MATHEMATICAL EVALUATION OF THE SAFETY EFFECTS OF EMBANKMENT ON CURVES FOR ROADS IN THE SOUTH AFRICAN CONTEXT

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ABSTRACT

Curves have always been a safety concern in roadway design. More crashes tend to occur on curved sections than on tangent sections. Factors influencing safety concerns on curves, such as design speed and, embankment angles, were investigated through mathematical models based on the design guidelines currently used in South Africa and the Green Book. The effect of gradient did not affect the maximum travel speed that can be obtained on a specific radius with an applied superelevation rate. Minimum radii were found to pose the highest risk, and it is recommended to impose desirable minimum limits rather than relying on theoretical minimum values to mitigate these risks.

1. INTRODUCTION

Every year, approximately 1.35 million individuals lose their lives on roads globally (Centers for Disease Control and Prevention, 2023). (https://www.cdc.gov/). Road crashes are a significant concern for road users in South Africa. In 2021, there were a total of 10,446 fatal crashes on South African roads (RTMC, 2021), resulting in 12,436 fatalities. These statistics highlight the severity of road crashes, emphasizing the importance of all efforts to reduce this number.

The goal of this study is to assess how each of these factors contributes to curved road risk and to develop mitigation strategies for crash prevention. The Green Book, 'A Policy on Geometric Design of Highways and Streets' (AASHTO, 2018), and the 'Geometric Design Manual' (SANRAL, 2002) serve as the foundation for this study, as they are the primary guidelines for geometric design used in South Africa and neighbouring countries.

From a geometric perspective, design elements such as curve radius, friction coefficient, superelevation, and design speed can significantly impact driver safety. By examining these elements individually, it is possible to identify potential safety limitations that may inform future road design practices.

2. LITERATURE REVIEW

The geometric design standards/guidelines are typically based on common sense, engineering judgment, and basic physics, with only a small percentage of standards grounded in actual research, and indeed most design standards being uninformed by crash frequency or safety research (Hauer, 1999). Designing a road to meet minimum standards does not necessarily ensure road safety. Instead, designers should ask, 'Is the road as safe as it could be?' It is worth noting that using values higher than the minimum standards does not always make the road safer, as higher design values can encourage higher speeds among road users.

The design parameters of horizontal curves, determined by factors such as travel speed, tyre friction, road radius, and road embankment (superelevation), are strongly underlain by physics and logic. Calculations consider the centrifugal force required to keep a vehicle within the lane and a conservative friction coefficient for the design speed, yielding a 'minimum safe curve.'

The Green Book (AASHTO, 2018) is widely referenced because AASHTO provides comprehensive guidance regarding geometric design and serves as the primary reference for all Southern African geometric design manuals. The Geometric Design Manual (SANRAL, 2002) was introduced in 2002 by the State-Owned Enterprise (SOE) SANRAL. This manual provides more theoretical information than preceding manuals in South Africa. It is based on the latest trends in geometric design at the time of publishing and includes sections based on academic studies where available. While this manual draws from the AASHTO manual, differences do exist. It is primarily used for National Roads, but when no design manual is prescribed by a client, this manual is the best go-to option due to its greater conciseness compared to preceding manuals.

2.1 Horizontal Alignment and Superelevation

The basic theory of vehicle dynamics on banked curves suggests that there is a relationship between the curve radius, angle of banking (referred to as superelevation), and the road friction coefficient. The Green Book (AASHTO, 2018) provides this relationship as follows:

Equation 3-6 (AASHTO, 2018, p 3-20) is given as follows:

$$\frac{\mathbf{0.01}e+f}{\mathbf{1-0.01}ef}=\frac{v^{2}}{g\mathbf{R}}=\frac{\mathbf{0.0079V^{2}}}{\mathbf{R}}=\frac{\mathbf{V^{2}}}{\mathbf{127R}} \qquad [1]$$

Where:

e = rate of superelevation, percent
f = side friction demand factor
v = vehicle speed, m/s
g = gravitational constant, 9.81m/s^2
V = vehicle speed, km/h
R = radius of curve measured to a vehicle’s centre of gravity, m
The value of 0.01e f is exceedingly small and omitted in calculations.

The forces acting on a moving object on a curve include the gravitational force due to the object’s weight, the normal force acting perpendicular to the road surface, the friction force along the surface and the centrifugal force due to the radius (UTA Physics Department - Technical Physics, Lecture Notes, Lecture 14: Banked Curves and Gravity, 2021).

The gravitational force is defined as the object mass times the gravitation acceleration, measured in the negative z-axis. The centrifugal force is defined as the square of the travel speed over curve radius (this force is in the direction of the curve centre point). The normal force is defined as the resulting force perpendicular to the road surface, measured in the direction away from the road. Finally, the friction force is perpendicular to the normal force multiplied by the friction factor. If the required friction factor exceeds the available side friction value, then the object will slip towards the outside of the curve. This force is measured along the degreasing slope of the embankment.

Using Newton's second law for the forces acting in the y and z-axis and solving the normal force component in these axis directions, and by setting the y-axis value for the normal force equal to the z-axis of the normal force, the formula for calculating the maximum travel speed can be derived as:

$$v_{max} = \sqrt{\frac{g\mathrm{R}(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta}} \qquad [2]$$

If $v_{car} > v_{max}$ then the car will slip up the incline.

From the equation derived above, when considering steady-state conditions, a vehicle's mass plays no role in the slip potential of a vehicle traveling along a curve.

The highest maximum superelevation rate allowed in the Geometric Design Manual is 10%.

2.2 Design Speed

It is worth noting that literature suggests that design speed does not have a direct influence on the design of a road but exerts an indirect effect (Harwood et al., 2014). The design speed determines the controlling criteria by defining the limits of these criteria. The criteria are directly aligned to the proposed design speed. Example of this is the minimum radius which is defined by the design speed. The minimum radius is dependent on the design speed. In this case the radius is the controlling criteria but is informed by the design speed.

3. RESEARCH METHODOLOGY

3.1 Excel Dynamic Particle Model

The primary objective of the dynamic particle model was to establish the relationship between speed and the embankment value when applying the design parameters from each selected design manual. Using the formulae obtained from the literature, the maximum travel speed for a curve-superelevation pairing was calculated and converted into a percentage value, indicating an increase or decrease in maximum speed relative to the design speed.

The correlation between the maximum speed and the design speed provides insight into the factor of safety available for the specific curve-superelevation pairing. It was anticipated that larger curve values (radii) within superelevated embankments would result in a higher maximum speed compared to the applied design speeds. Conversely, at the minimum curve radius, this would yield maximum speeds closer to the design speed.

Using the design speeds as defined by the Geometric Design Manual (SANRAL, 2002), starting at 40 km/h, and increasing the design speed by 10 km/h intervals, up to a maximum design speed of 130 km/h. The maximum superelevation values of 4%, 6%, 8%, and 10% were evaluated, as the Southern African design manuals do not account for superelevation values exceeding 10%.

Only the radii stated in the Geometric Design Manual (SANRAL, 2002) were used for all the various manuals included in this study.

Due to the nature of the design manuals, the following manuals was included in this study:

a) A Policy on Geometric Design of Highways and Streets (AASHTO, 2018).
b) Geometric Design Manual (SANRAL, 2002).
c) Gautrans Superelevation Parameters – Formula and Definitions (Gautrans, 1998).

Both the AASHTO and Gautrans design manuals include a calculation method to determining the superelevation rate that needs to be applied. The Geometric Design Manual does not provide calculated methods but instead lists the required superelevation rates in tabular form. The radii from these tables for each maximum superelevation class and for each corresponding design speed serves as the basis for the comparative mathematical model. This approach allows for a direct comparison between the various manual's methods.

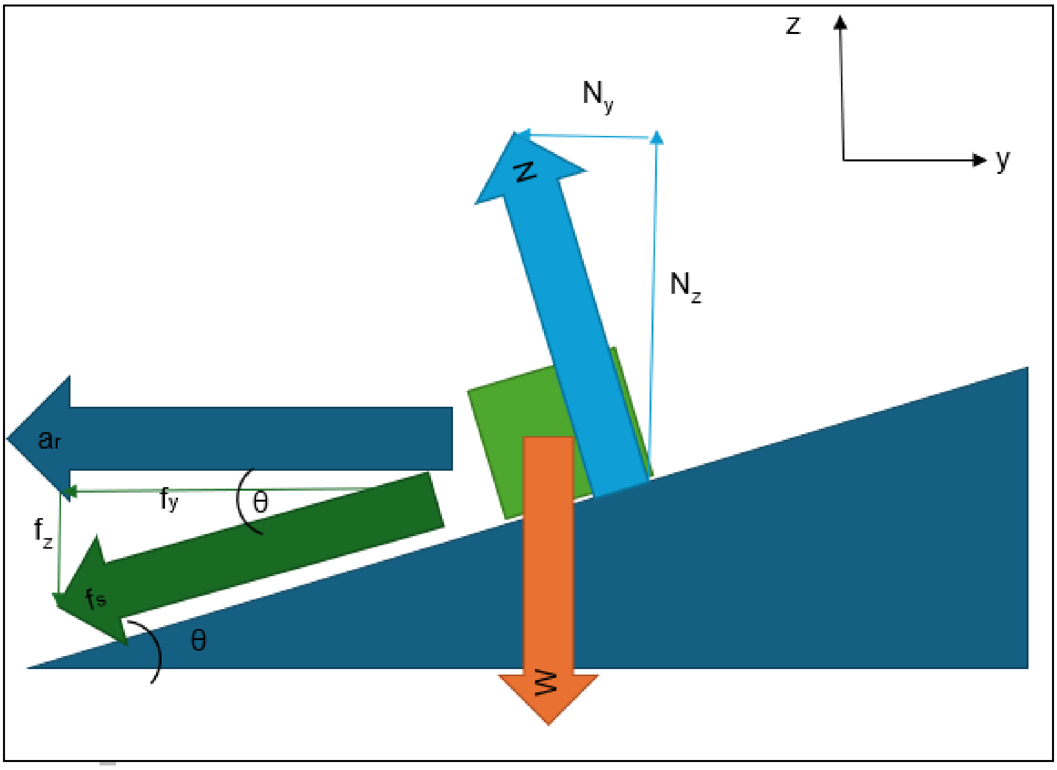

Figure 1: Force diagram for vehicle slipping to the outside of the curve

The mathematical model was based on the force diagram (Figure 1), the constituent elements of which were explained under the Horizontal and Superelevation section of the Literature Review. The annotated forces such as N_y refers to the Normal force component in the defined y-axis. After the initial mathematical assessment was conducted for flat curves, a third dimension in the form of gradient was introduced. The force diagram was reassessed using this additional variable, and the resulting formulas were used in the same manner as for the flat curves but included gradients ranging from -8% to 8% in 1% increments.

For this model, it was assumed that the vehicle was represented as a rigid block traveling at a constant speed on a banked curved road. Steady-state conditions applied, ensuring that all longitudinal forces acting on this block were in equilibrium.

The model utilizes Microsoft Excel to initially calculate the required embankment angle as per the manual being discussed. Once this calculation is completed, the model proceeds to determine the maximum velocity achievable at the calculated embankment angle, utilizing the variables for side friction from the relevant manual, which are dependent on the calculated velocity. The results of these calculations are then plotted, with the radii value on the x-axis and the maximum achievable velocity as a percentage of the design speed on the y-axis. A positive value indicates that a higher velocity can be achieved, while a negative value indicates that the maximum velocity achievable using the variables from the relevant manual is lower than the design speed.

3.2 Deriving 3D Variables

To determine whether gradient plays any significant role in cornering forces, it was necessary to introduce a third dimension. This was achieved through the use of vector algebra.

The weight of the particle was consistently directed along the negative z-axis, while the normal load remained perpendicular to the plane. In this scenario, the plane had a gradient causing a change in the x-axis direction and superelevation, leading to a change in the y-axis direction.

To determine the normal force, it was necessary to initially define three points on the road plane, after which the cross-product between these vectors then defines the perpendicular plane.

Using the same principles used for the initial force distribution, but with the inclusion of a new angle variable G, which is the angle of the longitudinal gradient.

Calculating the normal forces in both the y and z-axis, but using the normal force as defined by the perpendicular plane vector, the following equation for the maximum travel speed on a fixed embankment angle and longitudinal gradient was derived:

$$v_{max} = \sqrt{\frac{g\mathrm{R}(\cos \mathrm{G} \cdot \sin \theta + \mu_s \cos \theta)}{\cos \mathrm{G} \cdot \cos \theta - \mu_s \sin \theta}} \qquad [3]$$

3.3 Setup of Mathematical Models

The Geometric Design Manual (SANRAL, 2002) served as the baseline for these mathematical particle models.

To maintain consistency in the model results, it was decided to use the range of radii specified for each maximum superelevation range and their corresponding design speeds, as outlined in the Geometric Design Manual. The same radii and maximum superelevation rates were also applied in the other manuals, with the superelevation rates being determined using the calculation methods described in the relevant manuals.

Microsoft Excel was used to establish the mathematical model, primarily due to the friction coefficient's dependence on the velocity. The calculated maximum velocity was employed to determine the required friction coefficient. This process went through multiple iterations until the point was reached where the speed and friction coefficient no longer changed by more than 0.001 of their previous values. Given the complexity of these calculations, it was deemed preferable to utilise the goal seek function incorporated into a loop macro, ensuring that no numerical errors would be encountered during the iterative calculation.

For the sensitivity analysis, the AASHTO method for designing superelevation was employed, but with the use of more conservative side friction values from the Geometric Design Manual (SANRAL, 2002). In this context, the minimum curve radius was computed for each speed/e_{Max} group.

It was observed from the initial calculations that, in some cases, particularly those near the minimum radius conditions, the maximum curve velocities were less than the design speed. This implies that these curves are likely to induce slip as the vehicle approaches the design speed. However, it should be noted that the friction coefficient used in the calculations was lower than the actual friction coefficient, making the analysis conservative. Thus, a reduction in the maximum velocity compared to the design speed might not necessarily result in a slip event, but it does increase the likelihood of such an event. The actual friction coefficient is unknown and varies across different classes of roads, making it impossible to model accurately.

The superelevation data was converted to represent a percentage increase or decrease in relation to the maximum velocity achievable on an embanked curve. Another advantage of this transformation was that it allowed for a more direct comparison between higher and lower speed profiles.

The transformed data was plotted, with the radius on the x-axis and the percentage increase/decrease of the maximum velocity on the y-axis. The data related to each e_{Max} criterion was plotted separately to observe the combined effect.

4. RESULTS

4.1 Friction Coefficient and Design Speed

The different coefficients for friction assumed by the three design manuals investigated is shown in Figure 2 below.

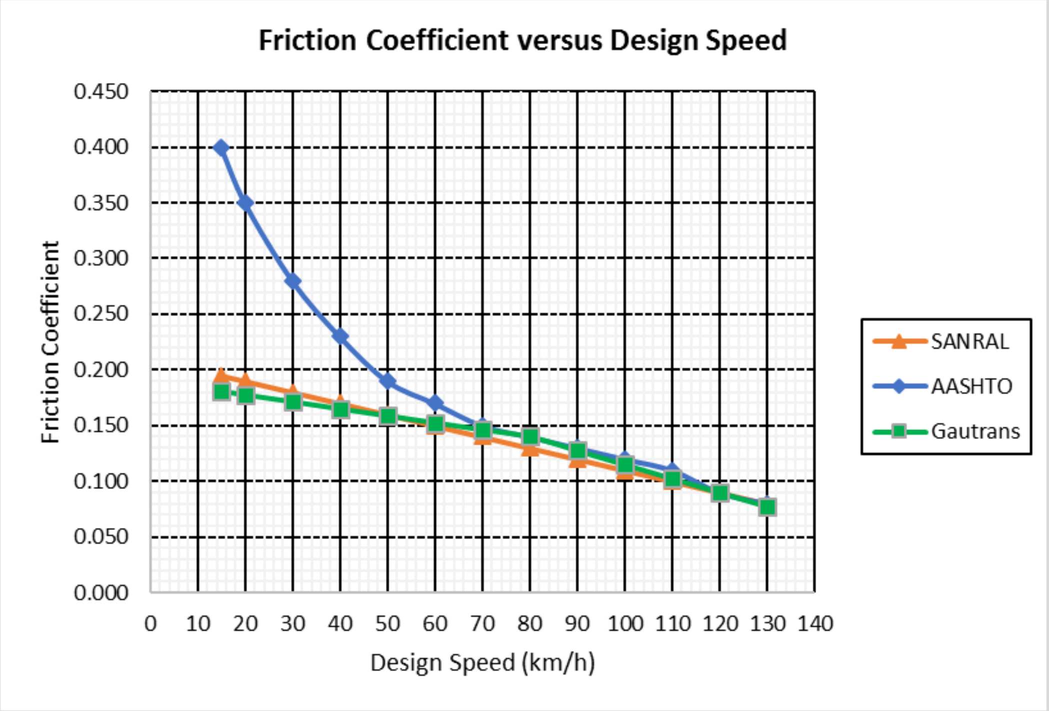

Figure 2: Friction Coefficient versus Design Speed for each Design Manual

From the graph it can be seen that starting at 70 km/h, the three manuals produce similar results, with the SANRAL model resulting in the most conservative values.

4.2 Superelevation

The differences in calculated superelevation rates as per the three design manuals under investigation are shown in Figure 3.

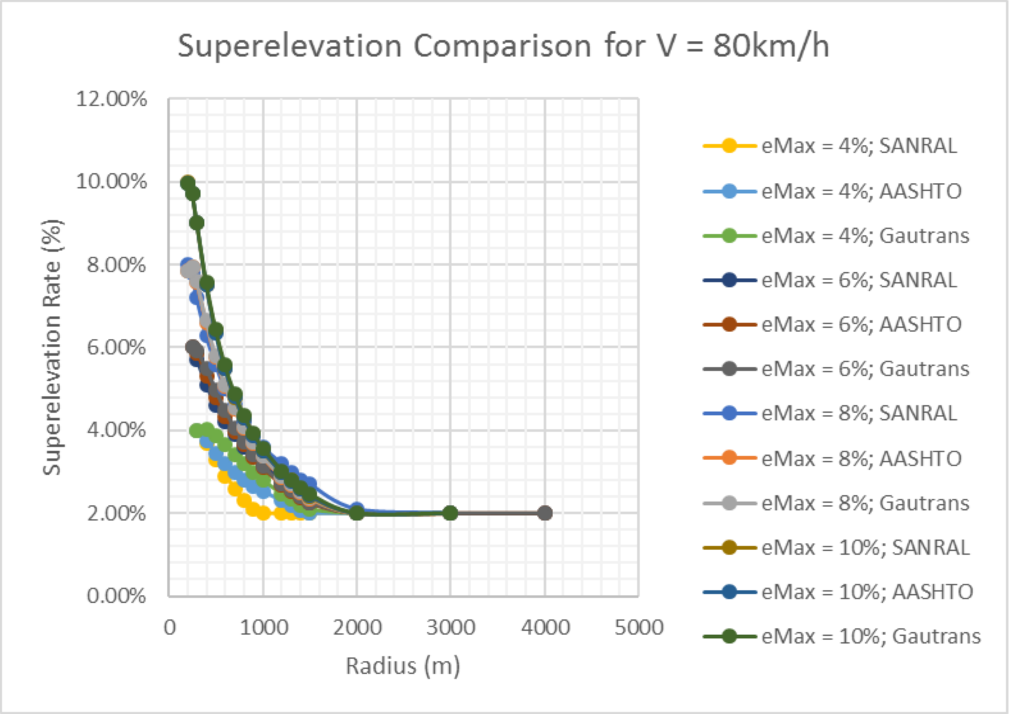

Figure 3: Superelevation Comparison for Design Speed of 80km/h for each Design Manual

Superelevation Comparison for V = 80km/h is shown in Figure 3 above.

From Figures 2 and 3, three distinct features are observed:

- At a maximum superelevation rate of 4%, there are significant differences in the design rates calculated by the various manuals. The Gautrans manual tends to produce the highest values, while the SANRAL manual tends to produce the lowest values, in general.
- With an increase in the maximum superelevation rate, the differences in calculated rates between these manuals decrease and become more consistent.
- As the design speed increases, the differences in calculated rates between the manuals also decrease.

Based on these observations, it can be concluded that at higher maximum superelevation rates and design speeds, the design superelevation rates between these manuals will be similar, and curve safety standards at these parameters will align. However, at lower design speeds and especially lower maximum superelevation rates, the design manuals will yield different design embankment rates. The Gautrans manual suggests higher embankment rates, indicating a more conservative approach in these situations.

4.3 Dynamic Particle Model Results

Figure 4 shows the typical dynamic particle model results from the SANRAL design manual for each maximum superelevation class, in this case the e_{Max} = 8%

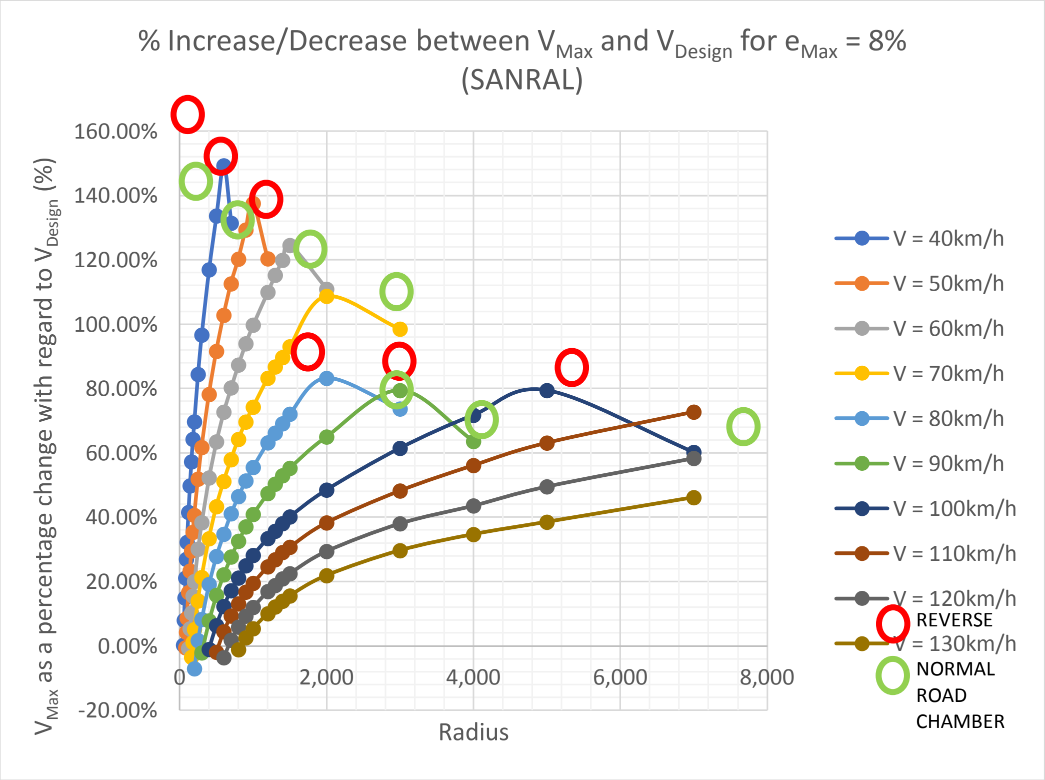

Figure 4: Design speed vs Maximum speed with not slipping

Figure 4 represents the relationship between the maximum obtainable velocity for the embankment angle calculated for the relevant manual's variables, based on a design speed. The x-axis gives the radii and is compared to the difference in obtainable velocity with regard to the design speed, given as a percentage. Positive values indicate an increase in obtainable velocity with regards to the design speed, and a negative value indicates that the maximum obtainable velocity is less that the design speed. From Figure 4 it is evident that the relationship between the maximum attainable velocity and the design speed increases as the radii increase. However, there is a drop in this ratio at the point where the superelevation rate transitions from a reverse superelevation to the normal tangent superelevation rate.

As anticipated, at the minimum radius, this ratio approaches the design speed. In some cases, the design speed exceeds the maximum attainable velocity. Negative values do not necessarily indicate a crash due to slippage but rather suggest a higher likelihood of a crash potential under these design conditions. As a result, it is advisable to avoid these design conditions. The application of a conservative friction coefficient provides an additional layer of safety. This is particularly important given the high variability in friction coefficients due to differences in road and vehicle tyre conditions.

4.4 Effect of Gradient

For the effect of gradient on the maximum obtainable embankment speed, only SANRAL manual's graphs were used because similar trends were observed in the other manuals. The effect of gradient was presented for all superelevation maximum rate classes but summarized by design speed. The values indicated the difference between the maximum design speed obtained at the evaluated grade and the maximum design speed obtained at level grade for the same design superelevation rate.

Figure 5 shows the typical speed differences per design speed for all gradient and superelevation classes. The y-axis shows the change in maximum obtainable velocity for sample on a grade, compared with the at grade sample utilizing the same geometry. The general trend shows an increase in maximum velocity as the absolute grade value increases, but the percentage of this change is less than 0.1%. Therefore, it can be concluded that working with a level gradient provides an accurate and conservative design approach, as the effects of gradients can be considered insignificant.

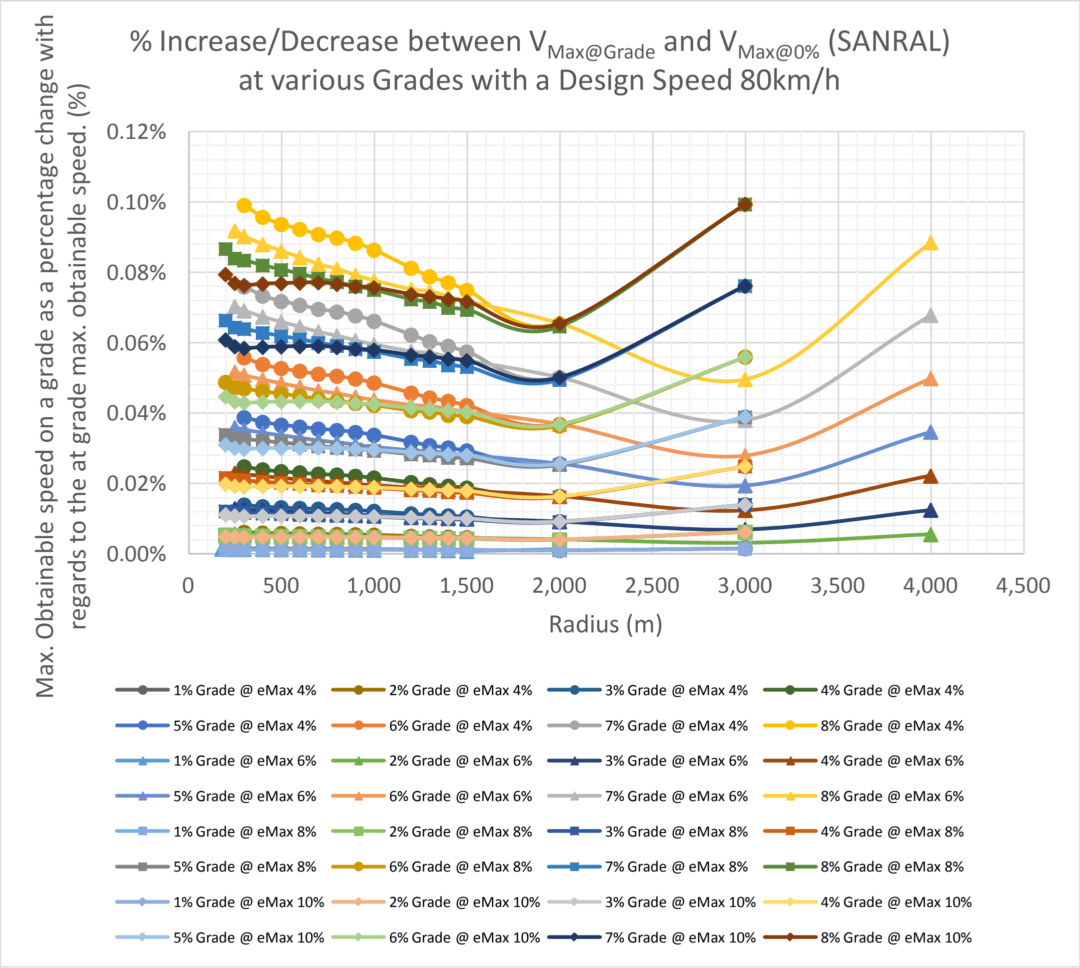

Figure 5: Influence of Gradient

After analysing the results obtained from the various manuals, the following trends were observed:

a) Superelevation design rates for sharper curves were conservative. However, in all the manuals, as the curve becomes sharper and the radius approaches the minimum defined values, the maximum attainable speed decreases, and the embankment superelevation rate becomes less conservative.
b) The higher the maximum superelevation rate designed for, the closer the results between the various manuals become. This trend is also observed with an increase in design speed.

c) It was established that the gradient does not significantly contribute to the reaction of the superelevation rates.

To determine the likelihood of a crash potential, it was necessary to refine the information based on a selected parameter that is easily understood and defendable.

Starting with an initial maximum obtainable velocity 5% higher than the design speed and solving for the required radius. This process was carried out for all eMax criteria and at all design speeds ranging from 40 km/h to 130 km/h, using the embankment calculation method outlined in the AASHTO manual, with friction criteria sourced from the SANRAL Manual.

From the mean and standard deviation statistical indicators, it becomes evident that a radius that is 10% larger than the minimum radius ensures that the maximum attainable speed exceeds the design speed by more than 5%. However, it is worth noting that only speeds exceeding 120 km/h will have a maximum attainable speed lower than 5% over the design speed. Therefore, based on these findings, it is recommended that the minimum desirable design radius should be 10% greater than the theoretical minimum radius. For design speeds exceeding 120 km/h, it is advisable to have a minimum desirable radius that is at least 15% greater than the theoretical minimum radius. This approach is aimed at reducing the likelihood of crashes caused by radial slip. The recommendations are shown in Table 1 below.

Table 1: Recommended Minimum Design Radius

Recommended Minimum Design Radius								
V_{Design}	e_{Max} = 4%		e_{Max} = 6%		e_{Max} = 8%		e_{Max} = 10%	
	R_{Min} Theoretical	$R_{Desirable}$	R_{Min} Theoretical	$R_{Desirable}$	R_{Min} Theoretical	$R_{Desirable}$	R_{Min} Theoretical	$R_{Desirable}$
40	60,57	70	55,26	70	50,80	60	47,01	60
50	99,68	110	90,52	100	82,88	100	76,44	90
60	151,56	170	136,96	160	124,86	140	114,75	130
70	218,56	250	196,38	220	178,18	200	163,12	180
80	303,50	340	270,99	300	244,58	270	222,94	250
90	410,04	460	363,51	400	326,16	360	295,90	330
100	543,22	600	477,36	530	425,51	470	384,01	430
110			616,45	680	545,82	610	489,71	540
120			787,07	910	691,12	800	616,03	710
130			996,30	1150	866,55	1000	766,69	890

5. CONCLUSIONS AND RECOMMENDATIONS

The dynamic particle evaluation aimed to assess the effects of design speed, curve radius, and the associated applied superelevation rates proposed by various design guidelines.

The maximum travel speed was then compared to the design speed to determine the factor of safety available for the design speed versus radius pair for a specific superelevation maximum rate.

It was concluded that vehicle mass does not have any influence of the risk of slipping.
The effect of gradient was neglectable and can therefore be ignored.

Based on the results of the particle model, it is recommended that for design speeds less than 120 km/h, the minimum desirable radii should be at least 10% larger than the theoretical minimum radius. For design speeds greater than or equal to 120 km/h, it is recommended that the desirable radii should be at least 15% larger than the theoretical minimum radius.

6. REFERENCES

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