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The impact of preservation technology investments on lot-sizing and shipment strategies in a three-echelon food supply chain involving growing and deteriorating items

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A R T I C L E I N F O

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A B S T R A C T

Food production systems are complex industrial operations that often involve multiple parties. This study proposes inventory management strategies for a multi-echelon perishable food supply chain with growing and deteriorating items. The upstream end of the proposed food supply chain is the farming echelon where newborn growing items are reared to maturity. Following this, the items are sent to the processing echelon for processing, a term that collectively describes activities such as slaughtering, cutting and packaging. The aim of the processing echelon is to transform live growing items into processed food products that are suitable for human consumption. The downstream end of the supply chain is the retail echelon where consumer demand for processed food products is met. Once the items are processed, they are subject to deterioration at both the processing and retail echelons. In light of this, an integrated inventory model aimed at optimising the performance of the entire food supply chain is formulated. The impact of investing in preservation technologies is also investigated due to the perishable nature of food products. To do this, a secondary model that incorporates an investment in preservation technologies is formulated. The model, representing a simplified industrial food production system, is aimed at jointly optimising the lot-size, number of shipments, growing cycle duration, processing cycle duration and the preservation technology investment amount. The results from the numerical example demonstrate that the preservation technology investment is worthwhile because it results in reduced inventory management costs across the supply chain.

1. Introduction

Food is one of the most important components of life as it supports a variety of human functions, activities and behaviours [1]. However, despite the importance of food to human life, food wastage and food losses have become important problems in recent years. Food waste and losses not only have a direct impact on food security in developing countries, but also on said countries' potential development (economic and otherwise). The Food and Agriculture Organisation (FAO) of the United Nations reported that about 1.3 billion tons of food, representing roughly a third of global food production, is wasted or lost globally [2]. In the United States (US) alone, this figure was estimated to be close to 103 million tons. Of that sum, 39.8 million tons of food waste in the US is generated by industrial food manufacturing and processing [3]. While there are different causes of food waste and losses, such as production yields, channel distribution and food use practices, to name a few [2], most of the waste is due to various inefficiencies in the food supply chain [4,5]. Consequently, food supply chains should be managed as efficiently as possible in order to minimise food waste and losses. To this end, multiple researchers have developed models for efficiently managing inventory in food supply chains. Case in point, Raut et al. [6] formulated a fuzzy multi-criteria decision making model for reducing food losses through the use of cold-third party logistics providers while Garre et al. [7] used machine learning algorithms for production planning and control in a food supply chain with the aim of reducing food waste. Other researchers have developed models for food supply chains with applications to specific food items such as fish [8,9], palm dates [10] and sugarcane [11] while other researchers have focused on the benefits of an opaque selling scheme [12] and the crop planning problem [13].

The primary input of most food supply chains is growing items such as crops, fish or livestock. In most cases, these items are reared (grown), harvested (in the case of crops) or slaughtered (in the case of livestock) and processed into different packaged food products before they are ready for human consumption. Lately, considerable attention has been paid to the development of inventory management policies for these items owing to some of their unique characteristics, the most important

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of which is the fact the items have the ability to grow (with growth being defined as increasing in weight over time). Other unique characteristics of these items include their susceptibility to illnesses (while they are still alive and growing) and their tendency to deteriorate (after they have been slaughtered). The latter two characteristics lead to food waste and losses and hence, efficiently managing growing inventory items in food supply chains can lead to improvements in food waste and losses.

Owing to the fact that one way of improving the operational and financial efficiency of a food supply chain is through the use of an effective inventory management policy, this paper investigates joint ordering and shipment strategies in a multi-echelon food supply chain involving growing and deteriorating items. The first echelon of the proposed food supply chain is the farming echelon where newborn items are procured at the start of a new growing cycle. The items are fed throughout the growing cycle in order to facilitate growth. At the end of the growing cycle, the items are transferred to the next echelon in the food supply chain which is the processing echelon where the items are slaughtered, stunned, cut, de-boned and packaged (collectively termed processing) during the processing cycle in preparation for sale at the next echelon in the food supply chain (i.e the retailer echelon). During the retail (or consumption) cycle, consumer demand for processed inventory is met via the retail echelon. This paper is aimed at investigating ordering and shipment strategies via an integrated inventory model that also accounts for the deterioration of the processed inventory.

Moreover, a second inventory model is presented with the aim of investigating the impact of investing in preservation technologies. The second model also deals with a three-echelon food supply chain with the major difference being the assumption that the supply chain parties have the option to invest in preservation technologies (such as more advanced refrigeration technologies) that can reduce the deterioration rate for the processed inventory. This particular model is also aimed at optimising the amount invested in preservation technologies (in addition to the ordering and shipment decisions).

Apart from the introduction, this paper has six more sections. Related inventory models in the literature are briefly reviewed in Section 2. In addition to providing a description of the problem at hand, Section 3 also details the notations used throughout the paper and the assumptions made when developing the models. The two inventory models are developed in Sections 4 and 5, respectively, with the latter section presenting the model that considers an investment in preservation technologies. Managerial insights are drawn from the results of numerical analyses conducted in Section 6. The paper is then rounded off in Section 7 via the provision of concluding remarks and suggestions for future research.

2. Literature review

2.1. Item growth in single- and multi-echelon supply chain systems

The first lot-sizing model that explicitly considers item growth was developed by Rezaei [14]. The major difference between that particularly model and the classic economic order quantity (EOQ) model, developed by Harris [15], is that the items under study are living organisms and thus, are able to grow (or increase in weight) during the course of the replenishment cycle. Growth of these items is facilitated by feeding and hence, Rezaei [14]'s model also included feeding costs. The model was aimed at optimising the number of newborn items that should be procured at the beginning of a replenishment cycle in order to meet a deterministic demand rate for slaughtered fully-grown items. Owing to the complexity brought by the feeding function, a closed-form solution could not be determined and hence, a bisection method, the Newton-Raphson method to be specific, was used to solve the model. Of late, Rezaei [14]'s work has been extended in various ways in order to account for different realistic situations that might occur in both company-level (or single-echelon) settings and multi-echelon supply chain settings.

Some of the company-level inventory models presented as extensions of Rezaei [14]'s work include Nobil et al. [16]'s model which was developed under the assumption that shortages are permitted and fully back-ordered. Sebatjane and Adetunji [17] relaxed the implicit assumption in Rezaei [14]'s model that all the slaughtered models are of good quality and developed a inventory model for growing that are subjected to a quality inspection process aimed at screening out the inferior quality slaughtered inventory from the lot that is used to meet consumer demand. Alfares and Afzal [18] combined the theory behind Nobil et al. [16] and Sebatjane and Adetunji [17]'s models and studied a growing items inventory system with imperfect quality and permissible shortages. De-La-Cruz-Márquez et al. [19] also used the theory behind Nobil et al. [16] and Sebatjane and Adetunji [17]'s models to develop an extension, aimed at jointly optimising both the EOQ and the selling price, that considers growing items with imperfect quality, permissible shortages, carbon emissions and price-dependent demand. Khalilpourazari and Pasandideh [20] applied sequential quadratic programming to an extension of Rezaei [14]'s model that considered multiple growing items with various constraints including a limited on-hand budget, allowable holding costs and a limited warehousing capacity. Owing to the fact that growing items are living organisms with the potential to die, Gharaei and Almehdawe [21] used uniform density functions to define the growing items' survival and mortality functions. Other notable (company-level) extensions to Rezaei [14]'s model include those that incorporate incremental quantity discounts [22] and the impact of over-breeding [23].

The theory behind Rezaei [14]'s model has also been extended to multi-echelon supply chain settings, ranging from two-echelon [24] to four-echelon settings [25]. Malekitabar et al. [24] developed a model for a two-echelon (supplier and retailer) supply chain for growing items with price-dependent demand and item deterioration. Malekitabar et al. [24]'s model assumed that the items are grown at the supplier's side and then sold at the retailer's side after they are slaughtered. Pourmohammad-Zia et al. [26] also considered a two-echelon supply chain for growing and deteriorating items with dynamic price-dependent demand and studied the problem under both centralised and decentralised supply chain policies. Based on the observation that in most cases the fully-gown items have to be processed further before they are sold, Sebatjane and Adetunji [27] formulated a model for three-echelon (farmer, processor and retailer) supply chain for growing items. Sebatjane and Adetunji [27]'s model assumed that the items are reared at farm, then processed into a saleable form at a processing plant and finally, the processed and packaged inventory is used to meet consumer demand at a retail outlet. The three-echelon setting introduced by Sebatjane and Adetunji [27]'s model served as a basis for two extensions, namely, Sebatjane and Adetunji [28] and Sebatjane and Adetunji [29], that considered item deterioration at the retail echelon by defining the demand rate as a function of the items' expiration dates (among other factors, with the demand rate in the former extension also being dependent on selling price while the latter extension considered stock-dependent demand). Pourmohammad-Zia et al. [30] applied game-theoretic techniques, namely, Nash and Stackelberg competitions, to study a three-echelon (farmer, manufacturer and multiple retailers) supply chain with price-dependent demand. Sebatjane and Adetunji [31] studied a farmer-processor-retailer supply chain for growing and deteriorating items whose rate of deterioration is dependent on the processed items' maximum shelf life or expiration date (i.e. age-dependent deterioration).

2.2. Item amelioration in single- and multi-echelon supply chain systems

In the context of lot-sizing, growing items are defined as items whose weight increases during the course of the replenishment cycle. In recent years, growing items have developed into an important class of inventory items. While the first lot-sizing model that explicitly considers item growth was developed by Rezaei [14], models for ameliorating items (defined as those whose utility increases over time) had been for some time and they laid the foundation for growing inventory items. For instance, using the Weibull distribution to describe the amelioration rate, Hwang [32] developed an inventory model for a system with ameliorating items. The combined effect of both amelioration and deterioration were accounted for in models presented by Hwang [33] and Mondal et al. [34], with the former model also considering a constant demand rate while the latter considered price-dependent demand.

Likewise, the single-echelon inventory models for ameliorating items have been extended to scenarios that consider multi-echelon supply chains. For instance, Law and Wee [35] extended Hwang [33] and Mondal et al. [34]'s models to a multi-echelon supply chain setting with a single manufacturer and a single retailer while also considering the time value of money. Singh and Vishnoi [36] and Sana and Vandana [37] developed integrated inventory models for both ameliorating and deteriorating items in a two-echelon supply chain, with the former researchers also considering price-dependent demand under a two-warehouse system at the consumption echelon and later researchers developing their model under the assumption that there are multiple buyers at the consumption echelon.

2.3. Item deterioration in multi-echelon supply chain systems

Item deterioration has been studied extensively in the context of inventory systems modelling, starting with the seminal work by Ghare and Schrader [38] who developed an EOQ model for an item deteriorating at a constant rate. A number of extensive literature reviews detailing developments in the modelling of deteriorating inventory systems have been published, most notably, Goyal and Giri [39], Bakker et al. [40] and Janssen et al. [41] which cover research papers published prior to 2001, those published between 2001 and 2012 and those published between 2012 and 2015, respectively.

Yang and Wee [42] extended the theory behind Ghare and Schrader [38]'s model' to a two-echelon supply chain with a single vendor and a single buyer. In their model, Yang and Wee [42] assumed that a vendor produces a deteriorating item (with a constant deterioration rate) at a specified finite production rate and then delivers the item to the buyer (or retailer) who meets consumer demand for the deteriorating item. One of the most notable features of Yang and Wee [42]'s model, which itself is based on Goyal [43]'s work, is that the vendor makes multiple delivers to the buyer during the course of a single production cycle. Wee at al. [44] corrected the holding cost expression in Yang and Wee [42]'s model so that when the vendor makes a single delivery to the buyer, the resulting holding cost is a positive number.

Yang and Wee [42]'s model has since been extended to suit various practical situations. For example, Yang and Wee [45] extended it to a situation where there are multiple buyers in the system as opposed to one. Rau et al. [46] developed a model for a three-echelon (supplier, producer and buyer) supply chain that tracks both the raw materials and finished goods inventory levels. Lee and Kim [47] studied a two-echelon deteriorating inventory system under the assumption that a given fraction of the items is of poorer quality and hence, the buyer subjects the items to a quality screening process prior to selling them. Recently, environmental issues have been incorporated in Yang and Wee [42]'s model, for example, Tiwari et al. [48] developed an extension that considers imperfect quality and carbon emissions in a two-echelon supply chain while Daryanto et al. [49] developed one that considers carbon emissions and a third-party logistics provider in a three-echelon supply chain setting.

2.4. Preservation technology investments in single- and multi-echelon supply chain systems

The seminal model that incorporated preservation technology investments into inventory theory was developed by Hsu et al. [50]. The authors studied a deteriorating inventory system with lost sales and back-ordering of shortages under the assumption that the rate of deterioration can be reduced by investing in preservation technologies. In addition to the order and back-ordering quantities, the amount invested in preservation technologies was also considered as a decision variable in Hsu et al. [50]'s model. Dye and Hsieh [51] formulated a model similar to Hsu et al. [50]'s with the major difference being that Dye and Hsieh [51] considered the amount invested in preservation technologies as a decision variable (in addition to the order quantity). The theory behind Hsu et al. [50] has since been extended to suit different practical situations such as in the case of an economic production quantity (EPQ) model where the demand rate is time-dependent [52], an EOQ model with non-instantaneous deteriorating items [53], to an EOQ model with price- and stock-dependent demand and both partial and full back-ordering of shortages [54] and to an EOQ model with price-dependent demand and partially back-ordering of shortages [55], to name a few. Other notable extensions include Yang et al. [56] who used dynamic programming to develop a model aimed at jointly optimising the lot size, the preservation technology investment amount and trade credit period for a deteriorating inventory system where delayed payments are permitted. Dye and Yang [57] studied the effect of a reference selling price, essentially a standard price that consumers use to gauge whether a product is priced fairly or not, on the inventory replenishment and preservation technology strategies adopted in a deteriorating inventory system where the demand rate is a function of the reference price. Zhang et al. [58] studied a deteriorating inventory system with price-dependent demand with the aim of optimising the selling price, the lot size and the preservation technology investment amount. Li et al. [59] extended Zhang et al. [58] by considering both the duration of the non-deterioration period and the deterioration rate to be affected by the preservation technology investment amount. Das et al. [55] used a variant of the particle swarm optimisation algorithm to optimise the preservation investment and lot size for a deteriorating inventory system where the deterioration rate is modelled by a three parameter Weibull distribution.

Extensions in multi-echelon supply chain settings have also been presented. As an example, Taya et al. [60] developed an inventory model for a two-echelon supply chain, with a single supplier and a single buyer, for deteriorating items with permissible delays in payments and preservation technology investments. Zhang et al. [61] used game theory, specifically a Stackelberg game, to model a two-echelon supply chain system (with a single manufacturer and a single retailer) for a deteriorating item under the assumption that the two supply chain members have a revenue sharing contract in place and they coordinate their investments in preservation technologies. Giri et al. [62] studied a vendor-buyer inventory system for a deteriorating item assuming that the retailer invests in preservation technologies and that the vendor's production process is unreliable in the sense that it can move from an in-control state to an out-of control state resulting in the production of nonconforming items. Dye et al. [63] developed a model for jointly optimising the selling price and the amount invested in preservation technologies (in addition to the lot-size) in a two-echelon deteriorating inventory system. Mohammadi et al. [64] developed an inventory model for a fresh product supply chain utilising a revenue sharing- and preservation technology sharing-contract as an incentive mechanism. Sepehri et al. [65] formulated a sustainable inventory model for deteriorating items with imperfect quality while considering investments in both preservation technologies and carbon reduction technologies.

Table 1
Summary of the major contributions made by closely related works on growing items in the current literature.

References	Item type		Supply chain structure			Deteriorating echelon(s)		Preservation
	Growing	Deteriorating	One echelon (or company-level)	Two echelons	Three or more echelons	Processing (or manufacturing)	Retail (or consumption)	technology investment
Rezaei [14]	✓		✓					
Khalilpourazari and Pasandideh [20]	✓		✓					
Malekitabar et al. [24]	✓	✓		✓			✓	
Nobil et al. [16]	✓		✓					
Sebatjane and Adetunji [17]	✓		✓					
Sebatjane and Adetunji [22]	✓		✓					
Gharaei and Almehdawe [21]	✓		✓					
Pourmohammad-Zia and Karimi [23]	✓	✓	✓				✓	
Sebatjane and Adetunji [28]	✓	✓			✓		✓	
Sebatjane and Adetunji [27]	✓				✓			
Alfares and Afzal [18]	✓		✓					
De-La-Cruz-Márquez et al. [19]	✓		✓					
Pourmohammad-Zia et al. [26]	✓	✓		✓			✓	
Pourmohammad-Zia et al. [30]	✓	✓			✓			
Sebatjane and Adetunji [29]	✓	✓			✓		✓	
This paper	✓	✓			✓	✓	✓	✓

2.5. Research gap and contribution

Compared with other classes of inventory items, such as deteriorating or conventional items, inventory modelling studies on growing items are few and far between. Table 1 provides an overview of some of the characteristics of a selection of previously published inventory models for growing items. When comparing the few published lot-sizing models for growing items, there are some gaps in the current literature. For instance, a vast majority of the models disregard the impact of deterioration. Moreover, most of the models were studied under company-level (i.e. one echelon) considerations and for these models, the authors assumed that once the items are slaughtered and they are instantaneously sold to customers. This is clearly not a realistic assumption because in reality, the items have to undergo some form of processing before they are suitable for human consumption. Even when considering the models that account for deterioration in multi-echelon supply chain settings, there are some minor deficiencies in some of the assumptions, with the most prominent one being that deterioration is only accounted for in the consumption (or retail) echelon of the supply chain. In reality, the items can deteriorate immediately following the processing (or production) stage. Finally, no previously published models investigated the impact of investing in preservation technologies aimed at slowing down the deterioration rate despite the importance of this in food supply chains.

This paper is aimed at filling the identified gaps in the literature. Hence, this paper proposes two integrated inventory models for a three-echelon food supply chain for growing and deteriorating items, with the first model disregarding preservation technology investments and the second accounting for it. Explicitly stated, the salient features of this paper are as follows:

- Simplifying a complex industrial food production system by modelling it as a three-echelon supply chain for growing and deteriorating items with farming, processing and retail echelons.
- Incorporating item deterioration at both the processing and the retail echelons of the supply chain.
- Joint optimisation of the number of shipments delivered to the retail echelon, the duration of the growing cycle (and by extension, the number of newborn items and the quantity of fully-grown items delivered to the processing echelon), the processing cycle duration (and by extension, the duration of the retail echelon and the quantity of processed inventory delivered to the retail echelon), and the amount invested in preservation technologies.
- Investigating the impact of an investment in preservation technologies on the performance of the food supply chain.

3. Problem description

This paper considers a three-echelon food supply chain with growing and deteriorating inventory items. Fig. 1 depicts the proposed supply chain with three distinct echelons, namely, farming, processing and retail. Newborn growing items are procured and reared to maturity at the farming echelon. Due to factors such as illnesses (which affect living organisms such as growing items), a certain fraction of the growing inventory items is unusable and this unusable fraction is discarded at the end of the growing cycle. The useable growing inventory items are then passed on to the processing echelon for slaughtering, stunning, cutting, de-boning and packaging (all these activities are collectively termed processing, for convenience and mathematical tractability). The processed inventory is perishable and therefore it deteriorates during the processing cycle. The processed inventory is then passed on to the retail echelon where consumer demand (for processed food products) is met. During the retail cycle, the processed inventory is depleted during to demand and deterioration. This type of supply chain setting represents a simplified industrial scale food supply chain for perishable food products. For example, consider chicken nuggets at the downstream (i.e. retail) end of the supply chain, whose primary input is live chickens (i.e. growing items) at the upstream end of the supply chain. The upstream and downstream ends of the supply chain are usually connected by a processing plant where the live chickens are transformed (via activities such as slaughtering, stunning, cutting, de-boning and packaging) into chicken nuggets.

3.1. Notations

This paper makes use of the following notations:

Input parameters:

D Demand rate at the retail echelon

K_r Retailer's fixed ordering cost per retail cycle

h_r Retailer's holding cost per weight unit per unit time

c_r Retailer's deterioration cost per weight unit

θ Deterioration rate at both the retail and processing echelons

P Processing rate at the processing echelon

K_p Processor's fixed setup cost per processing cycle

h_p Processor's holding cost per weight unit per unit time

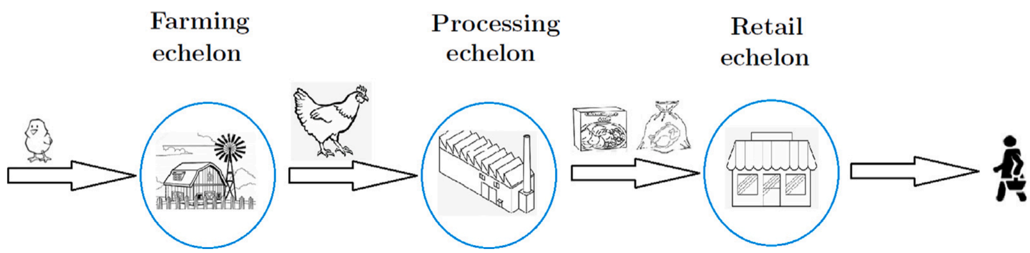

Fig. 1. Schematic representation of the proposed food supply chain.

c_p Processor's deterioration cost per weight unit

K_f Farmer's fixed setup cost per growing cycle

v_f Farmer's feeding cost per weight unit per unit time

δ Scale factor for the preservation technology function

Explicit decision variables:

N Number of shipments made (by the processor to the retailer) per processing cycle

T_f Farmer's growing period

T_{p_2} Processor's non-processing period per processing cycle

ξ Preservation technology cost per weight unit per unit time at both the retail and processing echelons

Implicit decision variables:

T_r Retailer's cycle time

Q_r Retailer's order quantity in weight units

T_p Processor's cycle time

Q_p Processor's processing quantity in weight units

T_{p_1} Processor's processing period per processing cycle

Q_0 Farmer's order quantity in weight units

y Number of newborn items purchased (by the farmer) at the beginning of each growing cycle

Functions:

$I_r(t)$ Retailer's inventory level in weight units at any time t

$I_p(t)$ Processor's inventory level in weight units at any time t

$I_f(t)$ Farmer's inventory level in weight units at any time t

$m(\xi)$ Preservation technology function

3.2. Assumptions

The model development process for the proposed three-echelon food supply chain, whose inventory system profile is depicted by Fig. 2, is guided by the following assumptions:

1. The inventory system has an infinite planning horizon and shortages are not permitted.
2. A single deteriorating and growing item is studied in the context of a three-echelon supply chain.
3. The supply chain has three echelons: a farming echelon where the growing items are reared; a processing echelons where the items are transformed into a saleable form that is suitable for human consumption; and a retail echelon used to meet consumer demand.
4. A new farming cycle commences upon receipt of an order for newborn items. Furthermore, all the items in the order grow at the same rate.
5. The items' growth, which continues throughout the farming cycle and ends when the items are ready for processing, is modelled by the Richards curve [66]. Hence, growth is modelled by the function $w(t) = A(1 + be^{-gt})^{-1}$, where $w(t)$ represents the items' weight at any time t, A is the items' asymptotic weight (i.e. the maximum possible weight that each items can grow to), b is the integration constant and g is the growth rate which determines the function's spread over time. The Richards curve is a commonly used biological function for modelling the weight increases experienced by various living organisms as a result of growth.
6. The cost of feeding the items increases as they grow (i.e. as their weight increases).
7. By virtue of the growing items being living organisms they are affected by illnesses. Hence, a fraction of the items will not be suitable for processing and eventually human consumption (this fraction is termed the unusable weight). At the end of the farming (or growing) cycle, the inventory is instantaneously quality controlled and the unusable weight fraction is disposed.
8. The farmer transfers a single shipment of useable items to the processor during the course of a single processing cycle.
9. A new processing cycle commences upon receipt of an order for matured items. The items are then processed (i.e. slaughtered, stunned, cut, de-boned and packaged) at a finite processing rate.
10. The processor transfers multiple shipments of processed inventory to the retailer during the course of a single processing cycle.
11. A new retail cycle commences upon receipt of an order for processed inventory. The processed inventory is used to meet consumer demand, with the demand rate being less than the processing rate.
12. The processed inventory (at both the processing and retail echelons) deteriorates at a constant rate. To safeguard consumer health, the deteriorated inventory cannot be sold.
13. The processing and retail echelons have the option to invest in preservation technologies that are aimed at reducing the rate of deterioration.

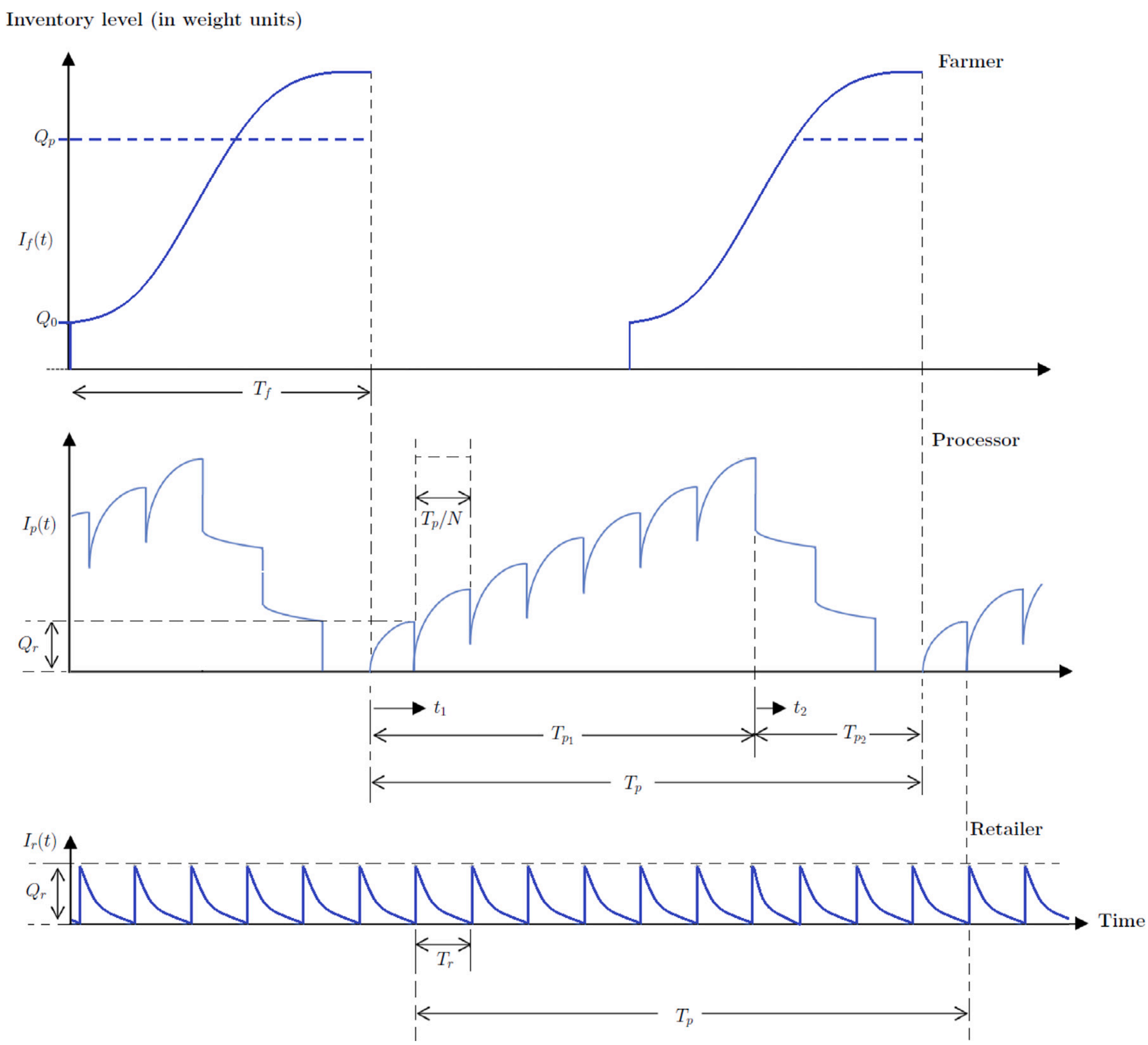

Fig. 2. Inventory system profile at each of the three echelons in the proposed food supply chain.

4. Model development

4.1. Retail echelon

The retailer receives an order of Q_r weight units of processed inventory every T_r time units as depicted in Fig. 3. The processed inventory deteriorates at a constant rate θ. Therefore, during the retail replenishment cycle the processed inventory is depleted due to both consumer demand and deterioration. Hence, over the time interval $[0, T_r]$, the processed inventory is governed by the differential equation

$$\frac{dI_r(t)}{dt} = -D - \theta I_r(t), \qquad 0 \leq t \leq T_r. \tag{1}$$

The retailer's inventory level at any time t is solved using the boundary condition $I_r(T_r) = 0$ and the result is

$$I_r(t) = \frac{D}{\theta}\left[e^{\theta(T_r - t)} - 1\right], \qquad 0 \leq t \leq T_r. \tag{2}$$

The initial order quantity that the retailer receives at the start of each retail cycle (i.e. at time $t = 0$) can be determined from Eq. (2) as

$$Q_r = I_r(0) = \frac{D}{\theta}\left(e^{\theta T_r} - 1\right). \tag{3}$$

Because the retailer incurs a fixed cost of K_r whenever a new order is placed, the retailer's ordering cost per unit time is thus

$$OC_r = \frac{K_r}{T_r}. \tag{4}$$

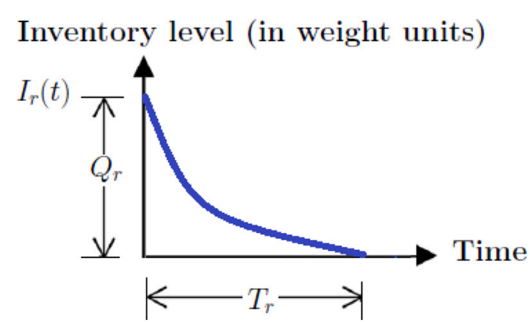

Fig. 3. Retail echelon inventory system profile.

The quantity of deteriorated inventory during the retailer's cycle, of duration T_r, is defined as the lot-size (i.e. Q_r) less the demand during T_r (i.e. DT_r). Hence, considering the retailer's deterioration cost of c_r per weight unit, the retailer's deterioration cost per unit time is

$$DC_r = \frac{c_r}{T_r}(Q_r - DT_r) = \frac{c_r}{T_r}\left[\frac{D}{\theta}\left(e^{\theta T_r} - 1\right) - DT_r\right]. \tag{5}$$

Considering the retailer's holding cost of h_r per weight unit and the average inventory level (i.e. the area under the inventory system profile

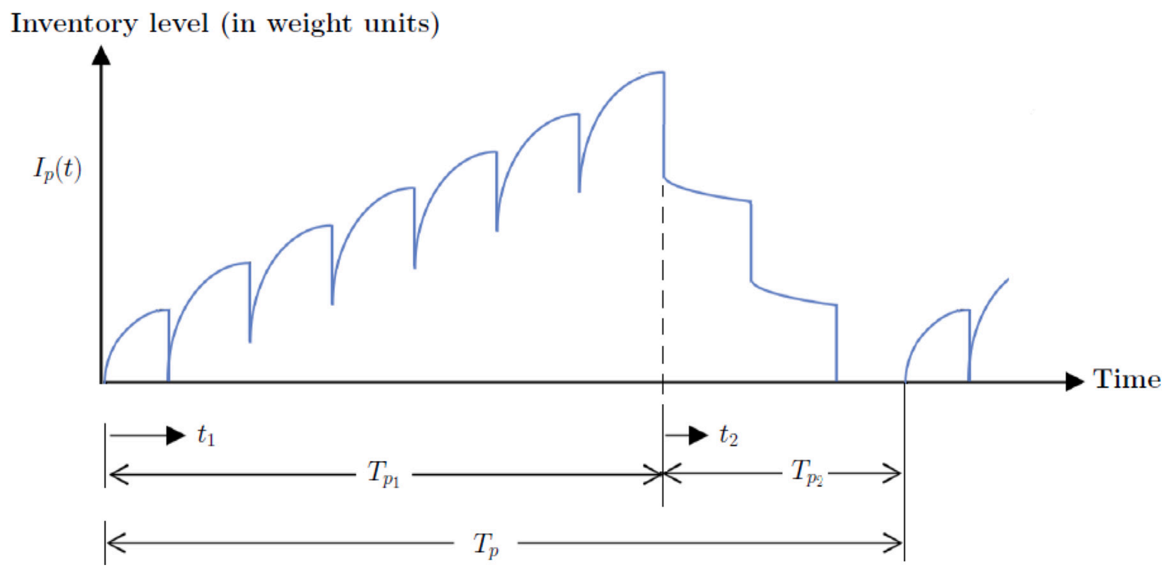

Fig. 4. Processing echelon inventory system profile.

as depicted by Fig. 3) the retailer's holding cost per unit time is

$$HC_r = \frac{h_r}{T_r}\int_0^{T_r} I_r(t)\,dt = \frac{h_r}{T_r}\left(\frac{D}{\theta^2}e^{\theta T_r} - \frac{DT_r}{\theta} - \frac{D}{\theta^2}\right). \tag{6}$$

The retailer's total cost is comprised of the ordering, deterioration and holding costs. Accordingly, the retailer's total cost per uni time is

$$TC_r = \frac{K_r}{T_r} + \frac{c_r}{T_r}\left[\frac{D}{\theta}\left(e^{\theta T_r}-1\right) - DT_r\right] + \frac{h_r}{T_r}\left(\frac{D}{\theta^2}e^{\theta T_r} - \frac{DT_r}{\theta} - \frac{D}{\theta^2}\right). \tag{7}$$

4.2. Processing echelon

The processor receives an order of Q_p weight units of matured growing inventory every T_p time units as depicted in Fig. 4. From this lot-size of Q_p weight units, the processor delivers N equally-sized shipments of processed inventory to the retailer (after processing) every T_r time units, hence, the relationship between T_p and T_r is

$$T_r = \frac{T_p}{N}. \tag{8}$$

Based on Fig. 4, the processor's cycle time (T_p) can be divided into two portions, namely, the processing portion (T_{p_1}) and the non-processing portion (T_{p_2}). During the processing portion, the processor simultaneously processes the matured growing items (into processed inventory) and delivers shipments of processed inventory to the retailer while during the non-processing portion, the processor only delivers shipments of processed inventory to the retailer. This is possible because the processing rate at the processing echelon, P, is greater than the demand rate at the retail echelon, D. This implies that the processed inventory can accumulate and hence, during the non-processing portions of the processor's cycle time, there is enough accumulated processed inventory that can be shipped to the retail echelon. Likewise, the processor's inventory level can be defined by two equations and thus,

$$I_p(t) = \int_0^{T_{p_1}} I_{p_1}(t_1)\,dt_1 + \int_0^{T_{p_2}} I_{p_2}(t_2)\,dt_2. \tag{9}$$

Given that the processed inventory is produced at a rate P, consumed at a rate D and deteriorated at a constant rate θ, during the processing portion of the processor's cycle time, the processed inventory is depleted due to both consumer demand and deterioration and it is accumulated due to processing. Hence, over the time interval $[0, T_{p_1}]$, the processed inventory is governed by the differential equation

$$\frac{dI_{p_1}(t_1)}{dt_1} = (P-D) - \theta I_{p_1}(t_1), \qquad 0 \le t_1 \le T_{p_1}. \tag{10}$$

Likewise, during the non-processing portion of the processor's cycle time, the processed inventory is depleted due to both consumer demand and deterioration, however, there is no accumulation of processed inventory. Hence, over the time interval $[0, T_{p_2}]$, the processed inventory is governed by the differential equation

$$\frac{dI_{p_2}(t_2)}{dt_2} = -D - \theta I_{p_2}(t_2), \qquad 0 \le t_2 \le T_{p_2}. \tag{11}$$

The boundary condition $I_{p_1}(0) = I_{p_2}(T_{p_2}) = 0$ is used to solve Eqs. (10) and (11), respectively, and the results are

$$I_{p_1}(t_1) = \frac{(P-D)}{\theta}\left(1 - e^{-\theta t_1}\right), \qquad 0 \le t_1 \le T_{p_1} \tag{12}$$

$$I_{p_2}(t_2) = \frac{D}{\theta}\left[e^{\theta(T_{p_2}-t_2)} - 1\right], \qquad 0 \le t_2 \le T_{p_2}. \tag{13}$$

The boundary condition $I_{p_1}(T_{p_1}) = I_{p_2}(0)$ is used to formulate the equation

$$(P-D)\left(1 - e^{-\theta T_{p_1}}\right) = D\left(e^{\theta T_{p_2}} - 1\right). \tag{14}$$

Using Taylor's series expansion and the assumption that $\theta \ll 1$, Eq. (14) can be simplified into

$$(P-D)T_{p_1}\left(1 - \frac{1}{2}\theta T_{p_1}\right) = DT_{p_2}\left(1 - \frac{1}{2}\theta T_{p_2}\right). \tag{15}$$

Using the results in Misra [67], the terms in Eq. (15) can be rearranged (by making T_{p_1} the subject of the equation) into

$$T_{p_1} \approx \frac{D}{(P-D)}(T_{p_2})\left(1 + \frac{1}{2}\theta T_{p_2}\right). \tag{16}$$

T_p can be made the subject of Eq. (16) by using the relation $T_p = T_{p_1} + T_{p_2}$, resulting in

$$T_p \approx \frac{T_{p_2}}{(P-D)}\left(P + \frac{1}{2}DT_{p_2}\right). \tag{17}$$

Considering that the processing rate is P and the duration of the processing portion of the processor's cycle time is T_{p_1}, the processing quantity Q_p (which is equal to the lot-size received by the processor from the farmer) is

$$Q_p = PT_{p_1}. \tag{18}$$

Owing to the processor incurring a fixed setup cost of K_p whenever a new processing cycle starts, the processor's setup cost per unit time is therefore

$$SC_p = \frac{K_p}{T_p}. \tag{19}$$

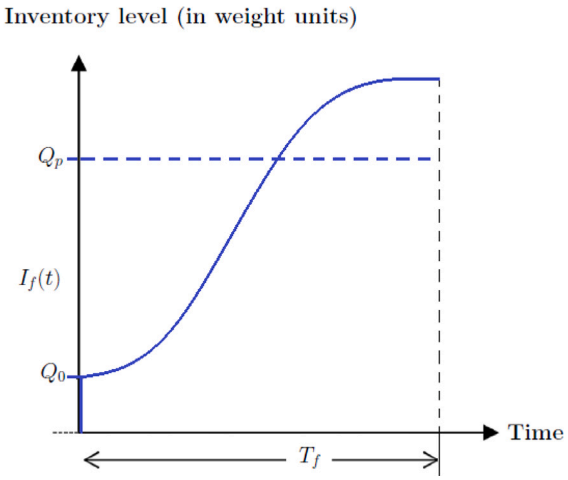

Fig. 5. Farming echelon inventory system profile.

The quantity of deteriorated inventory during the processor's cycle is defined as the processor's lot-size (i.e. Q_p) less the demand during T_p (i.e. DT_p). Therefore, when considering the processor's deterioration cost of c_p per weight unit, the processor's deterioration cost per unit time is

$$DC_p = \frac{c_p}{T_p}(Q_p - DT_p) = \frac{c_p}{T_p}\left(PT_{p_1} - DT_p\right). \tag{20}$$

Considering the processor's holding cost of h_p per weight unit and the area under the inventory system profile, as depicted by Fig. 4, the processor's holding cost per unit time is

$$\begin{aligned} HC_p &= \frac{h_p}{T_p}\left[\int_0^{T_{p_1}} I_{p_1}(t_1)\,dt_1 + \int_0^{T_{p_2}} I_{p_2}(t_2)\,dt_2\right] \\ &= \frac{h_p}{T_p}\left[\frac{(P-D)T_{p_1}^2}{2}\left(1-\frac{\theta T_{p_1}}{3}\right) + \frac{DT_{p_2}^2}{2}\left(1+\frac{\theta T_{p_2}}{3}\right)\right]. \end{aligned} \tag{21}$$

The processor's total cost is comprised of the setup, deterioration and holding costs. Accordingly, the processor's total cost per uni time is

$$\begin{aligned} TC_p &= \frac{K_p}{T_p} + \frac{c_p}{T_p}\left(PT_{p_1} - DT_p\right) \\ &+ \frac{h_p}{T_p}\left[\frac{(P-D)T_{p_1}^2}{2}\left(1-\frac{\theta T_{p_1}}{3}\right) + \frac{DT_{p_2}^2}{2}\left(1+\frac{\theta T_{p_2}}{3}\right)\right]. \end{aligned} \tag{22}$$

4.3. Farming echelon

The farmer receives an order of y live newborn items at the beginning of each farming cycle. The farmer rears the items throughout the growing period, of duration T_f time units, and at the end of the growing period, the farmer ships the entire lot of useable inventory to the processor. Fig. 5 depicts the farmer's inventory system profile showing changes to the weight of the items as they grow.

Since y newborn items are received when a new farming cycle commences and the weight of each item over time can be modelled by the Richards curve, given by $w(t) = A(1+be^{-gt})^{-1}$ [66], the weight of the farmer's growing inventory over the interval $[0, T_f]$ is governed by

$$I_f(t) = yw(t) = yA(1+be^{-gt})^{-1}, \qquad 0 \le t \le T_f. \tag{23}$$

Therefore, the weight of the farmer's initial lot-size at the start of the farming cycle (i.e. at $t = 0$) is

$$Q_0 = I_f(0) = yA(1+b)^{-1}. \tag{24}$$

From Eq. (24), $y = \frac{Q_0(1+b)}{A}$. By substituting y into Eq. (23), that equation can be rewritten (in terms of Q_0) as

$$I_f(t) = \frac{Q_0(1+b)}{(1+be^{-gt})}, \qquad 0 \le t \le T_f. \tag{25}$$

Since a fraction of items is not suitable for human consumption owing to the fact that the inventory items under study (i.e. growing items) are living organisms and as such, they are affected by illnesses. In accordance with Pourmohammad-Zia et al. [26], the unusable weight fraction is given by the function

$$\lambda(T_f) = 1 - e^{-\alpha T_f}, \tag{26}$$

where $\alpha > 0$. Pourmohammad-Zia et al. [26] chose that specific function to represent the unusable weight because of two properties that the function possesses, with the first property being that this fraction is negligible at the start of the farming cycle (i.e. $\lambda(0) = 0$) and the second being that this fraction approaches one as the duration of the growing period T_f takes very large values (i.e. $\lim_{T_f\to\infty} \lambda(T_f) = 1$). In simple terms, the practical implications of these two properties are that older items are more likely to be affected by factors such as illnesses and hence, the unusable weight fraction tends to be higher if the items are reared for longer periods of time and on the other hand, the unusable weight fraction tends to be lower for younger items because they are less likely to be affected by illnesses due to their relatively stronger immune systems.

Before shipping the inventory to the processor, the farmer instantaneously quality controls the inventory and disposes of the unusable inventory. This is done in order to safe guard consumer's health by ensuring that only the useable weight is sent to the processing plant. Let $I_f'(T_f) = \frac{Q_0(1+b)}{(1+be^{-gT_f})}$ represent the weight of the farmer's inventory level prior to the disposal of the unusable weight fraction. This means that the useable weight fraction is

$$I_f(T_f) = [1-\lambda(T_f)][I_f'(T_f)] = \frac{Q_0(1+b)e^{-\alpha T_f}}{\left(1+be^{-gT_f}\right)^{-1}}. \tag{27}$$

The useable inventory, as given in Eq. (27), is then shipped (by the farmer) to the processing echelon. This means that the useable weight should be equal to the processing quantity (i.e. the total inventory that is processed during a processing cycle), as given in Eq. (18). Hence, Eqs. (18) and (27) can be equated, resulting in

$$\frac{Q_0(1+b)e^{-\alpha T_f}}{\left(1+be^{-gT_f}\right)^{-1}} = PT_{p_1}. \tag{28}$$

From Eq. (28), a relationship can be established between Q_0, T_f and T_{p_1} by rewriting the equation into

$$Q_0 = \frac{\left(1+be^{-gT_f}\right)^{-1} PT_{p_1}}{(1+b)e^{-\alpha T_f}}. \tag{29}$$

Since the farmer incurs a fixed setup cost of K_f at the start of a new farming cycle, the farmer's setup cost per unit time is therefore

$$SC_f = \frac{K_f}{T_f}. \tag{30}$$

All y items received by the farmer are fed throughout the growing period at a cost of v_f per weight unit per unit time. Its necessary to account for the fact that as the items grow (i.e. increase in weight), they consume more feed material. To account for this, the exponential feeding function $F(t)$ which relates the feed consumption to the age of the items, adapted from Goliomytis et al. [68], is used to compute the feeding costs. Therefore, the feeding costs per unit time is

$$\begin{aligned} FC_f &= \frac{v_f y}{T_f}\int_0^{T_f} F(t)\,dt = \frac{v_f}{T_f}\left[\frac{Q_0(1+b)}{A}\right]\left(\int_0^{T_f} e^{\beta t}\,dt\right) \\ &= \frac{v_f}{T_f}\left[\frac{\left(1+be^{-gT_f}\right)^{-1}PT_{p_1}}{A(1+b)e^{-\alpha T_f}}\right]\left(\frac{e^{\beta T_f}-1}{\beta}\right). \end{aligned} \tag{31}$$

The farmer's total cost is comprised of the setup and feeding costs. Accordingly, the farmer's total cost per uni time is

$$TC_f = \frac{K_f}{T_f} + \frac{v_f}{T_f}\left[\frac{\left(1+be^{-gT_f}\right)^{-1}PT_{p_1}}{A(1+b)e^{-\alpha T_f}}\right]\left(\frac{e^{\beta T_f}-1}{\beta}\right). \tag{32}$$

4.4. Supply chain problem formulation

The total cost of managing inventory across the proposed food supply chain is comprised of the individual inventory management costs at each of the three echelons. Hence, the total inventory management costs for the supply chain per unit time, TC_{sc}, is defined as the sum of Eqs. (7), (22) and (32). After replacing T_r with $\frac{T_p}{N}$ as per Eq. (8), the mathematical problem for the proposed inventory system becomes

$$\begin{aligned}
\text{Minimise:}\quad & TC_{sc}(N, T_{p_2}, T_f) \\
&= \frac{K_r N}{T_p} + \frac{c_r N}{T_p}\left[\frac{D}{\theta}\left(e^{\frac{\theta T_p}{N}}-1\right) - \frac{DT_p}{N}\right] + \frac{h_r N}{T_p}\left(\frac{D}{\theta^2}e^{\frac{\theta T_p}{N}} - \frac{DT_p}{\theta N} - \frac{D}{\theta^2}\right) \\
&\quad + \frac{K_p}{T_p} + \frac{c_p}{T_p}\left(PT_{p_1} - DT_p\right) \\
&\quad + \frac{h_p}{T_p}\left[\frac{(P-D)T_{p_1}^2}{2}\left(1-\frac{\theta T_{p_1}}{3}\right) + \frac{DT_{p_2}^2}{2}\left(1+\frac{\theta T_{p_2}}{3}\right)\right] + \frac{K_f}{T_f} \\
&\quad + \frac{v_f}{T_f}\left[\frac{\left(1+be^{-gT_f}\right)^{-1}PT_{p_1}}{A(1+b)e^{-\alpha T_f}}\right]\left(\frac{e^{\beta T_f}-1}{\beta}\right) \\
\text{subject to:}\quad & T_p = \frac{T_{p_2}}{(P-D)}\left(P + \frac{1}{2}DT_{p_2}\right), \\
& T_{p_1} = \frac{D}{(P-D)}(T_{p_2})\left(1+\frac{1}{2}\theta T_{p_2}\right), \quad N \in \mathbb{Z}.
\end{aligned} \tag{33}$$

The problem is solved under the assumption of a centralised supply chain policy. Under a centralised policy, the aim is to find a global optimal solution for the entire supply chain and not the individual members.

Owing to N being a discrete variable, an iterative solution algorithm is utilised to solve the problem. The algorithm is based on the fact that the model's objective function, as given in Eq. (33), is a convex function of N, T_{p_2} and T_f, as proven in the Appendix. The solution algorithm is as follows:

Begin

Step 1: Let $N = 1$.

Step 2: Find the values of T_{p_2} and T_f that minimise TC_{sc} as given in Eq. (33).

Step 3: Increase N by 1 and find the values of T_{p_2} and T_f that minimise TC_{sc} as given in Eq. (33).

Step 4: If the latest computed value of TC_{sc} decreases, go back to Step 3. If the value of TC_{sc} increases, then the previously computed value of TC_{sc} (along with the corresponding N, T_{p_2} and T_f values) represents the best solution and this should be denoted as TC_{sc}^*, N^*, $T_{p_2}^*$ and T_f^*.

End.

5. Model 2: The impact of an investment in preservation technologies

The processed inventory in the processing and retail echelons of the proposed supply chain is subject to deterioration and hence, a given quantity of the inventory is lost due to deterioration. One way of reducing the quantity of deteriorated (and ultimately, wasted) inventory is by investing in preservation technologies such as advanced refrigeration and temperature controlled storage and transportation units. These preservation technologies essentially reduce the rate deterioration and consequently, this leads to a reduction in quantity of food wasted. Given that the implications of reduced food wastage have a direct impact on food security and economic development [2], investments in preservation technologies can have far reaching implications.

To investigate the impact of an investment in preservation technologies on the proposed three-echelon supply chain for growing and deteriorating items, an extension that incorporates preservation technology cost in presented. It is assumed that the processor and the retailer (i.e. the supply chain echelons where the processed inventory is subjected to deterioration) invest a certain amount in preservation technologies in order to reduce the deterioration effect. Based on Das et al. [55]'s model, the preservation technology function is defined as

$$m(\xi) = \frac{\delta\xi}{1+\delta\xi}, \tag{34}$$

where ξ is the preservation technology cost per weight unit per unit time invested and δ is a scale factor for the preservation technology function, with $\delta > 0$ and ξ being a decision variable. The preservation technology function $m(\xi)$ is an increasing function of the preservation technology cost ξ and in practical terms, this means that the more money is invested in preservation technologies the higher the preservation technology function and consequently, the lower the rate of deterioration (i.e. investing in preservation technologies results in a reduced rate of deterioration).

The reduced deterioration rate (due to investing in preservation technologies), denoted by θ', is a function of the preservation technology function and the regular deterioration rate (i.e the deterioration rate without any investments in preservation technologies). This relationship is defined as

$$\begin{aligned}
\theta' &= [1-m(\xi)]\theta \\
&= \left[\frac{\delta\xi}{1+\delta\xi}\right]\theta.
\end{aligned} \tag{35}$$

The reduced deterioration rate is only achievable by investing in preservation technologies and hence, there is an additional cost incurred. The retailer's preservation technology cost per unit time is a function of the retailer's average inventory level and it is defined as

$$PTC_r = \frac{\xi}{T_r}\int_0^{T_r} I_r(t)\,dt = \frac{\xi}{T_r}\left(\frac{D}{\theta'^2}e^{\theta' T_r} - \frac{DT_r}{\theta'} - \frac{D}{\theta'^2}\right). \tag{36}$$

Likewise, the processor's preservation technology cost per unit time is

$$\begin{aligned}
PTC_p &= \frac{\xi}{T_p}\left[\int_0^{T_{p_1}} I_{p_1}(t_1)\,dt_1 + \int_0^{T_{p_2}} I_{p_2}(t_2)\,dt_2\right] \\
&= \frac{\xi}{T_p}\left[\frac{(P-D)T_{p_1}^2}{2}\left(1-\frac{\theta' T_{p_1}}{3}\right) + \frac{DT_{p_2}^2}{2}\left(1+\frac{\theta' T_{p_2}}{3}\right)\right].
\end{aligned} \tag{37}$$

The new total cost functions for both the processor and the retailer will therefore incorporate preservation technology costs. Hence, when accounting for an investment in preservation technologies, the mathematical problem describing the proposed three-echelon food supply chain can be represented as

$$\begin{aligned}
\text{Minimise:}\quad & TC_{sc}(N, T_{p_2}, T_f, \xi) = \frac{K_r N}{T_p} + \frac{c_r N}{T_p}\left[\frac{D}{\theta}\left(e^{\frac{\theta' T_p}{N}}-1\right) - \frac{DT_p}{N}\right] \\
&\quad + \frac{(h_r+\xi)N}{T_p}\left(\frac{D}{\theta'^2}e^{\frac{\theta' T_p}{N}} - \frac{DT_p}{\theta' N} - \frac{D}{\theta'^2}\right) + \frac{K_p}{T_p} + \frac{c_p}{T_p}\left(PT_{p_1} - DT_p\right) \\
&\quad + \frac{(h_p+\xi)}{T_p}\left[\frac{(P-D)T_{p_1}^2}{2}\left(1-\frac{\theta' T_{p_1}}{3}\right) + \frac{DT_{p_2}^2}{2}\left(1+\frac{\theta' T_{p_2}}{3}\right)\right] + \frac{K_f}{T_f} \\
&\quad + \frac{v_f}{T_f}\left[\frac{\left(1+be^{-gT_f}\right)^{-1}PT_{p_1}}{A(1+b)e^{-\alpha T_f}}\right]\left(\frac{e^{\beta T_f}-1}{\beta}\right) \\
\text{subject to:}\quad & T_p = \frac{T_{p_2}}{(P-D)}\left(P + \frac{1}{2}DT_{p_2}\right), \\
& T_{p_1} = \frac{D}{(P-D)}(T_{p_2})\left(1+\frac{1}{2}\theta' T_{p_2}\right), \quad N \in \mathbb{Z}.
\end{aligned} \tag{38}$$

Table 2
Results from solving the model without preservation technology investments.

Decision variables and objective function	Optimal values
N^* (shipments)	9
T_f^* (years)	0.1228
$T_{p_2}^*$ (years)	0.7337
TC_{sc}^* ($/year)	68 655.19

Table 3
Results from solving the model incorporating preservation technology investments.

Decision variables and objective function	Optimal values
N^* (shipments)	9
T_f^* (years)	0.1222
$T_{p_2}^*$ (years)	0.7997
ξ^* ($/gram/year)	0.0007
TC_{sc}^* ($/year)	66 505.52

6. Results and analysis

In an effort to analyse and draw some managerial insights from the models, two numerical examples are solved. The two examples are essentially the same with the only difference being that the second example incorporates an investment in preservation technologies. The examples consider an integrated poultry production facility with a farming echelon where newborn broiler chickens are reared, a processing echelon where fully-grown broiler chickens are slaughtered, stunned, cut, de-boned and packaged (collectively termed processing) at a finite processing rate and a retail echelon where processed chicken products are sold to customers. The growth function of the broiler chickens is based on the Richards curve [66] and the input parameters for the curve are adapted from Pourmohammad-Zia et al. [26], with $A = 3200$, $b = 69.4$ and $g = 43.8$. Hence, the growth function for the broiler chickens is $w(t) = 3200(1 + 69.4e^{-43.8t})^{-1}$. In addition to the growth function, the feeding function of the chickens is also adapted from Pourmohammad-Zia [26] and it is $F(t) = e^{86t}$.

The rest of the input parameters (applicable to both examples) are: $D = 10000000$ g/year; $K_r = 400$ $/cycle; $h_r = 0.002$ $/grams/year; $c_r = 0.02$ $/grams; $\theta = 0.15$; $P = 50000000$ grams/year; $K_p = 10000$ $/cycle; $h_p = 0.001$ $/grams/year; $c_p = 0.01$ $/grams; $K_f = 5000$ $/cycle; $v_f = 0.02$ $/grams/year. Example 2 has $\delta = 1000$ grams.year/$ as an additional input parameter.

6.1. Example 1

The objective function and explicit decision variables are determined by solving Eq. (33) and the results are presented in Table 2.

The implicit decision variables can be derived from the explicit decision variables (using the relations derived when developing the model). Therefore, all the decision variables (i.e. inclusive of the implicit ones) at each of the three echelons are:

Farming echelon: $T_f^* = 0.1228$ years; $Q_0^* = 212259.57$ grams; $y^* \approx 4670$ broiler chicks.

Processing echelon: $T_{p_2}^* = 0.7337$ years; $T_{p_1}^* = 0.2003$ years; $T_p^* = 0.9845$ years; $Q_p^* = 10012926.78$ grams; $N^* = 9$ shipments.

Retail echelon: $T_r^* = 0.1094$ years; $Q_r^* = 1108947.71$ grams.

In practical terms, this means that the farmer should order $y^* \approx 4670$ newborn broiler chicks at the start of each growing cycle, with a combined weight of $Q_0^* = 212259.57$ grams. The chicks should be reared for a period of $T_f^* = 0.1228$ years. This growth period duration is roughly in line with common industry practice whereby broiler chickens are slaughtered after 40–42 days [69]. At the end of the growing period, the useable combined weight of the chickens would have increased to $Q_p^* = 10012926.78$ grams and this quantity is transferred to the processor for processing (which entails, among other activities, slaughtering, stunning, cutting, de-boning and packaging). The processing cycle has a duration of $T_p^* = 0.9845$ years, and during the first $T_{p_1}^* = 0.2003$ years of the processing cycle, the processor simultaneously processes the chickens and delivers processed chicken products to the retailer while during the last $T_{p_2}^* = 0.7337$ years, the processor only delivers processed chicken products to the retailer. During the course of a single processing cycle, the processor delivers $N^* = 9$ shipments of processed chicken products to the retailer (from the accumulated inventory). The retailer receives a shipment of chicken products weighing $Q_r = 1108947.71$ grams from the processor every $T_r^* = 0.1094$ years. By following this policy, the entire supply chain can expect to incur inventory management costs amounting to $TC_{sc}^* = \$68655.19$ per year.

6.2. Example 2

The objective function and explicit decision variables are determined by solving Eq. (38) and the results are presented in Table 3.

The implicit decision variables can be derived from the explicit decision variables (using the relations derived when developing the model). Therefore, all the decision variables (i.e. inclusive of the implicit ones) at each of the three echelons are:

Farming echelon: $T_f^* = 0.1222$ years; $Q_0^* = 225521.95$ grams; $y^* \approx 4961$ broiler chicks.

Processing echelon: $T_{p_2}^* = 0.7997$ years; $T_{p_1}^* = 0.2115$ years; $T_p^* = 1.0796$ years; $Q_p^* = 10576266.13$ grams; $N^* = 9$ shipments; $\xi = 0.0007$ $/gram/year.

Retail echelon: $T_r^* = 0.1200$ years; $Q_r^* = 1210006.85$ grams; $\xi = 0.0007$ $/gram/year.

This means that the farmer should order $y^* \approx 4961$ newborn broiler chicks at the start of each growing cycle, with a combined weight of $Q_0^* = 225521.95$ grams. The chicks should be reared for a period of $T_f^* = 0.1222$ years. At the end of the growing period, the useable combined weight of the chickens would have increased to $Q_p^* = 10576266.13$ grams and the farmer transfers this quantity is transferred to the processing plant. Each processing cycle has a duration of $T_p^* = 1.0796$ years, and during the first $T_{p_1}^* = 0.2115$ years of the processing cycle, the processor simultaneously processes the chickens and delivers processed chicken products to the retailer while during the last $T_{p_2}^* = 0.7997$ years, the processor only delivers processed chicken products to the retailer. During the course of a single processing cycle, the processor delivers $N^* = 9$ shipments of processed chicken products to the retailer. The retailer receives a shipment of chicken products weighing $Q_r = 1210006.85$ grams from the processor every $T_r^* = 0.1200$ years. In order to reduce the impact of deterioration, the processor and the retailer should invest $\xi^* = \$0.0007$ per gram (of processed inventory) per year in preservation technologies. By following this policy, the entire supply chain can expect to incur inventory management costs amounting to $TC_{sc}^* = \$66505.52$ per year. When compared to the scenario without any investments in preservation technologies, the total inventory management costs for this policy are slightly reduced which means that investing in preservation technologies has the potential to reduce supply chain costs despite the additional investment required. The investment in preservation technologies can pay off because it results in a reduced deterioration rate which consequently, results in lower quantities of wasted (or deteriorated) processed products (despite having a higher initial order quantity for newborn items).

A sensitivity analysis was conducted on Example 2 in order to investigate the response of the decision variables and the objective function to changes in some of the input variables. The results from the analysis are presented in Table 4 and a summary of the major findings is as follows:

- N^* is most sensitive to changes in K_r and K_f values, with N^* increasing with decreasing K_r values. This response can be attributed to the fact that when it becomes cheaper for the retailer to place an order (i.e. as K_r values decrease), the model responds by prompting the retailer to place smaller order more frequently because the costs of ordering are relatively lower. On the other hand, N^* increases with increasing K_f values. This is because as it becomes costlier for the farmer to set up a growing cycle (i.e. as K_f values decrease), the model responds by prompting the farmer to order larger quantities of newborn items so as to set up as fewer new growing cycles as possible. Consequently, the processor receives relatively larger quantities for processing which results in the processor delivering more shipments of processed inventory to the retailer during the course of a single processing cycle.
- T_f^* is most sensitive to changes in K_f and v_f values. In general, T_f^* increases with increasing K_f values. This is because if it becomes costlier for the farmer to set up new growing cycles, then the natural response of the model is to prompt the farmer to rear the items for longer periods of time so as to reduce the number of new setups. With reference to v_f, T_f^* decreases with increasing v_f values. This is because as it becomes costlier for the farmer to feed the growing items (i.e. as v_f values increase), the model responds by prompting the farmer to rear the items for shorter periods of time in an effort to reduce the amount of feed material consumed by the items.
- $T_{p_2}^*$ is most sensitive to changes in K_p values, with $T_{p_2}^*$ increasing with increasing K_p values. This is because if it becomes costlier for the processor to set up new processing cycles (i.e. as K_p values increase), then the model, in an effort to reduce costs, prolongs the duration of the processing cycle so that the processor processes as much inventory as possible during each cycle.
- ξ^* is most sensitive to changes in c_p and c_r values. This is because if the cost of deterioration is higher then the model responds by prompting the retailer and the processor to invest more in preservation technologies in an effort to reduce the quantity wasted or deteriorated inventory.
- y^* is most sensitive to changes in K_p values. Generally, y^* increases as K_p^* values increase. This is because if it becomes costlier for the processor to set up new processing cycles, then it is more economical to process as much inventory as possible during a processing cycle and this requires the receipt of a relatively large quantity of fully-grown items from the processor which implies a higher order of newborn items.

6.3. Managerial insights

After managerial insights can be drawn from the results of the sensitivity analysis and from solving the two examples. These insights, which can be used by operations and supply chain management practitioners to improve the overall supply chain performance, which in this context entail minimising the inventory management costs, include:

- Investments in preservation technologies were shown to have a significant impact on the performance of the supply chain, with an increase in the level of investment leading to a decrease in the total inventory management costs for the supply chain. Managers should therefore invest more in preservation technologies. In practical terms, managers can utilise more advanced refrigeration units at storage facilities and temperature-controlled trucks for transporting the inventory between the different echelons.
- Feeding costs were shown to have an impact on the supply chain performance, with an increase in feeding costs leading to increased total supply chain costs. It is recommended that managers take measures aimed at reducing the costs of feeding the items. This can be achieved by shopping around for the best supplier, with a particular focus on those who supply more nutritious feed material. If the specific nutritional value of the feed material is higher, then the same quantity can feed more items.
- The different fixed costs, specifically the retailer's ordering costs and the processor and the farmer's set up costs also have a sizeable impact on supply chain performance. In general, an increase in these fixed costs leads to an increase in the total supply chain costs and hence, managers should take measures to reduce these costs. In practical terms, managers can place larger order quantities which ensures that there are fewer order placements or farming and processing setups.

7. Conclusion

Inventory management in food supply chains is complicated because of some peculiar characteristics of food products. The first of these characteristics relates to the primary source of food products, which in most instances, is growing items such as crops or livestock. The second characteristic relates to the perishable nature of the final form (i.e. form at the time of consumption) of the products. The growing items are transformed into perishable food products via processing, a collective activity that encompasses slaughtering, cutting, de-boning and packaging. The changing nature of food products throughout the supply chain adds a level of complexity to the inventory management strategies adopted.

In light of this, this study developed an inventory model for a three-echelon food supply chain with farming, processing and retail echelons. Live growing items are reared to maturity at the farming echelon. These items are then processed into food products at the processing echelon and consumer demand (for processed food products) is met at the retail echelon. The model was then extended to a situation where an investment in preservation technologies is made in an effort to reduce the deterioration rate.

Through numerical analysis, it was shown that investing in preservation technologies is financially worthwhile. The total costs for managing inventory across the entire supply was just over 3% lower for model that incorporates preservation technologies. Consequently, it is advisable for operations management and supply chain practitioners in food production chains to make use of advanced refrigeration units when storing processed food inventory and to make use of temperature-controlled trucks for moving processed inventory between the processing and retail echelons.

Some of the assumptions made in this study limit its potential applicability to food supply chains. For instance, the constant rate demand rate and the no shortages assumptions are not representative of all real life situations. Moreover, perishable food products are sold at retail outlets that are often characterised by thin profit margins and hence in such, settings incentive mechanisms such as quantity discounts and permissible delays in payments might be used by retailers to stimulate consumer demand.

The aforementioned limitations can be explored further as potential future research directions. Case in point, the constant demand rate assumption can be relaxed by either considering the demand rate to be a function of the selling price or the stock level or incorporating uncertainty in the form of stochastic demand. The impact of offering quantity discounts and allowing delayed payments can also be explored. In addition, the impact of advertising and marketing strategies, which play an important role in grocery retail, on inventory management policies can also be studied. Other characteristics of food supply chains that can be explored as potential future extensions include quality control, advanced or delayed payment schemes, cold chain transportation, and expiration dates.

CRediT authorship contribution statement

Makoena Sebatjane: Conceptualisation, Methodology, Formal analysis, Writing – original draft, Writing – review & editing.

Table 4
Results from a sensitivity analysis performed on Example 2.

Parameters	% change	N^*		T_f^*		$T_{p_2}^*$		ξ^*		y^*		TC_{sc}^*	
Base case		**9**		**0.1222**		**0.7997**		**0.0007**		**4961**		**66 505.52**	
K_r	−40	12	+33.33	0.1221	−0.09	0.8771	+0.93	0.0007	−3.12	5 020	+1.17	64 984.32	−2.29
	−20	10	+11.11	0.1222	+0.01	0.7989	−0.12	0.0007	−1.24	4 956	−0.11	65 793.67	−1.07
	+20	8	−11.11	0.1223	+0.09	0.7926	−0.89	0.0007	+1.46	4 908	−1.08	67 153.50	+0.97
	+40	8	−11.11	0.1220	−0.15	0.8127	+1.63	0.0007	+1.55	5 054	+1.87	67 744.00	+1.86
c_r	−40	8	−11.11	0.1222	−0.02	0.7997	0	0.0006	−11.32	4 977	+0.3	65 756.40	−1.13
	−20	9	0	0.1220	−0.16	0.8126	+1.61	0.0007	−5.69	5 063	+2.04	66 148.00	−0.54
	+20	10	+11.11	0.1220	−0.16	0.8140	+1.79	0.0008	+3.97	5 060	+1.99	66 844.14	+0.51
	+40	10	+11.11	0.1220	−0.02	0.8032	+0.44	0.0008	+8.95	4 976	+0.29	67 154.46	+0.98
h_r	−40	8	−11.11	0.1224	+0.11	0.7905	−1.15	0.0007	+1.40	4 893	−1.38	66 016.79	−0.73
	−20	9	0	0.1221	−0.10	0.8085	+1.10	0.0007	−0.01	5 025	+1.29	66 262.80	−0.36
	+20	9	0	0.1223	+0.10	0.7912	−1.06	0.0007	−0.01	4 900	−1.24	66 745.43	+0.36
	+40	10	+11.11	0.1221	−0.12	0.8094	+1.22	0.0007	−1.15	5 034	+1.46	66 967.02	+0.69
K_p	−40	7	−22.22	0.1255	+2.71	0.6003	−24.93	0.0007	+2.35	3 561	−28.22	66 505.52	−6.48
	−20	8	−11.11	0.1237	+1.21	0.7038	−11.99	0.0007	+1.05	4 276	−13.82	64 526.87	−2.98
	+20	10	+11.11	0.1210	−1.02	0.8895	+11.23	0.0007	−0.87	5 624	+13.35	68 241.02	+2.61
	+40	11	+22.22	0.1199	−1.90	0.9743	+21.83	0.0007	−1.59	6 267	+26.32	69 796.79	+4.95
c_p	−40	9	0	0.1224	+0.15	0.7797	−2.50	0.0005	−37.62	4 871	−1.83	67 126.58	+0.93
	−20	9	0	0.1223	+0.10	0.7882	−1.44	0.0006	−18.01	4 902	−1.20	66 867.13	+0.54
	+20	9	0	0.1221	−0.14	0.8141	+1.80	0.0008	+16.75	5 045	+1.69	66 059.17	−0.67
	+40	10	+11.11	0.1215	−0.62	0.8590	+7.42	0.0010	+31.88	5 355	+7.93	65 533.73	−1.46
h_p	−40	10	+11.11	0.1210	−0.98	0.8854	+10.72	0.0007	−2.31	5 596	+12.78	64 859.08	−2.48
	−20	10	+11.11	0.1214	−0.63	0.8539	+6.78	0.0007	−1.70	5 361	+8.05	65 705.03	−1.20
	+20	9	0	0.1226	+0.29	0.7757	−3.0	0.0007	+0.51	4 787	−3.51	67 273.26	+1.15
	+40	8	−11.11	0.1233	+0.88	0.7292	−8.81	0.0007	+2.29	4 454	−10.23	68 009.89	+2.26
v_f	−40	10	+11.11	0.1272	+4.05	0.8470	+5.91	0.0007	−1.56	5 045	+1.68	64 130.22	−3.57
	−20	9	0	0.1245	+1.90	0.8089	+1.16	0.0007	−0.19	4 913	−0.97	65 435.04	−1.61
	+20	9	0	0.1203	−1.55	0.7917	−1.00	0.0007	+0.17	5 004	+0.87	67 422.80	+1.38
	+40	9	0	0.1187	−2.86	0.7845	−1.90	0.0007	+0.32	5 043	+1.65	68 230.39	+2.59
K_f	−40	10	+11.11	0.1154	−5.62	0.8855	+10.73	0.0007	−2.31	5 968	+20.29	49 690.97	−25.28
	−20	10	+11.11	0.1190	−2.65	0.8539	+6.78	0.0007	−1.70	5 503	+10.91	58 220.07	−12.47
	+20	9	0	0.1246	+1.94	0.7750	−3.09	0.0007	+0.53	4 694	−5.40	74 606.87	+12.18
	+40	8	−11.11	0.1270	+3.94	0.7269	−9.11	0.0007	+2.37	4 295	−13.44	82 555.00	+24.13

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Proof of the solution's optimality

The necessary conditions for TC_{sc} to be optimal are $\frac{\partial TC_{sc}}{\partial N} = 0$, $\frac{\partial TC_{sc}}{\partial T_{p_2}} = 0$ and $\frac{\partial TC_{sc}}{\partial T_f} = 0$. Therefore, it should be proven that these equations yield unique optimal solutions. Because of the complexity of the mathematical formulations in the proposed supply chain, the Hessian matrix will not be utilised to prove optimality, rather an approach adopted by [26] will be utilised. Lemmas 1, 2 and 3 are used to prove the solution's optimality.

Lemma 1. *For fixed T_{p_2} and T_f values, there exists a unique N^* value which minimises TC_{sc} where $\left.\frac{\partial TC_{sc}}{\partial N}\right|_{N=N^*} = 0$.*

Proof. The first order optimality condition for the number of shipments delivered by the processor to the retailer during a single processing cycle is

$$\frac{\partial TC_{sc}}{\partial N} = \frac{K_r}{T_p} + \frac{c_r D}{\theta T_p N}\left[(N-\theta T_p)e^{\frac{\theta T_p}{N}} - N\right] + \frac{h_r D}{\theta^2 T_p N}\left[(N-\theta T_p)e^{\frac{\theta T_p}{N}} - N\right] = 0. \tag{A.1}$$

To show that Eq. (A.1) yields a unique optimal N value when T_{p_2} and T_f are treated as fixed values, it is necessary to establish that TC_{sc} is convex with respect to N. Hence,

$$\frac{\partial^2 TC_{sc}}{\partial N^2} = \frac{\theta c_r D T_p e^{\frac{\theta T_p}{N}}}{N^3} + \frac{h_r D T_p e^{\frac{\theta T_p}{N}}}{N^3}. \tag{A.2}$$

Since $\frac{\partial^2 TC_{sc}}{\partial N^2} \geq 0$, TC_{sc} is convex with respect to N and thus, Lemma 1 is proven. □

Lemma 2. *For fixed N and T_f values, there exists a unique $T_{p_2}^*$ value which minimises TC_{sc} where $\left.\frac{\partial TC_{sc}}{\partial T_{p_2}}\right|_{T_{p_2}=T_{p_2}^*} = 0$.*

Proof. The first order optimality condition for the duration of the processor's non-processing period per processing cycle is

$$\frac{\partial TC_{sc}}{\partial T_{p_2}} = \frac{h_p T_{p_2}}{6T_p}\left[3\theta D T_{p_2} + (2\theta D - 2\theta P)T_p + 6P\right] = 0. \tag{A.3}$$

To show that Eq. (A.3) yields a unique optimal T_{p_2} value when N and T_f are treated as fixed values, it is necessary to establish that TC_{sc} is convex with respect to T_{p_2}. Hence,

$$\frac{\partial^2 TC_{sc}}{\partial T_{p_2}^2} = \frac{h_p}{3T_p}\left[3\theta D T_{p_2} + (\theta D - \theta P)T_p + 3P\right]. \tag{A.4}$$

Since $\frac{\partial^2 TC_{sc}}{\partial T_{p_2}^2} \geq 0$, TC_{sc} is convex with respect to T_{p_2} and thus, Lemma 2 is proven. □

Lemma 3. *For fixed N and T_{p_2} values, there exists a unique T_f^* value which minimises TC_{sc} where $\left.\frac{\partial TC_{sc}}{\partial T_f}\right|_{T_f=T_f^*} = 0$.*

Proof. The first order optimality condition for the duration of the farmer's growing period is

$$\frac{\partial TC_{sc}}{\partial T_f} = -\frac{K_f}{T_f^2} + \frac{v_f P T_{p_1}}{A\beta e^{-\alpha T_f}}\Big[\alpha\left(1+be^{-gT_f}\right)\left(e^{\beta T_f}-1\right) - bge^{-gT_f}\left(e^{\beta T_f}-1\right) + \beta\left(1+be^{-gT_f}\right)e^{\beta T_f}\Big] = 0. \tag{A.5}$$

To show that Eq. (A.5) yields a unique optimal T_f value when N and T_{p_2} are treated as fixed values, it is necessary to establish that TC_{sc} is convex with respect to T_f. Hence,

$$\frac{\partial^2 TC_{sc}}{\partial T_f^2} = \frac{2K_f}{T_f^3} + \frac{v_f P T_{p_1}}{A\beta e^{-\alpha T_f}} \Big[(\alpha+\beta)^2 e^{\beta T_f} + b(g-\alpha)(\alpha-g)e^{-gT_f} + b(\alpha-g+\beta)^2 e^{-gT_f} e^{\beta T_f} - \alpha^2\Big]. \quad \text{(A.6)}$$

Since $\frac{\partial^2 TC_{sc}}{\partial T_f^2} \geq 0$, TC_{sc} is convex with respect to T_f and thus, Lemma 3 is proven provided that $(2\alpha+\beta)(1+b) \geq 2bg$ [26]. □

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