A Vector Matroid-Theoretic Approach in the Study of Structural Controllability Over F(z)

YUPENG YUAN[1], ZHIXIONG LI[2,3], (Member, IEEE), REZA MALEKIAN[4], (Senior Member, IEEE), YONGZHI CHEN[1], AND YING CHEN[5]

[1]Reliability Engineering Institute, School of Energy and Power Engineering; Key Laboratory of Marine Power Engineering & Technology, (Ministry of Transport); National Engineering Research Center for Water Transport Safety, Wuhan University of Technology, Wuhan 430063,China
[2]School of Mechatronic Engineering, China University of Mining and Technology, Xuzhou 221116, China
[3]School of Mechanical and Manufacturing Engineering, University of New South Wales, Sydney 2052, Australia
[4]Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa
[5]Communication Staff Room of Communication Department, The Second Artillery Command College, Wuhan 430063, China

Corresponding author: Y. Chen (yzchen@whut.edu.cn)

ABSTRACT In this paper, the structural controllability of the systems over $F(z)$ is studied using a new mathematical method-matroids. First, a vector matroid is defined over $F(z)$. Second, the full rank conditions of $[sI - A|B](s \in \wp)$ are derived in terms of the concept related to matroid theory, such as rank, base, and union. Then, the sufficient condition for the linear system and composite system over $F(z)$ to be structurally controllable is obtained. Finally, this paper gives several examples to demonstrate that the married-theoretic approach is simpler than other existing approaches.

INDEX TERMS Matroid, structural controllability, rational function matrix, composite system.

I. INTRODUCTION

Kalman systematically introduced the state space method into systems and control theory, which is the landmark of modern control theory. In the state space method, two important concepts characterizing systems' properties were proposed, namely controllability and observability [1]. After Kalman first proposed the concepts of controllability and observability, many authors [2] studied the subject of linear systems over the field R of real numbers. In short, if the internal movement of all state variables in a system can be affected and controlled by the input, the system can be completely controlled; if all state variables within the system of any form of movement can be completely reflected by the output, the system can be completely observed. It is efficient to use the theory of controllability and observability in real number field for analyzing the controllability and observability jointly determined by system structures and the physical parameter values. The theory has also been successfully applied in many aspects of system analysis and design [2], [3]. However, in projects, due to the limitation of experimental conditions, manufacturing process, observational error, and approximate processing of data by human beings, the parameters of a real system are not accurate and even unknown. In such cases, by

using only a single index of controllability, it is impossible to decide whether the fact that the system does not satisfy completely controllable conditions is caused by structural reasons or incorrect parameter selection. Therefore, using the system in the real number field is not a direct approach to analyze the structural properties of physical systems [5], [6].

Take the bridge network shown in Fig. 1 as an example:

Set $x_1 = i_L, x_2 = u_C$ as two state variables; then the state equation of the system can be written as:

$$
\dot{x}_1 = -\frac{1}{L}\left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}\right) x_1
$$
$$
+ \frac{1}{L}\left(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4}\right) x_2 + \frac{1}{L} u
$$
$$
\dot{x}_2 = \frac{1}{C}\left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) x_1
$$
$$
- \frac{1}{C}\left(\frac{1}{R_1 + R_2} - \frac{1}{R_3 + R_4}\right) x_2 \tag{1}
$$

The controllability matrix of the system is

$$
[b, Ab] = \begin{bmatrix} \frac{1}{L} & -\frac{1}{L^2}\left(\frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}\right) \\ 0 & -\frac{1}{LC}\left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right) \end{bmatrix} \tag{2}
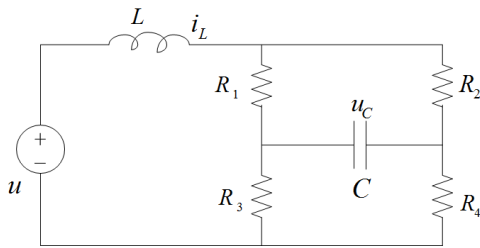$$

FIGURE 1. A bridge network.

For this system with given structure, only if all physical parameters L, R_1, R_2, R_3, R_4 and C are real-valued, A and b are real matrices and the system is a real number system. When $\frac{R_4}{R_3+R_4} \neq \frac{R_2}{R_1+R_2}$, no matter what values L, R_1, R_2, R_3, R_4 and C take, it is always true that rank$[b, Ab] = 2 = n$. Thus, system (1) is controllable everywhere in the real number field. However, when the bridge keeps a dynamic balance, in other words, when $R_1R_4 = R_2R_3$, there holds $\frac{R_1}{R_1+R_2} = \frac{R_3}{R_3+R_4}$ and $\frac{R_2}{R_1+R_2} = \frac{R_4}{R_3+R_4}$, then the controllability matrix $[b, Ab]$ is equal to $\begin{bmatrix} \frac{1}{L} & -\frac{1}{L^2}\left(\frac{R_1R_2}{R_1+R_2} + \frac{R_3R_4}{R_3+R_4}\right) \\ 0 & 0 \end{bmatrix}$. Since rank$[b, Ab] = 1 < n$, system (1) is not controllable in the real number field.

The above example demonstrates that the analytical results of real number system (such as the controllability of (A, B), the observability of (A^T, C^T), and the reducibility of eigen-polynomial $\det(\lambda - A)$ depend on two factors, namely the (physical) structure and values of the physical parameters of a system. However, we cannot determine what the separate role of system structures is.

In order to analyze the system structure, Lin [7] first introduced the concepts of structural controllability (SC) and structured matrix (SM) in 1974 and provided some structural controllability criteria by using graph theory. Later many researchers studied properties of structural controllability. Mayeda studied the controllability of multiple-input multiple-output system [8]. In 1976, the general rank of as tructural matrix is defined by Shield and Pearson [9] and Glover and Silverman [10] and the algebraic criterion was obtained. In recent decades, many scholars have studied the structural properties of a system by using the method of algebra or graph theory, and some valuable conclusions were drawn [11]–[20]. In fact, there exist an equivalence relation between the algebraic criterion and the graph criterion. The algebraic criterion and the graph theory criterion of structural controllability have an equivalent corresponding relationship as described in [21] and [22] with a rigorous proof. The above studies were based on SM proposed by Lin. Moreover, other forms of parameter matrices have also been proposed and used to study the structural properties of systems. For example, paper [23]–[26] introduced one-degree polynomial matrix, column-structured matrix (CSM) and mixed matrix (MM). K. Murota first studied the structural controllability of systems by the matroid theory [27]–[29].He used the union of matroids to analyze the structural controllable

conditions of descriptor system whose coefficient matrix is MM. The introduction of matroid further enriched the content of control theory. Although many significant conclusions with regard to structural controllability have been obtained, SM CSM and MM only described certain few types of systems, and the inverse matrix of the full rank square SM, CSM, one-degree polynomial matrix or mixed matrix is generally not SM, CSM, one-degree polynomial matrix or mixed matrix. To overcome this problem, papers [30]–[36] introduced rational function matrix (RFM)in multi-parameters to describe the coefficient matrices of systems and networks and described the systems and networks on RFM to study their structural properties.

The definition of RFM please see literature [30]–[36]. For example, if we consider a linear system:

$$\dot{X} = \bar{A}X + \bar{B}U, \quad Y = \bar{C}X + \bar{D}U \quad (3)$$

where $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$ are respectively $\bar{n} \times \bar{n}$, $\bar{n} \times \bar{m}$, $\bar{p} \times \bar{n}$, $\bar{p} \times \bar{m}$ matrices over $F(z)$, the system is called a rational function system (RFS) or a system over $F(z)$.

Since the entries of a rational function matrix are different from those of the SM, which are either fixed zero or independent free parameters, the conception of the RFM is more general. If all the physical parameters of a linear system are viewed as mutually independent parameter variables instead of constant real values, then the system is a linear system in $F(z)$, of which the controllability is irrelevant to system parameter values. If a system is controllable and observable in $F(z)$, i.e., structurally controllable and observable, then it is controllable and observable almost everywhere in parameter space R^q, which implies that the system is actually always controllable and observable in the real number field.

Currently, most studies on the structural property of systems in $F(z)$ are based on methods of algebra and graph theory, and many important conclusions have been obtained. However, the algebra or graph criterion for the structural controllability over $F(z)$ is more difficult and complicated than that of the systems which can be described as an SM, especially when the system dimension is very large, since a lot of symbolic operation is required for judging the structural controllability of systems by the criterions in time domain or frequency domain no matter which is obtained based on the state space method or polynomial matrix description method. In this paper, the matroid theory is firstly applied to analyze the system structure over $F(z)$. The equivalent condition that $[sI - \bar{A}|\bar{B}]$is full rank was explored with the concepts in matroid theory, such as base, rank, and union of matroids, and the structural controllability criteria of a system or a parallel composite system are obtained over $F(z)$. The matroids approach is generally easier than the traditional algebraic approach.

II. PRELIMINARIES
This section presents the basic concepts of matroid-theory. See [37] for the complete introduction of matroids [38]–[41]. We begin with the following definitions.

Definition 1: A matroid M is a pair $(E, \mathcal{I})$ where E is a finite set and collection $\mathcal{I} \subseteq 2^E$ is subsets of E such that:

1) $\phi \in \mathcal{I}$;

2) If $I \in \mathcal{I}$ and $I' \subseteq I$ then $I' \in \mathcal{I}$;

3) If $I_1, I_2 \in \mathcal{I}$ and $|I_1| < |I_2|$ then exist $e \in I_2 - I_1$ such that $I_1 \cup e \in \mathcal{I}$.

Thus, $(E, \mathcal{I})$ is a matroid which is denoted by $M = M(E, \mathcal{I})$. The entry of collection $\mathcal{I}$ is called an independent set of M. Axioms 1)-3) are called independent axioms. Generally, $E = E(M)$ and $\mathcal{I} = \mathcal{I}(M)$ are used to emphasize that$E(M)$ is the set of elements M and $\mathcal{I}(M)$ is the collection of independent sets M. If $I \in \mathcal{I}(M)$ and $|I| = k$, I is also called a k- independent set of M.

Definition 2: Let A be an $n \times m$ matrix over F and $E = E(A)$ is a set of labels of the column vectors of A. Define $\mathcal{I} \subseteq 2^E$ as a collection: $(E, \mathcal{I})$ is a matroid if and only if the column vectors marked by X are linearly independent in the vector space where $X \in \mathcal{I}$.

Definition 3: The maximal independent subset of a matroid M is called a base of M. Denote $\mathcal{B}(M)$ the family of all bases of M, then $\mathcal{B}(M) = Max(\mathcal{I})$.

Definition 4: The function $r_M : 2^E \rightarrow Z$, $r_M(X) = \max\{|I| : I \subseteq X, I \in \mathcal{I}(M)\}$ is called a rank function of the matroid M. Value $r_M(E)$ is called the rank of the matroid M and denoted as $r(M)$.

Definition 5: Let $M_1, M_2, \cdots, M_n$ be n matroids on the set E. Their rank functions are respectively $r_1, r_2, \cdots, r_n$. Let $\mathcal{I} = \{I_1 \cup I_2 \cup \cdots \cup I_n : \text{for each } 1 \leq i \leq n, I_i \in \mathcal{I}(M_i)\}$, then there exists a matroid M' on E such that

1) $\mathcal{I}(M') = \mathcal{I}$;

2) $r_{M'}(X) = \min\{\sum_{i=1}^{n} r_i(Y) + |X - Y| : Y \subseteq X\}$ for any subset $X \subseteq E$.

The matroid M' is usually denoted by $\bigvee_{i=1}^{n} M_i$ and called the union of $M_1, M_2, \cdots, M_n$. With the above definitions, we have the following lemmas.

Lemma 1: Let $M(E, \mathcal{I})$ be a matroid and $\mathcal{B} = \mathcal{B}(M)$. M The family $\mathcal{B}$ satisfies

1) $\mathcal{B}$ contains at least one element (which implies that M has at least one base);

2) For $B_1, B_2 \in \mathcal{B}$ and for $x \in B_1 - B_2$, there exists $y \in B_2 - B_1$ such that $(B_1 - x) \cup y \in \mathcal{B}$.

Lemma 2: Let M_1 and M_2 be two matroids both on set E. Then $\mathcal{B}(M_1 \vee M_2) = \max\{B_1 \cup B_2 : B_1 \in \mathcal{B}(M_1), B_2 \in \mathcal{B}(M_2)\}$.

A. STRUCTURAL CONTROLLABILITY OF SYSTEMS OVER *F(z)*

Let $A = \begin{pmatrix} A_1 & A_2 & \cdots & A_n \end{pmatrix}^T$ be matrices over $F(z)$; then we have $M[A] = M[A_1] \vee M[A_2] \vee \cdots \vee M[A_n]$ by the definition of union matroids. To proceed, we need the following concepts.

Definition 6: Let A be a matrix over $F(z)$ and $E = E(A)$ be a label set of the column vectors of A. $\mathcal{I} \in 2^E$ is defined

as a collection: $(E, \mathcal{I})$ is a matroid if and only if the column vectors marked by X are linearly independent in the vector space $V(n, F(z))$, where $X \in \mathcal{I}$. $(E, \mathcal{I})$ is also called vector matroid of A and denoted by $M_{F(z)}[A]$.

Definition 7: Let A and B be $n \times n$ and $n \times m$ matrices, respectively. $M[sI - A|B] = (E, \mathcal{I})$ is a vector matroid over $F(z)$. If there exist n column vectors of the matrix $[sI - A|B]$ ($\forall s \in \wp$) whose determinant is a unimodular matrix, the label set $\mathcal{B}$ of n column vectors will be a base of the matroid M. $\mathcal{B}$ is also called a unimodular base.

Lemma 3: Linear system (3) is structural controllability if and only if rank$[sI - A|B] = n, \forall s \in \wp$.

Our main results are as follows:

Theorem 1: Consider a linear system (3) over $F(z)$. Let $\mathcal{A} = [sI - A|B] = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \end{bmatrix}$. $M[\mathcal{A}_1]$ and $M[\mathcal{A}_2]$ be vector matrices respectively, where rank$(M[\mathcal{A}_1]) = n_1$, rank$(M[\mathcal{A}_2]) = n_2$. The system (A, B) is structural controllable if there exist two unimodular bases B_1 and B_2, satisfy $B_1 \cap B_2 = \phi$ and $n_1 + n_2 = n(\forall s \in \wp)$, where $B_1 \in \mathcal{B}(M[\mathcal{A}_1]), B_2 \in \mathcal{B}(M[\mathcal{A}_2])$.

Proof: Since $\mathcal{A} = [sI - A|B] = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \end{bmatrix}$, from definition 5 we have $M[\mathcal{A}] = M[\mathcal{A}_1] \vee M[\mathcal{A}_2]$. Additionally, since B_1 and B_2 are two unimodular bases, $|B_1| = n_1$ and $|B_2| = n_2$ can be established by Definition 7. Since $B_1 \cap B_2 = \phi$ and $|Max\{B_1 \cup B_2\}| = |B_1| + |B_2| = n_1 + n_2 = n$, where $B_1 \in \mathcal{B}(M[\mathcal{A}_1])$ and $B_2 \in \mathcal{B}(M[\mathcal{A}_2])$, we have $|\mathcal{B}(M[\mathcal{A}_1] \vee M[\mathcal{A}_2])| = n$ from lemma 2. Then rank$[sI - A|B] = $ rank$(M[\mathcal{A}]) = $ rank$(M[\mathcal{A}_1] \vee M[\mathcal{A}_2])|\mathcal{B}(M[\mathcal{A}_1] \vee M[\mathcal{A}_2])| = n$, so the linear system (A, B) is structural controllable.

Theorem 2: Consider a linear system (3) over $F(z)$. Let $\mathcal{A} = [sI - A|B] = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \\ \vdots \\ \mathcal{A}_n \end{bmatrix}$. $M[\mathcal{A}_1], M[\mathcal{A}_2], \cdots, M[\mathcal{A}_n]$ be vector matroids of matrices $\mathcal{A}_1, \mathcal{A}_2, \cdots, \mathcal{A}_n$, where rank$(M[\mathcal{A}_1]) = n_1$, rank$(M[\mathcal{A}_2]) = n_2, \cdots$, rank$(M[\mathcal{A}_n]) = n_n$. The system (A, B) is structural controllable if there exist n unimodular bases $B_1, B_2, \cdots, B_n$ satisfy that $B_i \cap B_j = \phi$, where $B_i \in \mathcal{B}(M[\mathcal{A}_i])$, $i, j = 1, 2, \cdots, n, i \neq j$ and $n_1 + n_2 + \cdots + n_n = n, \forall s \in \wp$.

Proof: Since $\mathcal{A} = [sI - A|B] = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \\ \vdots \\ \mathcal{A}_n \end{bmatrix}$, we have $M[\mathcal{A}] = $

$M[\mathcal{A}_1] \vee M[\mathcal{A}_2] \vee \cdots \vee M[\mathcal{A}_n]$ by Definition 5. It is straightforward to verify that $\mathcal{B}(M_1 \vee M_2 \vee \cdots \vee M_n) = Max\{B_1 \cup B_2 \cup \cdots \cup B_n\}$ by lemma 2, then $|\mathcal{B}(M_1 \vee M_2 \vee \cdots \vee M_n)| = |Max\{B_1 \cup B_2 \cup \cdots \cup B_n\}|$. Since $B_1, B_2, \cdots, B_n$ are unimodular bases, where $B_i \in \mathcal{B}(M[\mathcal{A}_i])$ and $B_i \cap B_j = \phi(i \neq j)$, we have $|\mathcal{B}(M_1 \vee M_2 \vee \cdots \vee M_n)| = |B_1| + |B_2| + \cdots + |B_n| = n_1 + n_2 + \cdots + n_n = n$. Hence, rank$[sI - A|B] = $ rank$(M[\mathcal{A}_1] \vee M[\mathcal{A}_2] \vee \cdots \vee M[\mathcal{A}_n]) = $

$|\mathcal{B}(M_1 \vee M_2 \vee \cdots \vee M_n)| = n$. So the system (A, B) is structural controllable.

Consider two linear subsystems over $F(z)$

$$\Sigma_i : \dot{X}_i = \bar{A}_i X_i + \bar{B}_i u_i, \ y_i = \bar{C}_i X_i + \bar{D}_i u_i \quad (4)$$

where $\dim \bar{A}_i = n_i \times n_i$, $\dim \bar{B}_i = n_i \times m$, $X_i \in R^{n_i}$, $u_i \in R^m$, $y_i \in R$, $i = 1, 2$. The state space description of the parallel composite system $\sum_p$ is:

$$\sum_p : \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_1 & 0 \\ 0 & \bar{A}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} \bar{C}_1 & \bar{C}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \bar{D}_1 + \bar{D}_2 \end{bmatrix} u \quad (5)$$

Let A_i, B_i, C_i and D_i be matrices over $F(z)(i = 1, 2)$. We have the following conclusions.

Theorem 3: A parallel composite system $\sum_p$ is structural controllable if $\sum_i$ is structural controllable and there exist two unimodular bases B_1 and B_2 of vector matroids $M[sI - \bar{A}_1 | 0_{n_1 \times n_2} | \bar{B}_1]$ and $M[0_{n_2 \times n_1} | sI - \bar{A}_2 | \bar{B}_2]$, such that $B_1 \cap B_2 = \phi (\forall s \in \wp)$.

Proof: Since $\sum_i$ is structural controllable, we have $\text{rank}[sI - \bar{A}_1 | \bar{B}_1] = \text{rank}(M[sI - \bar{A}_1 | \bar{B}_1]) = n_1$ and $\text{rank}[sI - \bar{A}_2 | \bar{B}_2] = \text{rank}(M[sI - \bar{A}_2 | \bar{B}_2]) = n_2$. Let $\mathcal{A}_1 = [sI - \bar{A}_1 | 0_{n_1 \times n_2} | \bar{B}_1]$ and $\mathcal{A}_2 = [0_{n_2 \times n_1} | sI - \bar{A}_2 | \bar{B}_2]$; then $\mathcal{A} = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \end{bmatrix} = \begin{bmatrix} sI - \bar{A}_1 | 0_{n_1 \times n_2} | \bar{B}_1 \\ 0_{n_2 \times n_1} | sI - \bar{A}_2 | \bar{B}_2 \end{bmatrix} = [sI - \bar{A} | \bar{B}]$, where $\bar{A} = \begin{bmatrix} \bar{A}_1 & 0 \\ 0 & \bar{A}_2 \end{bmatrix}$ and $\bar{B} = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}$. It is true that $M[\mathcal{A}] = M[\mathcal{A}_1] \vee M[\mathcal{A}_2]$ by Definition 5. So $\text{rank}(M[\mathcal{A}]) = \text{rank}(M[\mathcal{A}_1] \vee M[\mathcal{A}_2]) = |\mathcal{B}(M[\mathcal{A}_1] \vee M[\mathcal{A}_2])| = |Max\{B_1 \cup B_2\}|$, where $B_1 \in \mathcal{B}(M[\mathcal{A}_1])$ and $B_2 \in \mathcal{B}(M[\mathcal{A}_2])$. It is easy to show that $\text{rank}(M[sI - \bar{A}_1 | \bar{B}_1]) = \text{rank}(M[sI - \bar{A}_1 | 0_{n_1 \times n_2} | \bar{B}_1])$ and $\text{rank}(M[sI - \bar{A}_2 | \bar{B}_2]) = \text{rank}(M[0_{n_2 \times n_1} | sI - \bar{A}_2 | \bar{B}_2])$ by linear algebra theory. So $|B_1| = \text{rank}(M[sI - \bar{A}_1 | \bar{B}_1]) = n_1$ and $|B_2| = \text{rank}(M[sI - \bar{A}_2 | \bar{B}_2]) = n_2$. In addition, since $B_1 \cap B_2 = \phi$, we have $\text{rank}(M[\mathcal{A}]) = |Max\{B_1 \cup B_2\}| = |B_1| + |B_2| = n_1 + n_2 = \text{rank}[sI - \bar{A} | \bar{B}]$. Therefore, the parallel composite system $\sum_p$ is structural controllable.

We now study the structural controllability of parallel composite system consisting of N subsystems. Let $\sum_p$ be such a system whose state space description is:

$$\sum_p : \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} \bar{A}_1 & & \\ & \ddots & \\ & & \bar{A}_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_2 \end{bmatrix} + \begin{bmatrix} \bar{B}_1 \\ \vdots \\ \bar{B}_N \end{bmatrix} u$$
$$(6)$$

$$y = \begin{bmatrix} \bar{C}_1 & \cdots & \bar{C}_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$
$$+ \begin{bmatrix} \bar{D}_1 + \cdots + \bar{D}_N \end{bmatrix} u \quad (7)$$

where $\dim \bar{A}_i = n_i \times n_i$, $\dim \bar{B}_i = n_i \times m$, $i = 1, 2, \cdots, N$. Let A_i, B_i, C_i and D_i are matrices over $F(z)(i = 1, 2, \cdots, n)$. We have:

Theorem 4: A parallel composite system $\sum_p$ consisting of N subsystems is structural controllable if $\sum_i$ are structural controllable and there exist N unimodular bases $B_1, B_2, \cdots, B_N$ of vector matroids $M[sI - \bar{A}_1 | 0_{n_1 \times (n_2+n_3+\cdots+n_N)} | \bar{B}_1] M[0_{n_2 \times n_1} | sI - \bar{A}_2 | 0_{n_2 \times (n_3+n_4+\cdots+n_N)} | \bar{B}_2], \cdots, M[0_{n_N \times (n_1+n_2+\cdots+n_{N-1})} | sI - \bar{A}_N | \bar{B}_N]$ satisfying that $B_i \cap B_j = \phi$, $i, j = 1, 2, \cdots, N$, $i \neq j$, $\forall s \in \wp$.

Proof: Since $\sum_i$ are structural controllable, we have $\text{rank}[sI - \bar{A}_1 | \bar{B}_1] = \text{rank}(M[sI - \bar{A}_1 | \bar{B}_1]) = n_1$, $\text{rank}[sI - \bar{A}_2 | \bar{B}_2] = \text{rank}(M[sI - \bar{A}_2 | \bar{B}_2]) = n_2, \cdots, \text{rank}[sI - \bar{A}_N | \bar{B}_N] = \text{rank}(M[sI - \bar{A}_N | \bar{B}_N]) = n_N$. Let $\mathcal{A}_1 = [sI - \bar{A}_1 | 0_{n_1 \times (n_2+n_3+\cdots+n_N)} | \bar{B}_1]$, $\mathcal{A}_2 = [0_{n_2 \times n_1} | sI - \bar{A}_2 | 0_{n_2 \times (n_3+n_4+\cdots+n_N)} | \bar{B}_2], \cdots, \mathcal{A}_N = [0_{n_N \times (n_1+n_2+\cdots+n_{N-1})} | sI - \bar{A}_N | \bar{B}_N]$; then

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \\ \vdots \\ \mathcal{A}_N \end{bmatrix} = \begin{bmatrix} sI - \bar{A}_1 | 0_{n_1 \times (n_2+n_3+\cdots+n_N)} | \bar{B}_1 \\ 0_{n_2 \times n_1} | sI - \bar{A}_2 | 0_{n_2 \times (n_3+n_4+\cdots+n_N)} | \bar{B}_2 \\ \cdots \\ 0_{n_N \times (n_1+n_2+\cdots+n_{N-1})} | sI - \bar{A}_N | \bar{B}_N \end{bmatrix}$$
$$= [sI - \bar{A} | \bar{B}] \quad (8)$$

where

$$\bar{A} = \begin{bmatrix} \bar{A}_1 & & 0 \\ & \ddots & \\ 0 & & \bar{A}_N \end{bmatrix}, \bar{B} = \begin{bmatrix} \bar{B}_1 \\ \vdots \\ \bar{B}_N \end{bmatrix} \quad (9)$$

We have $M[\mathcal{A}] = M[\mathcal{A}_1] \vee M[\mathcal{A}_2] \vee \cdots \vee M[\mathcal{A}_N]$ by Definition 5.

So

$$\text{rank}(M[\mathcal{A}]) = \text{rank}(M[\mathcal{A}_1] \vee M[\mathcal{A}_2] \vee \cdots \vee M[\mathcal{A}_N])$$
$$= |Max\{B_1 \cup B_2 \cup \cdots \cup B_N\}|$$
$$= |\mathcal{B}(M[\mathcal{A}_1] \vee M[\mathcal{A}_2] \vee \cdots \vee M[\mathcal{A}_N]|$$

where $B_1 \in \mathcal{B}(M[\mathcal{A}_1])$, $B_2 \in \mathcal{B}(M[\mathcal{A}_2])$, $\cdots$, $B_N \in \mathcal{B}(M[\mathcal{A}_N])$

Since

$$\text{rank}(M[sI - \bar{A}_1 | \bar{B}_1])$$
$$= \text{rank}(M[sI - \bar{A}_1 | 0_{n_1 \times (n_2+n_3+\cdots+n_N)} | \bar{B}_1])$$
$$\text{rank}(M[sI - \bar{A}_2 | \bar{B}_2]) = \text{rank}(M[0_{n_2 \times n_1} | sI - \bar{A}_2$$
$$| 0_{n_2 \times (n_3+n_4+\cdots+n_N)} | \bar{B}_2])$$
$$\cdots$$
$$\text{rank}(M[sI - \bar{A}_N | \bar{B}_N])$$
$$= \text{rank}(M[0_{n_N \times (n_1+n_2+\cdots+n_{N-1})} | sI - \bar{A}_N | \bar{B}_N])[4pt]$$

So

$$|B_1| = \text{rank}(M[sI - \bar{A}_1 | \bar{B}_1]) = n_1,$$
$$|B_2| = \text{rank}(M[sI - \bar{A}_2 | \bar{B}_2]) = n_2, \cdots,$$
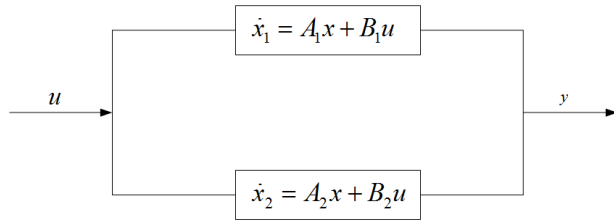$$|B_N| = \text{rank}(M[sI - \bar{A}_N | \bar{B}_N]) = n_N$$

FIGURE 2. A parallel composite system.

and, in addition,

$$B_i \cap B_j = \phi, \quad i, j = 1, 2, \cdots, N, i \neq j$$
$$\text{rank}(M[\mathcal{A}]) = |Max\{B_1 \cup B_2 \cup \cdots \cup B_N\}|$$
$$= |B_1| + |B_2| + \cdots + |B_N|$$
$$= n_1 + n_2 + \cdots + n_N$$
$$= \text{rank}[sI - \bar{A}|\bar{B}]$$

Therefore, the parallel composite system $\sum_p$ is structural controllable.

III. EXAMPLE

Example 1: Consider a parallel composite system [42-45] shown in figure 2.

where

$$A_1 = \begin{bmatrix} z_1 & 1 \\ 0 & z_2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ z_3 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} z_1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ are matrices over } F(z).$$

The controllability matrix of subsystem $\sum_1 : \dot{x}_1 = A_1x_1 + B_1u$ of parallel composite system is

$$\begin{bmatrix} B_1 & A_1B_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & z_3 & 1 \\ z_3 & 1 & z_2z_3 & z_2 \end{bmatrix} \quad (10)$$

Since $\text{rank}[B_1 \quad A_1B_1] = 2$, the subsystem $\sum_1 : \dot{x}_1 = A_1x_1 + B_1u$ is structural controllable. Similarly, the controllability matrix of subsystem $\sum_2 : \dot{x}_2 = A_2x_2 + B_2u$ is

$$\begin{bmatrix} B_2 & A_2B_2 & A_2^2B_2 \end{bmatrix} = \begin{bmatrix} z_1 & 0 & z_1 & 1 & z_1 & 1 \\ 0 & 1 & 0 & 0 & z_1 & 0 \\ 0 & 0 & z_1 & 0 & z_1 & 0 \end{bmatrix} \quad (11)$$

Then, $\text{rank}[B_2 \quad A_2B_2 \quad A_2^2B_2] = 3$, so the subsystem

$$\sum_p (p = 1, 2) : \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \quad (12)$$

Let $A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$ and $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$; then
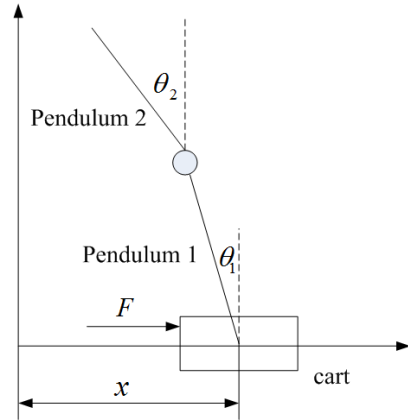
$[sI - A|B]$

FIGURE 3. Schematic diagram of double inverted pendulum system.

$$= \begin{bmatrix} sI - A_1|0|B_1 \\ 0|sI - A_2|B_2 \end{bmatrix}$$

$$= \begin{array}{c} \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7 \\ \begin{bmatrix} s - z_1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & s - z_2 & 0 & 0 & 0 & z_3 & 1 \\ 0 & 0 & s - 1 & -1 & 0 & z_1 & 0 \\ 0 & 0 & 0 & s & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & s & 0 & 0 \end{bmatrix} \end{array}$$
$$\quad (13)$$

where $E = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ is the label set of the column vector of the matrix $[sI - A|B]$. Let

$$\mathcal{A}_1 = [sI - A_1|0|B_1], \quad \mathcal{A}_2 = [0|sI - A_2|B_2] \quad (14)$$

then $M[\mathcal{A}_1]$ and $M[\mathcal{A}_2]$ are two vector matroids over $F(z)$. So

$$\text{rank}(M[\mathcal{A}_1]) = n_1 = 2, \quad \text{rank}(M[\mathcal{A}_2]) = n_2 = 3 \quad (15)$$

$B_1 = \{a_2, a_6\}$ and $B_2 = \{a_3, a_5, a_7\}$ are unimodular bases of vector matroid $M[\mathcal{A}_1]$ and $M[\mathcal{A}_2]$ respectively by linear algebra theory and the definition of matroid base. Moreover, since

$B_1 \cap B_2 = \{a_2, a_6\} \cap \{a_3, a_5, a_7\} = \phi$ and $n_1 + n_2 = 2 + 3 = 5$,

The parallel composite system is structural controllable by Theorem 3.

Example 2: The classical double inverted pendulum system shown in figure 3 is composed of a pair of rigid, homogenous pendulum rods which are interconnected in a joint, and one of them is attached to a stable mechanism (cart) which allows for movement alongside a single axis. To simplify the system, we ignore air resistance and all sorts of friction.

Here θ_1 and θ_2 respectively represent the angles of first and second pendulums with respect to the vertical axis, x denotes the position of the cart, M is the mass of the cart, m_1 is the mass of the first link, m_2 represents the mass of second link, m_3 is the mass of the hinge and encoder, l_1 and l_2 denote the length of the first and second pendulums, respectively.

State variables are taken as follows:
$$x_1 = x, x_2 = \theta_1, x_3 = \theta_2, x_4 = \dot{x}, x_5 = \dot{\theta}_1, x_6 = \dot{\theta}_2$$

Let $Z = (z_1, z_2, z_3, z_4, z_5) = (m_1, m_2, m_3, l_1, l_2)$ be five independent physical parameters. The state space equation of double inverted pendulum system can then be written as:

$$\dot{x} = Ax + Bu \qquad (16)$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{12} & K_{13} & 0 & 0 & 0 \\ 0 & K_{22} & K_{23} & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ K_{17} \\ K_{27} \end{bmatrix}$$

$$(17)$$

$$\frac{3g(z_1 + 2z_2 + 2z_3)}{z_4(4z_1 + 3z_2 + 12z_3)} = K_{12} \qquad (18)$$

$$-\frac{9z_2 g}{2z_4(4z_1 + 3z_2 + 12z_3)} = K_{13} \qquad (19)$$

$$-\frac{9g(z_1 + 2z_2 + 2z_3)}{2z_5(4z_1 + 3z_2 + 12z_3)} = K_{22} \qquad (20)$$

$$-\frac{3g(z_1 + 3z_2 + 3z_3)}{z_5(4z_1 + 3z_2 + 12z_3)} = K_{23} \qquad (21)$$

$$\frac{3(2z_1 + z_2 + 4z_3)}{2z_4(4z_1 + 3z_2 + 12z_3)} = K_{17} \qquad (22)$$

$$-\frac{3z_1}{2z_5(4z_1 + 3z_2 + 12z_3)} = K_{27} \qquad (23)$$

Let

$$\mathcal{A}_1 = \begin{bmatrix} s & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \qquad (24)$$

$$\mathcal{A}_2 = \begin{bmatrix} 0 & 0 & s & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & s & 0 & 0 & 1 \end{bmatrix} \qquad (25)$$

$$A_3 = \begin{bmatrix} 0 & -K_{12} & -K_{13} & 0 & s & 0 & K_{17} \\ 0 & -K_{22} & -K_{23} & 0 & 0 & s & K_{27} \end{bmatrix} \qquad (26)$$

We have

$$\mathcal{A} = [sI - A|B] = \begin{bmatrix} \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \end{bmatrix}$$

$$= \begin{array}{cccccccc} & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ & \begin{bmatrix} s & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & s & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & s & 0 & 0 & 1 \\ 0 & -K_{12} & -K_{13} & 0 & s & 0 & K_{17} \\ 0 & -K_{22} & -K_{23} & 0 & 0 & s & K_{27} \end{bmatrix} \end{array}$$

$$(27)$$

where $E = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ is the label set of the column vectors of matrix $\mathcal{A}$. Then $M[\mathcal{A}_1]$, $M[\mathcal{A}_2]$ and $M[\mathcal{A}_3]$ are all vector matroids over $F(z)$, and $M[\mathcal{A}] = M[\mathcal{A}_1] \vee M[\mathcal{A}_2] \vee M[\mathcal{A}_3]$. $B_1 = \{a_4, a_5\}$, $B_2 = \{a_6, a_7\}$ and $B_3 = \{a_2, a_3\}$ are unimodular bases of vector matroid $M[\mathcal{A}_1]$, $M[\mathcal{A}_2]$ and $M[\mathcal{A}_3]$ respectively by the definition of matroid base. Moreover, since

$$\text{rank}M[\mathcal{A}_1] = |B_1| = 2$$

$$\text{rank}M[\mathcal{A}_2] = |B_2| = 2$$

$$\text{rank}M[\mathcal{A}_3] = |B_3| = 3$$

and $|B_1| + |B_2| + |B_3| = 6 = n$, $B_i \cap B_j = \phi$, $i, j = 1, 2, 3, i \neq j$, $\forall s \in \wp$, the double inverted pendulum system is structural controllable by Theorem 4.

IV. CONCLUSION

The structural controllability of linear systems was studied with different mathematical methods and matrix structures, and some structural controllability criteria were derived. In this paper, the structural property has been studied using the concept of matroids over $F(z)$. Then, the structural controllability criteria have been obtained for linear systems and parallel composite systems. A novel approach to study the structural properties has also been provided. We have shown that the approach presented in this paper is substantially simpler than any other existing ones.

REFERENCES

[1] R. E. Kalman, "On the general theory of control systems," in *Proc. 1st Int. Congr. Autom. Control*, London, U.K., 1960, pp. 481–493.

[2] E. Gilbert, "Controllability and observability in multivariable control systems," *J. Soc. Ind. Appl. Math., Ser. A, Control*, vol. 1, no. 2, pp. 128–151, 1963.

[3] J. L. Casti, *Linear Dynamical Systems*. Orlando, FL, USA: Academic, 1987.

[4] C. T. Chen, *Linear System Theory and Design*. New York, NY, USA: Holt, Rinehart and Winston, 1984.

[5] K. S. Lu, "Investigation on structural properties of systems in terms of rational function matrices in multi-parameters," *Control Theory Appl.*, vol. 4, no. 5, pp. 770–772, 1997.

[6] L. A. Zadeh and C. A. Desoer, *Linear System Theory: The State Space Approach*. New York, NY, USA: McGraw-Hill, 1963.

[7] C.-T. Lin, "Structural controllability," *IEEE Trans. Autom. Control*, vol. 19, no. 3, pp. 201–208, Jun. 1974.

[8] H. Mayeda, "On structural controllability theorem," *IEEE Trans. Autom. Control*, vol. 26, no. 3, pp. 795–798, Jun. 1981.

[9] R. Shields and J. Pearson, "Structural controllability of multiinput linear systems," *IEEE Trans. Autom. Control*, vol. 21, no. 2, pp. 203–212, Apr. 1976.

[10] K. Glover and L. Silverman, "Characterization of structural controllability," *IEEE Trans. Autom. Control*, vol. 21, no. 4, pp. 534–537, Aug. 1976.

[11] B. Hou, X. Li, and G. Chen, "Structural controllability of temporally switching networks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 63, no. 10, pp. 1771–1781, Oct. 2016.

[12] S. Sun, S. Ma, Y. Wu, L. Wang, and C. Xia, "Towards structural controllability of local-world networks," *Phys. Lett. A*, vol. 380, nos. 22–23, pp. 1912–1917, 2016.

[13] H. Yin and S. Zhang, "Minimum structural controllability problems of complex networks," *Phys. A, Statist. Mech. Appl.*, vol. 443, no. 1, pp. 467–476, 2015.

[14] P. Liu, Y. Tian, and Y. Zhang, "Strong structural controllability and leader selection for multi-agent systems with unidirectional topology," in *Proc. Chin. Intell. Syst. Conf.*, Berlin, Germany, 2016, pp. 415–426.

[15] C. Alcaraz, E. E. Miciolino, and S. Wolthusen, "Multi-round attacks on structural controllability properties for non-complete random graphs," in *Information Security*. Switzerland: Springer, 2015, pp. 140–151.

[16] A. Qi, X. Ju, Q. Zhang, and Z. Chen, "Structural controllability of discretetime linear control systems with time-delay: A delay node inserting approach," *Math. Problems Eng.*, vol. 2016, Feb. 2016, Art. no. 1429164.

[17] H. Reza and R. N. Hamid. (2015). *Algorithm for Identifying Minimum Driver Nodes Based on Structural Controllability*. [Online]. Available: http://dx.doi.org/10.1155/2015/192307

[18] M. Pósfai and P. Hövel, "Structural controllability of temporal networks," *New J. Phys.*, vol. 16, no. 12, p. 123055, 2014.

[19] G. Reissig, C. Hartung, and F. Svaricek, "Strong structural controllability and observability of linear time-varying systems," *IEEE Trans. Autom. Control*, vol. 59, no. 11, pp. 3087–3092, Nov. 2014.

[20] J. C. Nacher and T. Akutsu, "Structural controllability of unidirectional bipartite networks," *Sci. Rep.*, vol. 3, no. 4, 2013, Art. no. 1647.

[21] T. Yamada, "Comments on 'On structural controllability theorem'," *IEEE Trans. Autom. Control*, vol. 29, no. 12, pp. 1130–1131, Dec. 1984.

[22] A. Linnemann, "A further simplification in the proof of the structural controllability theorem," *IEEE Trans. Autom. Control*, vol. 31, no. 7, pp. 638–639, Jul. 1986.

[23] B. D. O. Anderson and H.-M. Hong, "Structural controllability and matrix nets," *Int. J Control*, vol. 35, no. 3, pp. 397–416, 1982.

[24] T. Yamada, "Generic properties of column-structured matrices," *Linear Algebra Appl.*, vol. 65, pp. 186–206, Feb. 1985.

[25] K. Murota, *Systems Analysis by Graphs and Matroids: Structural Solvability and Controllability*. New York, NY, USA: Springer-Verlag, 1987.

[26] K. Murota, "On the Smith normal form of structured polynomial matrices," *SIAM J. Matrix Anal. Appl.*, vol. 12, no. 4, pp. 747–765, 1989.

[27] K. Murota, "Refined study on structural controllability of descriptor systems by means of matroids," *SIAM J. Control Optim.*, vol. 25, no. 4, pp. 967–989, 1987.

[28] K. Murota, "A matroid-theoretic approach to structurally fixed modes of control systems," *SIAM J. Control Optim.*, vol. 27, no. 6, pp. 1381–1402, 1989.

[29] K. Murota, *Matrices and Matroids for Systems Analysis*. New York, NY, USA: Springer, 2010.

[30] K. S. Lu and J. N. Wei, "Rational function matrices and structural controllability and observability," *IEE Proc. D, Control Theory Appl.*, vol. 138, no. 4, pp. 388–394, Jul. 1991.

[31] K.-S. Lu and J. N. Wei, "Reducibility condition of a class of rational function matrices," *SIAM J. Matrix Anal. Appl.*, vol. 15, no. 1, pp. 151–161, 1994.

[32] K.-S. Lu and K. Lu, "Controllability and observability criteria of RLC networks over $F(z)$," *Int. J. Circuit Theory Appl.*, vol. 29, no. 3, pp. 337–341, 2001.

[33] K.-S. Lu, "A sufficient condition of controllability of RLCM active networks over $F(z)$," *Int. J. Circuit Theory Appl.*, vol. 35, no. 2, pp. 205–210, 2007.

[34] K.-S. Lu, "Existence condition of state equations of linear active networks over $F(z)$," *Int. J. Circuit Theory Appl.*, vol. 36, no. 2, pp. 109–118, 2008.

[35] K.-S. Lu and G.-M. Liu, "Class of rational function matrices that satisfy two properties of linear systems and structural controllability," *IET Control Theory Appl.*, vol. 1, no. 1, pp. 113–118, 2007.

[36] K.-S. Lu and Y. J. T. Campus, "The controllability of linear systems over $F(z)$," in *Proc. 40th IEEE CDC*, Dec. 2001, pp. 2676–2681.

[37] J. H. Lai, *Matroid Theory*. BeiJing, China: Higher Education Press, 2002.

[38] Z. Xin, S. Jie, A. Wenwei, and Y. Tiantian, "An improved time-frequency representation based on nonlinear mode decomposition and adaptive optimal kernel," *Elektronika Elektrotechnika*, vol. 22, no. 4, pp. 52–57, 2016.

[39] X. Jin, J. Shao, X. Zhang, and W. An, "Modeling of nonlinear system based on deep learning framework," *Nonlinear Dyn.*, vol. 84, no. 3, pp. 1327–1340, 2016.

[40] Z. Wang, N. Ye, R. Wang, and P. Li, "TMicroscope: Behavior perception based on the slightest RFID tag motion," *Elektronika Elektrotechnika*, vol. 22, no. 2, pp. 114–122, 2016.

[41] R. Malekian, D. C. Bogatinoska, A. Karadimce, J. Trengoska, N. Ye, and W. A. Nyako, "A novel smart ECO model for energy consumption optimization," *Elektronika Elektrotechnika*, vol. 21, no. 6, pp. 75–80, 2015.

[42] K. Prabu, D. S. Kumar, and R. Malekian, "BER analysis of BPSK-SIM-based SISO and MIMO FSO systems in strong turbulence with pointing errors," *Opt.-Int. J. Light Electron Opt.*, vol. 125, no. 21, pp. 6413–6417, 2014.

[43] R. Malekian and A. H. Abdullah, "Traffic Engineering based on effective envelope algorithm on novel resource reservation method over mobile Internet protocol version 6," *Int. J. Innov. Comput., Inf. Control*, vol. 8, no. 9, pp. 6445–6459, 2012.

[44] Z. Wang, N. Ye, R. Malekian, F. Xiao, and R. Wang, "TrackT: Accurate tracking of RFID tags with mm-level accuracy using first-order taylor series approximation," *A d Hoc Netw.*, vol. 53, pp. 132–144, Dec. 2016.

[45] S. Jie, T. Peng, X. Yuan, J. Xiangjun, and R. Malekian, "An improved Synchronous Control Strategy based on fuzzy controller for PMSM," *Elektronika Elektrotechnika*, vol. 20, no. 6, pp. 17–23, 2014.

YUPENG YUAN received the bachelor's degree in thermal energy and power engineering and the Ph.D. degree in marine engineering from the Wuhan University of Technology, China, in 2005 and 2012, respectively. He is currently a Lecturer with the School of Energy and Power Engineering, Wuhan University of Technology. His research interests include control theory and application, new/clean energy aided marine power system, condition monitoring, and fault diagnosis in marine power system.

ZHIXIONG LI (M'16) is currently a Senior Lecturer with the China University of Mining and Technology, China, and also a Research Associate with the School of Mechanical and Manufacturing Engineering, UNSW, Australia. His research interests include mechanical systems and signal processing. He is an Associate Editor of the Journal of the IEEE Access.
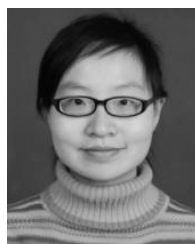
REZA MALEKIAN (M'10–SM'17) is currently an Associate Professor with the Department of Electrical, Electronic, and Computer Engineering, University of Pretoria, Pretoria, South Africa. His research interests include design and development of industrial applications using advanced sensor networks, Internet of Things. From 2013 to 2016, he was a Management Committee Member of the ICT COST Action IC1304 Autonomous Control for a Reliable Internet of Services. He is a Chartered Engineer registered in the Engineering Council of U.K. and a Professional Member of the British Computer Society.

YONGZHI CHEN is currently an Associate Researcher with the School of Energy and Power Engineering, Wuhan University of Technology. His current research interests include control and optimization, higher education.

YING CHEN received the B.S. degree in electronic engineering from the Wuhan University of Technology in 2006. She is currently a Teacher with the Department of communication, PLA Rocket Force Command College. Her current research interests include communication network, and communication and electronics systems.

● ● ●