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Hybrid retirement strategy in South Africa

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Background: Many retirees in South Africa face the challenge of either outliving their retirement savings or living below their means. Studies suggest a 'safe' withdrawal rate of between 4% and 5%, which is below the average fund size-weighted drawdown rate of approximately 6.66%.

Aim: To provide a scientific basis for the success rate of a 'hybrid' retirement strategy, whereby a retiree invests a proportion of their savings in a life annuity and the remaining proportion in a living annuity, to increase the success rate for South African retirees.

Setting: Historical asset class returns (equities, bonds and inflation) for South Africa were sourced for the period 1900–2020.

Method: Bootstrap sampling of historical asset returns was employed to simulate 10 000 random scenarios to investigate the success rate of various compositions of the 'hybrid' retirement strategy.

Results: The success rate of all 'hybrid' portfolio compositions is significantly greater than the success rate of a pure living annuity when the withdrawal rate is less than 8%.

Conclusion: In a South African context, a 'hybrid' retirement portfolio increases the probability of success for retirees withdrawing less than 8% from their portfolio – which constitutes approximately 50% of the current annuatised population – and may increase the inheritance of a retiree's heir.

Contribution: Where other studies have focussed solely on the success rate of a living annuity, we have shown that a 'hybrid' retirement strategy increases a South African retiree's likelihood of retiring successfully when the withdrawal rate is less than 8%, which is approximately 50% of the annuatised population.

Keywords: retirement; living annuity; life annuity; bootstrap simulation; portfolio management.

'The question isn't at what age I want to retire, it's at what income.' George Foreman

Introduction

The quote above, from George Foreman, is a great reminder that retirement ages are not fixed; they are unique to everyone. Some people can retire earlier than others. Hence, a retiree faces the following problem at retirement. They must decide on an appropriate amount to withdraw from their retirement investment, and 'optimise' between outliving their money and living below their means, wrote Maré (2016).

In a recent study by Allianz (2023), it was found that 61% of respondents are more afraid of depleting their savings during retirement than dying. The study was conducted on adults in the United States of America, aged between 44 and 75, and reiterates the fact that many retirees are more afraid of depleting their retirement resources than they are of death. Approximately 46% of respondents indicated that they were forced to reduce or stop saving for retirement because of inflation, market volatility and recurring financial crises. Furthermore, 40% of respondents admitted that their retirement strategy is flawed, and they are unsure when or how they will be able to get it back on track.

In South Africa, retirees are faced with emerging market conditions. A study conducted by the Association for Saving and Investments in South Africa (ASISA) suggests that only 6% of economically active South Africans will be able to retire comfortably, and many are at risk of living below their pre-retirement standard of living (ASISA 2022). Several factors contribute to the financial stress for those seeking to retire comfortably:

- **Inadequate savings:** Many South Africans start saving too late for retirement largely because of low incomes and a large informal labour market.

- **High levels of debt:** A significant portion of the population is burdened by debt in the form of credit cards, personal loans and home loans.
- **Lack of financial literacy:** The importance of retirement planning is not well understood by many South Africans, which results in poor investment decisions and inadequate savings.
- **Economic challenges:** High inflation, unemployment and slow economic growth exacerbate the financial stress that many South Africans face.

Retirees in South Africa have access to a variety of retirement strategies but face the risk of choosing an ineffective retirement strategy catering to their specific needs. Visser (2024) suggests that retirees in South Africa usually have a choice between two main types of annuities: guaranteed life annuities (also called life annuities) and living annuities. The former is an insurance contract that provides lifelong coverage to the insured retiree, offering a predetermined income. On the other hand, living annuities offer retirees the flexibility to choose from a broad range of investment options.

A life annuity is a financial product that guarantees the holder a stream of income for the rest of their life (see, e.g. Milevsky 2013). It is typically purchased from an insurance company using a lump-sum investment. The main feature of a life annuity is its ability to provide financial security by ensuring that the annuitant (the person who receives the payments) will receive a steady income, regardless of how long they live.

A living annuity is a type of retirement income product that allows retirees to invest their retirement funds in various investment portfolios and draw a regular income from them (Beinash 2008). Living annuities are popular for their flexibility and potential for growth, but they also carry more risk compared to life annuities, particularly because of market fluctuations and the responsibility placed on the retiree to manage their investment choices wisely so that they do not outlive their assets (National Treasury 2012). As such, they are often chosen by retirees who are more comfortable with investment risk and who desire greater control over their retirement funds (National Treasury 2012).

A typical retirement strategy requires a multifaceted approach, considering various products like annuities and a well-structured investment portfolio. Balancing risks such as mortality and longevity, alongside factors such as healthcare costs, inflation and tax implications, is essential for a secure retirement. Regularly reviewing and adjusting a retirement plan in response to life changes and the current economic climate will help ensure that a retiree's retirement years are not only financially secure but also fulfilling.

National Treasury (2012) conducted a study that indicated a sharp decline in the purchase of life annuities in South Africa. In 2003, 50% of retirement funds were utilised to buy life annuities and in 2011 this value had significantly decreased to only 14%. The study concluded that the decrease in the purchasing of life annuities can be attributed to sales incentives, particularly the higher commission earned by brokers for selling a living annuity compared to a life annuity (National Treasury 2012).

Blanchett (2014) conducted a study to identify the various risks that annuitants might face when investing their retirement savings in either a living annuity or a life annuity. Some of these risks include, but are not limited to:

- **Volatility risk:** This relates to investment risk assumed when investing in equities or other risky assets. A retiree who has sleepless nights over concerns about market volatility may derive increased utility from a life annuity that provides a guaranteed income until death.
- **Inflation risk:** The risk that inflation will erode a retiree's purchasing power throughout retirement. This risk can be hedged by purchasing a life annuity with an inflation protection feature.
- **Longevity risk:** The risk that a retiree will experience financial ruin (run out of money). One way to hedge against longevity risks is to purchase a life annuity that will provide a guaranteed income until death.
- **Bequest risk:** The risk associated with the implications that life annuities have on retirement decision-making. To a risk-averse retiree, a life annuity provides insurance against financial ruin. However, a retiree who seeks to maximise their wealth and leave an inheritance for their heirs may experience less utility from trading upside potential for safety.

Given the risks identified by Blanchett (2014), we investigate the merit of investing in a 'hybrid' retirement strategy, which consists of a combination of a life annuity and a living annuity, to increase the success rate of a retiree attaining their required withdrawal rate without outliving their retirement savings. The rationale for investing in a 'hybrid' strategy is that the life annuity will provide a guaranteed income until death (partially reducing the longevity, volatility and bequest risk). On the other hand, the living annuity provides flexibility, allows retirees to maximise their upside potential from investment returns and provides benefits for a retiree's heir, but is still subject to volatility risk and longevity risk. The aim of the 'hybrid' retirement strategy is to overall increase the success rate of retirement portfolios.

The rest of the article is structured as follows: we provide a detailed literature review as well as a problem statement and proposed extension of previous work with reference to the retirement landscape in South Africa. An overview of the data used to simulate portfolios is provided. The methodology and assumptions describe the general methodology followed, which is based on sampling with replacement. This is followed by the results which add to the existing literature by detailing the portfolio success rates for various 'hybrid' portfolio compositions. Lastly, we provide the conclusion and highlight other related areas for further research.

Literature review

Bengen (1994) introduced the '4% rule', by demonstrating that an inflation-adjusted withdrawal rate of 4% of the initial portfolio amount is most likely sustainable over a 30-year period. This formed the foundation upon which other researchers have built, by investigating various strategies and asset allocations under different economic conditions.

Cooley, Hubbard and Walz (1998) conducted a study to determine the impact of various withdrawal rates for US retirees investing in living annuities, and the necessary asset allocation to support the withdrawal rates. The framework proposed by Cooley et al. (1998) introduced a criterion for assessing the success rate of a retirement portfolio, which is defined by the probability of a portfolio 'outliving' the retiree over a predetermined period, withdrawal rate and asset allocation (e.g. 30 years, 6% withdrawal rate and a portfolio consists of 75% equity and 25% bonds).

The success rate metric was used to evaluate the performance of living annuities for a variety of portfolios and Cooley et al. (1998) found that an inflation-adjusted withdrawal rate of between 4% and 5% is 'safe' for portfolios consisting of at least 75% equity and 25% bonds over a 30-year period. By adjusting withdrawals for inflation, all withdrawals in the near term are substantially reduced to allow for larger withdrawals in the long term to protect a retiree from inflation.

Pfau (2011) examined over 100 years of historical stock, bond and inflation data from 17 developed countries and found that the 4% withdrawal rate was deemed to be too risky in 13 of the 17 countries. Further, a fixed asset allocation throughout the investment period was shown to fail in all countries.

Finke, Pfau and Williams (2012) considered the impact of risk tolerance on retirement decisions by investigating the relationship between risk tolerance and asset allocation (stock, bonds and bills) and withdrawal rates. They found that the 4% 'safe' withdrawal rate was appropriate for risk-averse retirees over a 30-year period. However, risk-tolerant retirees may prefer higher withdrawal rates from riskier retirement portfolios, albeit with a greater probability of ruin.

Maré (2016) conducted a study to investigate safe withdrawal rates for South African retirees investing in living annuities. The analysis is based on historical inflation-adjusted equity and bond returns from 1950 to 2014; similarly, as in Cooley, Hubbard and Daniel (1999), portfolio success rates were used to infer a safe withdrawal rate. Various fixed investment periods were considered, ranging between 15 years and 30 years, assuming no mortality or transaction fees.

Maré (2016) notes that spending patterns, mortality rates and asset returns tend to differ in South Africa, with which we concur. Remarkably, Maré's (2016) results suggest that a 5% withdrawal rate is sustainable over a relatively short period of 15 years or less in South Africa. For longer investment horizons, the results are comparable to those of Cooley et al. (1998).

Van Appel, Maré and Van Niekerk (2021) extended the research by investigating the effect of transactional fees and longevity using a dataset that spans from 1900 to 2020 for South African equities, bonds, cash and inflation. An analysis was performed to investigate the time until a living annuity depletes for various withdrawal rates, termed 'fugit'. Van Appel et al. (2021) suggested a strategy to hedge some of the equity risk in the portfolio by taking long positions in put options and short positions in call options on a rolling 1-month basis.

Van Appel et al. (2021) concluded that the portfolio success rate decreases rapidly when the withdrawal rate increases. For low withdrawal rates, the asset allocation does not have a significant impact on the portfolio success rate, and by making use of derivative instruments, retirees are able to hedge some of their risks and increase the success rate of the portfolio.

Anarkulova et al. (2023) examined retirement spending rules using historical stock and bond returns from 38 countries. Their findings suggest that a 65-year-old couple can withdraw only 2.26% of their retirement savings annually with a 5% probability of ruin over a 30-year period. This is significantly lower than the previously considered 4% 'safe' withdrawal rate.

Daraei and Sendova (2024) utilised a ruin-theory approach to analyse the inflows and outflows of a Canadian retiree's portfolio based on age, gender, initial wealth and transaction data. By employing a sophisticated ruin model, they determined the average and median time until the portfolio is depleted, assessed the likelihood of the funds being exhausted within the retiree's remaining lifetime and evaluated the deficit at the point of ruin.

Daraei and Sendova (2024) concluded that a withdrawal rate of approximately 4.5% is 'safe' over a 20-year period, but may not be enough to provide a sufficient income to ensure that a retiree can still live comfortably.

Structuring a 'safe' retirement and evaluating the relevant success rate of a certain retirement strategy is critically important. The consensus among the research done by Cooley et al. (1998), Finke et al. (2012), Maré (2016), Van Appel et al. (2021), and Daraei and Sendova (2024) is that staying within the 4%–5% spending band will significantly increase the probability of success of a retiree's retirement portfolio. Additional research done by, for example, Bengen (1994), Milevsky and Haung (2011), Butler and Van Zyl (2012), Warring and Siegel (2015), Maré (2016), Rusconi (2020), Klein and Sarpa (2020), and Visser (2024) also highlights the fact that each retiree is unique and will require a distinct retirement solution to their retirement situation (e.g. some may require medical care from the start of retirement, others may seek to maximise their heir's inheritance, etc.).

From our literature review, the research above has considered the success rate of investing in a living annuity as a retirement strategy, by looking at different compositions of bonds, bills and equities while taking inflation into account for different withdrawal rates. However, the novelty of our research looks at the success rate of a 'hybrid' strategy that entails investing in both a life annuity and a living annuity for different portfolio compositions in the living annuity while still attaining the required withdrawal rate.

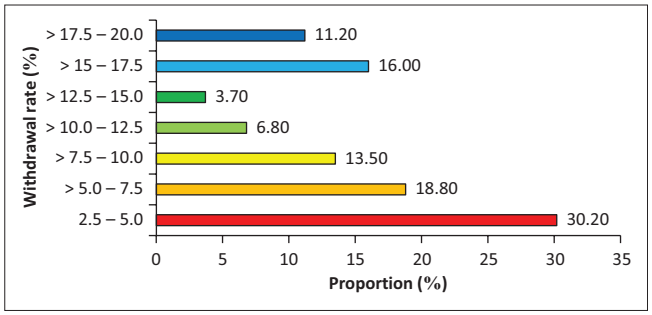
Problem statement

Bengen (1994), Cooley et al. (1998), Finke et al. (2012), Maré (2016), Van Appel et al. (2021), and Daraei and Sendova (2024) all suggest that a 4% withdrawal rate is deemed 'safe' over a 30-year period. Using a 30-year period to model the success rate of a living annuity aligns with the period used by Cooley et al. (1998) as it assumes death at age 90, which sits on the tail end of the conditional life expectancy for South African retirees derived in the data section.

The research conducted by Maré (2016) and Van Appel et al. (2021) is specific to the South African retirement landscape, where they found that withdrawal rates greater than 5% are not sustainable over a 30-year period.

It is alarming that a study conducted by ASISA (2022) suggests that approximately 69% of annuatised retirees have income drawdown rates greater than 5%.

Figure 1 suggests that approximately 50% of retirees in South Africa have an annual income drawdown rate of between 2.5% and 7.5%, where the average fund size-weighted annual drawdown rate is 6.66% (ASISA 2022). Given that the 'safe' withdrawal rates, determined by Bengen (1994), Cooley et al. (1998), Finke et al. (2012), Maré (2016), Van Appel et al. (2021), and Daraei and Sendova (2024), are lower than the average fund size-weighted annual drawdown rate determined by ASISA (2022), further research is required to determine whether a 'hybrid' retirement strategy may yield higher success rates for higher withdrawal rates.



Source: ASISA, 2022, *The association for savings and investment South Africa*, viewed 10 March 2024, from https://www.asisa.org.za/media/hfvhoous/20231128_asisa-living-annuities-drawdown-survey-statistics-report-2017-to-2022.xlsx&ved=2ahUKEwjW6trp_sGFAxUN-AIHHaELAbAQFnoECBUQAQ&usg=AOvVaw01xN-pn

FIGURE 1: Proportion of withdrawal rate by number of policies in South Africa for 2022.

In managing a retirement portfolio, it is common for institutions to charge various fees, including advisor fees, fund management fees and platform fees, which can significantly affect the portfolio's success rate; it is thus a crucial consideration for retirement planning. We consider the fee structure as seen in Table 1 for the living annuity by Van Appel et al. (2021).

The fees associated with a life annuity are typically lower than those associated with a living annuity, largely because once the investment is made into the life annuity, insurers are free to invest the money as they see fit. The insurance company will calculate the annuity rate by factoring in the costs of commission and investment charges that the insurance company will have to fund over the life of the product (National Treasury 2013). We assume that the initial costs associated with a life annuity amount to 1% of the initial investment.

To evaluate the effectiveness of a 'hybrid' retirement strategy, we consider different portfolio compositions with respect to the allocations in a life annuity and a living annuity. Our analysis of the success rate assumes that the retirement age is 60 and spans a 30-year period, which is conservative given the conditional median life expectancy in Table 3. It is crucial to acknowledge that our analysis does not include considerations for taxes; however, these could, in theory, be incorporated into our analysis.

We make the following assumptions to determine the success rate of our 'hybrid' retirement strategy:

- An initial investment of R1 million, where a portion of the initial investment is invested in a life annuity and the remaining portion is invested in a living annuity.
- The asset allocation between bonds and equities in the living annuity remains fixed over the life of the investment.
- Asset returns have been adjusted for inflation and are thus real returns.
- The mortality rates used for the life annuity calculation are for assured lives (Dorrington & Tootla 2007:161–184), with a 10% mortality adjustment factor.
- The interest rate curve used for the life annuity calculation is an inflation-adjusted curve, that is, real interest rates as of 29 December 2023 (South African Reserve Bank 2024).
- The life annuity includes a 5-year guarantee period, which means that if death occurs between age 60 and age 65, the remaining balance of the life annuity will be paid to the retiree's heirs.

TABLE 1: Fund management fees.

Variable	Fee per annum (%)
Bonds	0.50
Equities	0.75
Platform fee	0.50
Financial advisor fee	1.00

Source: Van Appel, V., Maré, E. & Van Niekerk, A., 2021, 'Quantitative guidelines for retiring (more safely) in South Africa', *South African Actuarial Journal* 21(1), 75–91. https://doi.org/10.4314/saaj.v21i1.4

- Withdrawals from both the life annuity and living annuity are monthly.
- A static withdrawal rate over the 30-year period.

In this study, we extend the research of Maré (2016) by investigating the success rate of a 'hybrid' strategy, whereby a portion of a retiree's portfolio is invested in a life annuity and the remaining portion is invested in a living annuity. The rationale behind this 'hybrid' approach is twofold. The life annuity diminishes longevity risk but provides no flexibility. On the other hand, the living annuity provides flexibility and benefits for a retiree's heir but has significant longevity risk. By investing in a combination of the two products, a retiree may find a suitable risk-adjusted hybrid strategy to meet their specific needs.

Data

The value of a living annuity is dependent on asset class returns, whereas a typical living annuity portfolio consists of a risky asset (equity) and a 'safe' asset (bonds). We highlight summary statistics for South African bonds and equities for the period from 1900 to 2020 in Table 2, which is based on data sourced from Firer and Stauton (2002), Dimson, Marsh and Staunton (2016), and Van Appel et al. (2021).

Throughout our research, we only consider real returns for equities and bonds, which means that the returns used in modelling the 'hybrid' retirement strategy have been adjusted for future inflation.

The value of a life annuity is dependent on the age of the insured, mortality rates and interest rates. We assume that the average retirement age in South Africa is 60 and derive conditional survival probabilities from the South African life tables for assured lives (Dorrington & Tootla 2007:161–184).

Given that these life tables are based on data from 1996 to 2000, a mortality adjustment factor of 10% is applied. The rationale for applying a mortality adjustment factor is that an analysis conducted by the World Health Organization (WHO) suggests that the life expectancy for both males and females has increased by 10.2% from the year 2000 to 2021 (World Health Organization 2021).

In South Africa, the World Health Organization (2021) suggests that the average life expectancy from birth is approximately 62 years of age for both males and females. However, using the South African life tables for assured lives (Dorrington & Tootla 2007:161–184), the expected lifespan extends to approximately 79 years for males and 84 years for females, provided that they have already reached the age of 60.

The conditional survival probabilities, with and without a mortality adjustment, are presented in Table 3.

After applying the mortality improvement factor of 10% to the South African life tables for assured lives, the conditional survival probabilities increase for both males and females. Further, the conditional median life expectancy increases to approximately 80 years for males and 85 years for females.

TABLE 2: Asset class statistics for South Africa over the period 1900 to 2020.

Variable	Nominal return (%)	Real return (%)	s.d. (%)	Skewness	Kurtosis
Equity	13.7	8.4	15.7	-0.15	4.19
Bonds	7.0	2.1	5.7	-0.05	12.05

Source: Firer, C. & Stauton, M., 2002, '102 Years of South African financial market history', *Investment Analysts Journal* 31(56), 57–65. https://doi.org/10.1080/10293523.2002.11082442; Van Appel, V., Maré, E. & Van Niekerk, A., 2021, 'Quantitative guidelines for retiring (more safely) in South Africa', *South African Actuarial Journal* 21(1), 75–91. https://doi.org/10.4314/saaj.v21i1.4

s.d., standard deviation.

Methodology and assumptions

The methodology used to simulate our portfolios is based on random sampling with replacement, also known as bootstrapping. The technique was first introduced by Efron (1979). This resampling technique is used to estimate the distribution of a statistic by resampling, with replacement from a dataset. It allows for estimating the precision of sample statistics by using subsets of accessible data or repeatedly drawing samples from the original dataset with replacement.

We created our bootstrapped samples by repeatedly resampling with replacements from our monthly return dataset for equities, bonds and inflation from January 1900 to April 2020. Let $Y_1^j, Y_2^j, Y_3^j, \ldots, Y_{1444}^j$ denote the historical return for the i^{th} month and j^{th} asset class. Based on the historical returns, we posit that each $Y_i^j, \forall i$ is equally likely to be chosen. Moreover, the i^{th} return sampled is then consistently applied across all asset classes to maintain the integrity of the correlation structure.

This produces a 30-year bootstrap sample that is composed of randomly selected observations, where each observation can be chosen more than once. This process is repeated many times (in our case 10000 samples) to build the distributional characteristics of our returns path process. Each individual path constitutes a random scenario based on the monthly bootstrapped returns. The process uses simulation by random sampling.

TABLE 3: Conditional survival probabilities for South African retirees given age 60.

Conditional survival probably from age 60 to	Unadjusted mortality (%)			Adjusted mortality (%)		
	Male	Female	Both	Male	Female	Both
65	89	95	92	90	95	93
70	77	88	82	79	89	84
75	62	78	70	65	80	72
80	46	64	54	49	67	58
85	28	45	35	32	49	39
90	12	24	17	15	28	21
95	3	8	5	5	11	7
100	0	2	1	1	2	1
Conditional median life expectancy given age 60	78.7	83.8	81.1	79.8	84.7	82.2

Source: Dorrington, R.E. & Tootla, S., 2007, 'South African annuitant standard mortality tables 1996–2000 (SAIML98 and SAIFL98)', *South African Actuarial Journal* 7, 161–184. https://doi.org/10.4314/saaj.v7i1.24512

A typical retirement strategy is to invest in either a living annuity, where the living annuity portfolio typically consists of bonds and equities, or a life annuity. Retirees who opt for the aforementioned are required to choose the composition of bonds and equities according to their risk appetite.

The 'hybrid' retirement strategy is multidimensional in the sense that a retiree needs to decide how much of their investment to invest in:

- A living annuity, which typically comprises holdings of equities, bonds and money market assets or
- A life annuity.

Furthermore, a retiree is also required to decide on an income drawdown rate to suit their specific spending requirements.

The value of the 'hybrid' retirement strategy, denoted by Π, at inception, is given by (Equation 1):

$$\Pi(0) = \Pi_{life}(0) + \Pi_{living}(0)$$
$$\Pi(0) = N \cdot \omega_{life} + N \cdot \omega_{living} \quad \text{[Eqn 1]}$$

where:

- N denotes the initial investment
- ω_{life} denotes the weight invested in the life annuity
- ω_{living} denotes the weight invested in the living annuity
- $\omega_{life} + \omega_{living} = 1$.

If $\omega_{life} = 1$, then the portfolio defaults to a living annuity; similarly, if $\omega_{living} = 1$, then the portfolio defaults to a life annuity.

The life annuity provides a predetermined income for the rest of the assured life and is known at the inception of the policy. Contrastingly, the value of the living annuity fluctuates with respect to asset class returns and withdrawals. Therefore, we consider the life annuity and living annuity in the 'hybrid' retirement strategy separately.

We denote the total annual withdrawal rate required by a retiree as $r_{portfolio}$, and the fixed withdrawal rate obtained from the life annuity as r_{life}, then the withdrawal rate required from the living annuity, r_{living}, is determined subject to the following conditions: $r_{portfolio} = r_{life} + r_{living}$.

The total withdrawal is based on the initial investment amount, that is Π(0). For example, if Π(0) = *R*1 000 000 and a retiree requires a total annual withdrawal of 5%, then an annual income of R50 000, paid monthly, is required for the remainder of the insured life.

The value of the retiree's living annuity at any time, t, $\Pi_{\text{living}}(t)$, is given by (Equation 2):

$$\Pi_{\text{living}}(t) = \Pi_{\text{living}}(t-1)\begin{bmatrix}\omega_{equity}\left(1 + r_{equity}(t)\right) \\ +\omega_{bond}\left(1 + r_{bond}(t)\right) - r_{living}\end{bmatrix} \quad \text{[Eqn 2]}$$

where:

- ω_{equity} denotes the weight invested in equities
- ω_{bond} denotes the weight invested in bonds
- r_{equity} denotes the monthly real return from equities
- r_{bond} denotes the monthly real return from bonds
- r_{living} denotes the monthly withdrawal required by a retiree from the living annuity.

The Gompertz Annuity Pricing Model (GAPM) is used to determine the price of a life annuity. This price – or the amount of income you can expect for a given premium deposit – is influenced by the competitive market interactions among insurance companies. Although market forces partly determine the actual price, a strict mathematical relationship links mortality expectations and interest rates to observed prices, in direct similarity to the concept of arbitrage in securities markets. Milevsky (2013) provides the following formula to price a life annuity (Equation 3):

$$a\left(x, g, R\right) = \sum_{i=1}^{g} \frac{1}{(1+R)^i} + \sum_{i=g+1}^{\varphi - x} \frac{p\,(x,i)}{(1+R)^i} \quad \text{[Eqn 3]}$$

where:

- $a\,(x, g, R)$ is the upfront cost of R1 per year for life, starting at age x, guaranteed for g periods, given annual nominal interest rate R
- $p(x, i)$ *is* the survival probability from age x to age $x + i$
- R is the interest rate
- φ denotes the oldest possible age attainable.

This formula includes two components: the guaranteed portion and the life-contingent portion. The guaranteed portion is the sum of the present values of payments for the guaranteed period, while the life-contingent portion considers survival probabilities.

Table 4 illustrates the contribution of the life annuity and living annuity to the total withdrawal rate for different proportions invested in the life annuity and living annuity. The life annuity withdrawal contribution is determined at inception, which is assumed to be at age 60. The required

TABLE 4: Life annuity withdrawal contribution.

Required withdrawal rate: Invested in life annuity (%)	Required withdrawal rate: Life annuity withdrawal contribution (%)	Living annuity withdrawal rate (%)		
		4%	6%	10%
0	0.00	4.00	6.00	10.00
10	0.71	3.29	5.29	9.29
20	1.42	2.58	4.58	8.58
30	2.13	1.87	3.87	7.87
40	2.85	1.15	3.15	7.15
50	3.58	0.42	2.42	6.42
60	4.31	0.00	1.69	5.69
70	5.04	0.00	0.96	4.96
80	5.78	0.00	0.22	4.22
90	6.52	0.00	0.00	3.48
95	6.89	0.00	0.00	3.11

contribution from the living annuity is determined by subtracting the life annuity withdrawal contribution from the required annual withdrawal rate.

To interpret the results in Table 4, we consider when the required withdrawal rate is 10%, which is illustrated in Figure 2.

When the initial investment in a life annuity is 0%, then the 'hybrid' retirement strategy defaults to a living annuity and the total annual income drawdown required from the living annuity is 10%. As the initial investment in the life annuity increases, the contribution from the life annuity to the total required withdrawal rate increases. As a result, the contribution required from the living annuity decreases – but the total withdrawal rate from the two annuities is always equal to the required withdrawal rate.

If you invest a portion of your retirement savings in a life annuity, even if your living annuity depletes, you will still be partially secured for the rest of your life from the predetermined income from the life annuity. This allows retirees to assume some level of safety from the life annuity while allowing for flexibility from the riskier living annuity.

To define the success rate, we first introduce an indicator function (Equation 4):

$$I = \begin{cases} 1, & \prod_{\text{living}}(t) > 0, \quad \forall t \\ 0, & \prod_{\text{living}}(t) \leq 0, \quad otherwise \end{cases} \qquad \text{[Eqn 4]}$$

The success rate (sr) is then defined as (Equation 5):

$$sr = \frac{1}{M}\sum_{j=1}^{M} I_j \qquad \text{[Eqn 5]}$$

where M denotes the number of simulations.

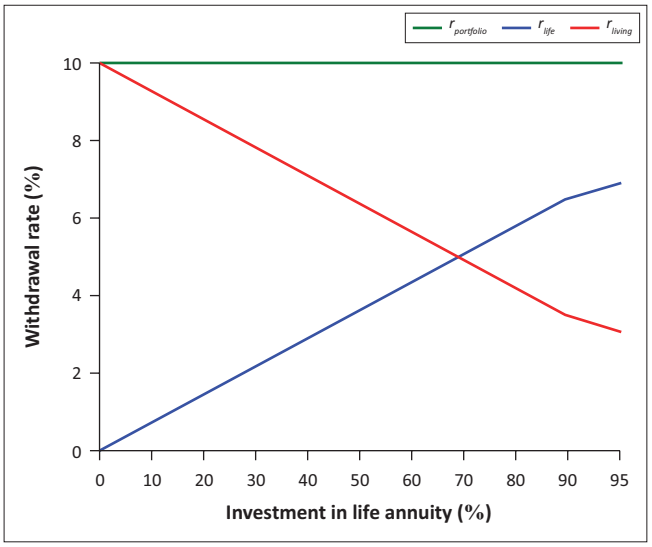


FIGURE 2: Withdrawal contributions from annuity products to the required withdrawal rate.

Results

As a preliminary analysis, we illustrate what the value of a living annuity may look like for two different random paths over 30 years, assuming an annual withdrawal rate of 6%, an initial investment amount of R1 million, and equal weights invested in bonds and equities. We perform the analysis over a period of 30 years given that the conditional median life expectancy, starting from age 60, is 82 years old.

From Figure 3, the value of the portfolio along path 2 reaches 0 at approximately 25 years. However, the value of the portfolio along path 1 does not reach 0. Therefore, we consider path 1 to be successful because a retiree will not run out of money. The success rate of this simple example is 50%.

Portfolio 1: 100% Investment in living annuity

To aid in analysing the results, we first consider a retirement strategy where 100% of a retiree's portfolio is invested in a living annuity, thus reducing the dimension of the 'hybrid' retirement strategy, that $\omega_{life} = 0$ and $\omega_{living} = 1$.

The success rate of the portfolio is determined using 10 000 simulations, where the annual withdrawal rates range from 3% to 12% and the asset allocation in bonds and equities vary.

Figure 4 illustrates the success rate for varying asset allocations and withdrawal rates for Portfolio 1.

Two important observations can be inferred from Figure 4:

1. The success rate increases as the withdrawal rate decreases.
2. The success rate increases as the equity exposure in the living annuity increases.

Our results indicate that increased withdrawal rates result in dramatically reduced success rates, regardless of the asset allocation. The greater the asset allocation to equity, the greater the success rate. This is expected because equities typically have higher returns compared to bonds, although they are more volatile.

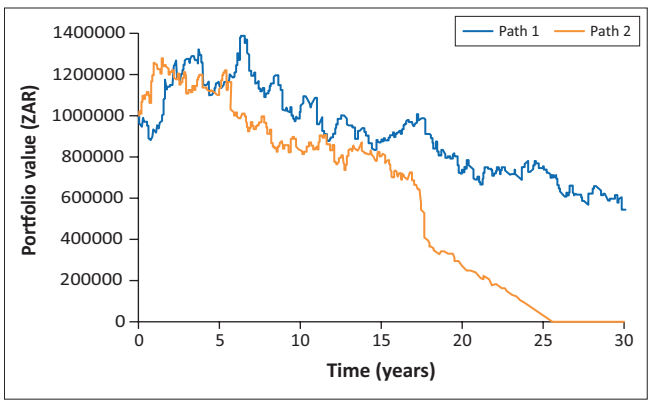


FIGURE 3: Two simulated paths of a living annuity.

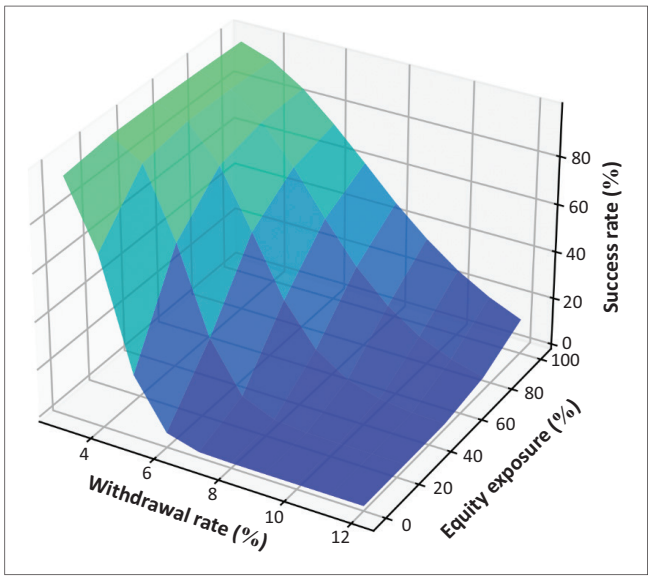


FIGURE 4: Success rates for portfolio 1 (ω_{living} = 1, ω_{life} = 0).

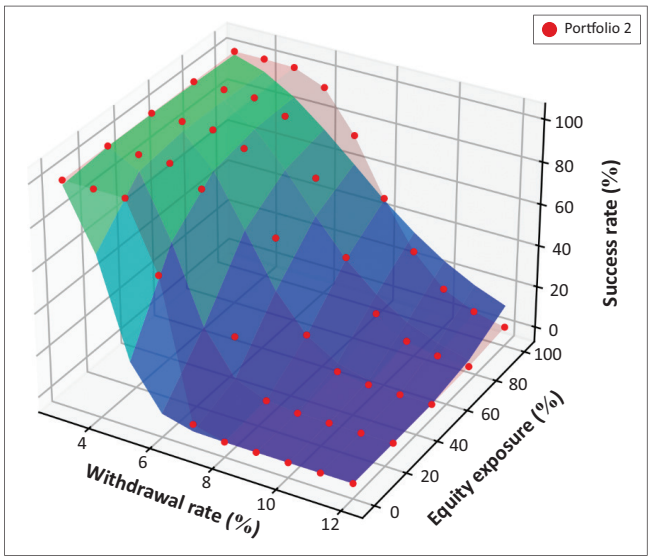


FIGURE 5: Success rates for portfolio 1 (ω_{living} = 1, ω_{life} = 0.) versus portfolio 2 (ω_{living} = 0.5 and ω_{life} = 0.5).

Portfolio 2: 50% investment in living annuity and 50% investment in life annuity

Our 'hybrid' retirement strategy introduces another dimension, an initial allocation in a life annuity. We consider the case when 50% of a retiree's initial investment is invested in a life annuity and the remaining 50% invested in a living annuity, that is. ω_{life} = 0.5 and ω_{living} = 0.5.

The annual withdrawal rate from the life annuity is 3.58% of the original investment amount. Therefore, for example, if a retiree requires a withdrawal rate of 6% annually, the remaining 2.42% will be withdrawn from the living annuity. As a result, the withdrawal from the living annuity is significantly reduced in comparison to Portfolio 1. This ensures that the retiree will always receive their required withdrawal rate, albeit from two different retirement products.

We superimpose a surface of the success rates using a 'hybrid' strategy over the surface of success rates for a pure living annuity in Figure 5.

There is a notable increase in the success rate of Portfolio 2 in comparison to Portfolio 1 when the withdrawal rate is less than or equal to 8% across all portfolio compositions for the living annuity. However, for higher withdrawal rates, the success rate of Portfolio 1 dominates that of Portfolio 2, specifically when the equity exposure is high.

The results from Portfolio 2 indicate that an initial investment in a life annuity increases the likelihood of success compared to retirement portfolios that consist of pure living annuities for withdrawal rates of 8% or less. This is a significant result because the study conducted by ASISA suggests that approximately 50% of South African retirees withdraw between 2.5% and 7.5% of their portfolios annually.

The success of approximately 50% of South Africans with living annuities can be increased by investing in a combination of a living annuity and a life annuity. Furthermore, retirees who have invested a portion of their portfolio in a life annuity will still receive an income drawdown until death, even if their living annuity depletes.

Until this point, we have only considered the 'hybrid' retirement strategy without fees. However, fees can have a large impact on the longevity and success of a retiree's portfolio. For example, if a retiree invested in a pure living annuity, where the asset allocation to bonds is 100%, a retiree may experience significant capital erosion as the average real return on bonds is approximately 2.1% and the fund management fees amount to approximately 2.25%. This results in an average negative real return of -0.15%.

To illustrate the impact of fees on a living annuity, we consider a pure living annuity with an annual drawdown rate of 6% and equal weights in bonds and equities. We plot the average portfolio value over 10000 simulations, including and excluding fees. Figure 6 shows that the impact of fees cannot be ignored as the portfolio value erodes significantly faster. The average value of the portfolio without fees is approximately R680000 with a success rate of 57%. However, once fees are introduced, the portfolio value decreases to approximately R120000 after 30 years, with a success rate of just 20%.

We illustrate the effect of fees on the success rate of Portfolio 1 for different asset allocations and withdrawal rates in Figure 7. We superimpose the success rate surface of Portfolio 1 with fees on the success rate surface of Portfolio 1 without fees.

It is evident that fees drastically reduce the success rate of a living annuity, especially when the equity exposure is low as the returns from bonds are not sufficient to offset the fees.

We now consider the effect of fees on the 'hybrid' retirement strategy, where equal portions are invested in a life annuity and a living annuity, that is, Portfolio 3. Figure 8 illustrates the success rate of Portfolio 1 (blue surface) and the success rate of Portfolio 2 (red surface), with fees taken into consideration for both portfolios.

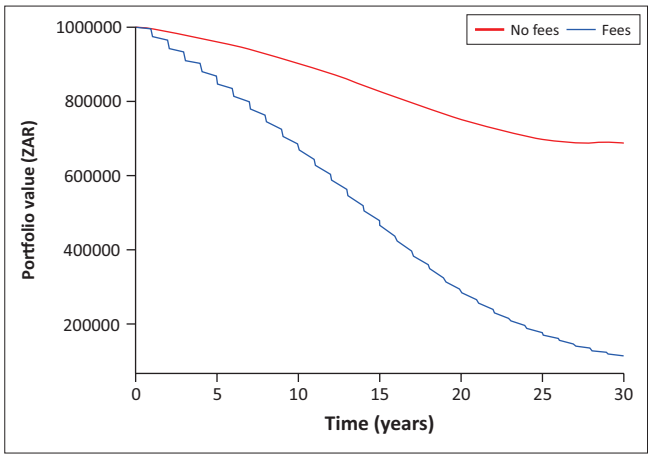


FIGURE 6: Impact of fees on portfolio value (ω_{living} = 1, ω_{life} = 0, ω_{equity} = 0.5, ω_{bond} = 0.5, $r_{portfolio}$ = 6%).

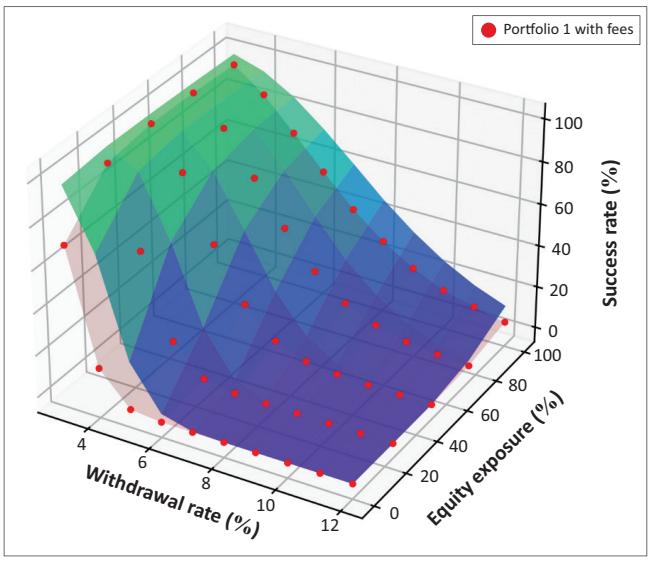


FIGURE 7: Impact of fees on the success rate of portfolio 1 (ω_{living} = 1, ω_{life} = 0).

The success rate of Portfolio 2 is still significantly higher than the success rate of Portfolio 1 when accounting for fees in both portfolios for withdrawal rates less than 8% per annum. This is expected as the fees typically associated with a life annuity are less than those associated with a living annuity.

Further, we note that although the success rates may be lower for Portfolio 2 compared to Portfolio 1 when the withdrawal rates are greater than 7% and fees are accounted for, a retiree will still receive an income until death from the life annuity.

The success rates for different portfolio compositions in life and living annuities, asset allocations and withdrawal rates including fees are provided in Table 5. Similarly, as noted previously, as the investment in a life annuity at inception increases, so does the success rate when the withdrawal rate is less than or equal to 7%.

Table 5 provides the portfolio success rates for different portfolio compositions in life and living annuities, asset allocations and withdrawal rates. The success rates for portfolios, excluding fees, are provided along with the

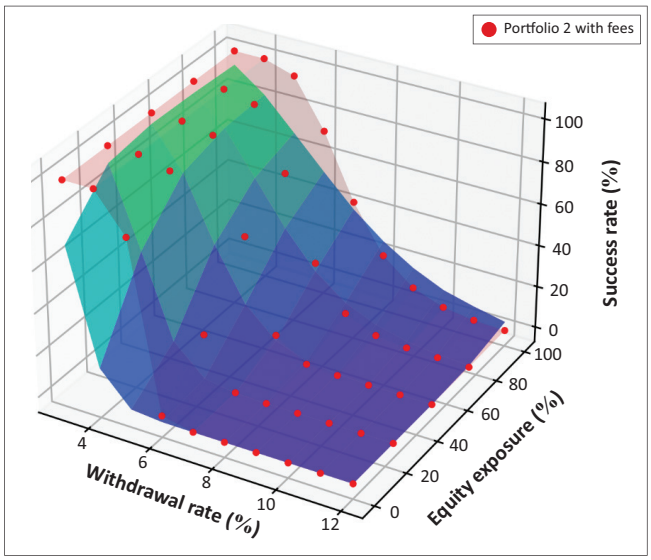


FIGURE 8: Impact of fees on the success rate of portfolio 1 (ω_{living} = 1, ω_{life} = 0) versus portfolio 2 (ω_{living} = 0.5, ω_{life} = 0.5).

success rates for portfolios with the associated portfolio management fees in square brackets.

From Table 5, we observe that the fee structure of a retirement portfolio significantly decreases the success rate for all portfolio compositions. For example, the success rate of Portfolio 1 with equal weights in bonds and equities and a 6% withdrawal rate is 57% without fees compared to a success rate of just 20% when fees are included.

With respect to Portfolio 2, with equal weights in bonds and equities, and a 6% withdrawal rate, we obtain a success rate of 95% without fees. This success rate declines to 53% when taking fees into account, but it is still significantly higher than the success rate of the pure living annuity (Portfolio 1).

For all different compositions of the 'hybrid' retirement strategy considered, we observe that the success rates of the 'hybrid' strategy dominate those of the pure living annuity for withdrawals of less than 8%. Portfolio management fees impact the success rate of all strategies but are more pronounced for portfolios that have a larger weight invested in a living annuity.

We observe that a larger investment in a life annuity at inception generally results in increasing success rates for withdrawal rates of less than 8%. However, each retiree is unique, and some may prefer the flexibility offered by investing a smaller portion in a life annuity.

Overall, an average decrease of 11% in the success rate is observed over the different portfolio compositions considered in Table 6.

Table 6 indicates that the success rates of retirement portfolios, which have a high percentage invested in a living annuity, are, on average, affected more by portfolio

TABLE 5: Portfolio success rates for different asset allocations, withdrawal rates and life annuity proportions over 30 years.

Equity exposure	Withdrawal rate as a percentage of initial investment																			
	3%		4%		5%		6%		7%		8%		9%		10%		11%		12%	
	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)	E.f. (%)	I.f. (%)
0% life annuity/100% living annuity																				
0%	98	70	70	16	24	1	4	0	0	0	0	0	0	0	0	0	0	0	0	0
25%	100	92	93	54	64	16	28	2	8	0	1	0	0	0	0	0	0	0	0	0
50%	100	95	95	76	81	46	57	20	32	7	16	2	7	1	2	0	1	0	0	0
75%	99	95	95	82	86	62	69	41	52	24	35	13	23	7	13	3	8	1	4	1
100%	98	93	94	83	86	69	74	53	61	38	48	27	36	17	26	11	19	7	13	4
25% life annuity/75% living annuity																				
0%	100	100	100	74	70	10	14	0	1	0	0	0	0	0	0	0	0	0	0	0
25%	100	100	100	94	93	43	52	6	12	0	1	0	0	0	0	0	0	0	0	0
50%	100	100	100	96	95	69	74	30	41	8	16	2	4	0	1	0	0	0	0	0
75%	100	100	100	95	95	77	80	50	59	26	36	11	19	4	9	2	4	1	1	0
100%	100	99	99	94	93	79	82	60	66	39	49	25	33	14	21	7	13	4	8	2
50% life annuity/50% living annuity																				
0%	100	100	100	100	100	82	68	3	3	0	0	0	0	0	0	0	0	0	0	0
25%	100	100	100	100	100	96	92	23	27	0	1	0	0	0	0	0	0	0	0	0
50%	100	100	100	100	100	97	95	53	57	10	16	1	2	0	0	0	0	0	0	0
75%	100	100	100	100	100	96	95	67	70	28	36	8	13	2	4	0	1	0	0	0
100%	100	100	100	100	100	95	94	72	75	41	49	20	27	8	13	1	5	0	2	0
75% life annuity/25% living annuity																				
0%	100	100	100	100	100	100	100	97	66	0	0	0	0	0	0	0	0	0	0	0
25%	100	100	100	100	100	100	100	100	91	2	1	0	0	0	0	0	0	0	0	0
50%	100	100	100	100	100	100	100	99	94	18	15	0	0	0	0	0	0	0	0	0
75%	100	100	100	100	100	100	100	99	94	38	35	3	4	0	0	0	0	0	0	0
100%	100	100	100	100	100	100	100	98	93	49	48	11	12	1	2	0	0	0	0	0
90% life annuity/10% living annuity																				
0%	100	100	100	100	100	100	100	100	100	4	0	0	0	0	0	0	0	0	0	0
25%	100	100	100	100	100	100	100	100	100	29	1	0	0	0	0	0	0	0	0	0
50%	100	100	100	100	100	100	100	100	100	58	14	0	0	0	0	0	0	0	0	0
75%	100	100	100	100	100	100	100	100	100	71	34	0	0	0	0	0	0	0	0	0
100%	100	100	100	100	100	100	100	100	100	75	46	1	1	0	0	0	0	0	0	0

E.f., excluding fees; I.f., including fees.

TABLE 6: The average impact of portfolio management costs on success rates.

Portfolio scenarios	Average success rate impact (%)
0% life annuity/100% living annuity	-14
25% life annuity/75% living annuity	-12
50% life annuity/50% living annuity	-11
75% life annuity/25% living annuity	-9
90% life annuity/10% living annuity	-7

management costs (Table 1). This is mainly because of the nature of the retirement products, where living annuities have annual management costs and life annuities only have once-off costs at inception.

The goal of the 'hybrid' strategy is twofold:

1. To increase the success rate of a retirement strategy for a given withdrawal rate.
2. To increase the inheritance of a retiree's heir.

Figure 9 illustrates the average value of a retiree's living annuity for different weights invested in a life annuity and a living annuity, assuming a 6% drawdown rate, portfolio management fees and equal weights invested in bonds and equities.

When investing purely in a living annuity, the value of the portfolio decreases rapidly, which is consistent with the low success rate of 20% observed for this portfolio. Although the value of the pure living annuity portfolio is the greatest at inception, it has the lowest average value after 30 years. The portfolio composition that maximises both the success rate and the living annuity value after 30 years is where there is an initial investment of 75% in a life annuity and 25% investment in a living annuity. The success rate for this portfolio composition is 99%, and the average living annuity value increases over time, allowing a retiree to leave an inheritance to their heir, while still achieving the required withdrawal rate of 6%.

In Figure 9, we have only considered a 6% withdrawal rate with an equal composition in bonds and equities. Our analysis can be extended to different withdrawal rates and living annuity compositions as each retiree is unique and some may require higher withdrawal rates than others, or some may seek to maximise their heir's inheritance. A 'hybrid' strategy increases both the success rate and the value of a retiree's living annuity when the withdrawal rate is less than 8%, which has the potential to increase the success rate of approximately 50% of annuatised South Africans.

Conclusion

> The choice of a withdrawal rate involves individual preference for current consumption, uncertainty of life expectancy, and variable financial needs, there is no single globally optimal withdrawal rate. (Cooley et al. 1999:47)

The quote by Cooley et al. (1999) is significant because every retiree has different needs and will have to adopt a strategy that meets their needs. In a South African context, we have shown that a 'hybridised' retirement portfolio increases the probability of success for retirees withdrawing less than 8% of their portfolio annually, which is approximately 50% of the annuatised population. We observed that portfolio management fees influence the success rate of our results significantly and cannot be ignored. However, the 'hybrid' strategy still has a higher success rate when portfolio management costs are accounted for. Further, we have shown that a 'hybrid' retirement strategy not only has an increased success rate but, on average, the living annuity portion has a higher value after 30 years (depending on the portfolio composition), providing a larger inheritance for a retiree's heir.

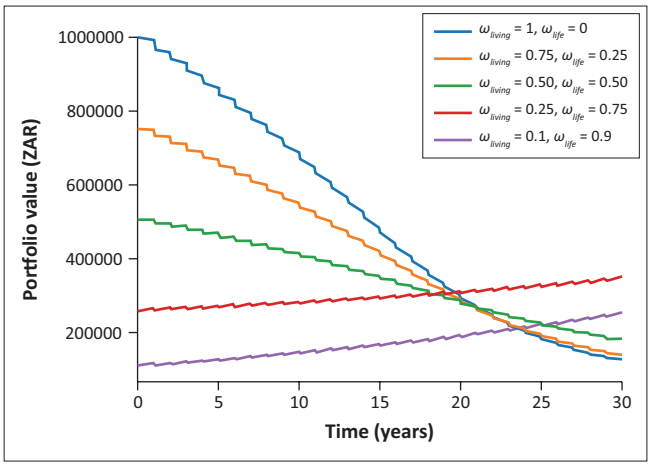


FIGURE 9: Average portfolio values for a pure living annuity versus various hybrid strategy exposures.

A 'hybrid' retirement strategy could provide 'safe' withdrawal rates of up to 6% when accounting for fees, which is a significant increase from the 4% 'safe' withdrawal rate without fees recommended by Maré (2016) for South African retirees.

Further research

Further research may include determining the 'optimal' portfolio composition that increases the success rate for a given withdrawal rate. Further, there may be merit in investigating the 'optimal' time to convert a living annuity (or a part thereof) to a life annuity, as a life annuity becomes cheaper the older a retiree becomes.

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Competing interests

The authors declare that they have no financial or personal relationship(s) that may have inappropriately influenced them in writing this article.

Authors' contributions

A.J.v.N., V.M. and E.M. contributed equally to this work.

Ethical considerations

This article followed all ethical standards for a research without direct contact with human or animal subjects.

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Data availability

Data are available from the corresponding author, A.J.v.N., under reasonable request.

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