Draft Article (For Mathematical and Computer Modelling)

Efficient optimisation of a vehicle suspension system, using a gradient-based approximation method. Part 2. Optimisation results

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12 May 2009

Abstract—Part 1 of this paper proposed a methodology for the efficient determination of gradient information, when optimising a vehicle's suspension characteristics for ride comfort and handling. The non-linear full vehicle model, and simplified models for gradient information has been discussed, and validated.

In this paper, the simplified models presented in Part 1 are used for gradient information simulations. The convergence histories of the optimisation are compared to those obtained when only the full, computationally expensive, vehicle model is used. For illustration of the proposed gradient-based optimisation methodology, up to four design variables are considered in modelling the suspension characteristics.

The proposed methodology is found to be an efficient alternative for the optimisation of the vehicle's suspension characteristics. The undesirable effects associated with noise in the gradient information is effectively reduced, using the simplified models. Substantial benefits are achieved in terms of computational time needed to reach a solution.

Keywords— Dynamic-Q, gradient-based mathematical optimisation, ride comfort, handling, vehicle suspension, semi-active.

1. INTRODUCTION

Part 1 of this paper presented a brief history of vehicle suspension optimisation, the general problem of numerical noise, and computationally expensive simulation models. It proposed the use of simplified mathematical models for calculating gradient information, and the full simulation model for determining the objective function value when optimising an off-road vehicle's suspension characteristics. Although this application uses the gradient-based optimisation algorithm Dynamic-Q [1], the principle can be applied to any gradient-based optimisation algorithm.

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The optimisation of an off-road vehicle's spring and damper characteristics for ride comfort and handling, is presented as a case study. The vehicle is to be fitted with a semi-active hydro-pneumatic suspension currently under development [2, 3], which will be refered to as the 4 State Semi-Active Suspension System ($4S_4$). Part 1 of this paper described the full vehicle model, developed in MSC.ADAMS [4], and the validation of the model with measured test data. This full vehicle model is highly accurate, but is computationally expensive, as a result of the detailed modelling of non-linear effects, which also introduce numerical noise.

The simplified vehicle models for handling and ride comfort, as described in Part 1, are used to decrease the computational complexity of the full vehicle simulation model, while still capturing the trends over the design space. For the determination of the gradient information when optimising for handling, a simplified non-linear four wheel model, that includes roll, is used. For the determination of the gradient information when optimising for ride comfort, a simplified non-linear pitch-plane model is used.

From Part 1 of this paper it is evident that the simplified models exhibit very similar trends as the full vehicle simulation model, however, the absolute values differ, necessitating the scaling of the simplified models to be more representative of the full vehicle model. The Dynamic-Q optimisation algorithm, to be used for the optimisation of the vehicle's handling and ride comfort, was introduced, and the design variables and objective functions defined.

2. OPTIMISATION PROCEDURE

This paper compares the optimisation results when using the full vehicle simulation model for objective function value and gradient information ($admsgrad$), as traditionally used in gradient-based optimisation, to the use of the full vehicle model only for the objective function value, and the simplified models for gradient information ($matgrad$). Central finite differences, at a computational cost of $2n + 1$ function evaluations per iteration (where n is the number of design variables), is used for the determination of the gradient information. The use of central finite differences for gradient information, was found to improve optimisation convergence in the presence of severe numerical noise by Els et. al. [5] and Thoresson [6].

The use of only the MSC.ADAMS full vehicle model in the optimisation ($admsgrad$) has a computational cost of $2n + 1$ computationally expensive simulations per iteration. The use of the MSC.ADAMS full vehicle model for only the objective function value, and the simplified MATLAB vehicle models for gradient information ($matgrad$), has a computational cost of one computationally expensive simulation per iteration, and $2n$ computationally inexpensive simulations per iteration. The simplified MATLAB models solve in approximately 10% of the full vehicle model's simulation time. Sufficient gradient information is obtained, after the simplified models have been scaled at a once-off cost of 30 computationally expensive simulations for ride comfort and handling separately.

With the proposed methodology, more starting points or design variables can be efficiently considered, in less computational time, making gradient-based approximation methods for optimisation of vehicle suspension systems more feasible. The simplified models also exhibit less numerical noise than the full simulation model, resulting in smoother gradient information.

2.1. Definition of the Design Variables

For the two design variable problem the design variables are explicitly defined as follows: (from Part 1)

$$x_1 = \tfrac{dpsf - 0.1}{3 - 0.1}, \;\; x_2 = \tfrac{gvol - 0.1}{0.6 - 0.1} \tag{1}$$

For the four design variable problem, the design variables are explicitly defined as follows:

$$\begin{aligned} x_1 &= \tfrac{dpsff - 0.1}{3 - 0.1}, \;\; x_2 = \tfrac{gvolf - 0.1}{0.6 - 0.1} \\ x_3 &= \tfrac{dpsfr - 0.1}{3 - 0.1}, \;\; x_4 = \tfrac{gvolr - 0.1}{0.6 - 0.1} \end{aligned} \tag{2}$$

Where $dpsf$ refers to the damper scale factors and $gvol$ refers to the static gas volumes. The additional f and r in equation (2) represent the front (f) and rear (r) values.

2.2. Definition of Objective Functions

For ride comfort the objective function, explained in Part 1, is defined as follows:

$$f_{ride}(x) = \frac{\sum(\frac{a_{zRMSd} - 0.7}{4.4 - 0.7}, \frac{a_{zRMSp} - 0.7}{4.4 - 0.7})}{2} \tag{3}$$

For handling the objective function is defined as follows:

$$f_{hand}(x) = \frac{\sum(\frac{(\dot{\varphi}_{RMS} - 0.8)0.9}{5.7 - 0.8} + 0.1, \frac{(\varphi_{1st_{peak}} - 1.4)0.9}{12.2 - 1.4} + 0.1)}{2} \tag{4}$$

3. HANDLING OPTIMISATION RESULTS

3.1. Two Design Variable Optimisation

The results for the comparison between the $admsgrad$ and the $matgrad$, when optimising handling for two design variables, are illustrated in Figure 1. It can be seen that the use of the simplified model for the gradient information ($matgrad$) converged to an optimum after 12 iterations and 13 expensive function evaluations. The use of the computationally expensive full vehicle model, for gradient information ($admsgrad$), converged to the same optimum point within 15 iterations, but took 80 computationally expensive function evaluations of the full vehicle model. The simplified model solves in approximately 10% of the solution time of the full MSC.ADAMS vehicle model. Central finite differences is used for the gradient determination, at a cost of $2n + 1$ function evaluations per iteration, where n is the number of design variables. When using only the MSC.ADAMS model for gradient and objective function evaluation ($admsgrad$), one iteration of two design variables costs the equivalent of 500% of the computational time of one MSC.ADAMS model simulation. When using the simplified models for gradient information, and only one full MSC.ADAMS simulation for the objective function value, the cost of one iteration is equivalent to $100\% + 2 \cdot 2 \cdot 10\%$, which is the equivalent of 140% of the computational time of one MSC.ADAMS simulation. The use of the simplified models for the determination of gradient information, is therefore approximately 3.5 times faster than using only the MSC.ADAMS model, when considering two design variables. This highlights the advantages in terms of simulation time achievable for just two design variables. It is also observed that the use of the simplified model for gradient information does not introduce instabilities in the optimisation convergence history. The simplified model produces sufficiently accurate gradient information to drive the optimisation to the same optimum.

3.2. Four Design Variable Optimisation

With the successful results obtained for two design variable handling optimisation, the problem was expanded to four design variables, thus allowing the front and rear suspension characteristics to be independent of each other. It is believed that the four design variable problem will exhibit more local minima, and the use of the simple model for gradient information needs to be tested for robustness. The results of the four design variable optimisation, where the full MSC.ADAMS model was used for gradient information is presented in Figure 2. From the figure it can be seen that the optimisation converged to a minimum identical to that for two design variables, considering the noise levels present in the numerical model. It is noted from the optimisation convergence history, that there are repeated equal local minima at iterations five,eight, and ten. Also to be noted is that if the optimisation had continued for a few more iterations, design variable $x2$, would most probably have moved to the same value as $x4$ as expected from Figure 1. It can be seen that the design variable $x1$ (front damper) takes on a value around 0.9, and $x3$ (rear damper design variable) takes on a value of 1, but they also have not converged on a final value. It is also evident that design variable $x4$ (rear gas volume design variable) moved to the boundary, and should be at the lowest value. However, interestingly the front gas volume, design variable $x2$ takes on a value around 0.27, but can also take on a value around 0.07. Considering the optimisation convergence history, when the simplified model is used for the gradient evaluations (Figure 3), it can be seen that the optimisation process converges to a minimum identical to that for two design variables and four design variables using the MSC.ADAMS model for gradient information. The design variable values converge to different values. Indicating the presence of multiple equivalent local minima. Yet the design variables $x1$ and $x3$ are very close to the average of the last few iterations of Figure 2, the results when using only the MSC.ADAMS model for gradient and function values. Design variable $x2$ went to the lower bound as would be expected from the two design variable results. However, when considering the results in Figure 2, it can be seen that design variable $x2$ takes on a value of around 0.27, but it appears not to have settled. From the results it is clear that no difficulties are experienced in obtaining a feasible optimum and that both the solutions are equally feasible. The four design variable optimisation for seven optimisation iterations, using the simplified model, is approximately five times faster than using only the full MSC.ADAMS vehicle model.

4. RIDE COMFORT OPTIMISATION RESULTS

4.1. Tyre Hop in the Optimisation Process

The ride comfort optimisation has to be performed considering tyre hop effects, as the vehicle can become unstable on the road should the tyres constantly loose contact with the road. The tyre hop constraints tend to exhibit a more prominent role, than the objective function, on the damping design variable's lower limit. An investigation was performed, to determine the most effective method of including the tyre hop effect within the optimisation process. The following conditions were considered:

- Constrained optimisation: (*constrained*) The objective function is defined as in equation (3). The tyre hop constraints are defined as: the individual tyre's vertical force F_{ztyre_i} may not be equal to zero for more than 10% of the total time t_{total}, when travelling on rough off-road terrain, and scaled as follows:

$$g_i(x) = 10(\frac{\sum t(F_{z\,tyre\,i}=0)}{t_{total}} - 0.1) \leq 0, \;\; i = 1, ..., 4 \quad (5)$$

Results are indicated in Figure 4.

- Unconstrained optimisation: The objective function is defined as in equation (3). The constraints, as defined in equation (5), are only monitored, but not considered by the optimisation algorithm, (*unconstrained*). The results are indicated in Figure 5.

The equivalent objective function $f(x)_{eq}$ values presented in Figures 4 and 5 is the ride comfort objective function defined by equation (3). The equivalent inequality constraint value $g(x)_{eq}$ is defined as:

$$g(x)_{eq} = max_{i=1,..4}(g_i(x)) \quad (6)$$

representing the maximum of the tyre hop constraint function of the four wheels. From the results it can be seen that the constrained optimisation (*constrained*, Figure 4), returns the lowest objective function value for the tyre hop inequality constraint being satisfied. In general the front tyres contributed most to the tyre hop, compared to the rear tyres, however, the rear tyres also contributed in the optimisation convergence history, making the inclusion of all tyres as constraints necessary. It was found that a tyre hop limit of 10% for the particular road in question is a reasonable constraint, as smaller limits tend to overconstrain the optimisation. It is thus decided that the tyre hop limit of 10% will be included as a constraint for all future ride comfort optimisation, when travelling over rough off-road terrain.

4.2. Two Design Variable Optimisation

The vehicle suspension settings were optimised for ride comfort, for two design variables, with the tyre hop constraint included, as defined in equation (5). The results of the optimisation process, for using only the MSC.ADAMS model for gradient information, compared to using the simplified pitch-plane model for gradient information are presented in Figure 6. It can be seen that the simplified gradients (*matgrad*) took approximately 24 iterations (25 expensive function evaluations) corresponding to an effective cost of 35 expensive function evaluations in terms of time, to reach an optimum. However, identical local minima, in terms of the objective function value, were repeatedly reached at iterations 10, 13, 17 and 20. The expensive gradients (*admsgrad*) effectively reached the optimum after 8 iterations at a cost of 45 expensive function evaluations, with an identical objective function value minimum repeated at iteration 19. Although the use of the simplified model for gradient information took more iterations, the total computing time is significantly less than using only the expensive numerical model for function values and gradient information. It is also apparent from the convergence histories that the use of the simplified model for gradient information, results in a much smoother convergence history, giving greater confidence in the computed results.

4.3. Two Design Variable Optimisation, MATLAB Model Only

With such reasonable results obtained using the simple model for the computation of gradient information, it is necessary to justify the use of the complete MSC.ADAMS vehicle model for the function value in the optimisation process. The same optimisation was done as above but using only the simple Matlab model for the optimisation procedure. From the results in Figure 7 it can be seen that the function values are not the same as the MSC.ADAMS simulation values (calculated at iteration 5 and 25) and that the optimisation algorithm will converge to an infeasible point, when considering the constraints. Thus the use of the full MSC.ADAMS vehicle model is necessary in order to ensure the optimisation algorithm terminates at a feasible minimum. Although the simplified model has very similar trends, the absolute values are not always the same, especially when considering the tyre hop constraints. This explains why the converged solution may not be feasible, when only using the simplified Matlab model for the optimisation procedure. The use of the complete MSC.ADAMS model for function values and the simplified Matlab model for gradient information is thus the most viable solution.

4.4. Four Design Variable Optimisation

The four design variable ride comfort optimisation, was started from the optimum achieved from the two design variable optimisation. The optimisation process worked equally well as in the previously considered cases, although only small improvements are visible from the starting point, as can be seen from the MSC.ADAMS gradient history in Figure 8, and the Matlab gradient history in Figure 9. It is observed that although both methods converge to equally feasible solutions, the front and rear spring characteristics should differ in absolute value as can be seen by design variables $x2$ and $x4$. The result of this is that if the front gas volume is larger the front seated passengers will experience better ride comfort than the rear passengers, and the opposite if the rear spring gas volume is larger.

From the above studies it is concluded that the optimisation process, making use of the simplified Matlab models for gradient information, produces equally feasible results in substantially less computational time. It will now be assumed that these models are sufficiently representative of the system for gradient information.

5. COMBINED RIDE COMFORT AND HANDLING OPTIMISATION

With the use of simplified models for gradient information validated, the models are combined to represent the vehicle performing a handling manœuvre on a rough terrain. For the combined ride comfort and handling optimisation, the vehicle performs the double lane change over the Belgian paving. The full simulation model will be used as before once per iteration for the exact objective function values and constraint values. The Matlab models will remain the same. However, the ride model will be used to observe the ride dynamics gradient tendencies, and the handling model for the handling dynamics gradient tendencies. A study was conducted as to how best to consider the optimisation of the compromise passive suspension. This is done to determine the methodology needed when including the control strategy of the $4S_4$ system for optimisation.

5.1. Handling Followed by Ride Comfort Optimisation

First the vehicle will be optimised for handling, subject to the tyre hop inequality constraints, and then optimised for ride comfort starting from the point where the handling optimisation converged. The ride comfort is optimised subject to the tyre hop inequality constraints, and an additional inequality constraint that the optimised handling f^*_{hand} may not decrease by more than 20% (compared to the optimised handling result) as stated below:

$$g(\mathbf{x})_{\mathbf{hand}} = \mathbf{10}(\mathbf{f_{hand}}(\mathbf{x}) - \mathbf{1.2f^*_{hand}}) \leq \mathbf{0} \tag{7}$$

The 20% parameter was selected as it was found that for optimisation runs where the handling constraint was 5 or 10 %, the handling constraint could not be satisfied, if improvements in ride comfort were achieved. The value of 20% was thus found to be a reasonable constraint value. The multiplication by 10 was used to better normalise the constraint values between -1 and 1.

The optimisation convergence history for two design variables is presented in Figure 10. The equivalent tyre hop constraint is plotted as defined in equation (6). The top graph refers to the handling optimisation where the objective function is defined as in equation (4), and the bottom graph is for the ride comfort optimisation, where the objective function is defined as in equation (3). It can be seen that the optimisation convergence history is well behaved for the handling optimisation, and results in an objective function value of approximately 0.21, which is equivalent to a body roll angle of 3 ^{o}, and a RMS body roll velocity of 1.3 $^o/s$. The ride comfort optimisation, subjected to the handling constraint, has a poorly behaved convergence history, and does not converge to a clear optimum. If iteration 18 is considered as the best minimum, the driver RMS vertical acceleration is approximately 2.2 m/s^2, which is considered as extremely uncomfortable [7], and needs to be improved.

5.2. Maximum of Ride Comfort and Handling

The results of handling followed by ride comfort optimisation, prompted the investigation into using the maximum value of the four normalised objective function parameters (roll angle, RMS roll velocity, driver comfort, passenger comfort) as the objective function value. The objective function is thus defined as follows:

$$f(x) = max(f(x)_{hand}, f(x)_{ride}) \quad (8)$$

The disadvantage of the nature of this objective function is the now inherent discontinuities due to the maximum function. However, very reasonable results were achieved as illustrated by Figure 11. In the figure, $f(x)hand$ is the handling objective function value as defined by equation (4), $f(x)ride$ is the ride comfort objective function as defined by equation (3), and the equivalent tyre hop constraint $g(x)eq$ defined by equation (6). Additionally it is observed that the overall optimum is the best of the two objectives. When considering the final design configuration iterations 3, 6 and 9, are repeated identical minima, and should be considered for the acceptable band of the design variables, to return objective function values of approximately 0.32. This results in vertical RMS accelerations of approximately 1.8 m/s^2, body roll angle of 4^o, and a RMS roll velocity of 1.9 $^o/s$. The optimisation convergence took fewer iterations than the optimisation of handling followed by ride comfort, even though the objective function is of a discontinuous nature, due to the maximum function.

The use of the maximum function for the objective function was expanded to four design variables, and started in the same place as for two design variables, the middle of the design space. The results, presented in Figure 12, illustrate the excellent convergence to the optimum, of identical magnitude as for two design variables, but the design variable values differ. Although it is evident that multiple local minima exist, the optimisation converges to identical objective function value minima.

6. SUMMARY OF RESULTS

Presented in Table 1 are the results for the optimisation runs. From the results it can be seen that the combined optimisation is a compromise between handling and ride comfort, especially when considering the use of the maximum function for the objective function. If reasonable handling is to be achieved, then the ride comfort suffers, while if good ride comfort is to be achieved then the handling suffers. This is the traditional compromise, that the $4S_4$ suspension avoids due to the ability to switch between the optimum handling and ride comfort settings. The resulting optimal damping multiplication factors and spring gas volumes are presented in Table 2. From the table it is clear that the ride comfort design parameters lie on the opposite corner of the design space to the handling design parameters. Also noticeable when observing the parameters of the combined optimisation, is that the gas volume lies in the middle of the design space at 0.3 l, but that the damping should be 50% of the current baseline characteristic. This however, severely affects the handling stability of the vehicle as can be observed by the higher RMS roll velocity value.

7. CONCLUSIONS

This paper has shown that the use of simplified mathematical models, of the computationally intensive full simulation model, for use in computing gradient information, can significantly improve the optimisation process. Firstly the optimisation process is significantly faster in terms of total optimisation time. Secondly the simplified models help to reduced numerical noise in the evaluation of the gradients, resulting in smoother convergence histories. Thirdly the simplified models are sufficiently representative of the vehicle system, when used for gradient information, although their absolute values may differ, and need to be properly scaled before use.

For the handling optimisation, it was found that the two methods gave identical optimum solutions, and that the optimal solutions lie along the maximum boundary of the damper design variable, and the lower boundary of the spring design variable.

For the ride comfort optimisation, the inclusion of the vehicle's tyre hop was investigated. It was found that the best results were achieved when including the tyre hop as an inequality constraint in the optimisation process. It was also found that the tyre hop tends to constrain the damping parameter from running towards it's lower boundary constraint.

Both ride comfort and handling were optimised simultaneously, and it was found that the maximum of the normalised ride comfort and handling objectives is a feasible objective function, for determining the compromise optimum. Both two and four design variable optimisation were successfully performed. Of particular interest is the fact that the spring design variable, lay close to the middle of the design space, while the damper design variable was decreased to 50% of the current baseline damper value. Work is still needed in the accurate description of the objective and constraint functions for the combined ride comfort and handling optimisation.

The methodology proposed is thus an efficient means of optimising a vehicle's suspension system for ride comfort and handling. This makes the use of deterministic gradient based optimisation algorithms most suitable, and competitive for suspension optimisation.

ACKNOWLEDGEMENTS

- Optimisation related investigations were performed under the auspices of the Multi-disciplinary Design Optimisation Group (MDOG) of the Department of Mechanical and Aeronautical Engineering of the University of Pretoria, and the National Research Foundation (NRF) under Thutuka Contract No. TTK-2004-081-0000-43.
- The vehicle dynamics simulation for the design of the controllable suspension system is based upon work supported by the European Research Office of the US Army under Contract's N68171-01-M-5852, N62558-02-M-6372 and N62558-04-P-6004.

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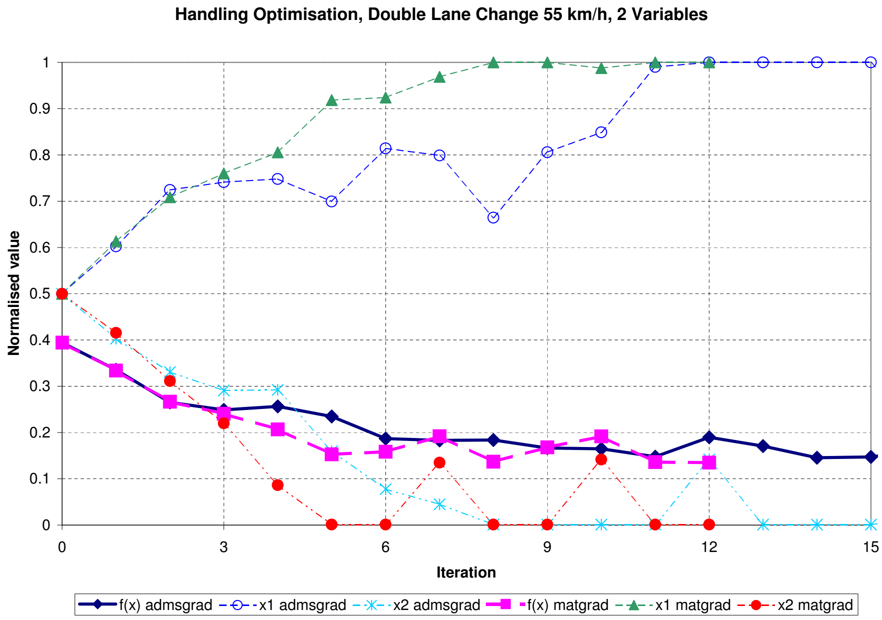


Fig. 1. Handling optimisation convergence histories for full MSC.ADAMS model, and using the simplified MATLAB model for gradient information, 2 design variables

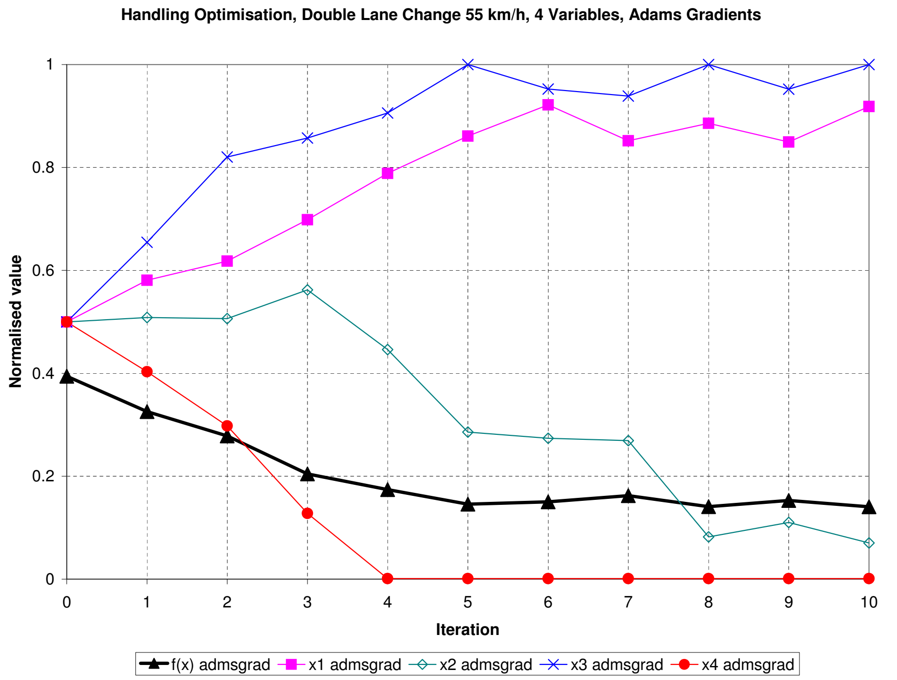


Fig. 2. Handling optimisation convergence history using the full MSC.ADAMS model for gradient information, 4 design variables

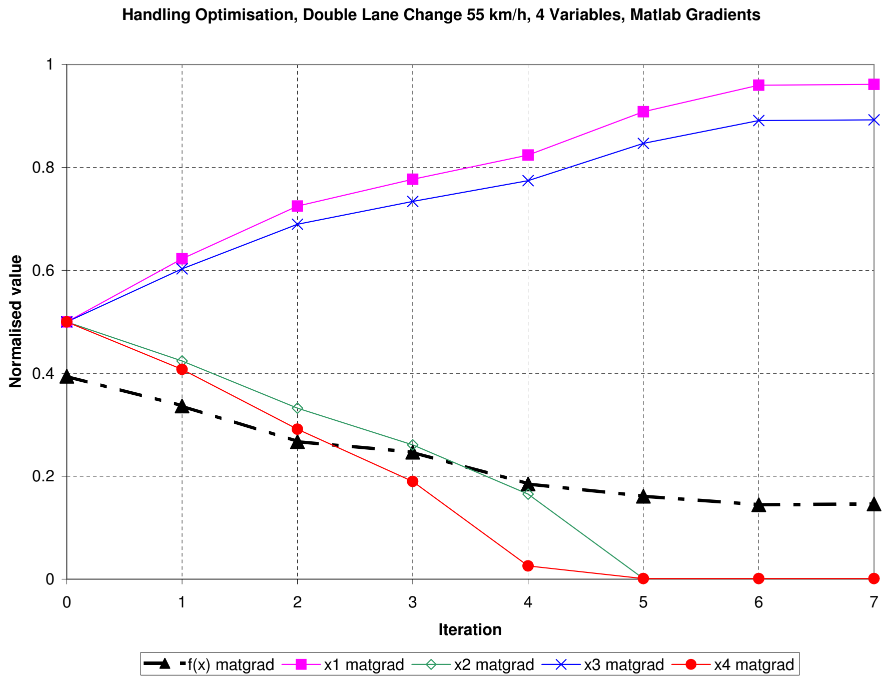


Fig. 3. Handling optimisation convergence histories using the simplified MATLAB model for gradient information, 4 design variables

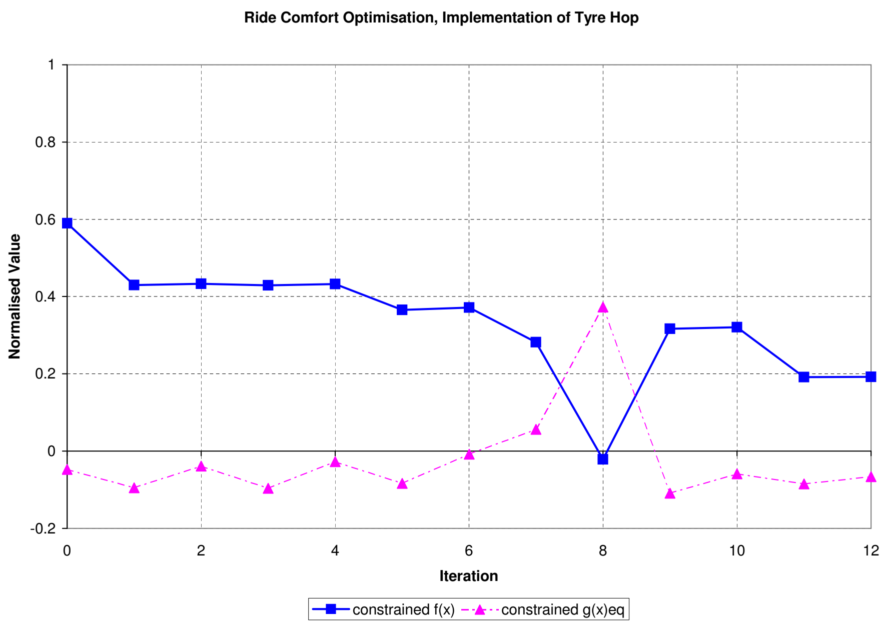


Fig. 4. Implementing tyre hop as a constraint in ride comfort optimisation

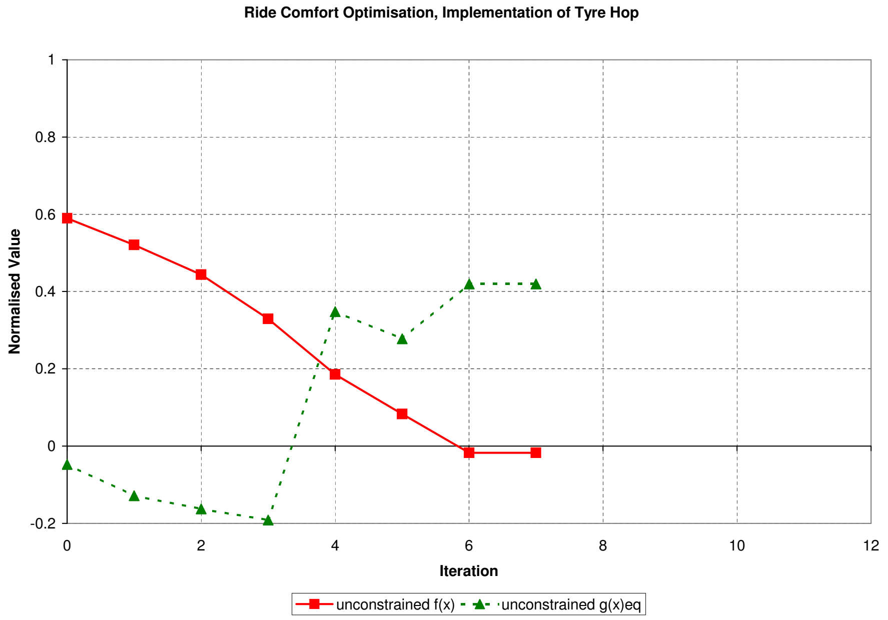
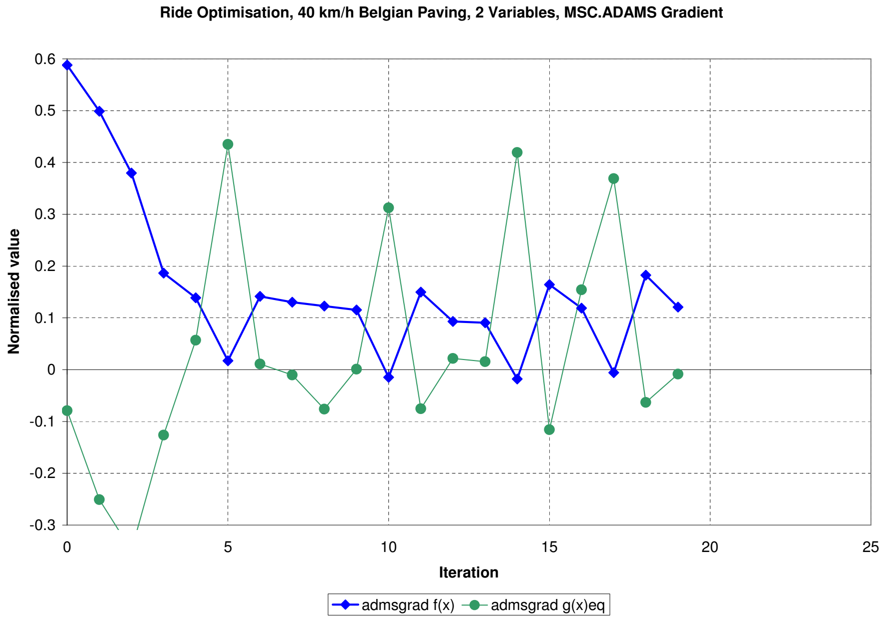


Fig. 5. Observing tyre hop value while performing ride comfort optimisation



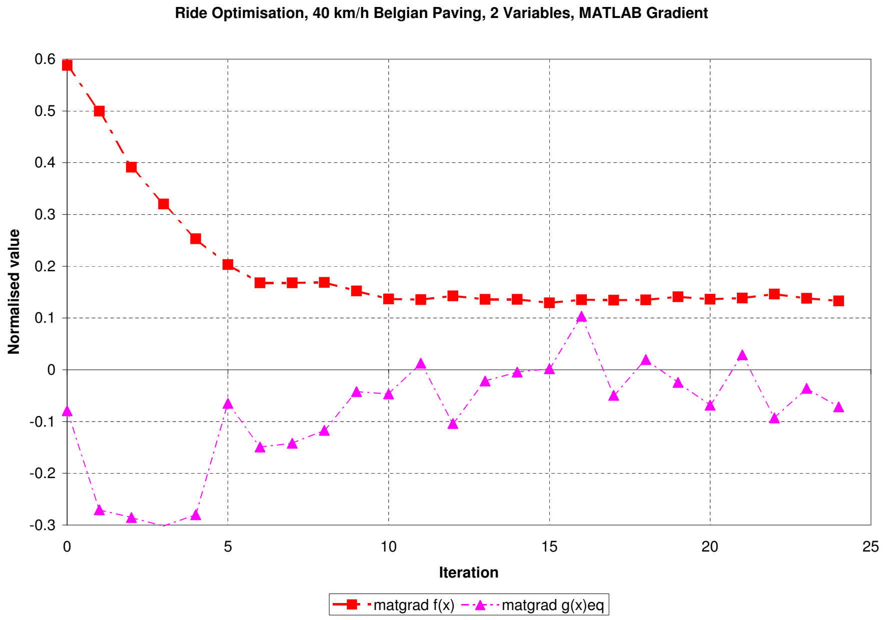


Fig. 6. Comparison of the optimisation histories for the MSC.ADAMS gradient and simple MATLAB model gradient methods for 2 design variable ride comfort optimisation

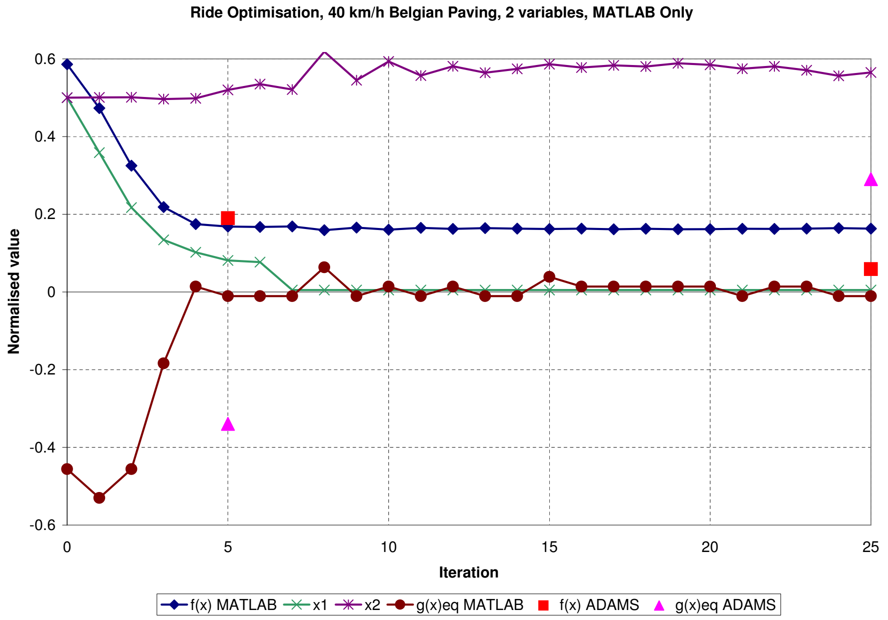


Fig. 7. Ride comfort optimisation convergence history for using only the simple Matlab based model, for objective function value, gradients, and tyre hop information.

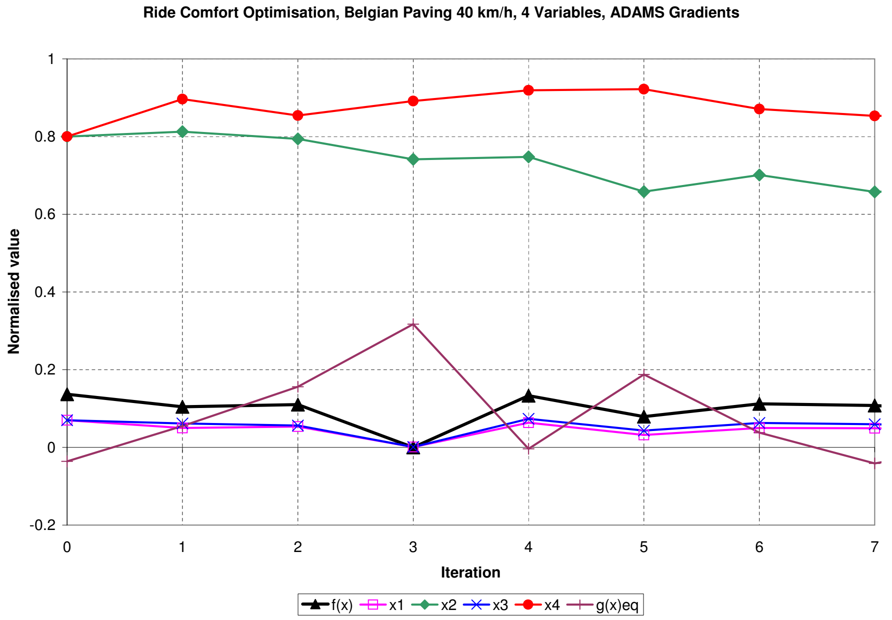


Fig. 8. Ride Comfort optimisation convergence history for 4 design variables using the full MSC.ADAMS model for gradient information, starting at the optimum from two design variables

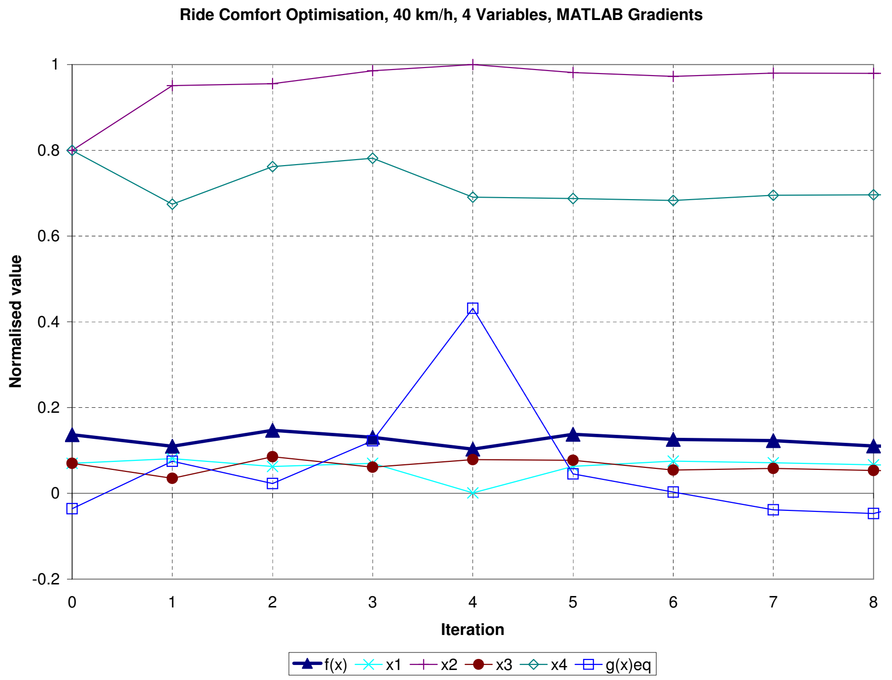
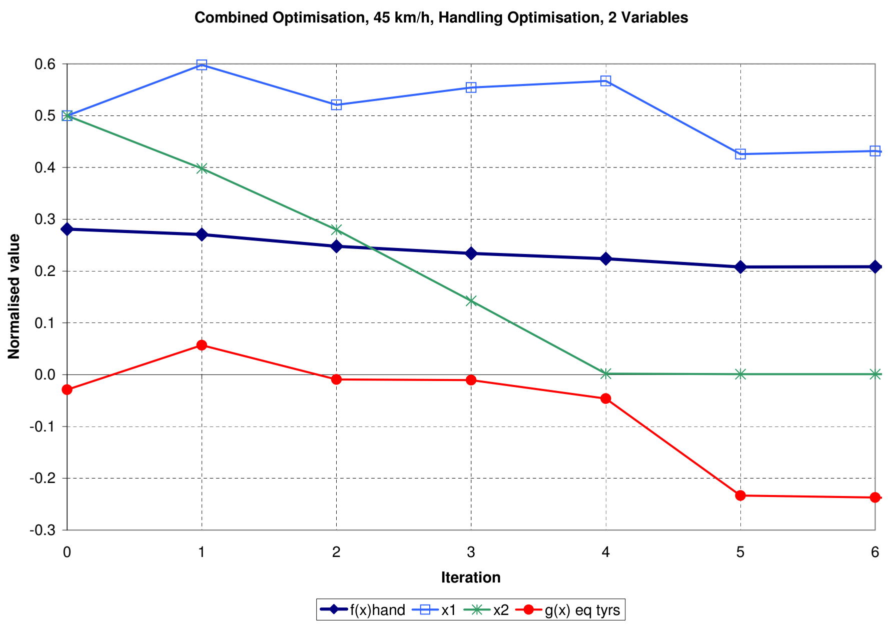


Fig. 9. Ride Comfort optimisation convergence history for 4 design variables using the Matlab model for gradient information, starting at the optimum from two design variables. Design variable x1 follows x3 very closely



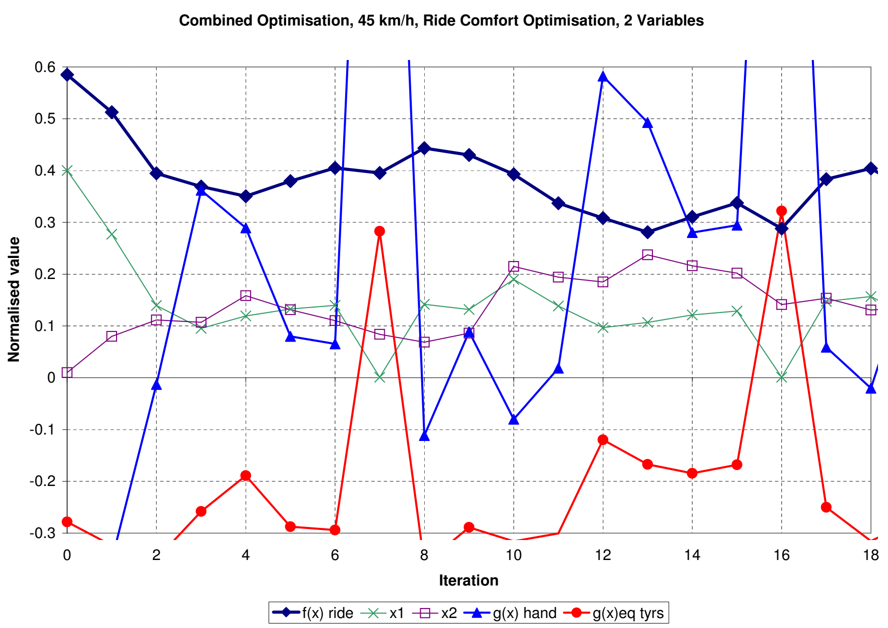


Fig. 10. Combined convergence history, first handling optimisation, then ride comfort.

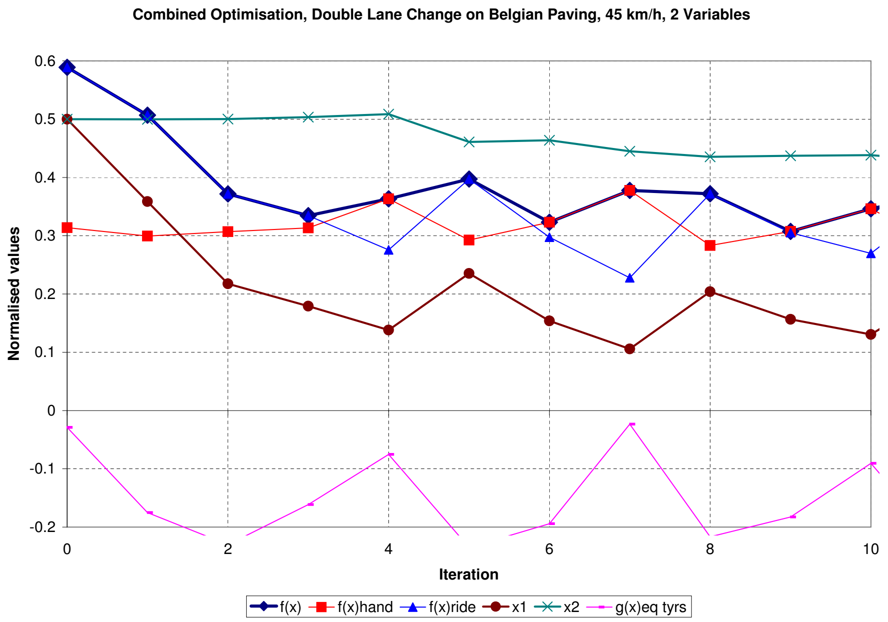


Fig. 11. Combined optimisation convergence history, maximum of handling and ride comfort objectives, 2 design variables.

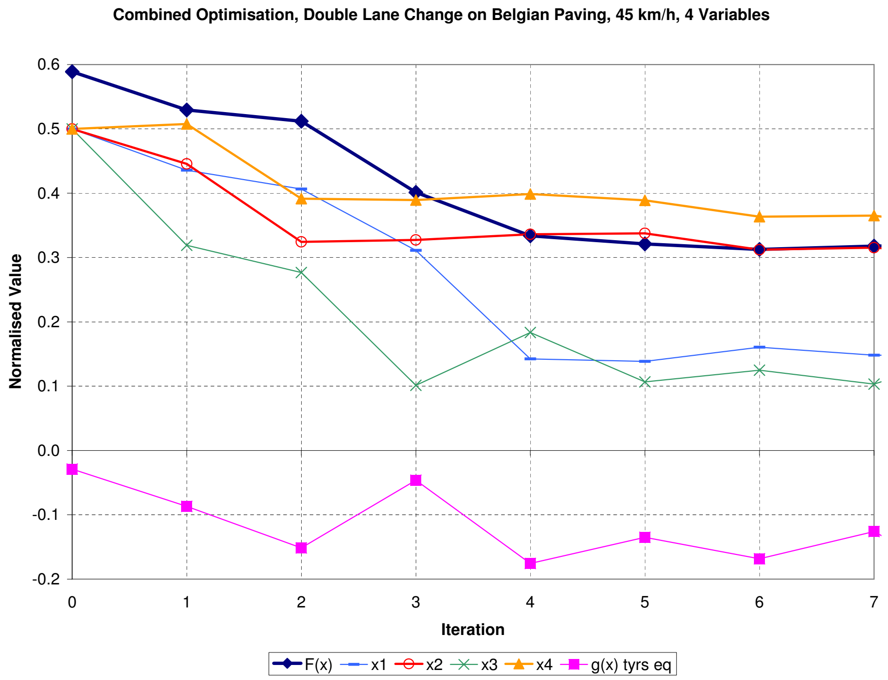


Fig. 12. Combined optimisation convergence history, maximum of handling and ride comfort objectives, 4 design variables.

Table 1. Summary of Results for Optimisation Objectives

variables, opt. run	Fig.	# iter. (eq evals)	$f^*(\mathbf{x})$ ± 0.01	$\dot{\varphi}_{RMS}$ $[^o/s]$	φ_{peak} $[^o]$	a_{RMS_d} $[m/s^2]$	a_{RMS_p} $[m/s^2]$
Handling							
2, matgrad	1	12 (18.2)	0.15	0.57	3.0	-	-
2, admsgrad	1	15 (80)	0.15	0.57	3.0	-	-
4, admsgrad	2	6 (63)	0.15	0.54	3.2	-	-
4, matgrad	3	7 (14.4)	0.15	0.55	3.1	-	-
Ride							
2, matgrad	6	24 (35)	0.13	-	-	1.20	1.18
2, admsgrad	6	19 (100)	0.12	-	-	1.16	1.14
4, matgrad	9	9 (18)	0.11	-	-	1.14	1.08
4, admsgrad	8	7 (72)	0.11	-	-	1.10	1.10
Combined							
2, handling 1^{st}	10	6 (9.8)	0.21	1.29	2.9	-	-
2, ride after	10	18 (34.2)	0.40	1.52	3.0	2.20	2.18
2, $f_{max}(x)$	11	6 (12.6)	0.32	1.86	4.0	1.78	1.78
4, $f_{max}(x)$	12	7 (20.8)	0.32	1.83	4.0	1.76	1.62

Table 2. Summary of optimum damper factors and gas volumes

opt. run	Fig.	$dpsff$	$gvolf$	$dpsfr$	$gvolr$
Handling					
2, matgrad	1	3.00	0.10	3.00	0.10
2, admsgrad	1	3.00	0.10	3.00	0.10
4, admsgrad	2	2.72	0.24	3.00	0.10
4, matgrad	3	2.89	0.10	2.69	0.10
Ride					
2, matgrad	6	0.30	0.51	0.30	0.51
2, admsgrad	6	0.29	0.54	0.29	0.54
4, matgrad	9	0.29	0.56	0.25	0.47
4, admsgrad	8	0.24	0.43	0.27	0.53
Combined					
2, handling 1^{st}	10	1.35	0.10	1.35	0.10
2, ride after	10	0.55	0.17	0.55	0.17
2, $f_{max}(x)$	11	0.51	0.30	0.51	0.30
4, $f_{max}(x)$	12	0.53	0.26	0.40	0.28