A LOT SIZING MODEL FOR TWO ITEMS WITH IMPERFECT MANUFACTURING PROCESS, TIME VARYING DEMAND AND RETURN RATES, DEPENDENT DEMAND AND DIFFERENT QUALITY GRADES

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Abstract: Natural resource scarcity has accelerated considerations of return logistics in manufacturing processes. Most supply chain designs now consider a closed-loop design where demand can be satisfied by both newly manufactured goods as well as remanufactured returns, allowing for maximum value creation over the entire life cycle of a product. This paper proposes an inventory system where returns are constrained and returns that cannot be remanufactured to an as-good-as-new state of the original product can be used to satisfy a secondary demand. It is also assumed that some items fail during manufacturing and these items are treated as returns that can be remanufactured to satisfy one of two types of demand. Returns are remanufactured to one of two states such that items that may not be remanufactured to an as-good-as-new state of the first product can satisfy a secondary customer demand at a lower grade. The remanufacturing processes require some other components to be procured in order to bring the returned items back to either of the two states of reuse. A modified Wagner/Whitin model for the alternate application of remanufacturing and manufacturing for the satisfaction of the top range item demand and supplemented by a modified reverse Wagner/Whitin model for the remanufacturing of the lower variety items is derived to solve the dynamic lot sizing model proposed in this paper. The model aims to minimize cost across a horizon and the total cost proves to be very sensitive to the manufacturing setup cost and the proportion of demand returned for remanufacturing.

Keywords: Inventory, Remanufacturing, Return logistics, Defective yield, Wagner/Whitin

1. DECLARATIONS

1.1 FUNDING

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1.2 CONFLICT OF INTEREST

The authors have no conflicts of interest to declare that are relevant to the content of this article.

1.3 AVAILABILITY OF DATA AND MATERIAL

Data used for testing was simulated based on the models studied as part of the research.

1.4 CODE AVAILABILITY

Custom code for the Wagner/Whitin algorithm written and tested in Microsoft Excel. Custom code for the linear programming model written and tested in Python. A copy of both could be made available.

1.5 AUTHORS' CONTRIBUTION

Model development and testing was done by Anneri van Zyl under the supervision of Olufemi Adetunji. The first draft of the manuscript was prepared by Anneri van Zyl, after which review and editing was done by Olufemi Adetunji and Anneri van Zyl.

2. INTRODUCTION

Reverse logistics is seen as an environmentally friendly way to deal with products at the end of their life span and has attracted an increasing amount of attention in the last couple of years (Zhalechian et al., 2016; Govindan and Soleimani, 2017; Rajeev et al., 2017; Govindan and Bouzon, 2018). Integrating the remanufacturing process into the manufacturing process is seen as an opportunity to improve profits and confirm sustainability by many companies (Wei and Zhao, 2014). Although traditional production planning and inventory management models do not typically consider the return and remanufacturing of an item to satisfy the same demand, remanufacturing of end-of-life products to an as-good-as-new state has attracted considerable attention in recent years (Jiang et al., 2016; Lee et al., 2017; Paterson et al., 2017; Zlamparet et al., 2017; Jin et al., 2018; Lu et al., 2018). Models that consider the use of remanufactured products to satisfy demand for a different product are still relatively uncommon. The paper industry is a typical example where returned products can be remanufactured to satisfy demand of the original product as well as a secondary demand. At the start of the paper recycling process, returned paper is divided into types and grades, where the higher-grade returns can be remanufactured to the same high-grade product such as office paper and lower grade returns are remanufactured to produce a lower grade paper product such as newsprint, tissues, toilet paper and cardboard. Returned paper is washed to remove any remaining film, glue, ink and other contaminants after which it is mixed with water to create the pulp from which recycled paper is made (Isustainrecycling.com, 2021). The objective of this paper is to develop a dynamic lot sizing model in return logistics, where two types of demand are satisfied. The first type is that of a top variety item that can be satisfied by newly manufactured items as well as remanufactured returns. The second demand type is that of a lower variety item that can be satisfied by the remanufacturing of the top variety returns that cannot be remanufactured to an as-good-as new top variety item as well as remanufactured lower variety items.

Reverse supply chain models can be broadly classified into two groups namely deterministic (Atasu and Çetinkaya, 2006; Feng et al., 2014; Konstantaras and Papachristos, 2008; Özceylan et al., 2014; Tang and Teunter, 2009; Teunter et al., 2009; Zanoni et al., 2012) and stochastic (Aras et al., 2004; Behret and Korugan, 2009; Fleischmann et al., 2002; Karaer and Lee, 2009; Timmer et al., 2013; van Donselaar and Broekmeulen, 2013; Vlachos and Dekker, 2003; Zolfagharinia and Haughton, 2012), where stochastic models can be further classified as continuous or periodically reviewed models. Akçali and Çetinkaya (2010) went even further in classifying periodic review models based on the number of stock points, dependency of return rates on demand, length of planning horizon and the consideration of lead time. Most of the models published thus far assume that demand and return rates are independent (Fleischmann and Kuik, 2003; Heisig and Fleischmann, 2001; Karaer and Lee, 2009; Kiesmüller and Scherer, 2003; Shi et al., 2011; Teunter and Vlachos, 2002). Although this assumption simplifies the model significantly, the dependency of the return rate on demand is a realistic assumption (Kim et al., 2013; Vercraene and Gayon, 2013). Kiesmüller and van der Laan (2001) studied a single stock model where the return rate is a function of product demand. The proposed problem was solved using a Markov-chain approach. Vlachos and Dekker (2003) proposed the first newsvendor model considering the probability of return and its dependency on demand. Sun et al. (2013) investigated a manufacturing and remanufacturing system where the return rate is dependent on demand and solved the proposed problem using a three-stage stochastic dynamic programming model. Schulz and Ferretti (2011) considered the meticulousness of the remanufacturing process itself, by including the disassembly process as an explicit recovery step. The remanufacturing process thus consists of two sub processes, the disassembly process in which returns are disassembled and a rework process, where items are reworked to an as-good-as-new state. Schulz and Ferretti considered random yield in components recovered from the disassembly process in conjunction with the demand dependency in the return rate. Zolfagharinia et al. (2014), was the first to explicitly consider the dependency of return rate on demand in a two-stock system where used or damaged products are returned to the original manufacturer and backordering is allowed. The two-stock system results in two types of inventories, namely serviceable and recoverable inventory. Serviceable inventory is used to satisfy demand and replenished either by a manufacturing batch or a remanufacturing batch with the assumption that remanufactured goods are as-good-as-new. Recoverable inventory is the returned goods inventory used for remanufacturing. By incorporating two stock points to separate serviceable and recoverable inventory, remanufacturing can be postponed as far as possible to take full advantage of the lower recoverable inventory holding cost. Zolfagharinia et al. used a simulation-based hybrid variable neighbourhood search to solve the proposed problem. Piñeyro and Viera (2015) considered a new theoretical approach that can be considered a generalisation of the well-known zero-inventory property. An optimized version of the existing Tabu Search algorithm was suggested to solve the problem. The model proposed in this paper takes into account that the return rate is dependent on demand in a two stock system.

The first deterministic lot sizing model was introduced by Richter and Sombrutzki (2000) as an extension to the well-known Wagner/Within model (Wagner and Whitin, 1958). Richter and Sombrutzki solved for the optimal manufacturing and remanufacturing lot size heuristically by first deriving a modified reverse Wagner/Whitin model. The model operates in a simplified environment where used products at the beginning of the decision period is sufficiently large to allow for a full remanufacturing batch. They then considered a modified Wagner/Whitin model to heuristically solve for the alternate application of manufacturing and remanufacturing. The reason for the modified heuristic is due to the weakness in the standard Wagner/Whitin model to consider two supply types. Richter and Sombrutzki made the assumption that a remanufacturing batch can be executed at any time because the returned item inventory at the start of any given period is greater than or equal to the demand in that same period. In each period, the cost of remanufacturing is weighed against the cost of manufacturing, where the less of the two drives the decision on whether to remanufacture or manufacture. The manufacturing cost considers the holding cost incurred on the returned items that will not be remanufactured in a period of manufacturing. The model was extended by Richter and Weber (2001) to consider variable costs in the manufacturing and remanufacturing processes. Dekker et al. (2004), formulated a mixed integer linear programming model where customers return used items in each period. Teunter et al. (2006) proposed two models, the first considers one setup cost for both manufacturing and remanufacturing and the second considers different setup costs for the two processes. Zouadi et al. (2014) proposed two metaheuristic-based approaches in a hybrid manufacturing and remanufacturing system. This work was later extended by Zouadi et al. (2017) to consider the returns collection and supplier selection phase. The proposed model here allows for the manufacturing and remanufacturing of the top variety item to share a resource as the end product is the same. A dedicated resource will be used in the remanufacturing of the lower variety items.

A mutual assumption across existing models is that all returned or defective items should be remanufactured to an as-good-as-new state, sold at a reduced rate or disposed of. The typical remanufacturing cost is between 40 to 60% of the manufacturing cost with 20% of the effort required (Dowlatshahi, 2000). In some cases, remanufactured products are perceived as a lower quality by the market compared to new products (Abbey et al., 2014). Hasanov et al. (2012), Helmrich (2013) and Jaber and El Saadany (2009) assumed the quality of remanufactured products to be incomparable with newly manufactured products, resulting in two demand streams. Piñeyro and Viera (2010) introduced a novel lot sizing model for a hybrid manufacturing and remanufacturing model. They proposed a one-way substitution option where demand for remanufactured items can be satisfied by newly manufactured items should there be a shortage of remanufactured products. Zouadi et al. (2019) investigated a joint pricing and lot sizing problem in a hybrid manufacturing and remanufacturing system with one-way substitution where the manufacturing and remanufacturing processes use the same resource. The remanufacturing process produces products of a lower quality, thus resulting in two demand streams for newly manufactured products and remanufactured products respectively. In the case of a shortage of remanufactured products, the demand for the lower quality product can be satisfied by newly manufactured products. Zouadi et al. proposed a mixed integer programming model to find the optimal production and pricing strategy over the planning horizon and a novel adaption of a cost benefit evaluation heuristic and memetic algorithm is proposed to find the near optimal solution. The model derived in this paper takes into consideration that the remanufacturing process does not always yield as-good-as-new items. These items that could not be remanufactured to an as-good-as-new state satisfy a lower variety product demand.

The possibility of producing defective items during the manufacturing process was introduced by Porteus (1986). A manufacturing process can go 'out of control' with a given probability every time a unit in a lot is produced, resulting in the production of defective items. These defective units are either scrapped, sold at a lower price, reworked or other remedial actions are taken. The defective yield models reviewed thus far either assume that once a process reaches an 'out of control' state, all items produced thereafter will be defective up until the point where the process is returned to an 'in control' state or alternatively, a process does not enter a complete 'out of control' state but rather produces a defective item occasionally. Defective items in this case can be identified by making use of a threshold policy where items are classified as good, reworkable or non-reworkable with known probabilities. Good or non-reworkable items leave the system completely after inspection, whereas reworkable items are returned to be remanufactured (So and Tang 1995a, 1995b; Liu and Yang, 1996; Chern and Yang, 1999). The latter is assumed for this work.

The model proposed in this paper considers that total returns consists of products returned by the customer as well as the proportion of items that fail during manufacturing. A comparison of the proposed inventory system and previously published relevant inventory models for return items in literature is provided in Table I.

TABLE I: GAP ANALYSIS OF RELATED LITERATURE WORKS AND THE CONTRIBUTION OF THIS PAPER

References	**Characteristics of the inventory system**					
	Remanufactured product satisfies original product demand	Remanufactured product satisfies secondary demand	Imperfect manufacturing yield	Return rate is a function of product demand	Shared resources	Remanufacturing process require dependent demand input
Chern and Yang, (1999)	✓		✓			
Dekker et al. (2004)	✓					
Hasanov et al. (2012)	✓	✓				
Helmrich (2013)	✓	✓				
Jaber and El Saadany (2009)	✓	✓				
Kiesmüller and van der Laan (2001)	✓			✓		
Liu and Yang (1996)	✓		✓			
Porteus (1986)	✓		✓			
Richter and Sombrutzki (2000)	✓					
Richter and Weber (2001)	✓					
So and Tang (1995a)	✓		✓			
So and Tang (1995b)	✓		✓			
Sun et al. (2013)	✓			✓		
Teunter et al. (2006)	✓				✓	
Vlachos and Dekker (2003)	✓			✓		
Zolfagharinia et al. (2014)	✓			✓		
Zouadi et al. (2014)	✓				✓	
Zouadi et al. (2017)	✓				✓	
Zouadi et al. (2019)	✓	✓			✓	
This paper	✓	✓	✓	✓	✓	✓

A review of current literature suggests that no work has been published on inventory modelling which considers remanufacturing to satisfy a primary and secondary demand, while also taking into consideration that a manufacturing process can yield defective items which are returned to be remanufactured. Additional dependent input items required in remanufacturing is also not considered in the extant literature. An attempt is made here to develop an inventory system that considers remanufacturing as an alternative to manufacturing for a top variety item. Where items cannot be remanufactured to a top variety item, they are remanufactured to a lower variety item to satisfy a secondary demand. Recoverable inventory consists of customer returns as well as a proportion of defective items produced during the manufacturing process, which is returned for remanufacturing to either one of the two item variety types depending on the extent of defect, which is a given proportion of new items manufactured. The model also considered that there may be dependent demand items for the remanufacturing processes, and such items need to be procured from outside the manufacturing system, and this process also needs optimisation. This integrated dependent demand structure is also not considered in extant literature.

The remainder of this paper is organised as follows: Section 3 provides a brief description of the proposed system. The notation adopted and mathematical representation of the inventory system considered is given in Section 4. Numerical results are presented in Section 5 to illustrate the proposed solution procedure and to provide managerial insights through a sensitivity analysis. The paper is then concluded in Section 0.

3. System Definition

The material flow of the proposed model is depicted in Fig. I, where the flow of the top range items is illustrated by a solid line and the flow of the lower range items is illustrated by the dotted line. Two inventory types are considered, namely serviceable and recoverable inventory. Serviceable inventory is inventory available to service the demand of the end user and can be further split into two types, namely Type A and Type B, where Type A is inventory of the top range product and Type B is inventory of the lower range product. Recoverable inventory consists of repairable products which includes items returned by the end user as well as defective products produced by an occasional 'out of control' manufacturing process. Recoverable inventory is also split into Type A and Type B items respectively. Demands for Type A and Type B are satisfied under the following conditions:

- The manufacturing and remanufacturing processes for Type A items are performed on the same resources and the manufacturing and remanufacturing batches are alternated in such a way as to minimize the total cost over the planning horizon;
- Items are remanufactured into Type B products on a separate resource;
- The manufacturing process produces Type A items only;
- Some items may fail during manufacturing and are sent to recoverable inventory for possible remanufacturing;
- Both Type A and Type B used items can be returned by the end user to recoverable inventory for possible remanufacturing;
- A reparable Type A item can either be remanufactured to an as-good-as-new state to satisfy Type A demand once again, or alternatively, a reparable Type A item can be remanufactured to a Type B item;
- Type B items can only be remanufactured to an as-good-as-new Type B item;
- Demand for both items is deterministic, but may vary over time;
- The rates of return are deterministic, but may vary over time;
- Reparable items need some other input items that need to be procured to bring the returned items back to one of two states of reuse;
- Lower variety input items are used to produce Type B items during the remanufacturing process;
- Top variety input items are used during the remanufacturing processes of Type A items;
- Ordering and setup costs are known and constant;
- Shortages in remanufactured and new products to fulfil demand is not allowed;
- Stock holding costs of Type A and Type B serviceable inventory as well as the holding costs of items waiting to be repaired are known;
- Lead times for both manufacturing and remanufacturing processes are negligible.

4. Model Formulation

The model of this inventory system is developed in this section, but the notations adopted for the model development is presented first.

4.1 List of Variables

4.1.1 List of decision variables

Q_{P_t} is the manufacturing batch size in time bucket t;

$Q_{P_{A_t}}$ is the product yield quantity for Type A items from a manufactured batch of size Q_{P_t} in time bucket t;

$Q_{r_{A_t}}$ is the remanufacturing batch size of Type A items in time bucket t;

$Q_{r_{B_t}}$ is the remanufacturing batch size of Type B items in time bucket t;

$I_{S_{A_t}}$ is the serviceable inventory level of Type A items at the start of time bucket t;

$I_{S_{B_t}}$ is the serviceable inventory level of Type B items at the start of time bucket t;

$I_{r_{A_t}}$ is the recoverable inventory level of Type A items at the start of time bucket t;

$I_{r_{B_t}}$ is the recoverable inventory level of Type B items at the start of time bucket t;

q_{A_t} is the Type A input item procurement batch size for use in the remanufacturing of Type A items in time bucket t;

q_{B_t} is the Type B input item procurement batch size for use in the remanufacturing of Type B items in time bucket t;

i_{A_t} is the Type A remanufacturing input item inventory level at the start of time bucket t;

i_{B_t} is the Type B remanufacturing input item inventory level at the start of time bucket t;

γ_{P_t} is the binary variable indicating the release of a manufacturing batch in time bucket t;

$\gamma_{r_{A_t}}$ is the binary variable indicating the release of a Type A remanufacturing batch in time bucket t;

$\gamma_{r_{B_t}}$ is the binary variable indicating the release of a Type B remanufacturing batch in time bucket t;

ω_{A_t} is the binary variable indicating the release of a Type A remanufacturing input item procurement batch in time bucket t;

ω_{B_t} is the binary variable indicating the release of a Type B remanufacturing input item procurement batch in time bucket t.

4.1.2 List of Parameters

D_A is the demand rate for Type A items;

D_B is the demand rate for Type B items;

r_{A_A} is the recovery rate of used Type A items that will be remanufactured to an as-good-as-new Type A item state, expressed as a percentage (or proportion) of the demand rate D_A;

r_{B_A} is the recovery rate of used Type A items that will be remanufactured to a Type B item, expressed as a percentage (or proportion) of the demand rate D_A;

r_{B_B} is the recovery rate of Type B items that will be remanufactured to an as-good-as-new Type B item state, expressed as a percentage (or proportion) of the demand rate D_B;

α is the failure rate of the manufacturing process for items that can be remanufactured to an as-good-as-new Type A item, expressed as a percentage (or proportion) of the manufacturing batch size Q_P;

β is the failure rate of the manufacturing process for items that can be remanufactured to a Type B item only, expressed as a percentage (or proportion) of the manufacturing batch size Q_P;

z_A is the number of Type A dependent demand items required in the remanufacturing of each Type A item;

z_B is the number of Type B dependent demand items required in the remanufacturing of each Type B item;

t is the length of a manufacturing/remanufacturing time (time bucket);

T is the total planning horizon (made up of t time buckets);

h_{s_A} is the holding cost rate of Type A serviceable inventory (per item per time);

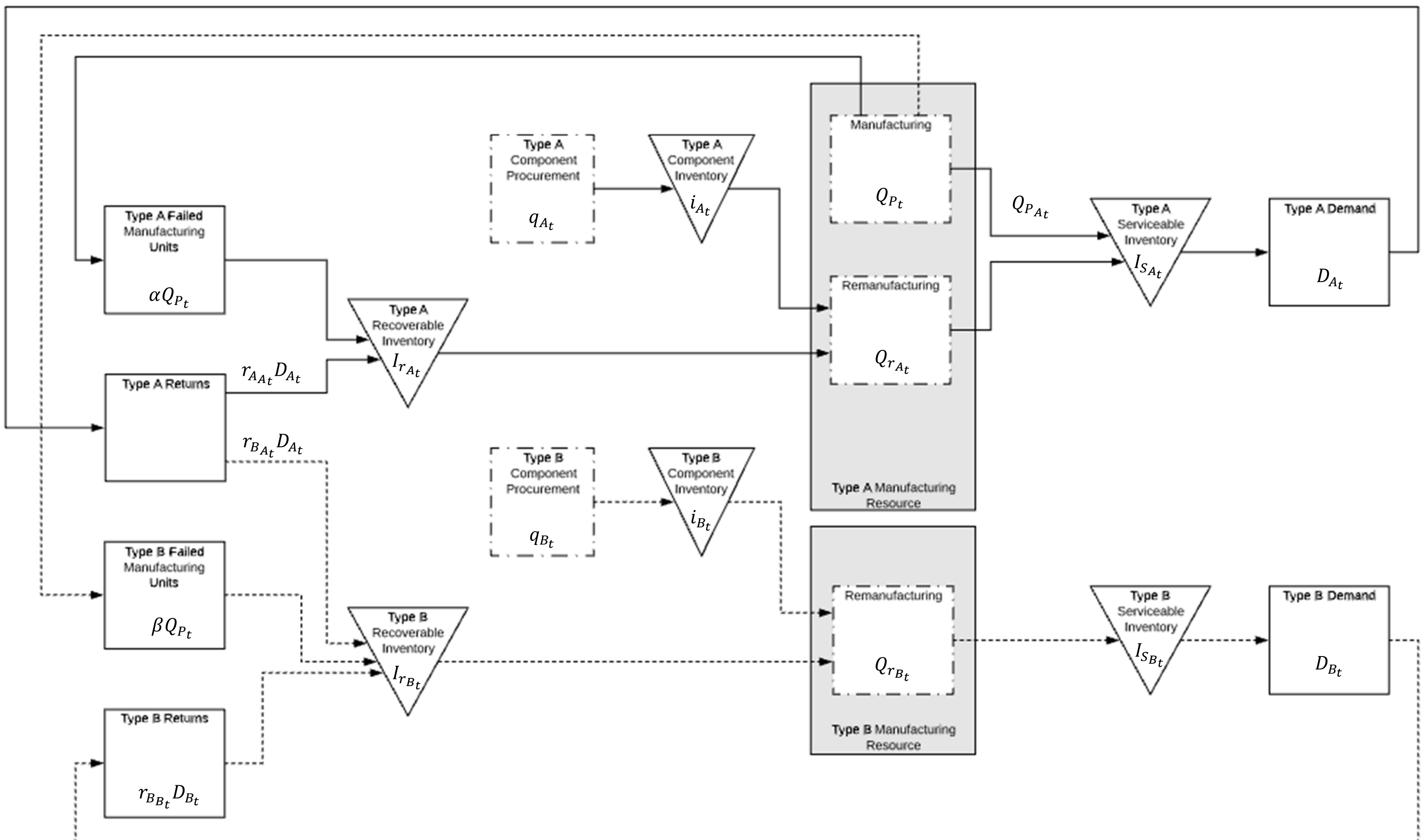

FIG. I MATERIAL FLOW DIAGRAM CONSIDERING REVERSE LOGISTICS IN SATISFYING THE DEMAND FOR A TOP RANGE ITEM AS WELL AS A LOWER RANGE ITEM

h_{s_B} is the holding cost rate of Type B serviceable inventory (per item per time);
h_{r_A} is the holding cost rate of Type A recoverable inventory (per item per time);
h_{r_B} is the holding cost rate of Type B recoverable inventory (per item per time);
v_A is the holding cost rate of Type A dependent demand item inventory (per item per time);
v_B is the holding cost rate of Type B dependent demand item inventory (per item per time);
K_p is the manufacturing batch setup cost;
K_{r_A} is the Type A remanufacturing batch setup cost;
K_{r_B} is the Type B remanufacturing batch setup cost;
P_A is the Type A remanufacturing input item ordering cost;
P_B is the Type B remanufacturing input item ordering cost;

4.2 The Total Cost Function

The cost of holding recoverable inventory h_r is assumed to be lower than or equal to the cost of holding serviceable inventory h_s. Serviceable and recoverable inventory for Type A and Type B items attract different costs, due to the different values of the items. The value of Type B serviceable inventory is also assumed higher than the value of Type A recoverable inventory, the holding cost relationship is expressed in (1).

$$h_{s_A} \geq h_{s_B} \geq h_{r_A} \geq h_{r_B} \tag{1}$$

Similarly, it is assumed that the cost of holding inventory of dependent demand items for remanufacturing of Type B items, v_B, is lower than or equal to the cost of holding inventory of dependent demand items used during the remanufacturing of Type A items, v_A since lower variety input items are used in the remanufacturing of Type B items. The holding cost relationship for the dependent demand items is expressed in (2).

$$v_A \geq v_B \tag{2}$$

A dynamic lot sizing model is proposed. The proposed model is expressed as an integer linear programming model and will be solved with a modified Wagner/Whitin dynamic programming algorithm. The mixed integer linear programming formulation with the objective of minimising the total cost over the planning horizon is given in (3) constrained by (4) to (16).

$$\min C = \sum_{\tau=1}^{T} \begin{pmatrix} K_p\gamma_{P_t} + K_{r_A}\gamma_{r_{A_t}} + K_{r_B}\gamma_{r_{B_t}} + h_{s_A}I_{S_{A_t}} + h_{s_B}I_{S_{B_t}} + h_{r_A}I_{r_{A_t}} + h_{r_B}I_{r_{B_t}} \\ +P_A\omega_{A_t} + P_B\omega_{B_t} + v_A i_{A_t} + v_B i_{B_t} \end{pmatrix} \tag{3}$$

subject to:

$$I_{S_{A_t}} = I_{S_{A_{t-1}}} + Q_{P_{A_t}} + Q_{r_{A_t}} - D_{A_t}\,;\; t = 1,2,\ldots T \tag{4}$$

$$I_{S_{B_t}} = I_{S_{B_{t-1}}} + Q_{r_{B_t}} - D_{B_t}\,; t = 1,2,\ldots T \tag{5}$$

$$I_{r_{A_t}} = I_{r_{A_{t-1}}} - Q_{r_{A_t}} + d_{A_t}\,; t = 1,2,\ldots T \tag{6}$$

$$d_{A_t} = r_{A_A}D_{A_t} + \alpha Q_{P_t}\,; t = 1,2,\ldots T \tag{7}$$

$$I_{r_{B_t}} = I_{r_{B_{t-1}}} - Q_{r_{B_t}} + d_{B_t}\,; t = 1,2,\ldots T \tag{8}$$

$$d_{B_t} = r_{B_A} D_{A_t} + r_{B_B} D_{B_t} + \beta Q_{P_t} \, ; t = 1,2, \dots T \tag{9}$$

$$Q_{P_{A_t}} = [1 - (\alpha + \beta \,)] \, Q_{P_t} \, ; t = 1,2, \dots T \tag{10}$$

$$\gamma_{P_t} + \gamma_{r_{A_t}} \leq 1 \tag{11}$$

$$i_{A_t} = i_{A_{t-1}} + q_{A_t} - z_A Q_{r_{A_t}} \, ; \; t = 1,2, \dots T \tag{12}$$

$$i_{r_{B_t}} = i_{r_{B_{t-1}}} + q_{r_{B_t}} - z_B Q_{r_{B_t}} \, ; t = 1,2, \dots T \tag{13}$$

$$Q_{P_t}, Q_{r_{A_t}}, Q_{r_{B_t}}, I_{S_{A_t}}, I_{S_{B_t}}, I_{r_{A_t}}, I_{r_{B_t}}, q_{A_t}, q_{B_t}, i_{A_t}, i_{B_t} \geq 0 \, ; \; t = 1,2, \dots T \tag{14}$$

$$\gamma_{P_t}, \gamma_{r_{A_t}}, \gamma_{r_{B_t}}, \omega_{A_t}, \omega_{B_t} \in \{0,1\} \, ; \; t = 1,2, \dots T \tag{15}$$

$$q_A \leq M\omega_A, q_B \leq M\omega_B, Q_P \leq M\gamma_P, Q_{r_A} \leq M\gamma_{r_A}, Q_{r_B} \leq M\gamma_{r_B}; t \in \{1, T\}$$
$$where\ M\ is\ a\ sufficiently\ large\ number \tag{16}$$

A finite planning horizon of T discrete time periods, $t = 1,2, \dots T$, is assumed. Customer demands D_{A_t} and D_{B_t} need to be satisfied in each time period t. Backordering is not allowed. Customers return $r_{A_A} D_{A_t}$, $r_{B_A} D_{A_t}$ and $r_{B_B} D_{B_t}$ used items in each period t. Although customer returns are expressed as a proportion of demand within a period, this does not mean that the exact item that has satisfied customer demand within period t is returned for remanufacturing in period t, this is simply a method of realistically constraining customer returns. Returns are thus, considered, to be available at the beginning of a period to be used in the remanufacturing process and not at the end. The feasibility of this assumption is guaranteed by the inclusion of sufficient opening balance at the beginning of the first period of the planning horizon. This can be construed to include the returns from the previous periods that have been carried over as the opening balance. This is a typical assumption of dynamic programming and aggregate planning models. A proportion of newly manufactured products α is returned to Type A recoverable inventory and a proportion β is returned to Type B recoverable inventory for remanufacturing due to imperfect manufacturing yield in each period t. The inventory balances of the four inventory types, namely Type A serviceable, Type B serviceable, Type A recoverable and Type B recoverable items at the end of each period, t, are expressed in (6) and (8) respectively.

The serviceable manufacturing quantity per manufactured batch size is given in (10). Initial inventory levels for $I_{S_{A_0}}$, $I_{r_{A_0}}$, $I_{S_{B_0}}$ and $I_{r_{B_0}}$ are given. Type A serviceable, Type A recoverable, Type B serviceable and Type B recoverable inventory are subject to holding costs of h_{s_A}, h_{r_A}, h_{s_B} and h_{r_B}, per unit item per unit of time respectively. Setup costs of K_p, K_{r_A} and K_{r_B} are associated with each manufacturing batch and remanufacturing batch of Type A and Type B items respectively. The release of a batch is indicated by the respective binary variables γ_{P_t}, $\gamma_{r_{A_t}}$ and $\gamma_{r_{B_t}}$. The variable is equal to one if a batch of the respective type is released at time t, otherwise zero. The manufacturing and remanufacturing processes for Type A items are performed on the same resources and only one of the two batches are, thus, allowed per period. The release of a Type A manufacturing or remanufacturing batch is constrained by (11) in every period t.

The Type A dependent demand component inventory level, i_{A_t} is, therefore, given by (12) and the Type B dependent demand component inventory level, i_{B_t} is given by (13).

Initial inventory levels, i_{A_0} and i_{B_0} are given. An ordering cost of P_A is applicable when purchasing a batch of input items used in the Type A remanufacturing processes. An ordering cost of P_B is applicable when purchasing a batch of input items used in the Type B remanufacturing process. The release of a purchasing batch is indicated by the respective binary variables ω_{A_t} and ω_{B_t}. The variable is equal to one if a batch of the respective type is released at time t, otherwise zero. Dependent demand items used in the Type A remanufacturing process is subject to holding costs of v_A per unit item per unit of time and dependent demand items used in the remanufacturing of Type B items are subject to holding costs of v_B per unit item per unit of time.

4.3 A Dynamic Programming Solution to the Problem

A dynamic programming algorithm is formulated to aid in solving the optimization problem in this paper. In doing this, the optimization problem is broken down into simpler sub problems utilizing the fact that the optimal solution to the overall problem depends on the optimal solution to its sub problems. In the heuristic based on a modified Wagner/Whitin algorithm presented by Richter and Sombrutzki (2000) for the alternate application of manufacturing and remanufacturing batches, the assumption is made that $d_{A_t} \geq D_{A_t}$ and thus every period has a chance of manufacturing or remanufacturing occurring. In the model presented in this paper, returns consists a combination of customer returns and items that fail during manufacturing. Customer returns are expressed as a proportion of demand and items that fail during manufacturing is expressed a proportion of the manufacturing batch size. The possibility of a remanufacturing batch is thus dependent on the accumulation of returns being enough to satisfy demand. The $d_{A_t} \geq D_{A_t}$ equation is thus modified for this paper and used to test whether a remanufacturing batch is possible in a period. Available recoverable inventory is calculated at the start of each period based on the manufacturing versus remanufacturing decision made in previous periods. Based on the available recoverable inventory at the start of the period, the possibility of a remanufacturing batch in that period can be determined as well as the possibility of remanufacturing to satisfy demand in future periods. The cost of manufacturing is compared to the cost of remanufacturing and the less of the two is selected as the preferred option.

A heuristic that builds on the modified Wagner/Whitin solution approach proposed by Richter and Sombrutzki is presented in (17) to (24).

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}, f_0 = 0 \tag{17}$$

$$f_{A_t} = \min_{1 \leq i < t} \{ f_{A_{it}} + c_{A_{it}} + f_{A_i} \} \tag{18}$$

$$f_{A_{it}} = min \left\{ \begin{array}{c} K_{P_i} + \sum_{j=1}^{i} \beta Q_{P_j} h_{r_{B_{j,t}}} + (\alpha Q_{P_i} + r_{A_A} D_{A_{i,t}}) h_{r_{A_{i,t}}} \\ + I_{r_{A_{i-1}}} h_{r_A}(t-i), K_{r_{A_i}} + I_{r_{A_i}} h_{r_{A_{i,t}}} + min \left\{ P_{A_i} + i_{A_i} v_{A_{i,t}} \right\} \end{array} \right\} \tag{19}$$

where $I_{r_{A_t}} = I_{r_{A_{t-1}}} - Q_{r_{A_t}} + d_{A_t}$ and $I_{r_{A_t}} \geq 0$ **(20)**

$$c_{A_{it}} = D_{A_{i,t}} h_{S_{A_{i,t}}} \tag{21}$$

$$f_{B_t} = \min_{1 \leq i < t} \{ f_{B_{it}} + f_{B_i} \} \tag{22}$$

$$f_{B_{it}} = K_{r_{B_i}} + D_{B_{i,t}} h_{S_{B_{i,t}}} + I_{r_{B_i}} h_{r_{B_{i,t}}} + min\left\{P_{B_i} + i_{B_i} v_{B_{i,t}}\right\}$$

(23)

where $I_{r_{B_t}} = I_{r_{B_{t-1}}} - Q_{r_{B_t}} + d_{B_t}$ and $I_{r_{B_t}} \geq 0$

(24)

The pseudo code for the main algorithm of the modified Wagner/Whitin algorithm is given in Fig. II**Error! Reference source not found.**. The pseudo code for each of the functions implementing the main algorithm can be seen in Appendix A: Algorithms for optimisation of item types and components plan

<u>Main Algorithm</u>
Optimise the manufacture/remanufacture of type A
Explode plan for type A to determine quantity and timing of input components of type A
Optimise the procurement plan for input components of type A
Optimise the remanufacture of type B
    (If recoverable stock is insufficient, sub-optimize type A to get a feasible plan for B)
Explode plan for type B to determine quantity and timing of input components of type B
Optimise the procurement plan for input components of type B

FIG. II PSEUDO CODE FOR MAIN ALGORITHM OF MODIFIED WAGNER/WHITIN HEURISTIC

The total cost to manufacture for Type A demand also considers the Type B recoverable inventory cost incurred due to items that fail during manufacturing. Total Type B recoverable inventory is also taken into account in the cost minimization of remanufacturing for Type B demand. The calculation of f_t in (17) thus needs to correct for the duplication in Type B recoverable inventory cost due to items that fail during manufacturing. After the optimization of f_{A_t} in (18), the total recoverable inventory holding cost incurred on items that failed during manufacturing and can only be remanufactured to an as-good-as-new Type B item is subtracted for the calculation of f_t in (17) as the recoverable inventory holding cost of these failed items is considered in the total recoverable inventory cost in the optimization of f_{B_t}, in (22), along with the usage of recoverable inventory to remanufacture.

The possibility of remanufacturing for Type A demand is dependent on the availability of sufficient recoverable stock to make at least a batch of remanufactured Type A items, this is expressed in (20). If Type A recoverable inventory is not up to the quantity required for a remanufacturing batch of Type A items, a manufacturing batch will have to be released. A Type B remanufacturing batch is dependent on the availability of recoverable Type B stock that is at least enough to make a batch of remanufactured Type B items, as expressed in (24). In the case of a shortage of Type B recoverable inventory, a Type A manufacturing batch will have to be released in order to replenish Type B recoverable inventory with defective yield items from the Type A manufacturing process. This could potentially lead to f_{A_t} being reoptimized to ensure that there are no shortages in satisfying Type A and Type B demand.

4.4 Time complexity analysis of the solution algorithm

The worst-case time complexity of each of the algorithms are: $O(n^2)$ for optimising the manufacturing/remanufacturing of type A, $O(n)$ for requirement planning for components of type A, $O(n^2)$ for optimising the procurement plan of components of type A, $O(n^2)$ for optimising the remanufacture of type B (this includes the reoptimisation algorithm, which is $O(n)$), $O(n)$ for requirement planning for components of type B, and $O(n^2)$ for optimising the procurement plan of components of type B. Overall, the algorithm is $O(n^2)$, and consequently, should be much faster than the Linear Programming solution when the size of the input data, n, gets very large, and would be quite advantageous if the quality of the solution produced is not too bad, compared to the LP solution approach which in its basic form may be exponential in the worst case.

5. Numerical Analysis

Data is simulated for the purpose of the numerical work. The assumptions about the structure of the data are guided by Dekker et al. (2004), Dowlatshahi (2000), Richter and Sombrutzki (2000) and Zolfagharinia et al. (2014). The numerical example using the modified Wagner/Whitin heuristic was solved using Microsoft Excel. The sensitivity analysis was however done using the linear programming model coded in Python 3.8 to calculate the result of numerous parameter value changes. Each run was timed, and an average runtime of 0.7 seconds was observed for the linear programming model over the same planning horizon of 5 time buckets. Results of the two respective models are also compared in Section 5.3. For the numerical experiments performed in Section 5.4, more than one parameter is varied in each scenario. Each scenario is performed 5 times in which the demand for each scenario is varied, the same distribution is used for the demand generation.

5.1. Numerical example

Let

$$K_P = 5\,000, K_{r_A} = 2\,000, K_{r_B} = 250, h_{S_A} = 1, h_{S_B} = 0.9, h_{r_A} = 0.8, h_{r_B} = 0.7, P_A = 2, P_B = 1,$$

$$v_A = 0.5, v_B = 0.2, \alpha = 0.1, \beta = 0.05, r_{A_A} = 0.5, r_{B_A} = 0.1, r_{B_B} = 0.25, z_A = 1, z_B = 1,$$

$$D_A = (1\,726, 1\,596, 1\,941, 1\,693, 1\,149), D_B = (199, 198, 193, 196, 141).$$

Then

$$r_{A_A} D_A = (863, 798, 971, 847, 575), r_{B_A} D_A = (173, 160, 194, 169, 115),$$

$$r_{B_B} D_B = (50, 50, 48, 49, 35).$$

A remanufacturing batch of Type A items is only possible in a period where Type A recoverable inventory is equal to or exceeds demand for Type A items in the same period. Based on the decision to manufacture or remanufacture in the previous period, Type A recoverable inventory is recalculated at the start of each period to determine if a remanufacturing batch is possible. Total recoverable inventory is shown in the first section of

Table II. The result of $f_{A_{it}} + c_{A_{it}}$ is provided in the second section of

Table II, where the cost of manufacturing and remanufacturing is compared in a period and the less of the two costs determine whether a manufacturing or remanufacturing batch will be executed in a period. The purchasing and holding cost of the dependent demand items required in the remanufacturing process is also minimised for each period in which a remanufacturing batch is executed and shown separately in the remanufacturing cost calculation in the second section of

Table II. The total cost of producing Type A items, f_{A_t}, is provided in the third section of

Table II.

Due to a constraint in Type A returns in period t = 1, the five-period cycle has to start with a manufacturing batch. The solution consists of a manufacturing batch of 3,322 units according to the Type A demand of the first two periods, including the provision for items that fail during manufacturing. The Type A demand in period t = 3 and t = 4 is satisfied by a remanufacturing batch of 1,941 and 1,693 units respectively, while the Type A demand in the last period is satisfied by a newly manufactured batch size of 1,149 units manufactured in period t = 5. Type A dependent demand for the remanufacturing batches in period t = 3 and t = 4 is satisfied by a purchasing batch of 3,634 units ordered in period t = 3.

A remanufacturing batch of Type B items is only possible in a period where Type B recoverable inventory is equal to or exceeds demand for Type B items in the same period. Type B recoverable inventory is calculated at the start of each period, based on the decision to manufacture or remanufacture Type A items to determine if a remanufacturing batch of Type B items is possible. Total recoverable inventory is shown in the first section of

Type B demand is satisfied by a remanufacturing batch in period t = 1,2 and 4. Type B dependent demand for the Type B remanufacturing batches in period t = 1 and t = 2 is satisfied by a purchasing batch of 590 units ordered in period t = 1. The Type B dependent demand for the Type B remanufacturing batch in period t = 4 is satisfied by a purchasing batch of 337 units ordered in period t = 4.

The total cost of production can now be calculated with the use of (17), where the holding cost of the β items that failed during the Type A manufacturing batches in period t = 1 and t = 5 needs to the subtracted to avoid double counting of these costs. This results in a total cost of R24,976.

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}$$
$$f_5 = f_{A_5} + f_{B_5} - \beta Q_{P_1} h_{r_{B_{1,5}}} - \beta Q_{P_5} h_{r_{B_{5,5}}}$$

$$f_5 = 23\ 627 + 2\ 080 - 0.05\ \times 3\ 322\ \times 0.7 \times 5 - 0.05 \times 1\ 149\ \times 0.7 \times 1$$

$$f_5 = R\ 24\ 976$$

Table III. The result of $f_{B_{it}}$ is provided in the second section of

Type B demand is satisfied by a remanufacturing batch in period t = 1,2 and 4. Type B dependent demand for the Type B remanufacturing batches in period t = 1 and t = 2 is satisfied by a purchasing batch of 590 units ordered in period t = 1. The Type B dependent demand for the Type B remanufacturing batch in period t = 4 is satisfied by a purchasing batch of 337 units ordered in period t = 4.

The total cost of production can now be calculated with the use of (17), where the holding cost of the β items that failed during the Type A manufacturing batches in period t = 1 and t = 5 needs to the subtracted to avoid double counting of these costs. This results in a total cost of R24,976.

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}$$
$$f_5 = f_{A_5} + f_{B_5} - \beta Q_{P_1} h_{r_{B_{1,5}}} - \beta Q_{P_5} h_{r_{B_{5,5}}}$$

$$f_5 = 23\ 627 + 2\ 080 - 0.05\ \times 3\ 322\ \times 0.7 \times 5 - 0.05 \times 1\ 149\ \times 0.7 \times 1$$

$$f_5 = R\ 24\ 976$$

Table III where the cost of remanufacturing Type B items is minimised in each period. The purchasing and holding cost of the dependent demand items required in the Type B remanufacturing process is also minimised for each period in which a remanufacturing batch is executed and shown separately when calculating the cost of remanufacturing in the second section of

Type B demand is satisfied by a remanufacturing batch in period t = 1,2 and 4. Type B dependent demand for the Type B remanufacturing batches in period t = 1 and t = 2 is satisfied by a purchasing batch of 590 units ordered in period t = 1. The Type B dependent demand for the Type B remanufacturing batch in period t = 4 is satisfied by a purchasing batch of 337 units ordered in period t = 4.

The total cost of production can now be calculated with the use of (17), where the holding cost of the β items that failed during the Type A manufacturing batches in period t = 1 and t = 5 needs to the subtracted to avoid double counting of these costs. This results in a total cost of R24,976.

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}$$

$$f_5 = f_{A_5} + f_{B_5} - \beta Q_{P_1} h_{r_{B_{1,5}}} - \beta Q_{P_5} h_{r_{B_{5,5}}}$$

$$f_5 = 23\ 627 + 2\ 080 - 0.05\ \times 3\ 322\ \times 0.7 \times 5 - 0.05 \times 1\ 149\ \times 0.7 \times 1$$

$$f_5 = R\ 24\ 976$$

Table III. The total cost for producing Type B items, f_{B_t}, is provided in the third section

Type B demand is satisfied by a remanufacturing batch in period t = 1,2 and 4. Type B dependent demand for the Type B remanufacturing batches in period t = 1 and t = 2 is satisfied by a purchasing batch of 590 units ordered in period t = 1. The Type B dependent demand for the Type B remanufacturing batch in period t = 4 is satisfied by a purchasing batch of 337 units ordered in period t = 4.

The total cost of production can now be calculated with the use of (17), where the holding cost of the β items that failed during the Type A manufacturing batches in period t = 1 and t = 5 needs to the subtracted to avoid double counting of these costs. This results in a total cost of R24,976.

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}$$

$$f_5 = f_{A_5} + f_{B_5} - \beta Q_{P_1} h_{r_{B_{1,5}}} - \beta Q_{P_5} h_{r_{B_{5,5}}}$$

$$f_5 = 23\ 627 + 2\ 080 - 0.05\ \times 3\ 322\ \times 0.7 \times 5 - 0.05 \times 1\ 149\ \times 0.7 \times 1$$

$$f_5 = R\ 24\ 976$$

Table III.

TABLE II: MODIFIED WAGNER/WHITIN HEURISTIC FOR THE CASE OF ALTERNATING MANUFACTURING AND REMANUFACTURING FOR TYPE A ITEM DEMAND

		$t =$				
	i	1	2	3	4	5
$I_{r_{A_t}}$	1					
	2		268			
	3			1 081		
	4				235	
	5					
$f_{A_{it}} + c_{A_{it}}$	1	5 924	**9 514**	16 739	26 228	35 548
	2		6 778 $>$	11 799	19 367	27 619
			4 214 $+$			
			2 000	2 971	4 664	6 387
	3			7 817 $>$	13 463	20 410
				2 865 $+$		
				2 000	**2 847**	
	4				7 045 $>$	10 755
					2 188 $+$	
					2 000	2 575
	5					**6 213**
f_{A_t}		5 924	9 514	14 379	17 414	23 627

Type B demand is satisfied by a remanufacturing batch in period t = 1,2 and 4. Type B dependent demand for the Type B remanufacturing batches in period t = 1 and t = 2 is satisfied by a purchasing batch of 590 units ordered in period t = 1. The Type B dependent demand for the Type B remanufacturing batch in period t = 4 is satisfied by a purchasing batch of 337 units ordered in period t = 4.

The total cost of production can now be calculated with the use of (17), where the holding cost of the β items that failed during the Type A manufacturing batches in period t = 1 and t = 5 needs to the subtracted to avoid double counting of these costs. This results in a total cost of R24,976.

$$f_t = f_{A_t} + f_{B_t} - \sum_{i=1}^{t} \beta Q_{P_i} h_{r_{B_{i,t}}}$$

$$f_5 = f_{A_5} + f_{B_5} - \beta Q_{P_1} h_{r_{B_{1,5}}} - \beta Q_{P_5} h_{r_{B_{5,5}}}$$

$$f_5 = 23\,627 + 2\,080 - 0.05 \times 3\,322 \times 0.7 \times 5 - 0.05 \times 1\,149 \times 0.7 \times 1$$

$$f_5 = R\ 24\,976$$

TABLE III: MODIFIED WAGNER/WHITIN HEURISTIC FOR THE CASE OF REMANUFACTURING FOR TYPE B ITEM DEMAND

		$\boldsymbol{t} =$				
	i	1	2	3	4	5
$\boldsymbol{I_{r_{B_t}}}$	1	219	21			
	2		230	37		
	3			279	83	
	4				302	161
	5					378
$\boldsymbol{f_{B_{it}}}$	1	**403 +** 100	604 + **178**			
	2		411 + 100	**645 +** 256		
	3			445	813	
	4				461 + **100**	**754**
$\boldsymbol{f_{B_{it}}}$	5					515
$\boldsymbol{f_{B_t}}$		503	992	1 226	1 787	2 080

5.2. Sensitivity analysis

Sensitivity analysis is conducted on the output of the linear programming model based on the small sample size used in the numerical analysis of the heuristic. The sensitivity analysis is conducted on selected input parameters considered relevant in order to investigate the effects that changes in those parameters have on the expected total cost and the number of Type A remanufacturing batches. The sensitivity analysis was conducted on 16 input parameters, serviceable and recoverable holding costs, setup costs, remanufacturing component ordering costs, recovery rates, remanufacturing component holding costs and manufacturing failure rates.

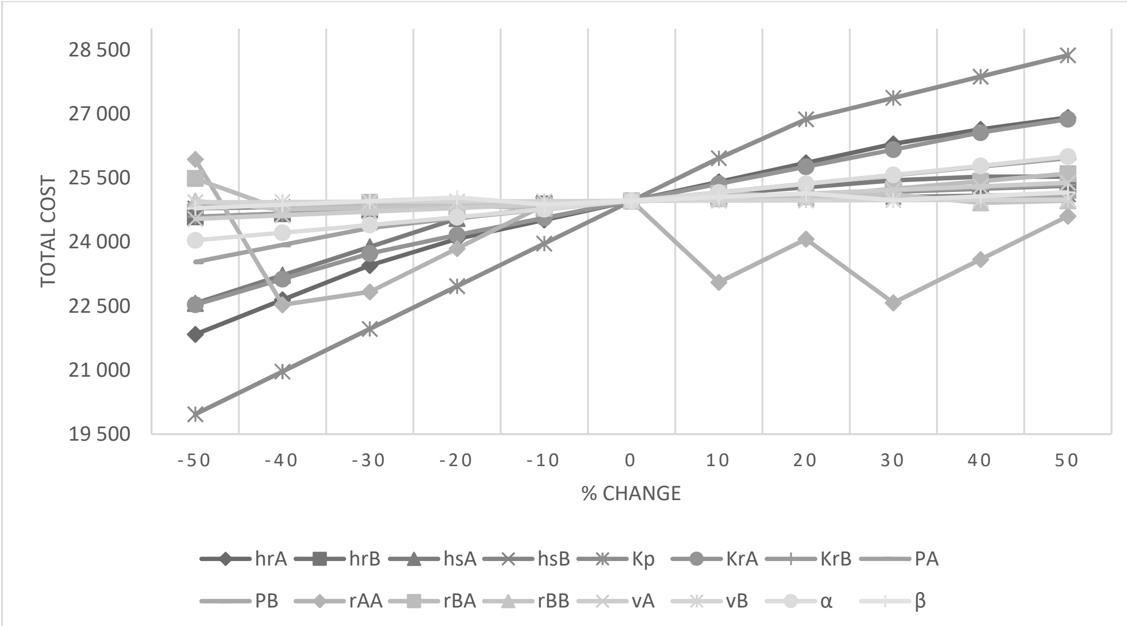

FIG. III TOTAL COST IMPACT DUE TO PERCENTAGE CHANGE IN INPUT PARAMETERS

The following observations are made based on Fig. III, which shows the results of the sensitivity analysis of the total cost:

- The total cost is most sensitive to the manufacturing setup cost. As the manufacturing setup cost increases/decreases, total cost increases/decreases. This is because if all other parameters remain unchanged, the number of remanufacturing batches, as a possible cost reduction, cannot change due to constrained returns. The sensitivity to a change in the manufacturing setup cost will thus remain, whether the remanufacturing setup cost is higher or lower than the manufacturing setup cost.
- The total cost sensitivity to the remanufacturing setup cost and Type A recoverable inventory holding cost is also significant. The percentage change in these two parameters respectively results in a similar increase in total cost. A decrease of more than thirty percent in Type A recoverable inventory holding cost results in a larger decrease in total cost compared to the same percentage decrease in remanufacturing setup cost.
- By reducing Type A serviceable inventory holding cost, total cost decreases.
- An increase in the Type A return rate leads to a reduction in total cost. This is not a linear relationship. This is because the increase in Type A return rate results in an increase in the number of periods in which demand is fulfilled by a remanufacturing batch. A reduction of thirty percent or more in the Type A return rate results in a reduced number of periods in which demand is fulfilled by remanufacturing batches. This holds true with the assumptions adopted from the literature on the relationship between manufacturing and remanufacturing setup costs in this paper. This also supports the sustainability of reverse logistics and how customers should be encouraged to return end-of-life products to enable the remanufacturing process.

The impact of the change in the Type A return rate on the total cost and the number of demand periods fulfilled by remanufacturing is shown in Fig IV.

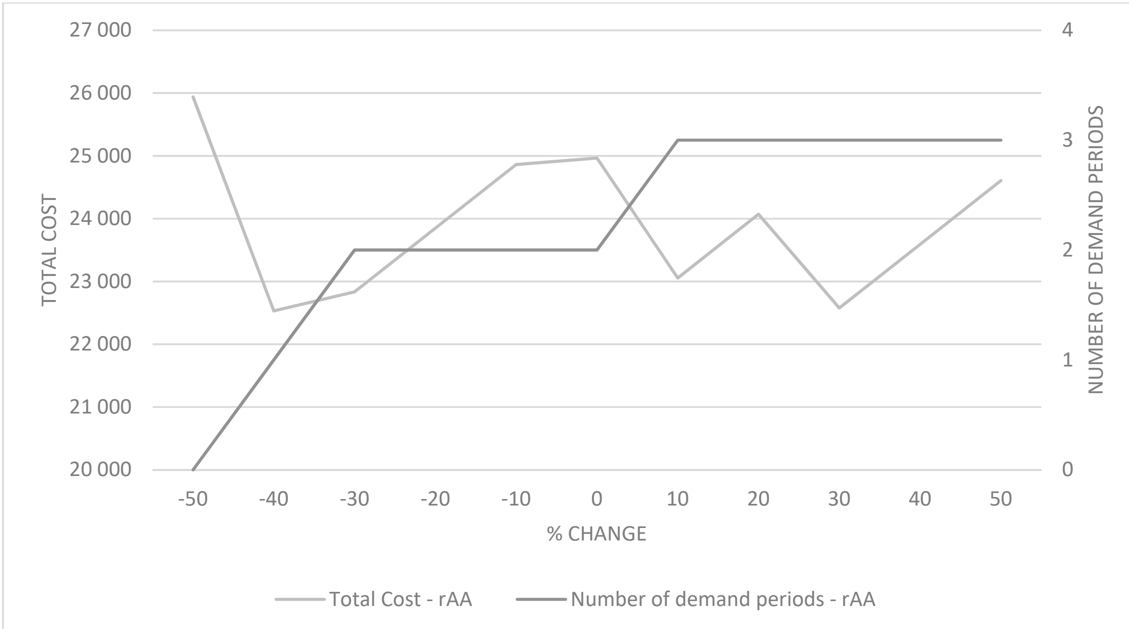

FIG IV: IMPACT OF CHANGE IN TYPE A RETURN RATE ON TOTAL COST AND NUMBER OF DEMAND PERIODS FULFILLED BY REMANUFACTURING

The following observations are made based on Fig IV:

- With an increase in the Type A return rate, there is an increase in the number of demand periods fulfilled by remanufactured items. This results in a lower total cost due to the reduction in total Type A and Type B recoverable inventory. As mentioned previously, this is not a linear relationship.
- With a decrease in the Type A return rate, there is a decrease in the number of demand periods fulfilled by remanufactured items. This results in an increase in the total cost due to the increase in both Type A and Type B recoverable inventory from items that fail during manufacturing.

5.3. MODEL RESULT COMPARISON

Richter and Sombrutzki (2000) compared the result of their modified Wagner/Whitin heuristic to the result obtained when using the well-known Silver/Meal heuristic, the results from the Silver/Meal heuristic coincided with the results of the modified Wagner/Whitin heuristic. For this paper the result of the modified Wagner/Whitin heuristic derived in this paper is compared to the linear programming model result for the small sample used in the numerical analysis. The comparison made in this paper is thus superior to the comparison made by Richter and Sombrutzki (2000). The LP results in an exact answer, whereas comparing to the Silver/Meal heuristic is simply comparing to another heuristic with known weaknesses. The linear programming model resulted in a total cost of R 24 966, a R10 difference to the total cost result obtained by this heuristic of R 24 976. The difference is due to more optimal decisions made in the remanufacturing of Type B items.

A further result comparison is conducted on selected input parameters that the linear programming model proved to be most sensitive to in order to investigate the effects that changes in those parameters have on the expected total cost output from the heuristic. This will determine whether the modified Wagner/Whitin model presented in this work is a suitable alternative to the LP in finding acceptable solution to the total cost function. The comparison was conducted on 4 input parameters, Type A recoverable holding costs, Type A remanufacturing setup cost, manufacturing setup cost and Type A return rates.

TABLE IV: LINEAR PROGRAMMING MODEL RESULT COMPARED TO HEURISTIC RESULT

Parameter	% Change	Parameter Value	Heuristic Total Cost	LP Total Cost	% Variance in Total Cost
r_{A_A}	-50%	0.25	25,939	25,939	0%
	0%	0.5	24,976	24,966	0%
	50%	0.75	24,607	24,607	0%
K_p	-50%	2500	19,976	19,966	0%
	0%	5000	24,976	24,966	0%
	50%	7500	29,966	28,378	5%
K_{r_A}	-50%	1000	25,128	22,533	10%
	0%	2000	24,976	24,966	0%
	50%	3000	26,966	26,878	0%
h_{r_A}	-50%	0.4	22,740	21,839	4%
	0%	0.8	24,976	24,966	0%
	50%	1.2	30,167	26,920	11%

The following observations are made based on Table IV:

- With no change in the parameter values of the numerical example, the linear programming model resulted in a total cost of R 24 966. With only a R10 difference due to more optimal decisions made in the remanufacturing of Type B items.
- A fifty percent decrease in the remanufacturing setup cost results in a ten percent variance in the total cost result of the two models. This is caused by a local optimum in period t = 2 of the modified Wagner/Whitin heuristic that recommends remanufacturing, this is however overridden by a lower cost decision to manufacture in period t = 1 for period t = 1,2 and 3. This increases the total cost for the full horizon, due to the increase in recoverable inventory.
- A fifty percent increase in the Type A recoverable inventory holding cost results in an eleven percent variance in the total cost result of the two models. This is again caused by a local optimum in period t = 2 of the modified Wagner/Whitin heuristic recommending a remanufacturing batch. This is once again overridden by a lower cost decision to manufacture in period t = 1 for period t = 1,2 and 3. This increases the total cost for the full horizon, due to the increase in recoverable inventory and the increase in the Type A recoverable inventory holding cost.
- On average the results of the two models differ by 3.33%, with the best-case scenario variance being as low as 0% and worst-case scenario variance being 11% on the small sample size evaluated.

The modified Wagner/Whitin heuristic seems to the be slightly lacking in considering the impact of holding costs incurred in future period due to manufacturing versus remanufacturing decisions made in a period. However, the heuristic output still produces the same trends in the total cost output. With an average variance of 3.33% in total cost, the heuristic proves to be a suitable alternative in calculating the total cost of manufacturing and remanufacturing for Type A and Type B items and determining the periods in which to manufacture and remanufacture for Type A and Type B items.

5.4. NUMERICAL EXPERIMENTS

To compare the performance of the heuristic to the simplex solution, the experiment was designed at two levels. The first is to test the quality of solution obtained by the heuristic against that of LP, and the other is to check the resolution time of the heuristic against that of LP. These two experiments were separated because large number of replications were made to test the heuristic against the LP solution, and the resolution time for LP was getting quite long as the planning horizon exceeded 10 time buckets.

To guarantee a fair comparison between the linear programming model and the proposed modified Wagner/Whitin heuristic, a representative number of instances were examined on 3 levels of input data for the return rates for item type A and item type B, 2 levels of setup costs for manufacturing of A, remanufacturing of A and remanufacturing of B, 1 level for the holding cost for serviceable stock A, 2 levels each for serviceable stock B, recoverable stock A, and recoverable stock B. Altogether, we created 24 possible scenarios from the combinations. For each scenario, a planning horizon of 5 was used and each scenario was replicated 5 times (scenario sample size of 5). The demand was varied for each sample using the same distribution to generate demand. This leads to 120 replications for both the heuristic approach and the LP solution (240 in total). To analyse the results, each replication of 5 instances for each scenario was analysed for the mean cost value and the variance of cost for the replications within each scenario. The sample average and standard deviation for the heuristic and LP solutions were then compared to understand how the heuristic differs from the exact LP solutions.

Type A manufacturing setup cost and Type A remanufacturing setup cost can take on values of 2000 and 5000, while Type B remanufacturing setup cost can take on values of 250 and 2000. While the rate of keeping a Type A serviceable item in stock is set to one, holding a Type B serviceable item for one period can cost 0.9 and 0.7, holding a Type A recoverable item for one period can cost 0.8 and 0.4 and holding a Type B recoverable item for one period can cost 0.7 and 0.3. The return rate of Type A items can take on values of 0.6, 0.5 and 0.3, the return rate of Type B items from Type A items can take on values of 0.15, 0.1 and 0.05, while the return rate of Type B items from Type B items can take on values of 0.5 and 0.25. These values allow for groupings of instances to be evaluated in the numerical experiments, these groupings consists of instances with high, medium and low return rates, instances with high and low holding cost variances, instances where the Type A manufacturing setup cost is greater than the Type A remanufacturing setup cost as well as instances where the Type A remanufacturing setup cost is greater than the Type A manufacturing setup cost and lastly instances of high and low variance in the Type A and Type B setup costs. The heuristic is evaluated by using the percentage gap to the optimal solution as a performance measure. The results of the numerical experiments are presented in Table V.

TABLE V: PERFORMANCE OF THE MODIFIED WAGNER/WHITIN HEURISTIC

	% cost error to the optimal solution		
	Average	**Standard deviation**	**Maximum**
All instances	**3,8%**	**2,6%**	**7,6%**
Return rate			
High return rates	5,5%	2,4%	7,2%
Medium return rates	1,4%	0,4%	1,8%
Low return rates	4,2%	2,9%	7,6%
Holding cost			
High holding cost variance	3,5%	3,1%	7,2%
Low holding cost variance	4,1%	2,5%	7,6%
Setup cost			
Type A manufacturing setup cost higher than remanufacturing’s	4,2%	2,9%	7,6%
Type A remanufacturing setup cost higher than manufacturing’s	3,5%	2,8%	7,2%
High variance in Type A and Type B setup costs	3,3%	3,2%	7,6%
Low variance in Type A and Type B setup costs	4,3%	2,4%	7,2%
$\boldsymbol{K_p}$			
5 000	4,2%	2,9%	7,6%
2 000	3,5%	2,8%	7,2%
$\boldsymbol{K_{r_A}}$			
5 000	3,5%	2,8%	7,2%
2 000	4,2%	2,9%	7,6%
$\boldsymbol{K_{r_B}}$			
2 000	4,3%	2,4%	7,2%
250	3,3%	3,2%	7,6%

The following observations are made based on Table V:

- With an average percentage gap of 3.8% in all instances and a maximum percentage gap of 7.6%, the heuristic proves to be a suitable alternative in calculating the total cost of manufacturing and remanufacturing for Type A and Type B items and determining the periods in which to manufacture and remanufacture for Type A and Type B items.
- The lowest percentage cost error is experienced with medium return rates
- The greatest percentage cost error is experienced with high return rates.

5.5. Resolution time of the models

To evaluate the resolution time of the two solution approaches, a sample size of 5 was run for time buckets varying from 5 to 15 for both the heuristic and the LP solutions from which it could be seen that the resolution time of the LP solution picked up rapidly after about 10 time buckets when compared to that of heuristic. This implies that as the planning horizon becomes longer, solving with LP may gradually become unrealistic.

The resolution time of the linear programming model is shown in Fig V.

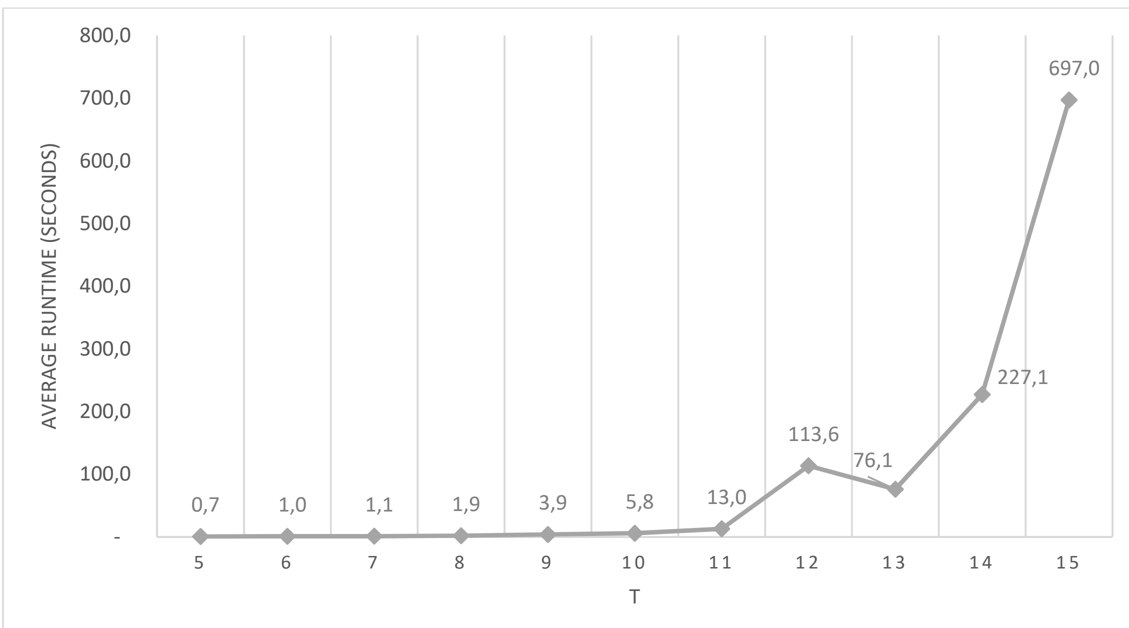

FIG V: RESOLUTION TIME OF THE LINEAR PROGRAMMING MODEL

The following observations can be made based on Fig V:

- The resolution time increases significantly from a total planning horizon of $T = 12$.
- The average runtime increases by 200% with each time bucket added from $T = 13$.

As the percentage cost error of the heuristic is relatively low and the resolution time of the linear programming model increases substantially from $T = 12$, the development and use of the modified Wagner/Whitin heuristic is supported. The runtime comparison of the two models is shown in Table VI.

TABLE VI: RESOLUTION TIME COMPARISON

T	Average LP runtime (seconds)	Average heuristic runtime (seconds)
5	0.7	0.0004
6	1.0	0.0012
7	1.1	0.0012
8	1.9	0.0020
9	3.9	0.0024
10	5.8	0.0024
11	13.0	0.0046
12	113.6	0.0040
13	76.1	0.0052
14	227.1	0.0058
15	697.0	0.0076

Comparing the quality of solutions obtained, it is apparent that the heuristic result should be quite close to optimal, while the solution continues to produce answers quite quickly as the horizon lengthens while LP needs a lot more time for resolution. This shows the quality of the solution approach to the problem, and makes it a good approach to consider when the planning horizon is long, which is not unrealistic in a number of planning environments, and even more so in instances where the organisation might need to analyse their plan in more granular time units, e.g. when moving from weekly to daily time buckets.

6. Conclusion

The major contribution made by the research presented in this paper is the incorporation of constrained returns, considering that the remanufacturing process requires additional input items and taking into consideration that not all items can be remanufactured to an as-good-as-new state of the original item. These items can be used to satisfy a lower variety secondary demand. The model also takes into consideration that items fail during manufacturing, these items are returned to be remanufactured to satisfy either one of the two demand types. The proportion of demand that is returned for remanufacturing has a significant impact on the total cost function. This finding should motivate production and operations managers to encourage the customers to return and recycle used products.

The manufacturing setup cost also has a significant impact on the total cost and can also be reduced by the increase in the proportion of demand that is returned. More returns result in more remanufacturing batches with a lower setup cost compared to that of a manufacturing batch. However, there will always be a need for manufacturing, as the items that fail during manufacturing are a vital input into the remanufacturing batches of both Type A and Type B items.

Future research considerations include partial manufacturing and remanufacturing batches to counter constraint returns within a period and shared remanufacturing resources for Type A and Type B items with a shortage cost in the case that demand for the respective items cannot be met within the same period.

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APPENDIX A: ALGORITHMS FOR OPTIMISATION OF ITEM TYPES AND COMPONENTS PLAN

Manufacturing/Remanufacturing planning algorithm for type A

```
For stage = 1 to planning horizon
    Optimise stage or manufacturing using WW algorithm
    Save all paths and the corresponding costs
    Set the minimum cost and the corresponding path as manufacturing optimum
    If there is sufficient recoverable stock for remanufacturing of A at stage
        Optimise stage for remanufacturing using WW algorithm
        Set the minimum cost and the corresponding path as remanufacturing optimum
        Set the minimum of manufacturing optimum and remanufacturing optimum as the stage optimum
    Else
        Set the manufacturing optimum as the stage optimum
    EndIf
EndFor
Return optimum cost and lot size plan for type A
```

Explosion of type A for its input components quantities

```
For i = 1 to planning horizon
    If a remanufacturing batch for A is scheduled for i
        Explode with BOM for component requirement for remanufacturing of type A
    EndIf
EndFor
Return component requirement schedule for type A
```

Procurement planning for input components of type A

```
For stage = 1 to planning horizon
    Optimise component purchase for A using WW algorithm
EndFor
Return optimal procurement cost and lot sizing plan for type A components
```

Remanufacturing planning algorithm for type B

```
For stage = 1 to planning horizon
    If there is sufficient recoverable stock to remanufacture B
        Use WW to optimise remanufacturing plan for B
    Else
        Reoptimise type A for feasible type B in stage
        Optimise B using WW algorithm
    EndIf
    Return reoptimized manufacturing cost and lot sizes for A
EndFor
Return remanufacturing cost and lot size plan for B
```

Reoptimisation/Suboptimisation Algorithm

```
Sort manufacturing plans at stage in non-decreasing order of costs
Select the first manufacturing plan that makes remanufacturing of B feasible
Set current manufacturing plan as the optimum for A for current stage
```

Explosion of type B for its input components quantities

```
For i = 1 to planning horizon
    If a remanufacturing batch for B is scheduled for i
        Explode with BOM for component requirement for remanufacturing of type B
    EndIf
EndFor
Return component requirement schedule for type B
```

Procurement planning for input components of type B

```
For stage = 1 to planning horizon
    Optimise component purchase for B using WW algorithm
EndFor
Return optimal procurement cost and lot sizing plan for type B components
```