Reconstruction of Deformed Bite Marks using Affine Transformations.

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RUNNING HEADING: Deformation of Bite marks

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ABSTRACT

Bite marks inflicted on animate and inanimate objects will undergo a certain degree of deformation. This fact remains one of the biggest stumbling blocks when analysing evidence for court presentation. It has been demonstrated that the reliability of pattern association analysis will not be affected by minimal degrees of warping, shrinkage and distortion. In this study affine transformations were applied to bite marks to establish if minimal distortions would affect the mathematically determined relationships of the defined features. In a real case study, it was then tested whether the distorted bite marks match the dentition of the perpetrator by applying an affine transformation. This was confirmed to be the case. Affine transformations will thus not affect the relationships of the individual features found in bite marks. The numerical calculations validate the reliability of pattern association analysis in the presence of minimal amounts of warping, shrinkage and distortion.

KEYWORDS: Forensic Odontology, bite mark deformations, affine transformations.

Many different techniques have been used to analyse bite marks found on animate and inanimate objects(1-5). Irrespective of the techniques used, the degree of warping, shrinkage and distortion present in bite mark patterns remains one of the biggest stumbling blocks when analysing evidence for court presentation(1, 6). For the purposes of this paper, deformation will include warping, shrinkage and distortion. The expert will never know the exact position of the victim during the biting episode, the exact degree of moisture loss as a result of dehydration of a bitten apple exhibit or the precise percentage of butterfat which oozed from a sample of cheese(4). Although the deformation may be microscopic in nature, it creates a degree of uncertainty when expert evidence is given in bite mark-associated court cases(7). Bernitz(1) has shown that a small degree of warping, shrinkage and distortion will not affect the pattern associated analysis of bite marks. He demonstrated the concept by comparing a set of facial portraits taken of Gary Player at 45 and 75 years of age, and altered with digital distortion. The golfer was easily identified despite the induced digital changes and the small amount of deformation that had taken place during the 30 year period. The ease of recognition was however due to the constant and unchanged relationships of his facial features. The purpose of this study is to analyse mathematically the

deformation of bite marks, so as to demonstrate the constant numerical relationships that exists during these deformation processes. The mathematical proof of the cognitive method as described by Bernitz(1) will be of invaluable help to expert witnesses when giving evidence in bite mark cases.

Materials and Methods

Affine transformations can be used to reconstruct a bite mark that was distorted as a result of stretching, shrinking, enlargement, reduction, rotation, shearing or any combination of the above. In order to reconstruct a transformed bite mark, three points on the suspect's dentition that correspond with three points on the inflicted bite mark are needed. If three points on a bite mark image are in a straight line, they will remain in a straight line after the transformation. This type of transformation preserves collinearity, midpoints of segments, and segment division ratios. Any affinity is the product of a shear, a strain and a similarity. A range of possible distortions that could occur in a bite mark are illustrated in Fig 1. These image distortions preserve both image relationships and key characteristics. The affine transformations do not, however, preserve distances. Applying an affine transformation to a uniformly deformed bite mark can correct for the enlargement, reduction, stretching and shearing of the mark(8). To demonstrate this concept the following mathematical explanation is offered.

AffineTransformation (Affinity)

Any affinity maps each point $P(x_P, y_P)$ of the Euclidean plane to a point

P' according to the equation $P' = AP$ where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$ and where the matrix

A is invertible.

Cederberg(9) explains that for any three non-collinear points P , Q and R , there is an affinity that maps the points to P' , Q' , and R' (Fig 2).

Using the three points P , Q and R on the inflicted bite mark and three corresponding points on the suspect's dentition P' , Q' and R' it is possible to determine matrix A with values a_{11} , a_{12} , a_{13} , a_{21} , a_{22} , and a_{23} . To determine if the bite mark is caused by the

suspect's dentition we apply the inverse of the matrix A , called A^{-1} on all the coordinates of the bite marks and determine if they corresponds with the suspect's dentition. The question, however, is how to find the matrix A of the affinity from the values of $P(x_P, y_P) \rightarrow P'(x_{P'}, y_{P'})$, $Q(x_Q, y_Q) \rightarrow Q'(x_{Q'}, y_{Q'})$, and $R(x_R, y_R) \rightarrow R'(x_{R'}, y_{R'})$.

To Find the Matrix Representation of an Affinity

To be able to define an affinity A we need to identify three pairs of corresponding points in both images. In Fig 2 we use the following pairs: $P(x_P, y_P) \rightarrow P'(x_{P'}, y_{P'})$, $Q(x_Q, y_Q) \rightarrow Q'(x_{Q'}, y_{Q'})$, and $R(x_R, y_R) \rightarrow R'(x_{R'}, y_{R'})$. Substituting these pairs of corresponding points into the equation $P' = AP$ will result in the following equations:

$$x_P a_{11} + y_P a_{12} + a_{13} = x_{P'} \quad \dots \text{Equation 1}$$

$$x_P a_{21} + y_P a_{22} + a_{23} = y_{P'} \quad \dots \text{Equation 2}$$

$$x_Q a_{11} + y_Q a_{12} + a_{13} = x_{Q'} \quad \dots \text{Equation 3}$$

$$x_Q a_{21} + y_Q a_{22} + a_{23} = y_{Q'} \quad \dots \text{Equation 4}$$

$$x_R a_{11} + y_R a_{12} + a_{13} = x_{R'} \quad \dots \text{Equation 5}$$

$$x_R a_{21} + y_R a_{22} + a_{23} = y_{R'} \quad \dots \text{Equation 6}$$

Using equations 1, 3 and 5 and applying Cramer's rule for a system in three variables we can find the values of a_{11} , a_{12} , and a_{13} :

$$a_{11} = \frac{x_{P'} y_Q + x_{Q'} y_R + x_{R'} y_P - x_{R'} y_Q - x_{P'} y_R - x_{Q'} y_P}{x_P y_Q + x_Q y_R + x_R y_P - x_R y_Q - x_P y_R - x_Q y_P}$$

$$a_{12} = \frac{x_P x_{Q'} + x_Q x_{R'} + x_R x_{P'} - x_R x_{Q'} - x_P x_{R'} - x_Q x_{P'}}{x_P y_Q + x_Q y_R + x_R y_P - x_R y_Q - x_P y_R - x_Q y_P}$$

$$a_{13} = \frac{x_P y_Q x_{R'} + x_Q y_R x_{P'} + x_R y_P x_{Q'} - x_R y_Q x_{P'} - x_P y_R x_{Q'} - x_Q y_P x_{R'}}{x_P y_Q + x_Q y_R + x_R y_P - x_R y_Q - x_P y_R - x_Q y_P}$$

Using equations 2, 4 and 6 and applying Cramer's rule for a system in three variables we can find the values of a_{21} , a_{22} , and a_{23} :

$$a_{21} = \frac{y_{P'} y_Q + y_{Q'} y_R + y_{R'} y_P - y_{R'} y_Q - y_{P'} y_R - y_{Q'} y_P}{x_P y_Q + x_Q y_R + x_R y_P - x_R y_Q - x_P y_R - x_Q y_P}$$

$$a_{22} = \frac{x_P y_{Q'} + x_Q y_{R'} + x_R y_{P'} - x_R y_{Q'} - x_P y_{R'} - x_Q y_{P'}}{x_P y_Q + x_Q y_R + x_R y_P - x_R y_Q - x_P y_R - x_Q y_P}$$

$$a_{23} = \frac{x_P y_Q y_{R'} + x_Q y_R y_{P'} + x_R y_P y_{Q'} - x_R y_Q y_{P'} - x_P y_R y_{Q'} - x_Q y_P y_{R'}}{x_P y_Q + x_Q y_R + x_R y_P - x_R y_Q - x_P y_R - x_Q y_P}$$

The matrix representation of the affinity that maps point P to P', Q to Q', and R to R' is

$$T = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Use the inverse of matrix T and multiply it with the coordinates of the end points of each segment of the bite mark. It will correspond with the endpoints of the coordinates of each segment of the suspect's dentition in the case of a uniformly distorted bite mark.

Example of an Actual Case

A real case study was used to demonstrate the degree of correlation that exists after the affine transformation of a bite mark. The lower dental study models of the suspect were used and represented the original image. The bite mark present on the breast of the victim represented the transformed image. The following analytical calculations were carried out to demonstrate that the transformations present in the bite mark on the victim did not significantly affect the relationships of the individual features.

We need three pairs of points to define an affinity, Fig 3. We can use the following pairs: A(4.05, 4.39) → A'(2.62, -3.04), E(10.5; 2.41) → E'(9.5, -6.93), and H(16.22, 4.63) → H'(17.59, -3.94).

Using the method as illustrated in the previous paragraph the matrix that represent this

$$\text{affinity is } T = \begin{bmatrix} 1.22021 & 0.500179 & -4.517638 \\ -0.105894 & 1.61969 & -9.721569 \\ 0 & 0 & 1 \end{bmatrix}.$$

This matrix that defines the affine transformation was determined by mapping the point A, E and H on the suspects dentition to the points A', E' and H' on the bite mark (Fig 3 & 4).

If the bite mark on the victim was inflicted by the suspect's dentition, the inverse of the affinity T^{-1} must map points A', B', C', D', F', G', and H' on the suspects dentition which is indicated by A, B, C, D, F, G, and H. When applying the inverse of the affinity T^{-1} to the points the calculations show that the coordinates of the transformed

bite mark are A"(4.05, 4.39), B"(5.91, 2.97), C"(6.94, 2.99), D"(9.7, 2.46), E"(10.5, 2.41), F"(13.43, 2.78), G"(13.85, 3.18), and H"(16.22, 4.63) (Fig 4).

To determine if the bite mark was caused by the suspect's dentition we compare the coordinates of the points A, B, C, D, F, G, and H of the suspect's dentition with A", B", C", D", E", F", G" and H" respectively.

Results

Affine transformations will not affect the relationships of the individual features found in bite marks. Mathematically defined points will maintain a constant relationship during minor deformations. The real case study clearly demonstrated that mathematically calculable affine transformation had taken place during the biting process and that the bite mark was in fact a distorted image of the original suspect's dentition.

Discussion

This study shows that the cognitive method described by Bernitz in 2005(1) can be mathematically validated. Minor distortions in the marks left by the suspect's dentition will be present in all bite marks inflicted on skin and inanimate objects during the biting process and during the analysis of the bite marks. Variations in tissue structure, dehydration and photographic technique will induce these distortions. The transformations observed in bite marks will thus never be perfectly uniform. The results of the mathematical model applied to the real case study showed that minor distortions did not affect the ability to show positive concordance between the suspect's dentition and the bite mark. The effect on the pattern association and affine transformation analysis is negligible as long as the deformities are minimal. Distortions encountered in routine bite mark analysis can generally be considered to be minimal. When analysing a minimally distorted bite mark the presence of a diastema between two central teeth might vary in width between the dentition of the perpetrator and the bite mark present on the victim, but the space between the central teeth will be present in both cases and have a similar relationship to the adjacent teeth.

This concept can be applied to rotated teeth, missing teeth, teeth out of the arch, intercanine widths, incisal grooves or any other recognisable tooth features(2). The real case study clearly demonstrated a high degree of concordance after the common points within the bite mark and suspect's dentition were evaluated mathematically and confirmed to be affine transformations. The expert witness is required to demonstrate that the tooth marks present on the victim's body or inanimate object and the suspect's dentition show similar dental features present in the same position, in relation to the same teeth, in the same shaped arches and have similar size ratios(1-5, 10). The difficulties experienced by expert witnesses regarding minimal amounts of warping, shrinkage and distortion present in bite marks can now be scientifically nullified by the above numerical explanation.

Conclusion

Small amounts of warping, shrinkage and distortion will not affect the relationships of features within a bite mark. The pattern association analysis of bite marks can be applied to skin and inanimate objects as long as the deformations are minimal. Affine transformations can be applied to skin and inanimate objects as long as the deformations are minimal. The mathematical limits of these deformations which will significantly affect the relationship of dental features have not yet been determined.

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FIG. 1- *Examples of affine transformations*

FIG.2- *Three pairs of corresponding points define an affinity*

FIG.3- Coordinates of tooth structure

FIG.4 - Transformed bite mark

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