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# Analysing the Impact of Brexit on Global Uncertainty Using Functional Linear Regression with Point of Impact: The Role of Currency and Equity Markets

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## Abstract

This paper studies the relationship between monthly economic uncertainty of 20 advanced and emerging markets, and two daily covariates, i.e., exchange rate and stock index, with particular emphasis to the relationship between the variables in response to the Brexit vote. We use a functional data approach supplemented with a point of impact structure to conduct a mixed-frequency analysis. We find that incorporating the point of impact, in this case the Brexit shock, is marginally important, relative to models that ignore it, and that the exchange rate played a more important role than the equity market in transmitting the Brexit shock to cause heightened uncertainty in the 20 countries considered. Our results have important policy implications.

Keywords: Functional Data Analysis, Point of Impact, Brexit, Uncertainty, Currency and Stock Markets

JEL Codes: C22, G10

# 1 Introduction

Since World War II, there has been a trend of increased political and economic integration across Europe, lately under the auspices of the European Union (EU). “Brexit”, the act of the United Kingdom (UK) leaving the EU, presents a clear departure from this trend. Days before the Brexit referendum, Gropp (2016) had observed that whenever the probability of Brexit moved above 50%, substantial depreciation was observed to the UK pound with respect to most major currencies. In addition, Dhingra et al. (2016) had posited that a vote for Brexit would be a negative signal for the prospects of the UK economy. The outcome of the UK referendum to leave the EU on 23 June 2016 thus sent shockwaves throughout Europe in particular, and throughout the world in general. In response to the results of the referendum, global stock markets declined sharply and lost more than 2 trillion dollars in value in response to the results of the Brexit vote – the largest single-day loss ever recorded in world markets (Raddant, 2016). The London Stock Exchange’s FTSE 100 index fell by nearly 11% in dollar terms, while many other European markets decreased by as much or even worse. Generally, the global equity markets fell by 4.7% on average, with European stock markets accounting for more of the losses, while the BRICS countries of Brazil, Russia, India, China and South Africa’s stock markets fell relatively less than the average (Burdekin et al., 2018).

The vote for Brexit increased uncertainty associated with financial markets. Indeed, a global measure of economic policy-related uncertainty, namely the Global Economic Policy Uncertainty Index (GEPU)<sup>1</sup>, indicated that the uncertainty levels following the Brexit vote were higher than those observed post-2008-2009, i.e., after the global financial crisis (Davis, 2016). This suggests that the Brexit vote was a significant shock within world markets and to the uncertainty levels in various countries. Investors tend to be risk averse, and base their current trading decisions on their expectations of the future performance of an asset. The movements of stock prices and exchange rates provide insights about how investors feel regarding the overall prospects of the financial markets (Apergis et al., 2018). In this regard, recent studies by Balli et al. (2017) and Gupta et al. (Forthcoming) have indeed shown that uncertainty spill-over through financial, currency and trade channels. Given this, the objective of this paper is to analyze the role of two fast-moving covariates, i.e., exchange rate and stock price indices in transmitting the Brexit shock on uncertainty levels of 20 developed and emerging economies of the world.

Given that the metric of uncertainty of these 20 countries is available monthly,

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<sup>1</sup>The GEPU is a weighted average of 16 national EPU indices, and each national index reflects the relative frequency of own-country newspaper articles that contain a trio of terms pertaining to the economy, uncertainty and policy-related matters.

and covariates used are observed daily to account for the the Brexit shock on a specific day, the functional data analysis (FDA) methods are preferred for this mixed-frequency analysis. Given a functional linear model with a scalar dependent variable and a functional independent variable, the relationship between the two variables can be explained using the model of Ramsay and Ramsey (2002), Malfait and Ramsay (2003) and Kosiorowski (2014), among others. However, within the trajectory of the independent variable, there may exist certain specific time points, like Brexit event in our case, that have an additional effect on the dependent variable that may not be captured by the standard functional linear regression model of Ramsay and Ramsey (2002) and others. In addition, results of the standard functional linear regression might be difficult to analyse and interpret since the model is a weighted average of the whole trajectory of the functional predictors. The question then is: how to capture these certain “points of impact” in the time variation of the independent variable that may not be easily explained by the traditional functional linear model? The model of Kneip et al. (2016) and Poß et al. (2018) presented in Section 3 answers the question outlined above by introducing an augmented model namely, the usual functional linear regression model with an additional term which captures the point(s) of impact on the trajectory of the independent variable that have an extra effect on the dependent variable. Such point(s) of impact may not necessarily be known, i.e., not necessarily fixed, and together with the regression coefficient function, must be estimated. While Mixed-data sampling (MIDAS) models can handle the mixed-frequency aspect of our data, it would not be able to account for the point of impact issue. Besides, FDA methods do not require us to work with stationary data, and hence is free from any transformation, which is likely to change the definition of the variables being investigated. So, even though the Brexit did occur on a specific date, it is not necessary that the impact is likely to be channelized to global uncertainty on that specific dates via stock and currency markets, since the agents operating in these markets are forward-looking, and because of to the fact that the Brexit vote was common knowledge ahead of the actual date.

To the best of our knowledge this is the first paper to analyze the role of currency and equity markets in transmitting the impact of the Brexit shock on uncertainty levels of 20 countries using an FDA analysis, which is supplemented with a point of impact structure. The remainder of the paper is organized as follows: Sections 2 and 3 outlines the literature and the details of the methodology respectively. Then Section 4 provides a discussion of the data and results, with Section 5 concluding the paper.

## 2 Literature Background on the Method

Kneip et al. (2016) posit that to their knowledge, McKeague and Sen (2010) were the first to explicitly study the identification and estimation of a point of impact in functional linear regression. The motivation for developing the model was based from genome-wide expression studies where it is of interest to locate genes associated with clinical outcomes. The model is given by

$$Y_i = \beta_0 + \beta_1 X_i(\tau) + \epsilon_i, \quad (1)$$

where  $Y_i$  is a scalar dependent variable,  $X_i(t)$  is a functional independent variable,  $\beta_0$  and  $\beta_1$  are the regression coefficients to be estimated,  $\tau$  is a single point of impact in the domain  $[a, b]$  of  $t$  and  $\epsilon_i$  is the error term with mean zero and a finite variance  $\sigma^2$ . McKeague and Sen (2010) state that their model complements, and is more interpretable than the functional linear regression approach, as it provides interpretable information about the influence of  $X$  at a specific location in  $[a, b]$ . Because in the classical functional linear regression model of Ramsay and Ramsey (2002), the influence of the independent variable, the  $X_i(t)$ , is spread continuously across the domain of  $t$ , such information, about the influence of the  $X_i(t)$  at a specific location, cannot be extracted. The parameters are estimated using the least squares method, where for a sample of size  $n$ , the mean squared error  $E[Y - \beta_0 - \beta_1 X(\tau)]^2$  is minimised.

Another solution to the problem of the influence of  $X_i(t)$  being spread continuously across the domain of  $t$  was suggested by James et al. (2009), when they proposed what they termed “Functional Linear Regression That’s Interpretable” (FLiRTI). Their method assumes that the regression coefficient of the functional linear regression is zero, that is,  $\beta(t) = 0$ , for most points  $t \in [a, b]$ , except at some identified subintervals of  $[a, b]$  where the coefficient is nonzero. They used the popular classical linear regression Canadian weather data set of Ramsay and Siverman (2005) as a practical application of the model. The objective was to predict annual rainfall using temperature curves. In the predicted regression coefficient of Ramsay and Siverman (2005), the curve suggests a positive linear relationship in the autumn months, a negative linear relationship in the spring months and no relationship in the summer months, but there are only two points where  $\beta(t) = 0$ . The FLiRTI approach provides better results with  $\beta(t) = 0$  in the summer and winter months, showing a positive relationship in autumn and a negative relationship in spring. Their method encourages sparsity in the  $\beta(t)$  for interpretability, accomplished via  $L_1$  penalisation to shrink the coefficients towards sparsity in a prespecified derivative. Piecewise constant basis functions for  $\beta(t)$  were used, although it was mentioned that other basis functions can be used.

Kneip et al. (2016) also used the Canadian weather data from Ramsay and Ramsey (2002), where the relative humidity is the response variable, and hourly temperature data was used as the explanatory variable. The linear regression as defined by Kneip et al. (2016) involves a scalar response  $Y$  and a functional predictor  $X(t)$ . The model is given by:

$$Y_i = \int_a^b \beta(t) X_i(t) dt + \sum_{r=1}^S \beta_r X_i(\tau_r) + \epsilon_i, \quad (2)$$

where  $\epsilon_i$ ,  $i = 1, 2, \dots, n$  are i.i.d. centred real random variables with  $\mathbb{E}(\epsilon_i^2) = \sigma^2 < \infty$ . The regression coefficient function  $\beta(t)$ , the number of points of impacts  $S \in \mathbb{N}$ , the location of the points of impact  $\tau_r$  and the coefficient of the points of impact term  $\beta_r$ ,  $r = 1, 2, \dots, S$  are all unknown and need to be estimated. Also,  $[a, b]$  is the domain of  $X_i(t)$ , although for computational sake, the domain is transformed into  $[0, 1]$ . The first term on the right hand-side of Equation 2,  $\int_a^b \beta(t) X_i(t) dt$ , is the usual regression term which describes a common effect of the whole trajectory of  $X_i(t)$  on  $Y_i$  (for instance, in Malfait and Ramsay (2003); Ramsay and Siverman (2005); Ramsay et al. (2009)). The hypothesis in this model is that there exists specific points, say,  $\tau_1, \tau_2, \dots, \tau_S$  at which the trajectory of  $X_i(t)$  may have an additional effect on the outcome of  $Y_i$ , which cannot be captured by  $\int_a^b \beta(t) X_i(t) dt$ . The introduction of the term  $\sum_{r=1}^S \beta_r X_i(\tau_r)$  in Equation 2 accounts for these effects, which are referred to as “points of impact”. What this means is that: firstly, the model incorporates an unknown number  $S$  of points of impact, namely,  $\tau_1, \tau_2, \dots, \tau_S$ , and secondly,  $\tau_1, \tau_2, \dots, \tau_S$  possess the property that their corresponding functional values,  $X(\tau_1), X(\tau_2), \dots, X(\tau_S)$  have some significant influence on the response variable  $Y_i$ .

Kneip et al. (2016) further compared three models, that is, augmented model (model consists of both the points of impact and a functional parts); points of impact model (model consists of only the points of impact part, that is,  $\beta(t)$  is set at zero); and functional linear regression model (model consists of only the functional part, that is, the number of points of impacts is set to be zero). They used the mean square prediction error (MSPE),  $MSPE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$ , where  $y_i$  is the scalar dependent variable and  $\hat{y}_i$  is the estimate of  $y_i$ , to compare the models. In addition, the median of  $(y_i - \hat{y}_i)^2$  was calculated as a more robust measure of error. It was found that the FLR model is outperformed by the model consisting of only points of impact, while the the augmented model performed better than the other models. More about these models will be discussed in the methodology section.

Poß et al. (2018)’s case study involved an experiment in which participants were asked to continuously rate their emotional state while watching a documentary about the persecution of African albinos. Their model is analogous to Equation 2.

The self-reported feeling trajectory,  $X_i(t)$ , are the explanatory variables in this scenario. In addition, after watching the videos, the participants were asked about their overall feeling about the video. This created a binary variable  $Y_i$ , where  $Y_i = 0$  meant participant was feeling negative, and  $Y_i = 1$  meant participant was feeling positive. The points of impact model was compared with two logit regression models. These logit regression models have what is called peak intensity predictors, and these were found to be evenly distributed across the whole domain. While the points of impact model identified two points of impact which linked the overall ratings  $Y_i$  with the two points of impact thus linking the overall rating with two emotionally charged phrases spoken during these points of impact.

Liebl et al. (2017b) further developed on the model of Kneip et al. (2016). In particular, their model is still the one given by Equation 2, but they propose a new algorithm (PES-ES) which they claim leads to better results. The algorithm involves pre-selecting the POIs, and estimating the slope parameters given the preselected POIs, and then sub-selecting the true PoIs from the pre-selected set. Sub selection is achieved by minimising the Bayesian information criterion (BIC) over subsets of the potential PoIs. In a practical application, they present data from Google AdWords where  $Y_i$  is the logarithmised number of clicks in a year and  $X_i(t)$  is the logarithmised number of “impressions”. We will not go into the details of the example, except to emphasize that Liebl et al. (2017b) proposed that within  $X_i(t)$ , there might be specific global seasonalities as well as local events (e.g. holidays) for some product being advertised. These functional linear regression models with points of impact methodologies are suitable for identifying and capturing such local and global events.

By no means is this a complete review on points of impact literature. But it provides enough leads on the motivation of using points of impact methods. In this paper, our interest is to investigate the relationship between monthly uncertainty and the two covariates of exchange rate and stock index, in the wake of the Brexit vote. In the process, we use the FDA-based point of impact model to study for the first time, the role of these two covariates in transmitting the uncertainty shock in one country (UK) to other economies around the world. In the process, we contribute to the burgeoning uncertainty spillover literature (Gupta et al., 2016; Antonakakis et al., 2018, 2019; Gabauer and Gupta, 2018; Çekin et al., 2019). This is an important line of research, since if uncertainties across economies are interrelated, as the above-mentioned studies provide ample evidence of, then even if there is no change in uncertainty at the domestic level, a particular economy will end up witnessing the negative impact of uncertainty through linkages that exist in a modern globalized world, following an increase in uncertainty in (a) foreign economies (economy).

The methodology is discussed in detail in the next section.

### 3 Methodology

We remind the reader of the functional regression model that involves a scalar response  $Y$  and a functional predictor  $X(t)$  as given by Equation 2, that is by:

$$Y_i = \int_a^b \beta(t) X_i(t) dt + \sum_{r=1}^S \beta_r X_i(\tau_r) + \epsilon_i, \quad (2 \text{ revisited})$$

where  $\epsilon_i$ ,  $i = 1, 2, \dots, n$  are i.i.d. centred real random variables with  $\mathbb{E}(\epsilon_i^2) = \sigma^2 < \infty$ . The regression coefficient function  $\beta(t)$ , the number of points of impacts  $S \in \mathbb{N}$ , the location of the points of impact  $\tau_r$  and the coefficient of the points of impact term  $\beta_r$ ,  $r = 1, 2, \dots, S$  are all unknown and have to be estimated.

The analysis of the model involves the central problem of estimating the number and locations of points of the impact. Kneip et al. (2016) developed conditions that ensure the identifiability of the components of the model in equation 2. The identifiability of a point of impact  $\tau_r$  means that some part of the local variation of  $X(t)$  in a small neighbourhood of  $[\tau - \epsilon, \tau + \epsilon]$  of  $\tau_r \in (a, b)$  is uncorrelated with the remainder of trajectories falling outside the interval  $[\tau - \epsilon, \tau + \epsilon]$ . This concept is formalised by Kneip et al. (2016) by referring to it as “specific local variation”. The main assumption here is that all variables have a mean of zero and so for practical applications this means that the data must be centred out by subtracting sample means from the data.

Instead of regressing on  $X_i(t)$ , the basic concept is to analyse the empirical correlation between  $Y_i$  and a process:

$$Z_{\partial,i}(t) := X_i(t) - \frac{1}{2}(X_i(t - \partial) + X_i(t + \partial)) \quad (3)$$

for some  $\partial > 0$ . Now,  $Z_{\partial,i}(t)$  is strongly correlated with  $X_i(t)$  and the correlation between  $Y_i$  and  $Z_{\partial,i}(t)$  will be high if and only if a particular point  $t$  is close to a point of impact. Further, Kneip et al. (2016) proves that points of impact may be estimated by using the locations of sufficiently large local maxima of  $|\frac{1}{2} \sum_{i=1}^n Z_{\partial,i}(t) Y_i|$ . The choice of  $\partial > 0$  is chosen in dependence to the sample size  $n$ . If  $\partial$  is too small compared to  $n$ , then it is difficult to find the points of impact as they “perish in a flood of random peaks” (Kneip et al., 2016). If  $\partial$  is too large, it is impossible to distinguish between the influence of points of impact close to each other. In order to consistently estimate  $S$ , the estimation procedure requires to exclude all points  $t$  in an interval of size  $\sqrt{\partial}$  around the local maxima of  $|\frac{1}{2} \sum_{i=1}^n Z_{\partial,i}(t) Y_i|$  from further consideration.



Liebl et al. (2017b) have further built on the model of Kneip et al. (2016) and propose a new algorithm called PES-ES. First, the algorithm involves pre-selecting the POIs using an algorithm from Kneip et al. (2016). Second, the slope parameters must be estimated (given the pre-selected POIs) using the penalized smoothing splines introduced by Crambes et al. (2009). Third, the true POIs must be selected from the pre-selected set. Sub-selection is achieved by minimising the Bayesian information criterion (BIC) over subsets of the potential POIs. Repetition of the last two steps leads to improved estimation results. Liebl et al. (2017b) emphasizes that the whole PES-ES algorithm depends on the initially selected POI's, and therefore on the choice of  $\partial$  (refer to Equation 3). The optimal value of  $\partial$  is found by minimising the BIC. For a detailed explanation of the full model and estimation, the reader is referred to Kneip et al. (2016) and Liebl et al. (2017b).

### 3.1 A Summary of the Models

In the interest of brevity, this subsection summarises the three possible regression models one may use to obtain the relationship between a scalar dependent variable and a functional independent variable, i.e. the Classical Functional Regression model; Points of Impact only model; and the Augmented model, which is a hybrid of the two models, as detailed below:

- **$M_{a.1}$ :** The Functional Linear Regression model can be defined by assuming that  $S = 0$  (i.e. there are no points of impact). Thus,

$$Y_i = \int_a^b \beta(t) X_i(t) dt + \epsilon_i(t). \quad (4)$$

We remind the reader that  $Y_i$  is the scalar dependent variable, while  $X_i(t)$  is the functional independent variable.

- **$M_{a.2}$ :** One can define the points of impact only model by assuming that  $\int_a^b \beta(t) X(t) dt = 0$  (specifically,  $\beta(t) = 0$ ). Thus,

$$Y_i = \sum_{r=1}^S \beta_r X_i(\tau_r) + \epsilon_i(t). \quad (5)$$

- **$M_{a.3}$ :** In a model where both the linear regression and points of impact

components are present. Thus,

$$Y_i = \int_a^b \beta(t) X_i(t) dt + \sum_{r=1}^S \beta_r X_i(\tau_r) + \epsilon_i. \quad (2 \text{ revisited})$$

The model is the one given in Equation 2. It may be noted now that the preceding two models are a special case of this model.

The notation above is analogous to that of Kneip et al. (2016) and Liebl et al. (2017b). The analysis was carried out using R Core Team (2018) software, in particular the R package *FunRegPoI* (Liebl et al., 2017a). In addition, the package *stats* which is part of R Core Team (2018) was also used. All analysis was carried within the R-Studio environment (RStudio Team, 2016).

## 4 Data and Results

### 4.1 Data

Uncertainty is a latent variable, and hence one requires ways to measure it. In this regard, besides the various alternative metrics of uncertainty associated with financial markets (such as the implied-volatility indices (popularly called the VIX), realized volatility, idiosyncratic volatility of equity returns, corporate spreads), there are primarily three broad approaches to quantify uncertainty (Gupta and Sun, 2020; Bilgin et al., 2019): (1) A news-based approach, with the main idea behind this method being to perform searches of major newspapers for terms related to economic and policy uncertainty, and then to use the results to construct indices of uncertainty; (2) Derive measures of uncertainty from stochastic-volatility estimates of various types of small and large-scale structural models related to macroeconomics and finance, and; (3) Uncertainty obtained from dispersion of professional forecaster disagreements.

As far as our metric of uncertainty is concerned, we use the first approach, i.e., news-based measure of Baker et al. (2016), primarily due to the fact that the measure does not require any complicated estimation of a large-scale model to generate it in the first place, and hence, is not model-specific. In addition, the data is available publicly for download.<sup>2</sup> While data on economic policy uncertainty (EPU) of the 20 countries (Australia, Brazil, Canada, Chile, China, France, Germany, Greece, India, Ireland, Italy, Japan, Mexico, The Netherlands, Singapore,

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<sup>2</sup>The data can be downloaded from: <http://policyuncertainty.com/>.

South Korea, Spain, Sweden, Russia, the United States (US)) are obtained at monthly frequency, in addition daily data on the stock indices of all the 20 countries and the British pound-based 20 bilateral nominal exchange rates are derived from Bloomberg and the Bank of International Settlements (BIS)<sup>3</sup>, respectively. Note that, we constrain our investigation to June 2016, i.e., the month the Brexit referendum was conducted in. Since the shock originated in the UK, its EPU is excluded from the model. In our case, the EPU of the various countries is the scalar dependent variable, while the the covariates of exchange rate and stock index are the functional independent variables.

## 4.2 Results

For the functional linear regression model with points of impact term, the variability of a scalar variable can be expressed in terms of variability in covariates, which must be functions (Kneip et al., 2016). As mentioned earlier, the dependent variable, i.e. the EPU, is a constant over a given month whereas the respective independent variables (exchange rates and stock price indices) are observed daily. At this point, it is worth noting that the exchange rate and the stock index fluctuations capture investor sentiment. Our primary objective is thus to study their impact (effectively investor sentiment) on global uncertainty. We remind the reader once more that we constrain the period of study to the month of June 2016, primarily because we are interested on the significance of the impact of the Brexit referendum on uncertainty of 20 advanced and emerging countries.

Table 1: Comparison of the model performance

	Model description	BIC
$M_{1.1}$	FLR with Exchange Rate	188.3229
$M_{1.2}$	POI only with Exchange Rate	185.3597
$M_{1.3}$	Hybrid with Exchange Rate	188.3818
$M_{2.1}$	FLR with Stock Index	188.9643
$M_{2.2}$	POI only with Stock Index	185.3602
$M_{2.3}$	Hybrid with Stock Index	190.7704
$M_{3.1}$	FLR with Interaction	193.1809
$M_{3.2}$	POI only with Interaction	186.4837
$M_{3.3}$	Hybrid with Interaction	193.6111

Note: Interaction implies that the two covariates are interacted together.

Table 1 presents results of the Bayesian information criterion (BIC) for the respective models, with the model with the least BIC preferred. Models 3.1 to 3.3

<sup>3</sup>The data is available at: <https://stats.bis.org/statx/toc/XR.html>.

are the interaction models of the two covariates, with the aim to capture the joint effect of currency and stock markets. In our case, the differences between the BIC values seem negligible to deserve more than a mere mention, except to point out that the POI only models (Models 1.2, 2.2, and 3.2) seem to perform better than the other models, and hence vindicates our decision to use them to capture the impact of the Brexit. In this regard, the POI only model with the exchange rate performs slightly better than its counterpart associated with the stock index. Also, the interaction models seem to be the poorest models across the board.

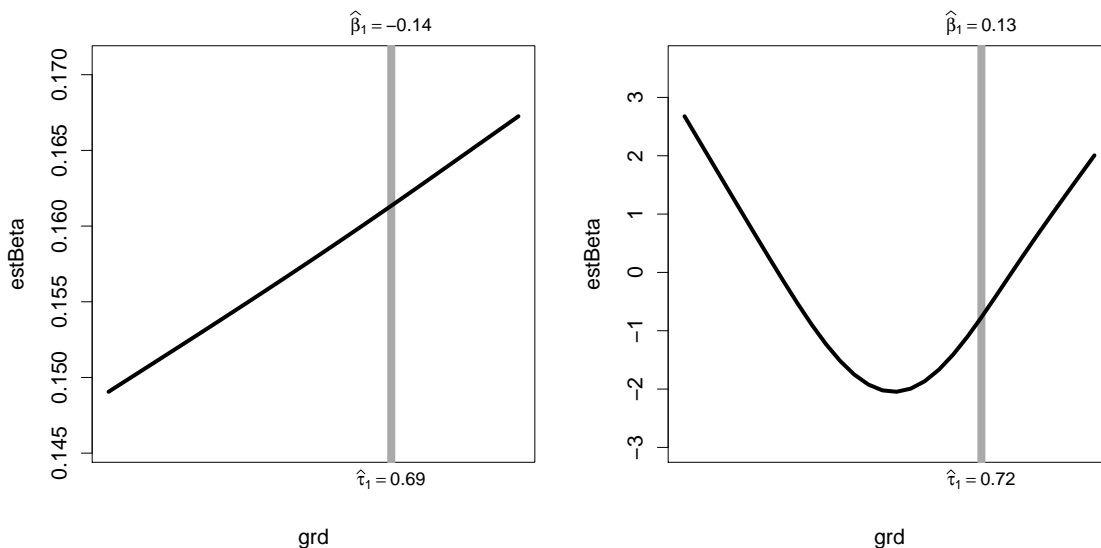


Figure 1: Estimated coefficients for the hybrid models. Left: Exchange rate, Right: Stock Index.

Note: *grd* is the domain of the coefficients, given by (0,1), and *estBeta* are values of the estimated coefficients functions.

Figure 1 presents the coefficient functions of all the hybrid (augmented) models. We include only the coefficient functions of the hybrid models since they are the only ones which provide the estimated point(s) of impact. The  $x$ -axis is given by the domain (0,1), but one can easily convert the domain to (1,30) days in June. The points of impact are given by the values  $\tau_i$ , while  $\beta_i$  gives the corresponding size and direction of the point of impact. The models capture the Brexit effect for all the models, albeit sometimes it estimates the impact earlier (i.e., 0.69 corresponds to 21 June and 0.72 corresponds to 22 June). Given the limited size of the sample, the fact that we do not get the desired precision is not surprising. The hybrid model with exchange rate as the covariate show a positive linear relationship, with the Brexit effect having a negative coefficient. These results imply, that in general, a currency depreciation (i.e., a rise in the domestic currency to pound value) is likely to increase

uncertainty, but on the Brexit date, the domestic currency appreciated (i.e., the domestic currency to pound value fell), and EPU still went up. At the same time, the hybrid model with stock index covariate seems to oscillate around zero, suggesting an insignificant relationship, and although the estimated point of impact is the most accurate in this case, the effect is positive. This result implies, that overall a decline in stock market results in enhanced uncertainty, at least initially, but on and from the Brexit date, the stock markets in other economies improved and was associated with heightened uncertainty. The counter intuitive results on the Brexit date on the currency and stock markets indicate that while the poor performance of these two assets in general, does enhance uncertainty, on the Brexit date, better activity in these two markets of other economies could not reduce the uncertainty faced by the traders, given the unclear picture of the future. Alternatively, heightened uncertainty on the event days could also have resulted from more trading in currencies and equities of other markets, resulting in greater volatility.

## 5 Conclusion

This paper examined the relationship between economic uncertainty of 20 advanced and emerging markets, and two covariates, i.e., exchange rate and stock index, with particular emphasis to the relationship between the variables in response to the Brexit vote. Econometrically, we use a functional data approach supplemented with a point of impact structure to conduct a mixed-frequency analysis, with uncertainty being measured monthly and the two covariates at daily frequency. In general, our results tend to show that incorporating the point of impact, in this case the Brexit shock, is marginally important, relative to models that ignore it. In addition, we also highlight that the exchange rate played a more important role than the equity market in transmitting the Brexit shock to result in heightened uncertainty in the 20 countries considered. Since, uncertainty shocks are transmitted to other countries via high-frequency variables, as highlighted by our results, this information is of tremendous value to policymakers in designing appropriate policies ahead of time to prevent heightened domestic uncertainty, which in turn is known to negatively impact the macroeconomy (Gupta et al., 2018, 2019).

As part of future research, it would be interesting to extend our analysis to check for the the possible role of commodity markets, in causing spillovers of uncertainty (Gozgor et al., 2016).

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