Nonfragile high-gain observers for nonlinear systems with output uncertainty

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Abstract

In this paper, we present the definitions of nonfragile high-gain observers and design method for lower-triangular nonlinear systems with output uncertainty. Radial basis function neural networks (RBFNNs) are used to approximate the output uncertainty. By inserting an output filter and an input-output filter, a new augmented adaptive observable canonical form is derived. Then, a corresponding observer with gain perturbations is designed to estimate the states and the coefficients of the RBFNNs, and a disturbance observer is designed to estimate the approximation error. The maximum allowable gain perturbation is also given. Then, the obtained results are extended to nonlinear systems in adaptive observer form with output uncertainty. Finally, some numerical simulations are offered to corroborate the theoretical results.

KEYWORDS

adaptive observer, augmented adaptive observable canonical form, high-gain observer, nonfragile, RBFNNs

1 | INTRODUCTION

It is a very hot topic in automation and control field to estimate unknown states for a nonlinear system based on measured outputs. Quite a number of design methods have been proposed in the literature. Among these methods, high-gain observers associated with the triangular structure have a higher important position in nonlinear observer design. Researchers have been carried out new developments in various directions.[1-9]

The above results are derived under the condition that the output can be accurately measured. However, in practice, the output sensitivity of some systems such as electrical devices[10] and mechanical systems,[11] may not be constant. In other words, there inevitably exists output uncertainty. In order to resist unknown measurement uncertainty, robust high-gain observer design methods were introduced and well discussed in References 12-16. Compared with a constant high-gain, a dynamic high-gain could achieve better balance between the speed of state reconstruction and the measurement noise sensitivity.[12,17,18] However, larger high-gain brought about the phenomena of oscillation and variation in the presence of measurement uncertainty.[19,20] Therefore, studies on the high-gain observer design with measurement disturbance by filtering technique have received more attention. In Reference 21, the states were estimated by inserting filtered measurements for single-output linear observable systems. Some researchers also analyzed the effect of output noise on high-gain observers when low-pass filter was used.[22,23] Motivated by the high sensitivity of high-gain observers to output disturbance, filtered high-gain observers were addressed in the presence of measurement uncertainty with saturation function.[19,24] In the latest work,[14] a low-pass filter on the measurement channel was inserted to reduce the sensitivity to measurement noise. It should be noted that the error of the output and its estimation was filtered. Then, the filtered error was used to construct an observer.

In order to estimate unknown states and parameters simultaneously, the adaptive technique has been developed for linear systems with unknown parameters,[25-30] and nonlinear systems with unknown parameters as well.[31-34] The unknown parameters appear in two different ways, that is, linear and nonlinear parameterized ways. Therefore, adaptive observer design has been studied for nonlinear systems with parameters in linear parameterized way,[25-28,35] and nonlinear parameterized way,[36-42] respectively. However, the observers based on adaptive technique show erratic performance if there exist measurement disturbances.[43,44]

Besides unknown parameters or uncertainties in the presence of system states and outputs, another significant issue in observer design field is that the observer gains are referred to fragility or nonresilience.[45] For example, the authors in Reference 46 pointed out that the closed-loop system presented ambiguous when the control gain involved some disturbances. In addition, it is revealed that observer gains may show insignificant drifts because sensor equipment gets aging since they are produced offline. More outcomes on nonfragile cases and states uncertainty cases for nonlinear systems can be found in References 47-54. However, there are some limitations in present works: (a) The observer gains are obtained by solving some linear matrix inequalities (LMIs). (b) The design methods may be unsuccessful if the LMIs are infeasible.

Motivated by above investigations, we firstly present the definition of nonfragile high-gain observer for lower-triangular systems with output uncertainty. Then, by inserting an output filter, an augmented nonlinear system is obtained and the output uncertainty is transformed into the state equation. Radial basis function neural networks (RBFNNs) are used to approximate the measurement uncertainty. By an input–output filter on the augmented system, a new augmented adaptive observable canonical form is derived. Then, a corresponding adaptive observer with gain perturbations is designed to estimate the states and the coefficients of the RBFNNs, and a disturbance observer is designed to estimate the approximation error. By constructing a Lyapunov function, it is shown that the estimation errors of the states and the approximation error are uniformly ultimately boundedness (UUB). If the measurement uncertainty can be exactly approximated by the RBFNNs, then, the estimation errors of the states will converge to origin. Moreover, under a persistent excitation condition, the estimation errors of the coefficients of the RBFNNs will converge to origin as well. At last, the obtained results are extended to nonlinear system with adaptive observer form and output disturbance.

The main contributions of this paper summarize as follows: (a) For the first time, definition of nonfragile high-gain observer is presented for lower-triangular nonlinear systems with output uncertainty. (b) By an output filter and an input–output filter and RBFNNs approximation theory, a new augmented adaptive observable canonical form is derived. The design of nonfragile high-gain observer is transformed into the design an adaptive observer with gain perturbations. (c) By constructing a Lyapunov function, the convergence of the estimation errors is analyzed. In addition, nonfragile adaptive observers are also extended to nonlinear systems with unknown parameters in the state equation and uncertainty in the measured output.

This paper is organized as follows. We introduce the problem description and some lemmas in Section 2. In Section 3, we propose a new augmented adaptive observable canonical form and nonfragile observers for nonlinear systems with the observer sensitivities and measurement disturbance. In Section 4, the results are extended to nonlinear systems in adaptive observer form with output uncertainty. The experimental simulation results are shown in Section 5. The last Section summarizes this paper.

2 | PRELIMINARIES AND PROBLEM DESCRIPTION

In this section, we introduce some fundamentals for later use.

Lemma 1. *(Reference 55): A continuous function $g(\upsilon)$ defined on a compact set $\Upsilon \subset R^q$, can be approximated by the following RBFNNs,*

$$\hat{g}(\upsilon) = \hat{O}^T \varphi(\upsilon) + \mu,$$

where $\upsilon = (\upsilon_1, \upsilon_2, \dots, \upsilon_q)^T \in \Upsilon$, $\hat{O} \in R^p$ and μ are the input vector, the weight vector and the approximation error, respectively. The approximation error μ satisfies $|\mu| \leq \overline{\mu}$ and $\overline{\mu} > 0$. The basis function vector $\varphi(\upsilon) = (\varphi_1(\upsilon), \varphi_2(\upsilon), \dots, \varphi_p(\upsilon))^T \in R^p$ and is continuous. The ideal weight O^* is given by

$$O^* = \arg\min_{\hat{O}\in h_g}\{\sup_{\upsilon\in\Upsilon}|\hat{g}(\upsilon|\hat{O}) - g(\upsilon)|\},$$

where $h_g = \{\hat{O} : \|\hat{O}\| \le \overline{h}\}$ *and* $\overline{h}$ *is a designed constant. Then,*

$$
\begin{aligned}
&g(v) = O^{*T}\varphi(v) + \mu^*, \\
&|\mu^*| \le \overline{\mu},
\end{aligned}
$$

where μ^* *is the optimal approximation error and* $\overline{\mu} > 0$ *is the upper bound of the approximation error.*

Lemma 2. *Meyer–Kalman–Yacubovich Lemma*[56]*: Let* H_1*,* H_2*, and* H_3 *are three matrices with appreciate dimensions. If and only if the triples* (H_1, H_2, H_3) *satisfy the strictly positive real condition, that is,* $Re\{H_3(j\omega I - H_1)^{-1}H_2\} > 0$*, then, there exist two positive definite matrices* P_0 *and* W_0*, a constant real vector* J_0*, and a positive real number* ϵ_0 *such that*

$$
\begin{aligned}
H_1^T P_0 + P_0 H_1 &= -J_0 J_0^T - \epsilon_0 W_0, \\
P_0 H_2 &= H_3^T.
\end{aligned}
$$

Lemma 3. *Let*

$$
A = \begin{pmatrix}
0 & 1 & 0 & \cdots & 0 & -k_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & -k_n \\
1 & 0 & 0 & \cdots & 0 & -k_{n+1}
\end{pmatrix}.
$$

Then the characteristic polynomial of the matrix A is $\lambda^{n+1} + k_{n+1}\lambda^n + k_1\lambda^{n-1} + \cdots + k_{n-1}\lambda + k_n$.

Proof. The characteristic polynomial of A is

$$
\det(\lambda I - A) = \begin{vmatrix}
\lambda & -1 & 0 & \cdots & 0 & k_1 \\
0 & \lambda & -1 & \cdots & 0 & k_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \lambda & k_n \\
-1 & 0 & 0 & \cdots & 0 & \lambda + k_{n+1}
\end{vmatrix}.
$$

Expanding by the last row of the determinant yields,

$$
\det(\lambda I - A) = \lambda^n(\lambda + k_{n+1}) - (-1)^{n+2} \begin{vmatrix}
-1 & 0 & \cdots & 0 & k_1 \\
\lambda & -1 & \cdots & 0 & k_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & \lambda & k_n
\end{vmatrix}.
$$

Then, we expand the above determinant by the last column. Therefore,

$$
\begin{aligned}
\det(\lambda I - A) &= \lambda^n(\lambda + k_{n+1}) + \sum_{i=1}^{n}(k_i \lambda^{n-i}) \\
&= \lambda^{n+1} + k_{n+1}\lambda^n + k_1\lambda^{n-1} + \cdots + k_{n-1}\lambda + k_n.
\end{aligned}
$$

The proof is completed. ∎

Next, we consider the following nonlinear system with measurement uncertainty,

$$
\begin{aligned}
\dot{x}(t) &= A_0 x(t) + B_0 u(t) + \phi_0(t, x), \\
y(t) &= C_0 x(t) + d(t),
\end{aligned} \tag{1}
$$

where

$$A_0 = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}, \quad B_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \text{ and } C_0 = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}, \quad x(t), \quad y(t) \text{ and } u(t),$$

are the state variable, the output variable and the control variable, respectively. The measurement uncertainty $d(t)$ is a continuous nonlinear function. The nonlinear function $\phi_0(t,x) = (\phi_1(t,x_1^\iota),\ \phi_2(t,x_2^\iota),\ \dots,\phi_n(t,x_n^\iota))^T \in \mathbb{R}^n$, $\phi_i(t,x_i^\iota) \in R$ are continuous nonlinear functions and satisfy

$$|\phi_i(t,x_i^\iota) - \phi_i(t,\hat{x}_i^\iota)| \le \varrho(|x_1(t) - \hat{x}_1(t)| + |x_2(t) - \hat{x}_2(t)| + \cdots + |x_i(t) - \hat{x}_i(t)|),$$

where $x_i^\iota = (x_1, x_2, \dots, x_i)$, $(i = 1, \dots, n)$ and $\varrho > 0$ is a constant. The measurement noise $d(t)$, by Lemma 1, can be approximated by RBFNNs,

$$d(t) = \sum_{i=1}^{m} a_i^* \varphi_i(t) + v^*(t), \tag{2}$$

where $\varphi_i(t)$ are the radial basis functions, a_i^* represent the optimal coefficients and $v^*(t)$ is the optimal approximation error. We assume that the optimal approximation error $v^*(t)$ and its derivative $\dot{v}^*(t)$ satisfy $|v^*(t)| \le \overline{v}^*$ and $|\dot{v}^*(t)| \le \bar{\bar{v}}^*$ where $\overline{v}^*$ and $\bar{\bar{v}}^*$ are two positive constants. Our aim is to design a nonfragile observer for the nonlinear system (1) with the measurement noise (2).

Remark 1. Except for RBFNNs, FLS (fuzzy logic system) is also able to approximate the measurement disturbance by implementing adaptive laws to identify the weights. It is proposed based on fuzziness characteristics of human brain thinking, and has superior adaptability and approximation ability.[57-59] RBFNNs are two-layer forward networks. The hidden nodes implement a set of radial basis functions. The output nodes implement linear summation functions. Whereas, FLS consists of a knowledge base, a fuzzifier, a fuzzy inference machine and a defuzzifier. In the point of estimating measurement disturbance, they share the same effectiveness.

3 | THE NONFRAGILE OBSERVER DESIGN STRATEGIES

Introduce the following filtering action to the output with disturbance. Then,

$$\dot{\overline{x}}_{n+1}(t) = -Lk_{n+1}\theta_{n+1}(t)\overline{x}_{n+1}(t) + y(t), \tag{3}$$

where $\theta_{n+1}(t)$ is an unknown function and denotes the gain perturbation, L and k_{n+1} are two positive parameters. We can obtain the following augmented system,

$$\begin{aligned} \dot{\overline{x}}(t) &= A_1\overline{x}(t) + B_1u(t) + \phi(t,\overline{x}) + B_2 d(t), \\ \overline{y}(t) &= \overline{x}_{n+1}(t), \end{aligned} \tag{4}$$

where $\overline{x}(t)$ and $\overline{y}(t)$ are the state variable and output of the system (4), respectively.

$$A_1 = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & -Lk_{n+1}\theta_{n+1}(t) \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \text{ and } \phi(t,\overline{x}) = \begin{pmatrix} \phi_0(t,\overline{x}) \\ 0 \end{pmatrix}.$$

Let $b_{n+1} = 1$, and choose n positive constants $b_1, \dots, b_n$ such that the polynomial $\lambda^n + b_1\lambda^{n-1} + \cdots + \lambda b_{n-1} + b_n$ is Hurwitz.

We insert the following input–output filter on the system (4),

$$\begin{aligned} \eta_j(t) &= \overline{x}_j(t) - \sum_{i=1}^{m} \alpha_j[i]a_i^*, \quad j = 1, \dots, n, \\ \eta_{n+1}(t) &= \overline{x}_{n+1}(t), \end{aligned} \tag{5}$$

where $\dot{\alpha}[i] = D\alpha[i] + E\varphi_i(t)$, $\alpha[i](0) = 0$ and $\alpha_j[i]$ is the jth element of the vector $\alpha[i]$.

The matrices D and E are given as,

$$D = \begin{pmatrix} -Lb_1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -L^{n-1}b_{n-1} & 0 & 0 & \cdots & 1 \\ -L^n b_n & 0 & 0 & \cdots & 0 \end{pmatrix} \text{ and } E = \begin{pmatrix} -Lb_1 \\ \vdots \\ -L^{n-1}b_{n-1} \\ -L^n b_n \end{pmatrix}.$$

Let $\varphi_{\max}$ be the upper bound of $|\varphi_i(t)|$ $(i = 1, \dots, m)$. Then, the following results can be obtained.

Lemma 4. *For the filter system $\dot{\alpha}[i] = D\alpha[i] + E\varphi_i(t)$, $\alpha[i](0) = 0$, the following inequalities hold,*

$$|\alpha_j[i]| \leq L^{j-1} \frac{2\lambda_1^{\frac{1}{2}} \|\overline{b}\|}{c_3 \lambda_2^{\frac{1}{2}}} \varphi_{\max}, j = 1, \dots, n, \ i = 1, \dots, m,$$

where $\overline{b} = \left(b_1, \dots, b_n\right)^T$, P_1 is a positive definite matrix, λ_1 and λ_2 are the maximum and the minimum eigenvalues of the matrix P_1, respectively, and c_3 is a positive constant,

Proof. Make the coordinate transformations as follows,

$$\vartheta_j[i](t) = \frac{\alpha_j[i]}{L^j}, \quad j = 1, \dots, n, \ i = 1, \dots, m.$$

Let $\vartheta[i](t) = (\vartheta_1[i](t), \dots, \vartheta_n[i](t))^T$. Then, we have

$$\dot{\vartheta}[i](t) = LA_2\vartheta[i](t) + \overline{b}\varphi_i(t),$$

where

$$A_2 = \begin{pmatrix} -b_1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -b_{n-1} & 0 & 0 & \cdots & 1 \\ -b_n & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

Since the polynomial $\lambda^n + b_1\lambda^{n-1} + \cdots + \lambda b_{n-1} + b_n$ is Hurwitz, there exists a positive definite matrix P_1 and a positive constant c_3 satisfying $A_2^T P_1 + P_1 A_2 \leq -c_3 P_1$.

Construct the Lyapunov function as $V_0(t) = \vartheta^T[i](t)P_1\vartheta[i](t)$. Then,

$$\begin{aligned} \dot{V}_0(t) &= L\vartheta^T[i](t)(A_2^T P_1 + P_1 A_2)\vartheta[i](t) + 2\vartheta^T[i](t)P_1\overline{b}\varphi_i(t) \\ &\leq -c_3 L\vartheta^T[i](t)P_1\vartheta[i](t) + 2(\vartheta^T[i](t)P_1\vartheta[i](t))^{\frac{1}{2}}(\overline{b}^T P_1 \overline{b})^{\frac{1}{2}}\varphi_{\max} \\ &\leq -c_3 LV_0(t) + 2\lambda_1^{\frac{1}{2}}\|\overline{b}\|\varphi_{\max}V_0^{\frac{1}{2}}(t). \end{aligned}$$

Since

$$\dot{V}_0^{\frac{1}{2}}(t) = \frac{1}{2}V_0^{-\frac{1}{2}}(t)\dot{V}_0(t) \leq -\frac{c_3}{2}LV_0^{\frac{1}{2}}(t) + \lambda_1^{\frac{1}{2}}\|\overline{b}\|\varphi_{\max},$$

thus,

$$V_0^{\frac{1}{2}}(t) \leq \frac{2\lambda_1^{\frac{1}{2}}\|\bar{b}\|}{c_3 L}\varphi_{\max}.$$

Moreover, we have

$$|\alpha_j[i]| = L^j\|\vartheta[i](t)\| \leq L^{j-1}\frac{2\lambda_1^{\frac{1}{2}}\|\bar{b}\|}{c_3\lambda_2^{\frac{1}{2}}}\varphi_{\max},$$

which completes the proof. ■

From (2) and (5), the system (4) can be rewritten as

$$\begin{aligned}\dot{\eta}(t) &= A_1\eta(t) + B_1u(t) + \phi(t,\eta) + B\beta^T(t)a + B_2v^*(t),\\ \gamma(t) &= C\eta(t),\end{aligned} \tag{6}$$

where $\gamma(t)$ is the output,

$$a = \begin{pmatrix} a_1^* \\ \vdots \\ a_m^* \end{pmatrix}, \quad \beta(t) = \begin{pmatrix} \alpha_1[1] + \varphi_1(t) \\ \vdots \\ \alpha_1[m] + \varphi_m(t) \end{pmatrix}, \quad C = (0 \ \cdots 0 \ 1), \quad B = \begin{pmatrix} Lb_1 \\ \vdots \\ L^n b_n \\ b_{n+1}, \end{pmatrix}.$$

The system (6) is called an **augmented adaptive observable canonical form**.

We will proceed the observer design under two cases. The first case is that the optimal approximation error $v^*(t) \neq 0$. The second case is that the optimal approximation error $v^*(t) \equiv 0$, that is, the output uncertainty $d(t)$ can be written as $d(t) = \sum_{i=1}^m a_i^* \varphi_i(t)$.

Under the first case, we propose the following observer,

$$\begin{aligned}\dot{\hat{\eta}}(t) &= A_1\hat{\eta}(t) + G(\eta_{n+1}(t) - \hat{\eta}_{n+1}(t)) + B_1u(t) + \phi(t,\hat{\eta}) + B\beta^T(t)\hat{a}(t) + B_2\hat{v}(t),\\ \dot{\hat{a}}(t) &= L\Lambda\beta(t)(\gamma(t) - C\hat{\eta}),\\ \dot{\hat{v}}(t) &= -\kappa_0\hat{v}(t) + \kappa_0(\gamma(t) - C\hat{\eta}),\\ \hat{\hat{x}}_j(t) &= \hat{\eta}_j(t) + \sum_{i=1}^m \alpha_j[i]\hat{a}_i, \quad j = 1, \ldots, n,\\ \hat{\hat{x}}_{n+1}(t) &= \hat{\eta}_{n+1}(t),\end{aligned} \tag{7}$$

where

$$G = \begin{pmatrix} L^2k_1\theta_1(t) \\ L^3k_2\theta_2(t) \\ \vdots \\ L^nk_{n-1}\theta_{n-1}(t) \\ L^{n+1}k_n\theta_n(t) \\ 0 \end{pmatrix}, \quad \theta_1(t), \theta_2(t), \ldots, \theta_{n+1}(t),$$

are the observer gain perturbations, Λ is a positive definite matrix and κ_0 is a constant satisfying $\kappa_0 > \frac{5}{2}$.

Remark 2. Due to manufacturing reasons, such as sensor equipments aging or round-off errors in numerical calculations, there always exist gain perturbations.[60] For example, in Reference 61, the authors pointed out that there exists ±10% sensitivity error in the displacement sensor of a magnetic bearing suspension system. Moreover, there are two basic forms of observer gain perturbations, that is, multiplicative perturbations[62] and additive perturbations.[45] In fact, these two forms of perturbations are interconvertible, for example, $\theta(t)x_1(t) = x_1(t) + (\theta(t) - 1)x_1(t)$, and $x_1(t) + d(t) = \left(1 + \frac{d(t)}{x_1(t)}\right)x_1(t)$ when $x_1(t) \neq 0$.

Under the second case, the following observer is presented,

$$\begin{aligned}\dot{\hat{\eta}}(t) &= A_1\hat{\eta}(t) + G(\eta_{n+1}(t) - \hat{\eta}_{n+1}(t)) + B_1u(t) + \phi(t,\hat{\eta}) + B\beta^T(t)\hat{a}(t),\\ \dot{\hat{a}}(t) &= L\Lambda\beta(t)(\gamma(t) - C\hat{\eta}),\\ \hat{\bar{x}}_j(t) &= \hat{\eta}_j(t) + \sum_{i=1}^{m}\alpha_j[i]\hat{a}_i, \quad j = 1, \ldots, n,\\ \hat{\bar{x}}_{n+1}(t) &= \hat{\eta}_{n+1}(t).\end{aligned} \tag{8}$$

Now, we give the following definitions.

Definition 1. For the nonlinear system (1), we construct the system (7) with unknown gain perturbations $\theta_i(t)$, $i = 1, \ldots, n$. If the gain perturbation $\theta_i(t) \in (1-\bar{\theta}, 1+\bar{\theta})$ ($\bar{\theta}$ is the maximum allowable gain perturbation), and there exist two positive real numbers t_1, d_1 such that

$$|\bar{x}_j(t) - \hat{\bar{x}}_j(t)| \leq d_1, \; j = 1, \ldots, n, \; t > t_1, \tag{9}$$

then, we call the system (7) is an **UUB nonfragile high-gain observer** of the nonlinear system (1).

Definition 2. For the nonlinear system (1), we establish the system (8) with unknown gain perturbations $\theta_i(t)$, $i = 1, \ldots, n$. If the gain perturbation $\theta_i(t) \in (1-\bar{\theta}, 1+\bar{\theta})$ ($\bar{\theta}$ is the maximum allowable gain perturbation), and the output uncertainty $d(t)$ can be exactly approximated by RBFNNs, and

$$\lim_{t\to\infty}|\bar{x}_j(t) - \hat{\bar{x}}_j(t)| = 0, \; j = 1, \ldots, n, \tag{10}$$

then, we call the system (8) is a **nonfragile high-gain observer** of the system (1).

In addition, if the unknown parameters a_i^* can be identified, that is

$$\lim_{t\to\infty}(\hat{a}(t) - a) = 0, \tag{11}$$

then, we call the system (8) is a **nonfragile high-gain observer with identification** of the nonlinear system (1).

For the first case, make the following coordinates transformation,

$$\begin{aligned}z_i(t) &= \frac{\eta_i(t) - \hat{\eta}_i(t)}{L^i}, \quad i = 1, \ldots, n,\\ z_{n+1}(t) &= \eta_{n+1}(t) - \hat{\eta}_{n+1}(t).\end{aligned} \tag{12}$$

From (6), (7), and (12), it is easy to infer the error system as follows,

$$\begin{aligned}\dot{z}(t) &= L\bar{A}z(t) + \tilde{\phi}(t,\eta) + b\beta^T(t)\tilde{a}(t) + B_2\tilde{v}(t),\\ \dot{\tilde{a}}(t) &= -L\Lambda\beta(t)Cw,\\ \dot{\tilde{v}}(t) &= \dot{v}^*(t) + \kappa_0\hat{v}(t) + \kappa_0 Cw,\end{aligned} \tag{13}$$

where

$$\bar{A} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & -k_1\theta_1(t)\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 0 & -k_n\theta_n(t)\\ 1 & 0 & 0 & \cdots & 0 & -k_{n+1}\theta_{n+1}(t)\end{pmatrix}, \quad w = \begin{pmatrix} Lz_1\\ \vdots\\ L^n z_n\\ z_{n+1},\end{pmatrix}, \quad b = \begin{pmatrix} b_1\\ \vdots\\ b_n\\ b_{n+1}\end{pmatrix}$$

$$\text{and } \tilde{\phi}(t,\eta) = \begin{pmatrix} \frac{1}{L}(\phi_1(t,\eta_1^l) - \phi_1(t,\hat{\eta}_1^l))\\ \vdots\\ \frac{1}{L^n}(\phi_n(t,\eta_n^l) - \phi_n(t,\hat{\eta}_n^l))\\ 0\end{pmatrix}.$$

The system (13) can be rewritten as

$$
\begin{aligned}
\dot{z}(t) &= LAz(t) + L\chi z_{n+1}(t) + \tilde{\phi}(t,\eta) + b\beta^T(t)\tilde{a}(t) + B_2\tilde{v}(t), \\
\dot{\tilde{a}}(t) &= -L\Lambda\beta(t)Cw, \\
\dot{\tilde{v}}(t) &= \dot{v}^*(t) + \kappa_0\hat{v}(t) + \kappa_0 Cw,
\end{aligned}
\tag{14}
$$

where

$$
A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & -k_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -k_n \\ 1 & 0 & 0 & \cdots & 0 & -k_{n+1} \end{pmatrix} \text{ and } \chi = \begin{pmatrix} (1-\theta_1(t))k_1 \\ (1-\theta_2(t))k_2 \\ \vdots \\ (1-\theta_{n+1}(t))k_{n+1} \end{pmatrix}.
$$

The observer gain vector $k = (k_1, \ \ldots \ , \ k_n, k_{n+1})^T$ is produced by

$$
k = A_3 b + sb,
$$

where

$$
A_3 = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{pmatrix},
$$

and s is a positive real number.

For the matrices A, b, and C given in (14), we have the following result.

Lemma 5. *The matrices A, b, and C given in (14) satisfy the strictly positive real condition. Moreover, there exist two positive definite matrices P and W, a constant real vector J, and a positive real number ϵ such that*

$$
A^T P + PA = -JJ^T - \epsilon W, \tag{15}
$$

and

$$
Pb = C^T. \tag{16}
$$

Proof. Note that

$$
\begin{aligned}
&\lambda^{n+1} + \lambda^{n-1}k_1 + \cdots + \lambda k_{n-1} + k_n + \lambda^n k_{n+1} \\
&\quad = (\lambda + s)(\lambda^n + b_1\lambda^{n-1} + \lambda^{n-2}b_2 + \cdots + \lambda b_{n-1} + b_n).
\end{aligned}
$$

By simple calculation, we have

$$
C(\lambda I - A)^{-1}b = \frac{1}{s+\lambda}.
$$

Therefore,

$$
C(j\omega I - A)^{-1}b = \frac{1}{s + j\omega} = \frac{s}{s^2+\omega^2} - \frac{j\omega}{s^2+\omega^2}.
$$

According to Lemma 2, the triples (A, b, C) are strictly positive real. Therefore, the conclusions (15) and (16) hold, which completes the proof.

Let λ_3 be the minimum eigenvalue of matrix $JJ^T + \epsilon W$, λ_4 and λ_5 be the maximum and minimum eigenvalues of the matrix P, respectively, and λ_6 be the minimum eigenvalue of matrix Λ^{-1}. Then, we have the following result of the UUB nonfragile high-gain observer. ■

Theorem 1. *If the high-gain L satisfies that $L > \max\left\{\frac{8\|P\|\varrho n}{\lambda_3}, \frac{4\|P\|^2}{\lambda_3}, \frac{4\kappa_0^2}{2\lambda_3}\right\}$, the observer gain perturbations $\theta_i(t)$ $(i = 1, \dots, n)$ fall in the interval $\left(1 - \frac{\lambda_3}{8\lambda_4\|k\|\sqrt{n+1}}, 1 + \frac{\lambda_3}{8\lambda_4\|k\|\sqrt{n+1}}\right)$ and $\beta(t)$ in (6) is an uniformly bounded function, then the system (7) is an UUB non-fragile high-gain observer for the system (1).*

Proof. Construct the following Lyapunov function

$$V(t) = V_1(t) + V_2(t),$$

and

$$\begin{aligned} V_1(t) &= \frac{1}{2}\tilde{v}^T(t)\tilde{v}(t), \\ V_2(t) &= z^T(t)Pz(t) + \tilde{a}^T(t)\Lambda^{-1}\tilde{a}(t). \end{aligned}$$

The derivative of $V_1(t)$ is given as follows,

$$\begin{aligned} \dot{V}_1(t) &= \tilde{v}^T(t)(\dot{v}^*(t) + \kappa_0\hat{v}(t) + \kappa_0 z_{n+1}(t)) \\ &= \tilde{v}^T(t)(\dot{v}^*(t) + \kappa_0 v^*(t) - \kappa_0\tilde{v}(t) + \kappa_0 z_{n+1}(t)) \\ &\leq -2\kappa_0 V_1(t) + \tilde{v}^T(t)\dot{v}^*(t) + \kappa_0\tilde{v}^T(t)v^*(t) + \kappa_0\tilde{v}^T(t)z_{n+1}(t) \\ &\leq (-2\kappa_0 + 3)V_1(t) + \frac{\kappa_0^2}{2}\|z(t)\|^2 + c_4, \end{aligned} \tag{17}$$

where $c_4 = \frac{1}{2}\bar{\dot{v}}^{*2} + \frac{\kappa_0^2}{2}\bar{v}^{*2}$.

The derivative of $V_2(t)$ can be calculated as,

$$\begin{aligned} \dot{V}_2(t) &= \dot{z}^T(t)Pz(t) + z^T(t)P\dot{z}(t) + \dot{\tilde{a}}^T(t)\Lambda^{-1}\tilde{a}(t) + \tilde{a}^T(t)\Lambda^{-1}\dot{\tilde{a}}(t) \\ &= Lz^T(t)(A^TP + PA)z(t) + 2Lz^T(t)Pb\beta^T(t)\tilde{a}(t) - 2L\tilde{a}^T(t)\Lambda\Lambda^{-1}\beta(t)Cw + 2z^T(t)P\tilde{\phi}(t,\eta) \\ &\quad + L\chi^TPz(t)z_{n+1}(t) + Lz^T(t)P\chi z_{n+1}(t) + 2z^T(t)PB_2\tilde{v}(t). \end{aligned} \tag{18}$$

Due to

$$\begin{aligned} \|\tilde{\phi}(t)\| &\leq \sqrt{\varrho^2(|z_1(t)|)^2 \cdots + \varrho^2\left(\left|\frac{z_1(t)}{L^{n-1}}\right| + \cdots + |z_n(t)|\right)^2} \\ &\leq \varrho\sqrt{n^2(z_1^2(t) + \cdots + z_n^2(t))} \leq \varrho n\|z(t)\|, \end{aligned} \tag{19}$$

and

$$2z^TPB_2\tilde{v}(t) \leq \|P\|^2\|z(t)\|^2 + 2V_1(t), \tag{20}$$

and

$$\begin{aligned} &L\chi^TPz(t)z_{n+1}(t) + Lz^T(t)P\chi z_{n+1}(t) \\ &\quad\leq 2L(z^T(t)Pz(t))^{\frac{1}{2}}\left(\chi^TP\chi\right)^{\frac{1}{2}}|z_{n+1}(t)| \\ &\quad\leq 2L\lambda_4\|z(t)\|^2\left(\sum_{i=1}^{n+1}(1-\theta_i)^2k_i^2\right)^{\frac{1}{2}} \\ &\quad\leq 2L\lambda_4\sqrt{n+1}\bar{\theta}_{\max}\|k\|\|z(t)\|^2, \end{aligned} \tag{21}$$

where $\bar{\theta}_{\max}$ is the maximum value of $|1-\theta_i|,\ i = 1, \dots, n+1$.

Then, by combining with (15)–(21), we can indicate that

$$\begin{aligned}\dot{V}(t) &\leq -\lambda_3 L\|z(t)\|^2 + \|P\|^2\|z(t)\|^2 + 2\|P\|\varrho n\|z(t)\|^2 + 2L\lambda_4\sqrt{n+1}\overline{\theta}_{\max}\|k\|\|z(t)\|^2 \\ &\quad + \frac{\kappa_0^2}{2}\|z(t)\|^2 + (-2\kappa_0+3)V_1(t) + 2V_1(t) + c_4 \\ &\leq -c_1 L\|z(t)\|^2 + c_4,\end{aligned}$$

where $c_1 = \frac{\lambda_3}{4} - 2\lambda_4\sqrt{n+1}\overline{\theta}_{\max}\|k\| > 0$.

If $c_1 L\|z(t)\|^2 \geq 2c_4$, we can imply

$$\int_{t_0}^{t} \dot{V}(t)dt = V(t) - V(t_0) \leq -c_4(t-t_0).$$

Let $t_1 = t_0 + \frac{V(t_0)}{c_4} - \frac{2\lambda_5}{c_1 L}$. When $t > t_1$, we have

$$V(t) \leq V(t_0) - c_4(t-t_0) \leq \lambda_5\frac{2c_4}{c_1 L}.$$

and

$$\begin{aligned}\|z(t)\| &\leq L^{-\frac{1}{2}}\sqrt{\frac{2c_4}{c_1}}, \\ \|\tilde{a}(t)\| &\leq L^{-\frac{1}{2}}\sqrt{\frac{2c_4\lambda_5}{c_1\lambda_6}}.\end{aligned} \tag{22}$$

From Lemma 4 and (22), when $t > t_1$, we have

$$\begin{aligned}|\overline{x}_j(t) - \hat{\overline{x}}_j(t)| &= |\eta_j(t) - \hat{\eta}_j(t) + \sum_{i=1}^{m}\alpha_j[i]\tilde{a}_i| \\ &\leq L^j\|z(t)\| + L^{j-1}\frac{2m\lambda_1^{\frac{1}{2}}\|\overline{b}\|}{c_3\lambda_2^{\frac{1}{2}}}\varphi_{\max}\|a\| \\ &\leq L^{j-\frac{1}{2}}c_6,\ j = 1, \ldots, n,\end{aligned} \tag{23}$$

where $c_6 = c_4^{\frac{1}{2}}\left(\sqrt{\frac{2}{c_1}} + L^{-1}\frac{2\sqrt{2}m\lambda_1^{\frac{1}{2}}\lambda_5^{\frac{1}{2}}\|\overline{b}\|}{c_3\sqrt{c_1}\lambda_2^{\frac{1}{2}}\lambda_6^{\frac{1}{2}}}\varphi_{\max}\right)$. The proof is completed. ∎

Remark 3. Note that $c_4 = \frac{1}{2}\overline{v}^{*2} + \frac{\kappa_0^2}{2}\overline{v}^{*2}$ is dependent on the optimal approximation error of the RBFNNs. Since RBFNNs can be able to approximate a continuous function with an arbitrary accuracy, thus c_4 can be arbitrarily small. Therefore, the estimation errors of the states can be arbitrarily small as well.

For the second case, from (6), (8), and (12), the dynamical error system is given by,

$$\begin{aligned}\dot{z}(t) &= LAz(t) + L\chi z(t) + \tilde{\phi}(t,\eta) + b\beta^T(t)\tilde{a}(t), \\ \dot{\tilde{a}}(t) &= -L\Lambda\beta(t)Cw.\end{aligned} \tag{24}$$

Next, we give the following result of the nonfragile high-gain observer.

Theorem 2. *If the high-gain L satisfies that $L > \frac{4\|P\|\varrho n}{\lambda_3}$, the observer gain perturbations $\theta_i(t)$ fall in the interval $\left(1 - \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}, 1 + \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}\right)$ and $\beta(t)$ in (6) is an uniformly bounded function, then the system (8) is a nonfragile high-gain observer for the system (1).*

Proof. We focus on the Lyapunov function $V_2(t)$ such as

$$V_2(t) = z^T(t)Pz(t) + \tilde{a}^T(t)\Lambda^{-1}\tilde{a}(t).$$

The derivative of $V_2(t)$ can be expressed as follows,

$$\begin{aligned}\dot{V}_2(t) &\le -\lambda_3 L\|z(t)\|^2 + 2L\lambda_4\sqrt{n}\|k\|\overline{\theta}_{\max}\|z(t)\|^2 + 2\|P\|\varrho n\|z(t)\|^2 \\ &\le -c_2 L\|z(t)\|^2,\end{aligned} \tag{25}$$

where $c_2 = \frac{\lambda_3}{2} - 2\lambda_4\sqrt{n+1}\overline{\theta}_{\max}\|k\| > 0$.

Therefore $\|z(t)\|$ and $\|\tilde{a}(t)\|$ are uniformly bounded for any $t > t_0$. Note that $\|z(t)\|$ and $\|\dot{z}(t)\|$ are uniformly bounded and

$$c_2 L\int_{t_0}^{t} z^T(\tau)z(\tau)d\tau < V_2(t_0) - V_2(\infty) < V_2(\infty).$$

By Barbalat's lemma,[63] we can imply that

$$\lim_{t\to\infty}\|z(t)\| = \lim_{t\to\infty}\|\eta(t)\| = \lim_{t\to\infty}\|x(t)\| = 0, \tag{26}$$

which confirms the correctness of Theorem 2.

If $\beta(t)$ in (6) satisfies the following persistent excitation condition

$$\int_{t}^{t+l_1} \beta(\tau)\beta^T(\tau)d\tau \ge l_2 I, \tag{27}$$

where l_1 and l_2 are two positive reals, then, the result of the non-fragile high-gain observer with identification is given as follows. ∎

Theorem 3. *If the high-gain L satisfies that $L > \frac{4\|P\|\varrho n}{\lambda_3}$, the observer gain perturbations $\theta_i(t)$ fall in the interval $\left(1 - \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}, 1 + \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}\right)$ and $\beta(t)$ in (6) satisfies the following persistent excitation condition (27), then, the system (8) is a non-fragile high-gain observer with identification of the system (1).*

Proof. According to Theorem 2, we have $\|z(t)\|$ and $\|\tilde{a}(t)\|$ are uniformly bounded $\forall t > t_0$.

From (13) and (26), we can also indicate

$$\lim_{t\to\infty}\dot{\tilde{a}}(t) = 0.$$

Then, there is a constant $\bar{a}$ such that $\lim_{t\to\infty}\tilde{a}(t) = \bar{a}$. Thus, for any $r > 0$, there exists $t_2 > 0$ to reveal that

$$\|\tilde{a}(t) - \bar{a}\| < r,\ \ t > t_2. \tag{28}$$

Next, we prove $\bar{a} = 0$ by contradiction. Assume that $\bar{a} \ne 0$.

Consider the following bounded function,

$$\Omega(\tilde{a}(t), t) = \frac{1}{2}\left(\tilde{a}^T(t+l_1)\Lambda^{-1}\tilde{a}(t+l_1) - \tilde{a}^T(t)\Lambda^{-1}\tilde{a}(t)\right).$$

Then,

$$\begin{aligned}\frac{d\Omega(\tilde{a}(t), t)}{dt} &= \tilde{a}^T(t+l_1)\Lambda^{-1}\dot{\tilde{a}}(t+l_1) - \tilde{a}^T(t)\Lambda^{-1}\dot{\tilde{a}}(t) \\ &= \int_t^{t+l_1}\frac{d}{d\tau}\tilde{a}^T(\tau)\Lambda^{-1}\dot{\tilde{a}}(\tau)d\tau \\ &= -L\int_t^{t+l_1}\frac{d}{d\tau}\tilde{a}^T(\tau)\beta(\tau)z_{n+1}(\tau)d\tau \\ &= L\int_t^{t+l_1} z_{n+1}(\tau)\beta^T(\tau)\Lambda\beta(\tau)z_{n+1}(\tau)d\tau - L\int_t^{t+l_1}\tilde{a}^T(\tau)\dot{\beta}(\tau)z_{n+1}(\tau)d\tau \\ &\quad - L\int_t^{t+l_1}\tilde{a}^T(\tau)\beta(\tau)(z_1(\tau) - k_{n+1}z_{n+1}(\tau))d\tau - L\int_t^{t+l_1}\tilde{a}^T(\tau)\beta(\tau)\beta^T(\tau)\tilde{a}(\tau)d\tau.\end{aligned} \tag{29}$$

From (27) and $\lim_{t\to\infty}\|z(t)\| = 0$, when $t > t_2$, there exists a positive constant R satisfying

$$\begin{aligned}\frac{d\Omega(\tilde{a}(t),t)}{dt} &\leq RL\int_t^{t+l_1} z_{n+1}^2(\tau)d\tau - L\int_t^{t+l_1}\bar{a}^T\beta(\tau)\beta^T(\tau)\bar{a}d\tau \\ &\quad - 2L\int_t^{t+l_1}\bar{a}^T\beta(\tau)\beta^T(\tilde{a}(\tau)-\bar{a})d\tau - L\int_t^{t+l_1}(\tilde{a}(\tau)-\bar{a})\beta(\tau)\beta^T(\tau)(\tilde{a}(\tau)-\bar{a})d\tau \\ &\leq RL\int_t^{t+l_1} z_{n+1}^2(\tau)d\tau - \frac{l_2L}{2}\bar{a}^T\bar{a} \\ &\leq -\frac{l_2L}{4}\bar{a}^T\bar{a}. \end{aligned} \tag{30}$$

The inequality (30) contradicts the boundedness of $\Omega(\tilde{a}(t),t)$. Therefore, we can conclude that $\lim_{t\to\infty}\tilde{a}(t) = 0$. The proof is completed. ∎

Remark 4. If $\theta_i(t) \equiv 1$ $(i = 1, \dots, n+1)$, which means that there don't exist observer gain perturbations, our conclusions also hold.

Remark 5. Based on the technology of LMIs, some authors investigated the observer design with non-fragile characteristics.[48-50] However, if the LMIs-based sufficient conditions are infeasible, then, the design method will fail. While, our nonfragile observers is always available.

Remark 6. In References 12-14 and 16, robust high gain observers were studied for nonlinear systems with unknown measurement uncertainties. In order to suppress the effect of the measurement uncertainties further, dynamic high-gain observers were investigated in References 12,17,18. However, larger high-gains may yield the problems, such as oscillations and variations.[19,20] Recently, it is shown that filtering technique is an effective way to resist unknown measurement uncertainty.[14,21-23] Especially in Reference 14, a low-pass filter on the measurement channel was inserted to reduce the sensitivity to measurement noise. For measurement noise such as a finite sum of sinusoids, the estimation errors have the low-pass filtering behavior. However, in all the literature mentioned above, the following problems are neglected: (i) estimation the unknown measurement uncertainties to suppress the effect of the measurement uncertainties and (ii) the effect of observer gain perturbations.

In this paper, for the first time, the definitions of nonfragile high-gain observers are presented for nonlinear systems with unknown measurement uncertainty. After an output filter, an augmented nonlinear system is derived and the output uncertainty is put into the state equation. Then, the measurement uncertainty is approximated by RBFNNs. An adaptive observer with gain perturbations and a disturbance estimator are designed to estimate the states, coefficients of the RBFNNs, and the approximation error, respectively. The estimation errors of the states are UUB and can be arbitrarily small if the RBFNNs have enough approximation accuracy and the observer gains can be varied in a prescribed range. If the measurement uncertainty can be exactly approximated by the RBFNNs and the observer gain perturbations do not exceed the maximum allowable range, then, the estimation errors of the states will converge to origin. Moreover, the estimation errors of the coefficients of the RBFNNs will converge to origin under a persistent excitation condition.

Remark 7. In Reference 64, an observer was designed for a nonlinear system with immeasurable states, multiplicative noises and measurement noise. However, the observer gains are required to guarantee a positive-definite matrix and a small convergence region. Since both the matrix and the convergence region are dependent on the observer gains, it may be difficult to select appropriate design parameters. However, by introducing the output filter in this paper, such a problem can be settled.

4 | FURTHER EXTENSIONS

With regard to the nonlinear systems with unknown parameters in the state equation and uncertainty in the measured output, we can also apply the above observer design strategies to resist the negative influence of the uncertainties. Consider the following classical nonlinear adaptive canonical form with the output disturbance,

$$\dot{x}(t) = A_0x(t) + B_0u(t) + \phi_0(t,x) + \sum_{i=1}^{p} \zeta_i\Gamma_i(t),$$
$$y(t) = C_0x(t) + d(t), \tag{31}$$

where ζ_i are unknown parameters, $\Gamma_i(t) = \begin{pmatrix} \varphi_{i,1}(t) \\ \vdots \\ \varphi_{i,n}(t) \end{pmatrix}$ and $\varphi_{i,j}(t)$ $(i = 1, \dots, p,\ j = 1, \dots, n)$ are continuous functions.

From (2), the system (31) becomes

$$\dot{x}(t) = A_0x(t) + B_0u(t) + \phi_0(t,x) + \sum_{i=1}^{p} \zeta_i\Gamma_i(t),$$
$$y(t) = C_0x(t) + \sum_{i=1}^{m} a_i^*\varphi_i(t) + v^*(t). \tag{32}$$

By inserting the output filter (3), we can obtain the following augmented system,

$$\dot{\bar{x}}(t) = A_1\bar{x}(t) + B_1u(t) + \phi(t,\bar{x}) + \sum_{i=1}^{p+m} \Theta_i(t)\xi_i + B_2v^*(t),$$
$$\bar{y}(t) = \bar{x}_{n+1}(t), \tag{33}$$

where $\xi = \left(\zeta_1, \dots, \zeta_p,\ a_1^*, \dots, a_m^*\right)^T$, $\Theta(t) = \begin{pmatrix} \Gamma_1 & \cdots & \Gamma_p & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \varphi_1(t) & \cdots & \varphi_m(t) \end{pmatrix}$ and $\Theta_i(t)$ is the ith column vector of matrix $\Theta(t)$.

Similar to the filtering transformation (5), we make the following transformation,

$$\eta_j(t) = \bar{x}_j(t) - \sum_{i=1}^{p+m} \delta_j[i]\xi_i, \quad j = 1, \dots, n,$$
$$\eta_{n+1}(t) = \bar{x}_{n+1}(t).$$

where $\dot{\delta}[i] = D\delta[i] + E_1\Theta_i(t)$, $\dot{\delta}[i](0) = 0$, $\delta_j[i]$ is the jth element of the vector $\delta[i]$ and the matrix

$$E_1 = \begin{pmatrix} 1 & 0 & \cdots & 0 & -Lb_1 \\ 0 & 1 & \cdots & 0 & -L^2b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -L^nb_n \end{pmatrix}.$$

Then, we can obtain the augmented adaptive observable canonical form as

$$\dot{\eta}(t) = A_1\eta(t) + B_1u(t) + \phi(t,\eta) + B\Delta^T(t)\xi + B_2v^*(t),$$
$$\gamma(t) = C\eta(t), \tag{34}$$

where

$$\Delta(t) = \begin{pmatrix} \delta_1[1] \\ \vdots \\ \delta_1[p] \\ \delta_1[p+1] + \varphi_1(t) \\ \vdots \\ \delta_1[p+m] + \varphi_m(t) \end{pmatrix}.$$

For the system (34), the UUB nonfragile high-gain observer can be designed as follows,

$$\begin{aligned}
\dot{\hat{\eta}}(t) &= A_1\hat{\eta}(t) + G(\eta_{n+1}(t) - \hat{\eta}_{n+1}(t)) + B_1 u(t) + \phi(t,\hat{\eta}) + B\Delta^T(t)\hat{\xi}(t) + B_2\hat{\nu}(t),\\
\dot{\hat{\xi}}(t) &= L\Lambda\Delta(t)(\gamma(t) - C\hat{\eta}),\\
\dot{\hat{\nu}}(t) &= -\kappa_0\hat{\nu}(t) + \kappa_0(\gamma(t) - C\hat{\eta}),\\
\hat{\bar{x}}_j(t) &= \hat{\eta}_j(t) + \sum_{i=1}^{p+m}\delta_j[i]\hat{\xi}_i,\quad j = 1, \ldots, n,\\
\hat{\bar{x}}_{n+1}(t) &= \hat{\eta}_{n+1}(t).
\end{aligned} \tag{35}$$

Similar to the observer (8), we establish the nonfragile high-gain observer (with identification) as,

$$\begin{aligned}
\dot{\hat{\eta}}(t) &= A_1\hat{\eta}(t) + G(\eta_{n+1}(t) - \hat{\eta}_{n+1}(t)) + B_1 u(t) + \phi(t,\hat{\eta}) + B\Delta^T(t)\hat{\xi}(t),\\
\dot{\hat{\xi}}(t) &= L\Lambda\Delta(t)(\gamma(t) - C\hat{\eta}),\\
\hat{\bar{x}}_j(t) &= \hat{\eta}_j(t) + \sum_{i=1}^{p+m}\delta_j[i]\hat{\xi}_i,\quad j = 1, \ldots, n,\\
\hat{\bar{x}}_{n+1}(t) &= \hat{\eta}_{n+1}(t).
\end{aligned} \tag{36}$$

Based on the results in Section 3, we have following corollaries.

Corollary 1. *If the high-gain L satisfies that $L > \max\left\{\frac{8\|P\|\varrho n}{\lambda_3}, \frac{4\|P\|^2}{\lambda_3}, \frac{4\kappa_{n+1}^2}{2\lambda_3}\right\}$, the observer gain perturbations $\theta_i(t)$ fall in the interval $\left(1 - \frac{\lambda_3}{8\lambda_4\|k\|\sqrt{n+1}}, 1 + \frac{\lambda_3}{8\lambda_4\|k\|\sqrt{n+1}}\right)$ and $\Delta(t)$ in (34) is an uniformly bounded function, then the system (35) is an UUB non-fragile high-gain observer for the system (31).*

Corollary 2. *If the high-gain L satisfies that $L > \frac{4\|P\|\varrho n}{\lambda_3}$, the observer gain perturbations $\theta_i(t)$ fall in the interval $\left(1 - \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}, 1 + \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}\right)$ and $\Delta(t)$ in (34) is an uniformly bounded function, then the system (36) is a non-fragile high-gain observer for the nonlinear system (31).*

If $\Delta(t)$ in (34) satisfies the persistent excitation condition

$$\int_t^{t+l_3}\Delta(\tau)\Delta^T(\tau)d\tau \geq l_4 I, \tag{37}$$

where l_3 and l_4 are two positive reals, then we can obtain the following corollary.

Corollary 3. *If the high-gain L satisfies that $L > \frac{4\|P\|\varrho n}{\lambda_3}$, the observer gain perturbations $\theta_i(t)$ fall in the interval $\left(1 - \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}, 1 + \frac{\lambda_3}{4\lambda_4\|k\|\sqrt{n+1}}\right)$ and $\Delta(t)$ in (34) satisfies the persistent excitation condition (37), then, the system (36) is a nonfragile high-gain observer with identification of the nonlinear system (31).*

The proofs of Corollary 1, Corollary 2, and Corollary 3 are similar to those of Theorem 1, Theorem 2, and Theorem 3, and omitted.

5 | SIMULATIONS

In this section, some numerical simulations are presented to illustrate the correctness and validity of our observer design strategies.

Example 1. Consider the following one-link manipulator system shown in Figure 1. The dynamic equation of such a system is described by[65-67]

$$\begin{cases} D\ddot{q} + B\dot{q} + N\sin q = \tau + \tau_d,\\ M\dot{\tau} + H\tau = u - K_m\dot{q}, \end{cases} \tag{38}$$

where $q, \dot{q}, \ddot{q}$ are the link position, velocity and acceleration, respectively. The torque τ is produced by the electrical system and τ_d denotes the torque disturbance. u is the system input. Set the mechanical inertia $D = 1$ kg m^2, the coefficient of viscous friction at the joint $B = 1$ Nm s/rad, $N = 10$ as the positive constant related to the coefficient of gravity and the mass of the load, the armature inductance $M = 0.1\ H$, the armature resistance $H = 1\ \Omega$ and the back electromotive force coefficient $K_m = 0.2\ Nm/A$.

By letting $x_1(t) = q$, $x_2(t) = \dot{q}$, $x_3(t) = \tau$ and $y = x_1(t) + d(t)$, the one-link manipulator system (38) can be represented as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\frac{B}{D}x_2(t) - \frac{N}{D}\sin x_1 + \frac{1}{D}(x_3(t) + \tau_d(t)), \\ \dot{x}_3(t) = -\frac{K_m}{M}x_2(t) - \frac{H}{M}x_3(t) + \frac{u}{M}, \\ y(t) = x_1(t) + d(t) \end{cases} \tag{39}$$

where $u = 5\cos 0.5t$.

For the purpose of comparison with Reference 14, we set the torque disturbance $\tau_d = 0$ and the observer gain perturbations $\theta(t) = (1,\ 1,\ 1,\ 1)^T$ in this scenario. Make an output filtering action and introduce RBFNNs to approximate the disturbance $d(t)$. Then, the following augmented system can be derived,

$$\begin{cases} \dot{\bar{x}}_1(t) = \bar{x}_2(t), \\ \dot{\bar{x}}_2(t) = \bar{x}_3(t) - \bar{x}_2(t) - 10\sin \bar{x}_1(t), \\ \dot{\bar{x}}_3(t) = 10u(t) - 2\bar{x}_2(t) - 10\bar{x}_3(t), \\ \dot{\bar{x}}_4(t) = \bar{x}_1(t) - Lk_3\bar{x}_4(t) + \sum_{i=1}^{4} a_i^*\varphi_i(t) + v^*(t), \\ \bar{y}(t) = \bar{x}_4(t). \end{cases}$$

Further, make an input–output filtering transformation. Thus,

$$\begin{cases} \dot{\eta}_1(t) = \eta_2(t) + Lb_1(a_1^*\alpha_1[1] + a_1^*\varphi_1(t) + a_2^*\alpha_1[2] + a_2^*\varphi_2(t) + a_3^*\alpha_1[3] + a_3^*\varphi_3(t) + a_4^*\alpha_1[4] + a_4^*\varphi_4(t)), \\ \dot{\eta}_2(t) = \eta_3(t) - \eta_2(t) - 10\sin \eta_1(t) + L^2b_2(a_1^*\alpha_1[1] + a_1^*\varphi_1(t) + a_2^*\alpha_1[2] + a_2^*\varphi_2(t) + a_3^*\alpha_1[3] \\ + a_3^*\varphi_3(t) + a_4^*\alpha_1[4] + a_4^*\varphi_4(t)), \\ \dot{\eta}_3(t) = 10u(t) - 2\eta_2(t) - 10\eta_3(t) + L^3b_3(a_1^*\alpha_1[1] + a_1^*\varphi_1(t) + a_2^*\alpha_1[2] + a_2^*\varphi_2(t) + a_3^*\alpha_1[3] \\ + a_3^*\varphi_3(t) + a_4^*\alpha_1[4] + a_4^*\varphi_4(t)), \\ \dot{\eta}_4(t) = \eta_1(t) - Lk_4\eta_4 + (a_1^*\alpha_1[1] + a_1^*\varphi_1(t) + a_2^*\alpha_1[2] + a_2^*\varphi_2(t) + a_3^*\alpha_1[3] + a_3^*\varphi_3(t) \\ + a_4^*\alpha_1[4] + a_4^*\varphi_4(t)) + v^*(t), \\ \gamma(t) = \eta_4(t), \end{cases}$$

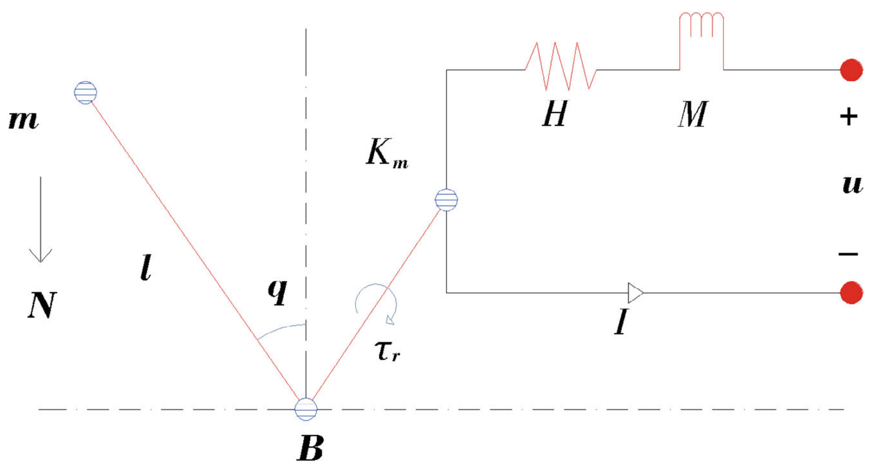

FIGURE 1 The model of one-link manipulator.

where $\alpha[i]$ are produced by

$$\begin{cases}\dot{\alpha}_1[1] = -Lb_1\alpha_1[1] + \alpha_2[1] - Lb_1\varphi_1(t),\\ \dot{\alpha}_2[1] = -L^2b_2\alpha_1[1] + \alpha_3[1] - L^2b_2\varphi_1(t),\\ \dot{\alpha}_3[1] = -L^3b_3\alpha_1[1] - L^3b_3\varphi_1(t),\\ \dot{\alpha}_1[2] = -Lb_1\alpha_1[2] + \alpha_2[2] - Lb_1\varphi_2(t),\\ \dot{\alpha}_2[2] = -L^2b_2\alpha_1[2] + \alpha_3[2] - L^2b_2\varphi_2(t),\\ \dot{\alpha}_3[2] = -L^3b_3\alpha_1[2] - L^3b_3\varphi_2(t),\\ \dot{\alpha}_1[3] = -Lb_1\alpha_1[3] + \alpha_2[3] - Lb_1\varphi_3(t),\\ \dot{\alpha}_2[3] = -L^2b_2\alpha_1[3] + \alpha_3[3] - L^2b_2\varphi_3(t),\\ \dot{\alpha}_3[3] = -L^3b_3\alpha_1[3] - L^3b_3\varphi_3(t),\\ \dot{\alpha}_1[4] = -Lb_1\alpha_1[4] + \alpha_2[4] - Lb_1\varphi_4(t),\\ \dot{\alpha}_2[4] = -L^2b_2\alpha_1[4] + \alpha_3[4] - L^2b_2\varphi_4(t),\\ \dot{\alpha}_3[4] = -L^3b_3\alpha_1[4] - L^3b_3\varphi_4(t).\end{cases}$$

Then, we design the UUB nonfragile high-gain observer as follows

$$\begin{cases}\dot{\hat{\eta}}_1(t) = \hat{\eta}_2(t) + L^2k_1\theta_1(t)(\eta_4(t) - \hat{\eta}_4(t)) + Lb_1(\hat{a}_1\alpha_1[1] + \hat{a}_1\varphi_1(t) + \hat{a}_2\alpha_1[2] + \hat{a}_2\varphi_2(t)\\ \quad + \hat{a}_3\alpha_1[3] + \hat{a}_3\varphi_3(t) + \hat{a}_4\alpha_1[4] + \hat{a}_4\varphi_4(t)),\\ \dot{\hat{\eta}}_2(t) = \hat{\eta}_3(t) - \hat{\eta}_2(t) - 10\sin\hat{\eta}_1(t) + L^3k_2\theta_2(t)(\eta_4(t) - \hat{\eta}_4(t)) + L^2b_2(\hat{a}_1\alpha_1[1]\\ \quad + \hat{a}_1\varphi_1(t) + \hat{a}_2\alpha_1[2] + \hat{a}_2\varphi_2(t) + \hat{a}_3\alpha_1[3] + \hat{a}_3\varphi_3(t) + \hat{a}_4\alpha_1[4] + \hat{a}_4\varphi_4(t)),\\ \dot{\hat{\eta}}_3(t) = 10u(t) - 2\hat{\eta}_2(t) - 10\hat{\eta}_3(t) + L^3k_3\theta_3(t)(\eta_4(t) - \hat{\eta}_4(t)) + L^3b_3(\hat{a}_1\alpha_1[1] + \hat{a}_1\varphi_1(t)\\ \quad + \hat{a}_2\alpha_1[2] + \hat{a}_2\varphi_2(t) + \hat{a}_3\alpha_1[3] + \hat{a}_3\varphi_3(t) + \hat{a}_4\alpha_1[4] + \hat{a}_4\varphi_4(t)),\\ \dot{\hat{\eta}}_4(t) = \hat{\eta}_1(t) - Lk_4\theta_4(t)\hat{\eta}_4 + (\hat{a}_1\alpha_1[1] + \hat{a}_1\varphi_1(t) + \hat{a}_2\alpha_1[2] + \hat{a}_2\varphi_2(t)\\ \quad + \hat{a}_3\alpha_1[3] + \hat{a}_3\varphi_3(t) + \hat{a}_4\alpha_1[4] + \hat{a}_4\varphi_4(t)) + \hat{v}(t),\\ \dot{\hat{a}}_1(t) = 50000L(\alpha_1[1] + \varphi_1(t))(\eta_4(t) - \hat{\eta}_4(t)),\\ \dot{\hat{a}}_2(t) = 50000L(\alpha_1[2] + \varphi_2(t))(\eta_4(t) - \hat{\eta}_4(t)),\\ \dot{\hat{a}}_3(t) = 50000L(\alpha_1[3] + \varphi_3(t))(\eta_4(t) - \hat{\eta}_4(t)),\\ \dot{\hat{a}}_4(t) = 50000L(\alpha_1[4] + \varphi_4(t))(\eta_4(t) - \hat{\eta}_4(t)),\\ \dot{\hat{v}}(t) = -\kappa_0\hat{v}(t) + \kappa_0(\eta_4(t) - \hat{\eta}_4(t)),\\ \hat{\hat{x}}_1(t) = \hat{\eta}_1(t) + \hat{a}_1\alpha_1[1] + \hat{a}_2\alpha_1[2] + \hat{a}_3\alpha_1[3] + \hat{a}_4\alpha_1[4],\\ \hat{\hat{x}}_2(t) = \hat{\eta}_2(t) + \hat{a}_1\alpha_2[1] + \hat{a}_2\alpha_2[2] + \hat{a}_3\alpha_2[3] + \hat{a}_4\alpha_2[4],\\ \hat{\hat{x}}_3(t) = \hat{\eta}_3(t) + \hat{a}_1\alpha_3[1] + \hat{a}_2\alpha_3[2] + \hat{a}_3\alpha_3[3] + \hat{a}_4\alpha_3[4],\\ \hat{\hat{x}}_4(t) = \hat{\eta}_4(t).\end{cases}$$

In Reference 14, a finite sum of sinusoids was applied to model the measurement noise and demonstrated that high-gain observers have the low-pass filtering behavior. Firstly, similar to Reference 14, we set the measurement disturbance $d(t)$ as,

$$d(t) = 3\sin(250t + 1) + 5\sin(433t + 2). \tag{40}$$

The initial conditions are given by $x(0) = (0.1,\ 0.2,\ 0.3,\ 0.4)^T$, $\hat{a}(0) = (1.5,\ 2.5,\ 1,\ 2)^T$ and $\hat{\eta}(0) = (1.1,\ 1.2,\ 1.3,\ 1.4)^T$. Set $L = 3$, $\kappa_0 = 200$, $b = (1,\ 2,\ 1.5,\ 1)^T$ and $s = 3$. The observer gain can be obtained as $k = (5,\ 7.5,\ 4.5,\ 4)^T$. The basis function vector is give by $\varphi(t) = \left(\frac{1}{1+e^{-h(t)}} + 3.5,\ \frac{1}{1+e^{-h(t)}} - 1.9,\ \frac{1}{1+e^{-h(t)}} - 2.6,\ \frac{1}{1+e^{-h(t)}} + 1.2\right)^T$, where $h(t) = \frac{150(\cos(500t)-5)}{1+80t} + \sin(500t)$. We plot the trajectories of the estimation errors of the UUB high-gain observer and the three degree low-pass filtered observer proposed in Reference 14 in Figures 2, 3, and 4, respectively. It is observed that our observer design methods have superior steady-state performance.

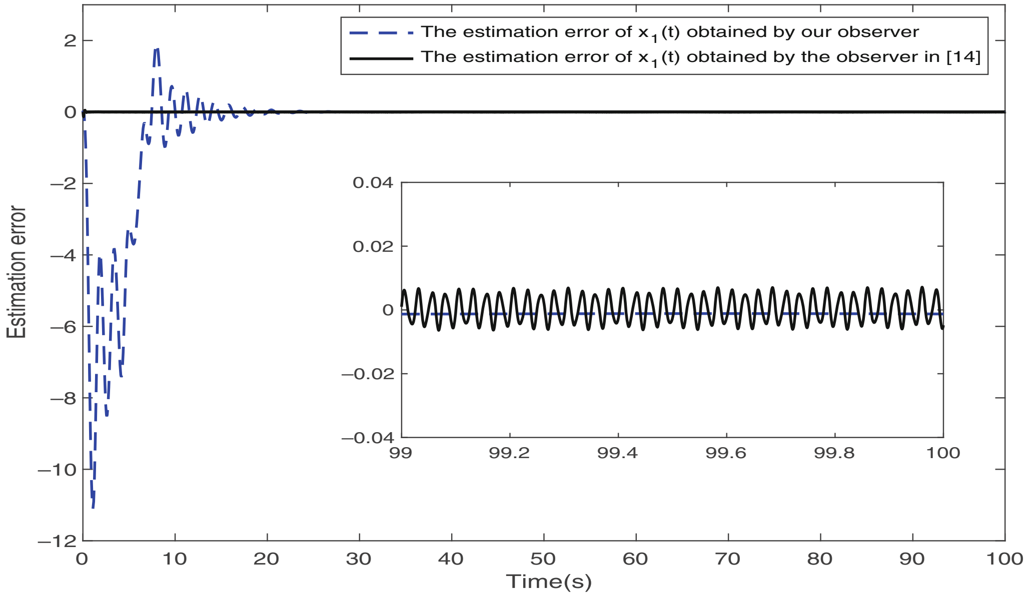

FIGURE 2 The comparison trajectories of the estimation errors of $x_1(t)$ with the disturbance (40).

Secondly, in order to reveal the performance of our proposed observer design further, we set the measurement disturbance as,

$$d(t) = 3\sin(0.2t + 1) + 5\sin(200t + 2), \tag{41}$$

which is a finite sum of both high- and low-frequency sinusoids. Under the same conditions, we plot the estimation error trajectories of the UUB high-gain observer and the three degree low-pass filtered observer[14] in Figures 5,6, and 7, respectively. Our observer design methods also yield more superior steady-state performance than the method in Reference 14.

Example 2. Consider the one-link manipulator system (39) with $d(t) = \sum_{i=1}^{2} a_i\varphi_i(t)$, and the torque disturbance $\tau_d = \sum_{i=1}^{2} \zeta_i\varphi_{i,2}(t)$, where ζ_i are unknown parameters, and $\varphi_{1,2}(t) = \sin(100t)$, $\varphi_{2,2}(t) = \sin(200t)$. By inserting the output filter, the one-link manipulator system (39) can be rewritten as,

$$\begin{cases} \dot{\overline{x}}_1(t) = \overline{x}_2(t), \\ \dot{\overline{x}}_2(t) = \overline{x}_3(t) - \overline{x}_2(t) - 10\sin x_1(t) + \sum_{i=1}^{2} \zeta_i\varphi_{i,2}(t), \\ \dot{\overline{x}}_3(t) = 10u(t) - 2\overline{x}_2(t) - 10\overline{x}_3(t), \\ \dot{\overline{x}}_4(t) = \overline{x}_1(t) - Lk_3\overline{x}_4(t) + \sum_{i=1}^{2} a_i\varphi_i(t), \\ \overline{y}(t) = \overline{x}_4(t). \end{cases}$$

Let $\xi = (\zeta_1,\ \zeta_2,\ a_1,\ a_2)^T$. After the input–output filtering transformation, we have the following augmented adaptive observable canonical form

$$\begin{cases} \dot{\eta}_1(t) = \eta_2(t) + Lb_1(\xi_1\delta_1[1] + \xi_2\delta_1[2] + \xi_3\delta_1[3] + \xi_3\varphi_1(t) + \xi_4\delta_1[4] + \xi_4\varphi_2(t)), \\ \dot{\eta}_2(t) = \eta_3(t) - \eta_2(t) - 10\sin\eta_1(t) + L^2b_2(\xi_1\delta_1[1] + \xi_2\delta_1[2] + \xi_3\delta_1[3] \\ + \xi_3\varphi_1(t) + \xi_4\delta_1[4] + \xi_4\varphi_2(t)), \\ \dot{\eta}_3(t) = 10u(t) - 2\eta_2(t) - 10\eta_3(t) + L^3b_3(\xi_1\delta_1[1] + \xi_2\delta_1[2] + \xi_3\delta_1[3] \\ + \xi_3\varphi_1(t) + \xi_4\delta_1[4] + \xi_4\varphi_2(t)), \\ \dot{\eta}_4(t) = \eta_1(t) - Lk_4\eta_4 + (\xi_1\delta_1[1] + \xi_2\delta_1[2] + \xi_3\delta_1[3] + \xi_3\varphi_1(t) + \xi_4\delta_1[4] + \xi_4\varphi_2(t)), \\ \gamma(t) = \eta_4(t), \end{cases}$$

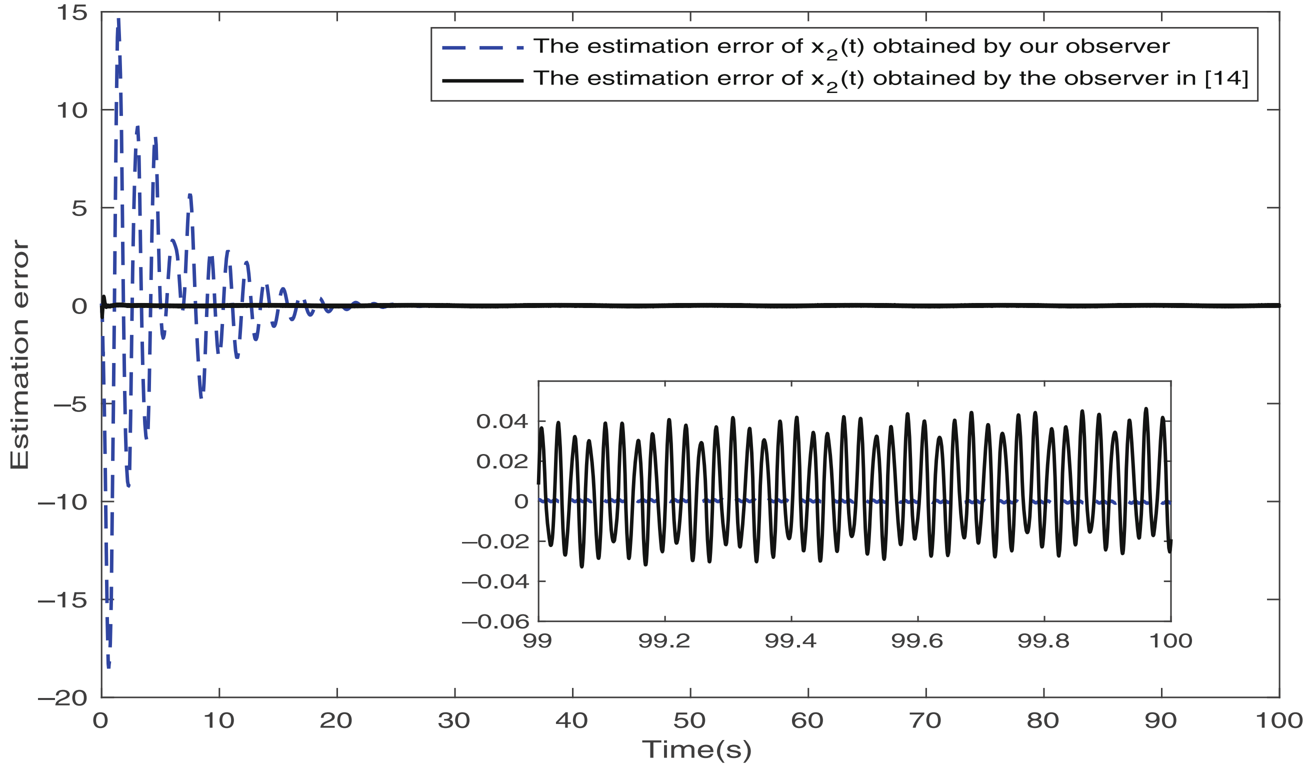

FIGURE 3 The comparison trajectories of the estimation errors of $x_2(t)$ with the disturbance (40).

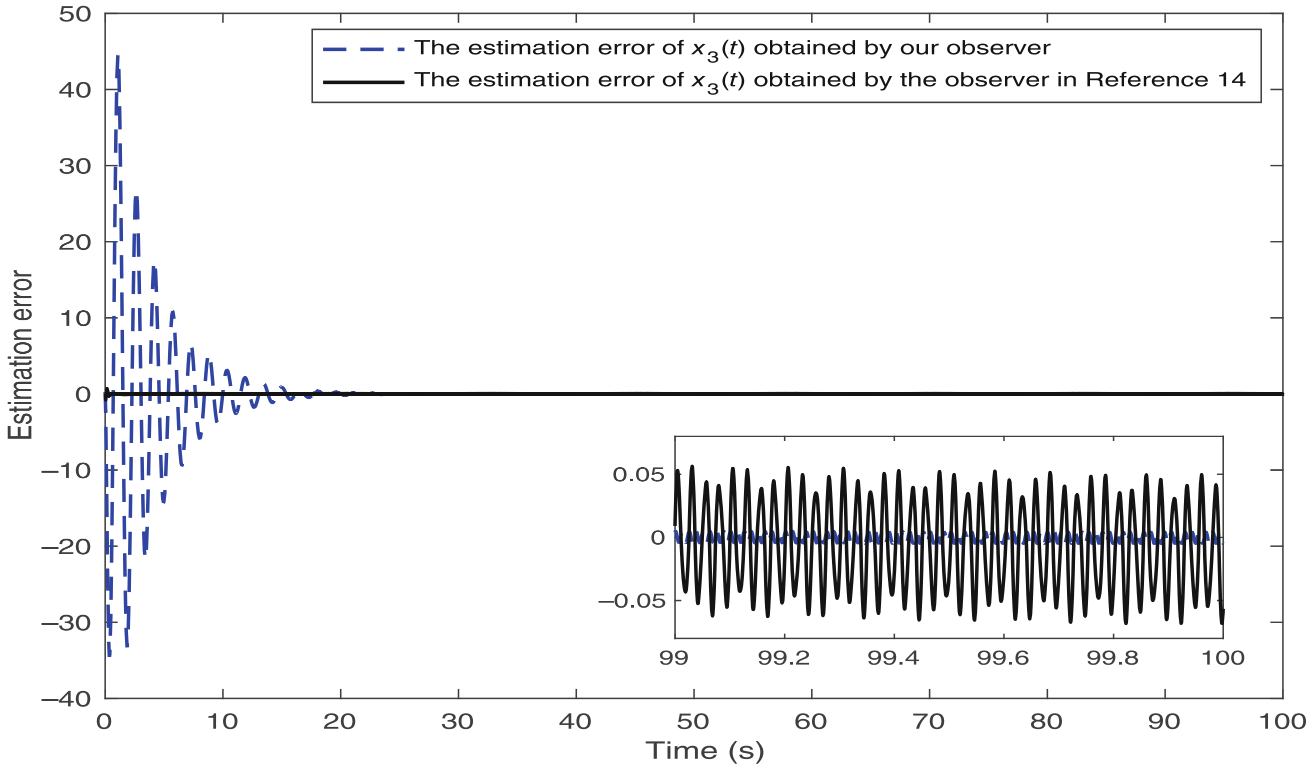

FIGURE 4 The comparison trajectories of the estimation errors of $x_3(t)$ with the disturbance (40).

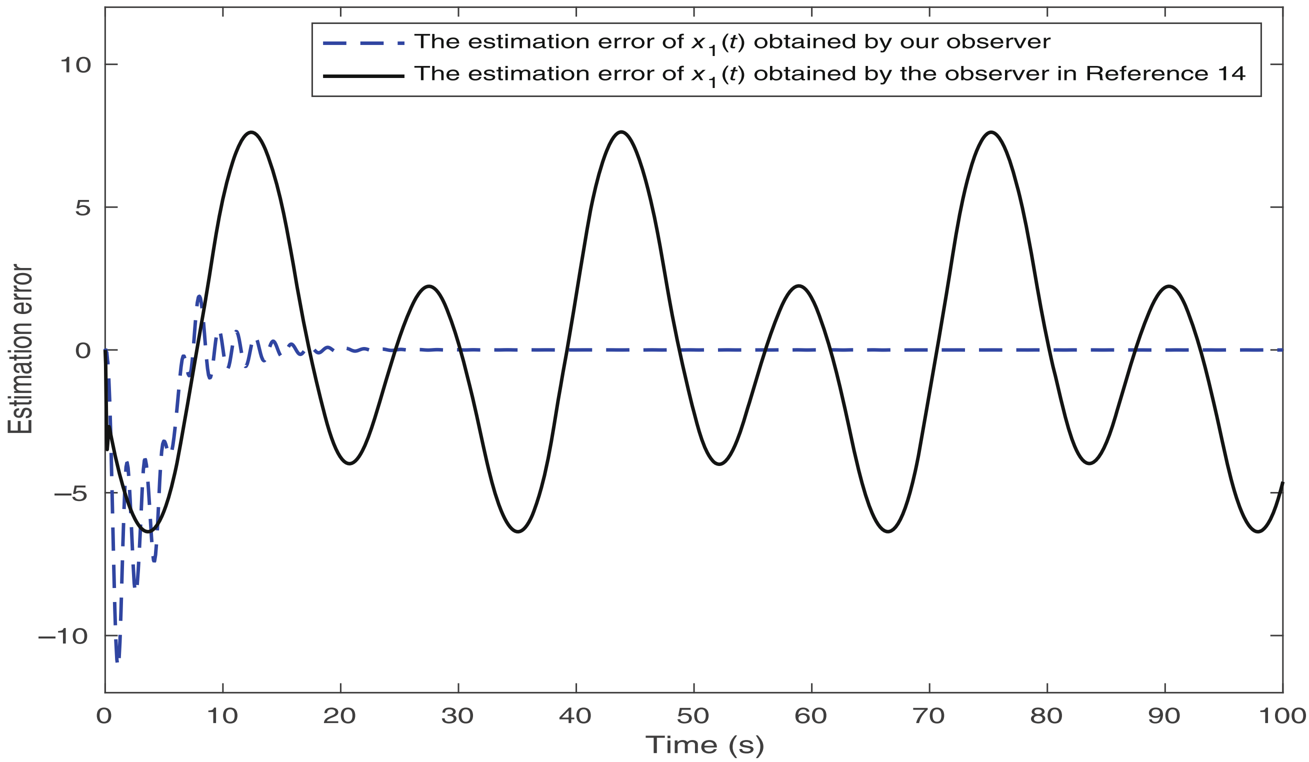

FIGURE 5 The comparison trajectories of the estimation errors of $x_1(t)$ with the disturbance (41).

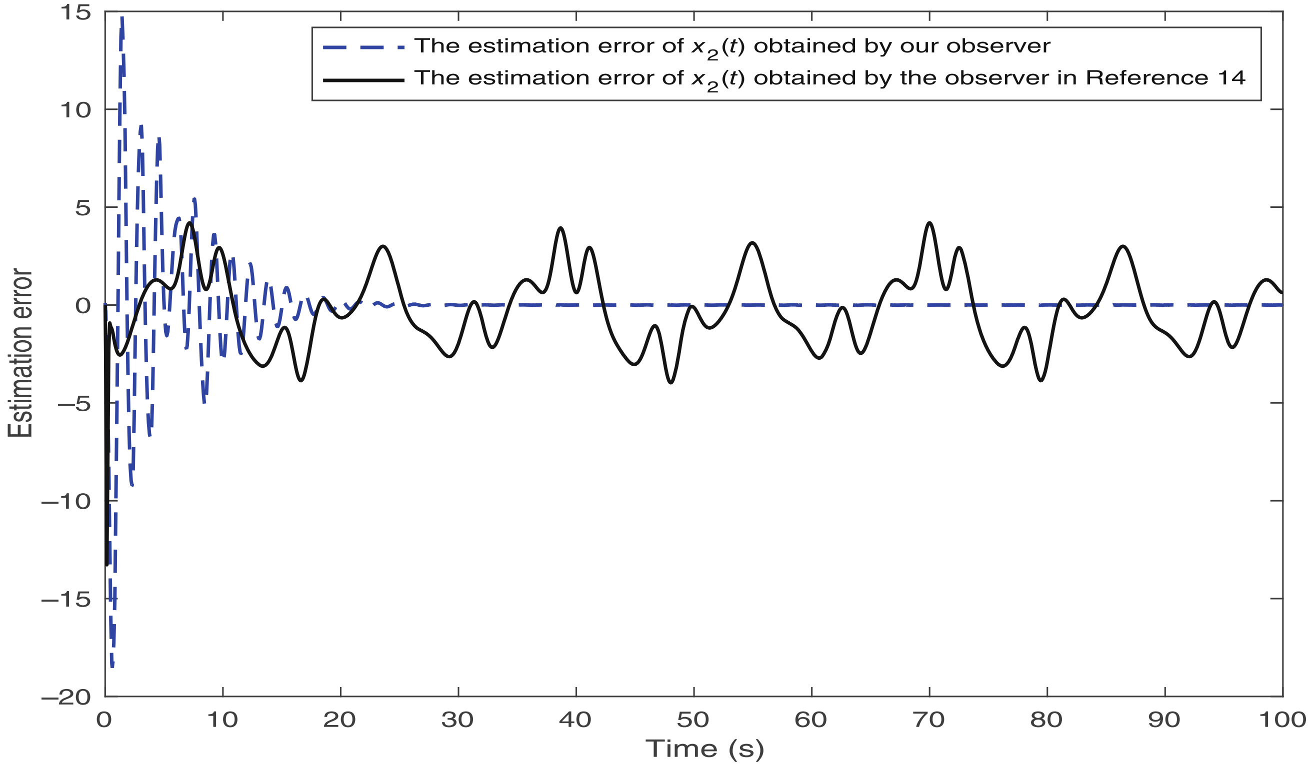

FIGURE 6 The comparison trajectories of the estimation errors of $x_2(t)$ with the disturbance (41).

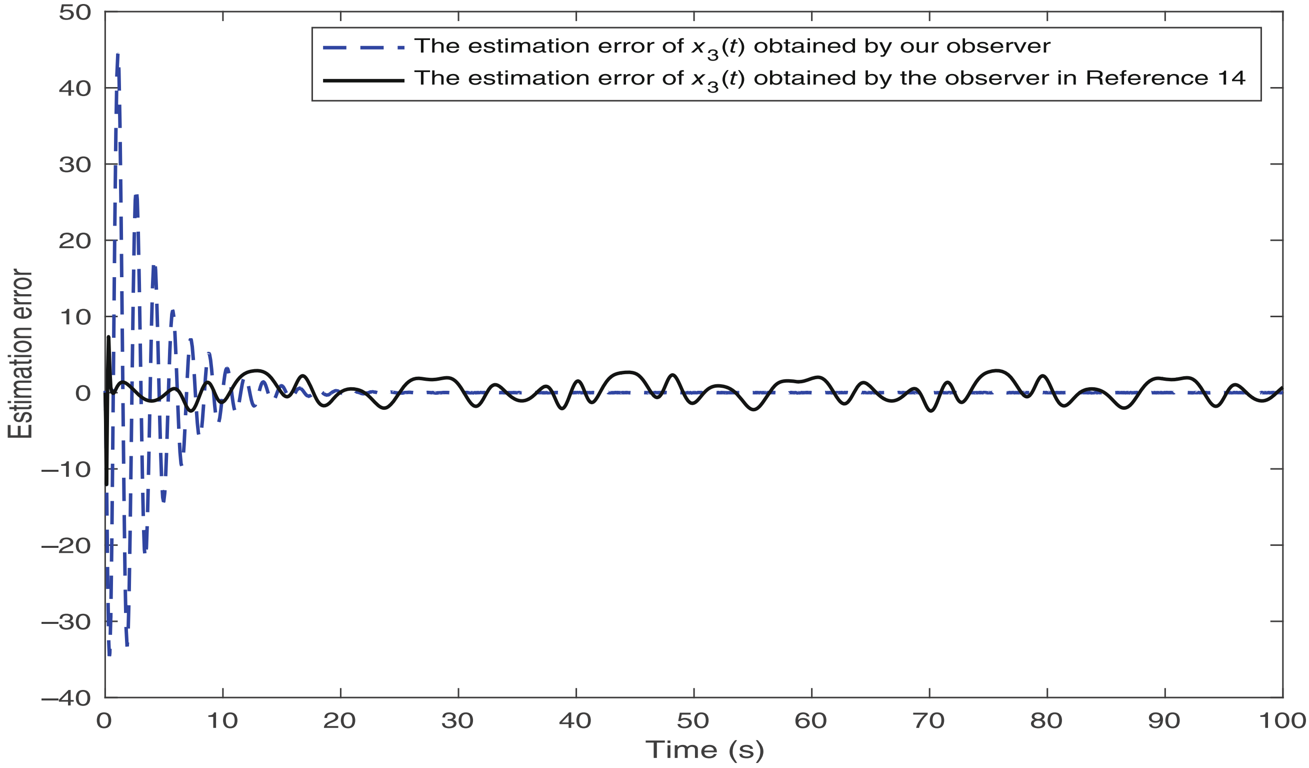

FIGURE 7 The comparison trajectories of the estimation errors of $x_3(t)$ with the disturbance (41).

where $\delta_j[i]$ are produced by

$$\begin{cases} \dot{\delta}_1[1] = -Lb_1\delta_1[1] + \delta_2[1], \\ \dot{\delta}_2[1] = -L^2b_2\delta_1[1] + \delta_3[1] + \varphi_{1,2}(t), \\ \dot{\delta}_3[1] = -L^3b_3\delta_1[1], \\ \dot{\delta}_1[2] = -Lb_1\delta_1[2] + \delta_2[2], \\ \dot{\delta}_2[2] = -L^2b_2\delta_1[2] + \delta_3[2] + \varphi_{2,2}(t), \\ \dot{\delta}_3[2] = -L^3b_3\delta_1[2], \\ \dot{\delta}_1[3] = -Lb_1\delta_1[3] + \delta_2[3] - Lb_1\varphi_1(t), \\ \dot{\delta}_2[3] = -L^2b_2\delta_1[3] + \delta_3[3] - L^2b_2\varphi_1(t), \\ \dot{\delta}_3[3] = -L^3b_3\delta_1[3] - L^3b_3\varphi_1(t), \\ \dot{\delta}_1[4] = -Lb_1\delta_1[4] + \delta_2[4] - Lb_1\varphi_2(t), \\ \dot{\delta}_2[4] = -L^2b_2\delta_1[4] + \delta_3[4] - L^2b_2\varphi_2(t), \\ \dot{\delta}_3[4] = -L^3b_3\delta_1[4] - L^3b_3\varphi_2(t). \end{cases}$$

Therefore, we can establish the nonfragile high-gain observer with identification as follows,

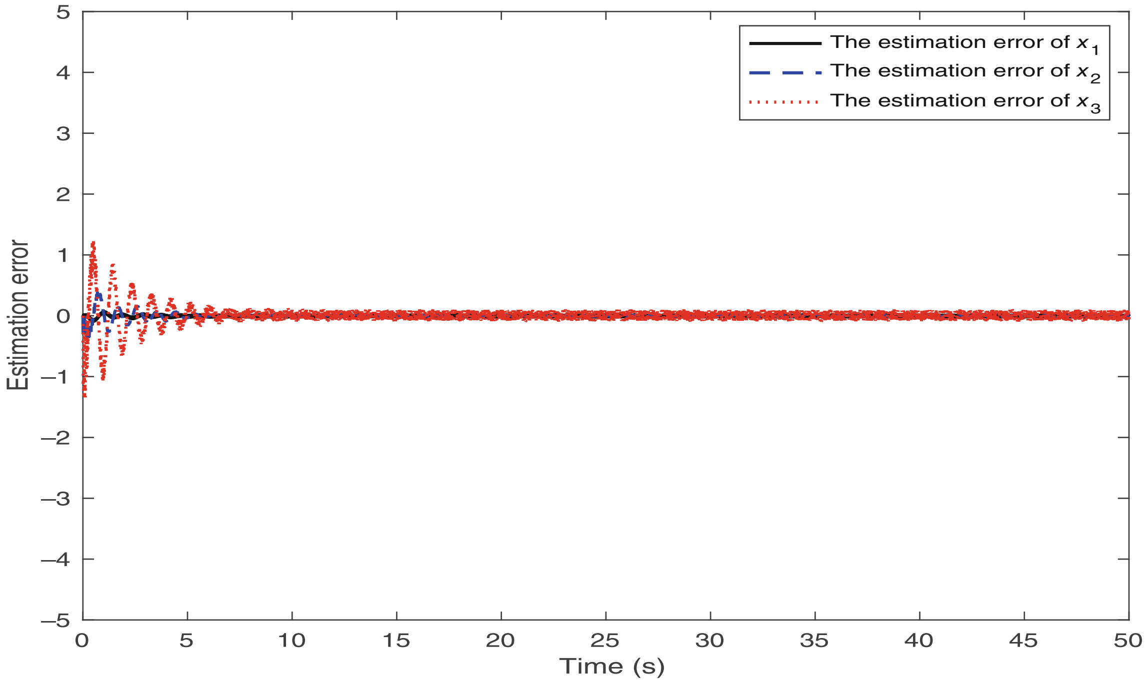

FIGURE 8 The trajectories of the estimation errors of the nonfragile high-gain observer.

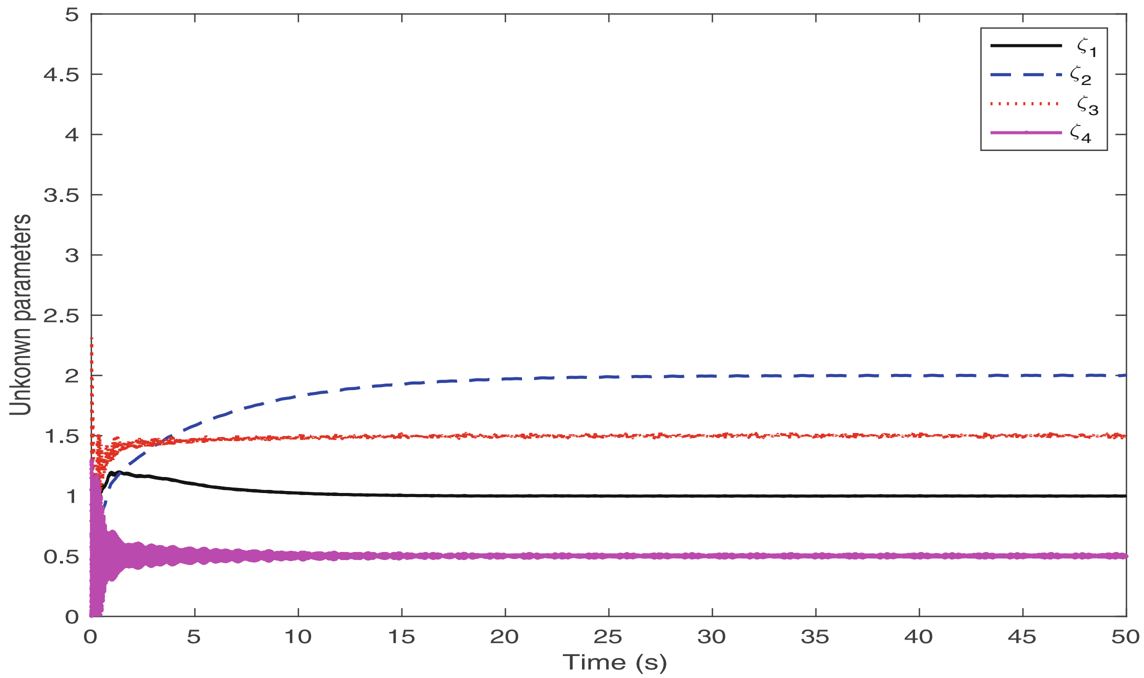

FIGURE 9 The trajectories of the unknown parameters identification of the nonfragile high-gain observer.

$$
\begin{cases}
\dot{\hat{\eta}}_1(t) = \hat{\eta}_2(t) + L^2 k_1 \theta_1(t)(\eta_4(t) - \hat{\eta}_4(t)) + Lb_1(\hat{\xi}_1\delta_1[1] + \hat{\xi}_2\delta_1[2] + \hat{\xi}_3\delta_1[3] + \hat{\xi}_3\varphi_1(t) \\
+ \hat{\xi}_4\delta_1[4] + \hat{\xi}_4\varphi_2(t)), \\
\dot{\hat{\eta}}_2(t) = \hat{\eta}_3(t) - \hat{\eta}_2(t) + 10\sin\hat{\eta}_1(t) + L^3 k_2\theta_2(t)(\eta_4(t) - \hat{\eta}_4(t)) + Lb_1(\hat{\xi}_1\delta_1[1] \\
+ \hat{\xi}_2\delta_1[2] + \hat{\xi}_3\delta_1[3] + \hat{\xi}_3\varphi_1(t) + \hat{\xi}_4\delta_1[4] + \hat{\xi}_4\varphi_2(t)), \\
\dot{\hat{\eta}}_3(t) = 10u(t) - 2\hat{\eta}_2(t) - 10\hat{\eta}_3(t) + L^4 k_3\theta_3(t)(\eta_4(t) - \hat{\eta}_4(t)) + L^3 b_3(\hat{\xi}_1\delta_1[1] \\
+ \hat{\xi}_2\delta_1[2] + \hat{\xi}_2\varphi_1(t) + \hat{\xi}_3\delta_1[3] + \hat{\xi}_3\varphi_2(t)), \\
\dot{\hat{\eta}}_4(t) = \hat{\eta}_1(t) - Lk_4\theta_4(t)\hat{\eta}_4 + (\eta_4(t) - \hat{\eta}_4(t)) + Lb_1(\hat{\xi}_1\delta_1[1] + \hat{\xi}_2\delta_1[2] + \hat{\xi}_3\delta_1[3] \\
+ \hat{\xi}_3\varphi_1(t) + \hat{\xi}_4\delta_1[4] + \hat{\xi}_4\varphi_2(t)), \\
\dot{\hat{\xi}}_1(t) = 50{,}000L(\delta_1[1])(\eta_4(t) - \hat{\eta}_4(t)), \\
\dot{\hat{\xi}}_2(t) = 50{,}000L(\delta_1[2])(\eta_4(t) - \hat{\eta}_4(t)), \\
\dot{\hat{\xi}}_3(t) = 10{,}000L(\delta_1[3] + \varphi_1(t))(\eta_4(t) - \hat{\eta}_4(t)), \\
\dot{\hat{\xi}}_4(t) = 10{,}000L(\delta_1[4] + \varphi_2(t))(\eta_4(t) - \hat{\eta}_4(t)), \\
\hat{\hat{x}}_1(t) = \hat{\eta}_1(t) + \hat{\xi}_1\delta_1[1] + \hat{\xi}_2\delta_1[2] + \hat{\xi}_3\delta_1[3] + \hat{\xi}_4\delta_1[4], \\
\hat{\hat{x}}_2(t) = \hat{\eta}_2(t) + \hat{\xi}_1\delta_2[1] + \hat{\xi}_2\delta_2[2] + \hat{\xi}_3\delta_2[3] + \hat{\xi}_4\delta_2[4], \\
\hat{\hat{x}}_3(t) = \hat{\eta}_3(t) + \hat{\xi}_1\delta_3[1] + \hat{\xi}_2\delta_3[2] + \hat{\xi}_3\delta_3[3] + \hat{\xi}_4\delta_3[4], \\
\hat{\hat{x}}_4(t) = \hat{\eta}_4(t).
\end{cases}
$$

Set $\varphi_1(t) = \sin(200t)$ and $\varphi_2(t) = \sin(300t)$, $\xi = (1,\ 2,\ 1.5,\ 0.5)^T$ and the observer gain perturbations $\theta(t) = (1 + 0.5\sin(t),\ 1 - 0.2\sin(t),\ 1 + 0.2\sin(t),\ 1 + 0.01\sin(t))^T$. The rest parameters are same to those in Example 1. In Figure 8, we can plot the trajectories of the estimation errors. The estimation errors of the unknown parameter are also shown in Figure 9. The simulation results demonstrate the validity and efficiency of our methods.

6 | CONCLUSION

For the first time, we presented the definitions of nonfragile high-gain observers and design method for lower-triangular nonlinear systems with output uncertainty. The output uncertainty was approximated by RBFNNs. After an output filter and an input–output filter, a new augmented adaptive observable canonical form was derived. Then, we designed an observer with gain perturbations to estimate the states and the coefficients of the RBFNNs. Besides, a disturbance observer was also designed to estimate the approximation error. We gave the maximum allowable gain perturbations as well. At last, the obtained results were extended to nonlinear systems in adaptive observer form with output uncertainty.

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CONFLICT OF INTEREST STATEMENT

The authors declared that they have no conflicts of interest to this work.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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