THE USE OF PERSONAL RESPONSE SYSTEMS TO RENEGOTIATE THE DIDACTICAL CONTRACT IN TERTIARY MATHEMATICS EDUCATION

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Abstract

Challenges experienced by first-year students transitioning from secondary to tertiary mathematics education are examined through the lens of the didactical contract. The didactical contract describes the expectations of both lecturer and students about their mutual obligations towards teaching and learning. First-year students' beliefs about the nature of mathematics and mathematics teaching/learning need to be challenged to renegotiate the didactical contract at tertiary level. The study focuses on how to elicit and confront transitioning students' beliefs in order to support their learning and influence a shift in the didactical contract. A Likert scale questionnaire was deployed at the beginning of students' first year to gauge their beliefs about mathematics and mathematics teaching/learning and redeployed near the end of the first semester (or term) to observe possible changes in their beliefs and hence the didactical contract. The intervention consisted of personal response system (PRS) sessions regularly incorporated into the traditional transmission mode lecture to flip the classroom and create a student-centred learning environment, aimed at influencing students' beliefs in order to make them aware of their own learning and their responsibility for learning. Questionnaire data were quantified and compared for the before and after surveys. There is evidence of a shift towards students taking ownership of their learning and a renegotiation of the didactical contract. Qualitative data generated by focus group interviews confirm the role of the PRS sessions in influencing student beliefs and the didactical contract.

Keywords: didactical contract, secondary-tertiary mathematics transition, personal response systems, flipped classroom

1. INTRODUCTION

Researchers acknowledge the existence of a widening gap between secondary and tertiary mathematics education globally, and attempt to examine the gap in order to assist students to make the transition (Benadé, 2013; Brandell, Hemmi & Thunberg, 2008; Clark & Lovric, 2009; Hourigan & O' Donoghue, 2007; Pepin, 2014; Selden, 2005). Hourigan and O' Donoghue (2007), Pepin (2014) and Yoon *et al*. (2011) identify the concept of the didactical contract as a lens through which the gap can be examined. The didactical contract is the implicit agreement about mutual roles and responsibilities in a teaching/learning event (D'Amore, 2008; Pepin, 2014). The contract comprises the lecturer and students' expectations about teaching and learning and the responsibility for learning, governed by the participants' beliefs about mathematics and mathematics teaching/learning, but directed by principles of effective teaching and learning (Hiebert *et al*., 1996; Hiebert & Carpenter, 2006; Schoenfeld, 2016). At tertiary level, the didactical contract becomes more complex because of the number of students in large classrooms but also because of the diverse mathematical abilities that students bring to the classroom (Biggs, 1999), especially at first-year level. The responsibility to successfully transition from secondary to tertiary education lies with the student, but the lecturer can and should assist by creating environments to support the transition (Clark & Lovric, 2009) and renegotiating the didactical contract (Yoon *et al*., 2009).

According to Beatty and Gerace (2009), students' "underlying" beliefs should be addressed in order to influence their classroom behaviour. Beatty and Gerace (2009: 151) define technology-enhanced formative assessment (TEFA) as pedagogy for utilising personal response systems such as clickers, to influence student beliefs in a physics classroom. Vicens (2017:17) reports on using "clicker-based peer instruction" to create a flipped classroom and Abeysekera and Dawson (2015) note that the flipped classroom model encourages students to take responsibility for their own learning by increasing student motivation.

The potential value of using personal response systems to create a flipped classroom, guide students towards self-directed learning and narrow the gap between secondary and tertiary mathematics education, inspired the study.

2. PROBLEM STATEMENT

Central to the study is the didactical contract in secondary mathematics education as compared to the contract in tertiary mathematics education, described in terms of beliefs about the nature of teaching in the mathematics classroom (whether it is lecturer-centred or student-centred), the nature of learning in a mathematics classroom (whether the understanding is more procedural or more conceptual) and the responsibility for student learning (whether the responsibility rests with the lecturer or students). In an attempt to use personal response systems to renegotiate the didactical contract in the mathematics classroom, the following research question is formulated:

How can personal response systems be utilised to renegotiate the didactical contract in the mathematics classroom through influencing student beliefs about (1) the centredness of the classroom, (2) the nature of mathematics learning and (3) the responsibility for learning?

3. LITERATURE REVIEW

3.1. Principles of effective teaching and learning

Bransford, Brown, and Cocking (2000: 14-19) identify three "key" principles that characterise student learning and underlie effective teaching: students' preconceptions must be challenged, deeper understanding must be encouraged, and metacognitive skills developed. Schoenfeld (2000) identifies a framework that includes equal access to rich content, mental effort, students taking ownership for their learning, and formative assessment as essential for effective teaching and learning.

He links the strategy of creating opportunities for students to take ownership for their own learning with the "development of positive identities as thinkers and learners" (Schoenfeld, 2000: 9). Hiebert *et al*. (1996: 17) advocate the facilitation of student learning through engagement in problem solving or "problematising mathematics". Students must identify and actively participate to resolve problem(s). This is accomplished through encouraging "deeper conceptual understanding", or meaningful connections between concepts (Hiebert *et al*., 1996), and creating a community of learners that practice mathematics, a responsibility shared by the teacher and students. Hiebert and Carpenter (2006) mention that the role of the teacher is to create learning opportunities situated in problems that the students find meaningful. According to Hiebert *et al*. (1996), evidence exists that the approach of problematising mathematics encourages positive beliefs about mathematics. According to Bransford *et al*. (2000), the teacher should facilitate learning in a learner-centred environment where close consideration is paid to the knowledge, skills and beliefs that learners bring to the classroom.

3.2. Theory of the didactical contract

The concept of the didactical contract, formulated as part of Brousseau's theory of didactical situations (TDS) (Artigue, 2009; Balachef, 1990; D'Amore, 2008), refers to the agreement about reciprocal responsibilities that govern the behaviour of the lecturer and students in the mathematics classroom. Three characteristics of TDS as identified by Grønbæk, Misfeldt and Winsløw (2009) are briefly discussed. The main characteristic of TDS is that the situation for learning takes centre stage. The epistemological meaning of knowledge and the learning environment or *milieu* are used to guide students towards an *adidactical* adaptation to learning, with learning based on the mathematical problem at stake and not on the direction of the lecturer or the didactical contract. Secondly, the role of the lecturer is restricted to facilitating learning, while opportunity is created for students to understand and create mathematics in the same way that a mathematician would understand and create mathematics, constituting the third characteristic.

Grønbæk *et al*. (2009: 88) identify four phases through which didactical situations created by the teacher are transformed into *adidactical* situations. The first phase is the "*devolution of adidactical situations of action*", where students actively explore the *milieu*. The second phase, the "*devolution of adidactical situations of formulation*", is where students are encouraged to express their observations of interactions with the *milieu*. The third phase, called *"situations of validation",* is where students attempt to validate statements generated through observations. The fourth phase is the phase of "*situations of institutionalisation"*, in which the knowledge to be retained is accentuated by the teacher.

The didactical contract in secondary mathematics education, as compared to tertiary mathematics education, is discussed.

3.2.1. The didactical contract: from secondary to tertiary mathematics education

A teacher's view of the nature of mathematics impacts on the model of teaching and learning enacted in the teacher's classroom (Ernest, 1988). In her investigation of the secondary-tertiary mathematics transition at a South African university, Benadé (2013: 20) distinguishes between the intended and the implemented curriculum, mentioning that a gap exists between the "formal approved guidelines" and what actually happens in secondary mathematics classrooms. She finds that students view the lecturer to be the source of all knowledge, learning is a process of transferral of knowledge, and mathematics is about procedural fluency. She notes that at secondary level the responsibility for learning lies mainly with the teacher, whereas at tertiary level the responsibility becomes the student's.

Hourigan and O' Donoghue (2007) find that the didactical contract in secondary mathematics education in Ireland is teacher-centred, with the primary concern of teaching/learning being preparation for the exam. As a result, learners are passive, and they expect the teacher to simplify the task of learning. Yoon *et al*. (2011) investigate the didactical contract in the traditional transmission mode mathematics classroom at tertiary level and find that students perceive good teaching to be captured by the lecturer's ability to break down and model procedures with clarity.

A summary of researchers' (Benadé, 2013; Biggs, 1999; Brandell *et al*., 2008; Hourigan & O' Donoghue, 2007; Pepin, 2014; Selden, 2005; Yoon *et al*., 2011) observations about the didactical contract in secondary mathematics education is provided in Table 1 as a generalised description that is not without exception. The summary in Table 1 also attempts to identify the didactical contract at tertiary level for renegotiation. The characteristics of a didactical contract beneficial to learning (at tertiary level) are elaborated subsequently.

3.2.2. From teacher-centred to student-centred education

From a constructivist point of view, students build or construct their understanding of new information on their understanding of previous knowledge. The construction of new knowledge in combination with previous knowledge characterises understanding and learning (Lantz, 2010). According to Biggs (1999), Clark and Lovric (2009) and Bransford *et al*. (2000) the construction should be guided by good teaching in a student-centred teaching/learning environment, where students are engaged in cognitive conflict on purpose, and learning activities are designed around "known misconceptions" (Bransford *et al.* 2000: 134) or misconceptions that the lecturer is aware of. According to Zian, Rasidi and Abidin (2012) a student-centred environment offers a teaching/learning approach where the focus is shifted from the lecturer to the student.

3.2.3. Focus on conceptual understanding, not only on procedural fluency

Mathematics is about "relational understanding" and not just about applying rules and procedures without understanding why, which is also referred to as instrumental understanding by Skemp (1976). Van de Walle *et al*. (2013) place instrumental and relational understanding on opposite ends of the continuum of understanding, where relational understanding is a meaningful and useful construct of interconnected concepts that supports learning. Kilpatrick, Swafford and Findell (2001) describe conceptual understanding as the comprehension of mathematical concepts and procedural fluency as the skill of performing procedures efficiently and appropriately. According to Usiskin (2012) conceptual understanding and procedural fluency can be mastered independently, but together constitute a meaningful understanding of mathematics. For the purpose of this study, procedural fluency in combination with conceptual understanding represents the relational end of the continuum of understanding as posited by Van de Walle *et al*. (2013), and procedural fluency without conceptual understanding represents the opposing or instrumental end of the continuum.

3.2.4. Students responsible for their own learning

In order to get students to take responsibility for their own learning, strategies have to be identified and incorporated to improve students' extrinsic and intrinsic motivation (Elton, 1996). According to researchers (Jones, 2007; Zian *et al*., 2012) students in a student-centred classroom are extrinsically motivated, because autonomy and independent learning are encouraged. In other words, students are encouraged to take responsibility for their own learning.

4. PEDAGOGY FOR NEGOTIATING THE CONTRACT

Yoon *et al*. (2011) mention that by including interactive learning activities in a teaching approach and focusing on conceptual understanding rather than procedural fluency, a new didactical contract can be negotiated. Pedagogy for the use of personal response systems within a flipped classroom model, aimed at renegotiating the didactical contract, is briefly discussed.

4.1. The flipped classroom educational model

Abeysekera and Dawson (2015) argue that the flipped classroom model has the potential to improve students' intrinsic and extrinsic motivation, as mentioned by Elton (1996). They condense the definition of a flipped classroom to imply all teaching approaches where the transferral of knowledge is moved outside the classroom, class time is utilised towards active learning, and students have to complete learning activities before and after the class.

4.2. Personal response systems

The pedagogy of technology enhanced formative assessment (Beatty & Gerace, 2009) advocates the use of deep and challenging PRS questions to introduce concepts and initiate discussion. Beatty and Gerace (2009) refer to Eric Mazur's peer instruction method as an effective technique for using personal response systems to improve student learning. Mazur utilises PRS questions throughout his lectures, students respond to a question by voting, time is allowed for peer discussion and then the students vote once more on the same question, followed by a class discussion. Bruff (2010) notes the value of peers explaining difficult concepts, but emphasises that the lecturer's closing statement(s) must focus on providing reasons behind correct or incorrect answers.

5. THEORETICAL FRAMEWORK

At the centre of the study is the goal of using personal response systems to flip the mathematics classroom, with the purpose of influencing first-year students' predominant beliefs about (1) the centredness of the mathematics classroom, (2) the nature of mathematics learning and (3) the responsibility for learning as summarised in Table 1. The summary is for the purpose of identifying and negotiating student beliefs and the didactical contract. The contract is mainly characterised but not defined by the said characteristics, and the mutual exclusivity of characteristics is not implied.

Table 1. The didactical contract in mathematics education

	Beliefs about mathematics	**Secondary level**	**Tertiary level**
1.	Centredness	Teacher-centred (Hourigan & O'Donoghue, 2007)	Student-centred (Biggs, 1999; Bransford *et al*., 2000)
2.	Mathematics learning	Learning is instrumental understanding: Focus on procedural fluency (Benadé, 2013)	Learning is relational understanding: Focus on conceptual understanding (Hiebert *et al*. 1996; Yoon *et al*., 2011)
3.	Responsibility for learning	Responsibility of the teacher (Benadé, 2013)	Responsibility of the student (Benadé, 2013)

The study aims to determine how personal response systems can be used to renegotiate the didactical contract by redirecting students' beliefs about mathematics teaching/learning from lecturer-centred to student-centred, from instrumental understanding to relational understanding, and from the lecturer being responsible for learning to the student being responsible for learning.

The pedagogy of Beatty and Gerace i.e. TEFA, (2009), Mazur's peer instruction and the flipped classroom educational model inspired the pedagogical design of PRS sessions, to be called Time-out sessions for the remainder of the report. The name is based on the idea that students take time out from the traditional mathematics lecture, and the mathematics classroom is flipped or shifted from lecturer-centred to student-centred.

6. RESEARCH DESIGN

The study was conducted in 2018 at the Department of Mathematics of University of Pretoria, where modules are presented per semester. The principal researcher was one of the four lecturers involved in a first-year applied calculus module, presented mainly to biological sciences students. The module is presented in the first semester of their first year and the lecture group involved is also referred to as the intervention group.

A pragmatic perspective determined the research approach, a design-based research approach or design experiment, where innovation aimed at improving student learning is engineered through the use of complex methodologies in the context of a classroom (Brown, 1992). In this study findings from qualitative and quantitative data were triangulated to gauge a possible shift in student beliefs and the didactical contract due to the intervention of Time-out sessions. Quantitative data were collected in the form of questionnaire responses on student beliefs deployed at the beginning of the semester (Survey 1) and redeployed near the end of the semester (Survey 2), with focus group interviews conducted around the same time as Survey 2 to generate qualitative data.

The intervention consisted of PRS sessions, or Time-out sessions, incorporated intermittently during the lectures of the intervention group. Figure 1 is included to demonstrate a typical PRS question with the bar chart of student responses included.

A control group or randomised sampling was out of the question, because the intervention group was one of four lecture groups taught by four different lecturers. Convenience sampling was used as sampling method, because students' timetables determined lecture group allocation.

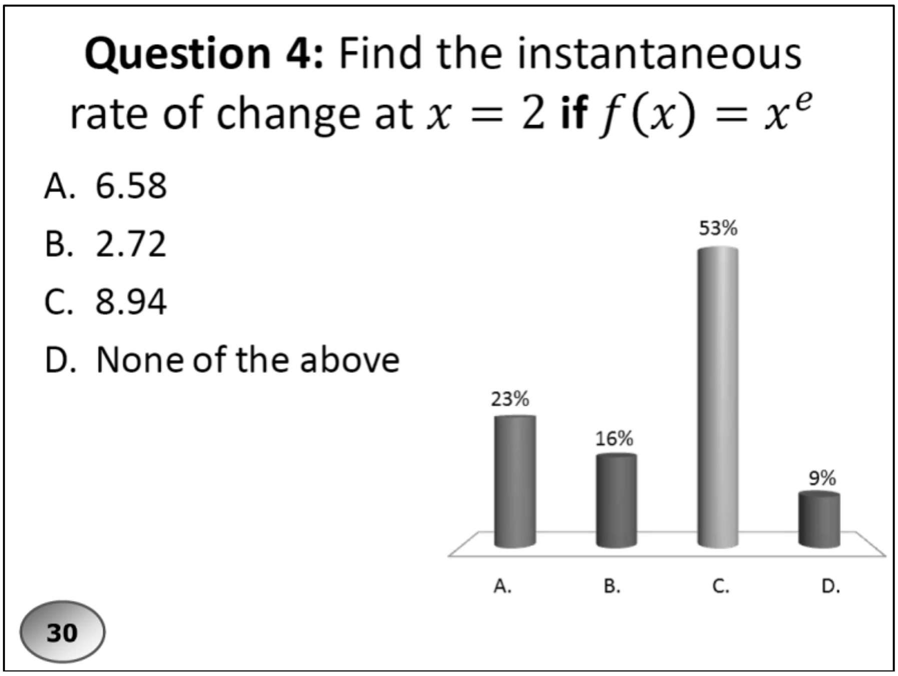

Figure 1. A typical PRS question with bar chart of student responses

6.1. Format of PRS sessions, or Time-out sessions

PRS questions were incorporated into the traditional transmission mode lecture. PRS questions were designed for deep conceptual learning and pedagogy aimed at influencing student beliefs to motivate them to take responsibility for their own learning, resulting in a shift in the didactical contract. For this purpose, the PRS questions were based on module-specific learning outcomes with possible student mistakes incorporated as distractors.

Students had to prepare the relevant content by means of a pre-class worksheet posted on the university's learning management system (clickUP). They had to complete a clickUP assessment, based on the worksheet, on the day before the lecture. During the lecture, no more than five PRS questions were presented for voting without revealing the bar chart of student responses or the correct answers. The first vote was followed by the opportunity to discuss solutions with peers before a second vote per question was allowed. During a Time-out session the lecturer's role was restricted to Socratic dialogue (Brogt, 2007), or eliciting understanding through questioning, and the lecturer concluded the session with a brief discussion about correct and incorrect answers. A second worksheet was distributed near the end of the lecture, to be completed and checked against a memorandum in the student's own time, but not to be discussed in class. The intervention consisted of six Time-out sessions, each incorporated into one of the four weekly lectures of the intervention group and distributed throughout the semester.

6.2. Data collection

A Likert scale questionnaire, consisting of 24 items initially, was designed to gauge students' beliefs about mathematics teaching/learning, with four options to choose from: strongly disagree, disagree, agree and strongly agree. The questionnaire items were categorised into the three categories of beliefs of the theoretical framework (Table 1) with items about centredness subcategorised either as describing student-centred learning (5 statements) or lecturer-centred learning (5 statements), items about mathematics learning subcategorised as either describing relational understanding (3 statements) or instrumental understanding (3 statements) and items about responsibility for learning subcategorised as either describing student responsibility (4 statements) or lecturer responsibility (4 statements).

The questionnaire was designed by the principal researcher to evaluate students' beliefs about mathematics, based on a questionnaire by Benadé (2013). The content validity of the questionnaire or the level to which the instrument measures all aspects of the targeted constructs (Pietersen & Maree, 2012a) was retained by having the instrument analysed by two experts in the field, and piloting the questionnaire in 2017. The questionnaire was first amended after being piloted, and again in 2018 based on the data of Survey 1. The Cronbach's alpha coefficients calculated for the three categories of the questionnaire were used to gauge the internal reliability of the questionnaire (Pietersen & Maree, 2012a). One of the items was consequently omitted and one was reverse scored, the implication being that 23 items of the questionnaire were earmarked for future analysis, and 2 (not 3) statements described mathematics learning as instrumental understanding. Items from each subcategory of the three categories of the questionnaire are included in Table 2 with the asterisk indicating items adapted from the questionnaire by Benadé (2013). 271 students of the cohort voluntarily completed Survey 1 and 59 students (of the intervention group) completed both surveys.

Table 2. Questionnaire items from the three categories of the questionnaire

Category	Description	Item
Centredness	Teaching as lecturer-centred	2.2* A good mathematics lecturer always demonstrates the correct method to solve problems.
	Teaching as student-centred	2.1* Students can find methods to solve problems without the help of the lecturer.
Mathematics learning	Instrumental understanding	1.1* Mathematics can be best described as a set of facts, rules and formulas that students have to learn.
	Relational understanding	1.3 To be able to do mathematics, a student has to understand mathematical concepts behind rules and formulas.
Responsibility for learning	Lecturer responsibility	2.6 The lecturer is responsible for the student's learning of mathematics through effective teaching.
	Student responsibility	1.8 It is the responsibility of the student to clarify confusion experienced with mathematics.

To judge the effectiveness of every Time-out session, the success of every PRS question was determined by comparing data of the second vote with data of the first vote. If the percentage of students that chose the correct response increased with the second vote to 70 percent or more, assuming the first vote had a score of less than 70 percent, enhanced learning was deemed evident (Crouch & Mazur, 2001). For every Time-out session a PRS question was used to determine whether students had prepared the relevant pre-class worksheet. On average 86 percent of the students acknowledged that they had prepared for the Time-out sessions.

Focus group interviews were conducted one week before the end of the semester, and thirteen students from the intervention group participated in two separate focus group interviews, the first consisting of five students (Respondent 1 to 5) and the second eight students (Respondents 6 to 13). The interviews were audio recorded, transcribed and analysed afterwards.

6.3. Data analysis

The questionnaire data for the 59 students that completed the questionnaire both times (Surveys 1 and 2) were quantified by means of indexes and compared through statistical methods.

Three beliefs indexes, inspired by Lucas (2009), were calculated from the two data sets generated by the two surveys for each of the 59 students, namely the Centredness index (C), Mathematics learning index (M) and Responsibility index (R). For this purpose, fixed values were associated with the four options of the Likert scale, a one with “strongly disagree”, a two with “disagree”, a three with “agree” and a four with “strongly agree”. Table 3 provides a summary of the three indexes. For the Centredness index (C) the ratio $\frac{sc}{lc}$ was calculated with *sc* the total score associated with a student’s responses to the five student-centred statements, and *lc* the total score associated with the student’s responses to the five lecturer-centred statements. In the same way the Mathematics learning index (M) is the ratio $\frac{ru}{iu}$ with *ru* the score for the three statements about relational understanding and *iu* the score for the two statements about instrumental understanding. Also, the Responsibility index (R) was calculated as the ratio $\frac{sr}{lr}$ with *sr* representing the score for the four statements about student responsibility and *lr* the score for the four statements about lecturer responsibility.

Table 3. Students’ Beliefs Indexes

Index	Pivotal values
Centredness index C= $\frac{sc}{lc}$ with sc: student-centred and lc: lecturer-centred	1
Mathematics learning index M= $\frac{ru}{iu}$ with ru: relational understanding and iu: instrumental understanding	1.5
Responsibility index R= $\frac{sr}{lr}$ with sr: student responsibility and lr: lecturer responsibility	1

In the case of the Centredness index, a value less than one ($C < 1$) implies that students’ beliefs lean towards the mathematics classroom being lecturer-centred, whereas a Responsibility index of less than one ($R < 1$) relates to the belief that the responsibility for learning is that of the lecturer. In the case of the Mathematics learning index a value less than 1.5 ($M < 1.5$) is interpreted as students believing mathematics learning to be about instrumental understanding. To explain, if a student agrees with all three statements (by assigning a score of 3) pertaining to relational understanding (*ru*) as well as the two statements pertaining to instrumental understanding (*iu*), hence demonstrating no bias, then $M = \frac{ru}{iu} = \frac{9}{6} = 1.5$. A bias towards relational understanding would be visible if at least $M = \frac{ru}{iu} = \frac{10}{6} \approx 1.7$, with one statement about relational understanding strongly agreed on, whereas a bias towards instrumental understanding can be construed if at most $M = \frac{ru}{iu} = \frac{8}{6} \approx 1.3$, with one statement about relational understanding disagreed on.

7. RESULTS

7.1. Quantitative data

The averages of the three beliefs indexes pertaining to the 59 students that completed both surveys, calculated from the data of Survey 1, are provided in Table 4. Because of the size of the sample and sampling methods these findings are not interpreted as representative of the population of the study. Therefore, the focus when analysing the results is the possibility of a shift in student beliefs.

To determine the influence of the Time-out sessions on student beliefs and the didactical contract, the averages of the three beliefs indexes for Survey 1 were compared to the averages of the three beliefs indexes calculated for Survey 2 (Table 4). The average of the Centredness index (C) decreased from 0.94 to 0.92, the average of the Mathematics learning index (M) increased from 1.83 to 1.91, and the average of the Responsibility index (R) increased from 1.10 to 1.19. The interpretation here is that after one semester of mathematics, students' beliefs about the mathematics classroom shifted towards lecturer-centred, whereas beliefs about mathematics learning shifted towards relational understanding and beliefs about responsibility for learning shifted towards student responsibility.

Table 4. Averages of the three beliefs indexes of Survey 1 and 2

Index	**Averages**			**Significance values**
	Survey 1 (Before the intervention)	**Survey 2** (After the intervention)	**Criteria**	• **p-value for one-sided hypothesis** (Significance level of 0.05)
Centredness (C)	0.94	0.92	C < 1	0.256
Mathematics learning (M)	1.83	1.91	M > 1.5	0.315
Responsibility (R)	1.10	1.19	R > 1	0.0315

From the statistical analysis of the data of Surveys 1 and 2 (in Table 4) the data distribution can be described as nonnormally, hence the decision to proceed with nonparametric statistical methods to confirm a possible shift in student beliefs. The nonparametric test for comparison of two variables in a single sample, the Wilcoxon signed rank test, was used to compare the Centredness Index (C), the Mathematics learning index (M) and the Responsibility index (R) for Survey 1 to that of Survey 2. With the Wilcoxon signed rank test "the null hypothesis is that the median of the difference scores is zero" (Pietersen & Maree, 2012a:231).

The significance (two-tailed) values (or p-values) for the three null hypotheses related to the three different beliefs indexes, with a significance level of 0.05, are discussed. In the case of all three indexes the hypotheses are one-sided, meaning that the particular index value is expected to be higher after the intervention than before the intervention. Since the significance (two tailed) values calculated for the three indexes are for a two-sided hypothesis, where the index values before and after the intervention are expected to differ, the significance (two-tailed) values are divided by two to represent the p-value for a one-sided hypothesis (Pietersen & Maree, 2012b). Based on a p-value of 0.0315, only the Responsibility index (R) shows a statistically significant increase.

7.2. Qualitative data

Extracts of student responses generated during the focus group interviews are provided here to elaborate student beliefs through triangulation with quantitative data.

When asked about whether they understood the purpose of the Time-out sessions, some respondents referred to a perceived purpose.

R3: *I think at first no, I did not understand, but as time went by I was like oh it works, because even my semester marks improved and I could understand like better.*

Hereby a shift in beliefs about the effectiveness of a student-centred mathematics classroom was demonstrated.

Three students from Focus group interview 1 and two from Focus group interview 2 used phrases like “forced you to work” or “they still felt that they needed to finish” when referring to the Time-out sessions. These phrases demonstrate that the students are motivated by external factors, but that the Time-out sessions did not influence their fundamental beliefs about mathematics teaching/learning.

R5: *I think the online worksheets actually almost to a certain extent, forced you to work or study ahead. It did not really give you much of a choice not to do it.*

When asked whether they thought that the Time-out sessions had altered their perceptions of mathematics and mathematics teaching/learning the students responded as follows, providing evidence of a shift in beliefs about mathematics learning.

R7: *I think that ... coming from high school, the teachers would teach us like everything, and now this, it shows me that there are a lot more work that we have to put in*.

R12: *I thought that she was trying to have us see how well we would do when we are doing it ourselves. Like you were saying it is easy when you see the lecturer doing it, but then it is a wake-up call for you if you are doing it and you realise that you are not doing as well as you would have expected or it’s not as easy as you would have thought.*

The following respondents mention an awareness of the responsibility for learning. One respondent referred to the Time-out sessions as the “non-learning learning sessions”. Other respondents referred to the LMS assessments written in preparation for the Time-out sessions as “LMS tests”.

R9: *I enjoyed the non-learning learning sessions, because now I have to take the initiative and make sure that I do the work.*

R6: *The-the LMS tests also show you that you shouldn’t be dependent on the teacher alone...I have to do it first, before I depend on somebody else.*

R13: *I would say LMS tests motivate me in such a way, only the lecturer does 10 percent, the 90 percent is your work.*

7.3. Triangulation of data

Brown (1992) mentions the Bartlett and Hawthorne effects as methodological concerns of design-based research. For the purpose of this article extracts of the data generated by focus group interviews are triangulated with the quantitative findings of the study, in order to compensate for the Bartlett effect. The use of phrases like “forced you to work” provides evidence of the Hawthorne effect, of the intervention eliciting a desired immediate response in students because of the need to appease the researcher. Brown (1992: 165) also mentions that the aim of design-based research is for students to become “coinvestigators of their own knowledge” and that evidence of the Hawthorne effect is proof of the intervention instigating change. The comment made by respondent 12 of the focus group interviews provides evidence that the intervention encourages students to reflect on and adapt their approach to learning, hence demonstrating metacognitive skills.

Statistical analysis of the quantitative data provided some evidence of a shift in student beliefs due to the influence of the Time-out sessions, although the shift was not always statistically significant. Students from the focus group interviews indicated that their beliefs about the centredness of the mathematics classroom were not necessarily influenced, hereby confirming the quantitative results. On the other hand, they provided evidence of the Time-out sessions impacting their beliefs about and approach to mathematics learning, and the responsibility for their learning, as observed through analysis of the quantitative data.

8. DISCUSSION

Analysing the Time-out sessions from the perspective of TDS implies a consideration of the three characteristics or principles of TDS mentioned earlier. From the perspective of TDS, the meaning behind knowledge needs to be emphasised in a teaching/learning event (Artigue, 2009; Grønbæk *et al*., 2009). For the Time-out sessions this first principle was maintained by designing deep challenging PRS questions aimed at addressing possible student mistakes. By encouraging students to engage with the content before and after the lecture and restricting the lecturer's dialogue to Socratic dialogue, prominence was given to the didactical situation and an attempt was made to inspire *adidactical* adaptations on the part of the students. Secondly, the lecturer's role was limited to inspiring a transfer of responsibility for learning and emphasising knowledge to be retained (Grønbæk *et al*., 2009). Through the Time-out sessions, opportunities were created for students to discover and institutionalise content in the context of didactical situations, in the same way that a mathematician discovers and institutionalises content, the third characteristic of TDS (Grønbæk *et al*., 2009). The design of the Time-out sessions allowed for the four phases of Grønbæk *et al*. (2009) to be demonstrated. The first two phases constituted the students initial voting, followed by peer discussion. Phase 3 was established by allowing students to vote again and validate their findings, while the closing statements of the lecturer allowed for institutionalisation of knowledge (phase 4).

The students proved to be motivated to participate in the Time-out sessions, since the majority of students acknowledged preparing the pre-class worksheets. Although the LMS assessments based on the pre-class worksheets did not contribute to students' marks, a pursuit of marks might have motivated some students to participate initially, while student preparation of the pre-class worksheets throughout the rest of the semester could be attributed to other motivational factors such as an increased sense of preparedness for the exam, as mentioned by Elton (1996).

Eric Mazur mentions (Lambert, 2012) that students are resistant to change and that they prefer to be passive and take notes, but he advocates that the focus in teaching/learning paradigms shift from lecturing to creating opportunities for student learning. Grønbæk *et al*. (2009) uphold the concept of didactically designed learning situations but mention that student-centred teaching/learning events should not dominate lectures at university.

9. CONCLUSION

The value of the study lies in highlighting the importance of supporting students to successfully transition from secondary to tertiary mathematics education by creating a student-centred learning environment focused on conceptual understanding and motivating students to accept responsibility for their own learning. Pedagogy for using PRS questions in a flipped mathematics classroom at tertiary level was explored and holds potential for future research.

When analysing the results of the study against the backdrop of the theoretical framework, it becomes evident that the intervention seemingly did not affect students' beliefs about the centredness of the mathematics classroom, but appeared to have impacted students' responsibility for learning and to some extent their beliefs about mathematics learning. A lack of change in student beliefs about centredness could be attributed to huge lecture groups (of up to 650 students) accommodated in huge lecture halls, in which the lecturer necessarily takes centre stage.

The study shows that the didactical contract can be influenced by purposefully incorporating PRS sessions into the traditional transmission style lectures of a large mathematics classroom. By using well-designed PRS questions and pedagogy to shift the mathematics classroom from lecturer-centred to student-centred, opportunity is created for students to compare active learning to passive learning and students are motivated to focus on and adjust their learning and learning strategies in the teaching/learning environment.

In this article, we have described our approach of using technology or clickers to create a learning environment conducive to influencing student beliefs about mathematics and mathematics learning in a large classroom at tertiary level. The study demonstrates the impact of designing classroom practice for meaningful learning, by providing students with equal access to rich content and encouraging mental effort and ownership through formative assessment, as mentioned by Schoenfeld (2000). The practice of teaching is informed, but the value of the study at design and theoretical level has to be observed. At design level, the study contributed to the development of statistical methods to examine student beliefs and the didactical contract in a mathematics classroom. At theoretical level, the study demonstrated how a lecturer "actively creates knowledge through experience" and "reflection on that experience" (Clarke, 1997: 283). This brings me to the theoretical framework of the study. Upon reflection it is realised that the responsibility for learning was overly simplified for the purpose of this study, and that the theoretical framework (and questionnaire) can be improved for use in future research. Having said that, the study lays the foundation for future research into classroom practice, but also design research at tertiary level. Although the terms teacher-centred and learner-centred may be questioned as being old-fashioned, due to the roles of teacher and learner evolving into that of knowledge facilitator and knowledge searcher, we venture to say that the terms are still actively used in literature, perhaps even more so in the science field. The centredness of a classroom in this study still adheres to this terminology but deserves rethinking in future research. A challenge of the study was my dual role as lecturer and researcher and a suggestion for future research is to separate the two roles in order to control bias even more and ensure the credibility of the research.

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