

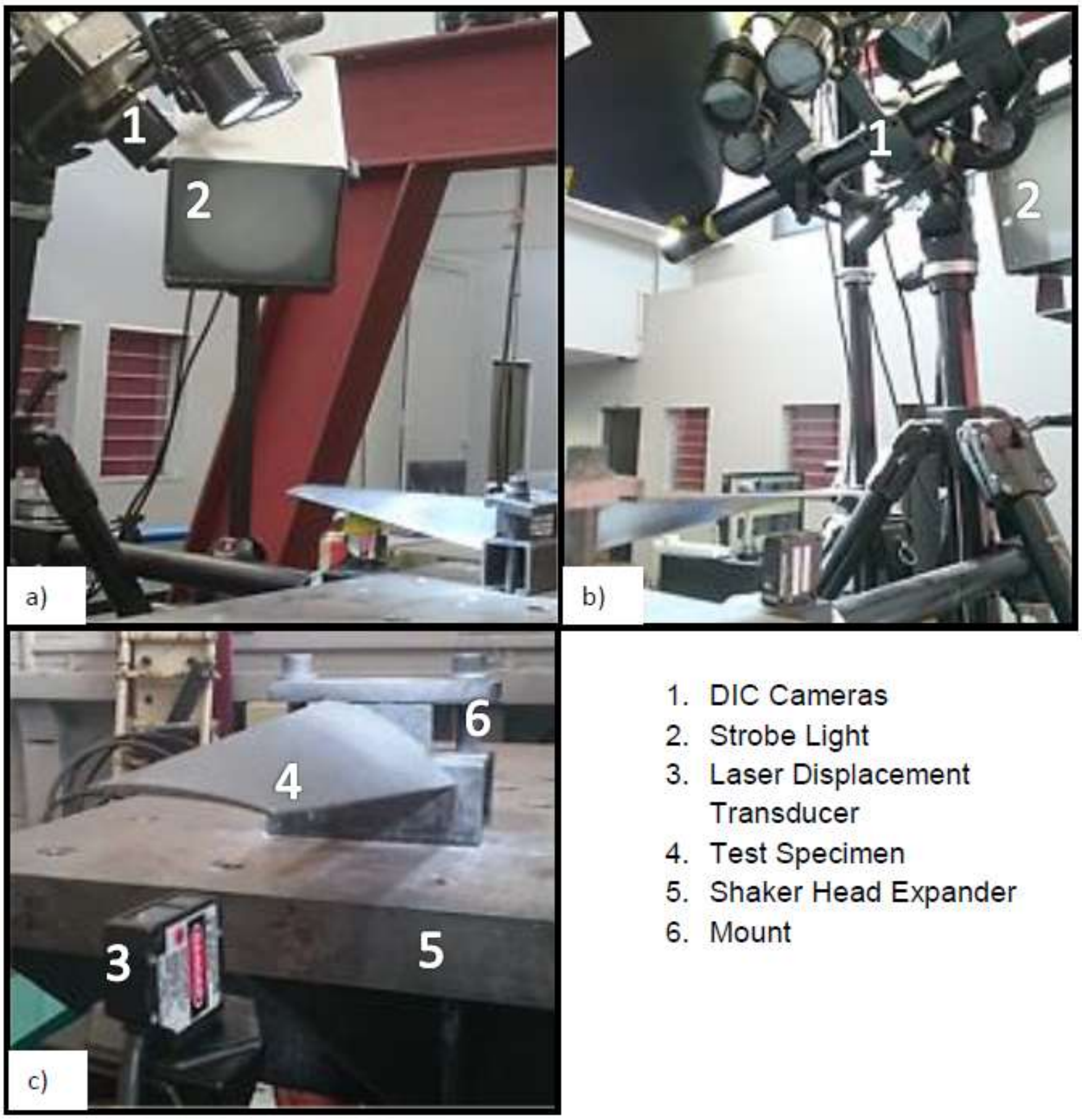


**Figure 1:** Experimental setup showing (a) Right side view, (b) Left side view, (c) The mounted test specimen. (Brits, 2016).

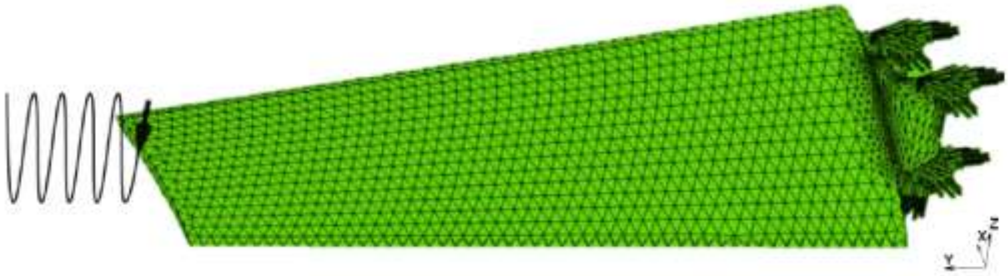

**Figure 2:** Finite Element Model of a 30-degree blade with boundary conditions (Brits, 2016)

Table 1 below records the material properties for the finite element model

**Table 1:** Material properties chosen for FE model

| Structural Property | Values |
|---|---|
| Elasticity Modulus E | 69 GPa |
| Tensile Strength | 220 MPa |
| Yield Stress | 165 MPa |
| Density | 2830 kg/m$^3$ |
| Poisson Ratio | 0.33 |

## 3.1 Computation

To determine the Weibull parameters $\beta, \eta, \gamma$ needed to construct the PHM, the likelihood equation (2) is solved numerically using the Newton-Raphson method which is the objective function that has been solved with the MATLAB script `fmincon`. `fmincon` is a nonlinear programming solver which allows finding the minimum of a constrained nonlinear multivariable function.

In this work the objective function was minimized using the syntax: $x = fmincon(fun, x_0, A, b, Aeq, beq, lb, ub)$. The results from the simulation give: (1) $\beta = 1.0012$ ; (2) $\eta = 7.10e + 05$; (3) $\gamma = 0.0293$.

The PHM construction obtained from the maximum likelihood output is:

$$h\big(t, z(t)\big) = \frac{1.0012}{7.1004e + 05} \left(\frac{t}{7.1004e + 05}\right)^{(1.0012-1)} \exp[0.0293 \times z(t)]$$

The K-S test was performed on the residual of the data for a 30- degree blade in the software R to evaluate how well the PHM fit the data, the output results obtained from R was:

$$D = 0.49659 \,, p - \text{value} = 0.06873$$

The above result shows that at 5 percent level of significance the null hypothesis is accepted for $D$ is less than $D_\alpha$ which is 0.565, and the $p -$ value being greater than 0.05, the null hypothesis is accepted which means that the PHM fit well the data.

## **3.2** Decision making with the PHM and its application to the spare parts

The PHM provides us with an approximation of the risk of failure for the blade based on the age and the blade natural frequency. The information from the PHM can be utilized to obtain economic benefits.

Vlok (1999) stated that economic benefits from a statistical failure analysis can be established with a high level of confidence if the minimum long-term life cycle cost (LCC) of the component is determined and pursued.

### 3.2.1 Long term life cycle cost (LCC) concept

In renewal analysis the LCC arise from two important quantities in practice:

- The cost of unexpected renewal (failure cost $C_f$)
- The cost of preventive replacement ($C_p$)

Equilibrium must be obtained between the risk of having to spend $C_f$ and the advantages in the cost difference between $C_f$ and $C_p$ without wasting useful life of a component.

Makis and Jardine (2013) made a model available for optimal decision making with PHM. The model specifies the optimal renewal policy in terms of an optimal hazard, leading to the minimum LCC. To be able to determine the hazard rate which leads to the minimum LCC, it is needed to predict the behaviour of covariates.

The Makis and Jardine (2013) model assumes the covariate behaviour to be stochastic and approximating it by a non-homogeneous Markov Chain in a finite space.

Referring to that model, the expected average cost per unit time is a function of the threshold risk level d and is given by

$$\emptyset(d) = \frac{C_p + KQ(d)}{W(d)} \qquad (3)$$

where, $Q(d) = P(T_d \geq T)$ represents the probability that failure replacement will occur and $W(d)$ the expected time until replacement and $K = C_f - C_p$.

Jardine et al. (1997) state that the calculation of the functions $Q(d)$ and $W(d)$, can sometimes be time consuming because of the number and structure of the covariates. Sometimes a simulation procedure could be used to determine the cost function (Jardine, 1997). In this paper such a simulation procedure is used to determine the cost function and the optimal risk point which minimizes the risk.

The next steps deal with the results and discussion of the simulation procedure used to blend the PHM with the economics. Furthermore, the application of the spare parts demand decision is also addressed.

### 3.2.2 The use of PHM to forecast spare parts demands

The diagram in Figure 3 expresses the use of PHM to forecast spare parts demand.

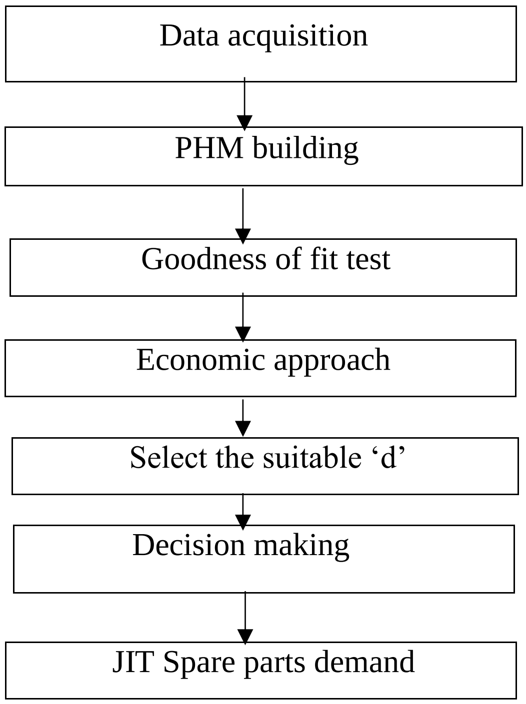


**Figure 3** PHM process to forecast spare parts demand

The process outlined in figure 3 can briefly be described as follows:

Step 1: Acquisition of Data (Event and Condition monitoring data).

Step 2: Building the PHM with the outcome from the maximum likelihood function.

Step 3: Perform the goodness of fit testing to assess how well the PHM fits the data, the Kolmogorov Smirnov is the statistical test used in this paper.

Step 4: Perform the blending of the PHM and economic considerations at this level. The outcome from this step is the optimal risk point that minimizes the cost during the simulation procedure d.

Step 5: The selected d point gives the critical number of loading cycle corresponding to each component.

Step 6: The information obtained from the previous step is used to make decisions about the right time to make the component replacement.

Step 7: The replacement is performed according to the critical point pre-defined, which means there is no need of stocking too much spares because the right time for replacement is known, means JIT (just-in-time) spare parts demand.

The integrated forecasting method being proposed in the flowchart above, the implementation of the given method in a case study brought to the results shown in the followings sections.

# 4. Results

## 4.1 Plotting of the resulting proportional hazard values for the seven experimental blades

The risk $h\big(t, z(t)\big)$ is an instantaneous conditional probability of failure for the blade at time $t$, given the value $Z(t)$. Figure 4 below indicates the plotting of risk versus the number of loading cycles as shown in the appendix from table 1 to table 7. For the obtained PHM in figure 4, risk curves are classified from blade 1 having a smallest failing load cycle to blade 7 which failed to the highest loading cycles:

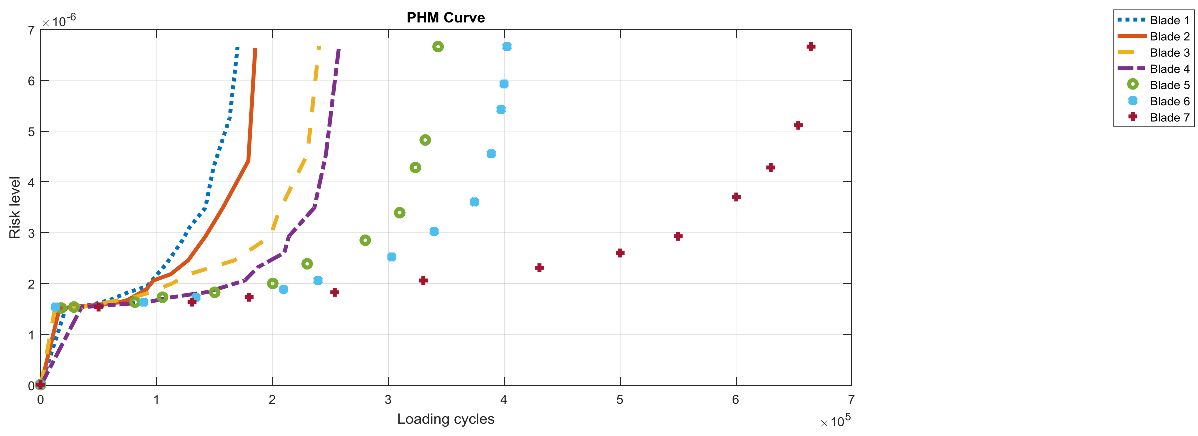


**Figure 4:** Risk versus loading cycles for all the blades.

After determining the PHM and plotting it as shown in Figure 4 for all the fan blades, the following step is to establish optimal replacement decisions, including economics considerations, based on the PHM values, where these values would be a function of both age (loading cycles) of the components as well as the natural frequency which is the condition parameter Z(t). This basically entails finding an optimum PHM risk value at which components would be replaced. It is normally expected that such optimisation would be achieved by balancing the risk of expensive failures (when the replacement PHM value is chosen at a too high a level and some components may fail before reaching this value) and wasting remaining useful life of components (when the replacement PHM value is chosen at a low level).

The trends of the PHM shown in Figure 3, however, show that, in this case, the optimisation problem seems to be trivial and that the optimal replacement PHM level would be at an almost constant PHM failure value (with some small safety factor) and that this result seems to be independent of age. With such a PHM trend, the application of PHM for spare parts demand becomes fruitless because time does not have an influence (see figure 4) and the forecast could be performed directly with the trend of the covariate.

The reason for this result is that, in this case, the covariate (natural frequency), is an accurate measure of crack size and that the critical crack size (at which final failure occurs) for all the blades were almost constant. The PHM approach is intended for situations where the covariates give some indication of approaching failure and, when combined with age, gives a good indication of risk of failure. When the covariate an accurate determinant of failure, irrespective of age, the PHM mathematics reduces the influence of the age (time) variable in the PHM equation, such that failure is predicted at an almost constant PHM value. Failure can therefore be predicted accurately based on the covariate alone (as it approaches the constant critical value), making the use of the PHM formulation unnecessary. This in turn means that the optimisation implied by choosing a level of risk (PHM value) for making a replacement decision, balancing the risk of catastrophic failure (if the risk level is chosen too high), with wasting useful life (if the risk level is chosen too low), is trivial. The chosen case study therefore, paradoxically, resulted in too accurate life prediction results, to be able to demonstrate the use of the PHM approach for economic optimisation.

To be able to demonstrate the ability to forecast spare parts demand with the PHM approach, it was decided to introduce noise in the PHM (to randomize the PHM) in such way that both age and covariate have an influence on the PHM. In practical applications (using field data, rather than laboratory data), this would be more representative. The following section illustrates and discusses the result of this approach.

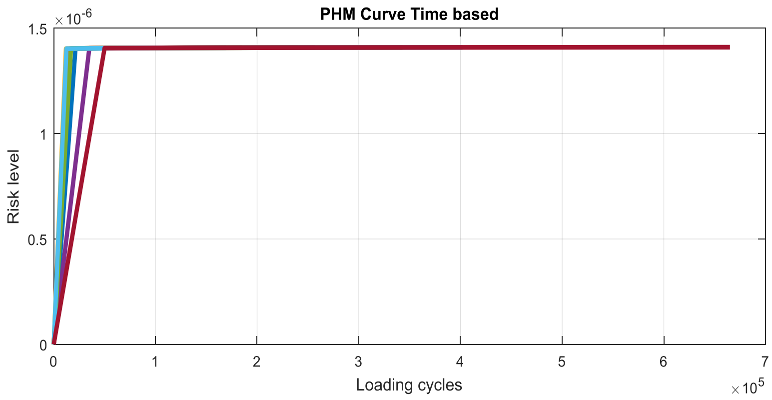


**Figure 5:** Hazard curve, without considering the covariate (Only baseline of the PHM).

## 4.2 Randomising the failure level

As the final purpose of this work is to use the PHM as a tool to make optimal decisions for fan blade replacement when managing spare parts, it is at least important to present a scenario which better approaches a practical scenario. Significant differences in the failure time of the blades are required and therefore we randomised the failure levels (critical crack sizes). This was done in a similar way than with the introduction of noise on the covariate (using the typical failure level as a mean and introducing a variance of a percentage of this mean. For each blade, the failure level is then randomly sampled from the arising normal distribution.

The result with 10% randomisation is illustrated by figure 6 below.

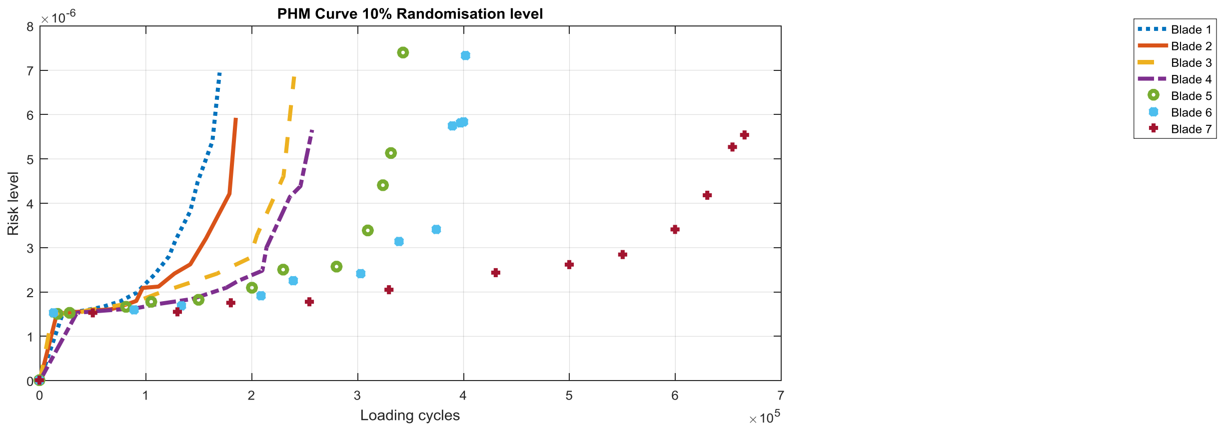


**Figure 6:** PHM at 10% failure level randomisation

Compared to Figure 3 and Figure 4, Figure 5 represents a situation where the PHM is no longer based on the covariate only with time being constant. In contrast it shows a situation where there is an influence of time and the covariate. This situation requires the economic approach to determine the optimal risk point because the blades are failing at different risk levels. If we set the replacement risk level (d) at $4 \times 10^{-6}$ all blades will be replaced before failure, but there will be some blades with significant remaining life. If we set the risk point at $6.5 \times 10^{-6}$ blade 2, blade 4 and blade 7 will fail before reaching the risk point that has been set, which will be expensive. The purpose is then to find an optimal choice for d.

### 4.2.1. Optimal decision policy with PHM using simulation procedure (with randomised failure data).

Jardine and Tsang (2013) successfully addressed optimal decision making with PHM. To build the cost function, the observed that the determination of the risk value which will lead to an optimal cost requires the prediction of the covariate behaviour. Their model is constructed based on the hypothesis that the covariate behaviour is stochastic and approximating a non-homogeneous Markov chain in a finite state space. The covariate behaviour was demonstrated using a Transition Probability Matrix (TPM), however, for the purpose of this research the cost function is built by means of simulation methods. The use of simulation approach in this paper is justify by the fact that it is less complex than the use of TPM where the covariate behaviour is investigated. Figure 7 below is the outcome from the simulation method. Figure 7: Cost per unit time versus risk point for 10% noise level.

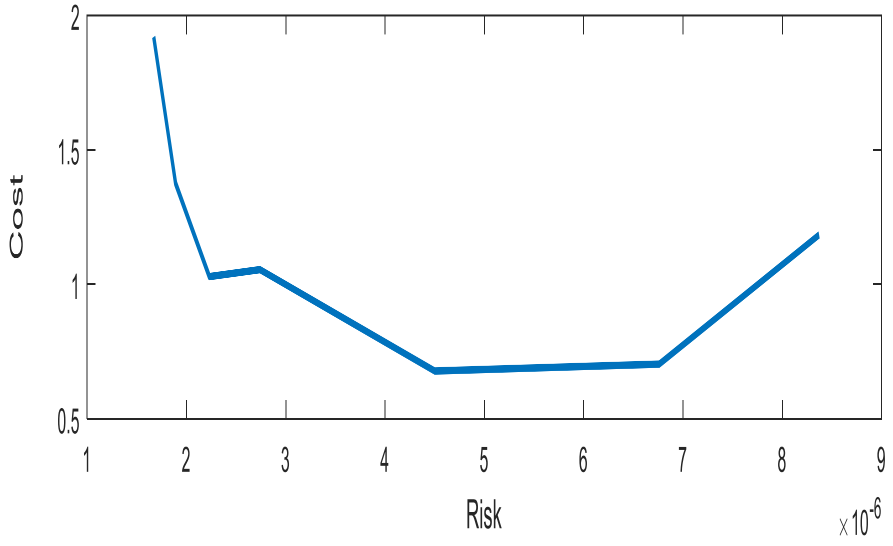


**Figure 7:** Cost per unit time versus risk point for 10% noise level.

Figure 7 above shows that the optimum risk level for replacement can be obtained between $4.5 \times 10^{-6}$ and $6.76 \times 10^{-6}$ because in that region the values of the optimal cost are almost the same. However, for less wasted life $6.76 \times 10^{-6}$ is better.

For the sample of data that was treated for 10% level of randomisation, the optimal risk point which minimizes the cost per unit time was found to be at $6.76 \times 10^{-6}$. Referring to the set risk point in terms of each blade, the following results are obtained:

- For blade 1 the optimal risk level corresponds to the crack length varying between 50 to 58.332 mm crack length and 165650 loading cycles. This implies less wasted life. The decision maker could adjust the replacement of the blade accordingly, then the management of the spare parts can be done efficiently.
- For blade 2, the optimal risk level corresponds to the crack length varying between 42.6 and 54.168 mm, 184620 loading cycles, therefore less wasted life.
- For blade 3 it failed before reaching the optimal risk level.
- For blade 4 it failed before reaching the optimal risk level.
- For blade 5 the optimal risk level corresponds to the crack length varying between 60 to 64.5 mm, 336870 loading cycles.
- For blade 6 it failed before reaching the optimal risk.

For blade 7 the optimal risk level corresponds to the crack length between 55 to 60 mm, 660680 loading cycles.

These results illustrate the replacement policy for each of the blades taken individually the decision maker managing the demand of the blades, can use these results and optimize the spare parts (blades) demand. As soon as the crack size which is linked to the natural frequency, or the number of cycles reach the mentioned value for each of the blades it will be known that replacement should be performed which is related to the demand of the blades.

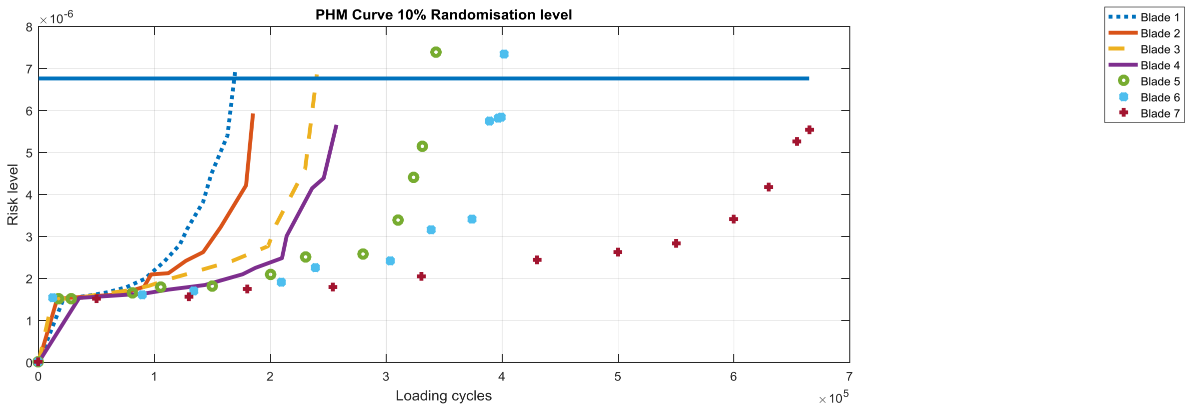


**Figure 8** presents the PHM curves with a cutting line at $6.76 \times 10^{-6}$ optimal risk value.

## 5. Conclusion

This research originated from an inventory management challenge, where the demand for spare parts is infrequent. The randomness of the demand when managing the inventory makes it difficult to forecast spare parts. Several forecasting methods have been developed over the years aiming to address the challenge. One of the more efficient traditional forecasting methods that tries to address the challenge is the Croston method. However, despite its performance, it does not consider the condition of the component to be replaced which is inefficient.

The purpose of this research was to develop an alternative forecasting method to the traditional method. To reach this goal, a PHM approach was suggested that integrates condition-based maintenance with to the spare parts forecasting method, so that the condition of the component is also considered for the demand. A PHM was used with condition monitoring data to calculate the risk of failure for the component under monitoring. The added value of this new method is that it tracks the failure arrival and makes the forecasting more accurate because of the condition of the component which is well known but also it is suitable for critical component where there is not enough historical data to forecast.

To demonstrate the expected solution from the PHM and to be able to determine the optimal risk point used to forecast the spare parts, an investigation was performed to calculate the demand for 30-degree fan axial blades from Fatigue Crack Life (FCL) data. Fatigue tests on fan blades resulted in cracks to develop and to grow until the blades failed. FCL data, consisting of crack length over the number of loading cycles, were acquired during the tests. In this work, a finite element model is used to estimate the natural frequencies of the blades over crack length and time from the FCL data. Both FCL data and natural frequency data served as inputs in a PHM to predict the failure arrival which is essential for forecasting spare parts.

The procedure used and the benefits of using it, are summarized as follows:

- We estimated the parameters needed to construct the PHM by means of maximizing the likelihood function. The maximization was performed with the Newton-Raphson method.
- To test how well the PHM fits the data, the K-S test was used with a 5% level of significance. The $D$-statistic and the $p$ −value obtained with the R package confirmed that the PHM fits the data well.
- The economic approach was investigated because the outcome from the PHM would be useless when applied without the context of economic considerations. A blending of the PHM with the economics allows us to determine the optimal risk level which minimizes the cost. The optimal risk point found was the main tool to define a spare parts management policy.
- The benefit of the proposed forecasting method is that it gives the ability to proactively generate information which will allow a just-in-time supply of spare parts. This implies that a component can be replaced without wasting useful life because the component replacement is no longer time-based only, but also condition-based.

## 6. Recommendations

From the observations and experiences obtained during this dissertation, the following recommendations are made for future investigation:

- As the spare parts approach in this paper was oriented to a single component replacement, it is required to extend the application to more than one component because most industrial machinery have more than one critical component.

Parameters such as lead time, stock holding, and cost related needs to be considered as well.

- Compare the PHM outcome with other regression models which also consider the condition of the component. An example is the Prentice William Peterson model (PWP) model which has additional benefits to the PHM because it considers also the previous replacement of the item under analysis as well. Vlok, (2006) briefly presented the benefits of this model in his work.
- Investigate on the influence of increasing the noise in the covariate and evaluate its impact on the three Weibull parameters and give physical meaning related to that. Because we assume that the noisy data are closer to the real situation than the experimental data which can be submitted to some constraint due to the measurement condition.

# Appendix

Tables of the resulting proportional hazard values for the seven experimental blades.

**Table A1:** Risk versus loading cycles for blade 1

| | N(cycles) | PHM (Risk) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 21400 | 1.51E-06 |
| 3 | 43200 | 1.58E-06 |
| 4 | 56400 | 1.68E-06 |
| 5 | 75700 | 1.83E-06 |
| 6 | 91200 | 1.94E-06 |
| 7 | 108200 | 2.38E-06 |
| 8 | 121900 | 2,84E-06 |
| 9 | 128200 | 3.10E-06 |
| 10 | 141900 | 3.49E-06 |
| 11 | 148900 | 4.29E-06 |
| 12 | 162890 | 5.26E-06 |
| 13 | 169700 | 6.65E-06 |

**Table A2:** Risk versus loading cycle for blade 2

| | N(cycles) | PHM(Risk) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 16000 | 1.52E-06 |
| 3 | 39000 | 1.56E-06 |
| 4 | 67000 | 1.63E-06 |
| 5 | 75000 | 1.68E-06 |
| 6 | 91000 | 1.89E-06 |
| 7 | 97000 | 2.06E-06 |
| 8 | 112000 | 2.18E-06 |
| 9 | 127000 | 2.46E-06 |
| 10 | 142000 | 2.93E-06 |
| 11 | 157000 | 3.49E-06 |
| 12 | 179000 | 4.41E-06 |
| 13 | 185000 | 6.63E-06 |

**Table A3:** Risk versus loading cycle for blade 3

| | N(cycles) | PHM (Risk) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 12000 | 1.50E-06 |
| 3 | 35000 | 1.53E-06 |
| 4 | 45000 | 1.58E-06 |
| 5 | 80000 | 1.73E-06 |
| 6 | 95000 | 1.83E-06 |
| 7 | 113000 | 2.00E-06 |
| 8 | 128000 | 2.18E-06 |
| 9 | 167000 | 2.46E-06 |
| 10 | 198000 | 2.93E-06 |
| 11 | 205000 | 3.39E-06 |
| 12 | 230000 | 4.55E-06 |
| 13 | 240000 | 6.67E-06 |

**Table A4**: Risk versus loading cycle for blade 4

| | N(cycles) | PHM (Risk) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 35000 | 1.53E-06 |
| 3 | 91000 | 1.63E-06 |
| 4 | 112000 | 1.73E-06 |
| 5 | 143500 | 1.83E-06 |
| 6 | 176000 | 2.06E-06 |
| 7 | 187000 | 2.32E-06 |
| 8 | 210000 | 2.61E-06 |
| 9 | 214000 | 2.93E-06 |
| 10 | 236000 | 3.49E-06 |
| 11 | 246000 | 4.55E-06 |
| 12 | 257000 | 6.62E-06 |

**Table A5:** Risk versus loading cycle for blade 5

| | N(cycles) | PHM(Risk) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 17000 | 1.51E-06 |
| 3 | 28500 | 1.53E-06 |
| 4 | 81000 | 1.63E-06 |
| 5 | 105000 | 1.73E-06 |
| 6 | 150000 | 1.83E-06 |
| 7 | 200000 | 2.00E-06 |
| 8 | 230000 | 2.39E-06 |
| 9 | 280000 | 2.85E-06 |
| 10 | 310000 | 3.39E-06 |
| 11 | 323520 | 4.29E-06 |
| 12 | 331500 | 4.82E-06 |
| 13 | 3433000 | 6.66E-06 |

**Table A6:** Risk versus loading cycle for blade 6

| | N(cycles) | PHM(Risk) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 12500 | 1.53E-06 |
| 3 | 89000 | 1.63E-06 |
| 4 | 134000 | 1.73E-06 |
| 5 | 209000 | 1.89E-06 |
| 6 | 239000 | 2.06E-06 |
| 7 | 303000 | 2.53E-06 |
| 8 | 339000 | 3.02E-06 |
| 9 | 374000 | 3.60E-06 |
| 10 | 389000 | 4.55E-06 |
| 11 | 397000 | 5.42E-06 |
| 12 | 399500 | 5.92E-06 |
| 13 | 402000 | 6.67E-06 |

**Table A7**: Risk versus loading cycle for blade 7

| | N(cycles) | PHM (Risk) |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 50000 | 1.54E-06 |
| 3 | 130000 | 1.63E-06 |
| 4 | 180000 | 1.73E-06 |
| 5 | 254000 | 1.83E-06 |
| 6 | 330000 | 2.06E-06 |
| 7 | 430000 | 2.32E-06 |
| 8 | 500000 | 2.61E-06 |
| 9 | 550000 | 2.93E-06 |
| 10 | 600000 | 3.31E-06 |
| 11 | 630000 | 4.29E-06 |
| 12 | 654000 | 5.12E-06 |
| 13 | 665000 | 6.65E-06 |

These tables represent the risk of failing for all the blades and the corresponding time expressed in term of number of loading cycle.

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