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THE IMPACTS OF MAGNETIC FIELDS ON THE THERMOCAPILLARY CONVECTION IN TWO LAYERS FLUID SYSTEM*

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ABSTRACT

Under a horizontal temperature gradient along the liquid-liquid interface, the developing processes of thermocapillary convection in two layers immiscible fluids system absent gravity were simulated numerically, where the upper layer fluid was encapsulant B_2O_3, the underlayer fluid was melting InP in this paper. The effects of different direction magnetic field on the developing behaviors of thermocapillary convection were investigated. The results showed that the flow pattern was changed obviously and the thermocapillary convection was damped in some extent and the temperature distributions became more uniform if magnetic fields in *X*, *Y* or *Z* direction were applied. *Z* direction magnetic field had a stronger effect on the thermocapillary convection and it was enough to suppress convection significantly at B_z between 0.15T and 0.2T. The simulation became numerically unstable when B_z was over 0.2T.

NOMENCLATURE

B	[T]	magnetic field intensity
b	[T]	induced magnetic field intensity
f_i	[N/m^3]	Lorentz force
F_s	[N/m]	surface tension
H_a	Hartmann number, $H_a = BL\sqrt{\sigma_m / \mu_2}$	
J	[A/m^2]	induced current
L	[m]	characteristic length
M_a	Marangoni number, $M_a = \frac{\sigma_T \Delta T L}{\mu_2 \lambda_2}$	
P	[Pa]	pressure
T	[K]	temperature
t	[s]	time
u,v,w	[-]	dimensionless velocity components
X, Y, Z	[-]	dimensionless Cartesian coordinates

Greek symbols

ρ	[kg/m^3]	density
α	[m^2/s]	thermal diffusivity
λ	[W/mK]	thermal conductivity
μ	[Nsm^{-2}]/[Hm^{-1}]	dynamic viscosity/magnetic conductivity
ν	[m^2/s]	kinematic viscosity
σ_T	[N/mK]	interface tension coefficient
σ_m	[s]	electrical conductivity

Subscripts

i	ith fluid layer (i=1,2)
m	magnetic

INTRODUCTION

Thermocapillary convection is driven by the surface-tension gradient that results from temperature gradient on a free surface or interface. Eyer [1] and Koler [2] had confirmed that thermocapillary convection was the predominant cause for those solidification processes and dopant inhomogeneities in space and on earth. Furthermore, recent studies indicated that some crystal growth system had to be taken it into account to control the flow pattern of convection and obtain an improved crystal quality. At present, the application of magnetic fields to suppress the thermal convection is considered as an effective method. Yang [3,4] investigated the impact of magnetic field on the natural convection of two layers fluids system. The direction of the interfacial velocity was determined by the melt circulation without magnetic field. Under horizontal magnetic field, B_x =0.4T, the interfacial velocity was negative, which displayed a dominance of the natural convection in the melt but this dominance decreased with the field intensity increasing. Yu [5] simulated numerically the flow of silicon melt under a cusp magnetic field and the results showed that the flow pattern was different from that flow without magnetic field. With the increasing of magnetic intensity the convection was damped obviously, which was beneficial for the crystal growth. Anwar [6] investigated the influences of the altering of direction of the external magnetic fields. The results showed the change of direction of the external magnetic force from horizontal to vertical leaded to decrease in the flow rates in both the primary

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and the secondary cells and that caused an increase in the effect of the thermocapillary force. The simulation results of Yildiz [7] indicated that the use of a static, vertical magnetic field was effective in suppressing natural convection in the solution. A stationary field intensity of 0.3T was sufficient to provide significant suppression. The use of rotating magnetic field was effective in providing sufficient mixing in the melt leading to more homogeneous SiGe crystals.

Rajiv [8] used a moving finite element technique to simulate the thermal convection of electrically conducting binary alloy driven by the combined action of buoyancy, surface tension, and electromagnetic forces. The results showed that the intensity of the convective flow decreased and was followed by a progressive change in the overall structure of the flow for increasing strength of the magnetic field. The characteristics of the final flow structure strongly depended on various factors such as orientation and strength of the applied magnetic field and gravity level. Morthland [9] investigated the effect of magnetic field on the thermocapillary convection of the floating zone in microgravity environment. The author thought a strong magnetic field parallel to the free surface of the floating zone could eliminate the unsteady convection associated with the hydrothermal rolls, and if *B* was increased, the unsteady thermocapillary convection that caused the periodic dopant striations in the crystal periphery would be eliminated. Pablo [10] applied an axial magnetic field to suppress the thermocapillary convection of melt during floating-zone crystal growth. The main feature of this flow pattern was the stagnant core that developed in the inner part of the melt, where the thermocapillary convection was effectively suppressed. The study of Li [11] described that thermocapillary convection was suppressed obviously under axial magnetic field in an annular melt pool, where the three-dimensional unsteady flow was transferred to the two dimensional steady flow.

Based on the fact that applying external magnetic field does suppress the thermal convection effectively, so, in this paper, the developments of thermocapillary convection in two layers immiscible fluid system were simulated under different direction magnetic fields in the absence of gravity and their effect on the stability of interface was discussed.

PHYSICAL AND MATHEMATICAL MODEL

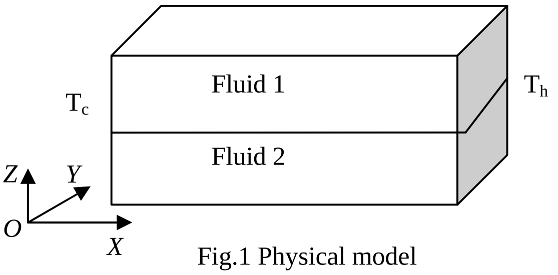


Fig.1 Physical model

The three-dimensional convective motions domain is a rectangular cavity as shown in Fig. 1 with a length (*X*) 40mm, depth (*Y*) 20mm, height (*Z*) 20mm, respectively. The characteristic length is L=40mm and the cavity is filled with the two layer immiscible liquids and each liquid layer height is 10mm. The upper layer fluid is encapsulant B_2O_3 and the underlayer fluid is melting InP. The right wall is maintained at a constant temperature T_h, while the left wall is at a lower temperature T_c ($T_h>T_c$). The other surfaces are considered to be adiabatic. Some assumptions are made in our model following as: (1) Two kinds of fluids are incompressible Newtonian fluid. (2) The flow is considered to be laminar flow. (3) The interface is flat and non-deformable. (4) All the walls are isolated.

With the above assumptions, the momentum and energy equations are expressed as following (subscript *i*=1,2, and 1 is denoted the upper layer, 2 is denoted the lower layer):

$$\nabla \cdot \upsilon_i = 0 \tag{1}$$

$$\frac{\partial \upsilon_i}{\partial t} + \upsilon_i \cdot \nabla \upsilon_i = \frac{1}{\rho_i}\nabla p_i + \nu_i \nabla^2 \upsilon_i + F_s + \frac{1}{\rho_i} f_i \tag{2}$$

$$\frac{\partial \boldsymbol{T_i}}{\partial \boldsymbol{t}} + \upsilon_i \cdot \nabla \boldsymbol{T_i} = \alpha_i \nabla^2 \boldsymbol{T_i} \tag{3}$$

where, Lorentz force is expressed as $f_i = J \times B$.

The Lorentz force is solved by Maxwell’s equations and Ohm’s law:

$$\frac{\partial \mathbf{b}}{\partial \boldsymbol{t}} + (\mathbf{V}\cdot\nabla)\mathbf{b} = \frac{1}{\boldsymbol{\sigma_m \mu_m}}\nabla^2\mathbf{b} + ((\mathbf{B_0}+\mathbf{b})\cdot\nabla)\mathbf{V} - (\mathbf{V}\cdot\nabla)\mathbf{B_0} \tag{4}$$

$$\nabla \cdot B = 0 \tag{5}$$

$$J = \frac{1}{\mu_m}(\nabla \times B) \tag{6}$$

$$\nabla \cdot J = 0 \tag{7}$$

where, B is magnetic intensity, including application magnetic field B_0 and induced magnetic field b.

EXTERNAL BOUNDARY CONDITIONS

All the walls are the no slip condition, and

$$T = T_c\,,\quad J_x = 0 \qquad \text{at } X=0 \tag{8a}$$

$$T = T_h\,,\quad J_x = 0 \qquad \text{at } X=\text{L} \tag{8b}$$

$$\frac{\partial T}{\partial t} = 0\,,\quad J_z = 0 \qquad \text{at } Z=0 \text{ and } L/2 \tag{8c}$$

$$\frac{\partial T}{\partial t} = 0\,,\quad J_y = 0 \qquad \text{at } Y=0 \text{ and } L/2 \tag{8d}$$

The boundary conditions at the interface (at *Z*=L/4):

$$u_1 = u_2\,,\quad \upsilon_1 = \upsilon_2\,,\quad w_1 = w_2 = 0 \tag{9a}$$

$$\frac{\partial u_2}{\partial z} - \frac{\mu_1}{\mu_2}\frac{\partial u_1}{\partial z} = -Ma\frac{\partial T_2}{\partial x} \tag{9b}$$

$$T_1 = T_2\,,\quad \lambda_1\frac{\partial T_1}{\partial z} = \lambda_2\frac{\partial T_2}{\partial z} \tag{9c}$$

$$J_z = 0 \tag{9d}$$

The initial conditions for fluids in the calculating domain are:

$$u_i = \upsilon_i = w_i = 0 \tag{9e}$$

$$T_i = \frac{T_c + T_h}{2} \tag{9h}$$

In this paper, $\Delta T = T_h - T_c$ =10， H_a =43-58. The physical properties of the fluid are according to reference [12].

COMPUTATIONAL METHOD

The PISO algorithm is used to handle the pressure-velocity coupling, the momentum and the energy equations are discretized by the first order upwind scheme. The total 128000 nodes with local refined regular mesh are adopted for the spatial discretization. According to Lebourcher’s study [13], to achieve a stable numerical solution throughout the simulation the time step should be reduced when magnetic fields are applied. So the adapt time step is used and varies between 1e^{-2} ~1e^{-4}. For the validation of the code, the simulation results of thermocapillary flow are compared with that of Schenkel [14] (see Fig.2). Fig.2 shows the velocity distribution at X=0.02m when Ma=150, where the solid line is Schenkel’s result and the dashed line is our simulation results, and the v is y direction velocity component. The two results have a well agreement.

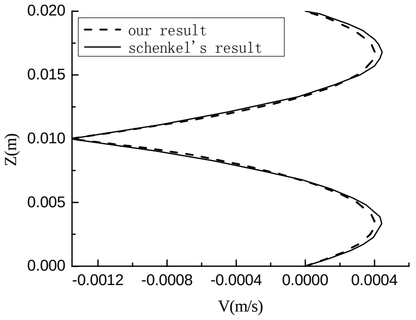
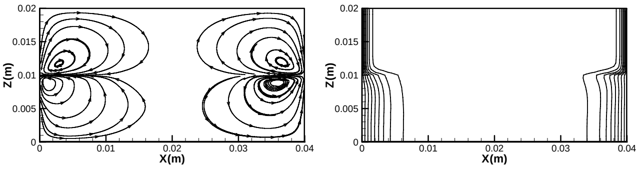
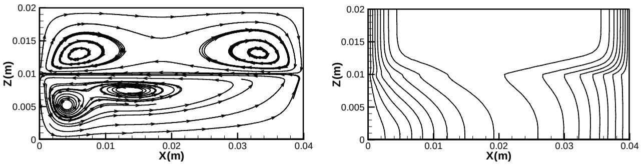


Fig.2 Comparison of the velocity distribution

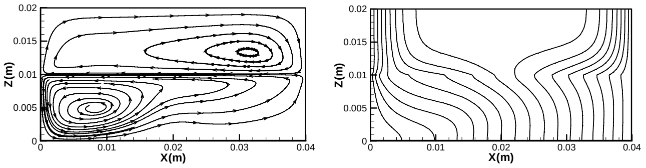
RESULTS AND DISCUSSION



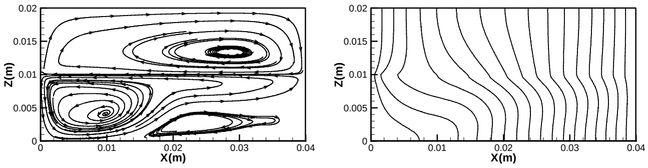
（a） t=0.5s



（b） t=4s



（c） t=10s



（d） fully developed

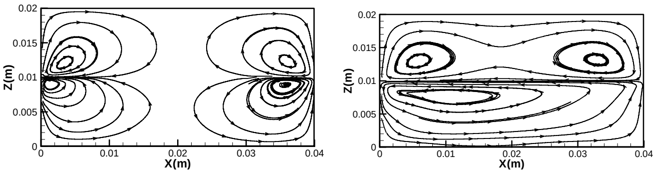
Fig.3 Variation of the streamline and the temperature distributions with time.

Figure 3 shows the projected streamline and the temperature distributions of the thermocapillary convection development at the location *y*=0.01m without magnetic field, in which, on the left side are streamlines shown and on the right side the isotherms. Fig. 3 reveals there are two convective eddies each layer fluid due to the impact induced by the thermocapillary force at t=0.5s and those eddies are departing from the cold or the hot wall. The upper eddies are obviously symmetric about the interface with that in the underlayer fluid. With the development of the thermocapillary convection the right vortex in the underlayer fluid moves to the left side firstly, and combines with the vortex near to the cold wall due to its higher conductivity and lower viscosity. However, the vortex near the cold wall migrates toward the bottom. The right vortex in the underlayer fluid has moved to the left side of the cavity and unites with the left vortex which moves down continuously at t=4s. In the upper layer fluid, two vortexes have moved to the central and have the tendency of mixture, also. At t=10s, the two vortexes in the underlayer have united to be a large convective cell which is similar to a ‘fly wheel’ pattern at the vicinity of the cold wall. Similar behaviour of convective cell was reported by other researcher[15,16]. At the same time, two vortexes in the upper layer combined to be a large convective cell near to the hot wall, also. The positions of two combined eddies in upper layer and underlayer fluids are opposite side of the cavity.

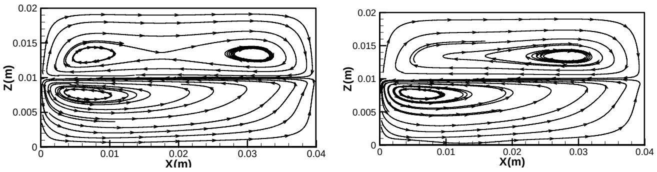
When the convection is fully developed, the vortex becomes asymmetric near the cold wall, and there is a secondary weaker counter-rotating vortex below the ‘fly wheel’ near to the right side of the cavity due to the intensity of convection becomes stronger for the underlayer fluid, but the flow pattern does not change for the upper layer fluid.

The temperature distribution depicts that the isotherms are nearly vertical and the gradient is uniform at the beginning because the heat transfer is mainly the heat conduction. The isotherms at the interface begin to become skew by the disturbance of thermocapillary convection and they become much more skew with the time because the thermocapillary convection becomes much stronger with the time. When the thermocapillary convection is fully developed, the isotherms in the upper layer fluid are still vertical due to its convection is weaker, but the isotherms' distortion is very obvious in the underlayer where the convection is stronger.

In order to understand the effect of the different magnetic fields on the development of thermocapillary convection, uniform magnetic field B_x, B_y or B_z was applied on the case, respectively. The intensity of B_x, B_y is all 0.2T, corresponding to Hartmann number 58 and the intensity of B_z=0.15T, corresponding to Hartmann number 43.



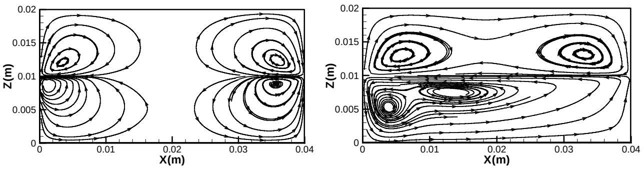
（a） t=0.5s （b） t=4s



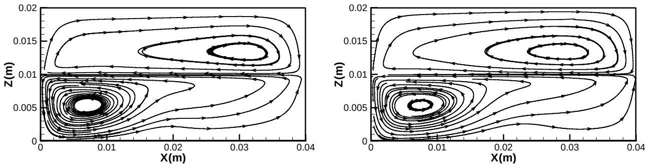
（c） t =10s （d） fully developed

Fig.4 The developing streamline of thermocapilary convection under the *X* direction magnetic fields.

Figure 4 shows the streamlines of the thermocapillary convection applying *X* direction magnetic field (at *Y*=0.01m). There are also two convective vortexes in the upperlayer and underlayer fluids, respectively at t=0.5s. With the development of the thermocapillary convection, the four vortexes are developing similar to the case without magnetic field. At t=4s, the two vortexes in underlayer fluid unite to be a long and narrow convective cell which locates at the underneath the interface and the development is a lit bit faster than that without magnetic field. The reason is that the Lorentz force induced by the *X* direction magnetic field damps the moving down of the left vortex, so their combining is earlier. At t=10s, the vortex intensity in the underlayer fluid increases continuously and the vortex center moves further toward the cold wall with the further developing of thermocapillary convection. However, the cell size is smaller compared with that without magnetic field. The vortexes developing in the upper fluid is slower than that without magnetic field because the magnetic field decreases obviously the intensity of the convection in underlayer fluid, which results in the intensity decrease of the convection in the upper fluid. When the thermocapillary convection is fully developed, a weaker counter-rotating vortex as shown in Fig.3(d) does not occur in the underlayer, which indicates the thermocapillary convection is suppressed effectively by magnetic field.



（a） t=0.5s （b） t=4s

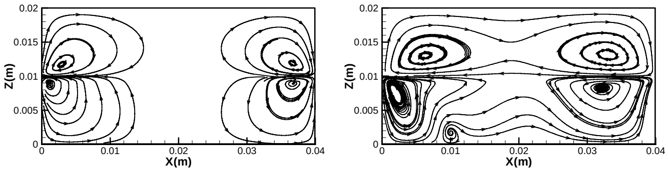


（c） t=10s （d） fully developed

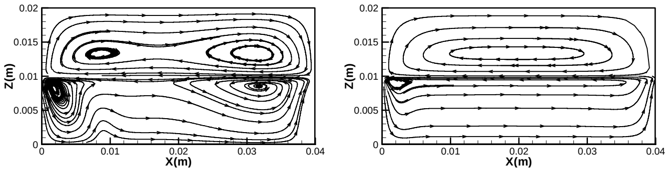
Fig.5 The developing streamline of the thermocapilary convection under the *Y* direction magnetic field

Figure 5 depicts the developing streamline of the thermocapillary convection under the *Y* direction magnetic field (at *Y*=0.01m). At t=0.5s, the streamlines are qualitatively same to that without magnetic field. However, comparing with the case without magnetic field, the center of the right vortex is much close to the hot wall in the underlayer fluid. The reason is that the direction of magnetic field is perpendicular to that of vortex moving, the direction of Lorentz force acting on the vortex is reverse to that of vortex's moving which suppresses the vortex moving. While t=4s, both vortexes in the upper fluid are transported to the middle of upper fluid, but the convection behavior of the underlayer fluid is almost same as that without magnetic field. At t=10s, there is a large convective cell and its center is closer to the hot wall in the upper fluid and two vortexes in the under layer unite to be one lager cell, also, which center is near to the cold wall. When the thermocapillary convection is fully developed, the flow pattern is similar to Fig.5(c), but the streamlines in the underlayer are much screwy, and the weaker counter-rotating convective cell dose not appear comparing with that of without magnetic field. That means the intensity of the convection is damped by the *y* direction magnetic field, however the influence is weak.

While applying the *Z* direction magnetic field B_z=0.2T which is equals to the intensity of B_x, B_y, it is sufficient to suppress the convection significantly but the simulating process is unstable numerically. Here, B_z=0.15T is applied to the system.



（a） t=0. 5s （b） t=4s



（c） t=10s （d）fully developed

Fig.6 The developing streamline of the thermocapilary convection under the *Z* direction magnetic field.

The development of the thermocapillary convection under the *Z* direction magnetic field is showed in Fig.6. At=0.5s, the vortexes in the underlayer are more close to the corner between the cold wall and the interface due to the influence of magnetic field, the streamlines are not smooth and the edges and corners occur. At t=4s, the shape of vortexes in the underlayer is changed obviously to a taper structure and the right vortex still stays near the hot wall. The vortex moving is slower than that without magnetic field. The left vortex is also stagnant at the corner, does not move down. At t=10s, two vortexes in the upper layer become one big cell and one small cell and the left vortex has the tendency to unite the right side vortex, but the development is slower. In the underlayer fluid, the left vortex becomes smaller and the right vortex becomes a flat and long shape due to the Lorentz force action, and both vortexes begin to unite at the periphery of the vortexes. Moreover, the convection developing in the case is slower than that without magnetic field, which means that the intensity of the convection is reduced and the developing is delayed significantly by the *Z* direction magnetic field. When the thermocapillary convection is fully developed, there is only one convective eddy in each layer fluid and occupies the entire domain, respectively. From Fig.6(d), it is found the vortex is smaller and closer to the corner between the interface and the cold wall and the streamlines become flat in underlayer comparing with that of the case applied *X* or *Y* direction magnetic field.

CONCLUSION

In this paper, the developments of the thermocapillary convection driven by the surface tension gradient were simulated and the effect of the magnetic field on the convection and the temperature distribution were investigated absent gravity. From the simulation results, the following conclusions were drawn:

1. At the beginning of the thermocapillary convection development, there are two convective cells in the upperlayer and underlayer fluid, respectively, which depart from the cold or the hot wall. Both vortexes in the upper fluid are symmetric about the interface with that in the underlayer fluid. The vortex in the underlayer fluid near to the hot wall moves toward the cold wall gradually with the time and unites the vortex near cold wall to be a lager convective cell which has a ‘fly wheel’ structure last. The convection intensity increases gradually with the development. In the upper layer fluid, the convection intensity increases gradually and forms a larger convective eddy, also, but the developing is obviously slower than that of the underlayer fluid.

2. Applying a magnetic field of any direction can suppress the thermocapillary convection and decrease its intensity, as well as delay the development process of the convection of upper layer fluid. The effect of the *Y* direction magnetic field on the convection is weakest. Applying the *X* direction magnetic field slows down the development in the upper layer fluid, but the developing is accelerated slightly in the underlayer fluid. The effect of the *Z* direction magnetic field on the thermocapillary convection is strongest and the developing is slowest. Furthermore, the development of the flow pattern is different from that of other cases.

3. The magnetic field intensity between 0.15T and 0.2T is sufficient to suppress convection significantly, and the simulating becomes numerically unstable when B_z is over 0.2T.

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