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Uniqueness of the Normal Consolidation Line for Gold Tailings

Reference

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ABSTRACT

Depending on the stress state, mine tailings are generally accepted to be susceptible to static liquefaction. A common method to assess the in situ stress state of tailings in relation to static liquefaction susceptibility involves the use of the state parameter. Because most tailings materials are normally consolidated (NC), this type of assessment requires knowledge about the normal consolidation line (NCL). It has been shown experimentally that the uniqueness of the NCL is vastly different for fine-grained and coarse-grained soils, with clays usually exhibiting a unique NCL and clean sands exhibiting an infinite number of parallel NCLs. Gold tailings, a sandy silt, fall between clays and clean sands, and there are limited experimental data regarding their compression behavior over a range of initial void ratios. This lack of data results in inconsistent interpretation of the uniqueness of the NCL for gold tailings in the industry. This can influence the results of designs and safety evaluations of tailings dams. In this study, a number of oedometer tests were conducted on gold tailings sourced from an active tailings dam in South Africa. Several specimens were prepared at various initial densities and were consolidated in small increments to a high effective stress. The oedometer tests were supplemented with triaxial compression tests, from which a unique critical state line was identified. Across the oedometer and triaxial tests, it was found that the behavior of the NC and overconsolidated samples was consistent with that typically observed for fine-grained soils. Therefore, for practical purposes, it appears that the gold tailings tested can be viewed in a framework with a unique NCL. No significant influence of particle crushing was noted.

Keywords

gold tailings, normal consolidation line, oedometer testing, critical state soil mechanics, wetting-induced collapse

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Introduction

Soil liquefaction is the process of strain-softening of contractive, saturated, cohesionless soils during undrained shear and is a major concern for structures constructed with or on saturated sandy soils (Robertson and Wride 1998). Depending on the in situ stress state, mine tailings are generally accepted to be susceptible to static liquefaction. A common method to assess the in situ stress state of mine tailings in relation to static liquefaction susceptibility involves the use of the state parameter. The state parameter is the difference between the current void ratio and the void ratio at the critical state, at the same mean effective stress (Been and Jefferies 1985).

Because tailings dams are continuously being constructed, it is often useful to be able to estimate the future state parameter based on expected stress state changes (e.g., due to routine raising or the construction of a stabilizing buttress). Because most tailings materials are normally consolidated (NC), this type of assessment requires knowledge about the normal consolidation line (NCL). However, it has been shown experimentally that the uniqueness of the NCL is vastly different for fine-grained and coarse-grained soils. Fine-grained soils such as clays tend to exhibit a unique NCL (e.g., Atkinson and Bransby 1978), whereas coarse-grained soils such as clean sands appear to have an infinite number of parallel NCLs (e.g., Jefferies and Been 2000). This difference in idealized compression behavior is depicted in **figure 1**.

Mine tailings, typically being silty sands, fall between clays and clean sands, and, as far as the authors are aware, there is little research on the uniqueness of the NCL for mine tailings. This lack of data results in inconsistent interpretation of the uniqueness of the NCL for mine tailings in the industry, and this can influence the results of designs and safety evaluations of tailings dams.

To address this lack of data, seven oedometer tests were conducted on gold tailings sourced from an active tailings dam in South Africa. Specimens were prepared at various initial densities and were consolidated in small increments to high effective stresses. These oedometer tests were supplemented by ten triaxial compression tests. The intent of this study is to investigate whether, from a practical perspective, the compression behavior of gold tailings can be viewed within the framework of a unique NCL.

Background

THEORETICAL FRAMEWORK AND TERMINOLOGY

Before investigating the uniqueness of the NCL, it is first necessary to define some common terminology and to discuss their importance in the interpretation of compression and swelling behavior of soils. Compression is the

FIG. 1 Comparison of idealized soil compression behavior for (*A*) fine-grained soils (clays) and (*B*) coarse-grained soils (sands) (after Jefferies and Been 2015).

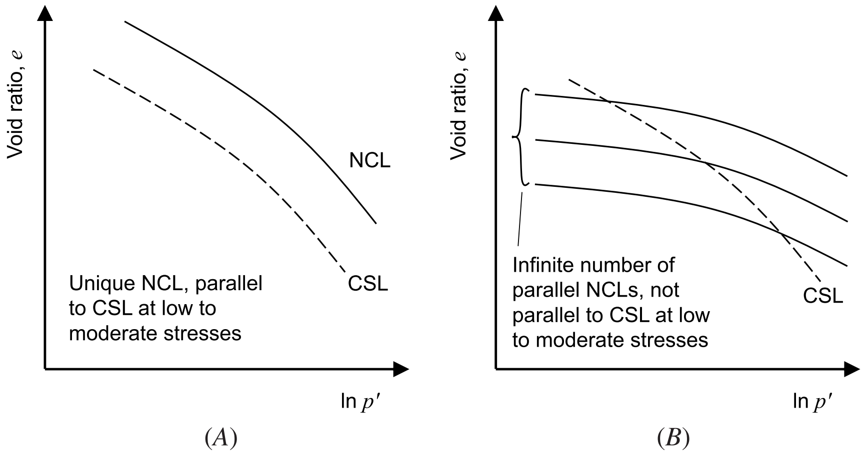


TABLE 1
Summary of key terminology used to describe soil compression curves

Parameter	Description
Virgin compression line	During compression along this line, the soil is subjected to plastic (irreversible) and elastic (recoverable) strains.
Unload–reload line	During compression (or swelling) along this line, the soil is subjected to only elastic (recoverable) strains.
LCC	Similar to the virgin compression line, but specified at high stresses at which particle breakage occurs. Typically defined for coarse-grained soils. The multiple NCLs for sands converge onto the LCC at high stresses.
NCL	The virgin compression line at which consolidation occurs under isotropic stress conditions. Typically defined for fine-grained soils. Also referred to as the isotropic compression line.
1D-NCL	The virgin compression line at which consolidation occurs under one-dimensional conditions (i.e., zero radial strain). It has been shown experimentally that the 1D-NCL is parallel to the NCL for fine-grained soils (Atkinson and Bransby 1978) and coarse-grained soils (Coop 1990).

Note: LCC = limiting compression curve; NCL = normal consolidation line; 1D-NCL = one-dimensional normal consolidation line.

process of gradual reduction in volume of a soil due to an increase in effective stress. Swelling is the reverse of compression; there is a gradual increase in volume of the soil due to a reduction in effective stress. Additional key terminology used to describe soil compression curves is provided in **Table 1**.

Transitional Soils

The terminology defined in **Table 1** is applicable for soils that generally agree with critical state soil mechanics principles. However, it has been known for some time that not all soils conform to these principles. These soils are classified as transitional soils. Transitional soils can be identified as those soils for which the initial void ratio continues to affect its compression and shearing behavior to large strains to the extent that unique NCLs and critical state lines (CSLs) cannot be found by means of conventional laboratory testing (Coop 2015). Initially it was thought that only soils with mixed mineralogy or mixed grading were susceptible to transitional behavior. However, it has since been shown that even uniform soils can be susceptible to transitional behavior, but this is highly dependent on the particle size distribution (Nocilla, Coop, and Colleselli 2006; Shipton and Coop 2012). Therefore, it can be expected that if the soil under consideration is transitional, it will not be possible to define a unique NCL. However, the authors are of the opinion that the gold tailings under consideration are not a transitional soil, as will be discussed in subsequent sections.

It is also known that structure can influence the NCL. For example, Burland (1990) showed how different virgin compression lines could be established based on the state of the soil structure. For this study, all samples were remolded to ensure that the influence of structure was minimal.

The Oedometer Test

The oedometer test is a popular soil test used to investigate the compression behavior of soil. The test works on the principle that mass pieces can be placed on a loading arm to apply a known force onto a soil sample of a known area. A dial gauge can be used to measure the change in height of the sample due to the applied loading. The standard test places the soil sample in a latex membrane with porous disks at the base and top to allow for two-way drainage.

Some limitations of the conventional test include the fact that there is no confirmation of saturation, no pore pressures are recorded, only a vertical stress can be calculated, and no shear phase is possible. However, these limitations are far outweighed by the simplicity of the test method and its ease of use. This test is also referred to as a one-dimensional compression test because the soil sample is placed in a stiff ring that restricts lateral displacement. This strain condition during consolidation, sometimes referred to as K_0 consolidation, is believed to best represent the consolidation of soils in the field under their own weight (e.g., Bishop and Henkel 1962).

It is common to plot the results from an oedometer test in an $e - \log \sigma_v'$ projection. The void ratio (e) of a soil is simply the ratio between the volume of voids and the volume of solids, and the vertical effective stress (σ_v') is simply the force applied during the loading increment divided by the soil sample surface area. Although the cell is filled with water, the pressure head in relation to the total applied stress is negligible and is usually ignored for moderate to high stress levels. The slope of the compression line in semi-log space is termed the compression index (C_c), and the slope of unload–reload lines is termed the expansion index (C_e).

The Triaxial Test

Like the oedometer test, the triaxial test is a popular soil test to investigate both the compression and shear behavior of soil. With the rapid advancements in technology, triaxial testing equipment has advanced significantly since the test was first introduced. However, even with the use of modern equipment, the test works on the principle that a membrane can be used to separate the stress conditions inside and outside the soil sample. This is important in terms of estimating the effective stress of the sample, for which total stresses are applied externally and pore pressures are measured internally. Water pressure is used to apply an equal radial and axial total stress to the soil specimen, and the loading ram can be used to apply an additional axial force. A dial gauge or displacement transducer can be used to measure the change in height of the sample due to the applied loading. The standard test places the soil sample a latex membrane with porous disks at the base and top. Two-way drainage is typically allowed during flushing and saturation, but one-way drainage is generally imposed during consolidation and shear.

A limitation of the conventional test is that the change in the sample's void ratio during flushing and saturation, which can be significant for loose sandy samples (Sladen and Handford 1987), is difficult to capture. However, there are several methods to avoid this uncertainty, such as the squeezing method (Verdugo and Ishihara 1996) and the freezing method (Jefferies and Been 2015). When these methods are employed, the triaxial test is considered a robust laboratory test and is commonly used in practice.

In terms of displaying results, it is common to plot both the compression and shear results from a triaxial test in an $e - \ln p'$ projection. The void ratio (e) was defined earlier, and the mean effective stress (p') is simply the average of the three principal effective stresses. The slope of the CSL in semi-ln space is represented by λ_e.

Isotropic versus One-Dimensional Compression

Isotropic compression occurs when the sample is consolidated under equal stress in all directions. This condition is typical for a standard triaxial test, in which the cell pressure is used to apply a radial confining stress equal to the axial confining stress. One-dimensional consolidation occurs when the sample is consolidated in such a manner that only strain in one dimension is possible. This condition is typical for a standard oedometer test in which the confining ring restrains strain in the radial direction.

It has been shown experimentally that the isotropic compression line is parallel to the one-dimensional compression line for both fine-grained and coarse-grained soils. For example, Atkinson and Bransby (1978) and Coop (1990) performed a series of isotropic and K_0-consolidated compression tests on kaolin clay and Dog's Bay sand, respectively. Both researchers found that the slopes of the one-dimensional normal consolidation line (1D-NCL) and NCL curves are parallel when projected in $e - \ln p'$ space. A key objective of this study is to investigate whether the 1D-NCL is parallel to the NCL for the gold tailings under consideration.

A key difference between isotropic consolidation and one-dimensional consolidation is that shear strains are induced in the soil sample because of the zero radial-strain boundary condition imposed during one-dimensional consolidation. Although it has been shown that these shear strains influence the mechanical response of the sample during undrained shear (e.g., Narainsamy and Jacobsz 2022), this is not explored further in this study.

PARTICLE CRUSHING

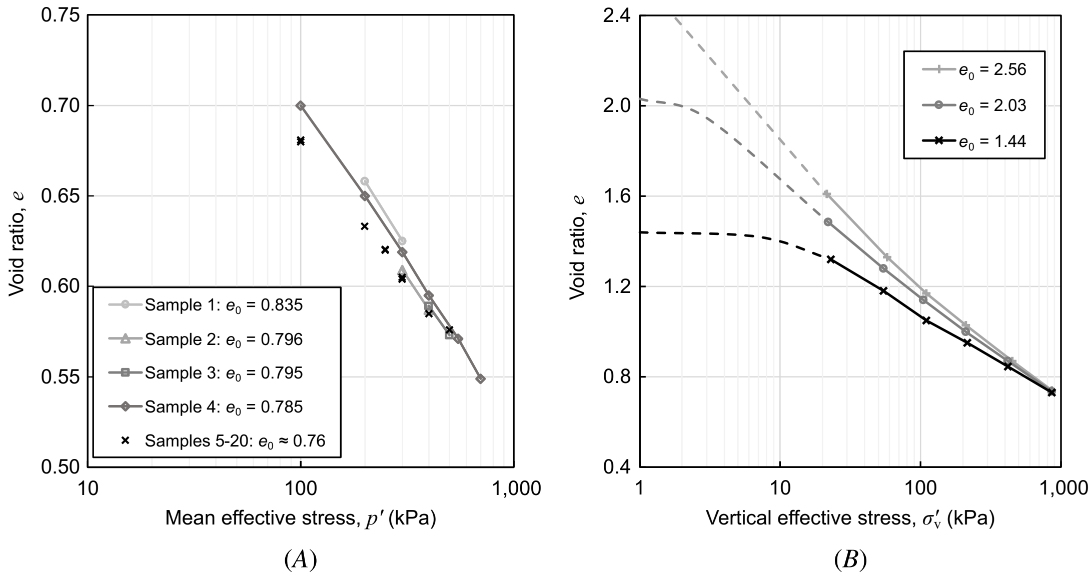
When subjecting soils to high stresses, there is a concern regarding particle crushing. Particle crushing is important because it results in a change in the particle size distribution of the soil, which will change the soil's true compression characteristics. For example, the debris caused by particle breakage continuously fills the pores between the particles, thereby reducing the compressibility of the new soil. It is therefore important to establish if particle crushing is occurring and, if so, to what extent. Zhang et al. (2020) conducted a series of triaxial compression tests on copper tailings to investigate the influence of the mechanical response during drained shear. It was found that particle breakage occurred as early as an applied effective stress of 200 kPa and continued all the way to 5 MPa. However, the overall magnitude of particle breakage was limited. In the study, the mean grain size (d_{50}) reduced from 0.165 mm per the initial grading to 0.131 mm as a result of an applied loading of 5 MPa.

PREVIOUS INVESTIGATIONS: CLAYS

Compression testing of clays has been conducted for many years. Terzaghi's theory of one-dimensional consolidation (Terzaghi 1925) was one of the first widely adopted theories regarding compression of soil. This theory works well for all soil types, but it is especially useful for fine-grained soils. A key assumption of this theory is that the relationship between void ratio and effective stress is unique for a given NC clay. Although not stated explicitly, this assumption implies a unique virgin compression line. Rendulic (1936, 1937) may have been the first to publish experimental evidence supporting this assumption. He conducted a series of triaxial tests on a remolded Wiener Tegel silty clay and postulated that unique relationships exist between void ratio and effective stress, both after isotropic consolidation and at failure. These relationships would today be referred to as the NCL and CSL, respectively. Henkel (1960) proved through a series of triaxial tests that Rendulic's postulation was also true for remolded NC Weald Clay and London Clay samples. The critical state concept and both the Original Cam-Clay and Modified Cam-Clay models (Schofield and Wroth 1968; Roscoe and Burland 1968; Atkinson and Bransby 1978) were developed in part on these findings. A unique NCL and a unique CSL are key assumptions of the Cam-Clay models. Some of Rendulic's results are presented in **figure 2*A***. Samples 1–4 were prepared at different initial void ratios, and isotropic consolidation stress paths were presented. Samples 5–20 were prepared at approximately equal void ratios, and only the void ratios after consolidation (prior to shearing) were provided. Rendulic noted that all these points tended toward a nearly identical NCL.

In another study, Skempton (1944) conducted oedometer tests on remolded samples of a blue estuarine clay from Gosport, prepared fully saturated over a wider range of initial void ratios. At low stresses, the differences in void ratio were noticeable, but as the applied stress increased, the differences became insignificant. This tends to suggest a unique relationship during one-dimensional consolidation. These results are provided in **figure 2*B***.

FIG. 2 Early experimental evidence of a unique NCL for remolded clays: (*A*) triaxial tests for unique NCL of remolded Wiener Tegel (after Rendulic 1936, 1937), and (*B*) oedometer tests for unique 1D-NCL of remolded Gosport clay (after Skempton 1944).



PREVIOUS INVESTIGATIONS: SANDS

Jefferies and Been (2000) conducted a series of triaxial tests on Erksak 330/0.7 sand to investigate the uniqueness of the NCL. The specific sand tested had a mean grain size (d_{50}) of 330 μm and had a fines content (material by mass finer than 0.075 mm) of 0.7 %. The results from the tests are shown in **figure 3**. A curved CSL as well as a pseudo NCL (PNCL) were indicated. The PNCL is based on the Cam-Clay proposition that the NCL is parallel to the CSL when presented in an $e - \log p'$ projection (Schofield and Wroth 1968). For the Erksak sand, a Cam-Clay spacing ratio of 2.73 (corresponding to the original Cam-Clay assumption) was used to define the offset of the PNCL from the CSL.

It was argued that each of the compression curves presented represent a true NCL for the Erksak 330/0.7 sand for the following reasons:

1. The samples were prepared under low stresses, either by gentle moist tamping or wet pluviation, and were thus never overconsolidated (OC) in their initial state. It is peculiar that such dense samples such as the load unload (LDUL) and air pluviated (PV) samples LDUL-3/PV and 667/PV could be created in an NC state, but this appears to a characteristic of clean sands.
2. The virgin compression and unload–reload loops formed a classic elastic–plastic form. These plots have not been reproduced in this paper but can be found in Jefferies and Been (2000).
3. Although no unique NCL was identified, a PNCL was defined using the Cam-Clay theory. If this were a true NCL, material consolidated at states looser than this should exhibit a rapid reduction in volume to reach the NCL. However, no collapse was observed for moist tamped (MT) tests 874/MT, 606/MT, or 601/MT. Rather, a smooth compression response was noted for these tests.

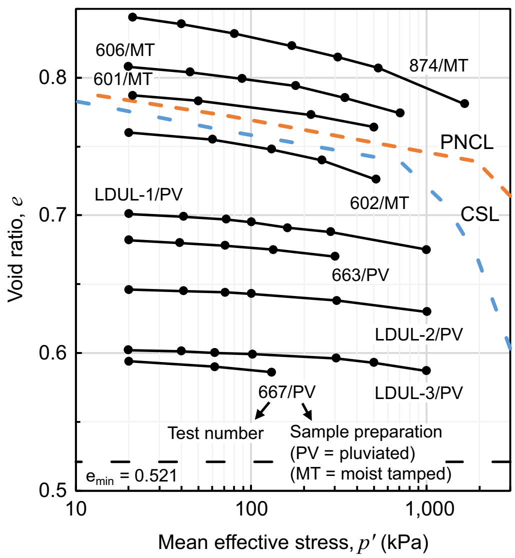
For this assessment, the review of the uniqueness of the NCL for gold tailings will be evaluated according to these aforementioned aspects.

PREVIOUS INVESTIGATIONS: GOLD TAILINGS

Perhaps the most well-known investigation into the shearing behavior of gold tailings is the study conducted by Reid et al. (2021). In the study, which involved 16 geotechnical laboratories around the world, each laboratory was provided with a large bulk sample of gold tailings and was requested to determine the CSL independently.

FIG. 3

Results from triaxial tests conducted on Erksak 330/0.7 sand (after Jefferies and Been 2000).



Remarkably, with a few exceptions, a unique CSL was identified to within a void ratio difference on 0.04. The study concluded that for the grading assessed, a unique CSL did exist for the gold tailings tested.

Although this study is useful for understanding the expected behavior of gold tailings in general, it would be more useful to understand the expected behavior of gold tailings mined from the same reef as the gold tailings under consideration for this study. Vermeulen (2001) and Fourie and Papageorgiou (2001) both studied the shearing behavior of gold tailings mined from the Witwatersrand Supergroup. Although it was concluded that the CSL varied depending on the grading of the gold tailings, it was found that for a given grading, a unique CSL exists. It is therefore expected that the gold tailings will not exhibit transitional behavior, at least in terms of the uniqueness of the CSL, for the grading being assessed.

Three additional studies are also of interest regarding gold tailings. Bedin et al. (2012), Schnaid et al. (2013), and Li (2017) all conducted a comprehensive laboratory testing program to investigate the compression behavior of a type of Brazilian gold tailings. Bedin et al. (2012) and Schnaid et al. (2013) found some evidence to suggest that a unique NCL exists but noted that the stress levels that were tested (<1,200 kPa) were too low to conclusively identify the NCL. Li (2017) was able to test up to 7 MPa but could also not identify a unique NCL. Convergence was eventually noted at 20 MPa, but at these high stress levels it is unclear whether this was indeed the NCL or simply the limiting compression curve (LCC).

Interestingly, vastly different conclusions were also drawn regarding particle crushing. Li (2017) found no evidence of particle crushing, even at stresses of up to 7 MPa, whereas Bedin et al. (2012) reported moderate particle crushing at stresses as low as 600 kPa. Although the tailings were reported to have been sampled from the same dam, it was specially noted that the gradings were different, which could perhaps explain the reason for these different findings.

Nonetheless, uncertainty regarding the compression behavior of the gold tailings under consideration still remains, and thus the objective of this study was to attempt to reduce this uncertainty.

Materials and Testing Procedures

Bulk samples of gold tailings were obtained from an active gold tailings dam in South Africa. The mine is located in the East Rand in Gauteng, and mining is from the Witwatersrand Supergroup. The tailings dam is an upstream-constructed facility, and material was sourced near the outer wall. A typical grading of the material is shown in figure 4. Additional gradings have been shown to provide context for the grain sizes of the soil under consideration and for comparison with the soils referenced in this study. The figure shows that the gold tailings are coarser than the fine-grained clays of high plasticity but finer than the coarse-grained clean sands, which supports the statement that there may be uncertainty regarding its compression characteristics.

The selected properties of the gold tailings are shown in Table 2. A mean grain size of 0.063 mm was determined using laser diffraction in accordance with ISO 13320:2020, *Particle Size Analysis — Laser Diffraction Methods*, and a specific gravity of 2.74 was determined using the gas pycnometer method described in ASTM D5550-14, *Standard Test Method for Specific Gravity of Soil Solids by Gas Pycnometer* (Superseded). Minimum and maximum void ratios of 0.47 and 1.57 were determined according to ASTM D4253-16e1, *Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table*, and ASTM D4254-16, *Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density*, respectively.

For the minimum void ratio, Test Method 1A was used, and an average void ratio for the two tests conducted was determined. For the maximum void ratio, Test Methods A and C were followed, and an average void ratio was determined. The material classifies as a nonplastic sandy silt, s(ML), according to ASTM D2487-17e1, *Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System)*. The parameters obtained are comparable to those published in literature for gold tailings mined from the Witwatersrand Supergroup (e.g., Vermeulen 2001; Papageorgiou 2004; Chang, Heymann, and Clayton 2011).

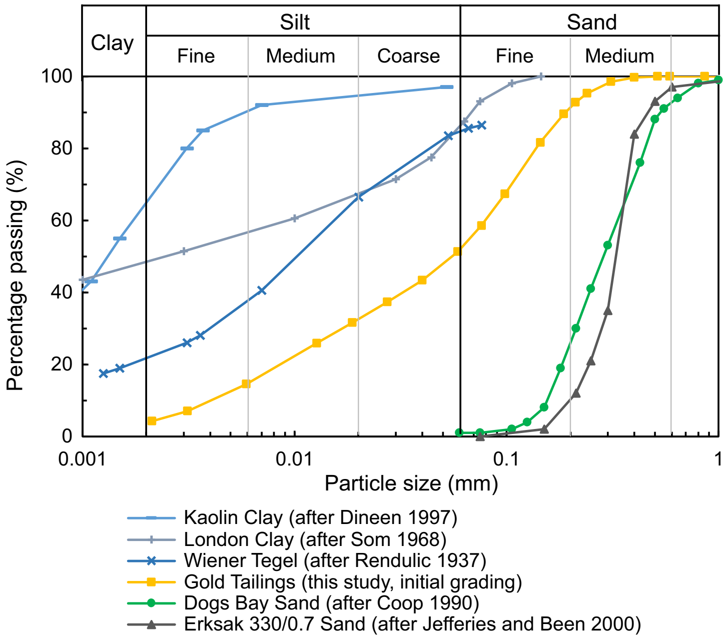


FIG. 4 Grading analysis for the gold tailings used in this study as well as other relevant soils (after Dineen 1997; Som 1968; Rendulic 1937; Coop 1990; Jefferies and Been 2000).

TABLE 2
Selected gold tailings properties

Parameter	Value	Test Method
Mean grain size (d_{50})	0.063 mm	ISO 13320:2020
Maximum void ratio (e_{max})	1.57	ASTM D4254-16
Minimum void ratio (e_{min})	0.47	ASTM D4253-16e1
Specific gravity (G_s)	2.74	ASTM D5550-14
Soil classification	s(ML), nonplastic sandy silt	ASTM D2487-17e1

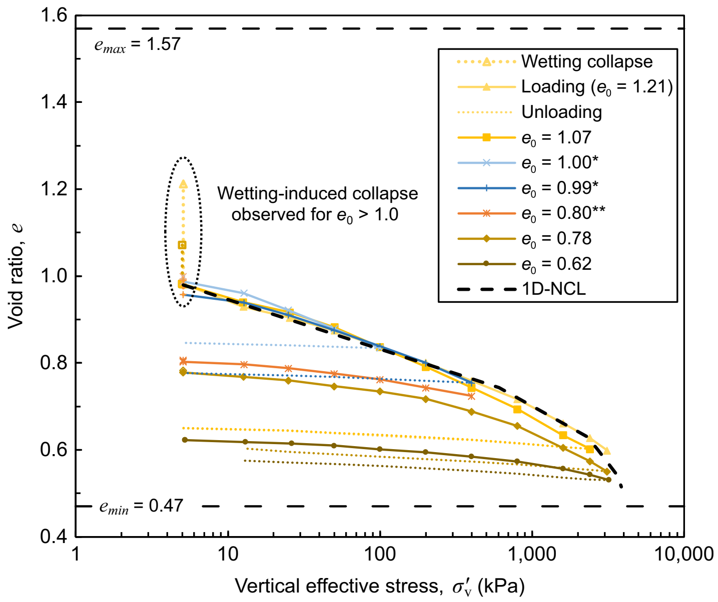
OEDOMETER TESTING

A total of seven oedometer tests were conducted at the geotechnical laboratory at the University of Pretoria. The tests were performed in accordance with ASTM D2435/D2435M-11(2020), *Standard Test Methods for One-Dimensional Consolidation Properties of Soils Using Incremental Loading*. Test Method B was followed, for which time-based deformation readings were obtained for all loading and unloading increments. Increments were considered complete when 100% of primary compression or swell had occurred. The masses of the weights used for the loading increments were confirmed using two different mass balances. For mass pieces less than 6 kg, a mass balance with a resolution of 0.01 g was used, and for mass pieces greater than 6 kg, a mass balance with a resolution of 1 g was used. The samples for the oedometer testing were prepared to the target densities at a moisture content of 15 % by moist tamping. Initial moisture contents were verified by taking end-of-test moisture contents, assuming full saturation, and performing a backward analysis. Standard 76.2-mm diameter, 18.9-mm high oedometer rings were used. A calibrated vernier caliper with a resolution of 0.01 mm was used to verify both the average height and diameter of the oedometer ring before and after each test. Care was taken to facilitate level surfaces that were flush with the top and bottom of the ring to minimize bedding errors.

Tests were performed on an ELE International 3,200 kPa consolidation frame with a lever arm ratio of 1:10. Sample settlement readings were recorded using an ELE International mechanical dial gauge with a range of

FIG. 5

Results from the oedometer testing (*tests for particle size distribution analysis; **test abandoned because of dial gauge malfunction).



25 mm and a resolution of 0.02 mm. Load increments continued until the end of primary consolidation, with a minimum consolidation time of 2 hours.

To investigate the behavior of the tailings upon saturation, the first load increment of 5 kPa was performed in two stages. First, the load was applied to the unsaturated soil, and settlement was allowed to take place. Once primary settlement had been completed, the cell was flooded with water (still under the applied load of 5 kPa) and compression was allowed to take place. The start and end points of these two phases are represented as “wetting-induced collapse” in **figure 5**.

TRIAXIAL TESTING

To supplement the oedometer testing, a total of ten triaxial compression tests were conducted on the gold tailings. The tests were also conducted at the geotechnical laboratories at the University of Pretoria. Testing was performed on modern Global Digital Systems (GDS) automated triaxial systems according to the guidelines provided by Jefferies and Been (2015). The samples were all prepared using the moist-tamping method at an initial water content of 15 %. Saturation was confirmed for all samples with a minimum B-value of 0.96. All void ratios were determined using backward calculation to eliminate errors associated with the collapse of loose sandy samples during saturation, as identified by Sladen and Handford (1987). The squeezing method as proposed by Verdugo and Ishihara (1996) was used to improve the accuracy of the end-of-test moisture content determination.

The under-compaction method as proposed by Ladd (1978) was used for sample preparation. Samples were created to be 70 mm in diameter and 140 mm in height. The mass for each layer was determined using a calibrated mass balance with a resolution of 0.001 g, and the height of each layer was controlled using a collar on the tamper, the height of which was set and verified using a vernier caliper with a resolution of 0.01 mm. After removal of the split mold, the initial sample geometry was determined using the same vernier caliper mentioned previously.

During the test, the sample height was tracked using the loading ram and an enforced nominal contact load of 20 N (approximately 5 kPa). This change in height was measured using a calibrated linear variable differential transformer with a resolution of 0.001 mm. The cell and back pressures and volumes were measured using GDS automated pressure controllers (V2), which have a resolution of 0.1 kPa and 1 mm^3 for the pressure and volume, respectively. Pore pressures were measured at the base of the sample using a calibrated 2-MPa pressure transducer with a resolution of 1 kPa. At the end of the test, the wet and dry sample masses were determined using a calibrated mass balance with a resolution of 0.1 g (approximately 0.01 % of the mass of a sample).

Compression Behavior

OEDOMETER TEST RESULTS

The intent of the oedometer testing was to investigate the uniqueness of the 1D-NCL of the gold tailings. To achieve this, samples were prepared at varying initial void ratios and were consolidated in small increments to high stresses. Care was taken during the sample preparation process to document any stress history of the sample because this would directly influence its initial state, which is key to the interpretation of the resulting compression curves.

The two loosest samples ($e_0 = 1.19$ and $e_0 = 1.03$) required minimal compaction effort and were deemed to be in an NC state at the start of the test. The two dense samples required additional compaction effort and were deemed to be in an OC state at the start of the test. The second densest sample ($e_0 = 0.78$) was compacted by hand using a metal tamper and a hammer, whereas the densest sample ($e_0 = 0.62$) required the use of a hydraulic press to achieve the desired compaction. A load cell was fitted to the frame to record the force applied to the sample during sample preparation, and a peak force of 25.5 kN (equivalent to a total stress of 5.7 MPa) was recorded, clearly indicating the sample was in an OC state.

Unfortunately, the dial gauge malfunctioned on one of the dense tests ($e_0 = 0.80$), so the test had to be abandoned during the 400-kPa load increment. To investigate the aspect of particle crushing, two additional tests were conducted ($e_0 = 1.00$ and $e_0 = 0.99$). Key details of the tests conducted are shown in **Table 3**.

The results from the oedometer testing are shown in **figure 5**. The minimum and maximum void ratio values are also presented for reference. In general, the primary consolidation was completed within a few minutes. Time for 90 % consolidation (t_{90}) was determined using Taylor's construction method (Taylor 1948) with an average time of less than 1 minute. Associated coefficient of vertical consolidation (c_v) values ranging between 239 m^2/year and 472 m^2/year were also determined. The estimated coefficient of permeability values ranged between 1×10^{-9} and 1×10^{-7} m/s. Again, these values match well with those found in literature for gold tailings from the Witwatersrand Supergroup.

A few aspects of the compression curves are of particular importance. First, wetting-induced collapse can be observed for the loosest two samples. The loosest sample ($e_0 = 1.19$) exhibited a reduction in void ratio of 0.23 during

TABLE 3

Summary of key aspects of the conducted oedometer tests

Test	Initial Void Ratio (e_0)	Relative Density, %	C_c: Final Three Load Increments	C_e: Unloading	C_e: Initial Loading	General Comments
1	1.19	34.5	0.198	0.0190	0.0814	Loosest specimen. Minimal compaction effort required during sample preparation. Large settlement upon wetting ($\Delta e = 0.23$).
2	1.03	49.1	0.188	0.0190	0.0756	Loose specimen. Low compaction effort required during preparation. Moderate settlement upon wetting ($\Delta e = 0.09$).
3	0.80	70.0	0.063	…	0.0224	Test was abandoned at the 400-kPa stress increment due to a dial gauge malfunction.
4	0.78	71.8	0.188	0.0194	0.0275	Sample was prepared by hand tamping. Moderate compaction effort was required during sample preparations.
5	0.62	86.4	0.103	0.0176	0.0189	Sample was statically compacted in a hydraulic press. A peak force of 25.5 kN was applied, equivalent to a total stress of 5.7 MPa.
6	1.00	51.8	0.147	0.0103	0.0653	Test performed to obtain a sample for grading analysis. Minor settlement upon wetting ($\Delta e = 0.01$).
7	0.99	52.7	0.151	0.0156	0.0473	Test performed to obtain a sample for grading analysis. Minor settlement upon wetting ($\Delta e = 0.03$).

saturation, whereas the second loosest sample ($e_0 = 1.03$) exhibited a reduction in void ratio of 0.09 during saturation. This behavior is consistent with the critical state soil mechanics framework of a soil in a looser state than the NCL.

Second, samples in an initially NC state ($e_0 = 1.19$, $e_0 = 1.03$, $e_0 = 1.00$, and $e_0 = 0.99$) showed consistent compression behavior at low stresses and tended sharply to an apparent unique NCL at moderate stresses. The slopes of these compression curves once they reached the NCL are fairly similar, as indicated in **Table 3**, varying between 0.15 and 0.20. There is, however, a slight vertical offset between the curves, which can perhaps be attributed to the inherent variability in particle size distribution of the gold tailings samples. Conversely, samples prepared to an initially OC state ($e_0 = 0.80$, $e_0 = 0.78$, and $e_0 = 0.62$) showed a much flatter compression curve initially and then gradually tended toward the apparent unique NCL. Interestingly, the slopes of initial loading curves of the OC soils (C_e between 0.019 and 0.028) were similar to the slopes of the unloading curves of the NC soils (C_e between 0.010 and 0.019). The second densest sample ($e_0 = 0.78$) appeared to reach the NCL at around 800 kPa, but the densest sample ($e_0 = 0.62$) seemed to still be approaching the NCL at the last stress increment of 3,200 kPa, which is consistent with the known stress applied during sample preparation.

This behavior of the NC and OC samples is entirely consistent with the idealized behavior of a fine-grained soil as shown in **figure 1*A***. Although the compression curves for the gold tailings presented in **figure 5** may look similar to the compression curves presented for Erksak 330/0.7 sand in **figure 3**, there are two key differences. First, the dense samples of Erksak sand were prepared in an NC state, whereas the gold tailings were prepared in an OC state. Second, the loose samples (i.e., those above the NCL) showed significant collapse during saturation at low applied stresses, indicating that they perhaps had a metastable structure prior to saturation.

Quantifying Convergence

Although convergence of compression curves can be evaluated visually, it is more useful to quantify the degree of convergence. Ponzoni et al. (2014) proposed a means of quantifying the degree of convergence for a number of one-dimensional compression curves being assessed for convergence, typically within a framework of identifying soils that may exhibit transitional behavior. To implement the method, the specific volume of each sample should be compared at two stress levels: the initial stress level, and the stress level at which convergence is being assessed. These values are plotted on a graph for each test, in which the vertical axis displays the specific volume at the stress level at which convergence is being assessed and the horizontal axis shows the initial specific volume. The slope of the linear trendline drawn between these values for all the samples is described as the parameter m. Using this approach, m varies between 0 and 1, where an m value of 0 indicates a unique compression curve and an m value of 1 indicates compression curves that are parallel.

For this study, the initial specific volume was taken at a vertical effective stress of 5 kPa, whereas the vertical effective stress at which convergence is assessed was taken at 2.4 MPa. **Figure 6** shows a plot of the specific volume at 2.4 MPa (υ_{2400}) versus the initial specific volume (υ_5) for the oedometer tests. An m value of 0.20 was determined for the gold tailings. As discussed earlier, the existence of a unique NCL would result in an m value of 0. However, this value of 0.20 is only slightly greater than the value of 0.13 obtained for the Brazilian gold tailings at 7 MPa by Li (2017), who found this value to reduce to 0 at 20 MPa and classified the tailings as having a unique 1D-NCL. Additionally, this value is lower than the value of 0.40, at which possible transitional behavior was noted for Venice Lagoon silts and silty clays (Ponzoni et al. 2014), and the value is significantly less than the 0.69 and 0.71 values found for Chinese copper tailings and Australian mineral sand tailings that were classified as transitional soils by Li (2017) and Cartwright, Coop, and Fourie (2022), respectively. Therefore, for practical purposes, it was concluded that there exists a unique 1D-NCL for the gold tailings.

TRIAXIAL COMPRESSION TEST RESULTS

The aim of the triaxial compression tests was to determine whether the NCL (defined during isotropic compression) is parallel to the 1D-NCL as obtained during one-dimensional compression when plotted in $e - \ln p'$ space. A summary of the key aspects of the triaxial tests conducted to evaluate the compression behavior of the gold tailings is provided in **Table 4**. A similar strategy as that adopted for the oedometer tests was used for the triaxial tests.

FIG. 6

Quantification of the degree of convergence of the compression curves.

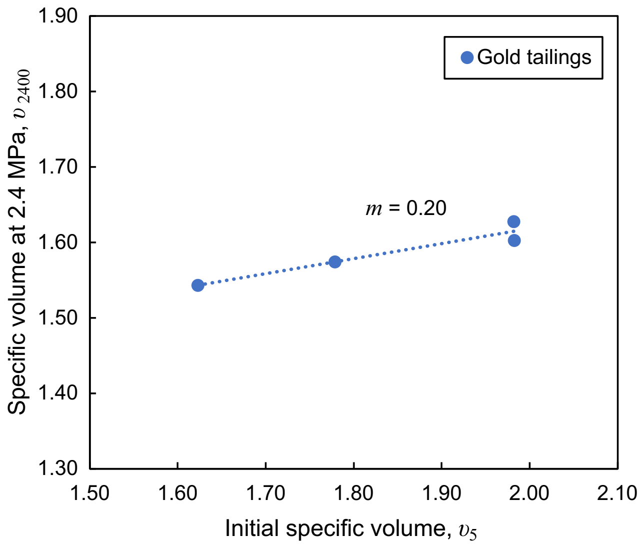


TABLE 4

Summary of key aspects of the triaxial tests for compression behavior

Test ID	Initial Void Ratio	Slope of 100–200-kPa Increment	Slope of 800–1,000-kPa Increment	Average Slope of Unloading Stages
txl ($e_0 = 1.18$)	1.18	0.056	0.062	0.009
txl ($e_0 = 1.10$)	1.10	0.050	0.070	0.009
txl ($e_0 = 0.84$)	0.84	0.027	0.071	0.009
txl ($e_0 = 0.74$)	0.74	0.016	0.043	0.008

A total of four tests were conducted; two samples were prepared in a loose state, and two samples were prepared in a dense state. As with the oedometer testing, little to no compaction effort was required to prepare the samples at an initial void ratio greater than 1.0. However, moderate to significant compaction effort was required to prepare the samples at initial void ratios of 0.84 and 0.74, respectively. At the start of the test, the two loosest samples were therefore considered to be in an NC state, and the two densest samples were considered to be in an OC state.

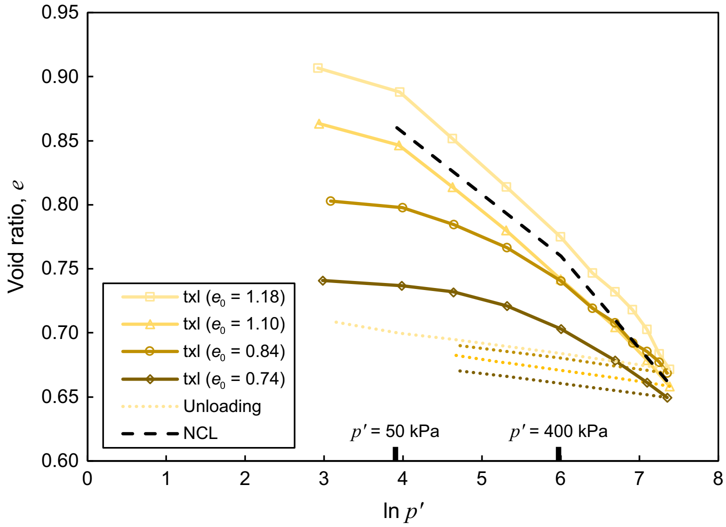
During testing, the samples were allowed to consolidate until the end of primary consolidation was observed. For the loading stages, a minimum consolidation time of 60 minutes was set, and a minimum time of 30 minutes was set for the unloading stages.

Also provided in Table 4 are the slopes of the compression curves at the 100–200-kPa loading increment and the 800–1,000-kPa loading increment. The average slopes of the unloading increments are also provided. It can be seen that the slopes at the 100–200-kPa increment are similar for the two loosest samples ($e_0 = 1.18$ and $e_0 = 1.10$), and these are significantly steeper than the slopes of the two densest samples ($e_0 = 0.84$ and $e_0 = 0.74$). At this point, it is suggested that the two loose tests lie on the NCL and the two dense tests are still approaching the NCL. However, at the 800–1,000-kPa increment, all samples except the densest sample ($e_0 = 0.74$) have similar slopes greater than the slope of the CSL. At this stage, it is suggested the loosest three samples ($e_0 = 1.18$, $e_0 = 1.10$, and $e_0 = 0.84$) lie on the NCL, whereas only the densest test ($e_0 = 0.74$) is still approaching the NCL. The average slope of the unloading curves was similar for all four tests.

Figure 7 shows the compression results from the triaxial tests in an $e - \ln p'$ projection. The two loose samples collapsed almost immediately into the range marked as the NCL region. This matches the behavior observed in the oedometer tests for the NC samples ($e_0 = 1.19$, $e_0 = 1.03$, $e_0 = 1.00$, and $e_0 = 0.99$). The dense

FIG. 7

Results from the triaxial (txl) testing for compression characterization.



samples, which were in an OC state, exhibited compression curves with flatter slopes than those observed for the NC samples. Again, this is consistent with the behavior observed from the oedometer tests for the OC samples ($e_0 = 0.80$, $e_0 = 0.78$, and $e_0 = 0.62$).

Unlike the oedometer test results, in which the compression curves for the NC samples converged rapidly, there is clearly an offset between the two NC samples for the triaxial tests ($e_0 = 1.18$ and $e_0 = 1.10$). The offset is approximately $e = 0.04$, which is the same value that was determined by Reid et al. (2021) to be the accuracy to which the CSL can reasonably be expected to be determined using modern triaxial testing techniques. Since the same approach that was used to determine the CSL was also used to determine the NCL in terms of void ratio estimates, it seems reasonable to assume that this accuracy applies to the NCL, as well. The NCL has therefore been drawn between these two curves with a slight bias toward the initially denser sample ($e_0 = 1.10$), which experienced less collapse during flushing and saturation. Considering the previous discussion, the authors are of the opinion that the results suggest that the gold tailings can be viewed as having a unique NCL, specifically for the intention of classifying the soil state for NC samples and for laboratory test planning purposes.

COMBINED OEDOMETER AND TRIAXIAL TEST RESULTS

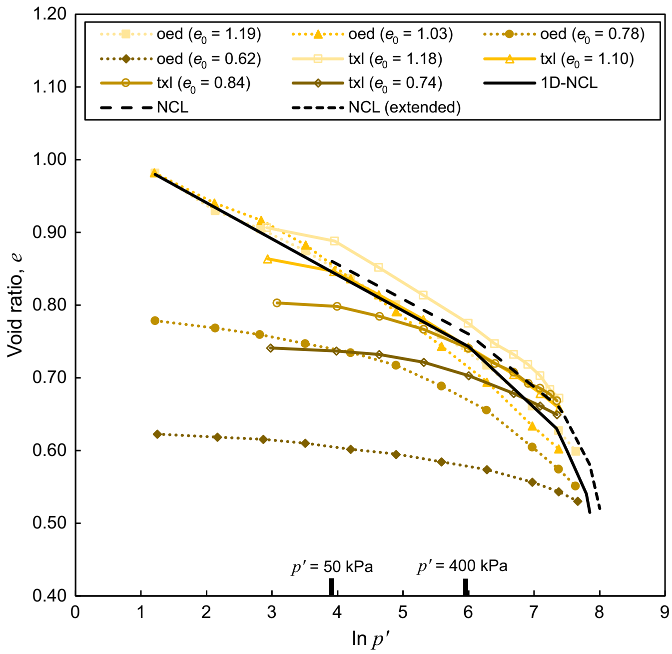
Although it is useful to plot the void ratio versus vertical effective stress (σ_v'), a more common way to present the data is to use the stress invariant, mean effective stress (p'). However, the use of this term requires knowledge of the horizontal effective stress, which is not known in the standard oedometer test. To address this, the coefficient of lateral earth pressure at rest (K_0), which is the ratio between the horizontal and vertical effective stress, can be used. For this study, a K_0 value of 0.6 was assumed based on the work by Fourie and Tshabalala (2005) on gold tailings from the same ore body and with similar grading to the material tested in this study. The mean effective stress was calculated as shown in equation (1).

$$p' = \frac{1}{3}[\sigma_v'(1 + 2K_0)] \tag{1}$$

The oedometer test results are plotted together with the triaxial test results in an $e - \ln p'$ projection in figure 8. As a result of system limitations, it was not possible to consolidate the triaxial samples to the same effective stress values as the oedometer tests, and therefore a direct comparison of the compression curves at high stresses was

FIG. 8

Results from the oedometer (oed) and triaxial testing, plotted in $e - \ln p'$ space.



not possible. To conclusively investigate the relationship between the 1D-NCL and NCL, a higher capacity triaxial testing system is required. However, with the data available, it does not seem unreasonable to assume that, for practical purposes, the 1D-NCL and NCL are parallel for the gold tailings under consideration at low to moderate stress levels. For example, between 50 and 400 kPa, the slope of the 1D-NCL is 0.049 and that of the NCL is 0.048.

PARTICLE CRUSHING

Figure 9 shows the results of the grading analyses conducted on the gold tailings before and after testing. Unfortunately, only a single grading was taken for the initial material. There is some evidence of the increase in fines with applied loading. For example, the d_{50} decreased from 0.063 mm per the initial grading to 0.055 mm as a result of an applied loading of 3.2 MPa. This is in line with findings by other researchers regarding particle crushing in tailings materials (e.g., Zhang et al. 2020).

For this material, however, this change in grading seems to be masked by the inherent variability of the particle sizes in the gold tailings sample that was used to create the oedometer samples. Although the bulk samples were obtained from the same location on the tailings dam, it is known that tailings material tends to be highly variable (e.g., Vermeulen 2001).

This may be further compounded by the fact that the gradings were determined using laser diffraction analysis for which only a small mass of sample was tested (approximately 15 g). A more robust study is recommended to investigate the variability in particle size of the large bulk sample. The study should consider multiple samples using the laser diffraction method as well as the use of traditional sieves and hydrometers.

In summary, some particle crushing was noted, but it appears that this was limited and that the grading of the material did not change significantly.

Shear Behavior

The aim of the CSL testing was to determine whether there exists a unique CSL for the gold tailings and to determine whether, for practical purposes, the NCL is parallel to the CSL at low to moderate stress levels. A total of six tests

FIG. 9
Grading analysis of the gold tailings after testing (samples labeled based on maximum loading during the test).

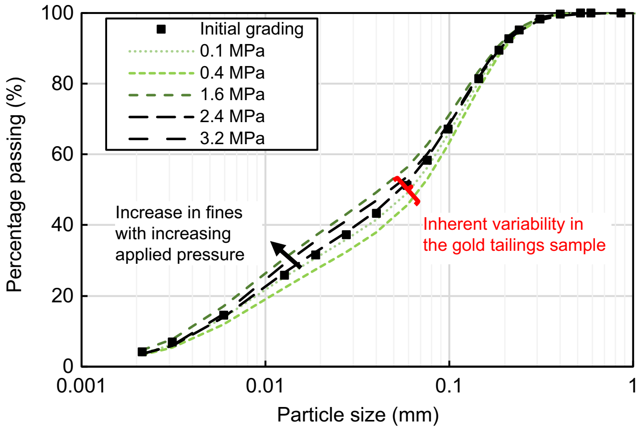


TABLE 5
Summary of key aspects of the triaxial tests conducted to determine the CSL

Test ID	Description	Void Ratio at Start of Shear	Void Ratio at End of Shear	Mean Effective Stress at Critical State, kPa	Deviator Stress at Critical State, kPa
CU100-L	Loose sample, sheared undrained from $p' = 100$ kPa	0.831	0.831	2.6	2.7
CD400-L	Loose sample, sheared drained from $p' = 400$ kPa	0.749	0.643	764.9	1,090.0
CU100-D	Dense sample, sheared undrained from $p' = 100$ kPa	0.751	0.751	33.0	51.2
CD100-D	Dense sample, sheared drained from $p' = 100$ kPa	0.755	0.676	217.2	348.6
CU100-D(4)	Dense sample, OC to $p' = 400$ kPa, sheared undrained from $p' = 100$ kPa	0.718	0.718	71.6	116.1
CU100-D(8)	Dense sample, OC to $p' = 800$ kPa, sheared undrained from $p' = 100$ kPa	0.686	0.686	179.3	255.0

Note: CD = consolidated and sheared drained, CU = consolidated and sheared undrained, L = loose sample, D = dense sample.

were conducted to determine the CSL for the gold tailings. Key aspects of the tests are provided in Table 5. A range of stress conditions, densities, and shearing drainage conditions were tested. This was done such that the CSL could be defined for a wide range of stress levels to assist with future calibration for numerical modeling purposes. The mean effective stress and deviator stress values at the critical state have also been provided for reference.

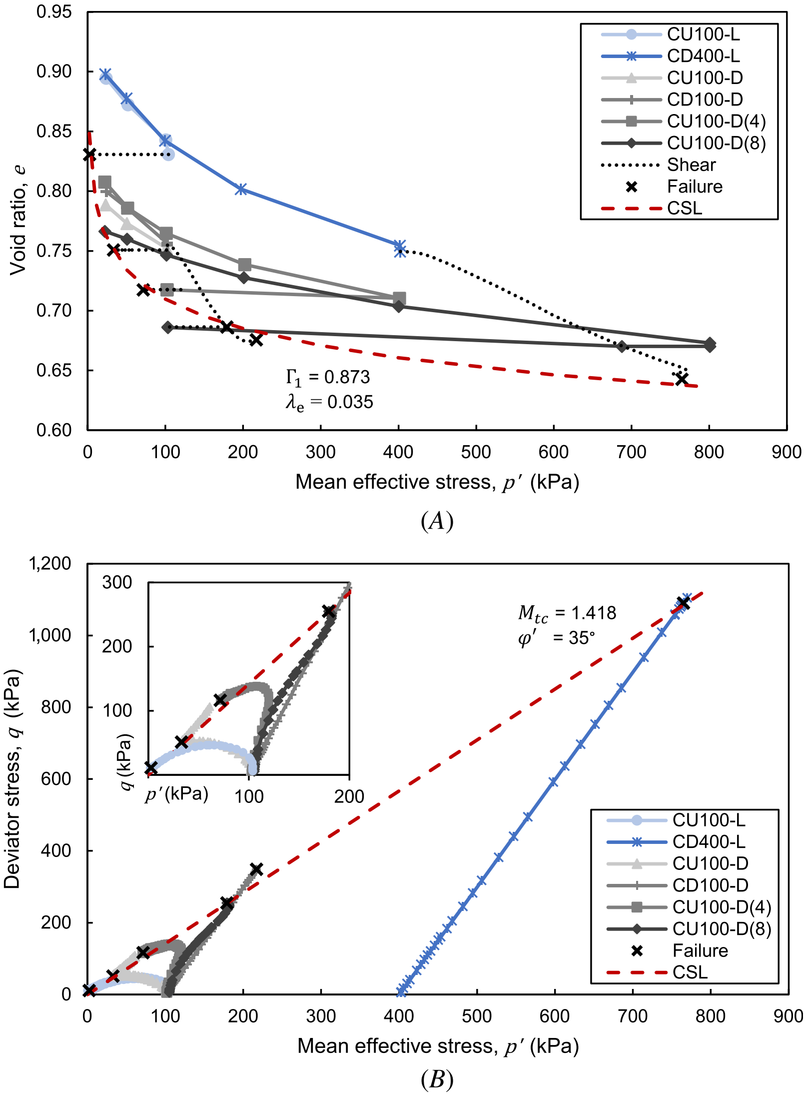
The results from the CSL testing are shown in figure 10. Both the consolidation and shear stress paths are presented in an $e - p'$ projection in figure 10*A*, whereas only the stress paths during shearing are presented in a $q - p'$ projection in figure 10*B*. The results cover a fair range of stress and density conditions, and the definition of a unique CSL seems appropriate. The CSL has been defined as a straight line in $e - \ln p'$ space, as shown with an intercept at 1 kPa of $\Gamma_1 = 0.873$ and a slope of $\lambda_e = 0.035$. In the $q - p'$ space, the CSL has been defined with an effective friction angle of 35°.

Discussion

The compression behavior of fine-grained soils (such as high-plasticity clays) and coarse-grained soils (such as clean sands) is distinctly different but fairly well understood. However, the compression behavior of mine tailings,

FIG. 10

Results from the CSL testing showing: (*A*) the definition of the CSL, and (*B*) the interpreted shear strength parameters.



which are generally silty soils, is still not as well understood, and there is some uncertainty regarding their behavior in compression. Although there is some theoretical and experimental evidence in the literature regarding the compression behavior of some mine tailings, it is known that the grading and mineralogical composition (which can vary substantially from mine to mine) play an important role in the compression behavior of the tailings. Therefore, it is of interest to investigate the behavior of the specific tailings material under consideration to improve the reliability of tailings dam designs and safety evaluations.

Knowledge of the uniqueness of the NCL is particularly useful for estimating changes in stress state due to increases in stress (either due to routine deposition as the dam is raised or due to buttressing activities, which are becoming more common to improve stability to modern safety standards). This is also useful to know when planning laboratory testing procedures, for which it is important to create soils at a desired state without inducing unwanted stress histories during sample preparation.

In this study, seven oedometer tests and ten triaxial tests were conducted to investigate the compression and shear characteristics of sandy silt gold tailings from an active tailings dam in the East Rand of Johannesburg, South Africa. It should be noted that all the samples were remolded using the moist-tamping sample preparation method and that these findings are therefore limited to this fabric. Key findings from this study are discussed subsequently.

UNIQUENESS OF THE 1D-NCL

There appears to be a unique 1D-NCL for the gold tailings. Quantifying the convergence of the compression curves up to 2.4 MPa, obtained from oedometer testing using the method proposed by Ponzoni et al. (2014), resulted in an m value of 0.20. A unique NCL would yield a value of 0. However, this value is significantly lower than the m values of 0.69 and 0.71 obtained by other researchers for transitional soils for which a unique NCL could not be found (Li 2017; Cartwright, Coop, and Fourie 2022). This indicates that for engineering purposes, the gold tailings can be considered to have a unique NCL. Further, it is likely that these compression curves will converge at higher stresses and that the associated m value will tend to 0. However, this was not tested as part of this study.

UNIQUENESS OF THE NCL

For moderate stresses between 50 and 400 kPa, there appears to be a unique NCL, and the 1D-NCL and NCL appear to be parallel for the gold tailings. By assuming a K_0 value for the tailings, it was possible to directly compare the 1D-NCL and NCL compression curves. Over a moderate stress range between 50 and 400 kPa, these lines can be considered parallel for engineering purposes. Over this range, the 1D-NCL has a slope of 0.049 and the NCL has a slope of 0.048. The existence of a unique 1D-NCL up to high stresses of 2.4 MPa, and the fact that the 1D-NCL is parallel to the NCL, suggests that a unique NCL exists for the gold tailings under consideration.

UNIQUENESS OF THE CSL

There appears to be a unique CSL for the gold tailings. Six triaxial compression tests were conducted over a range of initial void ratios, stress levels, and drainage conditions during shearing. The CSL was defined as a straight line in the $e - \ln p'$ space between the stress range of 50 and 800 kPa. Although not part of the scope of this testing plan, it is likely that the CSL will curve significantly at higher stresses, as was found for other gold tailings (e.g., Schnaid et al. 2013).

TRANSITIONAL BEHAVIOR

The fact that a unique NCL and CSL could be identified suggests that the gold tailings tested are not a transitional soil and that their behavior can be captured by traditional critical state soil mechanics concepts.

SLOPE OF THE CSL AND NCL

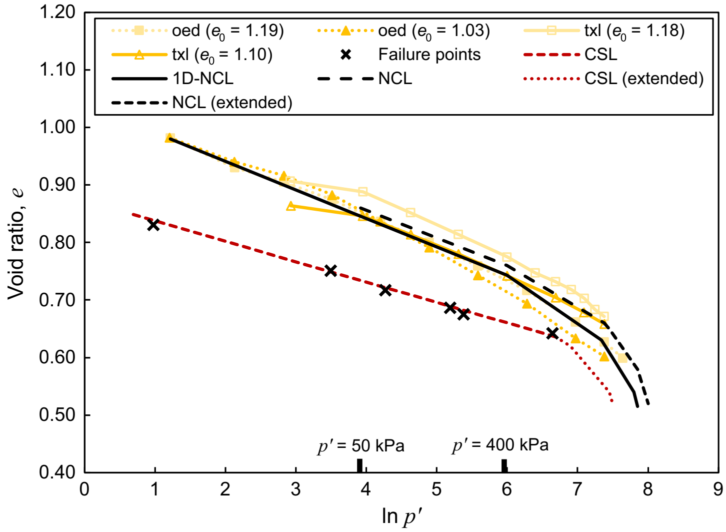
Figure 11 shows the combined results of the oedometer and triaxial tests. To match the 1D-NCL, the NCL and CSL have been extended to higher stress levels for illustrative purposes. The results suggest that at higher stresses, the CSL and NCL might in fact curve and will have a steeper slope, as was found by Bedin et al. (2012). It was assumed that these three curves would be parallel at higher stress, as is seen for sandy soils (e.g., Been, Jefferies, and Hachey 1991; Sully et al. n.d.).

At moderate stress levels between 50 and 400 kPa, the CSL and NCL of the gold tailings are not parallel; the slope of the CSL is 0.035, whereas the slope of the NCL is 0.048. However, the slopes are fairly similar, and they are sufficiently offset in the $e - \ln p'$ space such that, for the purposes of defining whether the soil is contractive or dilative at large strains, the NCL and CSL can be considered parallel.

For an NC sample, for example, the state parameter at 50 kPa is 0.126. This value changes to 0.100 at 400 kPa. In other words, at moderate stress between 50 and 400 kPa and if the soil element under consideration is NC, an increase in stress will not change the state parameter significantly and, more importantly, will not change the state of the soil in terms of a contractive or dilative tendency at large strains. Therefore, at moderate stress levels, the

FIG. 11

Combined oedometer and triaxial results.



concept of increasing the stress level such that the tailings move from a dilative state to a contractive state does not seem to be applicable for this material. This knowledge is particularly useful for future designs and safety evaluations of the tailings dam under consideration.

Conclusions

In this study, seven oedometer tests and ten triaxial tests were conducted to investigate the compression and shear characteristics of sandy silt gold tailings from an active tailings dam in the East Rand of Johannesburg, South Africa. It should be noted that all the samples were remolded using the moist-tamping sample preparation method and that these findings are therefore limited to this fabric. The following was concluded from the study:

1. There appears to be a unique 1D-NCL for the gold tailings. Quantifying the convergence of the compression curves up to 2.4 MPa, obtained from the oedometer testing using the method proposed by Ponzoni et al. (2014), resulted in an m value of 0.20. A unique NCL would yield a value of 0. However, this value is significantly lower than the m values of 0.69 and 0.71 obtained for transitional soils by other researchers, for which a unique NCL could not be found (Li 2017; Cartwright, Coop, and Fourie 2022). Therefore, for engineering purposes, the gold tailings can be considered to have a unique 1D-NCL.
2. For moderate stresses between 50 and 400 kPa, there appears to be a unique NCL, and the 1D-NCL and NCL appear to be parallel for the gold tailings. By assuming a K_0 value for the tailings, it was possible to directly compare the 1D-NCL and NCL compression curves. Over a moderate stress range between 50 and 400 kPa, these lines can be considered parallel for engineering purposes. Over this range, the 1D-NCL has a slope of 0.049 and the NCL has a slope of 0.048. The existence of a unique 1D-NCL up to high stresses of 2.4 MPa, and the fact that the 1D-NCL is parallel to the NCL, suggests that there exists a unique NCL for the gold tailings under consideration.
3. There appears to be a unique CSL for the gold tailings. Six triaxial compression tests were conducted over a range of initial void ratios, stress levels, and drainage conditions during shearing. The CSL was defined as a straight line in the $e - \ln p'$ space between the stress range of 50 and 800 kPa.
4. The fact that a unique NCL and CSL could be identified suggests that the gold tailings tested are not a transitional soil and that their behavior can be captured by traditional critical state soil mechanics concepts.
5. At moderate stress levels between 50 and 400 kPa, the CSL and NCL of the gold tailings are not parallel; the slope of the CSL is 0.035, whereas the slope of the NCL is 0.048. However, the slopes are fairly similar, and

they are sufficiently offset in the $e - \ln p'$ space that, for the purposes of defining whether the soil is contractive or dilative at large strains, the NCL and CSL can be considered parallel. For example, if the soil element under consideration is NC, an increase in stress from 50 to 400 kPa will only change the state parameter from 0.126 to 0.100. More importantly, this change in stress will not change the state of the soil in terms of contractive or dilative tendency at large strains. This knowledge is particularly useful for future designs and safety evaluations of the tailings dam under consideration.

Based on these conclusions, the authors are of the opinion that the gold tailings tested do not appear to exhibit an infinite number of parallel NCLs but can rather be viewed within the framework of having a unique NCL.

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