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Structural Breaks and GARCH Models of Stock Return Volatility: The Case of South Africa. [♦]

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ABSTRACT

This paper investigates the empirical relevance of structural breaks in forecasting stock return volatility using both in-sample and out-of-sample tests and daily returns for the Johannesburg Stock Exchange (JSE) All Share Index from 07/02/1995 to 08/25/2010. We find evidence of structural breaks in the unconditional variance of the stock returns series over the period, with high levels of persistence and variability in the parameter estimates of the GARCH (1, 1) model across the sub-samples defined by the structural breaks. This indicates that structural breaks are empirically relevant to stock return volatility in South Africa. In out-of-sample tests, we find that combining forecasts from different benchmark and competing models that accommodate structural breaks in volatility improves the accuracy of volatility forecasting. Furthermore, for shorter horizons, the MS-GARCH model better captures asymmetry in stock return volatility than the GJR-GARCH (1, 1) model, which better suited to longer horizons, but in general, the asymmetric models fail to outperform the GARCH (1,1) model.

Keywords: stock return volatility, structural breaks, in-sample tests, out-of-sample tests, GARCH Models.

JEL classification: C22, C53, G11, G12

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1. Introduction

The impact of structural breaks on the accuracy of volatility forecasts has largely been ignored in previous research. This is because researchers in using the generalised autoregressive conditional heteroscedastic (GARCH) model of Engle (1982) and Bollerslev (1986) often assume (both implicitly and explicitly) the existence of a stable GARCH process in volatility forecasting. As a result most researchers use a fixed or expanding window when estimating GARCH models used to generate out-of-sample volatility forecasts. This affects the accuracy of volatility forecasts using GARCH processes in several ways.

Failure to account for structural breaks in the unconditional variance of stock market returns can lead to sizeable upward biases in the degree of persistence in estimated GARCH models (Mikosh *et al.* 2004; Hillebrand, 2005; building on earlier work by Diebold, 1986; Hendry, 1986; Lamoureux *et al.* 1990). With structural breaks GARCH models do not accurately track changes in the unconditional variance leading to forecasts that underestimate or overestimate volatility on average for long stretches. This is because the fixed or expanding window mechanism as used under stable GARCH processes does not perform well in the presence of structural breaks (West *et al.* 1995). Again neglecting structural breaks in the unconditional variance may lead to over persistent GARCH models which have adverse effects on volatility forecasts (Rapach *et al.* 2008). Consequently long-horizon forecasts of stock return volatility generated by GARCH (1, 1) models that allow for periodic changes in the unconditional variance of stock returns have been found to yield better results than forecasts that assume parameter stability (Stărică *et al.* 2005).

Despite extensive work on volatility forecasting of asset returns, hardly any work is specific to South Africa in terms of forecasting the volatility of stock market returns. The only study we are aware of is by Samouilhan and Shannon (2008), who use a small data set of 682 observations (01/02/2004-28/09/2006) of daily data for the TOP40 index of the Johannesburg stock exchange (JSE). The authors investigated the comparative ability of three types of volatility forecasts namely different autoregressive conditional heteroscedasticity (ARCH) by Engle (1982), and as generalised by Bollerslev (1986) on one hand, a Safex Interbank Volatility Index (SAVI) for the options market, and measures of volatility based purely on historical volatility using a random

walk (naive) and 5-day moving average forecasts. Samouilhan and Shannon (2008) found that the GARCH (2, 2) specification provided the best in-sample fit of all the symmetric GARCH models. For their out-of-sample results the GARCH (1, 1) specification provided the best forecast of all the symmetric models as compared to GARCH (1, 2), (2, 1) and (2, 2) models.

However, Samouilhan and Shannon (2008) assume the existence of a stable GARCH process in volatility forecasting and do not take into consideration the impact of structural breaks on the accuracy of volatility forecasts. Additionally only one period ahead forecasts, whether using daily data or averaged out daily data to compute weekly data, were used in their paper to ascertain the accuracy of the three different volatility forecasting approaches.

To address these gaps in the South African literature, we investigate the empirical relevance of structural breaks for GARCH (1, 1) models of stock return volatility in South Africa using in-sample and out-of-sample tests. We again differ from Samouilhan and Shannon (2008) by using multi-period horizons to ascertain the accuracy of different forecasting approaches as compared to a one period ahead approach by Samouilhan and Shannon (2008). Note, we did not consider GARCH (p, q) models, because the GARCH (1, 1) model is essentially treated as the canonical specification in the literature on asset returns volatility. Further, the GARCH (1, 1) specification has been found to be sufficient in practice for such studies, even though the GARCH (p, q) model might be of theoretical interest (Bollerslev *et al.* 1992). More importantly, in our case, we found the GARCH (1, 1) model to fit the data better than the GARCH (2, 2) model, both in- and out-of-sample.¹ Given that, Samouilhan and Shannon (2008) observed the so-called “leverage effect” in the volatility of returns of the TOP40 index using the Glosten *et al.*, (1983)-GARCH (GJR-GARCH) model, we too look into the issue by considering not only the GJR-GARCH(1, 1) model, but also the Markov Switching-GARCH (MS-GARCH) framework (Klassen, 2002; Haas *et al.*, 2004) in terms of forecasting relative to our benchmark GARCH (1, 1) model. Note, the so-called leverage effect refers to the situation where negative returns shocks are correlated with larger increases in volatility than positive returns shocks. The rest of the paper is organised as follows; section 2 details the econometric methodology, section 3 the empirical results for the in-sample and out-of-sample tests. Section 4 concludes.

¹ The results from the GARCH (2, 2) model is available upon request from the authors.

2. Econometric Methodology

2.1 In-Sample Tests

For the in-sample tests we employ a modified version of the iterated cumulative sum of squares algorithm (Inclán *et al.* 1994) to test for the possibility of structural breaks in the unconditional variance of the daily Johannesburg Stock Exchange (JSE) all share index from 1995 to 2010.

Let $Z_t = 100[\ln(R_t) - \ln(R_{t-1})]$, be the returns on a stock index from time $t-1$ to period t . R_t denotes the value of the stock index at time t and $z_t = Z_t - \mu$, where μ is the constant (conditional and unconditional) mean of Z_t . Supposing z_t can be observed for $t = 1 \dots T$, the cumulative sum of squares statistic given by

$$IT = \sup_k \left| (T/2)^{0.5} D_k \right| \quad (1)$$

that tests the null hypothesis that the unconditional variance σ_z^2 is constant for $k = 1 \dots T$, against the alternative hypothesis of a break in the unconditional variance at some point in the sample. $D_k = (C_k/C_T) - (k/T)$ and $C_k = \sum_{t=1}^k z_t^2$ for $k = 1 \dots T$. When the null hypothesis is rejected, the value of k that maximises $\left| (T/2)^{0.5} D_k \right|$ serves as the estimate of the break date. When $z_t \sim iid N(0, \sigma_z^2)$, Inclán and Tiao (1994) show that under the null hypothesis the asymptotic distribution of the IT statistic is given by $\sup_r \left| W^*(r) \right|$ where $W^*(r) = W(r) - rW(1)$ is a Brownian bridge and $W(r)$ is a standard Brownian motion. Finite-sample critical values of IT are then generated using simulation methods.

Several studies (see Andreu *et al.* 2002; de Pooter *et al.* 2004; Sansó *et al.* 2004) have shown that the IT statistic can be substantially oversized when z_t follows a dependent process such as a GARCH process. This is because the IT statistic is designed for *i.i.d* processes. To address this deficiency of the IT statistic and allow z_t to follow a variety of dependent processes under the null hypothesis, including GARCH processes, a nonparametric adjustment based on the Bartlett Kernel is applied to the IT statistic given by

$$AIT = \sup_k \left| T^{-0.5} G_k \right| \quad (2)$$

where $G_k = \lambda^{-0.5} [C_k - (k/T) C_T]$, $\hat{\lambda} = \hat{\gamma}0 + 2 \sum_{l=1}^m [1 - l(m+1)^{-1}] \hat{\gamma}l$,

$\hat{\gamma}l = T^{-1} \sum_{t=l+1}^T (z_t^2 - \hat{\sigma}^2)(z_{t-1}^2 - \hat{\sigma}^2)$, $\hat{\sigma}^2 = T^{-1} C_T$. The lag truncation parameter m is selected using the procedure in Newey *et al.* 1994. The asymptotic distribution of AIT is also given by $\sup_r |W^*(r)|$ under general conditions, and finite-sample critical values can again be generated by simulation methods.

The IT statistic can also be used to test for multiple breaks in the unconditional variance using an iterative cumulative sum of squares (ICSS) algorithm also developed by Inclán *et al.* (1994). To avoid the size distortions that results with the use of the IT statistic, the ICSS procedure can alternatively be based on the AIT statistic in order to allow z_t to follow dependent processes under the null hypothesis. We then use a 5% level of significance to test for structural breaks in the unconditional volatility of the daily stock returns series for the JSE All Share Index.

The GARCH (1, 1) model for z_t with mean zero (conditional and unconditional) is expressed as

$$z_t = h_t^{0.5} \varepsilon_t \quad (3)$$

$$h_t = \omega + \alpha z_{t-1}^2 + \beta h_{t-1} \quad (4)$$

where h_t represents the conditional volatility of z_t and ε_t is *i.i.d.* with mean zero and unit variance. $\alpha + \beta$ measures the persistence of the GARCH (1.1) model and $\alpha + \beta < 1$ for the process to be covariance-stationary. When $\alpha + \beta = 1$ we have the integrated GARCH (1, 1) of model of Bollerslev (1986). In equation (4), β is unidentified and set to zero when $\alpha = 0$, so that $h_t = \omega$ and z_t is characterised by conditional homoscedasticity. For the GARCH (1, 1) process to be stationary the unconditional variance of z_t is given by $\omega / (1 - \alpha - \beta)$. The Quasi Maximum Likelihood Estimation (QMLE) is often used to estimate the GARCH (1, 1) because QMLE parameter estimates have been shown to be consistent and asymptotically normal (Berkes *et al.* 2003; Jensen *et al.* 2004; Straumann, 2005). It is however assumed that $\varepsilon_t \sim N(0,1)$, and the restrictions $\omega > 0$ and $\alpha, \beta \geq 0$ imposed. The in-sample tests enable us to analyse the empirical relevance of structural breaks in unconditional volatility for the JSE All share index

and the effect of structural breaks on GARCH (1, 1) models. The in-sample tests also provide a framework for analyzing the out-of-sample tests results.

2.2 Out-of-Sample Tests

To compare the out-of-sample forecasts of stock return volatility, we first divide the sample of stock returns into two portions; in-sample and out-of-sample, where the in-sample portion contains the first R observations and the out-of-sample portion contains the last P observations. We use three benchmark models and five competing models to compare the out-of-sample forecasts. The first benchmark model is a GARCH (1, 1) model estimated using an expanding window. The first out-of-sample forecast at the 1-period horizon ($s = 1$) is given by $\hat{h}_{R+1|R,EXP}$ = $\hat{\omega}_{R,EXP} + \hat{\alpha}_{R,EXP} z_R^2 + \hat{\beta}_{R,EXP} \hat{h}_{R,EXP}$, where $\hat{\omega}_{R,EXP}$, $\hat{\alpha}_{R,EXP}$, $\hat{\beta}_{R,EXP}$ and $\hat{h}_{R,EXP}$ are the estimates of ω , α , β and h_R respectively obtained from equation (4) using QMLE and data from the first observation through to observation R . For the second out-of-sample forecast $R+2$, we expand the estimation window by one observation using data from the first observation through observation $R+1$, $\hat{h}_{R+2|R+1,EXP}$. We continue this way through to the end of the available out-of-sample period, yielding a series of P one-step ahead out-of-sample forecasts given by $\{ \hat{h}_{t|t-1,EXP} \}_{t=R+1}^T$.

The RiskMetrics model is the second benchmark model based on an expanding window. It is easier to implement because it does not involve the estimation of any parameters. It is given by the exponential weighted moving average $\hat{h}_{t+1|t} = (1 - \lambda) \sum_{k=0}^{t-1} \lambda^k z_{t-k}^2$ where $\lambda = 0.94$ as recommended by the RiskMetrics Group (1996) for daily data. Consistent with the usual practice, we set the s -step-ahead forecast for $s > 1$ equal to the 1-step-ahead forecast for the RiskMetrics model. The s -step-ahead out-of-sample volatility forecasts for the RiskMetrics model is given by $\{ \hat{h}_{t|t-s,RM} \}_{t=R+1}^T$.

The fractionally integrated GARCH (1, d , 1) or FIGARCH (1, d , 1) model is the third benchmark model also estimated using an expanding window (see Baillie *et al.* 1996).

The FIGARCH (1, d , 1) specification is given by

$$h_t = \omega + \beta h_{t-1} + [1 - \beta L - (1 - \phi L)(1 - L)^d] z_t^2 \quad (5)$$

where L is lag operator and $(1-L)^d = 1 - dL - [d(1-d)/2]L^2 - [d(1-d)(2-d)/6]L^3 - \dots$ is fractional differencing operator. The parameter vector (ω, β, ϕ, d) is estimated using QMLE under the assumption that $\omega > 0$, $0 \leq d \leq 1 - 2\phi$, and $0 \leq \beta \leq \phi + d$ to ensure that the conditional variance is positive. The FIGARCH (1, d , 1) model is considered a relevant benchmark in forecasting volatility of asset returns. This is because autocorrelations of squared or (absolute) returns for many financial assets decay slower than exponentially as implied by GARCH models. Thus conditional heteroscedasticity may be better described by a long memory process as captured by the FIGARCH (1, d , 1) specification. The forecasts generated by the FIGARCH (1, d , 1) model is denoted by $\{\hat{h}_{t+1|t-1,FI}\}_{t=R+1}^T$.

The first forecasting competing model is a GARCH (1, 1) 0.5 rolling window model. This model generates forecasts using a rolling estimation window equal to one-half of the size of the in-sample period. The forecasts are generated similar to the GARCH(1,1) expanding window model, except that the parameter estimates for the first out-of-sample forecast are based on observations $(0.5R+1, \dots, R)$ and for the second out-of-sample forecast are based on observations $(0.5R+2, \dots, R+1)$ and so on. The forecasts for the GARCH (1, 1) 0.5 rolling window model is denoted by $\{\hat{h}_{t+1|t-1,ROLL(0.5)}\}_{t=R+1}^T$. The second competing model is a GARCH (1, 1) 0.25 rolling window. This model generates forecasts using a rolling estimation window equal to one-quarter of the size of the in-sample period so that the first out-of-sample forecast are based on observations $(0.75R+1, \dots, R)$ and for the second out-of-sample forecast are based on observations $(0.75R+2, \dots, R+1)$ and so on. We denote the forecasts for the GARCH (1, 1) 0.25 rolling window model by $\{\hat{h}_{t+1|t-1,ROLL(0.25)}\}_{t=R+1}^T$.

As in Mittnik *et al.* (2000), the third competing forecasting model is a GARCH (1, 1) model estimated using an expanding window and a weighted maximum likelihood procedure. This model is known to better handle structural instabilities in GARCH parameters (Mittnik *et al.* 2000). In forming the likelihood function used to estimate the GARCH (1, 1) model parameters, declining weights are assigned to observations in the more distant past. For the first out-of-sample forecast using data through R observations, a weight of ρ^{R-t} is attached to observation $t = 1, \dots, R$ in the log-likelihood function used to estimate the GARCH (1, 1) parameters. To

generate the second out-of-sample forecast the window is expanded by one observation and a weight of ρ^{R+1-t} attached to observations $t = 1, \dots, R + 1$ in the log-likelihood function. This procedure is continued through to the end of the available out-of-sample period. Mittnik *et al.* (2000) recommend $\rho = 0.994$, which they find work well in out-of-sample volatility forecasts. The forecasts generated by the GARCH (1, 1) weighted maximum likelihood model is denoted as $\{ \hat{h}_{t | t-1, WML} \}_{t=R+1}^T$

In the fourth competing model the modified ICSS algorithm is used to select the estimation window for the GARCH (1, 1) model. The modified ICSS algorithm is first of all applied to observation one through R. If there is evidence of one or more structural breaks, and the final break is expected to occur at time T_F , the GARCH (1, 1) model is estimated using observations

$T_F + 1$ through R to form an estimate of h_{R+1} . On the other hand if no evidence of a structural break is found the GARCH (1, 1) model is then estimated using observations one through R to form an estimate of h_{R+1} . For the second out-of-sample forecast, the modified ICSS algorithm is applied to observations one through $R+1$ and the same procedure as in the first out-of-sample forecast is followed. We proceed in this manner through the end of the available out-of-sample period, producing a series of forecasts corresponding to the GARCH (1, 1) with breaks model, given by $\{ \hat{h}_{t | t-1, BREAKS} \}_{t=R+1}^T$. The modified ICSS algorithm that determines the size of the estimation window only uses data available at the time of the forecast formation. As a result there is no “look ahead” bias involved in the generation of the forecasts for the GARCH (1, 1) with breaks model.

The final competing forecasting model is a simple moving average model that uses the average of the squared returns over the previous 250 days to form the volatility for day t : $\hat{h}_{t | t-1, MA} = (1/250) \sum_{i=1}^{250} z_{t-i}^2$. This model has been found to outperform GARCH (1, 1) models when forecasting daily stock return volatility over longer horizons especially for industrialised countries and also very useful in accommodating structural breaks (Stărică *et al.* 2005). Following Rapach *et al.* (2008), we set the s-step-ahead forecast for $s > 1$ equal to 1-step-ahead forecast for the moving average model. We denote the sequence of s-step-ahead out-of-sample forecasts for the moving average model by $\{ \hat{h}_{t | t-1, MA} \}_{t=R+s}^T$.

Additionally we consider a multi period volatility forecast over the out-of-sample period for horizons of 1, 20, 60 and 120 days with the aim of exploring the effects of structural breaks on volatility forecasting and the usefulness of various forecasting methods designed to accommodate potential structural breaks. Based on information available at period $t - s$ we denote the model i forecast² of h_t formed at period $t - s$ by $\tilde{h}_{t | t-s, i}$ yielding a series of $P - (s - 1)$ s -step ahead out-of-sample forecasts given by $\{\tilde{h}_{t | t-s, i}\}_{t=R+s}^T$. We then iterate forward by generating $\tilde{h}_{t | t-s, i}$ for $s > 1$ using the fitted GARCH (1, 1) process and the iterative procedure given by the equation

$$\tilde{h}_{t+s | t} = \omega + (\alpha + \varphi/2 + \beta) \tilde{h}_{t+s-1 | t} \text{ from Franses } et al. (2000).$$

To compare volatility forecasts across models we employ two loss functions; an aggregated version of the mean square forecast error (MSFE) metric by Stărică *et al.* (2005) and the value-at-risk (VaR) by Gonzalez-Rivera *et al.* (2004). The MSFE metric is given by

$$MSFE_{s, i}^* = [P - (s - 1)]^{-1} \sum_{t=R+s}^T (\tilde{z}_t^2 - \tilde{h}_{t | t-s, i})^2 \quad (6)$$

where $\tilde{z}_t^2 = \sum_{j=1}^s z_{t-(j-1)}^2$ and $\tilde{h}_{t | t-s, i} = \sum_{j=1}^s \tilde{h}_{t-(j-1) | t-s, i}$. Aggregation provides a more useful metric for comparing volatility forecasts because it reduces the idiosyncratic noise in squared returns at horizons beyond one period (Andersen *et al.* 1998). The MSFE loss function produces a consistent empirical ranking of forecasting models when squared returns serve as a proxy for measuring latent volatility (Awartani *et al.* 2004; Hansen *et al.* 2006). Thus using the aggregated MSFE metric we analyse volatility forecasts at horizons of 1, 20, 60 and 120 days ($s = 1, 20, 60, 120$).

With respect to the VaR loss function, let $VAR_{t | t-s, i}^{0.05}$ be the forecast of the 0.05 quartile of the cumulative distribution function for the cumulative return $\tilde{z}_t = \sum_{j=1}^s z_{t-(j-1)}$ generated by model i and formed at time $t - s$. The VaR loss mean function is given by

$$MVAR_{s, i} = [P - (s - 1)]^{-1} \sum_{t=R+s}^T (0.05 - d_{t, i}^{0.05})(\tilde{z}_t - VAR_{t | t-s, i}^{0.05}) \quad (7)$$

² Where $i = \text{EXP, RM FI, ROLL}(0.5), \text{ROLL}(0.25), \text{WML, BREAKS, MA}$

where $d_{t,i}^{0.05} = 1(\tilde{z}_t < VAR_{t|t-s,i}^{0.05})$ and $1(\cdot)$ is the indicator function that takes a value of unity when the argument is satisfied. When $\tilde{z}_t < VAR_{t|t-s,i}^{0.05}$, a large weight of 0.95 is attached by the loss function to the difference between \tilde{z}_t and $VAR_{t|t-s,i}^{0.05}$ indicating a relatively high cost associated with large losses. Conversely a smaller weight of 0.05 is attached by the loss function to the difference between \tilde{z}_t and $VAR_{t|t-s,i}^{0.05}$ when $\tilde{z}_t > VAR_{t|t-s,i}^{0.05}$. Although the weight is smaller in this case it is still positive thereby enabling the loss function to reflect the opportunity costs of the capital held to cover the potential losses indicated by $VAR_{t|t-s,i}^{0.05}$. The advantage of the $MVAR_{s,i}$ criterion is that it is motivated by the VaR as a risk management tool and does not require observations of the latent volatility h_t .

By assuming that $z_t \sim N(0, 1)$ we calculate $VAR_{t|t-s,i}^{0.05}$ as the simulated cumulative returns

($\tilde{z}_t^* = \sum_{j=1}^s z_{t-(j-1)}^*$) of a simulated sequence of returns based on estimates of the conditional volatility process available at the time of forecast formation ($\{z_{t-(j-1)}^*\}_{j=1}^s$). This process is repeated 2000 times yielding an empirical distribution of simulated cumulative returns. The 100th element of the ordered simulated cumulative returns is the $VAR_{t|t-s,i}^{0.05}$.

Besides ranking the forecasting models using the $MSFE_{s,i}^*$ and $MVAR_{s,i}$ loss functions, we test the null hypothesis that none of the competing models has superior predictive ability over the benchmark model in terms of expected loss, against the alternative one sided (upper-tail) hypothesis that at least one of the competing models has superior predictive ability over the benchmark model. That is we check whether the expected loss of the forecasts generated by at least one of the five competing models is significantly less than that generated by a given benchmark model using the White (2000) test. The loss at time t for forecasting model j relative to benchmark model i is defined as $f_{t,i,j} = L_{t,i} - L_{t,j}$, where L_t is given by the expression after the summation operator in equation (6) or (7) for each loss function, and $\bar{f}_{i,j} = [P - (s - 1)]^{-1} \sum_{t=R+s}^T f_{t,i,j}$. The White (2000) statistic for l competing models is given by

$$V_l = \max_{k=1, \dots, l} [P - (s - 1)]^{0.05} (\bar{f}_{i,1}, \dots, \bar{f}_{i,l}) \quad (8)$$

with $l = 5$ in this paper.

As in White (2000) a p-value corresponding to \bar{V}_t is generated using the stationary bootstrap method of Politis *et al.* (1994). The White (2000) reality check is performed by comparing each of the benchmark models (GARCH (1, 1) expanding window, RiskMetrics and FIGARCH (1,d,1)) to the five competing models to check whether any of the five competing models performs better than the given benchmark model in terms of real-time volatility forecasting. Additionally, the Hansen (2005) version of the White (2000) test which has a higher power in determining superior predictive ability is also conducted. The Hansen (2005) studentised version of the \bar{V}_t statistic, T_n^{SPA} is computed and the p-values again generated by the stationary bootstrap method of Politis *et al.* (1994).

One drawback of the GARCH (1, 1) model is its assumption that the response of the conditional variance to both positive and negative shocks is the same-symmetrical. Thus only the size and not the sign of the shock is relevant. There exists ample empirical evidence of the existence of leverage effects in stock returns, meaning negative return shocks results in higher volatility in subsequent periods than positive return shocks. We compare the performance of two asymmetric GARCH models in accurately capturing leverage effects in stock return volatility, namely the GJR-GARCH (1, 1) model and the MS-GARCH (1, 1). The GJR-GARCH (1, 1) model is the asymmetric GARCH (1, 1) model of Glosten *et al.* (1993) and is expressed as $z_t = h_t \varepsilon_t$ where $\varepsilon_t \sim iid N(0, 1)$, and $h_t = \omega + \alpha z_{t-1}^2(1 - I) + \gamma z_{t-1}^2 I + \beta h_{t-1}$. The dummy I takes on the value of unity when $\varepsilon_{t-1} < 0$ and zero otherwise. The MS-GARCH (1, 1) is the two-state Markov-switching GARCH(1,1) model of Haas *et al.* (2004) and is expressed as $z_t = h_t \varepsilon_t$ where $\varepsilon_t \sim iid N(0, 1)$, with $h_{s_t} = h_{1,t} = \omega_1 + \alpha_1 z_{t-1}^2 + \beta_1 h_{1,t-1}$ in state one, and in state two $h_{s_t} = h_{2,t} = \omega_2 + \alpha_2 z_{t-1}^2 + \beta_2 h_{2,t-1}$, with transition probabilities given by $p_{i,j} = [P(s_t = j | s_{t-1} = i)]$ for $j = 1, 2$. The ratio of the mean loss for each model to the mean loss of the benchmark GARCH (1, 1) expanding window model is compared under the $MSFE_{s,t}^*$ and $MVAR_{s,t}$ criteria, as above.

Recent literature has shown that comparing the relative predictive accuracy of different forecasting models need to take into consideration the relative sizes of the in-sample and out-of-sample periods (P/R), type of estimating window used (expanding, rolling or fixed) and whether the models being compared are nested or not. Since these requirements are not all necessarily

satisfied in our application we report the bootstrapped p-values for the White (2000) \bar{V}_1 and Hansen (2005) T_n^{SPA} statistics as a crude guide to assessing statistical significance of the various models used in this paper.

3. Empirical Results

3.1 Data and Descriptive Statistics

Daily data on the Johannesburg Stock Exchange All Share Index is used in this paper. The sample period is from 07/02/1995 to 08/25/2010 consisting of 3788 observations. The daily stock returns are based on the closing prices. The descriptive statistics are reported in Table 1. These statistics include heteroscedastic and autocorrelation consistent standard errors for the mean, standard deviation, skewness, and excess kurtosis. The computation of these statistics is based on the procedure in West *et al.* (1995). The mean is significantly different from zero at 5 % level of significance. Daily stock returns appear quite volatile and exhibit strong evidence of excess kurtosis. The modified Ljung-Box statistics are robust to conditional heteroscedasticity and show no evidence of autocorrelation of the daily stock returns. However there is strong evidence of serial correlations in the squared stock returns. The Lagrange multiplier statistics are significant at 1 % level confirming ARCH effects (Engle, 1982). These descriptives support the modelling of stock market returns in South Africa using GARCH processes.

Table 1: Summary statistics, SA JSE All Share Index returns (07/03/1995 to 08/25/2010)

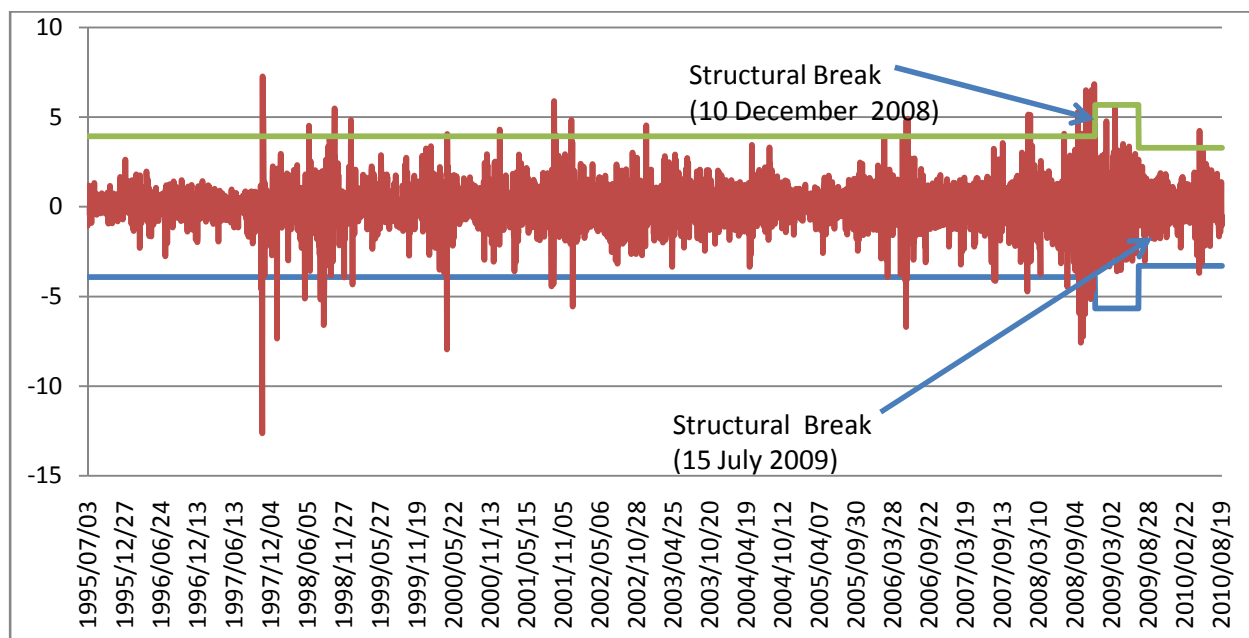
	Estimates	Std. err	p-values
Stock Market Price Return			
Mean	0.045	0.021	
Standard deviation	1.320	0.030	
Skewness	-0.479	0.253	
Excess kurtosis	5.944	1.852	
Minimum	-12.628		
Maximum	7.266		
Modified Ljung-Box (r=20)	26.137		0.161
Squared Stock Market Price Return			
Ljung-Box (r=20)	1781.218		0.000
ARCH Lagrange multiplier (q = 2)	411.880		0.000
ARCH Lagrange multiplier (q = 10)	560.927		0.000

Note: Returns are defined as 100 times the log-differences of the stock price indices. Ljung-Box statistics correspond to a test of the null hypothesis that the r autocorrelations are zero. Modified Ljung-Box statistics are robust to conditional heteroscedasticity. ARCH Lagrange multiplier statistics correspond to a test of the null hypothesis of no ARCH effects from lag 1 through q . 0.000 indicates the p values less than 0.0005.

3.2 In-sample results

The modified ICSS algorithm employed for our in-sample test revealed two structural breaks in the unconditional volatility of stock market return in South Africa, specifically on 10th of December, 2008 and 15th of July, 2009. Both structural breaks represent an increase in volatility due to the impact of the global financial crisis. Figure 1 below shows a plot of the stock returns series and three-standard-deviation bands defined by the structural breaks identified by the modified ICSS algorithm.

Figure 1: The JSE All Share Index and ± 3 standard deviation bands.



Most emerging markets like South Africa largely stood at the fringes of the global financial crisis for most part of 2007 and 2008. However its impact began to be felt end 2008 sending the South African economy into a technical recession in Q1 2009. The economic slowdown adversely affected the performance of most listed equities on the bourse. This aggravated volatility in most

stock market indices on the bourse including the JSE All Share index. Government revenue also declined significantly across the year worsening by Q3 2009. Hence the second structural break in July 2009. This created the need for Government to scale up domestic borrowing from Q3 2009. To address this government increased its net borrowing requirement R90.3 billion to R175.8 billion in October 2009. Both new and existing investors began to recomposed their portfolios, shifting away from listed equities which were already performing badly towards long term government bonds which peaked at R31.2 billion in Q4 2009 (JSE Review Report, 2010). Thus the South African financial market experienced two major structural breaks in December 2008 and July 2009 as a result of the impact of the global financial crisis.

Table 2 below shows the conditional volatility of the squared stock return series estimated using a standard QMLE GARCH (1, 1) model both over the full sample period and sub-sample periods.

Table 2: Quasi Maximum Likelihood Estimation Results for GARCH (1, 1) Models

	Estimates	Std.err
GARCH (1, 1) full sample estimation results		
ω	0.020	0.005
α	0.111	0.011
β	0.883	0.011
$\omega/(1 - \alpha - \beta)$	3.072	1.885
GARCH (1, 1) sub-sample 1 estimation results		
ω	0.020	0.005
α	0.115	0.012
β	0.880	0.012
$\omega/(1 - \alpha - \beta)$	4.386	4.704
GARCH (1, 1) sub-sample 2 estimation results		
ω	3.581	0.367
α	0	0
β	0	0
$\omega/(1 - \alpha - \beta)$	3.581	0.367
GARCH (1, 1) sub-sample 3 estimation results		
ω	0.058	0.041
α	0.089	0.036

β	0.863	0.307
$\omega/(1 - \alpha - \beta)$	1.191	0.307

Notes: Table 2 reports the GARCH (1, 1) model estimations for the squared stock return series for the full sample and those for the different sub-samples defined by the structural breaks. The table also includes standard deviations of the estimates.

The sub-samples are defined by the structural breaks identified by the modified ICSS algorithm. The fitted full sample GARCH (1, 1) model is highly persistent with an estimate of $\alpha + \beta$ of about 0.994. The first sub-sample GARCH (1, 1) model also exhibits high persistence with an estimate of $\alpha + \beta = 0.995$. The second sub-sample GARCH (1, 1) model shows absolutely no persistence, while the third sub-sample GARCH (1, 1) model also shows high persistence with an estimate of $\alpha + \beta = 0.953$. These high levels of persistence show that the sub-samples are generally characterized by conditional heteroscedasticity. Table 2 also shows some significant changes in the unconditional variance as reflected in $\omega/(1 - \alpha - \beta)$. These changes are due to the structural breaks bringing about substantial shift in the intercept defined by ω over the period under review. In addition the GARCH (1, 1) parameter estimates vary across sub-samples defined by the structural breaks. These in-sample results show highly persistent conditional volatility for the stock return and also confirm that structural breaks are an empirically relevant feature of stock market returns in South Africa.

3.3 Out-of-sample results

The out-of-sample period consists of the last 500 observations of the January 2 1995 to August 31 2010 full sample period and covers the September 2 2008 to August 31 2010 period for South Africa. Table 3 reports the out-of-sample volatility forecasting results over horizons of 1, 20, 60 and 120 days. The first row in each panel of the table reports the mean loss for the GARCH (1, 1) expanding window model, while the remaining rows present the ratio of the mean loss for each of the other models to the mean loss for the GARCH (1, 1) expanding window model. The model with the lowest mean loss ratio under both the MSFE and MVaR criteria performs better than the other models in forecasting volatility. The table also reports p -values corresponding to the White (2000) \overline{V}_T and Hansen (2005) $T_{\frac{SPA}{n}}$ statistics with the GARCH (1, 1) expanding window, RiskMetrics, and FIGARCH (1, d , 1) expanding window models serving as the

benchmark models and the two GARCH (1, 1) rolling window, GARCH (1, 1) weighted ML, GARCH (1, 1) with breaks and moving average models serving as the competing models.

From the out-of-sample results in Table 3, the competing models strictly perform better than all the benchmark models using the $MSFE_{s,t}^*$ loss function. None of the benchmark models delivers the lowest mean loss using the $MSFE_{s,t}^*$ loss function. Among the competing models, the GARCH (1, 1) 0.50 rolling window and the GARCH (1, 1) with breaks models report the lowest mean loss ratio over the 1-day horizon. The GARCH (1, 1) 0.25 rolling window model performs better than all the benchmark models over the 20, 60 and 120 days horizons under the $MSFE_{s,t}^*$ criterion. This proves that allowing for instabilities in GARCH (1, 1) models has benefits in out-of-sample volatility forecasting. The performance of GARCH (1, 1) with breaks worsens over higher forecasting horizons but performs relatively better than the GARCH (1, 1) weighted ML and the Moving average models. Among the benchmark models the RiskMetrics model performs better than the other benchmark models over the 1-day horizon under the $MSFE_{s,t}^*$ criterion. However as the forecasting horizons increase the FIGARCH (1, d , 1) outperforms the rest of the benchmark models in all cases using the $MSFE_{s,t}^*$ criterion. This confirms the literature that the FIGARCH (1, d , 1) model better captures conditional heteroscedasticity described by long memory processes. The p -values corresponding to the Hansen (2005) T_n^{SPA} statistics than the White (2000) \bar{V}_1 statistics rejects the null hypothesis in favour of the alternative hypothesis that at least one of the competing models have superior predictive ability over the benchmark models in forecasting volatility in stock market returns.

Using the $MVAR_{s,t}$ criterion, the benchmark models specifically the RiskMetrics Model (for $s = 60, 120$) deliver the minimum mean loss ratios thereby outperforming all the competing models for higher forecast horizons. However for shorter forecast horizons ($s = 1, 20$), the competing models outperform the benchmark models. At $s=1$, the GARCH (1, 1)0.50 rolling window model delivers the lowest mean loss ratio, while for $s=20$, the GARCH(1, 1) 0.25 rolling window model delivers the lowest mean loss ratio. Among the benchmark models, the RiskMetrics model performs better in all cases using the $MVAR_{s,t}$ criterion. The p -values corresponding to the Hansen (2005) T_n^{SPA} statistics and the White (2000) \bar{V}_1 statistics again rejects the null

hypothesis in favour of the alternative hypothesis that at least one of the competing models have superior predictive ability over the benchmark models in forecasting volatility in stock market returns. This is consistent with the inference from the results under the $MSFE_{s,t}^*$ criterion.

Table 3 also shows a summary of out-of-sample forecasting results comparing the GJR-GARCH (1, 1) expanding window model and MS-GARCH (1, 1) models. The GJR-GARCH (1, 1) model registers a higher mean loss ratio than the MS-GARCH (1, 1) model under both the $MSFE_{s,t}^*$ and $MVAR_{s,t}$ criteria at shorter forecast horizons. However as the forecast horizon increases, the MS-GARCH (1, 1) delivers a higher mean loss ratio than the GJR-GARCH (1, 1) under both $MSFE_{s,t}^*$ and $MVAR_{s,t}$ criteria. These results show that for shorter horizons the MS-GARCH (1, 1) better captures the leverage effect in stock market return volatility than the GJR-GARCH (1, 1). However for longer horizons, the GJR-GARCH (1, 1) model performs better than the MS-GARCH (1, 1) model in accurately capturing leverage effects in stock market return volatility.

Table 3: Summary of Out of Sample Forecasting Results

$s = 1$	MSFE	MVaR
GARCH (1, 1) expanding window	36.893 (0.879) [0.372]	0.173 (0.768) [0.558]
RiskMetrics	0.998 (0.867) [0.570]	1.012 (0.736) [0.382]
FIGARCH (1, d , 1) expanding window	1.015 (0.693) [0.097]	1.016 (0.705) [0.281]
GARCH (1, 1) 0.50 rolling window	0.995	0.995
GARCH (1, 1) 0.25 rolling window	1.011	1.007
GARCH (1, 1) weighted ML	1.029	1.017
GARCH (1, 1) with breaks	0.995	0.997
Moving average	1.290	1.262
GJR-GARCH (1, 1) expanding window	1.417	1.989
MS-GARCH (1, 1) expanding window	1.002	1.031
$s = 20$		
GARCH (1, 1) expanding window	2327.75 (0.962) [1.000]	0.837 (1.000) [1.000]
RiskMetrics	1.176 (0.689) [0.260]	1.068 (0.617) [0.106]
FIGARCH (1, d , 1) expanding window	1.125 (0.787) [0.254]	1.072 (0.682) [0.111]
GARCH (1, 1) 0.50 rolling window	1.044	1.032
GARCH (1, 1) 0.25 rolling window	1.005	1.024
GARCH (1, 1) weighted ML	1.635	1.046
GARCH (1, 1) with breaks	1.029	1.028
Moving average	2.888	1.302
GJR-GARCH (1, 1) expanding window	4.376	1.380
MS-GARCH (1, 1) expanding window	3704.21	0.997

s = 60

GARCH (1, 1) expanding window	27263.1 (0.758) [0.639]	1.504 (0.998) [1.000]
RiskMetrics	1.204 (0.663) [0.368]	1.001 (0.960) [1.000]
FIGARCH (1, <i>d</i> , 1) expanding window	0.954 (0.788) [0.039]	1.038 (0.755) [0.589]
GARCH (1, 1) 0.50 rolling window	0.939	1.010
GARCH (1, 1) 0.25 rolling window	0.861	1.019
GARCH (1, 1) weighted ML	3.852	1.073
GARCH (1, 1) with breaks	1.049	1.037
Moving average	1.944	1.153
GJR-GARCH (1, 1) expanding window	2.661	0.908
MS-GARCH (1, 1) expanding window	2.414e+16	0.978

MSFE

MVaR

Ratio

Ratio

s = 120

GARCH (1, 1) expanding window	92405 (0.635) [0.107]	2.106 (1.000) [1.000]
RiskMetrics	1.505 (0.503) [0.088]	1.003 (0.928) [1.000]
FIGARCH (1, <i>d</i> , 1) expanding window	0.787 (0.699) [0.000]	1.067 (0.450) [0.008]
GARCH (1, 1) 0.50 rolling window	0.682	1.010
GARCH (1, 1) 0.25 rolling window	0.619	1.016
GARCH (1, 1) weighted ML	16.353	1.133
GARCH (1, 1) with breaks	1.080	1.042
Moving average	1.888	1.188
GJR-GARCH (1, 1) expanding window	1.978	0.935
MS-GARCH (1, 1) expanding window	9.09640e+36	0.966

Note: Entries for the GARCH (1, 1) expanding window model give the mean loss for this model. Entries for the other models give the ratio of the mean loss for each model to the mean loss for the GARCH (1, 1) expanding window model. Bold entries denote the model with the smallest mean loss among all of the models. P-values for the White (2000) \overline{F}_1 (Hansen (2005) T_n^{SFA}) statistics are given in brackets (box brackets) and correspond to a test of the null hypothesis that none of the five competing models (two GARCH (1,1) rolling window, GARCH (1,1) weighted ML, GARCH (1,1) with breaks, and moving average models) has a lower expected loss than the benchmark model indicated on the left against the one sided (upper-tail) alternative hypothesis that at least one of the competing models have a lower expected loss than the benchmark model; 0.000 indicates less than 0.0005. The table reports on the GJR-GARCH (1, 1) and the MS-GARCH (1, 1) expanding windows models. The ratios in the table are the mean loss for each model to the mean loss for the GARCH (1, 1) expanding window model.

4. Conclusion

This paper investigates the empirical relevance of structural breaks in accurately forecasting the volatility of stock returns in South Africa using in-sample and out-of-sample tests and daily data on the JSE Allshare index from 1995 to 2010. Results from our in-sample tests using the modified ICSS algorithm identify two structural breaks in the unconditional volatility of the stock market return series in South Africa. These occurred in March and October 2009 attributable to the impact of the global financial crisis. In Q1 2009, the impact of the global

financial crisis began to be felt on the South African economy. This had a strong impact on most listed equities on the bourse and resulted in increased volatility in most stock market indices on the bourse including the JSE All Share index. This volatility peaked in March 2009 (end Q1/2009) after which some improvements began to register. The economic slowdown also adversely affect Government revenue collection across the year resulting in increased domestic borrowing by Government commencing in October 2009. Government increased its net borrowing requirement from R90.3 billion to R175.8 billion in October 2009. Both new and existing investors restructured their portfolios away from listed equities and towards long term government bonds which peaked at R31.2 billion in Q4 2009 (JSE Review Report, 2010). This explains the structural breaks identified by our in-sample results in March and October 2009. The fitted full sample GARCH (1, 1) model and the sub-samples also exhibit high levels of persistence with $\alpha + \beta$ ranging between 0.952 to 0.995 indicating that stock market returns in South Africa is characterised by conditional heteroscedasticity. This confirms that structural breaks are a relevant feature of stock market return volatility in South Africa and therefore needs to be accounted for to enhance accuracy in volatility forecasts of stock market returns in South Africa.

The out-of-sample tests also show that the quality of volatility forecasts of stock market returns improves when structural breaks are considered in the process of estimation. The GARCH (1, 1) rolling window models which accommodate structural breaks in most cases have superior predictive power under both the $MSFE_{s,t}$ and $MVAR_{s,t}$ criteria than the benchmark models. This is corroborated by the results of the White (2000) and Hansen (2005) hypothesis tests which reject the null hypothesis in favour of the alternative hypothesis that a least one of the competing models has superior predictive power over the benchmark models. The FIGARCH (1, d , 1) model that explicitly allows for long memory in stock returns volatility is not relevant in this case. The long memory evidence may therefore be spurious if structural breaks are not considered in the forecasting of volatility in stock market returns. Consistent with literature, stock return volatility is characterised by clustering and leverage effects sometimes resulting in an asymmetric response of conditional volatility to return shocks (Rapach *et al.* 2008). We found that the asymmetric GJR-GARCH (1, 1) model does not always outperform the MS-GARCH (1, 1) model in capturing leverage effect in stock return volatility. For shorter time horizons the MS-

GARCH (1, 1) model performed better, while the GJR-GARCH was better suited to longer horizons, but in general, the asymmetric models fail to outperform the GARCH (1,1) model.

Thus consistent with literature, structural breaks are a significant feature of volatility of stock market returns and need to be accounted for in empirical forecasts of volatility in stock market returns to enhance the accuracy of such forecasts. Finally, it is also relevant to employ multi-period forecasts across different horizons, as the performance of volatility forecasting approaches vary across different horizons.

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