S1 Appendix: Properties of the HWMA *W* scheme

This appendix contains the derivations of the mean and variance of the H_t statistic. The charting statistic of H_t is given by

$$H_t = \lambda W_t + (1-\lambda)\overline{W}_{t-1} \tag{A.1}$$

where

$$\overline{W}_{t-1} = \frac{\sum_k^{t-1} W_k}{t-1}.$$

- For $t = 1$,

$$H_1 = \lambda W_1 + (1-\lambda)\overline{W}_0.$$

Since $\overline{W}_0 = \mu_W$, when $t = 1$, Equation (A.1) becomes:

$$H_1 = \lambda W_1 + (1-\lambda)\mu_W. \tag{A.2}$$

Thus, the mean and variance of H_1 are given by

$$E(H_1) = \lambda E(W_1) + (1-\lambda)\mu_W = (\lambda + 1 - \lambda)\mu_W = \mu_W$$

and (A.3)

$$Var(H_1) = Var(\lambda W_1 + (1-\lambda)\mu_W) = \lambda^2 Var(W_1) = \lambda^2\sigma_W^2,$$

respectively.

- For $t > 1$,

Equation (A.1) can be written as:

$$H_t = \lambda W_t + (1-\lambda)\frac{\sum_k^{t-1} W_k}{t-1}. \tag{A.4}$$

From Equation (A.4), when $t > 1$, the mean of H_t can be derived as follows:

$$\begin{aligned} E(H_t) &= \lambda E(W_t) + (1-\lambda)\frac{\sum_k^{t-1} E(W_k)}{t-1} \\ &= \lambda\mu_W + (1-\lambda)\frac{E(W_1) + E(W_2) + \cdots + E(W_{t-1})}{t-1} \\ &= \lambda\mu_W + (1-\lambda)\frac{(t-1)\mu_W}{t-1} \\ &= \lambda\mu_W + (1-\lambda)\,\mu_W \\ &= \mu_W. \end{aligned}$$

Next, from Equation (A.4), when $t > 1$, the variance of H_t is derived as follows:

$$\begin{aligned} Var(H_t) &= Var\left[\lambda W_t + (1-\lambda)\frac{\sum_k^{t-1} W_k}{t-1}\right] \\ &= \lambda^2 Var(W_t) + (1-\lambda)^2\frac{\sum_k^{t-1} Var\,(W_k)}{t-1} \\ &= \lambda^2\sigma_W^2 + \left(\frac{1-\lambda}{t-1}\right)^2 (Var\,(W_1) + Var\,(W_2) + \cdots + Var\,(W_{t-1})) \\ &= \lambda^2\sigma_W^2 + \left(\frac{1-\lambda}{t-1}\right)^2 (t-1)\sigma_W^2 \\ &= \left(\lambda^2 + \frac{(1-\lambda)^2}{t-1}\right)\sigma_W^2. \end{aligned}$$

Hence, the mean and variance of the H_t statistic are given by

$$E(H_t) = \mu_{H_t} = \mu_W$$

and

$$Var(H_t) = \sigma_{H_t}^2 = \begin{cases} \lambda^2 \sigma_W^2 & for\ t = 1 \\ \left[\lambda^2 + \frac{(1-\lambda)^2}{t-1}\right]\sigma_W^2, & for\ t > 1, \end{cases} \tag{A.5}$$

respectively.