

Chaos in G7 Stock Markets using Over One Century of Data: A Note[#]

Aviral Kumar Tiwari^{*} and Rangan Gupta^{**}

Abstract

In our study, we tested for chaos in the historical daily and monthly datasets spanning over one century of stock returns for G7 countries. Applying the 0–1 test proposed by Gottwald and Melbourne (2005) and the recent test developed by BenSaïda and Litimi (2013), which is powerful in detecting chaotic dynamics, we found that (a) it is better to denoise the data before testing for chaos and (b), in general, chaos is observed for all countries, using both tests, when we denoised the data.

Keywords: Chaos, G7 countries, stock returns.

JEL Codes: C12, C45.

1. Introduction

Numerous studies have provided the evidence that stock returns do not follow random walks as they are characterized by nonlinearities and deterministic chaos, *i.e.*, stock returns follow erratic behavior (detailed literature reviews in this regard is provided in BenSaïda and Litimi 2013; BenSaïda 2014, 2015; Urquhart and Hudson 2013). Theoretically, a chaotic system is a random-looking nonlinear deterministic process with irregular periodicity and sensitivity to initial conditions.

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^{*} Corresponding author. Rajagiri Centre for Business Studies, Rajagiri Valley Campus, Kochi, India; Center for Energy and Sustainable Development (CESD), Montpellier Business School, Montpellier, France, Email: aviral.eco@gmail.com.

^{**} Department of Economics, University of Pretoria, Pretoria, 0002, South Africa. Email: rangan.gupta@up.ac.za.

Against this backdrop, the aim of this study is to investigate the presence of chaos using historical in monthly stock returns data for Canada (1915M02 to 2016M08), France (1898M01 to 2016M08), Germany (1870M01 to 2016M08), Italy (1905M02 to 2016M08), Japan (1914M08 to 2016M08), UK (1693M02 to 2016M08), and USA (1791M09 to 2016M08). To achieve our objective, we employ two tests to detect the presence of chaos. First, we use the 0-1 test proposed by Gottwald and Melbourne (2005) and second, a recent powerful test developed by BenSaïda and Litimi (2013). Further, as documented in Gottwald and Melbourne (2005) and BenSaïda and Litimi (2013), traditional tests of chaos are sensitive to presence/absence of noise in the data (as any measurement error causes systematic rejection of chaos), and currently available tests in the literature are based on the assumption that the data used is sans of such noises, i.e., noise-free.

In general, it is very difficult to measure the level and strength of noise in the data. Therefore, we proceed using several methods to denoise the data and then the denoised data series obtained from each method are evaluated based on certain criteria. Finally, the denoised series obtained from best method is again tested for chaos using the two test under consideration. To the best of our knowledge, this is the first study to use long time series data spanning for over a century for developed stock markets, based on methods that are robust in detecting chaos in the presence or absence of noise.

The remainder of the paper is organized as follows: in Section 2, we briefly discuss the various methodologies, while Section 3 presents the data and the results. Finally, Section 4 concludes the paper.

2. Methodology

In this study, six methods are used to denoise the data, namely, Hard, Mid, Soft, Moving Average method, Savitzky-Golay filter method and the Empirical mode decomposition de-trended fluctuation analysis (EMD-FDA). The first three methods of denosing i.e., Hard, Mid, and Soft, are based on Nonlinear denoising via the wavelet shrinkage method. In the Moving Average method, data has been smoothed by taking the median over a 25-element sliding window. The Savitzky-Golay filter smooth the data according to a quadratic polynomial that is fitted over each window of a returns series. This method is said to be more effective than other methods when the data varies rapidly. Finally, five methods are used to evaluate the performance of the denoised series. The used five methods are: mean squared error, mean absolute error, signal to noise ratio (DB), peak signal to noise ratio (DB) and cross correlation. The signal to noise ratio is defined as the ratio of signal power to the noise power, often expressed in decibels. A ratio higher than 1:1 (greater than 0 dB) indicates more signal than noise. The peak signal to noise ratio is the ratio between the maximum possible power of a signal and the power of corrupting the noise that affects the fidelity of its representation. Finally, once we identify the best denoising method we apply the 0-1 test of Gottwald and Melbourne (2005), and the artificial neural network based test of BenSaïda and Litimi (2013). In the following sub-section section, we provide only a brief description of these two approaches.

2.1. The Lyapunov exponent

In a chaotic system, if an infinitesimal change, $\delta x(0)$, appears in the initial conditions, the corresponding change iterated through the system will grow exponentially with time t . Technically, the largest Lyapunov exponent is widely-used,

and considered to be the only test explicitly devised for testing chaos, and measures the rate at which information is lost from a system (Mishra et al., 2011). A process shows chaotic behavior if the maximum Lyapunov exponent λ_{max} is positive (Mohammadi and Pouyanfar, 2011).

Given an infinitesimally small hypersphere of radius ϵ , the maximum Lyapunov exponent is measured by the extent of the deformation as follows (Wolf et al., 1985):

$$\lambda_i = \lim_{t \rightarrow \infty} \lim_{\epsilon(0) \rightarrow 0} \left\{ \frac{1}{T} \log_2 \left[\frac{\epsilon_i(t)}{\epsilon_i(0)} \right] \right\} \quad (1)$$

where: $\epsilon_i(t)$ represents the length of the i^{th} principal axis of the ellipsoid at time t .

The literature on chaos has considered several developments of the estimation of the Lyapunov exponent, based on both neural and non-neural networks. Briefly, Wolf et al. (1985) first proposed the idea of the dominant Lyapunov exponent which was advanced by Nychka et al. (1992). Nychka et al. (1992)'s proposed approach of dominant Lyapunov exponent is more powerful over the approach of Wolf et al. (1985) as it allows for the identification of chaotic dynamics in short noisy systems. Further, Lai and Chen (1998) and afterwards BenSaïda and Litimi (2013) adopt a Jacobian approach to estimate λ , with this new approach being called the Lyapunov exponent with minimum root mean square error (RMSE) neural network.

2.2. The 0–1 test for chaos

The 0–1 test for chaos proposed by Gottwald and Melbourne (2005) is based on an Euclidean extension, instead on a phase space reconstruction as the Lyapunov exponent. It tests for the presence of chaos in a deterministic dynamical system $\{x_t\}$, by studying the asymptotic behavior of the translation variables $p_c(n) = \sum_{j=0}^{n-1} \cos(jc) g(x_j)$ and $q_c(n) = \sum_{j=0}^{n-1} \sin(jc) g(x_j)$, with $n \in \mathbb{N}$ and $c \in (0, \pi)$

representing an arbitrary but fixed frequency. Both $p_c(n)$ and $q_c(n)$ remain bounded as $n \rightarrow \infty$ if the system does not exhibit chaos.

Gottwald and Melbourne (2005)'s test statistic is represented by the asymptotic Bravais–Pearson correlation coefficient between $n = (1, 2, \dots, n)^T$ and $\Delta_n = (D_c(1), \dots, D_c(n))^T$, with D_c being a modified mean square displacement:

$$K_c = \lim_{n \rightarrow \infty} \frac{\mathbf{n}^T \Delta_n - \frac{1}{n} \mathbf{1}_n^T \mathbf{n} \mathbf{1}_n^T \Delta_n}{\sqrt{\left[n^T n - \frac{1}{n} (\mathbf{1}_n^T n)^2 \right] \left[\Delta_n^T \Delta_n - \frac{1}{n} (\mathbf{1}_n^T \Delta_n)^2 \right]}} \quad (2)$$

where: $\mathbf{1}_n = (1, 1, \dots, 1)^T \in \mathbb{R}^n$.

2.3. Lyapunov exponent with minimum RMSE neural network¹

BenSaïda and Litimi (2013) adopted the Jacobian based approach to estimate λ , since the direct approach based on the Lyapunov exponent bounds the dynamics of the chaotic map, and is inefficient in presence of measurement noise. Briefly, the estimated Lyapunov exponent is

$$\hat{\lambda} = \frac{1}{2M} \ln(v_1) \quad (3)$$

where M is an arbitrary integer number of evaluation points $M \cong T^{2/3}$, and v_1 stands for the largest eigenvalue of the matrix $(T_M U_0)'(T_M U_0)$, such that $U_0 = (1, 0, \dots, 0)'$, and $T_M = \prod_{t=1}^{M-1} J_{M-t}$ the product of the Jacobian matrices J_t .

$$J_t = \begin{pmatrix} \frac{\partial f}{\partial x_{t-L}} & \frac{\partial f}{\partial x_{t-2L}} & \cdot & \cdot & \cdot & \frac{\partial f}{\partial x_{t-mL+L}} & \frac{\partial f}{\partial x_{t-mL}} \\ 1 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 1 & 0 \end{pmatrix}$$

¹More detail about the method can be found in BenSaïda and Litimi (2013).

To approximate the unknown chaotic map f for a given scalar time series $\{x_t\}_{t=1}^T$, BenSaïda and Litimi (2013) use a neural network function of a single q -dimensional hidden layer with a hyperbolic tangent activation function. The chaotic map can be approximated by the following equation (estimated by nonlinear least squares):

$$x_t \approx a_0 + \sum_{j=1}^q a_j \tanh(\beta_{0,j} + \sum_{i=1}^m \beta_{i,j} x_{t-iL}) + e_t \quad (4)$$

The orders (L, m, q) define how complex the chaotic map is. The L , m and q refer to time delay, embedding dimension and hidden layer, respectively. Theoretically, as the dimension of the hidden layer, q , increases toward infinity, the neural net function can approximate any nonlinear function, even the unknown chaotic map, to arbitrary accuracy. The time delay, L , must preserve time dependence in the time series, since an excessively large value causes a loss of information and a too small value makes the observations vector close in time and remote, giving rise to uncertainties. Consequently, low orders (L, m, q) may restrain the neural network from adequately approximating the map that generates the time series, and large orders increase computational time exponentially. The introduction of noise in the chaotic map will affect the accuracy of the computed Lyapunov exponent by rendering it stochastic, although it is a constant scalar by definition. Therefore, BenSaïda and Litimi (2013) approximate the variance of the largest Lyapunov exponent as:

$$\hat{\Sigma} = \frac{1}{M} \sum_{j=-M+1}^{M-1} \left[\zeta \left(\frac{j}{1.3221 \cdot M^{1/5}} \right) \sum_{t=|j|+1}^M \hat{\eta}_t \hat{\eta}_{t-|j|} \right] \quad (5)$$

where: $\hat{\eta}_t = \frac{1}{2} \ln \left[\frac{\max(\text{eig}(T'_t T_t))}{\max(\text{eig}(T'_{t-1} T_{t-1}))} \right] - \hat{\lambda}$, “eig” stands for the eigenvalue, and $\zeta(\cdot)$ is

the Quadratic Spectral kernel $\zeta_{QS}(Z) = \frac{25}{12\pi^2 Z^2} \left(\frac{\sin\left(\frac{6\pi Z}{5}\right)}{\frac{6\pi Z}{5}} - \cos\left(\frac{6\pi Z}{5}\right) \right)$.

The null hypothesis of the test is $H_0: \lambda \geq 0$ and its rejection provides a strong evidence of non-chaotic dynamics. Under H_0 , the test statistic, W , is asymptotically and normally distributed:

$$\hat{W} = \sqrt{M} \cdot \hat{\lambda}_M \sim N(0, \hat{\Sigma}) \quad (6)$$

The amplitude of λ will depend on the degree of divergence in the time series, *i.e.*, very low chaotic dynamics or near the transition to chaos will display λ of almost zero, with negative values of λ indicating the absence of chaos.

3. Data and Results

Our data set covers monthly data of the G7 stock markets, with all data obtained from the Global Financial Database, and converted into returns by taking first differences of natural logarithms of the data and multiplied by 100 to get the percentage. The various indices considered are: S&P/TSX 300 Composite (Canada, 1915M02-2016M08), CAC All-Tradable Index (France, 1802M01-2016M08), CDAX Composite Index (Germany, 1870M01-2016M08), Banca Commerciale Italiana Index (Italy, 1905M02-2016M08), Nikkei 225 (Japan, 1914M08-2016M08), FTSE All Share Index (UK, 1693M02-2016M08), and S&P500 (USA, 1791M09-2016M08). The data are plotted in Figure A1 in the Appendix. Table 1 provides the statistical properties for the return series. The skewness and kurtosis coefficients indicate asymmetric and fat-tail behaviors in the data. Moreover, the Jarque-Bera test strongly rejects the hypothesis of Gaussian distribution for all series.

Table 1: Descriptive Statistics of G7 stock returns

	UK	Canada	France	Japan	Germany	Italy	USA
Mean	0.12371	0.404298	0.514035	0.755135	0.218843	0.432622	0.248235
Median	0.151782	0.715464	0.151385	1.012605	0.321324	0	0.255555
Maximum	53.53325	20.58906	103.0344	50.87177	68.87212	46.81052	40.74591
Minimum	-73.5449	-33.4603	-27.6054	-27.2162	-145.996	-30.7573	-30.7528
Std. Dev.	3.989215	4.583921	5.592116	6.686965	7.230023	6.863842	3.842867
Skewness	-0.50945	-1.04948	4.332659	0.477769	-4.53688	0.903883	-0.58486
Kurtosis	57.08971	8.695183	78.79672	8.964642	108.5394	9.367104	14.75718
Jarque-Bera	472911.5*	1829.759*	411296.3*	1278.67*	833620*	2418.568*	15704.95*
Observations	3878	1192	1696	841	1783	1325	2700

Note: * denotes significance at 1% level.

Furthermore, we applied six denosing methods and evaluated their performance based on five criteria. Results of denosing evaluation are presented in Table 2. It is evident from Table 2, that the Hard denoising outperforms the other approaches for all countries except for UK, for which Mid denoising works well. Hence, the denoised column of Table 3 presents the results for outperforming denosing methods i.e., using Mid denosing method for UK and using Hard denosing method for all other countries.

Table 2: Denoising evaluation

	Hard denoised	Mid denoised	Soft denoised	Moving Average method	Savitzky-Golay filter method	EMD-FDA
Canada						
mean squared error	14.43164	16.83278	18.51026	19.424	18.5216	18.7532
mean absolute error	2.897223	3.055644	3.13646	3.1137	3.0348	3.0989
signal to noise ratio (DB)	1.661624	0.993221	0.580655	0.3714	0.578	0.524
peak signal to noise ratio (DB)	14.67958	14.01117	13.59861	13.3893	13.5959	13.542
cross correlation	0.595263	0.517012	0.505856	0.2772	0.3433	0.3395
France						
mean squared error	17.2422	19.9993	22.2197	29.0629	28.2189	30.0932
mean absolute error	3.1628	3.345	3.435	3.4259	3.4149	3.516
signal to noise ratio (DB)	2.6196	1.9754	1.5181	0.3521	0.4801	0.2008
peak signal to noise ratio (DB)	27.8937	27.2495	26.7923	25.6263	25.7542	25.475
cross correlation	0.6773	0.6095	0.5808	0.2666	0.3125	0.1965
Germany						
mean squared error	10.9854	14.9223	20.6764	48.3162	44.6218	0
mean absolute error	2.4929	2.8272	3.1582	3.5896	3.5385	0

signal to noise ratio (DB)	6.7762	5.446	4.0296	0.3434	0.6889	<u>Inf</u>
peak signal to noise ratio (DB)	26.3527	25.0225	23.6061	19.9199	20.2654	<u>Inf</u>
cross correlation	0.8907	0.8499	0.8193	0.2779	0.382	<u>1</u>
Italy						
mean squared error	27.8968	34.4411	38.9557	42.9028	41.723	43.7626
mean absolute error	3.9666	4.2815	4.445	4.4573	4.4374	4.5869
signal to noise ratio (DB)	2.2898	1.3745	0.8396	0.4204	0.5415	0.3343
peak signal to noise ratio (DB)	18.9513	18.0361	17.5012	17.082	17.2031	16.9958
cross correlation	0.6562	0.5656	0.5547	0.3053	0.3381	0.2835
Japan						
mean squared error	29.3862	34.5416	38.3432	41.8133	40.7946	40.872
mean absolute error	4.2169	4.4784	4.5953	4.5773	4.4609	4.5473
signal to noise ratio (DB)	1.8731	1.1711	0.7176	0.3414	0.4485	0.4403
peak signal to noise ratio (DB)	19.4481	18.7461	18.2927	17.9164	18.0235	18.0153
cross correlation	0.6017	0.524	0.4859	0.2534	0.296	0.2934
UK						
mean squared error	NaN	7.19628	8.920253	14.96845	14.04494	15.3642
mean absolute error	NaN	1.923337	2.058804	2.205398	2.184411	2.283933
signal to noise ratio (DB)	NaN	3.449724	2.517033	0.269037	0.545606	0.155706
peak signal to noise ratio (DB)	NaN	26.00139	25.0687	22.82071	23.09727	22.70737
cross correlation	NaN	0.74935	0.716846	0.244565	0.342472	0.194558
USA						
mean squared error	8.1326	10.0032	11.5268	13.3977	12.6575	14.0466
mean absolute error	2.1576	2.3326	2.4329	2.444	2.4009	2.5341
signal to noise ratio (DB)	2.6073	1.7082	1.0925	0.4393	0.6861	0.2339
peak signal to noise ratio (DB)	23.0994	22.2003	21.5846	20.9314	21.1782	20.726
cross correlation	0.6861	0.5961	0.5735	0.3088	0.3777	0.2211
<p>Note: (1) signal to noise (S/N) ratio is defined as the ratio of signal power to the noise power, often expressed in decibels. A ratio higher than 1:1 (greater than 0 dB) indicates more signal than noise. (2) The peak signal to noise ratio is the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. (3) Moving Average method-- smooth's the data by taking the median over a 25-element sliding window (4) Savitzky-Golay filter, which smooth's according to a quadratic polynomial that is fitted over each window of a returns series. This method can be more effective than other methods when the data varies rapidly, as in ours case. (5) Hard, Mid and Soft denosing are related to Nonlinear denosing via MODWT wave shrinking method where "universal" thresholding is used. (6) Italic values show the lowest values among all the denosing methods and bold value show the maximum values among all the denosing methods. (7) Values with underline show that method under consideration has not worked at all.</p> <p>Source: Authors' calculations</p>						

A positive Lyapunov exponent is considered as one of the operational definitions of chaotic behavior. While applying the 0-1 test, we, in each filter, draw 100 random frequencies from a uniform distribution on $\left[\frac{\pi}{5}, \frac{4\pi}{5}\right]$, and calculate the test statistic for

Table 3: Results of the 0–1 test and BenSaïda and Litimi (2013) test on returns and denoised returns

Series	0-1 test		BenSaïda and Litimi (2013) test							
	Raw Data	Denoised series {method}	Raw Data				Denoised series {method}			
	K_c	K_c	Triplet (L, m, q)	Accepted hypothesis	p -values	λ	Triplet (L, m, q)	Accepted hypothesis	p -values	λ
Canada	0.9985	0.9981 {1}	(1,6,5)	$H_1 = \lambda < 0$	0.000	-0.3606	(2,6,1)	$H_0 = \lambda \geq 0$	1	0.5273 {1}
France	0.9963	0.9826 {1}	(1,6,5)	$H_1 = \lambda < 0$	0.000	-0.3771	(1,5,4)	$H_0 = \lambda \geq 0$	1	0.9063 {1}
Germany	0.9984	0.9992 {1}	(1,6,5)	$H_1 = \lambda < 0$	0.000	-0.3293	(5,6,5)	$H_0 = \lambda \geq 0$	1	0.2722 {1}
Italy	0.9980	0.9967 {1}	(1,5,1)	$H_1 = \lambda < 0$	0.000	-0.4007	(1,6,3)	$H_0 = \lambda \geq 0$	1	0.1002 {1}
Japan	0.9984	0.9956 {1}	(2,6,2)	$H_1 = \lambda < 0$	0.000	-0.4100	(1,4,1)	$H_0 = \lambda \geq 0$	1	0.8097 {1}
UK	0.9978	0.9983 {2}	(1,6,5)	$H_1 = \lambda < 0$	0.000	-0.3369	(1,4,1)	$H_0 = \lambda \geq 0$	0.9998	0.0350 {2}
USA	0.9981	0.9981 {1}	(1,6,5)	$H_1 = \lambda < 0$	0.000	-0.3696	(3,6,1)	$H_0 = \lambda \geq 0$	1	0.3696 {1}

Note: (1) If $\lambda \geq 0$, and null hypothesis is $H_0 = 0$ with p -value = 1, it indicate that the presence of chaos is accepted and if the $\lambda < 0$ and alternative hypothesis $H_1 = 1$ with p -value = 0, it will indicate the absence of chaos; (2) In { } we mention the methods that are used for de-noising the returns series. Methods, 1,2,3,4,5, and 6 respectively denotes denosing is done via- Hard, Mid, Soft, Moving Average method, Savitzky-Golay filter method and EMD-FDA. (3) First three methods of denosing i.e., Hard denoised, Mid denoised, Soft denoised are based on Nonlinear denoising via wavelet shrinkage method.

each frequency. The results of the Gottwald and Melbourne (2005) test are presented in Table 3, where we report for each series the medians of these 100 single frequency test statistics. As can be seen, irrespective of whether we denoise the data or not, the Gottwald and Melbourne (2005) test provides strong evidence in favor of chaos in the G7 stock markets. In contrast, the BenSaïda and Litimi (2013) test supports the existence of chaotic behavior in all G7 stocks markets, only when denoised data is used.

4. Conclusions

This paper tests chaos in the stock returns of G7 countries, spanning long historical monthly and daily data. Chaos is tested both on raw data of returns, as well as on its noise-free counterpart, with noise being removed using outperforming denosing methods among the six denosing methods analyzed. The performance of denosing methods is calculated using five criteria. Interestingly, whereas the 0-1 test provides evidence of chaos in both noise-free and noisy data, the test proposed by BenSaïda and Litimi (2013), generally detects chaotic behavior only when noise-free data is used. Thus, noise in the return series is larger than the tolerable limit. In sum, based on our results, we can conclude that to detect the presence of chaos, it would perhaps be better to denoise the data. Also, since by construction, the 0-1 test is not capable of addressing the question of how weak or strong the chaotic behavior is, which BenSaïda and Litimi (2013)'s proposed test is able to, we can say that high level of chaos is detected for the stock returns of G7 markets.

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APPENDIX:

Figure A1: Data Plots







