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ORIGINAL RESEARCH

Adaptive finite-time control of multi-agent systems with partial state constraints and input saturation via event-triggered strategy

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Abstract

This paper focuses on the finite-time control problem of multi-agent systems with input saturation, unknown nonlinear dynamics, external disturbances and partial state constraints via output feedback. Fuzzy logic system and fuzzy state observer are introduced to approximate the uncertain nonlinearities and estimate the unmeasurable states, respectively. The partial state constraints are dealt with by using the barrier Lyapunov function, so that all states of the system do not exceed the preset boundary values. In order to reduce the computational complexity of the virtual controller and save communication resources, a first-order filter and an event-triggered mechanism are introduced, respectively. It is proved that the Zeno behavior does not occur via the proposed event-triggered controller. By stability analysis, the finite-time convergence of tracking error to a small neighborhood of the origin is proven. The effectiveness of the theoretical results is verified by examples.

1 | INTRODUCTION

In recent decades, coordination control for multi-agent systems (MASs) has been widely applied to formation control, cluster motion, sensor networks, ships, and so on [1–4]. More and more scholars in related fields began to study it. The key problem in coordination control of MASs is the consensus problem, which requires all agents' states to reach the same value. A large number of researches have been done in the field of coordination control of MASs, such as observer-based consensus of MASs [5] with uncertainties from a new perspective, where the interval observer of a MAS was developed to estimate the interval on which the state of each agent locates, average consensus of MASs [6–9], finite-time consensus [10–12], stochastic consensus [13–16], adaptive consensus [17–20], to name just a few.

In practical applications, such as robot control systems [21], vehicle guidance systems [22] and teleoperation systems [23], the system performance is often required to be reached in finite-time. Finite-time control has better control performance and faster convergence speed than infinite-time control, which attracted much attention of scholars [24–26]. In [24], heterogeneous nonlinear MASs with adaptive finite-time control was investigated. Finite-time adaptive neural networks controller for MASs was given in [25], and finally made the output of all followers consistent with the output of leaders. In [26], the authors addressed the finite-time consensus problem for high-order MASs with uncertain nonlinearities, where the uncertainties are assumed to satisfy the Hölder conditions.

In networked systems, continuous data communication may bring great burden of communication and result in waste of communication resources. Therefore, event-triggered control (ETC) strategies are increasingly being adopted. Compared with time-triggered strategy, ETC strategies can save the communication resources and improve efficiency [27–31]. For hybrid MASs, a novel event-triggered strategy was proposed in [27] to solve the second-order consensus problem. In [28], the authors proposed a novel distributed ETC strategy for heterogeneous linear MASs. In [29], the authors investigated the distributed adaptive ETC method for nonlinear MASs with output constraints. The prescribed performance adaptive finite-time control problem was studied in [30] for nonlinear systems with an event-triggered strategy. In [31], the authors considered the leader-following tracking finite-time controller for heterogeneous nonlinear MASs with output constraint, a fuzzy adaptive ETC strategy was proposed.

Due to the limitations of physical devices in practical systems, output and state constraints are becoming a major concern. An unsatisfactory control law will lead to poor transient performance and instability of the system. Therefore, the problem of full state constraints (FSCs) or partial state constraints (PSCs) in systems has received considerable attention from scholars [32–35]. By introducing the barrier Lyapunov function (BLF), the authors studied the control problem of strict feedback systems subject to output constraint in [32]. On the basis of [32], the BLF was also applied to systems with state constraints. In [33], the authors studied the adaptive finite-time control problem for non-triangular nonlinear systems with FSCs via output-feedback using BLF. The fuzzy control method for nonlinear systems with prescribed performance and FSCs was proposed in [34]. In [35], the authors mainly studied the adaptive ETC scheme of systems with nonlinearity and PSCs. Through the above literature analysis, it can be observed that few works are carried out on finite-time ETC problem of MASs subject to output and PSCs via output feedback, which prompts us to do the research of this paper.

In this work, we consider the problem of finite-time tracking for MASs nonlinearity subject to output and PSCs in network concluding a directed spanning tree (DST). An adaptive ETC mechanism is presented to solve the tracking control problem via output feedback, which can ensure the finite-time convergence of tracking error to a small neighborhood of the origin. Because system outputs are the only available information, a state observer is then constructed. The input saturation and external disturbances are also considered in this work. Compared with existing researches, our main contributions of this work can be summarized as follows:

(1) For MASs with unknown nonlinear dynamics, PSCs and input saturation, an ETC scheme is proposed via output feedback, which guarantees the boundedness of all resulting signals and the finite-time stability of MASs with a bounded error. Compared with [29, 35–38], in which asymptotic stability of systems were investigated, the finite-time stability of MASs is considered in this work. Compared with [24, 33, 39, 40], in which time-triggered strategy was studied, the ETC method is studied in this paper.
(2) The model in this paper is more general than those in [29–31, 33, 34, 41–44]. The consensus problem of MASs with external disturbance and unknown nonlinearity are considered in directed networks concluding a DST via output feedback. Fuzzy logic system (FLS) is utilized to approximate the uncertain nonlinearities. A state observer is designed to deal with the unmeasured states subject to output feedback. The adaptive control method is applied to estimate the unknown parameters. In the literatures mentioned above, some of them [29–31, 42] took external disturbance into consideration, while others [33, 41, 43, 44] did not. Compared with the time-triggered strategy for MASs [34, 41, 43, 44], the ETC scheme proposed in this paper can reduce the communication burden. Compared with [30, 31, 35], in which the case of state feedback was considered, an adaptive control method via output feedback is presented in this paper. Compared with [29, 42], in which the network topology was assumed to be undirected, a more general case of directed networks concluding a DST is considered in this paper.
(3) The BLF is introduced to handle the output and PSCs and ensure that the constrained partial states and outputs of the MAS will not exceed the time-varying boundaries. In [33, 37–39], the case of FSCs with constant boundaries was considered, which is a special case considered in this work.

The remainder of this paper is arranged as follows. Some preliminary knowledge and problem formulations are given in Section 2. In Section 3, a state observer is constructed. In Section 4, an adaptive fuzzy ETC strategy is given. The stability analysis is presented in Section 5. Two examples are given in Section 6. We conclude the paper in the last section.

Notations: $\mathcal{Z}^+$, $\mathcal{R}^n$ and $\mathcal{R}^{m\times n}$ denote the set of positive integers, n-dimensional Euclidean space and the set of $m\times n$ real matrix, respectively. For matrix A, denote by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ the largest and smallest eigenvalues, respectively. $1_N = (1,\dots,1)^T \in \mathcal{R}^N$. $\|\cdot\|_2$ and $\|\cdot\|_\infty$ are the 2-norm and ∞-norm of a matrix in $\mathcal{R}^{m\times n}$ or a vector in $\mathcal{R}^n$, respectively. $\mathrm{col}(x_i) = [x_1^T(t), x_2^T(t), \dots, x_N^T(t)]^T$ for $x_i \in \mathcal{R}^n$, $i = 1,\dots,N$.

2 | PRELIMINARIES AND PROBLEM FORMULATIONS

2.1 | Graph theory

For leader-following MASs with $N+1$ agents, we use a weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ to describe the interconnection relationship among agents. $\mathcal{V} = \{0,\dots,N\}$ is the node set of $N+1$ agents, in which the leader is labeled by 0. $\mathcal{E} \subset \mathcal{V}\times\mathcal{V}$ is the set of edges and $\mathcal{A}$ the weighted adjacency matrix. For an ordered pair $(j,i) \in \mathcal{E}$, we call agent j is agent i's neighbor, which means that i can get information from j. A directed graph $\mathcal{G}$ includes a DST if there is an agent called the root has at least one directed path to all other agents. Denote $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{(N+1)\times(N+1)}$ with $a_{ii} = 0$, $a_{ij} > 0$, if $(j,i)\in\mathcal{E}$, otherwise, $a_{ij} = 0$. The in-degree matrix $\mathcal{D}$ is a diagonal matrix with diagonal element $d_i = \sum_{j=0,j\neq i}^{N} a_{ij}$. The Laplacian matrix $\Xi = \mathcal{D} - \mathcal{A} = [\Xi_{ij}]_{(N+1)\times(N+1)}$ with $\Xi_{ii} = d_i$ and $\Xi_{ij} = -a_{ij}$, $i \neq j$.

Assumption 1. *Graph $\mathcal{G}$ contains a DST and the leader is its root.*

Lemma 1 [45]. *Ξ has zero eigenvalues with 1_N as its right eigenvector, its other eigenvalues have positive real parts. In addition, zero eigenvalue of Ξ is simple iff graph $\mathcal{G}$ has a DST.*

Under Assumption 1, the Laplacian Ξ can be rewritten as

$$\Xi = \begin{bmatrix} 0 & 0_{1\times N} \\ \Xi_2 & \Xi_1 \end{bmatrix}, \tag{1}$$

where $\Xi_1 \in \mathcal{R}^{N\times N}$, $\Xi_2 \in \mathcal{R}^{N\times 1}$. It follows from Assumption 1 and Lemma 1, each eigenvalue of Ξ_1 has a positive real part.

Lemma 2 [46]. *(Young's Inequality):*

$$a_1 a_2 \le \frac{\hat{a}_1^{\hat{a}_2}}{\hat{a}_2}|a_1|^{\hat{a}_2} + \frac{1}{\hat{a}_3 \hat{a}_1^{\hat{a}_3}}|a_2|^{\hat{a}_3}, \tag{2}$$

for $a_1, a_2 \in \mathcal{R}$, $\hat{a}_1 > 0$, $\hat{a}_2, \hat{a}_3 > 1$ *and* $(\hat{a}_2 - 1)(\hat{a}_3 - 1) = 1$.

Lemma 3 [47].

$$|\psi_1|^{\nu_1}|\psi_2|^{\nu_2} \le \frac{\nu_1}{\nu_1+\nu_2}\nu_3|\psi_1|^{\nu_1+\nu_2} + \frac{\nu_2}{\nu_1+\nu_2}\nu_3^{-\frac{\nu_1}{\nu_2}}|\psi_2|^{\nu_1+\nu_2}, \tag{3}$$

for $\psi_1, \psi_2 \in \mathcal{R}$, ν_1, ν_2 *and* ν_3 *are positive constants.*

Lemma 4 [48]. *Consider a nonlinear system* $\dot{X} = f(X)$, $X \in \mathcal{S}$. *If there exists a positive definite function* $V(X) : \mathcal{S} \to \mathcal{R}$ *with constants* $\hat{d} > 0$, $0 < \flat < 1$ *and* $\hat{M} > 0$, *such that*

$$\dot{V} \le -\hat{d}V^{\flat}(X) + \hat{M}, t \ge 0, \tag{4}$$

then $\dot{X} = f(X)$ *is semiglobal practical finite-time stable (SGPFS).*

Lemma 5 [49].

$$\left(\sum_{i=1}^{n}|\hat{b}_i|\right)^q \le \sum_{i=1}^{n}|\hat{b}_i|^q \le n^{1-q}\left(\sum_{i=1}^{n}|\hat{b}_i|\right)^q, \tag{5}$$

for $0 < q \le 1$, $\hat{b}_i \in \mathcal{R}$, $i = 1, \dots, n$.

Lemma 6 [35]. *Let* $\Pi = \{\omega = [\omega_1, \dots, \omega_{n_1}]^T \in \mathcal{R}^{n_1} : |\omega_i| < 1\} \subset \mathcal{R}^{n_1}$, $i = 1, \dots, n_1$ *and* $\Lambda = \Pi \times \mathcal{R}^{n_2} \subset \mathcal{R}^{n_1+n_2}$ *be open sets.* $\Pi_i = \{\omega_i \in \mathcal{R} : |\omega_i| < 1\} \subset \mathcal{R}$. *For the system*

$$\dot{X} = f(X, u), \tag{6}$$

where $X = (\omega, v)^T \in \Lambda$ *and* $f : \mathcal{R}_+ \times \Lambda \to \mathcal{R}^{n_1+n_2}$ *are locally Lipschitz in* X, *piecewise continuous in* t *and uniformly in* t, *on* $\mathcal{R}_+ \times \Lambda$. *If there exist* $V_i : \Pi_i \to \mathcal{R}_+$ *and* $U : \mathcal{R}^{n_2} \to \mathcal{R}_+$, *which are positive definite, and continuously differentiable in* Π_i *and* $\mathcal{R}^{n_2}$, *respectively, such that*

$$V_i(\omega_i) \to \infty \quad as \quad |\omega_i| \to 1 \quad, i = 1, 2, \dots, n_1, \tag{7}$$

and

$$\gamma_1(\|v\|) \le U(v) \le \gamma_2(\|v\|), \tag{8}$$

with $\gamma_1, \gamma_2 \in \mathcal{K}_\infty$. *Let* $V(\check{X}) = \sum_{i=1}^{n_1} V_i(\omega_i) + U(X)$ *and* $\omega_i(0) \in \Pi_i$. *If*

$$\dot{V} = \frac{\partial V}{\partial X} f \le -\hat{d}V^{\flat} + \hat{M}, \tag{9}$$

holds, where $\hat{d} > 0$, $0 < \flat < 1$ *and* $\hat{M} > 0$, *then* $\omega_i(t) \in \Pi_i$, $\forall\, t \in [0, \infty)$.

Lemma 7 [50]. *For any* $\Delta_1 > 0$ *and* Δ_2,

$$0 < |\Delta_2| - \Delta_2 \tanh\left(\frac{\Delta_2}{\Delta_1}\right) \le 0.2785\Delta_1. \tag{10}$$

2.2 | Problem formulations

Consider MASs consisting of N follower agents and a leader with external disturbances. The ith follower agent's dynamics is

$$\begin{cases} \dot{x}_p^i = x_{p+1}^i + f_p^i(\bar{x}_p^i) + w_p^i(t), \\ \dot{x}_n^i = \mathrm{sat}_i(u_i) + f_n^i(\bar{x}_n^i) + w_n^i(t), \\ y^i = x_1^i, i = 1, 2, \dots, N, p = 1, 2, \dots, n-1, \end{cases} \tag{11}$$

where $\bar{x}_p^i = (x_1^i, x_2^i, \dots, x_p^i)^T \in \mathcal{R}^p$, $p = 1, 2, \dots, n$, $y^i \in \mathcal{R}^n$ are system state and output, respectively. The full state $\bar{x}_n^i = (x_1^i, x_2^i, \dots, x_n^i)^T$ is divided into the constrained state $\bar{x}_\ell^i = (x_1^i, x_2^i, \dots, x_\ell^i)^T$ and the unconstrained state $\underline{x}_{\ell+1}^i = (x_{\ell+1}^i, x_{\ell+2}^i, \dots, x_n^i)^T$. The constrained states x_p^i, $p = 1, 2, \dots, \ell$, are restricted by $|x_p^i| < \bar{k}_p^i(t)$, where the boundary function $\bar{k}_p^i(t) > 0$. Nonlinear functions $f_p^i(\bar{x}_p^i)$ are smooth and unknown. $w_p^i(t) \in \mathcal{R}$ is external disturbance satisfying $|w_p^i(t)| \le w_p^{i*}$ with $w_p^{i*} > 0$ being an unknown constant. Note that only the output $y^i = x_1^i$ is the available information. $\mathrm{sat}_i(u_i)$ is the saturation controller given as

$$\mathrm{sat}_i(u_i) = \begin{cases} u_M^i, u_i > u_M^i, \\ u_i, -u_m^i \le u_i \le u_M^i, \\ -u_m^i, u_i < -u_m^i, \end{cases} \tag{12}$$

where $u_m^i > 0$ and $u_M^i > 0$ are constants. The saturation control function $\mathrm{sat}_i(u_i)$ can be transformed into the following form:

$$\mathrm{sat}_i(u_i) = \zeta(u_i)u_i, \tag{13}$$

where

$$\zeta(u_i) = \begin{cases} \dfrac{u_M^i}{u_i}, u_i > u_M^i, \\ 1, -u_m^i \le u_i \le u_M^i, \\ -\dfrac{u_m^i}{u_i}, u_i < -u_m^i. \end{cases} \tag{14}$$

The coefficient $\zeta(u_i) \in (0, 1]$ represents the saturation degree of the control signal. In particular, when $\zeta(u_i) = 1$, there is no saturation phenomenon. As discussed in [51],

$$0 < \varrho_i \le \min_t[\zeta(u_i(t))] \le 1, \tag{15}$$

for some constant ϱ_i and any u_i not going to infinity. This assumption is reasonable for practical applications. Meanwhile, the unknown lower bound of $\zeta(u_i)$ will be estimated adaptively.

The BLF will be applied to deal with the constrained output and states. A BLF is defined as

$$B(t) = \frac{1}{2}\log\frac{k^2(t)}{k^2(t)-e^2(t)}, \tag{16}$$

where $e(t)$ is a error variable, which is constrained by boundary function $k(t)$.

Lemma 8 [52].

$$\log\frac{k^2(t)}{k^2(t)-e^2(t)} < \frac{e^2(t)}{k^2(t)-e^2(t)}, \tag{17}$$

for $|e(t)| < k(t)$ *and* $\forall\ k(t) > 0$.

The control objectives are to design a fuzzy adaptive ETC mechanism for MASs (11) such that

(i) System outputs y^i can converge to the given reference signal y_r with bounded errors in finite-time.
(ii) All resulting system signals are bounded in finite-time.
(iii) Output and partial state constraints are never violated, that is $|x_p^i| < \bar{k}_p^i(t), \forall\ t > 0, p = 1, 2, \dots, \ell$.
(iv) The Zeno behavior does not occur.

For the reference signal y_r, we make the following assumption.

Assumption 2. *$y_r \in \mathcal{R}$ is smooth and bounded, $\dot{y}_r$ and $\ddot{y}_r$ are bounded and continuous, that is, $|y_r| \le \hat{r}_0$, $|\dot{y}_r| \le \hat{r}_1$, and $|\ddot{y}_r| \le \hat{r}_2$, for some positive constants $\hat{r}_0$, $\hat{r}_1$ and $\hat{r}_2$.*

2.3 | FLS

FLS plays a key role in function approximation, which is established by fuzzy IF-Then rule base, a fuzzifier and a defuzzifier [53]. A fuzzy rule is expressed as

R^ι: If $\mathfrak{I}_1$ is G_1^ι, $\mathfrak{I}_2$ is G_2^ι, ..., $\mathfrak{I}_n$ is G_n^ι,

Then: Y is H^ι, $\iota = 1, 2, \dots, \Gamma$.

where vector $\mathfrak{I} = (\mathfrak{I}_1, \mathfrak{I}_2, \dots, \mathfrak{I}_n)^T \in \mathcal{R}^n$ and Y are input and output of FLS, respectively. G_p^ι and H^ι are fuzzy sets. The output Y of FLS can be calculated by

$$Y = \frac{\sum_{\iota=1}^{\Gamma}\bar{Y}^\iota\prod_{p=1}^{n}\mu_{G_p^\iota}(\mathfrak{I}_p)}{\sum_{\iota=1}^{\Gamma}\prod_{p=1}^{n}\mu_{G_p^\iota}(\mathfrak{I}_p)}, \tag{18}$$

where $\mu_{G_p^\iota}$ and μ_{H^ι} are fuzzy membership functions associated with G_p^ι and H^ι, and $\bar{Y}^\iota$ is the maximum value of $\mu_{H^\iota}(Y)$. Define

$$\Phi^\iota(\mathfrak{I}) = \frac{\prod_{p=1}^{n}\mu_{G_p^\iota}(\mathfrak{I}_p)}{\sum_{\iota=1}^{\Gamma}\prod_{p=1}^{n}\mu_{G_p^\iota}(\mathfrak{I}_p)}. \tag{19}$$

Let $\theta = (\theta^1, \theta^2, \dots, \theta^\Gamma)^T = (\bar{Y}^1, \bar{Y}^2, \dots, \bar{Y}^\Gamma)^T$ and $\Phi = (\Phi^1, \Phi^2, \dots, \Phi^\Gamma)^T$. A FLS has the following form

$$Y = \theta^T\Phi(\mathfrak{I}). \tag{20}$$

Lemma 9 [54]. *Suppose that $f(\mathfrak{I})$ is continuous in a compact set Ω. For $\forall\ \varepsilon > 0$, there exists a proper FLS such that*

$$\sup_{\mathfrak{I}\in\Omega}|f(\mathfrak{I}) - \theta^T\Phi(\mathfrak{I})| \le \varepsilon. \tag{21}$$

Remark 1. It follows from the definition and properties of Φ^ι and Φ that $0 < \Phi^\iota \le 1$ and $0 < \Phi^T\Phi \le 1$, which will be applied to the proof process of our theoretical results.

3 | OBSERVER DESIGN

In order to estimate the unmeasurable states, a state observer is constructed in this section. Let $x^i = \bar{x}_n^i$, system (11) is rewritten as follows

$$\begin{cases} \dot{x}^i = A_i x^i + L_i y^i + \sum_{p=1}^{n} E_p f_p^i(\bar{x}_p^i) + b\,\mathrm{sat}_i(u_i) + w^i(t), \\ y^i = Cx^i, \end{cases} \tag{22}$$

where

$$A_i = \begin{pmatrix} -l_1^i & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -l_{n-1}^i & 0 & \cdots & 1 \\ -l_n^i & 0 & \cdots & 0 \end{pmatrix}, \tag{23}$$

$L_i = (l_1^i, \dots, l_n^i)^T$, $b = (0, \dots, 0, 1)^T$, $E_p = (\underbrace{0, \dots, 0, 1}_{p}, 0, \dots, 0)^T$,

$w^i(t) = [w_1^i(t), \dots, w_n^i(t)]^T$ and $C = (1, 0, \dots, 0)$. A_i is a strict Hurwitz matrix by selecting parameters $l_1^i, \dots, l_n^i$. Thus, for given matrix $Q_i = Q_i^T > 0$, there exists a matrix $F_i = F_i^T > 0$, satisfying

$$A_i^T F_i + F_i A_i = -2Q_i. \tag{24}$$

Let $\hat{\bar{x}}_p^i = (\hat{x}_1^i, \dots, \hat{x}_p^i)^T$ and $\hat{x}^i = \hat{\bar{x}}_n^i = (\hat{x}_1^i, \dots, \hat{x}_n^i)^T$ be the estimation of $\bar{x}_p^i$ and x^i, respectively. From Lemma 9, the uncertain function $f_p^i(\bar{x}_p^i)$, $1 \le p \le n$, in (22) is approximated by a FLS as

$$\hat{f}_p^i(\hat{\bar{x}}_p^i|\hat{\theta}_p^i) = \hat{\theta}_p^{iT}\Phi_p^i(\hat{\bar{x}}_p^i). \tag{25}$$

Let

$$\theta_p^i = \arg\min_{\hat{\theta}_p^i \in \Omega_{\hat{\theta}_p^i}} [\sup_{\hat{\bar{x}}_p^i \in \Omega_{\hat{\bar{x}}_p^i}, \bar{x}_p^i \in \Omega_{\bar{x}_p^i}} |\hat{f}_p^i(\hat{\bar{x}}_p^i|\hat{\theta}_p^i) - f_p^i(\bar{x}_p^i)|], \tag{26}$$

be the optimal parameter vector, where $\Omega_{\hat{\theta}_p^i}$, $\Omega_{\bar{x}_p^i}$ and $\Omega_{\hat{\bar{x}}_p^i}$ are some compact regions for $\hat{\theta}_p^i$, $\bar{x}_p^i$ and $\hat{\bar{x}}_p^i$, respectively, $\hat{\theta}_p^i$ is the estimation of θ_p^i.

Let

$$\eta_p^i = f_p^i(\bar{x}_p^i) - f_p^i(\hat{\bar{x}}_p^i|\theta_p^i), \tag{27}$$

be the minimum approximation error, which is assumed to be bounded, that is, $|\eta_p^i| \le \eta_p^{i*}$ with constant $\eta_p^{i*} > 0$.

Since the states of (22) are unmeasurable, an observer is constructed as

$$\dot{\hat{x}}^i = A_i\hat{x}^i + L_i y^i + \sum_{p=1}^{n} E_p\hat{f}_p^i(\hat{\bar{x}}_p^i|\hat{\theta}_p^i) + b\,\mathrm{sat}_i(u_i). \tag{28}$$

Denote by $\tilde{x}^i = (\tilde{x}_1^i, \tilde{x}_2^i, \dots, \tilde{x}_n^i)^T = x^i - \hat{x}^i$ the observer error vector. From (22) and (28), one gets

$$\begin{aligned}\dot{\tilde{x}}^i &= \dot{x}^i - \dot{\hat{x}}^i \\ &= A_i\tilde{x}^i + \sum_{p=1}^{n} E_p\left[f_p^i(\bar{x}_p^i) - \hat{f}_p^i(\hat{\bar{x}}_p^i|\hat{\theta}_p^i)\right] + w^i \\ &= A_i\tilde{x}^i + \sum_{p=1}^{n} E_p\tilde{\theta}_p^{iT}\Phi_p^i(\hat{\bar{x}}_p^i) + w^i + \eta^i,\end{aligned} \tag{29}$$

where $\eta^i = (\eta_1^i, \eta_2^i, \dots, \eta_n^i)^T$ and $\tilde{\theta}_p^i = \theta_p^i - \hat{\theta}_p^i$.

Define

$$V_0^i = \frac{1}{2}\tilde{x}^{iT}F_i\tilde{x}^i, \tag{30}$$

and choose the Lyapunov function as

$$V_0 = \sum_{i=1}^{N} V_0^i. \tag{31}$$

Taking the time derivative of V_0^i yields

$$\begin{aligned}\dot{V}_0^i &= \tilde{x}^{iT}F_i\dot{\tilde{x}}^i \\ &= \tilde{x}^{iT}F_i\left[A_i\tilde{x}^i + \sum_{p=1}^{n} E_p\tilde{\theta}_p^{iT}\Phi_p^i(\hat{\bar{x}}_p^i) + \eta^i + w^i\right] \\ &= -\tilde{x}^{iT}Q_i\tilde{x}^i + \tilde{x}^{iT}F_i\left[\sum_{p=1}^{n} E_p\tilde{\theta}_p^{iT}\Phi_p^i(\hat{\bar{x}}_p^i) + \eta^i + w^i\right].\end{aligned} \tag{32}$$

By employing Lemma 2 and noting that $0 < \Phi_p^i(\cdot)^T\Phi_p^i(\cdot) \le 1$, we have

$$\begin{aligned}\tilde{x}^{iT}F_i\left[\sum_{p=1}^{n} E_p\tilde{\theta}_p^{iT}\Phi_p^i(\hat{\bar{x}}_p^i) + \eta^i + w^i\right] &\le \frac{3}{2}\|\tilde{x}^i\|^2 + \frac{\|F_i\|^2}{2}\sum_{p=1}^{n}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i \\ &+ \frac{1}{2}\|F_i\|^2 \cdot (\|\eta^{i*}\|^2 + \|w^{i*}\|^2),\end{aligned} \tag{33}$$

where $\eta^{i*} = (\eta_1^{i*}, \eta_2^{i*}, \dots, \eta_n^{i*})^T$ and $w^{i*} = (w_1^{i*}, w_2^{i*}, \dots, w_n^{i*})^T$.

Combining (32) and (33), yields

$$\begin{aligned}\dot{V}_0^i &\le -\tilde{x}^{iT}Q_i\tilde{x}^i + \frac{3}{2}\|\tilde{x}^i\|^2 + \frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i \\ &+ \frac{1}{2}\|F_i\|^2 \cdot (\|\eta^{i*}\|^2 + \|w^{i*}\|^2).\end{aligned} \tag{34}$$

Invoking (31) and (34), we have

$$\begin{aligned}\dot{V}_0 &= \sum_{i=1}^{N}\dot{V}_0^i \\ &\le \sum_{i=1}^{N}\Bigg[-\tilde{x}^{iT}Q_i\tilde{x}^i + \frac{3}{2}\|\tilde{x}^i\|^2 + \frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i \\ &\quad + \frac{1}{2}\|F_i\|^2 \cdot (\|\eta^{i*}\|^2 + \|w^{i*}\|^2)\Bigg] \\ &\le -\sum_{i=1}^{N}\lambda_0\|\tilde{x}\|^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i + M_0,\end{aligned} \tag{35}$$

where $\lambda_0 = \lambda_{\min}(Q_i) - \frac{3}{2}$ and $M_0 = \frac{1}{2}\sum_{i=1}^{N}\|F_i\|^2 \cdot (\|\eta^{i*}\|^2 + \|w^{i*}\|^2)$.

4 | CONTROL DESIGN

4.1 | Back-stepping design

In this subsection, the back-stepping method is utilized to construct the finite-time controller. For MASs, the distributed characteristics of the controller is a very important aspect, so we first do the following coordinate transformation.

Let

$$\begin{aligned}e_1^i &= \sum_{j=1}^{N} a_{ij}(y^i - y^j) + a_{i0}(y^i - y_r), \\ e_p^i &= \hat{x}_p^i - \bar{\alpha}_p^i, \\ \xi_p^i &= \bar{\alpha}_p^i - \alpha_{p-1}^i, p = 2, \dots, n,\end{aligned} \tag{36}$$

where e_1^i is the local tracking error, e_p^i the error surface, $\bar{\alpha}_p^i$ the output of the following first-order filter

$$\begin{cases}\upsilon_p^i\dot{\bar{\alpha}}_p^i + \bar{\alpha}_p^i = \alpha_{p-1}^i, \\ \bar{\alpha}_p^i(0) = \alpha_{p-1}^i(0), p = 2, \dots, n,\end{cases} \tag{37}$$

where α_{p-1}^i is a virtual control function, υ_p^i a positive constant and ξ_p^i the error between $\bar{\alpha}_p^i$ and α_{p-1}^i.

From (36) and (37), we have

$$\dot{\bar{\alpha}}_p^i = -\frac{\xi_p^i}{\upsilon_p^i}, p = 2, \ldots, n. \tag{38}$$

Remark 2. In the traditional backstepping approach, computing the high-order derivatives of virtual control signal α_{p-1}^i will increase computational complexity. Thus, a first-order auxiliary filter (37) with backstepping technique is introduced to overcome this weakness. The virtual control signal α_{p-1}^i generates a new signal $\bar{\alpha}_p^i$ through the filter and is used in the design of the next controller. Therefore, repeated differentiation is avoided.

Step 1: According to (36), the time derivative of e_1^i is

$$\begin{aligned}
\dot{e}_1^i &= \sum_{j\in N_i} a_{ij}\dot{y}^i - \sum_{j=1}^N a_{ij}\dot{y}^j - a_{i0}\dot{y}_r \\
&= d_i\left[e_2^i + \xi_2^i + \alpha_1^i + \tilde{x}_2^i + \eta_1^i + \theta_1^{iT}\Phi_1^i(\hat{x}_1^i) + w_1^i\right] \\
&\quad - \sum_{j=1}^N a_{ij}\left[e_2^j + \xi_2^j + \alpha_1^j + \tilde{x}_2^j + \eta_1^j + \theta_1^{jT}\Phi_1^j(\hat{x}_1^j)\right. \\
&\quad \left. + w_1^j\right] - a_{i0}\dot{y}_r.
\end{aligned} \tag{39}$$

We assume that the local consensus error e_1^i is constrained by $|e_1^i| < k_1^i(t)$, where $k_1^i(t)$ is a boundary function. The BLF defined in (16) is used to construct the Lyapunov function candidate:

$$\begin{aligned}
V_1 &= V_0 + \sum_{i=1}^N V_1^i \\
&= V_0 + \frac{1}{2}\sum_{i=1}^N\left[\log\frac{(k_1^i(t))^2}{(k_1^i(t))^2 - (e_1^i)^2} + \frac{1}{\varsigma_1^i}\tilde{\theta}_1^{iT}\tilde{\theta}_1^i + \sum_{j=1}^N \frac{a_{ij}}{\varpi_1^j}\tilde{\theta}_1^{jT}\tilde{\theta}_1^j\right],
\end{aligned} \tag{40}$$

where $\varsigma_1^i > 0$ and $\varpi_1^j > 0$ are constants.

Thus,

$$\begin{aligned}
\dot{V}_1 &= \dot{V}_0 + \sum_{i=1}^N\left[\frac{e_1^i\dot{e}_1^i}{(k_1^i)^2 - (e_1^i)^2} - \frac{1}{\varsigma_1^i}\tilde{\theta}_1^{iT}\dot{\hat{\theta}}_1^i - \sum_{j=1}^N\frac{a_{ij}}{\varpi_1^j}\tilde{\theta}_1^{jT}\dot{\hat{\theta}}_1^j \right. \\
&\quad \left. - \frac{\dot{k}_1^i(e_1^i)^2}{k_1^i((k_1^i)^2 - (e_1^i)^2)}\right] \\
&= \dot{V}_0 + \sum_{i=1}^N\left\{\frac{d_i e_1^i}{(k_1^i)^2 - (e_1^i)^2}\left[e_2^i + \xi_2^i + \alpha_1^i + \tilde{x}_2^i + \eta_1^i + w_1^i + \hat{\theta}_1^{iT}\Phi_1^i(\hat{x}_1^i)\right.\right. \\
&\quad \left. - \frac{a_{i0}}{d_i}\dot{y}_r - \frac{\dot{k}_1^i e_1^i}{d_i k_1^i}\right] - \frac{1}{\varsigma_1^i}\tilde{\theta}_1^{iT}\left[\dot{\hat{\theta}}_1^i - \frac{d_i\varsigma_1^i e_1^i}{(k_1^i)^2 - (e_1^i)^2}\Phi_1^i(\hat{x}_1^i)\right] \\
&\quad - \sum_{j=1}^N\frac{a_{ij}}{\varpi_1^j}\tilde{\theta}_1^{jT}\left[\dot{\hat{\theta}}_1^j + \frac{\varpi_1^j e_1^i}{(k_1^i)^2 - (e_1^i)^2}\Phi_1^j(\hat{x}_1^j)\right] \\
&\quad \left. - \frac{e_1^i}{(k_1^i)^2 - (e_1^i)^2}\sum_{j=1}^N a_{ij}\left[\hat{x}_2^j + \tilde{x}_2^j + \eta_1^j + w_1^j + \hat{\theta}_1^{jT}\Phi_1^j(\hat{x}_1^j)\right]\right\}.
\end{aligned} \tag{41}$$

Noting that $0 < \Phi_p^i(\cdot)^T\Phi_p^i(\cdot) \le 1$ and from Lemma 2, we have

$$\begin{aligned}
\frac{d_i e_1^i}{(k_1^i)^2 - (e_1^i)^2}(\tilde{x}_2^i + \eta_1^i + w_1^i) \le\ & \frac{3d_i^2}{2}\frac{(e_1^i)^2}{((k_1^i)^2 - (e_1^i)^2)^2} \\
& + \frac{1}{2}\left[\|\tilde{x}^i\|^2 + (\eta_1^{i*})^2 + (w_1^{i*})^2\right],
\end{aligned} \tag{42}$$

$$\frac{d_i e_1^i \xi_2^i}{(k_1^i)^2 - (e_1^i)^2} \le \frac{d_i^2}{2}\frac{{e_1^i}^2}{((k_1^i)^2 - (e_1^i)^2)^2} + \frac{1}{2}(\xi_2^i)^2, \tag{43}$$

and

$$\begin{aligned}
& -\frac{e_1^i}{(k_1^i)^2 - (e_1^i)^2}\sum_{j=1}^N a_{ij}(\tilde{x}_2^j + \eta_1^j + w_1^j) \\
& \le \frac{3}{2}\frac{(e_1^i)^2}{((k_1^i)^2 - (e_1^i)^2)^2} + \frac{1}{2}\sum_{j=1}^N a_{ij}^2\left[\|\tilde{x}\|^2 + (\eta_1^{j*})^2 + (w_1^{j*})^2\right].
\end{aligned} \tag{44}$$

Substituting (35), (42)–(44) into (41), one gets

$$\begin{aligned}
\dot{V}_1 \le\ & \sum_{i=1}^N\left(-\lambda_0\|\tilde{x}\|^2 + \frac{1}{2}\sum_{p=1}^n\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i\right) + M_0 \\
& + \sum_{i=1}^N |\left\{\frac{d_i e_1^i}{(k_1^i)^2 - (e_1^i)^2}\left[\alpha_1^i + \hat{\theta}_1^{iT}\Phi_1^i(\hat{x}_1^i) - \frac{a_{i0}}{d_i}\dot{y}_r + \frac{2d_i e_1^i}{\left(k_1^i\right)^2 - \left(e_1^i\right)^2}\right.\right. \\
& \left. - \frac{1}{d_i}\sum_{j=1}^N a_{ij}\left(\hat{x}_2^j + \hat{\theta}_1^{jT}\Phi_1^j(\hat{x}_1^j)\right) + \frac{3}{2d_i}\frac{e_1^i}{(k_1^i)^2 - (e_1^i)^2} - \frac{\dot{k}_1^i e_1^i}{d_i k_1^i}\right] \\
& + \frac{d_i e_1^i e_2^i}{(k_1^i)^2 - (e_1^i)^2} - \frac{1}{\varsigma_1^i}\tilde{\theta}_1^{iT}\left[\dot{\hat{\theta}}_1^i - \frac{d_i\varsigma_1^i e_1^i}{(k_1^i)^2 - (e_1^i)^2}\Phi_1^i(\hat{x}_1^i)\right] \\
& - \sum_{j=1}^N\frac{a_{ij}}{\varpi_1^j}\tilde{\theta}_1^{jT}\left[\dot{\hat{\theta}}_1^j + \frac{\varpi_1^j e_1^i}{(k_1^i)^2 - (e_1^i)^2}\Phi_1^j(\hat{x}_1^j)\right] + \frac{1}{2}\left[\|\tilde{x}^i\|^2\right. \\
& \left. + (\eta_1^{i*})^2 + (w_1^{i*})^2\right] + \frac{1}{2}(\xi_2^i)^2 + \frac{1}{2}\sum_{j=1}^N a_{ij}^2\left[\|\tilde{x}\|^2\right. \\
& \left.\left. + (\eta_1^{j*})^2 + (w_1^{j*})^2\right]\right\}.
\end{aligned} \tag{45}$$

Select the virtual controller α_1^i and the adaptive laws $\dot{\hat{\theta}}_1^i$ and $\dot{\hat{\theta}}_1^j$ as

$$\begin{aligned}
\alpha_1^i =\ & -\hat{\theta}_1^{iT}\Phi_1^i(\hat{x}_1^i) + \frac{a_{i0}}{d_i}\dot{y}_r - \frac{2d_i e_1^i}{(k_1^i)^2 - (e_1^i)^2} + \frac{1}{d_i}\sum_{j=1}^N a_{ij}[\hat{x}_2^j \\
& + \hat{\theta}_1^{jT}\Phi_1^j(\hat{x}_1^j)] - \frac{3}{2d_i}\frac{e_1^i}{(k_1^i)^2 - (e_1^i)^2} - \frac{c_1^i(e_1^i)^{2\beta-1}}{d_i((k_1^i)^2 - (e_1^i)^2)^{\beta-1}} + \frac{\dot{k}_1^i e_1^i}{d_i k_1^i},
\end{aligned} \tag{46}$$

$$\dot{\hat{\theta}}_1^i = \frac{d_i \varsigma_1^i e_1^i}{(k_1^i)^2 - (e_1^i)^2}\Phi_1^i(\hat{x}_1^i) - \delta_1^i \hat{\theta}_1^i, \tag{47}$$

and

$$\dot{\hat{\theta}}_1^j = -\frac{\varpi_1^j e_1^j}{(k_1^j)^2 - (e_1^j)^2}\Phi_1^j(\hat{x}_1^j) - \gamma_1^j \hat{\theta}_1^j, \tag{48}$$

where $c_1^i > 0$, $\delta_1^i > 0$ and $\gamma_1^j > 0$ are constants.

From (45)-(48), one has

$$\begin{aligned}\dot{V}_1 \leq & -\sum_{i=1}^{N} \lambda_1 \|\tilde{x}\|^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{p=1}^{n} \|F_i\|^2 \tilde{\theta}_p^{iT}\tilde{\theta}_p^i \\ & + \sum_{i=1}^{N}\left[-\frac{c_1^i (e_1^i)^{2\beta}}{((k_1^i)^2 - (e_1^i)^2)^\beta} + \frac{d_i e_1^i e_2^i}{(k_1^i)^2 - (e_1^i)^2} + \frac{\delta_1^i}{\varsigma_1^i}\tilde{\theta}_1^{iT}\hat{\theta}_1^i \right. \\ & \left. + \sum_{j=1}^{N} a_{ij}\frac{\gamma_1^j}{\varpi_1^j}\tilde{\theta}_1^{jT}\hat{\theta}_1^j + \frac{1}{2}(\xi_2^i)^2\right] + M_1, \end{aligned} \tag{49}$$

where $\lambda_1 = -\frac{1}{2}\sum_{j=1}^{N} a_{ij}^2 - \frac{1}{2} + \lambda_0$ and $M_1 = M_0 + \frac{1}{2}\sum_{i=1}^{N}[(\eta_1^{i*})^2 + (w_1^{i*})^2 + \sum_{j=1}^{N} a_{ij}^2((\eta_1^{j*})^2 + (w_1^{j*})^2)]$.

Step 2: The time derivative of e_2^i is

$$\dot{e}_2^i = \dot{\hat{x}}_2^i - \dot{\alpha}_2^i \tag{50}$$

$$\begin{aligned} &= e_3^i + \xi_3^i + \alpha_2^i + l_2^i\tilde{x}_1^i + \hat{\theta}_2^{iT}\Phi_2^i(\hat{x}_2^i) \\ & + \tilde{\theta}_2^{iT}\Phi_2^i(\hat{x}_2^i) - \tilde{\theta}_2^{iT}\Phi_2^i(\hat{x}_2^i) - \dot{\alpha}_2^i. \end{aligned} \tag{50}$$

Define

$$V_2^i = \frac{1}{2}\left[\log\frac{(k_2^i(t))^2}{(k_2^i(t))^2 - (e_2^i)^2} + \frac{1}{\varsigma_2^i}\tilde{\theta}_2^{iT}\tilde{\theta}_2^i + (\xi_2^i)^2\right], \tag{51}$$

and construct the Lyapunov function as

$$V_2 = V_1 + \sum_{i=1}^{N} V_2^i, \tag{52}$$

where $\varsigma_2^i > 0$ is a constant and the error variable e_2^i is constrained by $|e_2^i| < k_2^i(t)$ with $k_2^i(t) > 0$ being a boundary function.

Thus,

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \sum_{i=1}^{N}\left[\frac{e_2^i \dot{e}_2^i}{(k_2^i)^2 - (e_2^i)^2} - \frac{1}{\varsigma_2^i}\tilde{\theta}_2^{iT}\dot{\hat{\theta}}_2^i + \xi_2^i\dot{\xi}_2^i - \frac{\dot{k}_2^i (e_2^i)^2}{k_2^i((k_2^i)^2 - (e_2^i)^2)}\right] \\ &= \dot{V}_1 + \sum_{i=1}^{N}\left\{\frac{e_2^i}{(k_2^i)^2 - (e_2^i)^2}\left[e_3^i + \xi_3^i + \alpha_2^i + l_2^i(y^i - \hat{x}_1^i) + \hat{\theta}_2^{iT}\Phi_2^i(\hat{x}_2^i)\right.\right. \\ & \left.\left. - \dot{\alpha}_2^i - \frac{\dot{k}_2^i e_2^i}{k_2^i}\right] - \frac{e_2^i\tilde{\theta}_2^{iT}\Phi_2^i(\hat{x}_2^i)}{(k_2^i)^2 - (e_2^i)^2} - \frac{1}{\varsigma_2^i}\tilde{\theta}_2^{iT}\left[\dot{\hat{\theta}}_2^i - \frac{\varsigma_2^i e_2^i}{(k_2^i)^2 - (e_2^i)^2}\Phi_2^i(\hat{x}_2^i)\right] + \xi_2^i\dot{\xi}_2^i\right\}. \end{aligned} \tag{53}$$

In view of Young's inequality, one gets

$$\frac{e_2^i \xi_3^i}{(k_2^i)^2 - (e_2^i)^2} \leq \frac{(e_2^i)^2}{2((k_2^i)^2 - (e_2^i)^2)^2} + \frac{1}{2}(\xi_3^i)^2, \tag{54}$$

and

$$-\frac{e_2^i \tilde{\theta}_2^{iT}\Phi_2^i(\hat{x}_2^i)}{(k_2^i)^2 - (e_2^i)^2} \leq \frac{(e_2^i)^2}{2((k_2^i)^2 - (e_2^i)^2)^2} + \frac{1}{2}\tilde{\theta}_2^{iT}\tilde{\theta}_2^i. \tag{55}$$

From (36) to (38), one has

$$\dot{\xi}_2^i = \dot{\alpha}_2^i - \dot{\alpha}_1^i = -\frac{\xi_2^i}{\upsilon_2^i} + \varepsilon_2^i(\cdot), \tag{56}$$

where $\varepsilon_2^i(\cdot)$ is a bounded function satisfying $|\varepsilon_2^i| \leq \bar{\varepsilon}_2^i$ with constant $\bar{\varepsilon}_2^i > 0$.

Choose the virtual controller α_2^i as

$$\begin{aligned}\alpha_2^i = & -l_2^i(y^i - \hat{x}_1^i) - \hat{\theta}_2^{iT}\Phi_2^i(\hat{x}_2^i) + \dot{\alpha}_2^i - \frac{d_i e_1^i((k_2^i)^2 - (e_2^i)^2)}{(k_1^i)^2 - (e_1^i)^2} \\ & - \frac{e_2^i}{(k_2^i)^2 - (e_2^i)^2} - \frac{c_2^i (e_2^i)^{2\beta - 1}}{((k_2^i)^2 - (e_2^i)^2)^{\beta - 1}} + \frac{\dot{k}_2^i e_2^i}{k_2^i}, \end{aligned} \tag{57}$$

and the adaptive law $\dot{\hat{\theta}}_2^i$ as

$$\dot{\hat{\theta}}_2^i = \frac{\varsigma_2^i e_2^i}{(k_2^i)^2 - (e_2^i)^2}\Phi_2^i(\hat{x}_2^i) - \delta_2^i\hat{\theta}_2^i, \tag{58}$$

where $c_2^i > 0$ and $\delta_2^i > 0$ are constants. From (53) to (58), one has

$$\begin{aligned}\dot{V}_2 \leq \sum_{i=1}^{N}\Bigg[& -\lambda_1\|\tilde{x}\|^2 + \frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i \\ & - \frac{c_1^i (e_1^i)^{2\beta}}{((k_1^i)^2 - (e_1^i)^2)^\beta} + \frac{\delta_1^i}{\varsigma_1^i}\tilde{\theta}_1^{iT}\hat{\theta}_1^i + \sum_{j=1}^{N} a_{ij}\frac{\gamma_1^j}{\varpi_1^j}\tilde{\theta}_1^{jT}\hat{\theta}_1^j \\ & + \frac{1}{2}(\xi_2^i)^2 - \frac{c_2^i (e_2^i)^{2\beta}}{((k_2^i)^2 - (e_2^i)^2)^\beta} + \frac{1}{2}(\xi_3^i)^2 + \frac{1}{2}\tilde{\theta}_2^{iT}\tilde{\theta}_2^i \\ & + \frac{e_2^i e_3^i}{(k_2^i)^2 - (e_2^i)^2} + \frac{\delta_2^i}{\varsigma_2^i}\tilde{\theta}_2^{iT}\hat{\theta}_2^i - \frac{(\xi_2^i)^2}{\upsilon_2^i} + \xi_2^i\varepsilon_2^i(\cdot)\Bigg] + M_2, \end{aligned} \tag{59}$$

where $M_2 = M_1$.

Step s ($3 \leq s \leq \ell$): The derivative of e_s^i is

$$\begin{aligned}\dot{e}_s^i &= \dot{\hat{x}}_s^i - \dot{\alpha}_s^i \\ &= e_{s+1}^i + \xi_{s+1}^i + \alpha_s^i + l_s^i\tilde{x}_1^i + \hat{\theta}_s^{iT}\Phi_s^i(\hat{x}_s^i) \\ & -\tilde{\theta}_s^{iT}\Phi_s^i(\hat{x}_s^i) + \tilde{\theta}_s^{iT}\Phi_s^i(\hat{x}_s^i) - \dot{\alpha}_s^i. \end{aligned} \tag{60}$$

Choose the Lyapunov function as

$$V_s = V_{s-1} + \frac{1}{2}\sum_{i=1}^{N}\left[\log\frac{(k_s^i(t))^2}{(k_s^i(t))^2-(e_s^i)^2} + \frac{1}{\varsigma_s^i}\tilde{\theta}_s^{iT}\tilde{\theta}_s^i + (\xi_s^i)^2\right], \tag{61}$$

where $\varsigma_s^i > 0$ is a constant, the error variable e_s^i is constrained by $|e_s^i| < k_s^i(t)$, where $k_s^i(t)$ is a boundary function. Similarly, one has

$$\frac{e_s^i\xi_{s+1}^i}{(k_s^i)^2-(e_s^i)^2} \le \frac{(e_s^i)^2}{2((k_s^i)^2-(e_s^i)^2)^2} + \frac{1}{2}(\xi_{s+1}^i)^2, \tag{62}$$

and

$$-\frac{e_s^i\tilde{\theta}_s^{iT}\Phi_s^i(\hat{\bar{x}}_s^i)}{(k_s^i)^2-(e_s^i)^2} \le \frac{(e_s^i)^2}{2((k_s^i)^2-(e_s^i)^2)^2} + \frac{1}{2}\tilde{\theta}_s^{iT}\tilde{\theta}_s^i. \tag{63}$$

Bearing in mind (36)-(38), one gets

$$\dot{\xi}_s^i = \dot{\bar{\alpha}}_s^i - \dot{\alpha}_{s-1}^i = -\frac{\xi_s^i}{\upsilon_s^i} + \varepsilon_s^i(\cdot), \tag{64}$$

where $\varepsilon_s^i(\cdot)$ is a bounded function satisfying $|\varepsilon_s^i| \le \bar{\varepsilon}_s^i$ with constant $\bar{\varepsilon}_s^i > 0$.

Choose the virtual controller α_s^i as

$$\alpha_s^i = -l_s^i(y^i - \hat{x}_1^i) - \hat{\theta}_s^{iT}\Phi_s^i(\hat{\bar{x}}_s^i) + \dot{\bar{\alpha}}_s^i - \frac{e_{s-1}^i((k_s^i)^2-(e_s^i)^2)}{(k_{s-1}^i)^2-(e_{s-1}^i)^2}$$
$$-\frac{e_s^i}{(k_s^i)^2-(e_s^i)^2} - \frac{c_s^i(e_s^i)^{2\beta-1}}{((k_s^i)^2-(e_s^i)^2)^{\beta-1}} + \frac{\dot{k}_s^i e_s^i}{k_s^i}, \tag{65}$$

and the adaptive law $\dot{\hat{\theta}}_s^i$ as

$$\dot{\hat{\theta}}_s^i = \frac{\varsigma_s^i e_s^i}{(k_s^i)^2-(e_s^i)^2}\Phi_s^i(\hat{\bar{x}}_s^i) - \delta_s^i\hat{\theta}_s^i, \tag{66}$$

where $c_s^i > 0$ and $\delta_s^i > 0$ are constants.

Thus,

$$\dot{V}_s \le \sum_{i=1}^{N}\Bigg[-\lambda_1\|\tilde{x}\|^2 + \frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i$$
$$+\sum_{p=1}^{s}\frac{1}{2}(\xi_{p+1}^i)^2 - \sum_{p=1}^{s}\frac{c_p^i(e_p^i)^{2\beta}}{((k_p^i)^2-(e_p^i)^2)^{\beta}} + \sum_{p=1}^{s}\frac{\delta_p^i}{\varsigma_p^i}\tilde{\theta}_p^{iT}\hat{\theta}_p^i$$
$$+\sum_{j=1}^{N}a_{ij}\frac{\gamma_1^j}{\varpi_1^j}\tilde{\theta}_1^{jT}\hat{\theta}_1^j + \sum_{p=2}^{s}\frac{1}{2}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i + \frac{e_s^ie_{s+1}^i}{(k_s^i)^2-(e_s^i)^2}$$
$$-\sum_{p=2}^{s}\frac{(\xi_p^i)^2}{\upsilon_p^i} + \sum_{p=2}^{s}\xi_p^i\varepsilon_p^i(\cdot)\Bigg] + M_s, \tag{67}$$

where $M_s = M_{s-1}$.

Step $\ell + 1$: According to (28) and (36), we have

$$\dot{e}_{\ell+1}^i = \dot{\hat{x}}_{\ell+1}^i - \dot{\alpha}_{\ell+1}^i$$
$$= e_{\ell+2}^i + \xi_{\ell+2}^i + \alpha_{\ell+1}^i - \dot{\bar{\alpha}}_{\ell+1}^i + l_{\ell+1}^i\tilde{x}_1^i + \hat{\theta}_{\ell+1}^{iT}\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i)$$
$$-\tilde{\theta}_{\ell+1}^{iT}\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i) + \tilde{\theta}_{\ell+1}^{iT}\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i). \tag{68}$$

Choose the Lyapunov function as

$$V_{\ell+1} = V_\ell + \frac{1}{2}\sum_{i=1}^{N}\left[(e_{\ell+1}^i)^2 + \frac{1}{\varsigma_{\ell+1}^i}\tilde{\theta}_{\ell+1}^{iT}\tilde{\theta}_{\ell+1}^i + (\xi_{\ell+1}^i)^2\right], \tag{69}$$

where $\varsigma_{\ell+1}^i > 0$ is a constant.

The time derivative of $V_{\ell+1}^i$ is

$$\dot{V}_{\ell+1} = \dot{V}_\ell + \sum_{i=1}^{N}\left(e_{\ell+1}^i\dot{e}_{\ell+1}^i - \frac{1}{\varsigma_{\ell+1}^i}\tilde{\theta}_{\ell+1}^{iT}\dot{\hat{\theta}}_{\ell+1}^i + \xi_{\ell+1}^i\dot{\xi}_{\ell+1}^i\right)$$
$$= \dot{V}_\ell + \sum_{i=1}^{N}\Bigg\{e_{\ell+1}^i\Big[e_{\ell+2}^i + \xi_{\ell+2}^i + \alpha_{\ell+1}^i + l_{\ell+1}^i(y^i - \hat{x}_1^i)$$
$$+\hat{\theta}_{\ell+1}^{iT}\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i) - \dot{\bar{\alpha}}_{\ell+1}^i\Big] - e_{\ell+1}^i\tilde{\theta}_{\ell+1}^{iT}\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i)$$
$$-\frac{1}{\varsigma_{\ell+1}^i}\tilde{\theta}_{\ell+1}^{iT}\left[\dot{\hat{\theta}}_{\ell+1}^i - \varsigma_{\ell+1}^ie_{\ell+1}^i\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i)\right] + \xi_{\ell+1}^i\dot{\xi}_{\ell+1}^i\Bigg\}. \tag{70}$$

Similarly, one gets

$$e_{\ell+1}^i\xi_{\ell+2}^i \le \frac{1}{2}(e_{\ell+1}^i)^2 + \frac{1}{2}(\xi_{\ell+2}^i)^2, \tag{71}$$

$$-e_{\ell+1}^i\tilde{\theta}_{\ell+1}^{iT}\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i) \le \frac{1}{2}(e_{\ell+1}^i)^2 + \frac{1}{2}\tilde{\theta}_{\ell+1}^{iT}\tilde{\theta}_{\ell+1}^i, \tag{72}$$

and

$$\dot{\xi}_{\ell+1}^i = \dot{\bar{\alpha}}_{\ell+1}^i - \dot{\alpha}_\ell^i = -\frac{\xi_{\ell+1}^i}{\upsilon_{\ell+1}^i} + \varepsilon_{\ell+1}^i(\cdot), \tag{73}$$

where $\varepsilon_{\ell+1}^i(\cdot)$ is a bounded function satisfying $|\varepsilon_{\ell+1}^i| \le \bar{\varepsilon}_{\ell+1}^i$ with constant $\bar{\varepsilon}_{\ell+1}^i > 0$.

Choose the virtual controller $\alpha_{\ell+1}^i$ as

$$\alpha_{\ell+1}^i = -l_{\ell+1}^i(y^i - \hat{x}_1^i) - \hat{\theta}_{\ell+1}^{iT}\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i) + \dot{\bar{\alpha}}_{\ell+1}^i$$
$$-\frac{e_\ell^i}{(k_\ell^i)^2-(e_\ell^i)^2} - e_{\ell+1}^i - c_{\ell+1}^i(e_{\ell+1}^i)^{2\beta-1}, \tag{74}$$

and the adaptive law $\dot{\hat{\theta}}_{\ell+1}^i$ as

$$\dot{\hat{\theta}}_{\ell+1}^i = \varsigma_{\ell+1}^ie_{\ell+1}^i\Phi_{\ell+1}^i(\hat{\bar{x}}_{\ell+1}^i) - \delta_{\ell+1}^i\hat{\theta}_{\ell+1}^i, \tag{75}$$

where $c_{\ell+1}^i > 0$ and $\delta_{\ell+1}^i > 0$ are constants.

Thus,

$$\begin{aligned}\dot{V}_{\ell+1} \leq \sum_{i=1}^{N}\Bigg[&-\lambda_1\|\tilde{x}\|^2+\frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i\\&-\sum_{p=1}^{\ell}\frac{c_p^i(e_p^i)^{2\beta}}{((k_p^i)^2-(e_p^i)^2)^{\beta}}+\sum_{p=1}^{\ell+1}\frac{\delta_p^i}{\varsigma_p^i}\tilde{\theta}_p^{iT}\hat{\theta}_p^i\\&+\sum_{p=1}^{\ell+1}\frac{1}{2}(\xi_{p+1}^i)^2+\sum_{j=1}^{N}a_{ij}\frac{\gamma_1^j}{\varpi_1^j}\tilde{\theta}_1^{jT}\hat{\theta}_1^j\\&+\sum_{p=2}^{\ell+1}\frac{1}{2}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i+e_{\ell+1}^ie_{\ell+2}^i-c_{\ell+1}^i(e_{\ell+1}^i)^{2\beta}\\&-\sum_{p=2}^{\ell+1}\frac{(\xi_p^i)^2}{\upsilon_p^i}+\sum_{p=2}^{\ell+1}\xi_p^i\varepsilon_p^i(\cdot)\Bigg]+M_{\ell+1},\end{aligned}\tag{76}$$

where $M_{\ell+1}=M_\ell$.

Step m ($m=\ell+2,\dots,n-1$): The derivative of e_m^i is

$$\begin{aligned}\dot{e}_m^i&=\dot{\hat{x}}_m^i-\dot{\alpha}_m^i\\&=e_{m+1}^i+\xi_{m+1}^i+\alpha_m^i+l_m^i\tilde{x}_1^i+\hat{\theta}_m^{iT}\Phi_m^i(\hat{\bar{x}}_m^i)\\&\quad-\tilde{\theta}_m^{iT}\Phi_m^i(\hat{\bar{x}}_m^i)+\tilde{\theta}_m^{iT}\Phi_m^i(\hat{\bar{x}}_m^i)-\dot{\bar{\alpha}}_m^i.\end{aligned}\tag{77}$$

The Lyapunov function is chosen as

$$V_m=V_{m-1}+\frac{1}{2}\sum_{i=1}^{N}[(e_m^i)^2+\frac{1}{\varsigma_m^i}\tilde{\theta}_m^{iT}\tilde{\theta}_m^i+(\xi_m^i)^2],\tag{78}$$

where $\varsigma_m^i>0$ is a constant. Similarly, we have

$$e_m^i\xi_{m+1}^i\leq\frac{1}{2}(e_m^i)^2+\frac{1}{2}(\xi_{m+1}^i)^2,\tag{79}$$

$$-e_m^i\tilde{\theta}_m^{iT}\Phi_m^i(\hat{\bar{x}}_m^i)\leq\frac{1}{2}(e_m^i)^2+\frac{1}{2}\tilde{\theta}_m^{iT}\tilde{\theta}_m^i.\tag{80}$$

and

$$\dot{\xi}_m^i=\dot{\bar{\alpha}}_m^i-\dot{\alpha}_{m-1}^i=-\frac{\xi_m^i}{\upsilon_m^i}+\varepsilon_m^i(\cdot),\tag{81}$$

where $\varepsilon_m^i(\cdot)$ is a bounded function satisfying $|\varepsilon_m^i|\leq\bar{\varepsilon}_m^i$ with constant $\bar{\varepsilon}_m^i>0$.

Choose the virtual controller α_m^i as

$$\begin{aligned}\alpha_m^i=&-l_m^i(y^i-\hat{x}_1^i)-\hat{\theta}_m^{iT}\Phi_m^i(\hat{\bar{x}}_m^i)+\dot{\bar{\alpha}}_m^i\\&-e_{m-1}^i-e_m^i-c_m^i(e_m^i)^{2\beta-1},\end{aligned}\tag{82}$$

and the adaptive law $\dot{\hat{\theta}}_m^i$ as

$$\dot{\hat{\theta}}_m^i=\varsigma_m^ie_m^i\Phi_m^i(\hat{\bar{x}}_m^i)-\delta_m^i\hat{\theta}_m^i,\tag{83}$$

where $c_m^i>0$ and $\delta_m^i>0$ are constants.

Thus,

$$\begin{aligned}\dot{V}_m\leq\sum_{i=1}^{N}\Bigg[&-\lambda_1\|\tilde{x}\|^2+\frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i\\&-\sum_{p=1}^{\ell}\frac{c_p^i(e_p^i)^{2\beta}}{((k_p^i)^2-(e_p^i)^2)^{\beta}}+\sum_{p=1}^{m}\frac{\delta_p^i}{\varsigma_p^i}\tilde{\theta}_p^{iT}\hat{\theta}_p^i\\&+\sum_{p=1}^{m}\frac{1}{2}(\xi_{p+1}^i)^2+\sum_{j=1}^{N}a_{ij}\frac{\gamma_1^j}{\varpi_1^j}\tilde{\theta}_1^{jT}\hat{\theta}_1^j\\&+\sum_{p=2}^{m}\frac{1}{2}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i+e_m^ie_{m+1}^i-\sum_{p=\ell+1}^{m}c_p^i(e_p^i)^{2\beta}\\&-\sum_{p=2}^{m}\frac{(\xi_p^i)^2}{\upsilon_p^i}+\sum_{p=2}^{m}\xi_p^i\varepsilon_p^i(\cdot)\Bigg]+M_m,\end{aligned}\tag{84}$$

where $M_m=M_{m-1}$.

Step n: The derivative of e_n^i is

$$\begin{aligned}\dot{e}_n^i&=\dot{\hat{x}}_n^i-\dot{\bar{\alpha}}_n^i\\&=\mathrm{sat}_i(u_i)+l_n^i\tilde{x}_1^i+\hat{\theta}_n^{iT}\Phi_n^i(\hat{x}^i)-\tilde{\theta}_n^{iT}\Phi_n^i(\hat{x}^i)\\&\quad+\tilde{\theta}_n^{iT}\Phi_n^i(\hat{x}^i)-\dot{\bar{\alpha}}_n^i.\end{aligned}\tag{85}$$

Select the Lyapunov function as

$$\begin{aligned}V_n&=V_{n-1}+\frac{1}{2}\sum_{i=1}^{N}V_n^i\\&=V_{n-1}+\frac{1}{2}\sum_{i=1}^{N}\left[(e_n^i)^2+\frac{1}{\varsigma_n^i}\tilde{\theta}_n^{iT}\tilde{\theta}_n^i+\frac{\varrho_i}{r_i}\tilde{\hbar}_i^2+(\xi_n^i)^2\right],\end{aligned}\tag{86}$$

where $\varsigma_n^i>0$ and $r_i>0$ are constants, $\tilde{\hbar}_i=\hbar_i-\hat{\hbar}_i$ denotes the estimation error between $\hbar_i=\frac{1}{\varrho_i}$ and $\hat{\hbar}_i$, in which $\hbar_i$ is unknown and will be estimated adaptively.

Thus,

$$\begin{aligned}\dot{V}_n&=\dot{V}_{n-1}+\sum_{i=1}^{N}(e_n^i\dot{e}_n^i-\frac{1}{\varsigma_n^i}\tilde{\theta}_n^{iT}\dot{\hat{\theta}}_n^i-\frac{\varrho_i}{r_i}\tilde{\hbar}_i\dot{\hat{\hbar}}_i+\xi_n^i\dot{\xi}_n^i)\\&=\dot{V}_{n-1}+\sum_{i=1}^{N}\{e_n^i[\mathrm{sat}_i(u_i)+l_n^i\tilde{x}_1^i+\hat{\theta}_n^{iT}\Phi_n^i(\hat{x}^i)-\dot{\bar{\alpha}}_n^i]\\&\quad-e_n^i\tilde{\theta}_n^{iT}\Phi_n^i(\hat{x}^i)-\frac{1}{\varsigma_n^i}\tilde{\theta}_n^{iT}[\dot{\hat{\theta}}_n^i-\varsigma_n^ie_n^i\Phi_n^i(\hat{x}^i)]+\xi_n^i\dot{\xi}_n^i-\frac{\varrho_i}{r_i}\tilde{\hbar}_i\dot{\hat{\hbar}}_i\}.\end{aligned}\tag{87}$$

4.2 | ETC design

In this subsection, an adaptive ETC scheme will be established for system (11) to track the reference signal $y_r(t)$ and ensure SGPFS.

Let

$$u_i(t) = \tau_i(t_k^i), \forall t \in [t_k^i, t_{k+1}^i), \tag{88}$$

where $\{t_k^i, k \in \mathcal{Z}^+\}$ is a trigger time instant sequence. The trigger time instant will be updated as t_{k+1}^i, when the following designed trigger condition

$$t_{k+1}^i = \inf\{t \in R \| \Theta_i(t)| \geq q_i|u_i(t)| + h_i\}, \tag{89}$$

is satisfied, where $\Theta_i(t) = \tau_i(t) - u_i(t)$, $0 < q_i < 1$ and $h_i > 0$ are constants. Note that the control input $u_i(t)$ will remain constant in time interval $[t_k^i, t_{k+1}^i)$.

Remark 3. By selecting appropriate parameters q_i and h_i, the number of transmissions can be adjusted via the ETC scheme. The smaller the parameter values, the more times the trigger occurs. Instead, the trigger times will decrease. Specially, it becomes time-triggered when $q_i = 0$ and $h_i = 0$.

Remark 4. The ETC mechanism presented in (88) and (89) can be updated according to the value of the control signal. When the control input $u_i(t)$ is large, a larger threshold will be generated, which can reduce the times of information transmission. When the control input $u_i(t)$ is small, a smaller threshold will be generated, so as to obtain better system performance. If the measurement error of the system meets the event-triggered conditions, the controller will be updated.

Similar to the discussion in [55], one gets

$$\tau_i(t) = (1 + \kappa_1(t)q_i)u_i(t) + \kappa_2(t)h_i, t \in \left[t_k^i, t_{k+1}^i\right], \tag{90}$$

where $\kappa_1(t)$ and $\kappa_2(t)$ satisfy $|\kappa_1(t)| \leq 1$ and $|\kappa_2(t)| \leq 1$, respectively.

Further, one has

$$u_i(t) = \frac{\tau_i(t)}{1 + \kappa_1(t)q_i} - \frac{\kappa_2(t)h_i}{1 + \kappa_1(t)q_i}. \tag{91}$$

The control function $\tau_i(t)$ in (88) is designed as

$$\tau_i(t) = -(1 + q_i)\left[\alpha_n^i \hat{\hbar}_i \tanh\left(\frac{e_n^i \alpha_n^i \hat{\hbar}_i}{\epsilon_i}\right) + \frac{h_i}{1 - q_i}\tanh\left(\frac{h_i e_n^i}{(1 - q_i)\epsilon_i}\right)\right], \tag{92}$$

where $\epsilon_i > 0$ is a constant.

According to Lemma 7, we have

$$\frac{\zeta(u_i)\tau_i(t)e_n^i}{1 + \kappa_1(t)q_i} \leq -e_n^i \alpha_n^i + \varrho_i \tilde{\hbar}_i |e_n^i \alpha_n^i| + 0.2785\epsilon_i$$
$$- \frac{h_i \zeta(u_i)e_n^i}{1 - q_i}\tanh\left(\frac{h_i e_n^i}{(1 - q_i)\epsilon_i}\right), \tag{93}$$

and

$$-\frac{h_i \zeta(u_i)\kappa_2(t)e_n^i}{1 + \kappa_1(t)q_i} \leq \frac{h_i \zeta(u_i)\kappa_2(t)|e_n^i|}{1 + \kappa_1(t)q_i} \leq \frac{h_i \zeta(u_i)|e_n^i|}{1 - q_i}. \tag{94}$$

It follows (91)–(94), (87) that

$$\dot{V}_n \leq \dot{V}_{n-1} + \sum_{i=1}^{N}\Bigg\{e_n^i\left[l_n^i \tilde{x}_1^i + \hat{\theta}_n^{iT}\Phi_n^i(\hat{x}^i) - \dot{\bar{\alpha}}_n^i - \alpha_n^i\right]$$
$$-e_n^i \tilde{\theta}_n^{iT}\Phi_n^i(\hat{x}^i) - \frac{1}{\varsigma_n^i}\tilde{\theta}_n^{iT}\left[\dot{\hat{\theta}}_n^i - \varsigma_n^i e_n^i \Phi_n^i(\hat{x}^i)\right] + \xi_n^i \dot{\xi}_n^i$$
$$+ \frac{\varrho_i \tilde{\hbar}_i}{r_i}\left(r_i|e_n^i \alpha_n^i| - \dot{\hat{\hbar}}_i\right) + 0.557\epsilon_i\Bigg\}. \tag{95}$$

Similarly, we have

$$-e_n^i \tilde{\theta}_n^{iT}\Phi_n^i(\hat{x}^i) \leq \frac{1}{2}(e_n^i)^2 + \frac{1}{2}\tilde{\theta}_n^{iT}\tilde{\theta}_n^i, \tag{96}$$

and

$$\dot{\xi}_n^i = \dot{\bar{\alpha}}_n^i - \dot{\alpha}_{n-1}^i = -\frac{\xi_n^i}{\upsilon_n^i} + \varepsilon_n^i(\cdot), \tag{97}$$

where $\varepsilon_n^i(\cdot)$ is a bounded function satisfying $|\varepsilon_n^i| \leq \bar{\varepsilon}_n^i$ with constant $\bar{\varepsilon}_n^i > 0$.

The tuning function α_n^i is designed as

$$\alpha_n^i = l_n^i(y^i - \hat{x}_1^i) + \hat{\theta}_n^{iT}\Phi_n^i(\hat{x}^i) - \dot{\bar{\alpha}}_n^i + e_{n-1}^i$$
$$+\frac{1}{2}e_n^i + c_n^i(e_n^i)^{2\beta-1}, \tag{98}$$

where c_n^i is a positive constant.

Select $\dot{\hat{\theta}}_n^i$ as

$$\dot{\hat{\theta}}_n^i = \varsigma_n^i e_n^i \Phi_n^i(\hat{x}^i) - \delta_n^i \hat{\theta}_n^i, \tag{99}$$

and $\dot{\hat{\hbar}}_i$ as

$$\dot{\hat{\hbar}}_i = r_i|e_n^i \alpha_n^i| - \iota_i \hat{\hbar}_i, \tag{100}$$

where $\delta_n^i > 0$, $\iota_i > 0$ are constants.

From (95)-(100), we have

$$\dot{V}_n \leq \sum_{i=1}^{N}\Bigg[-\lambda_1\|\tilde{x}\|^2 + \frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2 \tilde{\theta}_p^{iT}\tilde{\theta}_p^i$$
$$-\sum_{p=1}^{\ell}\frac{c_p^i(e_p^i)^{2\beta}}{((k_p^i)^2 - (e_p^i)^2)^\beta} + \sum_{p=1}^{n}\frac{\delta_p^i}{\varsigma_p^i}\tilde{\theta}_p^{iT}\hat{\theta}_p^i + \sum_{p=1}^{n-1}\frac{1}{2}(\xi_{p+1}^i)^2$$
$$+\sum_{j=1}^{N}a_{ij}\frac{\gamma_1^j}{\varpi_1^j}\tilde{\theta}_1^{jT}\hat{\theta}_1^j + 0.557\epsilon_i + \sum_{p=2}^{n}\frac{1}{2}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i - \sum_{p=2}^{n}\frac{(\xi_p^i)^2}{\upsilon_p^i}$$
$$+\sum_{p=2}^{n}\xi_p^i\varepsilon_p^i(\cdot) - \sum_{p=\ell+1}^{n}c_p^i(e_p^i)^{2\beta} + \frac{\iota_i\varrho_i\tilde{\hbar}_i\hat{\hbar}_i}{r_i}\Bigg] + M_{n-1}. \tag{101}$$

By Young's inequality, we have

$$\frac{\iota_i\varrho_i\tilde{\hbar}_i\hat{\hbar}_i}{r_i} \leq -\frac{\iota_i\varrho_i\tilde{\hbar}_i^2}{2r_i} + \frac{\iota_i\varrho_i\hbar_i^2}{2r_i}. \tag{102}$$

From (101) and (102), one gets

$$\begin{aligned}\dot{V}_n \leq \sum_{i=1}^{N}\Bigg[&-\lambda_1\|\tilde{x}\|^2 + \frac{1}{2}\sum_{p=1}^{n}\|F_i\|^2\tilde{\theta}_p^{iT}\tilde{\theta}_p^i\\ &-\sum_{p=1}^{\ell}\frac{c_p^i(e_p^i)^{2\beta}}{((k_p^i)^2-(e_p^i)^2)^{\beta}} + \sum_{p=1}^{n}\frac{\delta_p^i}{\varsigma_p^i}\tilde{\theta}_p^{iT}\hat{\theta}_p^i\\ &+\sum_{p=1}^{n-1}\frac{1}{2}(\xi_{p+1}^i)^2 + \sum_{j=1}^{N}a_{ij}\frac{\gamma_1^j}{\varpi_1^j}\tilde{\theta}_1^{jT}\hat{\theta}_1^j\\ &+\sum_{p=2}^{n}\frac{1}{2}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i - \sum_{p=2}^{n}\frac{(\xi_p^i)^2}{\upsilon_p^i} + \sum_{p=2}^{n}\xi_p^i\varepsilon_p^i(\cdot)\\ &-\sum_{p=\ell+1}^{n}c_p^i(e_p^i)^{2\beta} - \frac{\iota_i\varrho_i\tilde{\hbar}_i^2}{2r_i}\Bigg] + M_n,\end{aligned} \tag{103}$$

where $M_n = 0.557\epsilon_i + \dfrac{\iota_i\varrho_i\hbar_i^2}{2r_i} + M_{n-1}$.

5 | STABILITY ANALYSIS

In this section, we give our main results in Theorem 1, which shows that the proposed scheme can achieve the control objectives mentioned in Subsection 2.2.

Theorem 1. *For MASs (11) with virtual controller (46), (57), (65), (74), (82), (98) and adaptive laws (47), (48), (58), (66), (75), (83), (99), (100) subject to ETC mechanism (88), (89), under Assumptions 1, the closed-loop system is SGPFS and the finite-time convergence of tracking error to a small neighborhood of the origin is ensured. The settling time is given as*

$$T^* = \frac{1}{d(1-\beta)\rho}\left[V_n^{1-\beta}(0) - \left(\frac{M_n^{(2)}}{d(1-\rho)}\right)^{\frac{1-\beta}{\beta}}\right]. \tag{104}$$

In addition, the following results are guaranteed.

(i) The error signals e_p^i, the adaptive parameter errors $\tilde{\theta}_p^i$, $\tilde{\hbar}_i$, the observer errors $\tilde{x}^i$ and the filter errors ξ_p^i for $i = 1, \ldots, N$, are bounded in finite-time and satisfy

$$|e_p^i| \leq k_p^i(t)\left[1 - e^{-2\left(\frac{M_n^{(2)}}{d(1-\rho)}\right)^{\frac{1}{\beta}}}\right]^{\frac{1}{2}}, p = 1, 2, \ldots, \ell, \tag{105}$$

$$|e_p^i| \leq \sqrt{2\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, p = \ell+1, \ldots, n, \tag{106}$$

$$\|\tilde{\theta}_p^i\|_2 \leq \sqrt{2\varsigma_p^i\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, p = 1, 2, \ldots, n, \tag{107}$$

$$|\tilde{\hbar}_i| \leq \sqrt{\frac{r_i}{\varrho_i}\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, \tag{108}$$

$$\|\tilde{x}^i\|_2 \leq \sqrt{\frac{2}{\lambda_{\min}(F_i)}\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}} \tag{109}$$

and

$$|\xi_p^i| \leq \sqrt{2\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, p = 2, \ldots, n, \tag{110}$$

for $t > T^*$.

(ii) The constrained partial states do not exceed the preset boundary, that is, $|x_p^i| < \bar{k}_p^i(t)$, $\forall\, t > 0$, $p = 1, \ldots, \ell$.

(iii) All the system signals are bounded.

(iv) The Zeno behavior does not occur.

Proof. Since

$$\tilde{\theta}_p^{iT}\hat{\theta}_p^i \leq \frac{1}{2}\theta_p^{iT}\theta_p^i - \frac{1}{2}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i, \tag{111}$$

thus

$$\sum_{p=1}^{n}\frac{\delta_p^i}{\varsigma_p^i}\tilde{\theta}_p^{iT}\hat{\theta}_p^i \leq \sum_{p=1}^{n}\frac{\delta_p^i}{2\varsigma_p^i}\theta_p^{iT}\theta_p^i - \sum_{p=1}^{n}\frac{\delta_p^i}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i. \tag{112}$$

In addition, $\varepsilon_p^i(\cdot)$ is a bounded function satisfying $|\varepsilon_p^i| \leq \bar{\varepsilon}_p^i$. From Lemma 2, we have

$$|\xi_p^i\varepsilon_p^i| \leq \frac{(\xi_p^i)^2(\bar{\varepsilon}_p^i)^2}{2J} + \frac{J}{2}, \tag{113}$$

where $J > 0$ is a constant.

Thus,

$$\begin{aligned}\dot{V}_n \leq \sum_{i=1}^{N}\Bigg\{&-\frac{\lambda_1}{\lambda_{max(F_i)}}\tilde{x}^{iT}F_i\tilde{x}^i - \sum_{p=1}^{\ell}2^{\beta}c_p^i\left[\frac{(e_p^i)^2}{2((k_p^i)^2-(e_p^i)^2)}\right]^{\beta}\\ &-\Bigg[(\delta_1^i - \|F_i\|^2\varsigma_1^i)\frac{1}{2\varsigma_1^i}\tilde{\theta}_1^{iT}\tilde{\theta}_1^i + \sum_{p=2}^{n}(\delta_p^i - \varsigma_p^i\\ &-\|F_i\|^2\varsigma_p^i)\frac{1}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i + \sum_{p=2}^{n}\left[\frac{2}{\upsilon_p^i} - \frac{(\bar{\varepsilon}_p^i)^2}{J} - 1\right]\frac{1}{2}(\xi_p^i)^2\\ &+\sum_{j=1}^{N}a_{ij}\frac{\gamma_1^j}{2\varpi_1^j}\tilde{\theta}_1^{jT}\tilde{\theta}_1^j + \frac{\iota_i\varrho_i\tilde{\hbar}_i^2}{2r_i} + \sum_{p=\ell+1}^{n}c_p^i(e_p^i)^{2\beta}\Bigg]\Bigg\} + M_n^{(1)},\end{aligned} \tag{114}$$

where $M_n^{(1)} = M_n + \sum_{i=1}^{N}[\sum_{p=1}^{n} \frac{\delta_p^i}{2\varsigma_p^i}\theta_p^{iT}\theta_p^i + \frac{J}{2}(n-1)] + \sum_{j=1}^{N} a_{ij}\frac{\gamma_1^j}{2\varpi_1^j}\theta_1^{jT}\theta_1^j$.

Let

$$\sigma = \min_{\substack{i=1,\dots,N\\ p=2,\dots,n}} \left\{ 2^\beta c_1^i, 2^\beta c_p^i, \delta_1^i - \|F_i\|^2\varsigma_1^i, \delta_p^i - \varsigma_p^i - \|F_i\|^2\varsigma_p^i, \frac{2}{\upsilon_p^i} - \frac{(\bar{\varepsilon}_p^i)^2}{J} - 1, \frac{\gamma_1^j}{\varpi_1^j}, \frac{\iota_i\varrho_i}{r_i} \right\}, \tag{115}$$

where $\delta_1^i - \|F_i\|^2\varsigma_1^i > 0$, $\delta_p^i - \varsigma_p^i - \|F_i\|^2\varsigma_p^i > 0$, $\frac{2}{\upsilon_p^i} - \frac{(\bar{\varepsilon}_p^i)^2}{J} - 1 > 0$, by selecting appropriate parameters.

Thus,

$$\begin{aligned}\dot{V}_n \le & -\sum_{i=1}^{N}\frac{2\lambda_1}{\lambda_{max(F_i)}}\frac{1}{2}\tilde{x}^{iT}F_i\tilde{x}^i - \sigma\sum_{i=1}^{N}\left\{\sum_{p=1}^{\ell}\left[\frac{(e_p^i)^2}{2((k_p^i)^2-(e_p^i)^2)}\right]^\beta\right. \\ & + \sum_{p=1}^{n}\frac{1}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i + \sum_{p=1}^{n-1}\frac{1}{2}(\xi_{p+1}^i)^2 + \sum_{j=1}^{N}\frac{a_{ij}}{2}\tilde{\theta}_1^{jT}\tilde{\theta}_1^j + \frac{\tilde{\hbar}_i^2}{2} \\ & \left. + \sum_{p=\ell+1}^{n}\frac{(e_p^i)^{2\beta}}{2^\beta}\right\} + M_n^{(1)}. \end{aligned} \tag{116}$$

According to Lemma 3, choose $\psi_1 = 1$, $\psi_2 = \sum_{p=1}^{n}\frac{1}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i$, $\nu_1 = 1-\beta$, $\nu_2 = \beta$ and $\nu_3 = \beta^{\frac{\beta}{1-\beta}}$, one gets

$$\left(\sum_{p=1}^{n}\frac{1}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i\right)^\beta \le (1-\beta)\nu_3 + \sum_{p=1}^{n}\frac{1}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i. \tag{117}$$

Similar to (117), the following inequalities hold

$$\left[\sum_{p=1}^{n-1}\frac{1}{2}(\xi_{p+1}^i)^2\right]^\beta \le (1-\beta)\nu_3 + \sum_{p=1}^{n-1}\frac{1}{2}(\xi_{p+1}^i)^2, \tag{118}$$

$$\left(\frac{1}{2}\tilde{x}^{iT}F_i\tilde{x}^i\right)^\beta \le (1-\beta)\nu_3 + \frac{1}{2}\tilde{x}^{iT}F_i\tilde{x}^i, \tag{119}$$

$$\left(\sum_{j=1}^{N}\frac{a_{ij}}{2}\tilde{\theta}_1^{jT}\tilde{\theta}_1^j\right)^\beta \le (1-\beta)\nu_3 + \sum_{j=1}^{N}\frac{a_{ij}}{2}\tilde{\theta}_1^{jT}\tilde{\theta}_1^j, \tag{120}$$

and

$$\left(\frac{\tilde{\hbar}_i^2}{2}\right)^\beta \le (1-\beta)\nu_3 + \frac{\tilde{\hbar}_i^2}{2}. \tag{121}$$

Noting that $\log\frac{(k_p^i)^2}{(k_p^i)^2-(e_p^i)^2} \le \frac{(e_p^i)^2}{(k_p^i)^2-(e_p^i)^2}$ when $|e_p^i| < k_p^i$ and from (117)-(121), we have

$$\begin{aligned}\dot{V}_n \le & -\sum_{i=1}^{N}\frac{2\lambda_1}{\lambda_{\max(F_i)}}\left(\frac{1}{2}\tilde{x}^{iT}F_i\tilde{x}^i\right)^\beta - \sum_{i=1}^{N}\left\{\sigma\left[\sum_{p=1}^{\ell}\frac{1}{2}\frac{(e_p^i)^2}{(k_p^i)^2-(e_p^i)^2}\right]^\beta\right. \\ & + \sigma\left(\sum_{p=1}^{n}\frac{1}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i\right)^\beta + \sigma\left[\sum_{p=1}^{n-1}\frac{1}{2}(\xi_{p+1}^i)^2\right]^\beta + \sigma\left(\sum_{j=1}^{N}\frac{a_{ij}}{2}\tilde{\theta}_1^{jT}\tilde{\theta}_1^j\right)^\beta \\ & \left. + \sigma\left(\frac{\tilde{\hbar}_i^2}{2}\right)^\beta + \sigma\left[\sum_{p=\ell+1}^{n}\frac{(e_p^i)^2}{2}\right]^\beta\right\} + M_n^{(2)}, \end{aligned} \tag{122}$$

where $M_n^{(2)} = M_n^{(1)} + \sum_{i=1}^{N}[4\sigma(1-\beta)\nu_3 + \frac{2\lambda_1}{\lambda_{max(F_i)}}(1-\beta)\nu_3] > 0$. Let $d = \min\{\frac{2\lambda_1}{\lambda_{\max(F_i)}}, \sigma\}$, we have

$$\dot{V}_n \le -dV_n^\beta + M_n^{(2)}. \tag{123}$$

Therefore, from (123) and Lemma 4, the closed-loop system is SGPFS.

Now, we obtain the settling time T^*. Let $e^i = (e_1^i, \dots, e_n^i)^T$, $\tilde{\theta}^i = (\tilde{\theta}_1^i, \dots, \tilde{\theta}_n^i)^T$, $\xi^i = (\xi_2^i, \dots, \xi_n^i)^T$ and $X = [\mathrm{col}^T(\tilde{x}^i), \mathrm{col}^T(e^i), \mathrm{col}^T(\tilde{\theta}^i), \mathrm{col}^T(\xi^i), \mathrm{col}^T(\tilde{\hbar}_i)]^T$ for $i = 1, \dots, N$. From (123), one gets

$$\dot{V}_n(X) \le -d\rho V_n^\beta(X) - d(1-\rho)V_n^\beta(X) + M_n^{(2)}, \tag{124}$$

where $0 < \rho < 1$.

Let $\Omega_X = \{X | V_n^\beta(X) \le \frac{M_n^{(2)}}{d(1-\rho)}\}$ and $\bar{\Omega}_X = \{X | V_n^\beta(X) > \frac{M_n^{(2)}}{d(1-\rho)}\}$. If $X \in \bar{\Omega}_X$, then

$$\dot{V}_n(X) \le -d\rho V_n^\beta(X). \tag{125}$$

Integrating on both sides of (125) from 0 to T, one has

$$\int_0^T \frac{\dot{V}_n(X)}{V_n^\beta(X)}dt \le -d\int_0^T \rho dt, \tag{126}$$

which yields

$$\frac{1}{1-\beta}V_n^{1-\beta}(X(T)) - \frac{1}{1-\beta}V_n^{1-\beta}(X(0)) \le -d\rho T. \tag{127}$$

Let

$$T^* = \frac{1}{d(1-\beta)\rho}\left[V_n^{1-\beta}(0) - \left(\frac{M_n^{(2)}}{d(1-\rho)}\right)^{\frac{1-\beta}{\beta}}\right]. \tag{128}$$

From (127), one has $x \in \Omega_X$ for $\forall t \ge T^*$.

By the definitions of V_n and Ω_X, for $t \geq T^*$, one has

$$V_n^{\beta} \leq \frac{M_n^{(2)}}{d(1-\rho)}. \tag{129}$$

For the error variable e_p^i, $p = 1, \ldots, \ell$, it follows from (61) and (129) that

$$\frac{1}{2}\log\frac{(k_p^i)^2}{(k_p^i)^2 - (e_p^i)^2} \leq V_n \leq \left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}. \tag{130}$$

Thus,

$$|e_p^i| \leq k_p^i(t)\left[1 - e^{-2\left(\frac{M_n^{(2)}}{d(1-\rho)}\right)^{\frac{1}{\beta}}}\right]^{\frac{1}{2}} < k_p^i(t), p = 1, \ldots, \ell. \tag{131}$$

For the error variable e_p^i, $p = \ell + 1, \ldots, n$, it follows from (86) and (129) that

$$\frac{1}{2}(e_p^i)^2 \leq V_n \leq \left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}. \tag{132}$$

Thus,

$$|e_p^i| \leq \sqrt{2\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, p = \ell + 1, \ldots, n. \tag{133}$$

Let $\bar{e}_1 = [e_1^1, \ldots, e_1^N]^T$. It is obvious that $\bar{e}_1 = \Xi_1(y - 1_N y_r)$. From Lemma 1, one has

$$\|y - 1_N y_r\|_2 \leq \frac{1}{\underline{\sigma}(\Xi_1)} k_1^i(t)\left[1 - e^{-2\left(\frac{M_n^{(2)}}{d(1-\rho)}\right)^{\frac{1}{\beta}}}\right]^{\frac{1}{2}}, \tag{134}$$

where $\underline{\sigma}(\Xi_1) = \sqrt{\lambda_{\min}(\Xi_1^H \Xi_1)}$ and Ξ_1^H is the conjugate transpose of Ξ_1.

Therefore, the finite-time convergence of tracking error to a small neighborhood of the origin with settling time T^* is ensured.

(i) For the adaptive parameters estimation errors $\tilde{\theta}_p^i$, $i = 1, \ldots, N$, $p = 1, \ldots, n$, it follows from (86) and (129) that

$$\frac{1}{2\varsigma_p^i}\tilde{\theta}_p^{iT}\tilde{\theta}_p^i \leq V_n \leq \left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}, \tag{135}$$

which means that $\tilde{\theta}_p^i$ is bounded and

$$\|\tilde{\theta}_p^i\|_2 \leq \sqrt{2\varsigma_p^i\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, t > T^*. \tag{136}$$

For the estimation errors $\tilde{\hbar}_i$, $i = 1, \ldots, N$, it follows from (86) and (129) that

$$\frac{\varrho_i}{r_i}\tilde{\hbar}_i^2 \leq V_n \leq \left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}, \tag{137}$$

which means that $\tilde{\hbar}_i$ is bounded and

$$|\tilde{\hbar}_i| \leq \sqrt{\frac{r_i}{\varrho_i}\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, t > T^*. \tag{138}$$

For observer errors $\tilde{x}^i$, according to (31) and (129), one has

$$\frac{1}{2}\tilde{x}^{iT}F_i\tilde{x}^i \leq V_n \leq \left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}. \tag{139}$$

Thus, for $i = 1, \ldots, N$,

$$\|\tilde{x}^i\|_2 \leq \sqrt{\frac{2}{\lambda_{\min}(F_i)}\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, t > T^*. \tag{140}$$

Therefore, the observer error $\tilde{x}^i$ is bounded.
For the filter errors ξ_p^i, $i = 1, \ldots, N$, $p = 2, \ldots, n$, it follows from (86) and (129) that

$$\frac{1}{2}(\xi_p^i)^2 \leq V_n \leq \left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}, \tag{141}$$

which means that ξ_p^i is bounded and

$$|\xi_p^i| \leq \sqrt{2\left[\frac{M_n^{(2)}}{d(1-\rho)}\right]^{\frac{1}{\beta}}}, t > T^*. \tag{142}$$

From (131), (133), (140), (136), (138) and (142), the boundedness of the error signals e_p^i, $\tilde{\theta}_p^i$, $\tilde{\hbar}_i$, $\tilde{x}^i$, ξ_p^i and the bound conditions given in (i) are ensured.

(ii) It follows from Lemma 8 that $|e_p^i| < k_p^i(t)$. Since $y = \Xi_1^{-1}\bar{e}_1 + 1_N y_r$, thus $|x_1^i| \leq \|y\|_\infty \leq \|\Xi_1^{-1}\|_\infty \|\bar{e}_1\|_\infty + \hat{r}_0 \leq \|\Xi_1^{-1}\|_\infty \cdot \max_{i=1,\ldots,N}\{k_1^i(t)\} + \hat{r}_0$. Choose $k_1^i(t) \leq \frac{\bar{k}_1^i(t) - \hat{r}_0}{\|\Xi_1^{-1}\|_\infty}$, we have $|x_1^i| \leq \bar{k}_1^i(t)$, which means that the state x_1^i do not exceed the constraint boundary. Since $e_2^i = \hat{x}_2^i - \bar{\alpha}_2^i = x_2^i - \tilde{x}_2^i - \xi_2^i - \alpha_1^i$, thus

$|x_2^i| \leq |e_2^i| + |\tilde{x}_2^i| + |\xi_2^i| + |\alpha_1^i|$. Since α_1^i is a continuous function in a compact set, thus $|\alpha_1^i| < \breve{\alpha}_1^i$ with $\breve{\alpha}_1^i > 0$ being a constant. It follows from (140) and (142) that $|\tilde{x}_2^i| < \|\tilde{x}^i\|_2 \leq \sqrt{\frac{2}{\lambda_{\min}(F_i)}[\frac{M_n^{(2)}}{d(1-\rho)}]^{\frac{1}{\beta}}} = \hat{\Upsilon}_2^i$ and $|\xi_p^i| \leq \sqrt{2[\frac{M_n^{(2)}}{d(1-\rho)}]^{\frac{1}{\beta}}} = \check{\Upsilon}_2^i$. Thus, $|x_2^i| \leq k_2^i(t) + \hat{\Upsilon}_2^i + \check{\Upsilon}_2^i + \breve{\alpha}_1^i$. Choose $k_2^i(t) = \bar{k}_2^i(t) - \hat{\Upsilon}_2^i - \check{\Upsilon}_2^i - \breve{\alpha}_1^i$, one gets $|x_2^i| < \bar{k}_2^i(t)$, thus state x_2^i do not exceed the constraint boundary. Similar to the deduction of x_2^i, and choosing $k_p^i(t) = \bar{k}_p^i(t) - \hat{\Upsilon}_p^i - \check{\Upsilon}_p^i - \breve{\alpha}_{p-1}^i$, we have $|x_p^i| < \bar{k}_p^i(t)$, $p = 3, \ldots, \ell, \forall\, t > 0$.

Therefore, the constrained partial states do not exceed the preset boundary, that is, $|x_p^i| < \bar{k}_p^i(t)$, $p = 1, 2, \ldots, \ell, \forall\, t > 0$.

(iii) Since $e_p^i = \hat{x}_p^i - \bar{\alpha}_p^i = x_p^i - \tilde{x}_p^i - \xi_p^i - \alpha_{p-1}^i$, $p = \ell + 1, \ldots, n$, thus the unconstrained state x_p^i satisfy $|x_p^i| \leq |e_p^i| + |\tilde{x}_p^i| + |\xi_p^i| + |\alpha_{p-1}^i|$. Similar to the deduction in (ii), we have e_p^i, $\tilde{x}_p^i$, ξ_p^i and α_{p-1}^i are bounded, thus x_p^i is bounded. That is, the unconstrained states are bounded. Since $|\hat{x}_p^i| \leq |x_p^i| + |\tilde{x}_p^i| \leq \bar{k}_p^i(t) + \hat{\Upsilon}_p^i$, $p = 1, \ldots, n$, thus observer state $\hat{x}_p^i$ is bounded. Since $\|\hat{\theta}_p^i\|_2 \leq \|\theta_p^i\|_2 + \|\tilde{\theta}_p^i\|_2 \leq \|\theta_p^i\|_2 + \sqrt{2\varsigma_p^i[\frac{M_n^{(2)}}{d(1-\rho)}]^{\frac{1}{\beta}}}$, $p = 1, \ldots, n$, and θ_p^i is a constant vector, thus $\hat{\theta}_p^i$ is bounded. Since $|\bar{\alpha}_p^i| \leq |\xi_p^i| + |\alpha_{p-1}^i| \leq \sqrt{2[\frac{M_n^{(2)}}{d(1-\rho)}]^{\frac{1}{\beta}}} + |\alpha_{p-1}^i|$, $p = 2, \ldots, n$, and α_{p-1}^i is bounded, thus $\bar{\alpha}_p^i$ is bounded. Since $|\hat{\hbar}_i| \leq |\hbar_i| + |\tilde{\hbar}_i| \leq |\hbar_i| + \sqrt{\frac{r_i}{\varrho_i}[\frac{M_n^{(2)}}{d(1-\rho)}]^{\frac{1}{\beta}}}$ and $\hbar_i$ is a positive constant, thus $\hat{\hbar}_i$ is bounded.

Therefore, (iii) is guaranteed.

(iv) Now, we prove that the inter-event time $t_{k+1}^i - t_k^i \geq \check{t}^i > 0$, $\forall\, k \in \mathcal{Z}^+$. For $t \in [t_k^i, t_{k+1}^i)$,

$$\frac{d}{dt}|\Theta_i| = \frac{d}{dt}(\Theta_i \times \Theta_i)^{\frac{1}{2}} = \mathrm{sign}(\Theta_i)\dot{\Theta}_i \leq |\dot{\tau}_i|. \tag{143}$$

From (92), one gets

$$\dot{\tau}_i = -(1+q_i)\left[\dot{\alpha}_n^i \hat{\hbar}_i \tanh\left(\frac{e_n^i \alpha_n^i \hat{\hbar}_i}{\epsilon_i}\right) + \frac{\alpha_n^i \hat{\hbar}_i}{\epsilon_i} \frac{\left(\dot{e}_n^i \alpha_n^i \hat{\hbar}_i + e_n^i \dot{\alpha}_n^i \hat{\hbar}_i + e_n^i \alpha_n^i \dot{\hat{\hbar}}_i\right)}{\cosh^2\left(\frac{e_n^i \alpha_n^i \hat{\hbar}_i}{\epsilon_i}\right)} \right.$$
$$\left. + \alpha_n^i \dot{\hat{\hbar}}_i \tanh\left(\frac{e_n^i \alpha_n^i \hat{\hbar}_i}{\epsilon_i}\right) + \frac{h_i^2}{(1-q_i)^2 \epsilon_i} \frac{\dot{e}_n^i}{\cosh^2\left(\frac{h_i e_n^i}{(1-q_i)\epsilon_i}\right)}\right]. \tag{144}$$

From (144) and Theorem 1, one gets $\dot{\tau}_i$ is bounded and continuous. Thus, $|\dot{\tau}_i| \leq \breve{\tau}$ with $\breve{\tau}$ being some positive constant. Since $\lim_{t \to t_{k+1}^i} \Theta_i(t) = q_i|u_i(t)| + h_i$ and $\Theta_i(t_k^i) = 0$, thus, the lower bound $\check{t}^i$ satisfies $\check{t}^i \geq \frac{q_i|u_i(t)| + h_i}{\breve{\tau}} > 0$.

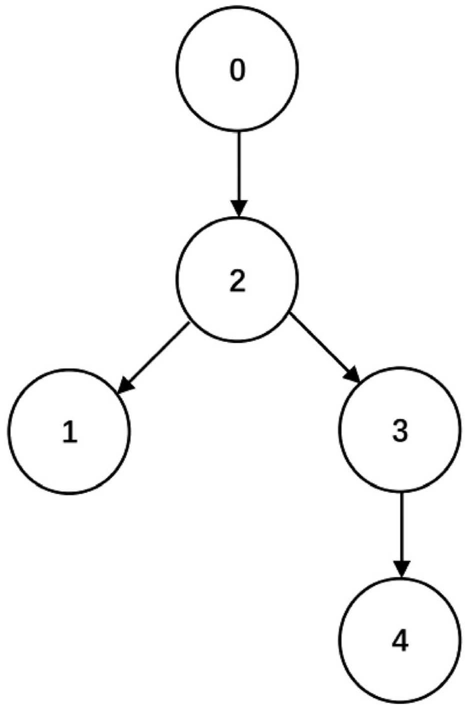

FIGURE 1 Communication graph

As a consequence, the Zeno behavior can be avoided successfully. □

Remark 5. The stability of the closed-loop system (123) is guaranteed by choosing appropriate parameters. From (123), larger d and smaller $M_n^{(2)}$ can make the system have a good performance. According to (115), smaller parameters $\varsigma_1^i, \varsigma_p^i, \bar{\varepsilon}_p^i, \varpi_1^j$ and r_i, and larger parameters $c_1^i, c_p^i, \delta_1^i, \delta_p^i, \gamma_1^j, J$ and $\iota_i \varrho_i$, $i = 1, \ldots, N$, $p = 2, \ldots, n$, can accelerate the convergence of the system, but may result in larger error bounds. As a result, it is important to acquire a balance between tolerable errors and convergence rate.

6 | SIMULATIONS

In this section, the validity of our theoretical results is verified by two examples.

Example 1:

The dynamic equation of four agents are described by

$$\begin{cases} \dot{x}_1^j = x_2^j + (x_1^j)^2 \sin(x_1^j) + 0.1\cos(2t), \\ \dot{x}_2^j = \mathrm{sat}_i(u_i) + [1 + (x_1^j)^2](x_2^j)^2 + 0.1\sin(t), \\ y^j = x_1^j, i = 1, \ldots, 4. \end{cases} \tag{145}$$

The leader signal is given as $y_r(t) = 0.5\sin(t)$. Set $\hat{x}_1^j(0) = 0.8$, $\hat{x}_2^j(0) = 0.1$, $x_1^j(0) = 0.3$, the other initial values are set to zeros. The interconnected relationships of a leader and four followers are shown in Figure 1. Figure 1 shows that a directed graph concludes a DST satisfying Assumption 1. The corresponding adjacency matrix, in-degree matrix and the Laplacian matrix of

Figure 1 are

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \mathcal{D} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{146}$$

and

$$\Xi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}, \tag{147}$$

respectively.

The fuzzy membership functions are given as

$$\mu_{i,F_1^{\iota}}(\hat{x}_1^j) = \exp\left[-\frac{(\hat{x}_1^j + \iota)^2}{4}\right], \tag{148}$$

and

$$\mu_{i,F_2^{\iota}}(\hat{x}_1^j, \hat{x}_2^j) = \exp\left[-\frac{(\hat{x}_1^j + \iota)^2}{4}\right] \times \exp\left[-\frac{(\hat{x}_2^j + \iota)^2}{4}\right], \tag{149}$$

where $i = 1, \dots, 4$, $\iota = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$.

The parameters are chosen as $\beta = \frac{99}{101}$, $c_1^i = 9$, $c_2^i = 16$, $l_1^i = 30$, $l_2^i = 80$, $\delta_1^i = \delta_2^i = 0.01$, $\varsigma_1^i = \varsigma_2^i = 0.01$, $r_i = 0.5$, $\iota_i = 2$, $q_i = 0.5$, $b_i = 0.5$, for $i = 1, \dots, 4$, $u_M^1 = u_M^2 = u_M^3 = 4$ and $u_M^4 = 5$. The time-varying boundary functions of the constrained local consensus errors e_1^i are given as $k_1^i(t) = 2e^{-t} + 1$, for $i = 1, \dots, 4$. The time-varying boundary functions of the constrained states x_1^1, x_1^2, x_1^3 and x_1^4 are chosen as $\bar{k}_1^1(t) = 4e^{-t} + 2$, $\bar{k}_1^2(t) = 2e^{-t} + 1$, $\bar{k}_1^3(t) = 4e^{-t} + 2$ and $\bar{k}_1^4(t) = 6e^{-t} + 3$, respectively.

Figures 2–12 show the simulation results. The responses of the output y^i, $i = 1, \dots, 4$ and the reference signal $y_r(t)$ are shown in Figure 2. It is clear that the finite-time convergence of tracking error to a small neighborhood around the origin is guaranteed. The curves of the local consensus errors e_1^i, $i = 1, \dots, 4$ are shown in Figure 3, which are constrained within time-varying boundaries. From Figure 3, we can see that the local consensus errors do not exceed the preset boundary functions. The trajectories of the states $x_1^j, \hat{x}_1^j, x_2^j, \hat{x}_2^j$, $i = 1, \dots, 4$ and the observer errors $\tilde{x}_1^j, \tilde{x}_2^j$, $i = 1, \dots, 4$ are shown in Figures 4–7. It can be seen from Figures 4–7 that a small observer errors is achieved as time evolving and the constrained outputs $x_1^j, \hat{x}_1^j$ do not exceed the time-varying boundary. Figures 8–11 show the trajectories of the saturation controller $\mathrm{sat}_i(u_i)$ and controller u_i, $i = 1, \dots, 4$. From the enlarged view of Figures 8–11, it can be seen that the saturation control input does not exceed saturation boundary. The inter-event time $t_{k+1}^i - t_k^i$, $i = 1, \dots, 4$ of the ith

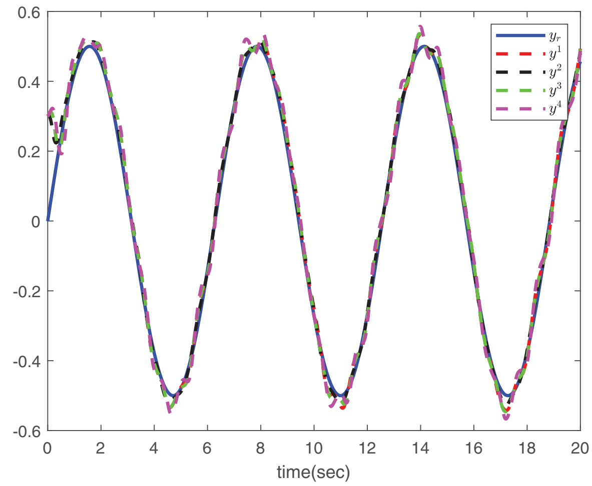

FIGURE 2 The trajectories of y^i and y_r in (145) ($i = 1, \dots, 4$)

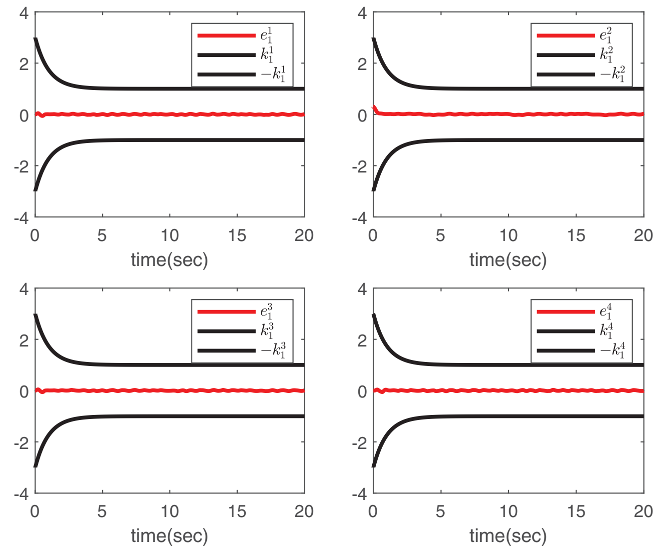

FIGURE 3 The trajectories of the errors e_1^i in Example 1 ($i = 1, \dots, 4$)

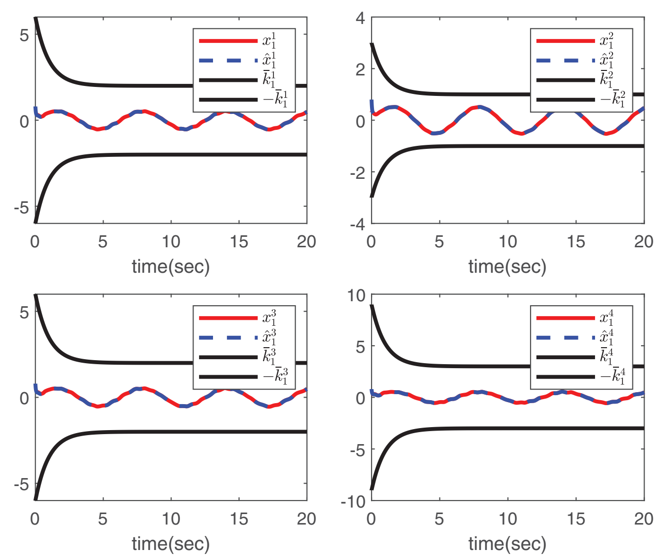

FIGURE 4 The trajectories of x_1^i and $\hat{x}_1^i$ in Example 1 ($i = 1, \dots, 4$)

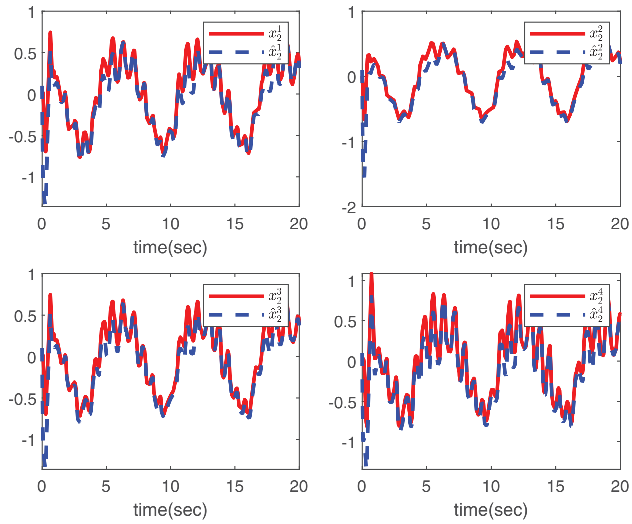

FIGURE 5 The trajectories of x_2^j and $\hat{x}_2^j$ in Example 1 ($i = 1, \ldots, 4$)

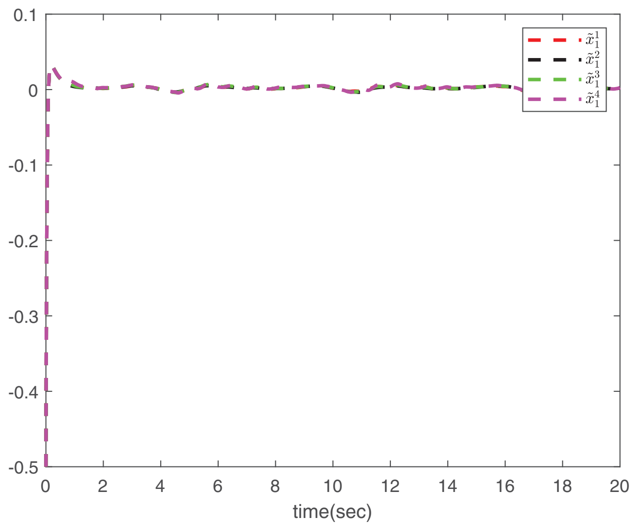

FIGURE 6 The observer errors $\tilde{x}_1^j$ in Example 1 ($i = 1, \ldots, 4$)

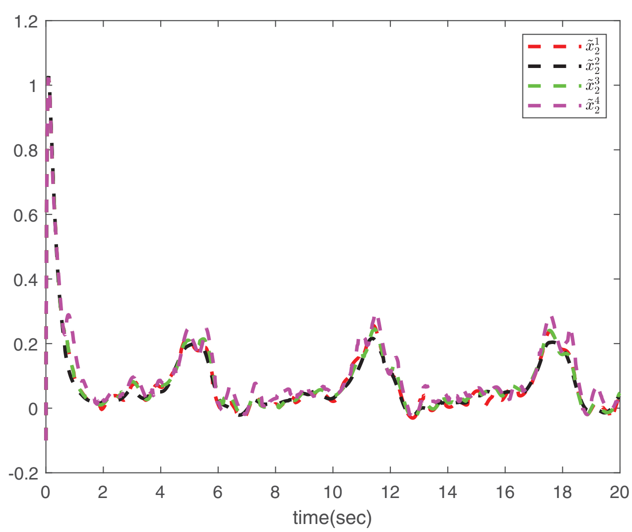

FIGURE 7 The observer errors $\tilde{x}_2^j$ in Example 1 ($i = 1, \ldots, 4$)

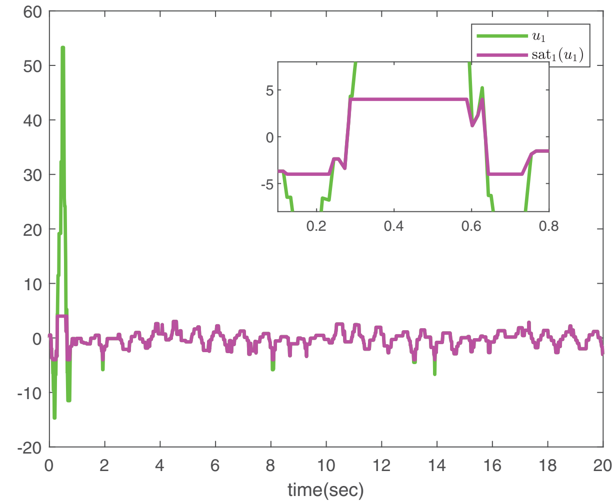

FIGURE 8 The trajectories of the saturation controller $\mathrm{sat}_1(u_1)$ and controller u_1 in Example 1

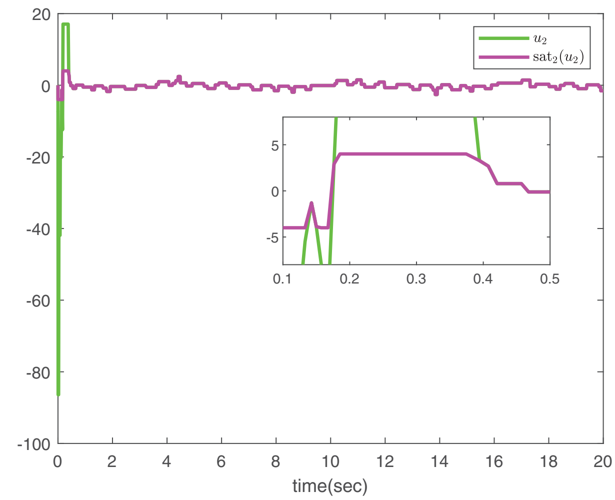

FIGURE 9 The trajectories of the saturation controller $\mathrm{sat}_2(u_2)$ and controller u_2 in Example 1

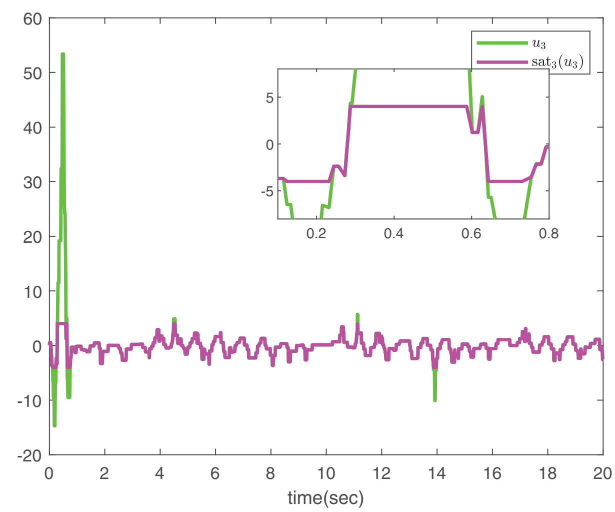

FIGURE 10 The trajectories of the saturation controller $\mathrm{sat}_3(u_3)$ and controller u_3 in Example 1

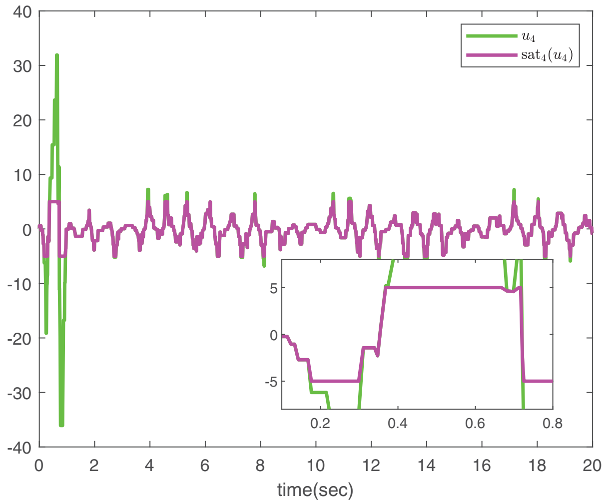

FIGURE 11 The trajectories of the saturation controller $\mathrm{sat}_4(u_4)$ and controller u_4 in Example 1

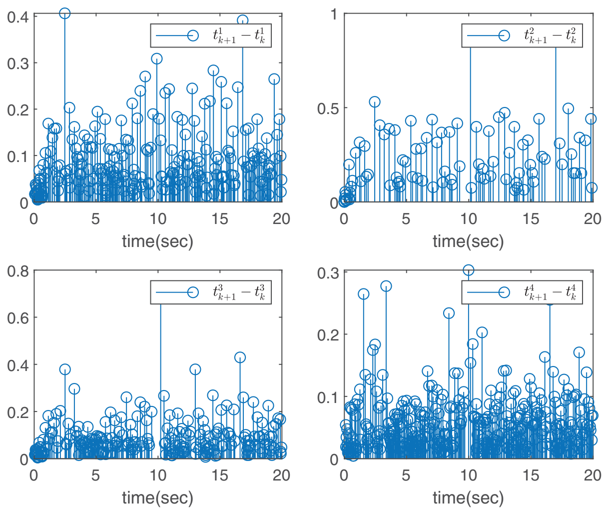

FIGURE 12 The inter-event time $t^i_{k+1} - t^i_k$ in Example 1 ($i = 1, \dots, 4$)

agents are displayed in Figure 12. It is clear that the ETC scheme only updates when an event is triggered and can avoid the occurrence of the Zeno behavior. In this manner, the proposed ETC scheme can save communication resources.

Example 2: A practical application example is given to verify the validity of our theoretical results. Consider four one-link manipulators with motor described by

$$\begin{cases} D_i\ddot{\hbar}_i + I_i\dot{\hbar}_i + N_i\sin(\hbar_i) = \vartheta_i + \vartheta_{id}, \\ K_i\dot{\vartheta}_i + P_i\vartheta_i = \mathrm{sat}_i(u_i) - J_i\dot{\hbar}_i, \end{cases} \tag{150}$$

where $\hbar$, $\dot{\hbar}_i$ and $\ddot{\hbar}_i$ are the link position, velocity and acceleration, respectively. ϑ_i and ϑ_{id} are the torque and torque disturbance, respectively. The parameters are set as D_i=1 kg m^2, I_i=1 Nm s/rad, $N_i = 10$, $K_i = 1$, $P_i = 1\Omega$, J_i=0.2 Nm A and $\vartheta_{id} = 0.1\sin(10t)$, for $i = 1, \dots, 4$.

Let $x^j_1 = \hbar_i$, $x^j_2 = \dot{\hbar}_i$, and $x^j_3 = \vartheta_i$, the above systems become

$$\begin{cases} \dot{x}^j_1 = x^j_2, \\ \dot{x}^j_2 = \frac{1}{D_i}x^j_3 - \frac{I_i}{D_i}x^j_2 - \frac{N_i}{D_i}\sin\left(x^j_1\right) + \frac{1}{D_i}\vartheta_{id}, \\ \dot{x}^j_3 = \frac{1}{K_i}\mathrm{sat}_i(u_i) - \frac{J_i}{K_i}x^j_2 - \frac{P_i}{K_i}x^j_3. \end{cases} \tag{151}$$

The reference signal is given as $y_r(t) = -\cos(1.5t) + \sin(0.5t)$. Set all initial states to zeros except for $\hat{x}^j_1(0) = -0.9$, $\hat{x}^j_2(0) = 0.5$ and $x^j_2(0) = 0.6$.

The fuzzy membership functions are selected as

$$\mu_{i,F^t_1}(\hat{x}^j_1) = \exp\left[-\frac{(\hat{x}^j_1 + \iota)^2}{16}\right], \tag{152}$$

$$\mu_{i,F^t_2}(\hat{x}^j_1, \hat{x}^j_2) = \prod_{p=1}^{2}\exp\left[-\frac{(\hat{x}^j_p + \iota)^2}{16}\right], \tag{153}$$

and

$$\mu_{i,F^t_3}(\hat{x}^j_1, \hat{x}^j_2, \hat{x}^j_3) = \prod_{p=1}^{3}\exp\left[-\frac{(\hat{x}^j_p + \iota)^2}{16}\right], \tag{154}$$

where $i = 1, \dots, 4$, $\iota = -4, -3, -2, -1, 0, 1, 2, 3, 4$.

The parameters are defined as $\beta = \frac{99}{101}$, $c^i_1 = 3$, $c^i_2 = 5$, $c^i_3 = 5$, $l^i_1 = 3$, $l^i_2 = 3$, $l^1_3 = l^3_3 = l^4_3 = 80$, $l^2_3 = 60$, $\delta^i_1 = \delta^i_2 = \delta^i_3 = 0.01$, $\varsigma^i_1 = \varsigma^i_2 = \varsigma^i_3 = 0.01$, $r_i = 0.5$, $\iota_i = 5$, $q_i = 0.1$, $b_i = 0.5$ and $u^i_M = 40$, for $i = 1, \dots, 4$. The time-varying boundary functions of the constrained local consensus errors e^i_1 are given as $k^i_1(t) = 0.5e^{-t} + 4$, for $i = 1, \dots, 4$. The time-varying boundary functions of the constrained states x^1_1, x^2_1, x^3_1, x^4_1 and x^i_2 are given as $\bar{k}^1_1(t) = e^{-t} + 8$, $\bar{k}^2_1(t) = 0.5e^{-t} + 4$, $\bar{k}^3_1(t) = e^{-t} + 8$, $\bar{k}^4_1(t) = 1.5e^{-t} + 12$ and $\bar{k}^i_2(t) = 2e^{-t} + 8.1$, for $i = 1, \dots, 4$, respectively.

Figures 13–22 show the simulation results. The curves of the output y^i, $i = 1, \dots, 4$ and the reference signal $y_r(t)$ are depicted in Figure 13. The curves of the local consensus errors e^i_1, $i = 1, \dots, 4$ are shown in Figure 14, which are constrained within time-varying boundaries. From Figures 13 to 14, it can be seen that the system outputs y^i can converge to the given reference signal $y_r(t)$ with bounded errors. Figures 15–17 show the trajectories of the states x^i_1, x^i_2, x^i_3 and the corresponding observer states $\hat{x}^i_1$, $\hat{x}^i_2$, $\hat{x}^i_3$, $i = 1, \dots, 4$. It is obvious that the constrained states x^i_1, $\hat{x}^i_1$, x^i_2, $\hat{x}^i_2$ do not exceed the constraint boundary and the state observer works well. Figures 18–21 exhibit the trajectories of the saturation controller $\mathrm{sat}_i(u_i)$ and controller u_i, $i = 1, \dots, 4$. As you can see from Figures 18 to 21 that the saturation controllers work well. The inter-event time $t^i_{k+1} - t^i_k$, $i = 1, \dots, 4$ are shown in Figure 22. As you can see from Figure 22 that the ETC mechanism works well and the Zeno behavior does not occur.

The simulation results of the two examples demonstrate the effectiveness of the algorithm proposed in this paper.

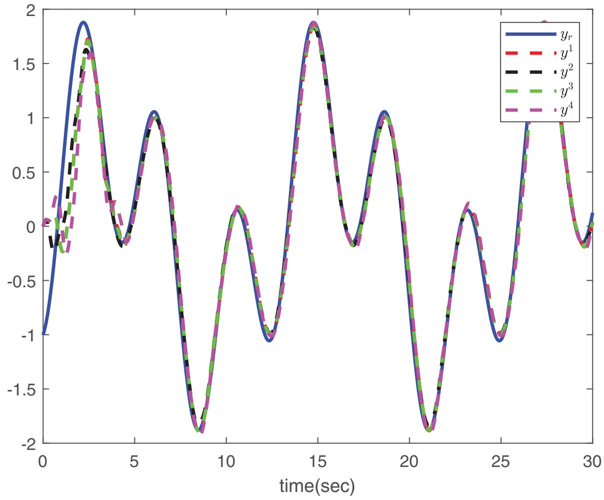

FIGURE 13 The trajectories of y^i and y_r in (151) $(i = 1, \dots, 4)$

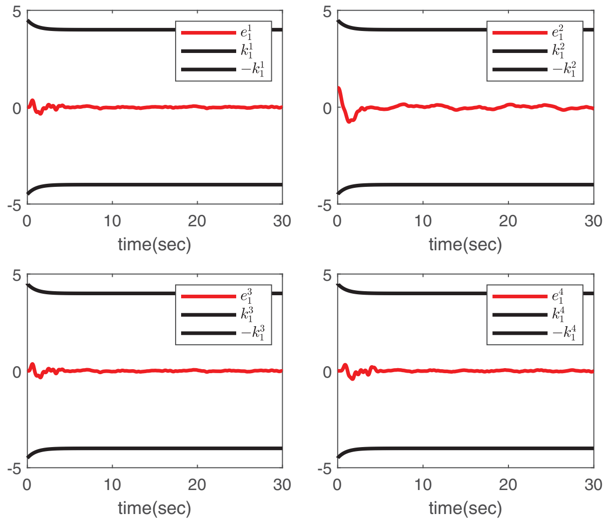

FIGURE 14 The trajectories of the errors e_1^i in Example 2 $(i = 1, \dots, 4)$

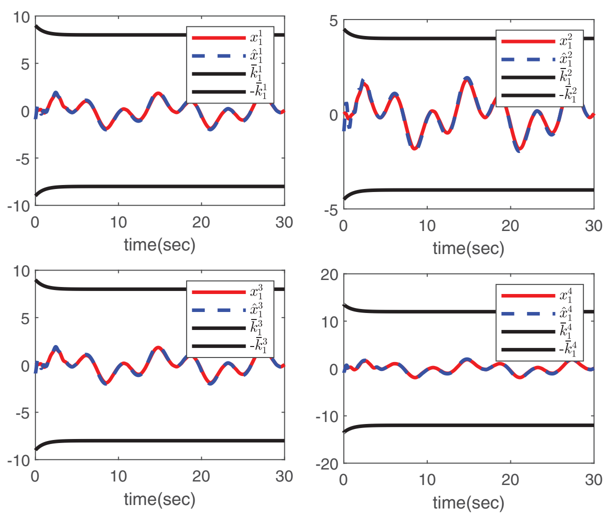

FIGURE 15 The trajectories of x_1^i and $\hat{x}_1^i$ in Example 2 $(i = 1, \dots, 4)$

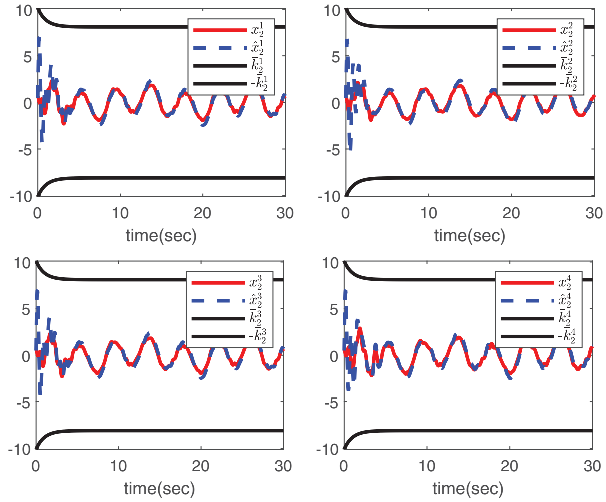

FIGURE 16 The trajectories of x_2^i and $\hat{x}_2^i$ in Example 2 $(i = 1, \dots, 4)$

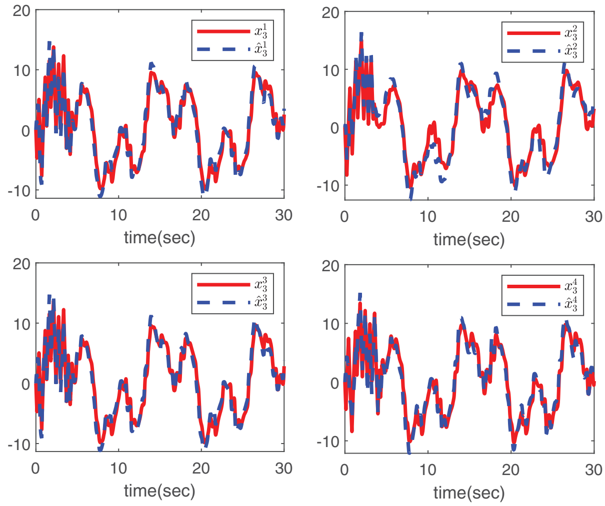

FIGURE 17 The trajectories of x_3^i and $\hat{x}_3^i$ in Example 2 $(i = 1, \dots, 4)$

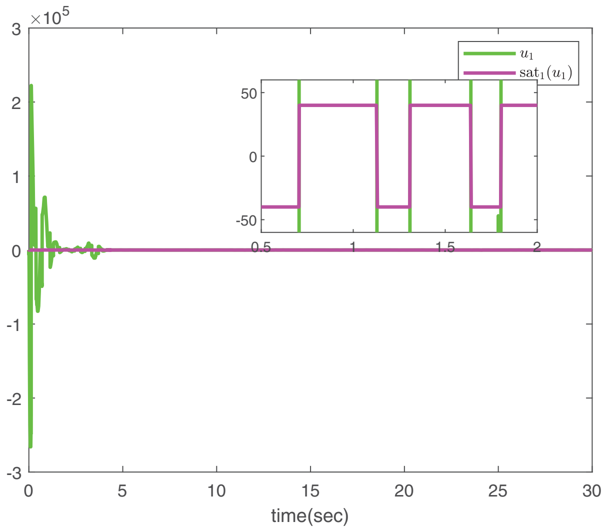

FIGURE 18 The trajectories of the saturation controller $\mathrm{sat}_1(u_1)$ and controller u_1 in Example 2

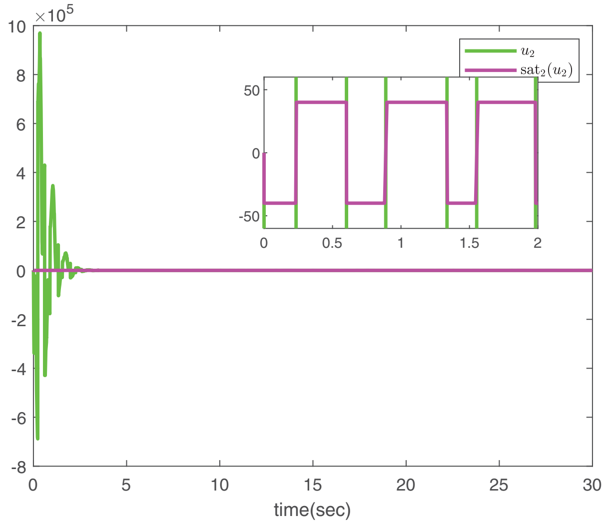

FIGURE 19 The trajectories of the saturation controller $\mathrm{sat}_2(u_2)$ and controller u_2 in Example 2

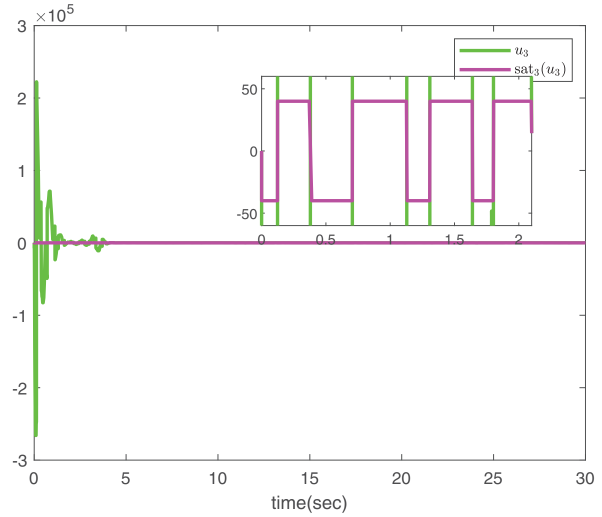

FIGURE 20 The trajectories of the saturation controller $\mathrm{sat}_3(u_3)$ and controller u_3 in Example 2

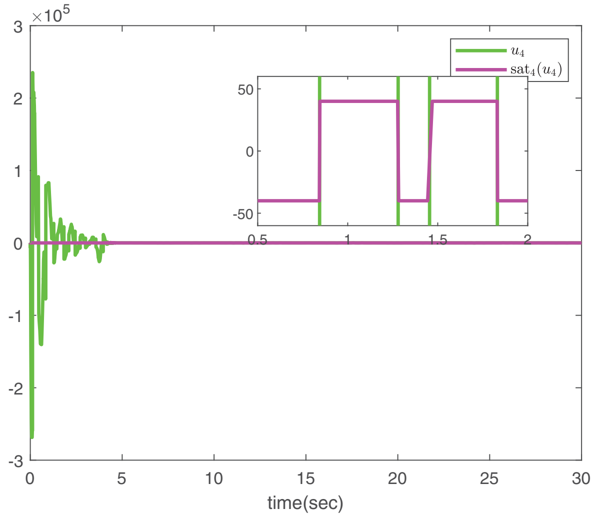

FIGURE 21 The trajectories of the saturation controller $\mathrm{sat}_4(u_4)$ and controller u_4 in Example 2

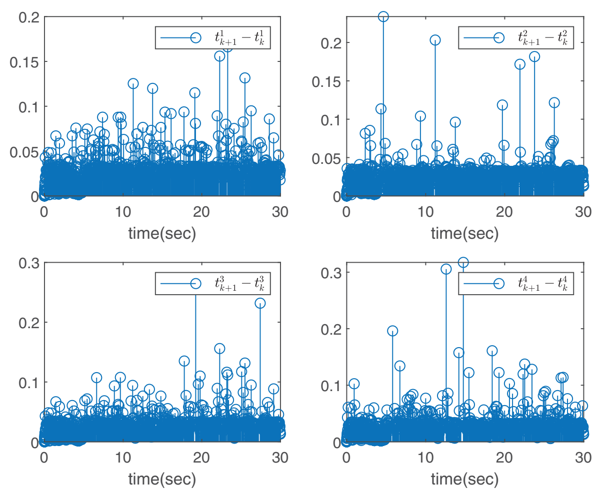

FIGURE 22 The inter-event time $t^i_{k+1} - t^i_k$ in Example 2 ($i = 1, \ldots, 4$)

7 | CONCLUSION

This work mainly investigates the finite-time ETC problem for MASs with input saturation, unknown nonlinear dynamics, external disturbances and partial state constraints via output feedback. FLS and fuzzy state observer are utilized to approximate the uncertain nonlinearities and obtain the unmeasured states, respectively. The BLF is introduced to deal with PSCs and ensure that all states will not exceed the preset boundary values. A fuzzy adaptive tracking controller is proposed by using backstepping technique with a filter. In order to save communication resource, the ETC strategy is also considered, which can avoid the occurrence of the Zeno behavior. By stability analysis, the presented ETC mechanism guarantees the finite-time convergence of tracking error to a small neighborhood of the origin, partial states can be constrained within preset time-varying boundaries and all resulting system signals are bounded. Finally, the simulation results verify the rationality of the theoretical results. In the future, the above problem for stochastic MASs will be considered.

AUTHOR CONTRIBUTIONS

Xiaoting Huang: Investigation, methodology, software, writing - original draft. Hui Yu: Conceptualization, resources, supervision, visualization. Xiaohua Xia: Writing - review and editing.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable - no new data generated, or the article describes entirely theoretical research

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