Periodically intermittent controlling for finite-time synchronization of complex dynamical networks

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Abstract In this paper, we consider finite-time synchronization between two complex dynamical networks by using periodically intermittent control. Based on finite-time stability theory, some novel and effective finite-time synchronization criteria are derived by applying stability analysis technique. The derivative of the Lyapunov function $V(t)$ is smaller than $\beta V(t)$ (β is an arbitrary positive constant) when no controllers are added into networks. This means that networks can be self-synchronized without control inputs. As a result, the application scope of synchronization greatly enlarged. Finally, a numerical example is given to verify the effectiveness and correctness of the synchronization criteria.

Keywords Complex dynamical networks · finite-time synchronization · periodically intermittent control · Lyapunov-Krasovskii functional

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1 Introduction

Complex dynamical networks consist of a number of nodes, which represent dynamic systems, and links between the nodes. Complex networks [1-4] exist in various fields of science, engineering and society, such as the World Wide Web, ecological networks, neural networks and electrical power grids, etc. As the major collective behavior [5-6], synchronization of complex networks is one of the key issues that has been extensively addressed [7-11] because the synchronization mechanism can explain well many significant and interesting natural phenomenons, including the synchronous information exchange in the Internet and the WWW, the synchronous transfer of digital or analog signals in communication networks and the synchronous cooperative behavior of close relationship in the nature and society.

Up to now, the synchronization for nonlinear systems especially dynamical networks [12-13] has been widely studied, and many control schemes, such as adaptive control [14-15], pinning control [10, 16], observer based control [17], integral control [18], and hybrid feedback control [19], have been focused on this topic. The above effective control approaches are continuous control. The discontinuous control methods which include impulsive control and intermittent control have attracted much interest due to its practical and easy implementation in engineering fields. The intermittent control, as a special form of switching control [20], was first introduced to control linear econometric models in [21]. On the other hand, the intermittent control [22-31] is different from the impulsive control [32-37] since impulsive control is activated only at some isolated instants, while intermittent control has a nonzero control width. What's more, intermittent control is more effective and robust [22]. In recent years, several synchronization criteria for complex dynamical networks with or without time delays via intermittent control have been presented, see [23-24, 29-30].

Nevertheless, to our best knowledge, the previous results focus on asymptotical or exponential synchronization of networks through intermittent control [22-30], and there are also many results concerning finite-time synchronization or stability [38-47]. However, there are very few results concerned with finite-time synchronization via intermittent control [48]. Compared with exponential synchronization of networks via intermittent control, the finite-time synchronization of networks by using intermittent control is realized in a finite time. Furthermore, the finite-time control strategies have demonstrated better robustness and disturbance rejection properties [49]. Therefore, it is worth studying the finite-time synchronization of complex networks via periodically intermittent control.

The main purpose of this paper is to achieve finite-time synchronization by adding an intermittent controller. Based on the finite-time stability, some novel criteria for finite-time synchronization between two complex networks are derived by using a central lemma. On the other hand, the conditions for finite-time synchronization via periodically intermittent control are expressed in terms of linear matrix inequalities (LMIs), which is easy to be verified. Besides, the derivative of the Lyapunov function $V(t)$ is smaller than $\beta V(t)$ (β is an arbitrary positive constant), which enriches the previous results in Ref. [48], when no controllers are added into networks. Finally, a numerical example is given to demonstrate the effectiveness of the proposed approach.

The paper is organized as follows. In Section 2, the synchronization problem to be considered and some necessary preliminaries are restated. In Section 3, a sufficient condition for the synchronization scheme being uniformly finite-time synchronized is obtained. In section 4, some simulation results are presented. Conclusions are drawn in Section 5.

2 Preliminaries

Consider a complex dynamical network consisting of N nodes, in which each node is a n-dimensional dynamical system. The state equation of the entire network is given as:

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), \quad i = 1, 2, \cdots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in R^n$ is the state vector of the ith dynamical node, $f : R^n \to R^n$ is a continuous vector function, the constant $c > 0$ is a coupling strength. $\Gamma \in R^{n\times n}$ is a positive definite diagonal matrix which describe the inner-coupling matrix and $A = (a_{ij}) \in R^{N\times N}$ is the coupling configuration matrix. If there is a connection from the nodes i to j $(j \neq i)$, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$ $(j \neq i)$, and the diagonal elements of matrices A is defined as

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}.$$

For simplicity, regarding model (1) as the master (or drive) system, and the response (or slave) system is given by:

$$\dot{y}_i(t) = f(y_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma y_j(t) + u_i(t), \quad i = 1, 2, \cdots, N, \tag{2}$$

where $y_i(t) = (y_{i1}(t), y_{i2}, \ldots, y_{in}(t))^T \in R^n$, $i = 1, 2, \ldots, N$, is the response state vector of the node i. $u(t) = (u_1(t), u_2(t), \ldots, u_N(t))^T$ is an intermittent controller defined by

$$\begin{cases} u_i(t) = -\eta_i e_i(t) - \overline{k}\frac{(\lambda_{\max}(P))^{\frac{1+\mu}{2}}}{\lambda_{\min}(P)} sign(e_i)|e_i|^{\mu}, \\ \qquad lT \le t < lT + \delta, \qquad i = 1, 2, \cdots, N, \\ u_i(t) = 0, \qquad lT + \delta \le t < (l+1)T, \\ \qquad i = 1, 2, \cdots, N, \end{cases} \tag{3}$$

where $|e_i|^{\mu} = (|e_{i1}|^{\mu}, |e_{i2}|^{\mu}, \ldots, |e_{in}|^{\mu})^T$, $sign(e_i) = (sign(e_{i1}), sign(e_{i2}), \ldots, sign(e_{in}))^T$, and $e_i(t)$ is synchronization error, $\eta_i > 0$ is a positive constant called control gain, $\overline{k} > 0$ is a tunable real constant, the real number μ satisfies $0 < \mu < 1$. Denote $\lambda_{\max}(P)(\lambda_{\min}(P))$ as the maximum (minimum) eigenvalue of the positive definite diagonal matrix P. $T > 0$ is the control period, $\delta > 0$ is called the control width (control duration). $\jmath = \{1, 2, \ldots, p\}$ is a finite natural number set and $l \in \jmath$. $\theta = \delta/T$ be the ratio of the control width δ to the control period T called control rate.

Remark 1. Obviously, when $\theta = 1$, the intermittent control (3) is degenerated to a continuous control input which has been extensively proposed in previous work (see [40-41]). However, this trivial case not being discussed in this paper.

Let $e_i(t) = y_i(t) - x_i(t)$ $(1 \le i \le N)$ be synchronization errors, according to the intermittent controller (3), then the error dynamical system can be derived as

$$\begin{cases} \dot{e}_i(t) = f(y_i(t)) - f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) \\ +u_i(t), \; lT \le t < lT + \theta T, \quad i = 1, 2, \cdots, N, \\ \dot{e}_i(t) = f(y_i(t)) - f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t), \\ \qquad lT + \theta T \le t < (l+1)T, \quad i = 1, 2, \cdots, N, \end{cases} \tag{4}$$

To obtain the results of this paper, we have the following assumption and some important lemmas.

Assumption 1. ([54]) Assume that there exists a positive definite diagonal matrix $P = \text{diag}(p_1, \ldots, p_n)$ and a diagonal matrix $\Theta = \text{diag}(\theta_1, \ldots, \theta_n)$, such that $f(\cdot)$ satisfies the following inequality:

$$(y-x)^T P(f(y)-f(x)-\Theta(y-x)) \le -\xi(y-x)^T(y-x), \quad (5)$$

for some $\xi > 0$, all $x, y \in R^n$ and $t > 0$.

The function $f(\cdot) \in QUAD(P, \Theta)$. it can be shown that QUAD assumption holds for several well-known chaotic oscillators, such as the Lorenz's systems, Chua's systems, Rössler's systems, and so on. This assumption has been widely given in the synchronization literature ([31],[42],[48],[53]).

Lemma 1 ([50]). Assume that a continuous, positive-definite function $V(t)$ satisfying the following differential inequality:

$$\dot{V}(t) \le -\alpha V^{\eta}(t), \quad \forall\, t \ge t_0, \quad V(t_0) \ge 0, \quad (6)$$

where $\alpha > 0, 0 < \eta < 1$ are two constants. Then, for any given $t_0, V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - \alpha(1-\eta)(t-t_0), \ t_0 \le t \le t_1, \quad (7)$$

and

$$V(t) \equiv 0, \quad \forall\, t \ge t_1, \quad (8)$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)}. \quad (9)$$

Lemma 2 ([51]). Suppose there exist a continuous, positive-definite function $V(t)$ satisfies the following inequality:

$$\dot{V}(t) \le -\alpha V^{\eta}(t) + \kappa V(t), \quad \forall t \ge t_0, \quad V^{1-\eta}(t_0) \le \frac{\alpha}{\kappa},$$

where $\alpha,\ \kappa > 0, 0 < \eta < 1$ are three constants. Then, the settling time t_2 satisfies

$$t_2 \le \frac{\ln(1 - \frac{\kappa}{\alpha} V^{1-\eta}(t_0))}{\kappa(\eta - 1)}.$$

Remark 2. Let $0 < \eta < 1,\ 0 < \theta < 1,\ \beta > 0$ and suppose that the time t_2 as the setting time of the above Lemma 2, there exists a positive real M, such that

$$\int_{t_0}^{t_2} e^{\beta(1-\eta)(1-\theta)s} ds < M$$

for any given initial value t_0.

Lemma 3. Suppose that function $V(t)$ is continuous and non-negative when $t \in [0, \infty)$ and satisfies the following conditions:

$$\begin{cases} \dot{V}(t) \le -\alpha V^{\eta}(t), & lT \le t \le lT + \theta T, \\ \dot{V}(t) \le \beta V(t), & lT + \theta T \le t < (l+1)T, \end{cases} \quad (10)$$

where $\alpha,\ \beta > 0,\ T > 0,\ 0 < \eta,\ \theta < 1$, then the following inequality holds:

$$V^{1-\eta}(t) \le V^{1-\eta}(0) e^{(1-\eta)\beta(1-\theta)t} - \alpha\theta(1-\eta)t, \quad 0 \le t \le t_4. \quad (11)$$

Proof. Take $M_0 = V^{1-\eta}(0)$ and $W(t) = V^{1-\eta}(t) + \alpha(1-\eta)t$, where $t \ge 0$. Let $Q(t) = W(t) - hM_0$, where $h > 1$ is a constant. It is easy to see that

$$Q(t) < 0, \qquad \text{for } t = 0. \quad (12)$$

First, we will prove that

$$Q(t) < 0, \qquad \text{for all } t \in [0, \theta T). \quad (13)$$

Otherwise, there exists a $t_0 \in [0, \theta T)$ such that

$$Q(t_0) = 0, \qquad \dot{Q}(t_0) > 0, \quad (14)$$
$$Q(t) < 0, \qquad 0 \le t < t_0. \quad (15)$$

Using Eqs. (12), (14) and (15), we obtain

$$\begin{aligned} \dot{Q}(t_0) &= (1-\eta)V^{-\eta}(t_0)\dot{V}(t_0) + \alpha(1-\eta) \\ &\le (1-\eta)V^{-\eta}(t_0)(-\alpha V^{\eta}(t_0)) + \alpha(1-\eta) \\ &= -\alpha(1-\eta) + \alpha(1-\eta) \\ &= 0 \end{aligned} \quad (16)$$

This contradicts the second inequality in (14), and so (13) holds.

Let $W_1(t) = [V^{1-\eta}(t) + \alpha(1-\eta)t]e^{-(1-\eta)\beta(t-\theta T)}$, and $H(t) = W_1(t) - hM_0 - \alpha(1-\eta)(t-\theta T)e^{-(1-\eta)\beta(t-\theta T)}$, $t \ge \theta T$. Next, we will prove that for $t \in [\theta T, T)$

$$H(t) \le 0, \qquad \text{for all } \ t \in [\theta T, T). \quad (17)$$

Otherwise, there exists a $t_1 \in [\theta T, T)$ such that

$$H(t_1) = 0, \qquad \dot{H}(t_1) > 0, \quad (18)$$
$$H(t) < 0, \qquad \theta T \le t < t_1. \quad (19)$$

Using Eqs. (18) and (19), we have

$$\begin{aligned} \dot{H}(t_1) &= (1-\eta)V^{-\eta}(t_1)\dot{V}(t_1)e^{-(1-\eta)\beta(t-\theta T)} \\ &\quad -\beta(1-\eta)e^{-(1-\eta)\beta(t-\theta T)}V^{1-\eta}(t_1) + \\ &\quad \alpha(1-\eta)e^{-(1-\eta)\beta(t-\theta T)} - \beta(1-\eta)\cdot \\ &\quad e^{-(1-\eta)\beta(t-\theta T)} \cdot \alpha(1-\eta)t - \alpha(1- \\ &\quad \eta)e^{-(1-\eta)\beta(t-\theta T)} + \beta(1-\eta)e^{-(1-\eta)\beta(t-\theta T)}\cdot \\ &\quad \alpha(1-\eta)(t-\theta T) \\ &\le \beta(1-\eta)V^{1-\eta}(t_1)e^{-(1-\eta)\beta(t-\theta T)} - \\ &\quad \beta(1-\eta)e^{-(1-\eta)\beta(t-\theta T)}V^{1-\eta}(t_1) - \\ &\quad \beta(1-\eta)e^{-(1-\eta)\beta(t-\theta T)} \cdot \alpha(1-\eta)t + \\ &\quad \beta(1-\eta)e^{-(1-\eta)\beta(t-\theta T)} \cdot \alpha(1-\eta)(t-\theta T) \\ &= -\beta\theta T(1-\eta)e^{-(1-\eta)\beta(t-\theta T)} \cdot \alpha(1-\eta) \\ &< 0, \end{aligned} \quad (20)$$

which contradicts the second inequality in (18). Hence (17) holds. From Eqs. (17), it is easy to see that

$$W(t) \le e^{(1-\eta)\beta(t-\theta T)}[hM_0 + \alpha(1-\eta)(t-\theta T)e^{-(1-\eta)\beta(t-\theta T)}]$$
$$= e^{(1-\eta)\beta(t-\theta T)}hM_0 + \alpha(1-\eta)(t-\theta T)$$
$$\le hM_0 e^{(1-\eta)\beta(1-\theta)T} + \alpha(1-\eta)(1-\theta)T. \quad (21)$$

Consequently, on the one hand, for $t \in [\theta T, T)$,

$$W(t) \le e^{(1-\eta)\beta(t-\theta T)}hM_0 + \alpha(1-\eta)(t-\theta T)$$
$$< hM_0 e^{(1-\eta)\beta(1-\theta)T} + \alpha(1-\eta)(1-\theta)T.$$

On the other hand, it follows from Eqs. (12) and (13) that for $t \in [0, \theta T)$

$$W(t) \le hM_0$$
$$< hM_0 e^{(1-\eta)\beta(1-\theta)T} + \alpha(1-\eta)(1-\theta)T.$$

So

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)T} + \alpha(1-\eta)(1-\theta)T, \quad \text{for all } t \in [0, T).$$

Similar to the proof of Eq. (13), we can proof that

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)T} + \alpha(1-\eta)(1-\theta)T$$

is true for $t \in [T, (1+\theta)T)$. Suppose $Q_1(t) = W(t) - hM_0 e^{(1-\eta)\beta(1-\theta)T} - \alpha(1-\eta)(1-\theta)T$, it is easy to see that $\dot{Q}_1(t) < 0$, for $t \in [T, (1+\theta)T)$. Similar to the proof of Eq. (17), we can verify

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)T+\beta(1-\eta)(t-\theta T-T)} + \alpha(1-\eta)(1-\theta)T + \alpha(1-\eta)(t-\theta T-T)$$
$$= hM_0 e^{(1-\eta)\beta(t-2\theta T)} + \alpha(1-\eta)(t-2\theta T)$$

for $t \in [T+\theta T, 2T)$. Take $W_2(t) = [V^{1-\eta}(t) + \alpha(1-\eta)t]e^{-(1-\eta)\beta(1-\theta)T-\beta(1-\eta)(t-\theta T-T)}$ and

$$H_1(t) = W_2(t) - hM_0 - [\alpha(1-\eta)(1-\theta)T + \alpha(1-\eta)(t-\theta T-T)]e^{-(1-\eta)\beta(1-\theta)T-\beta(1-\eta)(t-\theta T-T)},$$

then, according to the Eqs. (20), we can easy obtain that $\dot{H}_1(t) < 0$, for $t \in [T+\theta T, 2T)$.

In the following, we will use mathematical induction method to derive that the following statement are true.

For $nT \le t < (n+\theta)T$,

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)nT} + \alpha(1-\eta)(1-\theta)nT, \quad (22)$$

and for $(n+\theta)T \le t < (n+1)T$,

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)nT+\beta(1-\eta)(t-(n+\theta)T)} + \alpha(1-\eta)(1-\theta)nT + \alpha(1-\eta)(t-(n+\theta)T)$$
$$= hM_0 e^{(1-\eta)\beta[t-(n+1)\theta T]} + \alpha(1-\eta)[t-(n+1)\theta T]. \quad (23)$$

Assume that inequalities (22) and (23) are true for $n \le k-1$, where k is a positive integer. Then, for any integer m satisfying $0 \le m \le k-1$, if $mT \le t < (m+\theta)T$,

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)mT} + \alpha(1-\eta)(1-\theta)mT$$
$$< hM_0 e^{(1-\eta)\beta(1-\theta)kT} + \alpha(1-\eta)(1-\theta)kT,$$

and if $(m+\theta)T \le t < (m+1)T$,

$$W(t) < hM_0 e^{(1-\eta)\beta[t-(m+1)\theta T]} + \alpha(1-\eta)[t-(m+1)\theta T]$$
$$< hM_0 e^{(1-\eta)\beta(m+1)(1-\theta)T} + \alpha(1-\eta)(1-\theta)(m+1)T$$
$$\le hM_0 e^{(1-\eta)\beta(1-\theta)kT} + \alpha(1-\eta)(1-\theta)kT.$$

Then, together with Eq. (12), for any $t \in [0, kT)$, we have

$$W(t) < hM_0 e^{(1-\eta)\beta k(1-\theta)T} + \alpha(1-\eta)(1-\theta)kT. \quad (24)$$

Similar to the proof of Eq. (13), we can prove that inequality (24) holds for $kT \le t < (k+\theta)T$. And similar to Eq. (17), we can verify the fact that

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)kT+\beta(1-\eta)(t-(k+\theta)T)} + \alpha(1-\eta)(1-\theta)kT + \alpha(1-\eta)(t-(k+\theta)T)$$
$$= hM_0 e^{(1-\eta)\beta[t-(k+1)\theta T]} + \alpha(1-\eta)[t-(k+1)\theta T]$$

for $(k+\theta)T \le t < (k+1)T$. Hence the detail of these proofs is omitted.

Therefore, from mathematical induction, we can conclude that the inequalities (22) and (23) hold for any positive integer n such that $nT \le t < (n+\theta)T$. If $(n+\theta)T \le t < (n+1)T$, we obtain $n \le t/T$, then

$$W(t) < hM_0 e^{(1-\eta)\beta(1-\theta)nT} + \alpha(1-\eta)(1-\theta)nT$$
$$\le hM_0 e^{(1-\eta)\beta(1-\theta)t} + \alpha(1-\eta)(1-\theta)t.$$

If $(n+\theta)T \le t < (n+1)T$, then it yields $n+1 > t/T$ and

$$W(t) < hM_0 e^{(1-\eta)\beta[t-(n+1)\theta T]} + \alpha(1-\eta)[t-(n+1)\theta T]$$
$$< hM_0 e^{(1-\eta)\beta(1-\theta)t} + \alpha(1-\eta)(1-\theta)t$$

Let $h \to 1$, from the definition of $W(t)$, we obtain

$$V^{1-\eta}(t) \le V^{1-\eta}(0)e^{(1-\eta)\beta(1-\theta)t} - \alpha(1-\eta)t + \alpha(1-\eta)(1-\theta)t$$
$$= V^{1-\eta}(0)e^{(1-\eta)\beta(1-\theta)t} - \alpha\theta(1-\eta)t$$

for any $t \ge 0$. The proof of Lemma 3 is completed.

Remark 3. Lemma 3 plays an important role in the finite-time synchronization analysis of dynamical networks via intermittent control in this brief, because it shows the utilization of finite-time intermittent control.

Lemma 4 (Jesen inequality [53]). If $a_1, a_2, \cdots, a_n$ are any positive numbers and $0 < r < p$, then

$$(\sum_{i=1}^{n} a_i^p)^{1/p} \leq (\sum_{i=1}^{n} a_i^r)^{1/r}.$$

3. Main results

In this section, with the help of Lemma 3, some novel finite-time synchronization criteria via periodically intermittent control are rigorously derived. The main results are stated as follows.

Theorem 1. Under Assumption 1, if there exist positive constants $\eta_1, \eta_2, \ldots, \eta_N,\ \xi,\ \beta$ and a positive definite diagonal matrix $P > 0$ such that the following conditions hold:

$$\theta_j I_N - \Xi + c\gamma_j A - \frac{\xi}{\lambda_{\max}(P)} I_N \leq 0, \tag{25}$$

$$\theta_j I_N - \frac{\xi}{\lambda_{\max}(P)} I_N + c\gamma_j A - \beta I_N \leq 0, \tag{26}$$

where $j = 1, 2, \ldots, n, \Gamma = diag(\gamma_1, \gamma_2, \ldots, \gamma_n), \Xi = diag(\eta_1, \eta_2, \ldots, \eta_N), \Theta = diag(\theta_1, \theta_2, \ldots, \theta_n)$ and I_N is the $N \times N$ identity matrix. Then the error system (4) is synchronized under the periodically intermittent controllers (3) in a finite time:

$$t \leq \frac{V^{\frac{1-\mu}{2}}(0) e^{\frac{1-\mu}{2}\beta(1-\theta)t}}{\bar{k}\theta(1-\mu)} = T_1, \tag{27}$$

where $V(0) = \sum_{i=1}^{N} e_i^T(0) P e_i(0),\ \ e_i(0)$ is the initial condition of $e_i(t)$.

Proof. Construct the following Lyapunov function:

$$V(t) = \sum_{i=1}^{N} e_i^T(t) P e_i(t). \tag{28}$$

Then the derivative of $V(t)$ with respect to time t along the solutions of Eq. (4) can be calculated as follows:

When $lT \leq t < (l+\theta)T$, for $l \in \jmath$,

$$\begin{aligned}
\dot{V}(t) = &\ 2\sum_{i=1}^{N} e_i^T(t) P \dot{e}_i(t) \\
= &\ 2\sum_{i=1}^{N} e_i^T(t) P[f(y_i(t)) - f(x_i(t)) + \\
&\ c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + u_i(t)] \\
= &\ 2\sum_{i=1}^{N} \{e_i^T(t) P[f(y_i(t)) - f(x_i(t)) - \Theta e_i(t)] + \\
&\ e_i^T(t) P\Theta e_i(t) + e_i^T(t) P u_i(t) + \\
&\ c e_i^T(t) P\Gamma \sum_{j=1}^{N} a_{ij} e_j(t)\} \\
\leq &\ -2\xi \sum_{i=1}^{N} e_i^T(t) e_i(t) + 2\sum_{i=1}^{N} e_i^T(t)(\Theta P - \eta_i P) e_i(t) \\
&\ +2c\sum_{i=1}^{N} e_i^T(t)\Gamma \sum_{j=1}^{N} a_{ij} P e_j(t) \\
&\ -2\bar{k}\sum_{i=1}^{N} \frac{(\lambda_{\max}(P))^{\frac{1+\mu}{2}}}{\lambda_{\min}(P)} e_i^T(t) P sign(e_i)|e_i|^{\mu} \\
\leq &\ -\frac{2\xi}{\max(P)} \sum_{i=1}^{N} e_i^T(t) P e_i(t) + 2\sum_{j=1}^{n} p_j \tilde{e}_j^T(t)(\theta_j I_N \\
&\ -\Xi + c\gamma_j A)\tilde{e}_j(t) - \\
&\ 2\bar{k}\sum_{i=1}^{N} \frac{(\lambda_{\max}(P))^{\frac{1+\mu}{2}}}{\lambda_{\min}(P)} |e_i(t)|^T P |e_i|^{\mu} \\
\leq &\ 2\sum_{j=1}^{n} p_j \tilde{e}_j^T(t)(\theta_j I_N - \Xi + c\gamma_j A - \\
&\ \frac{\xi}{\max(P)} I_N)\tilde{e}_j(t) - \\
&\ 2\bar{k}\sum_{i=1}^{N} \frac{(\lambda_{\max}(P))^{\frac{1+\mu}{2}}}{\lambda_{\min}(P)} |e_i(t)|^T P |e_i|^{\mu}.
\end{aligned}$$

Since $\sum_{i=1}^{N} |e_i(t)|^T |e_i|^{\mu} = \sum_{i=1}^{N}\sum_{j=1}^{n} |e_{ij}|^{1+\mu}$ and using Lemma 3, it can be implied that

$$(\sum_{i=1}^{N}\sum_{j=1}^{n} |e_{ij}|^{\mu+1})^{\frac{1}{1+\mu}} \geq (\sum_{i=1}^{N}\sum_{j=1}^{n} |e_{ij}|^2)^{\frac{1}{2}}.$$

Hence,

$$\begin{aligned}
\sum_{i=1}^{N}\sum_{j=1}^{n} |e_{ij}|^{1+\mu} \geq &\ (\sum_{i=1}^{N}\sum_{j=1}^{n} |e_{ij}|^2)^{\frac{1+\mu}{2}} \\
= &\ (\sum_{i=1}^{N} e_i^T(t) e_i(t))^{\frac{1+\mu}{2}}.
\end{aligned}$$

Then $\dot{V}$ becomes

$$\begin{aligned}
\dot{V}(t) \leq &\ 2\sum_{j=1}^{n} p_j \tilde{e}_j^T(t) Z_j \tilde{e}_j^T(t) - \\
&\ 2\bar{k}(\sum_{i=1}^{N} \lambda_{\max}(P) e_i^T(t) e_i(t))^{\frac{1+\mu}{2}}.
\end{aligned}$$

where $\tilde{e}_j(t) = [\tilde{e}_{j1}, \tilde{e}_{j2}, \ldots, \tilde{e}_{jN}]^T$ is a column vector of $e_j(t)$, Z_j is defined as

$$Z_j = \theta_j I_N - \Xi + c\gamma_j A - \frac{\xi}{\max(P)} I_N, \quad j = 1, 2, \ldots, n.$$

It follows from inequality (25) that

$$Z_j \leq 0,$$

which shows that $\dot{V}(t) \leq -2\overline{k}V^{\frac{1+\mu}{2}}(t)$.

When $(l+\theta)T \leq t < (l+1)T$, for $l \in \jmath$, we have

$$\begin{aligned}
\dot{V}(t) = & \sum_{i=1}^{N} e_i^T(t)P\dot{e}_i(t) \\
= & \sum_{i=1}^{N}\{e_i^T(t)P[f(y_i(t)) - f(x_i(t)) - \Theta e_i(t)] \\
& + e_i^T(t)P\Theta e_i(t) + c\sum_{j=1}^{N} a_{ij}e_i^T(t)P\Gamma e_j(t)\} \\
\leq & -\xi\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{j=1}^{n} p_j\widetilde{e}_j^T(t)\theta_j I_N\widetilde{e}_j(t) \\
& + c\sum_{j=1}^{n} p_j\widetilde{e}_j^T(t)\gamma_j A\widetilde{e}_j(t) - \\
& \beta\sum_{i=1}^{N} e_i^T(t)Pe_i(t) + \beta\sum_{i=1}^{N} e_i^T(t)Pe_i(t) \\
\leq & -\sum_{j=1}^{n} p_j\widetilde{e}_j^T(t)\frac{\xi}{\lambda\max(P)}I_N\widetilde{e}_j(t) + \\
& \sum_{j=1}^{n} p_j\widetilde{e}_j^T(t)\theta_j I_N\widetilde{e}_j(t) + c\sum_{j=1}^{n} p_j\widetilde{e}_j^T(t)\gamma_j A\widetilde{e}_j(t) \\
& -\sum_{i=1}^{n} p_j e_i^T(t)\beta I_N e_i(t) + \beta\sum_{i=1}^{N} e_i^T(t)Pe_i(t) \\
= & \sum_{j=1}^{n} p_j\widetilde{e}_j^T(t)S_j(\widetilde{e}_j^T(t) + \beta\sum_{i=1}^{N} e_i^T(t)Pe_i(t),
\end{aligned}$$

where $\widetilde{e}_j(t) = [\widetilde{e}_{j1}, \widetilde{e}_{j2}, \ldots, \widetilde{e}_{jN}]^T$ is a column vector of $e_j(t)$, S_j is defined as

$$S_j = \theta_j I_N - \frac{\xi}{\lambda\max(P)}I_N + c\gamma_j A - \beta I_N,\ j = 1, 2, \ldots, n.$$

It follows from inequality (26) that

$$S_j \leq 0,$$

which shows that $\dot{V}(t) \leq \beta V(t)$.

Namely, we have

$$\begin{cases} \dot{V}(t) \leq -2\overline{k}V^{\frac{1+\mu}{2}}(t), & lT \leq t < lT + \theta T, \\ \dot{V}(t) \leq \beta V(t), & lT + \theta T \leq t < (l+1)T. \end{cases} \tag{29}$$

Using Lemma 3, we obtain

$$V^{\frac{1-\mu}{2}}(t) \leq\ V^{\frac{1-\mu}{2}}(0)e^{\frac{1-\mu}{2}\beta(1-\theta)t} - \overline{k}\theta(1-\mu)t. \tag{30}$$

By Lemma 2, we have

$$t \leq \frac{V^{\frac{1-\mu}{2}}(0)e^{\frac{1-\mu}{2}\beta(1-\theta)t}}{\overline{k}\theta(1-\mu)} = T_1.$$

This implies the conclusion and the proofs is completed.

Remark 4. From condition (4.5) of Theorem 2 in Ref. [48], one can known that the derivative of the Lyapunov function $\dot{V}(t) \leq 0$ when no controllers are added into the network. This means that network can be self-synchronization without control inputs. This strong inequality condition has limited the application. In this paper, a novel differential inequality (see Lemma 3) is established, based on which some new and useful results are then derived, that is, the derivative of the Lyapunov function $\dot{V}(t) \leq \beta V(t)$ when no controllers are added into networks. Hence, in such sense the results derived here generalize the results in Ref. [48].

If the Lyapunov function $\dot{V}(t) \leq 0$ $(\beta = 0)$ when no controllers are added into networks, that is, we can clear the term $\beta V(t)$. Then, it is easy to see that Theorem 1 can be restate as the following form.

Corollary 1. Under Assumption 1, if there exist positive constants $\eta_1, \eta_2, \ldots, \eta_N,\ \xi$ and a positive define diagonal matrix $P > 0$ such that the following conditions hold:

$$\theta_j I_N - \Xi + c\gamma_j A - \frac{\xi}{\lambda\max(P)}I_N \leq 0, \tag{31}$$

$$\theta_j I_N - \frac{\xi}{\lambda\max(P)}I_N + c\gamma_j A \leq 0, \tag{32}$$

where $j = 1, 2, \ldots, n, \Gamma = diag(\gamma_1, \gamma_2, \ldots, \gamma_n), \Xi = diag(\eta_1, \eta_2, \ldots, \eta_N), \Theta = diag(\theta_1, \theta_2, \ldots, \theta_n)$ and I_N is the $N \times N$ identity matrix. Then the error system (4) is synchronized under the periodically intermittent controllers (3) in a finite time:

$$t \leq \frac{V^{1-\eta}(0)}{\overline{k}\theta(1-\mu)} = T_2, \tag{33}$$

where $V(0) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(0)Pe_i(0),\ e_i(0)$ is the initial condition of $e_i(t)$.

Remark 5. Corollary 1 in this paper is the main result of Theorem 2 in Ref. [48].

Suppose $\theta = 1$, the periodically intermittent control problem becomes a general control problem, then based on Theorem 1, the following Corollary 1 is immediate.

Corollary 2. Under Assumption 1, if there exist positive constants $\eta_1, \eta_2, \ldots, \eta_N,\ \xi$ and a positive definite diagonal matrix $P > 0$ such that the following conditions hold:

$$\theta_j I_N - \Xi + c\gamma_j A - \frac{\xi}{\lambda\max(P)}I_N \leq 0, \tag{34}$$

where $j = 1, 2, \ldots, n, \Gamma = diag(\gamma_1, \gamma_2, \ldots, \gamma_n), \Xi = diag(\eta_1, \eta_2, \ldots, \eta_N), \Theta = diag(\theta_1, \theta_2, \ldots, \theta_n)$ and I_N is the $N \times N$ identity matrix. Then the error system (4) is synchronized under the controllers (3) in a finite time:

$$t \leq \frac{V^{1-\eta}(0)}{\overline{k}(1-\mu)} = T_3, \tag{35}$$

where $V(0) = \sum_{i=1}^{N} e_i^T(0)Pe_i(0)$, $e_i(0)$ is the initial condition of $e_i(t)$.

Remark 6. According to the Eqs. (27), (33) and (35), and the convergence time T_1, T_2, T_3, we can conclude that the convergence time satisfies: $T_3 \leq T_2 \leq T_1$. Hence, the term $\beta V(t)$ should cost the setting time.

Let $\vartheta = \theta_1 = \ldots = \theta_n$ and $\gamma = \gamma_1 = \ldots = \gamma_n$ in Theorem 1, then we have the following corollary.

Corollary 3. Under Assumption 1, if there exist positive constants $\eta_1, \eta_2, \ldots, \eta_N$, ξ, β, and a positive define diagonal matrix $P > 0$ such that the following conditions hold:

$$\vartheta I_N - \varXi + c\gamma A - \frac{\xi}{\lambda_{\max}(P)} I_N \leq 0, \tag{36}$$

$$\vartheta I_N - \frac{\xi}{\lambda_{\max}(P)} I_N + c\gamma A - \beta I_N \leq 0, \tag{37}$$

where $\varGamma = diag(\gamma, \gamma, \ldots, \gamma), \varXi = diag(\eta_1, \eta_2, \ldots, \eta_N)$, $\varTheta = diag(\vartheta, \vartheta, \ldots, \vartheta)$ and I_N is the $N \times N$ identity matrix. Then the error system (4) is synchronized under the periodically intermittent controllers (3) in a finite time:

$$t \leq \frac{V^{\frac{1-\mu}{2}}(0)e^{\frac{1-\mu}{2}\beta(1-\theta)t}}{\overline{k}\theta(1-\mu)} = T_4, \tag{38}$$

where $V(0) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(0)Pe_i(0)$, $e_i(0)$ is the initial condition of $e_i(t)$.

Remark 7. Our objective is to seek an appropriate control gain η for which synchronization happens. In inequalities (36) and (37), the control gain is required to be large enough $[\eta \geq \vartheta + c\gamma\lambda_{\max}(A) - \frac{\xi}{\max(P)}]$, but it may be much large than the needed value. So, some previous works adopted adaptive control approach [14-15], but here we can choose a parameter ξ to adjust the appropriate control gain η.

4. Simulation example

In this section, an example is presented to show the validity and effectiveness of the derived results.

Example 4.1. Consider the network:

$$\dot{x}_i = f(x_i) + \sum_{j=1}^{100} a_{ij}\varGamma x_j, \quad i = 1, 2, \ldots, 100, \tag{39}$$

for

$$x_i = \begin{bmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i2} \end{bmatrix}, \ f(x_i) = C \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix} + \begin{bmatrix} 0 \\ -x_{i1}x_{i3} \\ x_{i1}x_{i2} \end{bmatrix},$$

$$C = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix}, \ \varGamma = I_3, \ c = 1,$$

where the parameters are selected as $a = 10$, $c = 28$, $b = 8/3$, the single Lorenz system has a chaotic attractor (see Ref. [48]). And from [53], we take $P = \mathrm{diag}(24, 0.002, 12)$ and $\varTheta = \mathrm{diag}(3, 3, 3)$, the Lorenz system is easily satisfies Assumption 1. $A = (a_{ij})_{100\times100}$ is a symmetrically diffusive coupling configuration matrix with $a_{ij} = 0$ or 1 $(j \neq i)$.

System (39) is considered as a drive system, the controlled response system is given by

$$\dot{y}_i = f(y_i) + \sum_{j=1}^{100} a_{ij}\varGamma y_j + u_i, \quad i = 1, 2, \cdots, 10, \tag{40}$$

where c, a_{ij}, and $f_i(\cdot)$ are the same as (39) and the controllers are given as (3) and $\overline{k} = 10$.

In the simulation, the values of the parameters for the controllers (3) are taken as $T = 0.2$ and $\delta = 0.16$. Since $\theta_j = 3$ and $\lambda_{\max}(A) = -1.6414$. By using LMI toolbox in Matlab, it is easy to verify that inequalities (25) and (26) in Theorem 1 is satisfied and we can obtain $\eta_i = 6.4142$.

The initial conditions of the numerical simulation are as follows: $x_i(0) = (-8 + 0.5i, -5 + 0.5i, -10 + 0.5i)^T$, $y_i(0) = (2 + 0.5i, 0.2 + 0.5i, 0.3 + 0.5i)$, where $1 \leq i \leq 100$. The synchronous errors $e_i(t)$ are illustrated in Figs. 1-3.

5 Conclusion

In this paper, we have investigated the finite-time synchronization between two complex networks by using intermittent control. Some novel and useful synchronization criteria ensuring the systems to synchronize up to zero in a given time are obtained. In the previous works, the derivative of the Lyapunov function $V(t)$ is smaller than zero, but the strong results have released for the $\dot{V}(t) \leq \beta V(t)$ (β is an arbitrary positive constant) in this paper. An example is presented to verify the effectiveness of the proposed synchronization criteria finally. Currently, semi-global finite-time synchronization criteria of complex dynamical networks via periodically intermittent control have been discussed in this paper, the global problem will be investigated and it will be

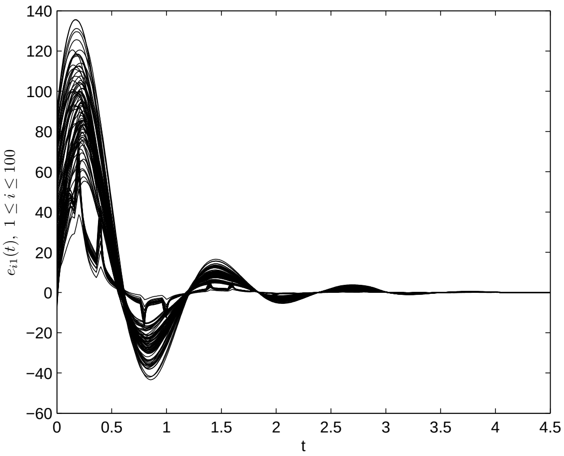

Fig. 1 The synchronization errors e_{i1} $(1 \leq i \leq 100)$ with periodically intermittent controllers (3) under parameters $T = 0.2,\ \theta = 0.8$

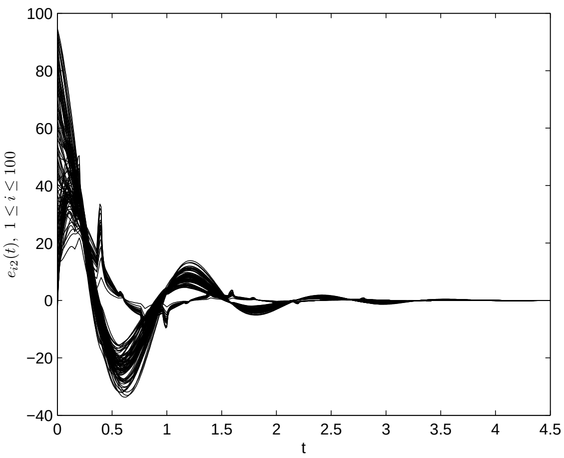

Fig. 2 The synchronization errors e_{i2} $(1 \leq i \leq 100)$ with periodically intermittent controllers (3) under parameters $T = 0.2,\ \theta = 0.8$

worth to solve in the future.

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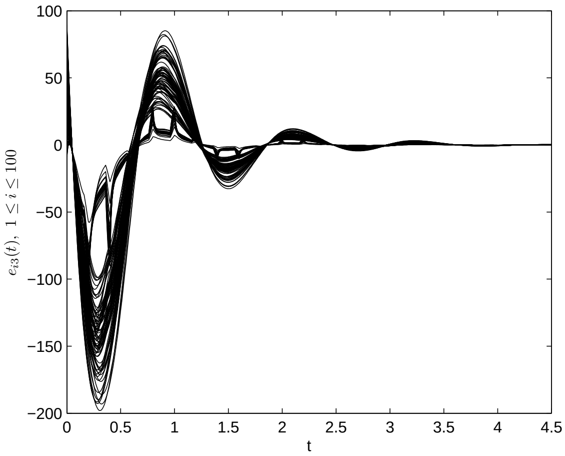

Fig. 3 The synchronization errors e_{i3} $(1 \leq i \leq 100)$ with periodically intermittent controllers (3) under parameters $T = 0.2,\ \theta = 0.8$

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