USING MACHINE LEARNING TECHNIQUES AS TRACK GEOMETRY PREDICTORS FOR RAILWAY TRACK

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ABSTRACT

This paper investigates utilising machine learning (ML) techniques to predict the five major parameters of track geometry in railway infrastructure. Track geometry and incurred vehicle acceleration measurements were collected on a railway line and matched according to their GPS coordinates. The data were then split into test/train and validation datasets and processed using various ML methods. The predictive results of each ML method were compared for each track geometry parameter and the best methods were highlighted. The quality of the results was mixed with accurate results obtained for cant and alignment but inaccurate results for gauge, profile and twist. Overall, this research paper contributes to the field of railway engineering by demonstrating the potential to utilise ML in the field by predicting track geometry parameters. The findings have practical implications for improving track maintenance and ensuring passenger safety and comfort in railway operations. The promising results of this paper warrant more research being conducted and potential methods for improvement are highlighted.

1. INTRODUCTION

Maintenance of railway infrastructure typically comprises over 70% of a track's lifecycle costs (Heyns, 2006). One of the most important aspects of railway track maintenance is the identification and correction of poor track geometry conditions, known as track geometry irregularities (Li et al., 2006). These irregularities incur high vehicle vibrations which lead to poor ride quality as well as a greater risk of vehicle derailment (Hao et al., 2023). Although it is understood that vehicle vibrations and track geometry have an underlying correlation, there has long been a history of unsuccessful research attempting to find a correlation between these metrics (Haigermoser et al., 2014). However, recent advancements in algorithms and computational power have enabled some success in this endeavour. For example, Odashima et al. (2016) have shown that by using a Kalman filter, vertical track geometry can be estimated within 1 mm numerical accuracy by knowing the speed and car-body accelerations of the train.

ML is one of these recent technological innovations enabling advancement in engineering (AlHamaydeh et al., 2022) and in particular, railway engineering. These applications include, but are not limited to: image-based condition monitoring of track superstructure (Broekman & Gräbe, 2021); prediction of rail contact fatigue using image processing (Sysyn et al., 2019) and track geometry deterioration prediction using periodic measurement data (Lee et al., 2020). Hao et al. (2023) showed that by using a combination of ML techniques, the measured vertical and lateral accelerations as well as speed of the train could be used as inputs to predict the vertical profile of the rail as an output. However, research is still required outlining how the five major track geometry parameters (profile, alignment, gauge, cant and twist) can be predicted using the train's ride quality, as most research focuses on either predicting the combined track quality or identifying individual defects in the track (Liao et al., 2022). This research therefore aims to utilise ride quality data processed through ML techniques to predict the five major parameters of track geometry.

1.1 Objectives

The main objective of this study is to utilise ML techniques to determine algorithms that can utilise incurred vehicle accelerations and speed of the vehicle as inputs to predict the railway's track geometry.

In addition, the different algorithms derived from the different ML methods will be evaluated to determine which formula results in the smallest error when presented with a previously unseen set of input data, known as a validation dataset.

2. LITERATURE REVIEW

2.1 Track Geometry

Track geometry is utilised to determine the roughness of the track and plays a crucial role in evaluating a track's safety (Tsunashima et al., 2014). Track geometry is categorised in terms of the rail's shape projected into four planes: horizontal; longitudinal vertical; transverse vertical and track. These projected shapes enable standardisation of track geometry for both design and maintenance operations. A summary of the planes utilised and the corresponding names of the track geometry measurements are shown in Table 1 with a graphical description of the track geometry measurements shown in Figure 1.

Table 1: Track geometry measurement descriptions

Plane	Name of Measurement(s)	Description
Longitudinal Vertical	Vertical profile	Vertical deviation in rail shape
Horizontal	Alignment	Horizontal deviation in rail shape
Transverse Vertical	Cant	Difference in elevation of rails
	Twist	Change in cant over a distance known as a twist-base
Track	Gauge	Distance between inside of rail heads, 15 mm below top of rail head

The measurements shown in Table 1 and Figure 1 enable characterisation of the track's geometry either by analysing each measurement individually or by combining the measurements into a Track Quality Index (El-Sibaie & Zhang, 2004).

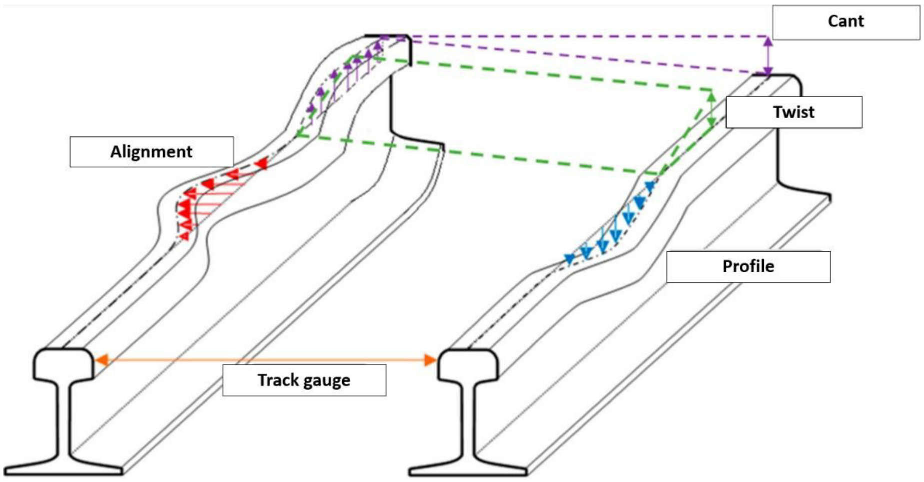


Figure 1: Track geometry measurements depiction (adapted from Liao et al., 2022)

2.2 Vehicle Dynamics

Together with track geometry, train dynamics play an important role in evaluating the safety of a railway operation. The dynamic behaviour of trains is significantly influenced by track geometry quality (Haigermoser et al., 2014) and thus track geometry contributes to passenger comfort and safety (Sresakoolchai & Kaewunruen, 2022). Higher train speeds incur higher vibrations, both wheel-axle and axle-body, in turn reducing ride quality (Liu et al., 2020).

2.3 Machine Learning

ML is a subset of artificial intelligence based on constructing predictive algorithms that describe a complex interaction between input and output variables in a dataset (Markou and Bakas, 2021). It can be explained as an algorithm that learns directly from a given dataset, without human assumptions or theory (Christodoulou et al., 2019).

A commonly used type of ML technique is multiple linear regression, which aims to model the relationship between many input variables (features) and an output variable (target) using a linear relationship (Pandit et al., 2021). The underlying principle is based on the minimisation of residuals (bias) between predicted values and actual data. Multiple nonlinear regression is based on the same principle of multiple linear regression however, feature variables utilise nonlinear relationships to predict the target variables.

Empirically-based ML algorithms also exist, such as random forests, decision trees and artificial neural networks (ANN) (Christodoulou et al., 2019). These algorithms are sometimes referred to as "black-box" models as it is not evidently clear how the algorithms model predictions. For example, the random forest algorithm is derived from a forest of decision trees which can utilise classification or regression to predict an output at a leaf basedon decisions occurring at a node. ANNs utilise input and output layerswith hidden layers in between which transform the features to match the target by passing the features through functions at the nodes (Markou et al., 2023). Deep learning is an extension of ANNs but is typically best suited for large datasets and where relationships between data are complex (Kelleher, 2019). This arises from deep learning's ability to utilise more hidden layers between the input and output nodes, thus enabling modelling of more complex relationships between the feature and target variables (Muniasamy & Alasiry, 2020).

3. EXPERIMENTAL SETUP

Data were collected on the railway line between Phalaborwa and Hoedspruit in South Africa’s Limpopo province. The data acquisition consisted of a road-rail vehicle (RRV), track geometry trolley (Krab) and inertial measurement and high-accuracy unit (IMU). An RRV is a dual-purpose vehicle that can travel on both roadways and railways and is typically utilised for track maintenance purposes such as collecting track geometry data. Inside the cab of the RRV, the IMU was positioned in the place of a seat to capture accelerations reflecting the passenger ride quality, shown in Figure 2(a). Behind the RRV, a track geometry trolley was towed to capture the track geometry, shown in Figure 2(b).

(a) (b)

Figure 2: Experimental setup of (a) IMU in RRV cab and (b) track geometry trolley behind RRV

The experimental setup allows for capturing incurred vehicle accelerations and subsequently capturing the corresponding track geometry causing these accelerations. All data for this research was captured at a low speed of approximately 10 km/h.

4. DATA PREPROCESSING

4.1 Matching Accelerations and Track Geometry Data

As the data were captured from two separate apparatus, the data would have to be matched to ensure that the track geometry measurements were matched exactly with the incurred accelerations. The different sampling frequencies of the two apparatus led to a potential problem – one track geometry measurement had roughly 10 acceleration values associated with it. As both apparatus had built-in GPS capabilities, it was decided to match the track geometry measurement with the single closest acceleration value geographically using the Haversine formula.

In an effort to negate erroneous acceleration and track geometry data experienced at turnouts, the matched accelerations and track geometry data were filtered to include all values above the 2nd percentile and below the 98th percentile of the final matched values. An example of the matched values for cant and accelerations in the x- and y-directions is shown in Figure 3.

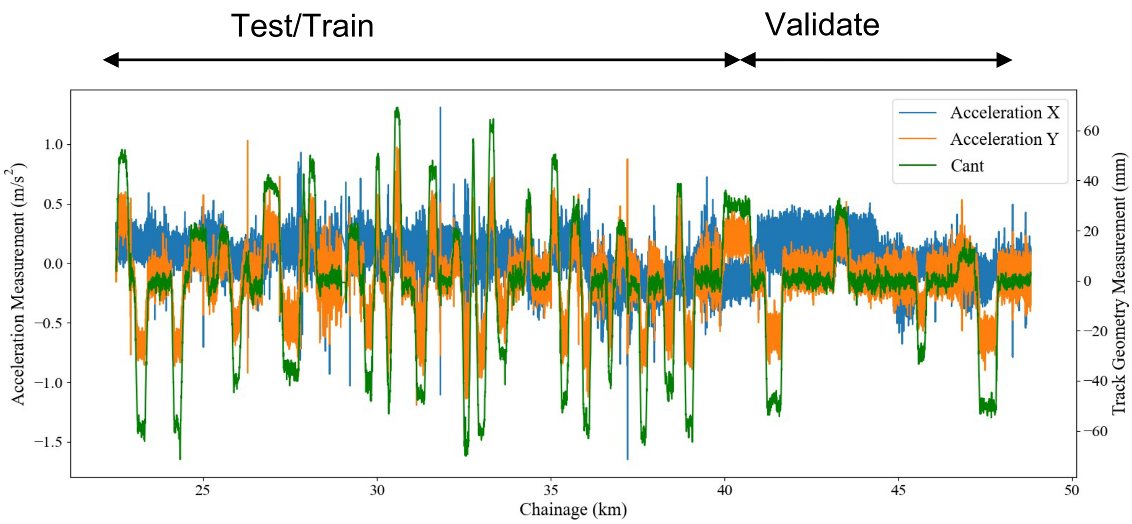


Figure 3: Example of matched track geometry and acceleration values

Figure 3 confirms good matching of the datasets when comparing Acceleration Y and cant in the curves, represented by the peaks of the plots. These two curves exhibit peaks at the same track distance, both during and after the curve's development in superelevation. These peaks are however of different magnitudes.

The dataset was then split into two datasets – the test/train and validation datasets. The test/train dataset utilised data collected from kilometres 22 to 41 and was randomly split into a 20:80 ratio respectively. The test/train dataset is used to train the algorithms and subsequently evaluate the performance of the algorithms by utilising cross-validation so that the algorithm can learn from errors and improve on the following iteration. The validation dataset utilised data from kilometres 41-49 and represents the out-of-sample metrics for this experiment. The use of a validation dataset is important because as shown in Figure 3, the curves (represented by peaks in cant) in the test/train and validation portions of the dataset exhibit different lengths and rates of change of superelevation. Thus, the performance of the validation dataset is a good indicator of the robustness of the algorithms as it can potentially prove the algorithms' abilities to adapt to track characteristics it was not explicitly trained on.

4.2 Descriptive Statistics

Before training the different ML algorithms, the feature and target variables of the dataset were compared using Pearson's correlation coefficient, r to determine if any underlying relationships existed between the feature and target variables. The formula to determine Pearson's correlation coefficient is given in Equation 1.

$$r = \frac{n(\sum_{i=1}^{n} xy) - (\sum_{i=1}^{n} x)(\sum_{i=1}^{n} y)}{\sqrt{n(\sum_{i=1}^{n} x^2) - (\sum_{i=1}^{n} x)^2)(n\sum_{i=1}^{n} y^2 - \sum_{i=1}^{n} y)^2}} \quad (1)$$

To visualise the correlations of many feature and target variables, it is helpful to utilise correlation matrices. Figure 4 shows the Pearson correlations between all feature and target variables individually. As target and feature variables names have been abbreviated, a full list of explanations for these variables is given in Appendix A.

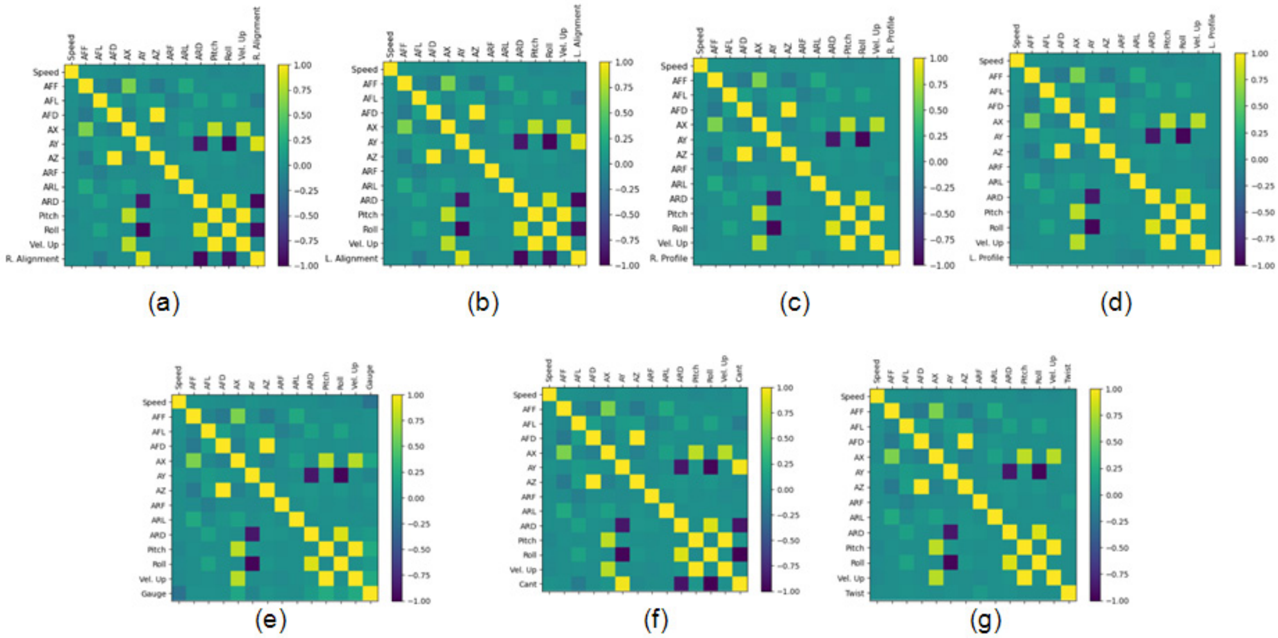


Figure 4: Correlation matrices of (a) right alignment (b) left alignment (c) right profile (d) leftprofile (e) gauge (f) cant and (h) twist

The yellow and purple blocks in Figure 4 show very strong correlations between feature and target variables. For example, in Figure 4(a), the target variable (right alignment) shows a strong positive correlation with AY (acceleration in the y-direction) and strong negative correlations with ARD (angular rate down) and the vehicle's roll, respectively. These strong correlations are indicative of good results when the data are processed using the ML algorithms. However, results shown in Figure 4(c) show a very weak correlation between features and the target variable of left profile, indicating that the ML algorithms will incur difficulty in predicting the target variable of right profile.

5. APPLIED MACHINE LEARNING

5.1 Error Metrics

Various error metrics are used to evaluate the performance of a ML algorithm. These metrics compare the predicted target values with the actual target values to find the accuracy of the prediction. The different error metrics used for the purpose of this research work are shown in Appendix A.

5.2 Results

The ML algorithms used in this paper as proposed by Markou et al. (2023) are multiple linear regression, multiple nonlinear regression, random forest (RF), XGBoost, artificial neural networks by neighbour (ANNBN) and deep artificial neural networks (DANN). The use of many algorithms allows for the comparison of their respective accuracies and ability to predict target values. All utilised algorithms are courtesy of open-source code from Bakas et al. (2023) called nbml. The statistical results for the validation datasets for the various algorithms are shown in Table 2 and the graphical depiction of the results are depicted in Figure 5.

Table 2: Statistical errors of ML algorithms for validation dataset

Target	Model	r	MAPE	MAMPE	MAE	RMSE
Right Alignment	ANNBN	0.915	Inf	0.353	5.277	6.839
	DANN	**0.934**	Inf	0.302	4.517	5.949
	Linear Reg.	0.906	Inf	0.261	3.897	**4.997**
	Nonlinear Reg.	0.923	Inf	0.256	4.530	6.388
	RF	0.930	Inf	**0.254**	**3.793**	5.356
	XGBoost	0.905	Inf	0.360	5.386	7.765
Left Alignment	ANNBN	0.907	Inf	0.368	5.500	7.243
	DANN	**0.934**	Inf	0.277	4.136	5.752
	Linear Reg.	0.908	Inf	0.259	3.870	**4.968**
	Nonlinear Reg.	0.928	Inf	0.320	4.636	6.462
	RF	0.930	Inf	**0.256**	**3.824**	5.402
	XGBoost	0.910	Inf	0.354	5.293	7.514
Right Profile	ANNBN	0.088	Inf	1.037	1.193	1.589
	DANN	0.001	Inf	1.000	1.151	1.549
	Linear Reg.	**0.137**	Inf	**0.994**	**1.143**	**1.534**
	Nonlinear Reg.	0.118	Inf	0.998	1.149	1.540
	RF	0.077	Inf	1.003	1.155	1.547
	XGBoost	0.032	Inf	1.221	1.405	1.849
Left Profile	ANNBN	0.123	Inf	1.023	1.327	1.721
	DANN	0.004	Inf	1.012	1.312	1.703
	Linear Reg.	**0.190**	Inf	**0.983**	**1.275**	**1.656**
	Nonlinear Reg.	0.158	Inf	0.988	1.294	1.677
	RF	0.012	Inf	1.001	1.298	1.687
	XGBoost	0.067	Inf	1.133	1.469	1.892
Gauge	ANNBN	0.293	Inf	0.687	2.330	3.139
	DANN	0.355	Inf	**0.586**	1.987	2.700
	Linear Reg.	**0.613**	Inf	0.633	2.146	2.585
	Nonlinear Reg.	0.320	Inf	0.662	**1.918**	**2.522**
	RF	0.419	Inf	0.623	2.113	2.779
	XGBoost	0.416	Inf	0.692	2.345	3.002
Cant	ANNBN	0.990	Inf	0.210	2.148	2.823
	DANN	**0.994**	Inf	0.174	1.783	**2.197**
	Linear Reg.	0.993	Inf	0.180	1.842	2.269
	Nonlinear Reg.	0.993	Inf	0.179	1.840	2.250
	RF	0.994	Inf	0.177	1.814	2.257
	XGBoost	**0.993**	Inf	**0.173**	**1.770**	2.234
Twist	ANNBN	0.012	Inf	1.220	1.495	1.962
	DANN	0.023	Inf	**1.002**	**1.228**	**1.598**
	Linear Reg.	**0.040**	Inf	1.005	1.231	1.600
	Nonlinear Reg.	0.024	Inf	1.004	1.261	1.630
	RF	0.030	Inf	1.094	1.340	1.730
	XGBoost	0.026	inf	1.247	1.527	1.941

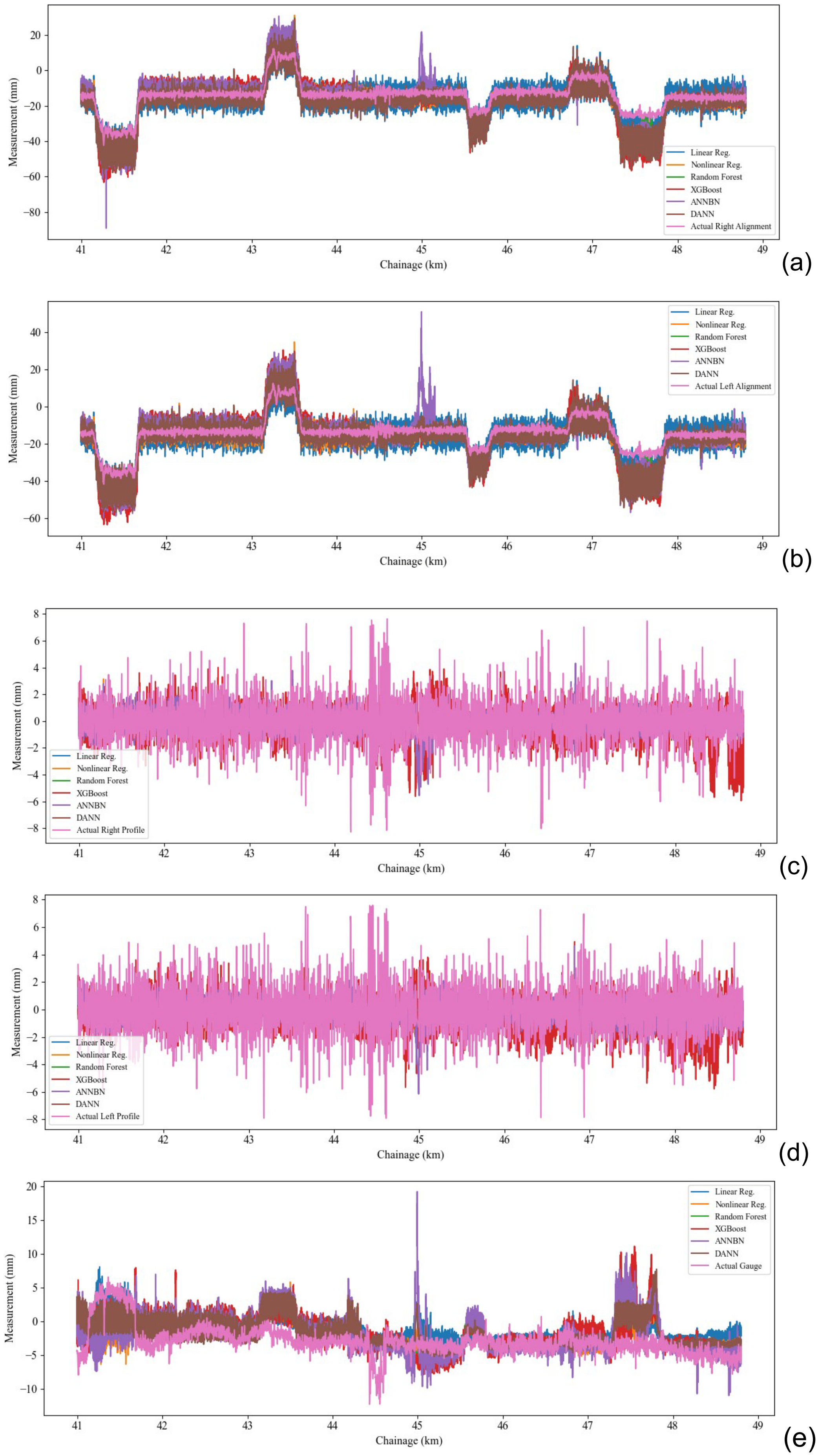
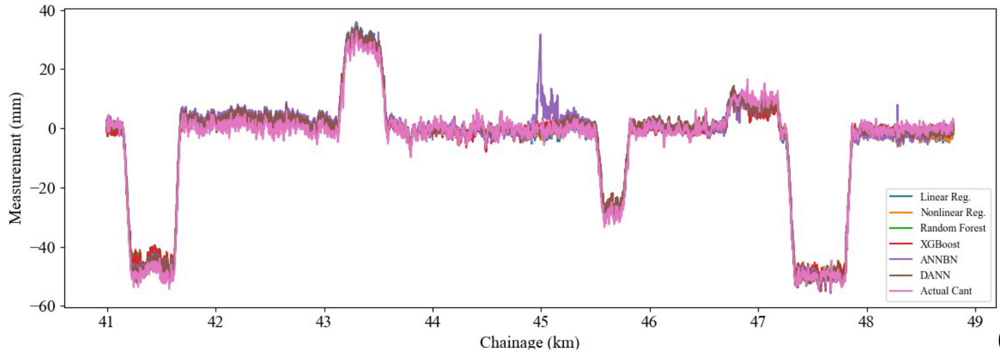
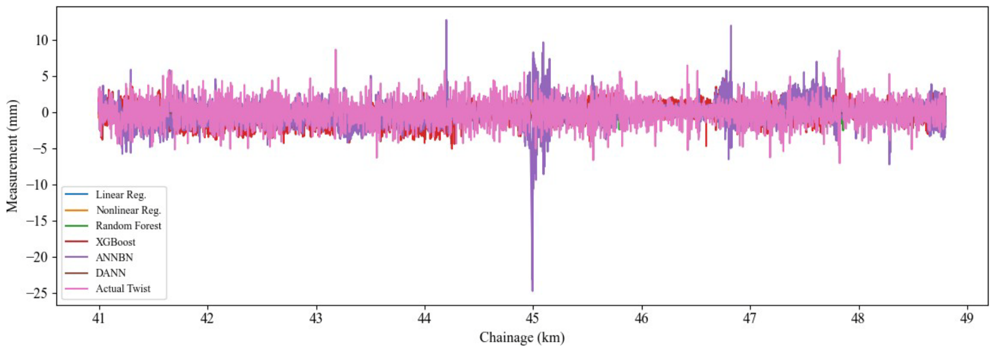


Figure 5: Graphical depiction of validation dataset's ML results for (a) right alignment (b) left alignment (c) right profile (d) left profile (e) gauge (f) cant and (h) twist



(f)



(g)

Figure 5: Cont'd

The results shown in Table 2 and Figure 5 are indicative of promising results for utilising ML to predict track geometry. The predicted alignments and cant show very accurate results, with low errors and graphs resembling the actual measured values. However, due to the lack of statistical correlations shown in Figure 4 - profile, gauge and twist produce very inaccurate results both statistically and visually. These sets of measurements are all centred about the x-axis, and therefore correlation can potentially be found by including more features centred about a common value. The simpler methods of multiple linear and nonlinear regression perform better for these metrics as the empirical methods incur difficulty in making accurate decision trees/hidden layers for values that vary about the x-axis.

The poor results for twist are expected as twist is defined as the change in cant along a distance of track. As the ML models only learn from each record of data and therefore cannot learn from previous entries, it is logical that the models cannot predict twist accurately.

The infinite errors for the MAPE metric arise from its formula which has the actual measurement, ytr_i as its denominator. Due to the fact that this experimental data contains values which cross the x-axis, the denominator tends to zero and thus division by zero occurs, leading to a mathematical error.

In summary, the model performance varies across different target categories. DANN consistently achieves high correlations and relatively low MAE values across most categories, making it a strong performer. Multiple linear regression also shows strong correlations but can have slightly higher MAE values in some cases. ANNBN tends to have lower correlations and higher MAE values, making it less effective for these target categories. The high spikes of ANNBN around kilometre 45 indicate potential overfitting of the test/train dataset. RF and XGBoost generally perform well with strong correlations and moderate MAE values, while multiple nonlinear regression's performance falls in between.

6. CONCLUSIONS

The main research aim of utilising ML techniques to predict track geometry showed mixed quality results. It has been shown that due to the strong statistical correlation between parameters, it is possible to yield a good prediction of alignment and cant values respectively. As little correlation exists between the selected features and the target variables of profile, gauge and twist, the ML techniques experienced difficulty in accurately predicting these track geometry parameters. The secondary research aim of comparing the different ML techniques shows that the two best techniques in terms of statistical correlation are linear regression or deep artificial neural networks (DANN), depending on the target variable being predicted. Further research is required to evaluate the training time and hyperparameter optimisation of the different methods, as these topics were beyond the scope of this study.

This research contributes to the field of railway engineering through enabling easier operations of track maintenance. The use of a model trained on both vehicular ride quality data and corresponding track geometry data can enable track maintenance teams to perform future condition monitoring using ride quality data only as opposed to the combination of both ride quality data and track geometry data. This is beneficial in a time-saving and operational efficiency aspect as track geometry data is typically more difficult to collect than ride quality data.

As this paper forms part of ground-breaking research in the railway environment, this research will be extended in an attempt to decrease errors and improve predictions of gauge, cant and twist by utilising more feature variables as inputs and exploring data manipulation through the use of fast Fourier transforms and Kalman filters. As this research was not conducted in an actual train but rather an RRV, data captured from a moving train at high speeds (up to 160 km/h) is another area in which this project will be extended. In summation, this study presents encouraging preliminary findings, indicating significant potential for advancements in this field.

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APPENDIX A

Table 3: All feature and target variables used for ML

Variable Abbreviation	Variable Name	Description
Speed	Speed	Speed of the RRV $[m/s]$
AFF	Acceleration filtered forward	Moving average filter of accelerations in x-direction $[m/s^2]$
AFL	Acceleration filtered lateral	Moving average filter of accelerations in y-direction $[m/s^2]$
AFD	Acceleration filtered down	Moving average filter of accelerations in z-direction $[m/s^2]$
AX	Acceleration X	Accelerations in direction of rail $[m/s^2]$
AY	Acceleration Y	Accelerations in direction normal to rail $[m/s^2]$
AZ	Acceleration Z	Accelerations in vertical direction $[m/s^2]$
ARF	Angular rate forward	Angular forward velocity $[deg/s]$
ARL	Angular rate lateral	Angular lateral velocity $[deg/s]$
ARD	Angular rate down	Angular downwards velocity $[deg/s]$
Pitch	Pitch	Rotation about y-axis [°]
Roll	Roll	Rotation about x-axis [°]
Vel. Up	Velocity up	Upwards velocity in vertical direction $[m/s]$
L. Alignment	Alignment of left rail	Vertical deviation of left rail $[mm]$
R. Alignment	Alignment of right rail	Vertical deviation of left rail $[mm]$
L. Profile	Profile of left rail	Vertical deviation of left rail $[mm]$
R. Profile	Profile of right rail	Vertical deviation of right rail $[mm]$
Gauge	Track gauge	Distance between inside of rail heads $[mm]$
Cant	Cant	Difference in elevation of rails $[mm]$
Twist	Twist	Change in cant along a twist-base of 3 m $[mm]$

Table 4: Error formulae and descriptions

Name	Formula	Description
Root Mean Squared Error	$RMSE = \sqrt{\frac{\sum_{i=1}^{(m} \hat{ytr}_i - ytr_i)^2}{m}} \#(2)$	Square root of sum of biases squared divided bynumber of samples
Mean Absolute Error	$MAE = \frac{\sum_{i=1}^{m} \lvert \hat{ytr}_i - ytr_i \rvert}{m} \#(3)$	Sum of absolute biases divided by number of samples
Mean Absolute Percentage Error	$MAPE = \frac{1}{m} \sum_{i=1}^{m} \frac{\lvert \hat{ytr}_i - ytr_i \rvert}{ytr_i} \#(4)$	Sum of absolute percentage errors of dataset divided by number of samples
Mean Absolute MeanPercentage Error	$MAMPE = \frac{1}{m} \sum_{i=1}^{m} \frac{\lvert \hat{ytr}_i - ytr_i \rvert}{\frac{1}{m} \sum_{i=1}^{m} ytr_i} \quad (5)$	Sum of biases divided by average feature value,divided by number of samples