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A diversity of patterns to new (3 + 1)-dimensional Hirota bilinear equation that models dynamics of waves in fluids

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ABSTRACT

This article discusses the behavior of specific dispersive waves to new (3+1)-dimensional Hirota bilinear equation (3D-HBE). The 3D-HBE is used as a governing equation for the propagation of waves in fluid dynamics. The Hirota bilinear method (HBM) is successfully applied together with various test strategies for securing a class of results in the forms of lump-periodic, breather-type, and two-wave solutions. Solitons for nonlinear partial differential equations (NLPDEs) can be identified via the well-known mathematical methodology known as the Hirota method. However, this requires for bilinearization of nonlinear PDEs. The method employed provides a comprehensive explanation of NLPDEs by extracting and also generating innovative exact solutions by merging the outcomes of various procedures. To further illustrate the impact of the parameters, we also include a few numerical visualizations of the results. These findings validate the usefulness of the used method in improving the nonlinear dynamical behavior of selected systems. These results are used to illustrate the physical properties of lump solutions and the collision-related components of various nonlinear physical processes. The outcomes demonstrate the efficiency, rapidity, simplicity, and adaptability of the applied algorithm.

Introduction

The motion of fluid below a pressure surface is described by a set of hyperbolic partial differential equations called the shallow-water equations. Saint-Venant equations are a special case of transverse-form shallow-water equations that arise when the diagonal length scale is much larger than the vertical length scale. The Navier–Stokes equations are converted into these equations by integrating them in depth. For this reason, the principle of conservation of mass dictates that the fluid's vertical velocity scale is smaller than its horizontal velocity scale [1]. As the mobility equation demonstrates, the transverse velocity field is constant throughout the depth of the fluid, with vertical pressure gradients being almost hydrodynamic and horizontal pressure gradients created by the displacement of the pressure surface. By combining, we may ignore the vertical velocity and get more accurate results. Because of this, the equations for shallow water are formulated. The shallow-water equations are applicable in many situations of fluid dynamics where the transverse length scale is much bigger than the vertical length scale. They are used in combination with Coriolis forces to modify the continuity equation of airflow in oceanic and atmospheric modeling. Shallow-water equation models cannot directly account for variables that vary with depth because there is just one vertical level. In cases where the mean state is exceptionally straightforward, one can use distinct sets of shallow-water equations to characterize the vertical and horizontal fluctuations explicitly [2].

Moreover, there has been a resurgence of interest in classical physics in the latter half of the twentieth century. When nonlinear effects

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in dynamical equations became apparent, scientists took notice [3–5]. Two unique manifestations of this nonlinearity were found: chaos, the apparent randomness in the behavior of absolutely deterministic systems, and solitons, confined, stable moving particles that scatter elastically. Both of these areas have advanced to the point that they can be considered paradigms, having robust mathematical underpinnings and numerous physical discoveries and applications. The chaotic behavior of nature is fascinating, and many scientists see it as the key to unlocking the mysteries of the universe [6–10]. Researching various types of NLPDE is crucial for mathematically characterizing complex systems with time-varying characteristics [11,12]. Simplifying complex physical processes down to NLPDEs has been an objective of researchers [13,14].

Investigating the solutions and properties of nonlinear PDEs helps one gain a more complete understanding of the structure they explain. There are many potential responses, including ones involving solitary waves [15,16]. Solitary wave phenomena have connections to the diverse fields of study. As a result, solving the coupled PDEs has become an important goal. Several effective methods such as the truncated Painlevé approach [17], Riccati equation mapping method [18], improved F-expansion function method [19], Lie symmetry technique [20], bifurcation analysis [21], modified simple equation technique [22], iterative transform method [23], new sub equation method and modified Khater''s method [24], new extended direct algebraic method [25], for discussing exact solutions of NLPDEs were devised with the help of symbolic calculations. Regarding fixes, every strategy has its drawbacks and needs to be applied to control models in a certain way.

NLPDEs have the ability to produce a diverse range of outcomes. For non-linear PDEs, lump solutions are extremely common. According to the data, all rational functions in all possible directions comprise lump solutions. lump solutions are crucial for both linear and nonlinear PDEs [26,27]. The solutions of nonlinear interactions can be understood through the use of lump solutions. The existence of lump solutions to integrable equations has been the subject of extensive research. Lump solutions exist for some non-integrable equations. Interactions between lumps have also been shown in numerous studies to yield reliable solutions to nonlinear integrable equations. Moreover, Lump solutions to mathematical equations are crucial for comprehending the qualitative properties of multiple occurrences and processes in numerous branches of natural science. Numerous complex nonlinear phenomena, such as the spatial localization of transfer processes, the existence of peaking regimes, and the multiplicity or absence of stable states under various conditions, are graphically illustrated and explained by lump solutions of nonlinear differential equations. In addition, uncomplicated solutions are frequently used as specific examples illuminating fundamental concepts of a theory that can be explained mathematically in numerous courses. Noting that many equations in physics, chemistry, and biology contain empirical parameters or empirical functions is essential. Exact solutions enable scientists to design and conduct experiments to determine these parameters or functions by simulating natural conditions that are optimal [28–30].

However, the reviewed study shows that no HBM has been utilized to the analyzed equation. So, our focus is on developing a wide range of solutions by employing this holistic tactic. The article is arranged as follows: the studied model with its bäcklund transformation, Hirota bilinear form and solutions in Section "Extraction of solutions", while conclusion is presented in Section "Concluding remarks".

Extraction of solutions

Waves have long been recognized for their importance in the natural world. Mechanical waves, including earthquakes, sound waves traveling through the air, and ocean waves, have existed alongside electromagnetic and quantum waves. Water waves have been observed in streams, pools, and oceans. Nonlinear evolution equations have been used as models for some nonlinear processes. Fluid dynamics is a specialized branch of fluid mechanics that is extensively employed in the fields of physics and engineering. It is concerned with the examination and analysis of the movement and behavior of liquids as they flow. There exist numerous subfields within this discipline, encompassing areas such as aerodynamics and hydrodynamics. Fluid dynamics has multiple applications, particularly in the quantification of forces and moments.

The three dimensional HB equation in fluids has the following form [31]:

$$u_{yt}+\delta_1(u_{xxxy}+6u_xu_y+3u_{xy}u+3u_{xx}\int u_y dx)+\delta_2u_{yy}+\delta_3u_{zz}=0. \tag{1}$$

The Eq. (1) may also be written as:

$$u_{yt}+\delta_1(u_{xxxy}+6u_xu_y+3u_{xy}u+3u_{xx}v)+\delta_2u_{yy}+\delta_3u_{zz}=0,$$
$$u_y=v_x. \tag{2}$$

On taking the $\delta_3=0$ the Eq. (2) takes the form of two dimensional HB equation, which has been discussed by Hua et al. by applying a series of ansatz techniques [32]. In [33], Hosseini et al. also discussed the two dimensional HB equation and secured the different wave structures by the assistance of useful techniques. So, we are interested to discuss the a variety of wave structures to the Eq. (2).

For securing the different types of solutions of the governing system, we proceed with the following transformation

$$u=2(lnf)_{xx},\ v=2(lnf)_{xy}. \tag{3}$$

So, the Hirota bilinear form based on the Eq. (3) to the studied equation is expressed as:

$$-f_y\left(f_t+\delta_1f_{\rm xxx}+\delta_2f_y\right)+f\left(f_{\rm ty}+\delta_1f_{\rm xxxy}+\delta_2f_{\rm yy}\right)$$
$$+3\delta_1\left(f_{\rm xx}f_{\rm xy}-f_xf_{\rm xxy}\right)+\delta_3\left(ff_{\rm zz}-f_z^2\right)=0. \tag{4}$$

The Eq. (4) may be written as:

$$(D_yD_t+\delta_1D_x^3D_y+\delta_2D_y^2+\delta_3D_z^2)f.f=0, \tag{5}$$

where D_x, D_y, D_z and D_t are the derivative operators of Hirota's bilinear.

Lump-periodic solutions

Suppose

$$f(x,y,z,t)=q_1\cosh\left(b_1x+b_2y+b_3z+b_4t\right)$$
$$+q_2\cos\left(b_5x+b_6y+b_7z+b_8t\right)+q_3\cosh\left(b_9x+b_{10}y+b_{11}z+b_{12}t\right). \tag{6}$$

On solving the Eq. (4) and (6), we get

Set-1:

$$\delta_1=0,\delta_2=-\frac{b_7^2\delta_3}{b_6^2}-\frac{b_4}{b_2},b_3=\frac{b_2b_7}{b_6},b_8=\frac{b_4b_6}{b_2},q_3=0. \tag{7}$$

The Eqs. (6) and (7) provide

$$f(x,y,z,t)=q_2\cos\left(\frac{b_4b_6t}{b_2}+b_5x+b_6y+b_7z\right)$$
$$+q_1\cosh\left(b_4t+b_1x+b_2y+\frac{b_2b_7z}{b_6}\right), \tag{8}$$

The following solutions are secured on managing the Eqs. (3) and (8) as:

$$u(x,y,z,t)$$
$$=2\left(\frac{b_1^2q_1\cosh\left(\Xi_1\right)-b_5^2q_2\cos\left(\Xi_2\right)}{q_2\cos\left(\Xi_2\right)+q_1\cosh\left(\Xi_1\right)}-\frac{\left(b_1q_1\sinh\left(\Xi_1\right)-b_5q_2\sin\left(\Xi_2\right)\right)^2}{\left(q_2\cos\left(\Xi_2\right)+q_1\cosh\left(\Xi_1\right)\right)^2}\right), \tag{9}$$

$$v(x,y,z,t)=2\left(\frac{b_1b_2q_1\cosh\left(\Xi_1\right)-b_5b_6q_2\cos\left(\Xi_2\right)}{q_2\cos\left(\Xi_2\right)+q_1\cosh\left(\Xi_1\right)}-\frac{\left(b_1q_1\sinh\left(\Xi_1\right)-b_5q_2\sin\left(\Xi_2\right)\right)\left(b_2q_1\sinh\left(\Xi_1\right)-b_6q_2\sin\left(\Xi_2\right)\right)}{\left(q_2\cos\left(\Xi_2\right)+q_1\cosh\left(\Xi_1\right)\right)^2}\right), \quad (10)$$

where $\Xi_1=b_4t+b_1x+b_2y+\frac{b_2b_7z}{b_6}$, $\Xi_2=\frac{b_4b_6t}{b_2}+b_5x+b_6y+b_7z$.

Set-2:

$$b_4=\frac{b_2b_{12}}{b_{10}},b_{11}=\frac{b_3b_{10}}{b_2},q_2=0,\delta_2=-\frac{b_3^2\delta_3}{b_2^2}-\frac{b_{12}}{b_{10}},\delta_1=0. \quad (11)$$

The Eqs. (6) and (11) yield

$$f(x,y,z,t)=q_1\cosh\left(\frac{b_2b_{12}t}{b_{10}}+b_1x+b_2y+b_3z\right)+q_3\cosh\left(b_{12}t+b_9x+b_{10}y+\frac{b_3b_{10}z}{b_2}\right). \quad (12)$$

Hence

$$u(x,y,z,t)=2\left(\frac{b_1^2q_1\cosh\left(\Xi_3\right)+b_9^2q_3\cosh\left(\Xi_4\right)}{q_1\cosh\left(\Xi_3\right)+q_3\cosh\left(\Xi_4\right)}-\frac{\left(b_1q_1\sinh\left(\Xi_3\right)+b_9q_3\sinh\left(\Xi_4\right)\right)^2}{\left(q_1\cosh\left(\Xi_3\right)+q_3\cosh\left(\Xi_4\right)\right)^2}\right), \quad (13)$$

$$v(x,y,z,t)=2\left(\frac{b_1b_2q_1\cosh\left(\Xi_3\right)+b_9b_{10}q_3\cosh\left(\Xi_4\right)}{q_1\cosh\left(\Xi_3\right)+q_3\cosh\left(\Xi_4\right)}-\frac{\left(b_1q_1\sinh\left(\Xi_3\right)+b_9q_3\sinh\left(\Xi_4\right)\right)\left(b_2q_1\sinh\left(\Xi_3\right)+b_{10}q_3\sinh\left(\Xi_4\right)\right)}{\left(q_1\cosh\left(\Xi_3\right)+q_3\cosh\left(\Xi_4\right)\right)^2}\right), \quad (14)$$

where $\Xi_3=\frac{b_2b_{12}t}{b_{10}}+b_1x+b_2y+b_3z$, $\Xi_4=b_{12}t+b_9x+b_{10}y+\frac{b_3b_{10}z}{b_2}$.

Set-3:

$$b_1=b_9,b_4=-4b_9^3\delta_1-b_2\delta_2-\frac{b_3^2\delta_3}{b_2},b_{11}=\frac{b_3b_{10}}{b_2},$$
$$b_{12}=b_{10}\left(-\frac{b_3^2\delta_3}{b_2^2}-\delta_2\right)-4b_9^3\delta_1,q_2=0. \quad (15)$$

The Eqs. (6) and (15) yield

$$f(x,y,z,t)=q_1\cosh\left(t\left(-4b_9^3\delta_1-b_2\delta_2-\frac{b_3^2\delta_3}{b_2}\right)+b_9x+b_2y+b_3z\right)+q_3\cosh\left(t\left(b_{10}\left(-\frac{b_3^2\delta_3}{b_2^2}-\delta_2\right)-4b_9^3\delta_1\right)+b_9x+b_{10}y+\frac{b_3b_{10}z}{b_2}\right) \quad (16)$$

Hence

$$u(x,y,z,t)=2\left(\frac{b_9^2q_1\cosh\left(\Xi_5\right)+b_9^2q_3\cosh\left(\Xi_6\right)}{q_1\cosh\left(\Xi_5\right)+q_3\cosh\left(\Xi_6\right)}-\frac{\left(b_9q_1\sinh\left(\Xi_5\right)+b_9q_3\sinh\left(\Xi_6\right)\right)^2}{\left(q_1\cosh\left(\Xi_5\right)+q_3\cosh\left(\Xi_6\right)\right)^2}\right), \quad (17)$$

$$v(x,y,z,t)=2\left(\frac{b_2b_9q_1\cosh\left(\Xi_5\right)+b_9b_{10}q_3\cosh\left(\Xi_6\right)}{q_1\cosh\left(\Xi_5\right)+q_3\cosh\left(\Xi_6\right)}-\frac{\left(b_9q_1\sinh\left(\Xi_5\right)+b_9q_3\sinh\left(\Xi_6\right)\right)\left(b_2q_1\sinh\left(\Xi_5\right)+b_{10}q_3\sinh\left(\Xi_6\right)\right)}{\left(q_1\cosh\left(\Xi_5\right)+q_3\cosh\left(\Xi_6\right)\right)^2}\right), \quad (18)$$

where $\Xi_5=t\left(-4b_9^3\delta_1-b_2\delta_2-\frac{b_3^2\delta_3}{b_2}\right)+b_9x+b_2y+b_3z$,
$\Xi_6=t\left(b_{10}\left(-\frac{b_3^2\delta_3}{b_2^2}-\delta_2\right)-4b_9^3\delta_1\right)+b_9x+b_{10}y+\frac{b_3b_{10}z}{b_2}$.

Set-4:

$$b_2=0,b_3=0,b_4=-b_1^3\delta_1,b_6=0,b_7=0,b_8=b_5^3\delta_1,b_9=0,$$
$$b_{12}=-\frac{b_{11}^2\delta_3}{b_{10}}-b_{10}\delta_2. \quad (19)$$

The Eqs. (6) and (19) yield

$$f(x,y,z,t)=q_2\cos\left(b_5^3\delta_1t+b_5x\right)+q_1\cosh\left(b_1x-b_1^3\delta_1t\right)+q_3\cosh\left(t\left(-\frac{b_{11}^2\delta_3}{b_{10}}-b_{10}\delta_2\right)+b_{10}y+b_{11}z\right). \quad (20)$$

Hence (see Box I).

- **Graphs**

Breather-type solutions

Consider

$$f(x,y,z,t)=\exp\left(\xi_1\right)+a_1\cos\left(\xi_2\right)+a_2\exp\left(\xi_3\right), \quad (23)$$

where,

$$\xi_1=-p_1\left(x+y+z+m_0t\right),\xi_2=p_0\left(x+y+z+d_0t\right),$$
$$\xi_3=p_1\left(x+y+z+m_0t\right).$$

By solving Eqs. (4) and (23), yields

Set-1:

$$m_0=-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2,d_0=-\delta_2-\delta_3+\delta_1p_0^2-3\delta_1p_1^2,b_2=-\frac{b_1^2p_0^2}{4p_1^2}. \quad (24)$$

On manipulating the Eqs. (23) and (24) provides

$$f(x,y,z,t)=-\frac{b_1^2p_0^2\exp\left(p_1\left(t\left(-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2\right)+x+y+z\right)\right)}{4p_1^2}+b_1\cos\left(p_0\left(t\left(-\delta_2-\delta_3+\delta_1p_0^2-3\delta_1p_1^2\right)+x+y+z\right)\right)+\exp\left(-p_1\left(t\left(-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2\right)+x+y+z\right)\right). \quad (25)$$

So,

$$u(x,y,z,t)=2\left(\frac{-\frac{1}{4}b_1^2p_0^2e^{\Xi_9p_1}-b_1p_0^2\cos\left(\Xi_8p_0\right)+p_1^2e^{\Xi_9(-p_1)}}{-\frac{b_1^2p_0^2e^{\Xi_9p_1}}{4p_1^2}+b_1\cos\left(\Xi_8p_0\right)+e^{\Xi_9(-p_1)}}-\frac{\left(-\frac{b_1^2p_0^2e^{\Xi_9p_1}}{4p_1}-b_1p_0\sin\left(\Xi_8p_0\right)-p_1e^{\Xi_9(-p_1)}\right)^2}{\left(-\frac{b_1^2p_0^2e^{\Xi_9p_1}}{4p_1^2}+b_1\cos\left(\Xi_8p_0\right)+e^{\Xi_9(-p_1)}\right)^2}\right), \quad (26)$$

$$v(x,y,z,t)=2\left(\frac{-\frac{1}{4}b_1^2p_0^2e^{\Xi_9p_1}-b_1p_0^2\cos\left(\Xi_8p_0\right)+p_1^2e^{\Xi_9(-p_1)}}{-\frac{b_1^2p_0^2e^{\Xi_9p_1}}{4p_1^2}+b_1\cos\left(\Xi_8p_0\right)+e^{\Xi_9(-p_1)}}-\frac{\left(-\frac{b_1^2p_0^2e^{\Xi_9p_1}}{4p_1}-b_1p_0\sin\left(\Xi_8p_0\right)-p_1e^{\Xi_9(-p_1)}\right)^2}{\left(-\frac{b_1^2p_0^2e^{\Xi_9p_1}}{4p_1^2}+b_1\cos\left(\Xi_8p_0\right)+e^{\Xi_9(-p_1)}\right)^2}\right), \quad (27)$$

where $\Xi_8=t\left(-\delta_2-\delta_3+\delta_1p_0^2-3\delta_1p_1^2\right)+x+y+z$, $\Xi_9=t\left(-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2\right)+x+y+z$

Set-2:

$$p_0=-ip_1,d_0=-2\delta_2-2\delta_3-m_0-8\delta_1p_1^2,b_2=\frac{b_1^2}{4}. \quad (28)$$

For set-2, we secure

$$f(x,y,z,t)=b_1\cosh\left(p_1\left(t\left(-2\delta_2-2\delta_3-m_0-8\delta_1p_1^2\right)+x+y+z\right)\right)+\frac{1}{4}b_1^2e^{p_1\left(m_0t+x+y+z\right)}+e^{p_1\left(-\left(m_0t+x+y+z\right)\right)}. \quad (29)$$

Finally, we have

$$u(x,y,z,t)=2\left(\frac{\frac{1}{4}b_1^2p_1^2e^{\Xi_{12}p_1}+b_1p_1^2\cosh\left(\Xi_{11}p_1\right)+p_1^2e^{\Xi_{10}p_1}}{\frac{1}{4}b_1^2e^{\Xi_{12}p_1}+b_1\cosh\left(\Xi_{11}p_1\right)+e^{\Xi_{10}p_1}}\right.$$

$$u(x,y,z,t)=2\left(\frac{b_1^2 q_1\cosh\left(b_1x-b_1^3\delta_1t\right)-b_5^2q_2\cos\left(b_5^3\delta_1t+b_5x\right)}{q_2\cos\left(b_5^3\delta_1t+b_5x\right)+q_1\cosh\left(b_1x-b_1^3\delta_1t\right)+q_3\cosh\left(\Xi_7\right)}-\frac{\left(b_1q_1\sinh\left(b_1x-b_1^3\delta_1t\right)-b_5q_2\sin\left(b_5^3\delta_1t+b_5x\right)\right)^2}{\left(q_2\cos\left(b_5^3\delta_1t+b_5x\right)+q_1\cosh\left(b_1x-b_1^3\delta_1t\right)+q_3\cosh\left(\Xi_7\right)\right)^2}\right), \tag{21}$$

$$v(x,y,z,t)=-\frac{2b_{10}q_3\sinh\left(t\left(-\frac{b_{11}^2\delta_3}{b_{10}}-b_{10}\delta_2\right)+b_{10}y+b_{11}z\right)\left(b_1q_1\sinh\left(b_1x-b_1^3\delta_1t\right)-b_5q_2\sin\left(b_5^3\delta_1t+b_5x\right)\right)}{\left(q_2\cos\left(b_5^3\delta_1t+b_5x\right)+q_1\cosh\left(b_1x-b_1^3\delta_1t\right)+q_3\cosh\left(\Xi_7\right)\right)^2}, \tag{22}$$

where $\Xi_7=t\left(-\frac{b_{11}^2\delta_3}{b_{10}}-b_{10}\delta_2\right)+b_{10}y+b_{11}z.$

Box I.

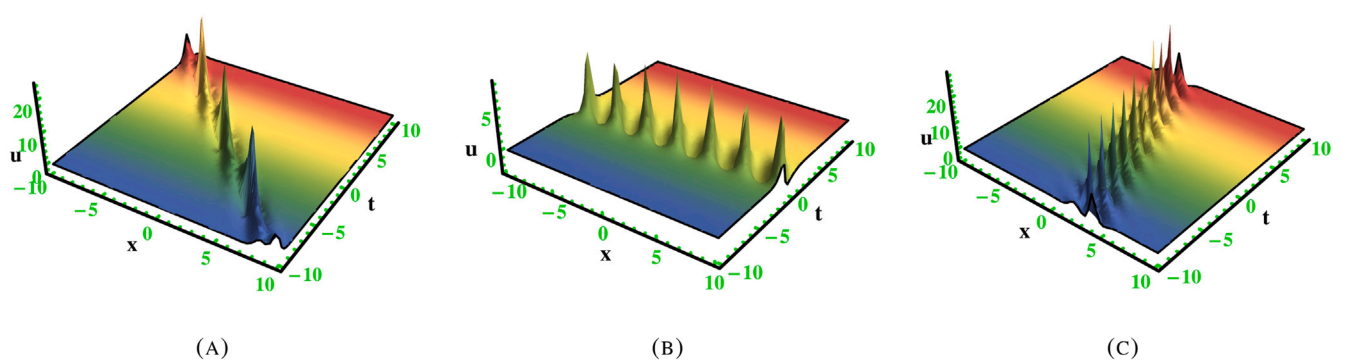

Fig. 1. Graphs of solution (9) with parameters, $b_1=0.2, b_2=0.72, b_4=0.08, b_5=2, b_6=0.92, b_7=0.92, q_1=0.3, q_2=1.02$, (A) for $y=0.2, z=0.7$, (B) for $y=0.21, z=2.7$, (C) for $y=.23, z=3$.

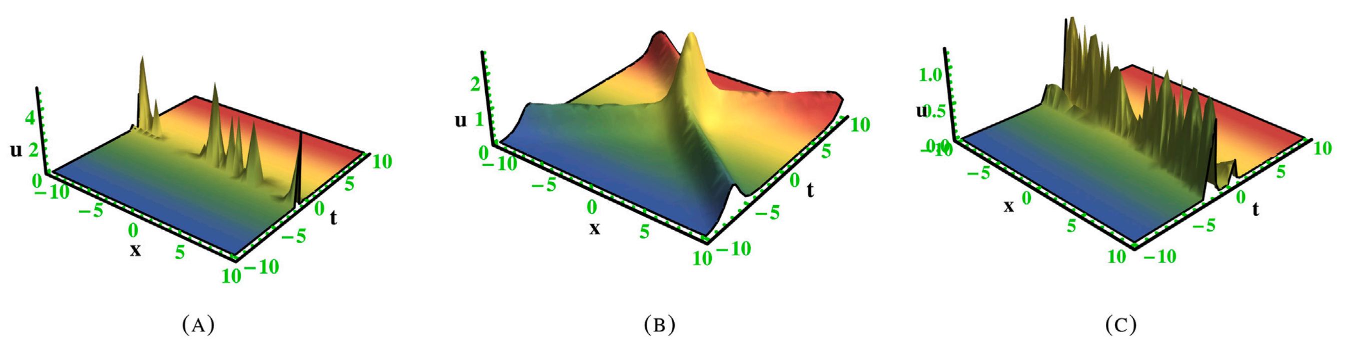

Fig. 2. Graphs of solution (13) with parameters, $q_3=2.02, b_1=2, b_2=1, b_3=1.8, b_{12}=2.03, b_{10}=0.02, b_9=1.2, q_1=0.3$, (A) for $y=2, z=1$, (B) for $y=2.2, z=1.3$, (C) for $y=2.1, z=.03$.

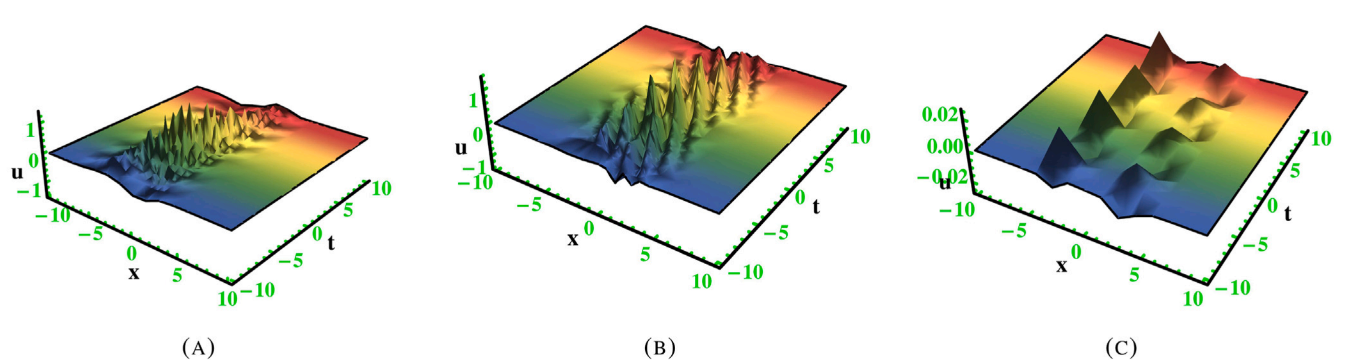

Fig. 3. Graphs of solution (21) with parameters, $q_3=0.02, b_1=2, b_5=0.02, b_{10}=1.3, b_{11}=0.03, \delta_1=3.2, \delta_2=1.2, \delta_3=0.3, q_1=2, q_2=2.2$, (A) for $y=2, z=3$, (B) for $y=.2, z=1.3$, (C) for $y=.03, z=2$.

$$-\frac{\left(\frac{1}{4}b_1^2 p_1 e^{\Xi_{12}p_1}+b_1 p_1 \sinh\left(\Xi_{11}p_1\right)-p_1 e^{\Xi_{10}p_1}\right)^2}{\left(\frac{1}{4}b_1^2 e^{\Xi_{12}p_1}+b_1\cosh\left(\Xi_{11}p_1\right)+e^{\Xi_{10}p_1}\right)^2}\Bigg), \tag{30}$$

$$v(x,y,z,t)=2\left(\frac{\frac{1}{4}b_1^2 p_1^2 e^{\Xi_{12}p_1}+b_1 p_1^2\cosh\left(\Xi_{11}p_1\right)+p_1^2 e^{\Xi_{10}p_1}}{\frac{1}{4}b_1^2 e^{\Xi_{12}p_1}+b_1\cosh\left(\Xi_{11}p_1\right)+e^{\Xi_{10}p_1}}-\frac{\left(\frac{1}{4}b_1^2 p_1 e^{\Xi_{12}p_1}+b_1 p_1 \sinh\left(\Xi_{11}p_1\right)-p_1 e^{\Xi_{10}p_1}\right)^2}{\left(\frac{1}{4}b_1^2 e^{\Xi_{12}p_1}+b_1\cosh\left(\Xi_{11}p_1\right)+e^{\Xi_{10}p_1}\right)^2}\right), \tag{31}$$

where $\Xi_{10}=-x-y-z-m_0t, \Xi_{11}=\left(-2\delta_2-2\delta_3-m_0-8\delta_1p_1^2\right)t+x+y+z, \Xi_{12}=x+y+z+m_0t$.

Set-3:

$$p_0=-\frac{\sqrt{\delta_2+\delta_3+m_0+\delta_1p_1^2}}{\sqrt{3}\sqrt{\delta_1}}, d_0=\frac{1}{3}\left(-2\delta_2-2\delta_3+m_0-8\delta_1p_1^2\right),$$
$$b_2=-\frac{b_1^2\left(\delta_2+\delta_3+m_0+\delta_1p_1^2\right)}{12\delta_1p_1^2}. \tag{32}$$

We get result on tackling

$$f(x,y,z,t)=e^{p_1\left(-\left(m_0t+x+y+z\right)\right)}+b_1\cos\left(\frac{\sqrt{\delta_2+\delta_3+m_0+\delta_1p_1^2}\left(\frac{1}{3}t\left(-2\delta_2-2\delta_3+m_0-8\delta_1p_1^2\right)+x+y+z\right)}{\sqrt{3}\sqrt{\delta_1}}\right)-\frac{b_1^2\left(\delta_2+\delta_3+m_0+\delta_1p_1^2\right)e^{p_1\left(m_0t+x+y+z\right)}}{12\delta_1p_1^2}. \tag{33}$$

Hence, we secure

$$u(x,y,z,t)=2\left(\frac{-\frac{b_1\Xi_{14}\cos(\Xi_{13})}{3\delta_1}-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1}+p_1^2e^{\Xi_{10}p_1}}{b_1\cos\left(\Xi_{13}\right)-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1p_1^2}+e^{\Xi_{10}p_1}}-\frac{\left(-\frac{b_1\sqrt{\Xi_{14}}\sin(\Xi_{13})}{\sqrt{3}\sqrt{\delta_1}}-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1p_1}-p_1e^{\Xi_{10}p_1}\right)^2}{\left(b_1\cos\left(\Xi_{13}\right)-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1p_1^2}+e^{\Xi_{10}p_1}\right)^2}\right), \tag{34}$$

$$v(x,y,z,t)=2\left(\frac{-\frac{b_1\Xi_{14}\cos(\Xi_{13})}{3\delta_1}-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1}+p_1^2e^{\Xi_{10}p_1}}{b_1\cos\left(\Xi_{13}\right)-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1p_1^2}+e^{\Xi_{10}p_1}}-\frac{\left(-\frac{b_1\sqrt{\Xi_{14}}\sin(\Xi_{13})}{\sqrt{3}\sqrt{\delta_1}}-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1p_1}-p_1e^{\Xi_{10}p_1}\right)^2}{\left(b_1\cos\left(\Xi_{13}\right)-\frac{b_1^2\Xi_{14}e^{\Xi_{12}p_1}}{12\delta_1p_1^2}+e^{\Xi_{10}p_1}\right)^2}\right), \tag{35}$$

where $\Xi_{10}=-x-y-z-m_0t, \Xi_{12}=x+y+z+m_0t, \Xi_{13}=\frac{\sqrt{\delta_2+\delta_3+m_0+\delta_1p_1^2}\left(\frac{1}{3}t\left(-2\delta_2-2\delta_3+m_0-8\delta_1p_1^2\right)+x+y+z\right)}{\sqrt{3}\sqrt{\delta_1}}$,
$\Xi_{14}=\delta_2+\delta_3+m_0+\delta_1p_1^2$.

Set-4:

$$m_0=-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2, d_0=-\delta_2-\delta_3+\delta_1p_0^2-3\delta_1p_1^2, b_1=\frac{2\sqrt{-b_2}p_1}{p_0}. \tag{36}$$

On solving, we secure

$$f(x,y,z,t)=b_2e^{\left(p_1\left(t\left(-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2\right)+x+y+z\right)\right)}+e^{\left(-p_1\left(t\left(-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2\right)+x+y+z\right)\right)}+\frac{2\sqrt{-b_2}p_1\cos\left(p_0\left(t\left(-\delta_2-\delta_3+\delta_1p_0^2-3\delta_1p_1^2\right)+x+y+z\right)\right)}{p_0}. \tag{37}$$

Therefore, we get

$$u(x,y,z,t)=2\left(\frac{b_2p_1^2e^{\Xi_9p_1}-2\sqrt{-b_2}p_0p_1\cos\left(\Xi_8p_0\right)+p_1^2e^{\Xi_9(-p_1)}}{b_2e^{\Xi_9p_1}+\frac{2\sqrt{-b_2}p_1\cos(\Xi_8p_0)}{p_0}+e^{\Xi_9(-p_1)}}-\frac{\left(b_2p_1e^{\Xi_9p_1}-2\sqrt{-b_2}p_1\sin\left(\Xi_8p_0\right)+p_1\left(-e^{\Xi_9(-p_1)}\right)\right)^2}{\left(b_2e^{\Xi_9p_1}+\frac{2\sqrt{-b_2}p_1\cos(\Xi_8p_0)}{p_0}+e^{\Xi_9(-p_1)}\right)^2}\right), \tag{38}$$

$$v(x,y,z,t)=2\left(\frac{b_2p_1^2e^{\Xi_9p_1}-2\sqrt{-b_2}p_0p_1\cos\left(\Xi_8p_0\right)+p_1^2e^{\Xi_9(-p_1)}}{b_2e^{\Xi_9p_1}+\frac{2\sqrt{-b_2}p_1\cos(\Xi_8p_0)}{p_0}+e^{\Xi_9(-p_1)}}-\frac{\left(b_2p_1e^{\Xi_9p_1}-2\sqrt{-b_2}p_1\sin\left(\Xi_8p_0\right)+p_1\left(-e^{\Xi_9(-p_1)}\right)\right)^2}{\left(b_2e^{\Xi_9p_1}+\frac{2\sqrt{-b_2}p_1\cos(\Xi_8p_0)}{p_0}+e^{\Xi_9(-p_1)}\right)^2}\right), \tag{39}$$

where $\Xi_8=\left(-\delta_2-\delta_3+\delta_1p_0^2-3\delta_1p_1^2\right)t+x+y+z, \Xi_9=\left(-\delta_2-\delta_3+3\delta_1p_0^2-\delta_1p_1^2\right)t+x+y+z$.

- **Graphs**

Two-wave solutions

Suppose

$$f(x,y,z,t)=\chi_1\exp\left(x+y+z+\phi_1t\right)+\chi_2\exp\left(-(x+y+z+\phi_2t)\right)+\chi_3\sin\left(x+y+z+\phi_3t\right)+\chi_4\sinh\left(x+y+z+\phi_4t\right). \tag{40}$$

We have the following results on solving the system (4) and (40)

Set-1:

$$\phi_3=-2\left(4\delta_1+\delta_2+\delta_3\right)-\phi_1, \chi_2=-\frac{\chi_4^2}{4\chi_1}, \chi_3=0. \tag{41}$$

The Eqs. (40) and (41) give

$$f(x,y,z,t)=\chi_4\sinh\left(t\left(-2\left(4\delta_1+\delta_2+\delta_3\right)-\phi_1\right)+x+y+z\right)-\frac{\chi_4^2e^{-t\phi_1-x-y-z}}{4\chi_1}+\chi_1e^{t\phi_1+x+y+z}. \tag{42}$$

Therefore, we extract

$$u(x,y,z,t)=2\left(1-\frac{\left(\chi_4\cosh\left(t\left(-2\left(4\delta_1+\delta_2+\delta_3\right)-\phi_1\right)+x+y+z\right)+\frac{\chi_4^2e^{-t\phi_1-x-y-z}}{4\chi_1}+\chi_1e^{t\phi_1+x+y+z}\right)^2}{\left(\chi_4\sinh\left(t\left(-2\left(4\delta_1+\delta_2+\delta_3\right)-\phi_1\right)+x+y+z\right)-\frac{\chi_4^2e^{-t\phi_1-x-y-z}}{4\chi_1}+\chi_1e^{t\phi_1+x+y+z}\right)^2}\right). \tag{43}$$

$$v(x,y,z,t)=2\left(1-\frac{\left(\chi_4\cosh\left(t\left(-2\left(4\delta_1+\delta_2+\delta_3\right)-\phi_1\right)+x+y+z\right)+\frac{\chi_4^2e^{-t\phi_1-x-y-z}}{4\chi_1}+\chi_1e^{t\phi_1+x+y+z}\right)^2}{\left(\chi_4\sinh\left(t\left(-2\left(4\delta_1+\delta_2+\delta_3\right)-\phi_1\right)+x+y+z\right)-\frac{\chi_4^2e^{-t\phi_1-x-y-z}}{4\chi_1}+\chi_1e^{t\phi_1+x+y+z}\right)^2}\right). \tag{44}$$

Set-2:

$$\phi_3=2\delta_1-\delta_2-\delta_3, \phi_2=-2\delta_1-\delta_2-\delta_3, \chi_1=0, \chi_2=0, \chi_4=\chi_3. \tag{45}$$

The Eqs. (40) and (45) provide

$$f(x,y,z,t)=\chi_3\sin\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)+\chi_3\sinh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right). \tag{46}$$

So, we extract

$$u(x,y,z,t)=2\left(\frac{\chi_3\sinh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)-\chi_3\sin\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)}{\chi_3\sin\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)+\chi_3\sinh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)}-\frac{\left(\chi_3\cos\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)+\chi_3\cosh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)\right)^2}{\left(\chi_3\sin\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)+\chi_3\sinh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)\right)^2}\right). \tag{47}$$

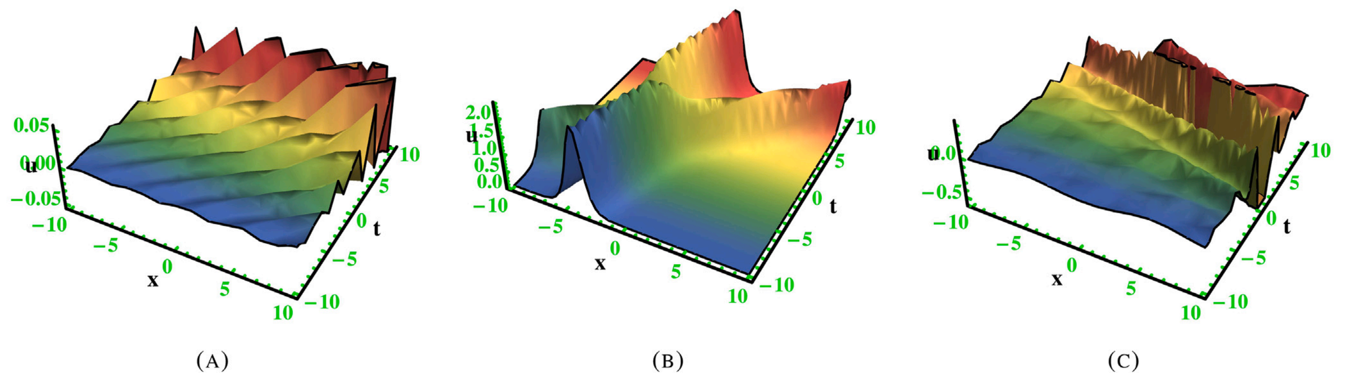

Fig. 4. Graphs of solution (26) with parameters, $b_1 = 0.558, \delta_1 = 1.9, \delta_2 = 0.2, \delta_3 = 1.899, p_0 = 0.0091, p_1 = 0.2$, (A) for $y = 0.009, z = .2$, (B) for $y = 2.009, z = 2.2$, (C) for $y = 2, z = 0.276$.

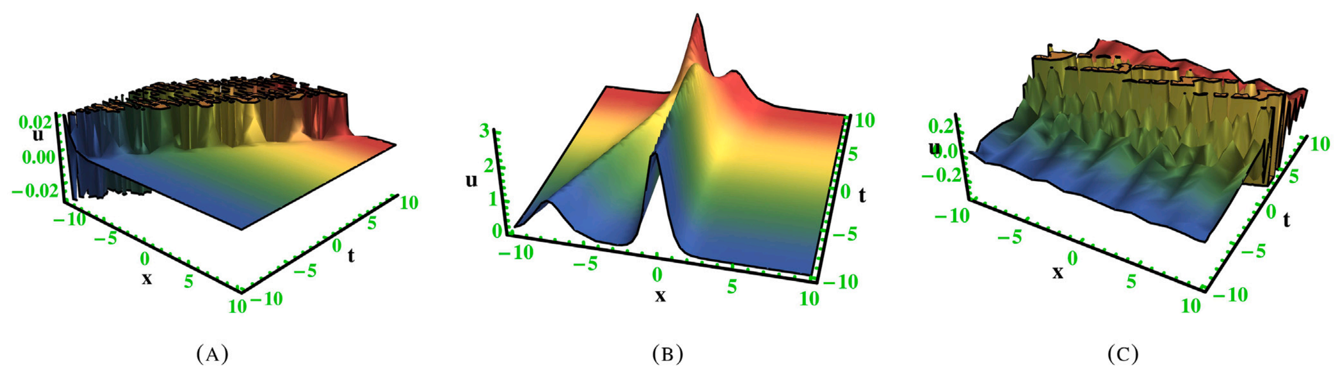

Fig. 5. Graphs of solution (34) with parameters, $b_1 = 3.558, \delta_1 = 0.9, \delta_2 = 2.2, \delta_3 = 0.899, m_0 = 2, p_1 = 3.2$, (A) for $y = 1, z = 2$, (B) for $y = 1.9, z = .02$, (C) for $y = 1.009, z = 2.02$.

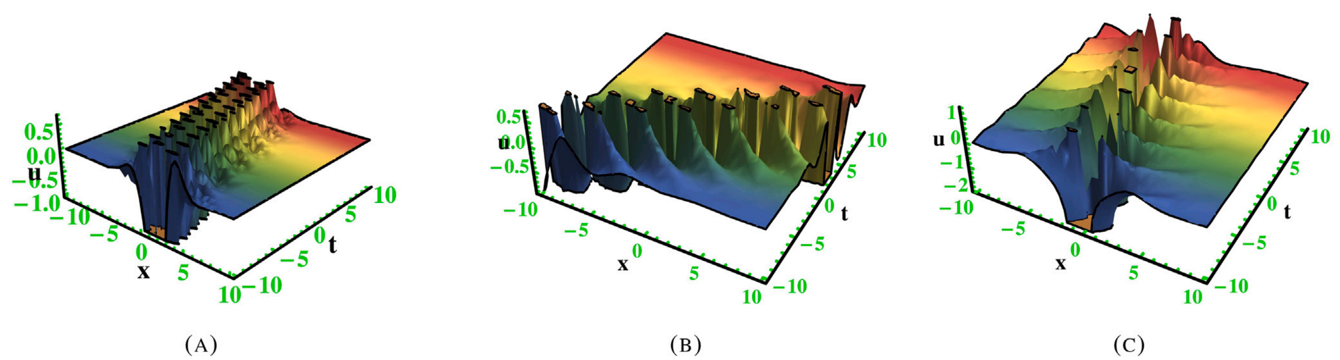

Fig. 6. Graphs of solution (38) with parameters, $b_2 = 3.3, \delta_1 = 3.9, \delta_2 = 0.2, \delta_3 = 1.899, p_0 = 1.2, p_1 = 3.22$, (A) for $y = .9, z = .02$, (B) for $y = 1, z = .02$, (C) for $y = .66, z = .22$.

$$v(x,y,z,t) = 2\left(\frac{\chi_3 \sinh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)-\chi_3 \sin\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)}{\chi_3 \sin\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)+\chi_3 \sinh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)} - \frac{\left(\chi_3 \cos\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)+\chi_3 \cosh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)\right)^2}{\left(\chi_3 \sin\left(\left(-2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)+\chi_3 \sinh\left(\left(2\delta_1-\delta_2-\delta_3\right)t+x+y+z\right)\right)^2}\right). \quad (48)$$

Concluding remarks

A soliton refers to a certain type of wave pattern that maintains its structural integrity as it propagates at a consistent velocity. Solitons arise due to the suppression of nonlinear and inhomogeneous representations in media. Solitons are outcomes arising from a broad range of parameter estimation in dispersive nonlinear models that accurately represent various physical processes. This study extensively examined a novel (3+1)-dimensional Hirota bilinear equation, which serves as a mathematical model for describing nonlinear waves in fluid dynamics and oceanography. The current goal has been achieved by employing the simplified Hirota's method with test functions as a trustworthy strategy. Different solutions, such as lump-periodic, breather-type, and two-wave solutions, have been successfully extracted. The results could be used to shed light on some of the more elusive aspects of physical processes. Three-dimensional graphs have been sketched in figures (1–6) to aid in the explanation of the solutions. Lump solutions are commonly used to interpret nonlinear interaction solutions. For comprehending the qualitative properties of multiple occurrences and processes in various branches of natural science, lump solutions to mathematical equations are essential. Numerous complicated nonlinear phenomena, including the spatial localization of transfer processes, the existence of peaking regimes, the multiplicity or lack of steady states under diverse conditions, are visually illustrated and explained by lump solutions of nonlinear differential equations. A breather is a nonlinear wave in physics that has energy concentrated in a focused, oscillating manner. Breathers are autonomous structures. There are two different kinds of breathers: standing ones and traveling ones. Standing breathers relate to localized solutions whose amplitude varies over time (they are sometimes called oscillons). The main frequency of the breather and all of its multipliers must be outside of the lattice''s phonon spectrum in order for breathers to exist in discrete lattices. The approach described is applicable to a wide class of NLPDEs. The considered model has never been done before by this approach, and the results reported here are entirely original. According to the results we obtained, our method provides a quick and efficient means of dealing with nonlinear dynamical systems.

CRediT authorship contribution statement

U. Younas: Writing – original draft, Formal analysis, Conceptualization. **Hajar F. Ismael:** Writing – original draft, Conceptualization. **T.A. Sulaiman:** Writing – original draft, Conceptualization. **Muhammad Amin S. Murad:** Writing – review & editing. **Nehad Ali Shah:** Writing – review & editing. **Mohsen Sharifpur:** Writing – review & editing.

Declaration of competing interest

The authors declare no conflict of interest.

Data availability

Data will be made available on request.

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