**S2 Appendix: Properties of the DHWMA *W* scheme**

This Appendix contains the derivations of the mean and variance of the $DH_t$ statistic. The charting statistic of $DH_t$ is given by

$$\begin{cases} H_t = \lambda W_t + (1-\lambda)\bar{W}_{t-1} \\ DH_t = \lambda H_t + (1-\lambda)\bar{H}_{t-1} \end{cases}$$

with (B.1)

$$\bar{W}_{t-1} = \frac{\sum_k^{t-1} W_k}{t-1} \text{ and } \bar{H}_{t-1} = \frac{\sum_k^{t-1} H_k}{t-1}.$$

- For $t = 1$,

$$DH_1 = \lambda H_1 + (1-\lambda)\bar{H}_0 = \lambda^2 W_1 + \lambda(1-\lambda)\bar{W}_0 + (1-\lambda)\bar{H}_0. \quad \text{(B.2)}$$

Since $\bar{W}_0 = \bar{H}_0 = \mu_W$, when $t = 1$, Equation (B.2) becomes:

$$DH_1 = \lambda^2 W_1 + [\lambda(1-\lambda) + 1 - \lambda]\mu_W = \lambda^2 W_1 + (1-\lambda^2)\mu_W. \quad \text{(B.3)}$$

Thus, the mean and variance of $DH_1$ are given by

$$E(DH_1) = \lambda^2 E(W_1) + (1-\lambda^2)\mu_W = \lambda^2\mu_W + (1-\lambda^2)\mu_W = \mu_W$$

and (B.4)

$$Var(DH_1) = Var[\lambda^2 W_1 + (1-\lambda^2)\mu_W] = \lambda^4 Var(W_1) = \lambda^4\sigma_W^2,$$

respectively.

- For $t = 2$,

$$\begin{aligned} DH_2 &= \lambda H_2 + (1-\lambda)\bar{H}_1 \\ &= \lambda(\lambda W_2 + (1-\lambda)\bar{W}_1) + (1-\lambda)\bar{H}_1 \\ &= \lambda^2 W_2 + \lambda(1-\lambda)W_1 + (1-\lambda)[\lambda W_1 + (1-\lambda)\mu_W] \\ &= \lambda^2 W_2 + 2\lambda(1-\lambda)W_1 + (1-\lambda)^2\mu_W. \end{aligned}$$

Thus, the expression of $DH_2$ is given by

$$DH_2 = \lambda^2 W_2 + 2\lambda(1-\lambda)W_1 + (1-\lambda)^2\mu_W. \quad \text{(B.5)}$$

From Equation (B.5), when $t = 2$, the mean of $DH_t$ can be derived as follows:

$$\begin{aligned} E(DH_2) &= \lambda^2 \mathrm{E}(W_2) + 2\lambda(1-\lambda)E(W_1) + (1-\lambda)^2\mu_W \\ &= [\lambda^2 + 2\lambda(1-\lambda) + (1-\lambda)^2]\mu_W \\ &= [\lambda^2 + 2\lambda - 2\lambda^2 + 1 - 2\lambda + \lambda^2]\mu_W \\ &= \mu_W. \end{aligned}$$

From Equation (B.5), when $t = 2$, the variance of $DH_t$ can be derived as follows:

$$\begin{aligned} Var(DH_2) &= Var[\lambda^2 W_2 + 2\lambda(1-\lambda)W_1 + (1-\lambda)^2\mu_W] \\ &= \lambda^4 Var(W_2) + 4\lambda^2(1-\lambda)^2 Var(W_1) \\ &= \lambda^2[\lambda^2 + 4(1-\lambda)^2]\sigma_W^2. \end{aligned}$$

Thus, the mean and variance of $DH_2$ are given by

$$E(DH_2) = \mu_W$$

and (B.6)

$$Var(DH_2) = \lambda^2[\lambda^2 + 4(1-\lambda)^2]\sigma_W^2,$$

respectively.

- For $t > 2$,

$$\begin{aligned}
DH_t &= \lambda H_t + (1-\lambda)\bar{H}_{t-1} \\
&= \lambda(\lambda W_t + (1-\lambda)\bar{W}_{t-1}) + \frac{(1-\lambda)}{t-1}\sum_{k=1}^{t-1} H_k \\
&= \lambda^2 W_t + \lambda(1-\lambda)\bar{W}_{t-1} + \frac{(1-\lambda)}{t-1}\sum_{k=1}^{t-1}(\lambda W_k + (1-\lambda)\bar{W}_{k-1}) \\
&= \lambda^2 W_t + \lambda(1-\lambda)\bar{W}_{t-1} + \frac{\lambda(1-\lambda)}{t-1}\sum_{k=1}^{t-1} W_k + \frac{(1-\lambda)^2}{t-1}\sum_{k=1}^{t-1}\bar{W}_{k-1} \\
&= \lambda^2 W_t + 2\lambda(1-\lambda)\bar{W}_{t-1} + \frac{(1-\lambda)^2}{t-1}\sum_{k=0}^{t-2}\bar{W}_k \\
&= \lambda^2 W_t + 2\lambda(1-\lambda)\bar{W}_{t-1} + \frac{(1-\lambda)^2}{t-1}\sum_{k=1}^{t-2}\bar{W}_k + \frac{(1-\lambda)^2}{t-1}\mu_W \\
&= \lambda^2 W_t + 2\lambda(1-\lambda)\bar{W}_{t-1} + \frac{(1-\lambda)^2}{t-1}\sum_{k=1}^{t-2}\frac{1}{k}\sum_{u=1}^{t-2} W_u + \frac{(1-\lambda)^2}{t-1}\mu_W \\
&= \lambda^2 W_t + \frac{2\lambda(1-\lambda)}{t-1}W_{t-1} + \frac{2\lambda(1-\lambda)}{t-1}\sum_{u=1}^{t-2} W_u + \frac{(1-\lambda)^2}{t-1}\sum_{u=1}^{t-2}\left(\sum_{k=u}^{t-2}\frac{1}{k}\right)W_u + \frac{(1-\lambda)^2}{t-1}\mu_W \\
&= \lambda^2 W_t + \frac{2\lambda(1-\lambda)}{t-1}W_{t-1} + \frac{(1-\lambda)}{t-1}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right)W_u + \frac{(1-\lambda)^2}{t-1}\mu_W.
\end{aligned}$$

Thus, the expression of $DH_t$ is given by

$$DH_t = \lambda^2 W_t + \frac{2\lambda(1-\lambda)}{t-1}W_{t-1} + \frac{(1-\lambda)}{t-1}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right)W_u + \frac{(1-\lambda)^2}{t-1}\mu_W. \quad \text{(B.7)}$$

From Equation (B.7), when $t > 2$, the mean of $DH_t$ can be derived as follows:

$$\begin{aligned}
E(DH_t) &= \left[\lambda^2 + \frac{2\lambda(1-\lambda)}{t-1} + \frac{(1-\lambda)}{t-1}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right) + \frac{(1-\lambda)^2}{t-1}\right]\mu_W \\
&= (\lambda^2 + \frac{2\lambda(1-\lambda)}{t-1} + \frac{2\lambda(1-\lambda)(t-2)}{t-1} + (1-\lambda)^2)\mu_W \\
&= (\lambda^2 + 2\lambda(1-\lambda) + (1-\lambda)^2)\mu_W \\
&= \mu_W.
\end{aligned}$$

From Equation (B.7), when $t > 2$, the variance of $DH_t$ can be derived as follows:

$$\begin{aligned}
Var(DH_t) &= Var\left[\lambda^2 W_t + \frac{2\lambda(1-\lambda)}{t-1}W_{t-1} + \frac{(1-\lambda)}{t-1}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right)W_u + \frac{(1-\lambda)^2}{t-1}\mu_W\right] \\
&= \lambda^4 Var(W_t) + \frac{4\lambda^2(1-\lambda)^2}{(t-1)^2}Var(W_{t-1}) + \frac{(1-\lambda)^2}{(t-1)^2}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right)^2 \mathrm{Var}(W_u) \\
&= \left[\lambda^4 + \frac{4\lambda^2(1-\lambda)^2}{(t-1)^2} + \frac{(1-\lambda)^2}{(t-1)^2}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right)^2\right]\sigma_W^2.
\end{aligned}$$

Thus, when $t > 2$, the mean and variance of $DH_t$ are given by

$$E(DH_t) = \mu_W \quad \text{(B.8)}$$

and

$$Var(DH_t) = \left[\lambda^4 + \frac{4\lambda^2(1-\lambda)^2}{(t-1)^2} + \frac{(1-\lambda)^2}{(t-1)^2}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right)^2\right]\sigma_W^2,$$

respectively.

Therefore, at the sampling time $t$, the mean and variance of $DH_t$ statistic are defined by

$$E(DH_t) = \mu_W$$

and

$$Var(DH_t) = \begin{cases} \lambda^4\sigma_W^2 & for\ t = 1 \\ \lambda^2(\lambda^2 + 4(1-\lambda)^2)\sigma_W^2 & for\ t = 2 \\ \left[\lambda^4 + \frac{4\lambda^2(1-\lambda)^2}{(t-1)^2} + \frac{(1-\lambda)^2}{(t-1)^2}\sum_{u=1}^{t-2}\left(2\lambda + (1-\lambda)\sum_{k=u}^{t-2}\frac{1}{k}\right)^2\right]\sigma_W^2 & for\ t > 2, \end{cases} \quad \text{(B.9)}$$

respectively.