Measurement of pile uplift forces due to soil heave in expansive clays: Supplementary material

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Abstract

Xiao et al. (2011) and Fan et al. (2007) developed an analytical method to calculate the tension developed in a pile in a heaving clay. The method uses the movement of the soil against the pile to determine the axial force in the pile. The coefficients indicated in the solution for the pile displacement, and consequently the tension in the pile are not consistent with between the two publications, and furthermore, neither results in the ability

to reproduce the results in the figures and parametric studies presented in these papers. This supplementary information presents the recalculation of these coefficients.

1 Introduction

Free field heave is used to calculate the modified soil displacement at the pile-soil interface; this is influenced by the radius and length of the pile, r_0 and L respectively, and the shear modulus and Poisson ratio of the soil, G_s and ν respectively.

Assuming that there is no slip between the soil and the pile, the pile displacement, $w(z)$ is set to equal the soil displacement at the interface. Using the relationship between pile displacement and axial force, P, (i.e. $P/(-EA) = w'(z)$) and calculating the shaft friction on the pile (i.e. $P'(z)$) and setting this to equal the induced friction by the soil movement, and the following differential equation is set up:

$$\text{(1)} \quad \frac{d^2w(z)}{dz^2} - \alpha^2 w(z) = \alpha^2 s(z)$$

Where $\alpha^2 = 2\pi/\lambda_p A_p \zeta$ [$\zeta = \ln(r_m/r_0)$; $r_m = 2.5L(1-\nu)$; $\lambda_p = E_p/G_s$] and $s(z)$ is the heave profile with depth. The sign convention is that upward movement of the soil or pile is negative, and compression in the pile is positive.

Assuming that the soil heave profile varies linearly from s_0 at the surface, to 0 at the base of the active layer at a depth h_0 [Note: h_0 is used in place of H in the main paper for consistency with Xiao et al. (2011) and Fan et al. (2007)], the following differential equations are determined:

$$\text{(2)} \quad \begin{aligned} &\frac{d^2w_1(z)}{dz^2} - \alpha^2 w_1(z) = \alpha^2 \frac{s_0(h_0 - z)}{h_0}; \qquad && 0 \le z \le h_0 \\ &\frac{d^2w_1(z)}{dz^2} - \alpha^2 w_2(z) = 0; && h_0 \le z \le L \end{aligned}$$

The solutions for the displacement of the pile and tension developed in the piles are presented below [Note: C_3, C_4, C_5, and C_6 are used to replace C_1, C_2, C_3, and C_4 in the paper respectively for consistency with Xiao et al.

(2011) and Fan et al. (2007)]:

$$
(3)\quad \begin{aligned}
w_1(z) =& C_3 \sinh(\alpha z) + C_4 \cosh(\alpha z) - \frac{s_0(h_0 - z)}{h_0}; \quad && 0 \le z \le h_0 \\
w_2(z) =& C_5 \sinh(\alpha z) + C_6 \cosh(\alpha z); && h_0 \le z \le L
\end{aligned}
$$

$$
(4)\quad \begin{aligned}
&P_1(z) = -E_p A_p \left(\alpha C_3 \cosh(\alpha z) + \alpha C_4 \sinh(\alpha z) + \frac{s_0}{h_0} \right); \\
&\qquad\qquad\qquad\qquad 0 \le z \le h_0 \\
&P_2(z) = -E_p A_p \left(\alpha C_5 \cosh(\alpha z) + \alpha C_6 \sinh(\alpha z) \right); \\
&\qquad\qquad\qquad\qquad h_0 \le z \le L
\end{aligned}
$$

The solutions for the coefficients in the above equation are different in Fan et al. (2007) and Xiao et al. (2011), and do not result in the ability to reproduce the results shown in the figures and parametric analysis in these papers. These solutions are presented in the table below. The calculations for the boundary conditions (BC) of the problem were used to provide the correct calculation of these coefficients; these are also included in the table, and the derivations are included in detail below the table. It appears that the solutions presented by Fan et al. (2007) were more correct and had an apparent error in the display of the equation for C_4. The coefficients derived below were successfully used to replicate the figures in the indicated papers.

	Fan et al. (2007)	Xiao et al. (2011)	Current derivation
C_3	$-\frac{s_0}{\alpha h_0}$	$-\frac{s_0}{\alpha h_0}$	$-\frac{s_0}{\alpha h_0}$
C_4	$C_6 - \frac{s_0 \sinh(\alpha h_0)}{\alpha h_0}$	$\frac{C_6 - s_0 \sinh(\alpha h_0)}{\alpha h_0}$	$C_6 + \frac{s_0 \sinh(\alpha h_0)}{\alpha h_0}$
C_5	$C_3 + \frac{s_0 \cosh(\alpha h_0)}{\alpha h_0}$	$\frac{C_3 + s_0 \cosh(\alpha h_0)}{\alpha h_0}$	$C_3 + \frac{s_0 \cosh(\alpha h_0)}{\alpha h_0}$
C_6	$\frac{-s_0 \cosh(\alpha L)(\cosh(\alpha h_0) - 1)}{\alpha h_0 \sinh(\alpha L)}$	$\frac{-s_0 \cosh(\alpha L)(\cosh(\alpha h_0) - 1)}{\alpha h_0 \sinh(\alpha L)}$	$-\frac{\cosh(\alpha L)}{\sinh(\alpha L)} C_5 = \frac{-s_0 \cosh(\alpha L)(\cosh(\alpha h_0) - 1)}{\alpha h_0 \sinh(\alpha L)}$

The strain is given by:

$$w_1'(z) = \alpha C_3 \cosh(\alpha z) + \alpha C_4 \sinh(\alpha z) + \frac{s_0}{h_0}; \quad 0 \le z \le h_0$$
$$w_2'(z) = \alpha C_5 \cosh(\alpha z) + \alpha C_6 \sinh(\alpha z); \qquad h_0 \le z \le L$$

BC1: Axial force (and thus strain) at surface is zero: $w_1'(0) = 0$

$$w_1'(0) = \alpha C_3 \cosh(0) + \alpha C_4 \sinh(0) + \frac{s_0}{h_0} = 0$$
$$\alpha C_3 + \frac{s_0}{h_0} = 0$$
$$C_3 = -\frac{s_0}{\alpha h_0}$$

BC2: Axial force (and thus strain) at boundary is equal: $w_1'(h_0) = w_2'(h_0)$

$$w_1'(h_0) = \alpha C_3 \cosh(\alpha h_0) + \alpha C_4 \sinh(\alpha h_0) + \frac{s_0}{h_0}$$
$$w_2'(h_0) = \alpha C_5 \cosh(\alpha h_0) + \alpha C_6 \sinh(\alpha h_0)$$
$$C_3 \cosh(\alpha h_0) + C_4 \sinh(\alpha h_0) + \frac{s_0}{h_0 \alpha} =$$
$$C_5 \cosh(\alpha h_0) + C_6 \sinh(\alpha h_0)$$
$$\therefore (C_3 - C_5) \cosh(\alpha h_0) + \frac{s_0}{h_0 \alpha} = (C_6 - C_4) \sinh(\alpha h_0)$$

BC3: Displacement at boundary is equal: $w_1(h_0) = w_2(h_0)$

$$w_1(h_0) = C_3 \sinh(\alpha h_0) + C_4 \cosh(\alpha h_0) - \frac{s_0(h_0 - h_0)}{h_0}$$
$$w_2(h_0) = C_5 \sinh(\alpha h_0) + C_6 \cosh(\alpha h_0)$$
$$C_3 \sinh(\alpha h_0) + C_4 \cosh(\alpha h_0) + \frac{s_0}{h_0} =$$
$$C_5 \sinh(\alpha h_0) + C_6 \cosh(\alpha h_0)$$
$$\therefore (C_3 - C_5) \sinh(\alpha h_0) = (C_6 - C_4) \cosh(\alpha h_0)$$

BC4: Axial force (and thus strain) at the base is zero: $w_L'(0) = 0$

$$w_2'(L) = \alpha C_5 \cosh(\alpha L) + \alpha C_6 \sinh(\alpha L) = 0$$
$$C_6 = -\frac{\cosh(\alpha L)}{\sinh(\alpha L)} C_5$$

Solving for coefficients C_4 and C_5:

$$
\begin{aligned}
(C_3 - C_5) =& \frac{(C_6 - C_4)\sinh(\alpha h_0) - \frac{s_0}{\alpha h_0}}{\cosh(\alpha h_0)} \quad [BC2] \\
(C_3 - C_5) =& \frac{(C_6 - C_4)\cosh(\alpha h_0)}{\sinh(\alpha h_0)} \quad [BC3] \\
\therefore (C_6 - C_4)\sinh^2(\alpha h_0) - \frac{s_0}{\alpha h_0}\sinh(\alpha h_0) &= (C_6 - C_4)\cosh^2(\alpha h_0) \\
(C_6 - C_4)(\sinh^2(\alpha h_0) - \cosh^2(\alpha h_0)) &= \frac{s_0}{\alpha h_0}\sinh(\alpha h_0) \\
(C_6 - C_4)(-1) &= \frac{s_0}{\alpha h_0}\sinh(\alpha h_0) \\
C_4 - C_6 &= \frac{s_0}{\alpha h_0}\sinh(\alpha h_0) \\
C_4 &= C_6 + \frac{s_0}{\alpha h_0}\sinh(\alpha h_0)
\end{aligned}
$$

$$
\begin{aligned}
(C_6 - C_4) =& \frac{(C_3 - C_5)\cosh(\alpha h_0) + \frac{s_0}{\alpha h_0}}{\sinh(\alpha h_0)} \quad [BC2] \\
(C_6 - C_4) =& \frac{(C_3 - C_5)\sinh(\alpha h_0)}{\cosh(\alpha h_0)} \quad [BC3] \\
\therefore (C_3 - C_5)\cosh^2(\alpha h_0) + \frac{s_0}{\alpha h_0}\cosh(\alpha h_0) &= (C_3 - C_5)\sinh^2(\alpha h_0) \\
(C_3 - C_5)(\cosh^2(\alpha h_0) - \sinh^2(\alpha h_0)) &= -\frac{s_0}{\alpha h_0}\cosh(\alpha h_0) \\
C_3 - C_5 &= -\frac{s_0}{\alpha h_0}\cosh(\alpha h_0) \\
C_5 &= C_3 + \frac{s_0}{\alpha h_0}\cosh(\alpha h_0)
\end{aligned}
$$

References

Fan, Z.h., Wang, Y.h., Xiao, H.b., Zhang, C.s., 2007. Analytical method of load-transfer of single pile under expansive soil swelling. Journal of Central South University of Technology 14, 575–579.

Xiao, H.b., Zhang, C.s., Wang, Y.h., Fan, Z.h., 2011. Pile-soil interaction in expansive soil foundation: Analytical solution and numerical simulation. International journal of geomechanics 11, 159–166.

List of symbols

A	Cross sectional area of the pile
$C_{1,2,3,4}$	Coefficients for elastic pile force solution in main paper
$C_{3,4,5,6}$	Corresponding coefficients for elastic pile force solution in reference papers
E	Modulus of elasticity of the pile
G_s	Shear modulus of the soil
h_0	Thickness of active soil layer
L	Length of pile
P	Axial force in the pile
r_m	Maximum effective radius around the pile
r_0	Pile radius
$s(z)$	Soil displacement as a function of depth
$w(z)$	Pile displacement as a function of depth
z	Depth of soil from the surface
α	Simplification factor, $\alpha^2 = 2\pi/\lambda_p A_p \zeta$
ν	Poisson ratio of soil
λ_p	Pile to soil stiffness ratio, $\lambda_p = E/G_s$
ζ	Effective parameter of the pile radius, $\zeta = \ln(r_m/r_0)$