Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 54-21 (2021) 139–144

Decentralised Model Predictive Control For Parallel Unit Operation Optimisation

Andries J. Wiid [*,1,2] **Johan D. le Roux** [*] **Ian K. Craig** [*]

[*] *Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria, South Africa.*

Abstract: This paper describes the application of decentralised model predictive control (DMPC) to parallel unit operations. A decentralised approach may provide advantages in terms of maintenance, online time, and tuning complexity. Total flow control and linearisation, flow biasing and mass balance baselayer schemes are discussed as well as the DMPC structures. Finally, an industrial case study is presented where a DMPC approach is applied to steam header pressure control and steam generation optimisation.

Keywords: decentralized control, decoupling problems, process control applications

1. INTRODUCTION

Parallel unit operations are common in many industrial facilities due to the mechanical design constraints of processing large amounts of feedstock as well as maintenance and turn-down requirements. However, these parallel unit operations are not identical in terms of performance metrics due to design considerations or small deviations in the physical layout. Examples of performance metrics include: yield, efficiency, and throughput. It is desired to not only optimise individual unit operations but also the overall combined process. These optimisation objectives can be realised through the design and implementation of suitable control strategies.

The optimisation and control of parallel unit operations as a whole has not been given much attention in literature although it is common in industry. Examples in literature are restricted to specialised equipment applications such as parallel pumps (Wu et al., 2015), compressors (Paparella et al., 2013; Xenos et al., 2015), heat exchangers, and cooling towers (Viljoen et al., 2020). However, the potential applications are abundant in industry such as reactors, gasifiers, reformers, distillation columns, boilers, and generation turbines.

Model predictive control (MPC) is widely used in industry to deal with large multivariable interacting constrained control problems. Industrial MPCs are usually implemented from a centralised perspective where all available manipulated variables (MVs), controlled variables (CVs) and disturbance variables (DVs) are included in a single constrained optimisation problem. However, the numerous interacting control and optimisation objectives, varying dynamics, and the number of variables when optimising parallel processing units makes decentralised model predictive control (DMPC) an attractive solution (Christofides et al., 2013).

[1] The author would like to thank Gideon Slabbert for his assistance in the implementation of the industrial DMPC application and data gathering.

[2] Corresponding author. E-mail: `drieswiid@gmail.com`.

The contribution of this paper is a generalised presentation of typical baselayer and DMPC schemes when applied to parallel unit operation optimisation. The paper details the design of the DMPC structure and decoupling interaction. Furthermore, a DMPC scheme is applied to an industrial case study and compared with baselayer control.

2. BASELAYER CONTROL

The baselayer control structures are described in this section, specifically, mass balance control, total flow linearisation and control, and flow biasing.

2.1 Mass Balance Control

The total flow to parallel unit operations is usually governed by a mass balance such as reactor streams pulling from a communal storage tank or steam generation turbines supplied from a common steam header. Therefore, it is common for baselayer control structures to have the form as shown in Fig. 1. The mass balance controller (M) is typically a level controller for liquids and a pressure controller for gasses. The mass balance controller will require a certain total flow to meet the mass balance requirements.

Total flow controllers are used in Fig. 1 to linearise the overall response and reject local disturbances in the slave loops such as individual unit trips, shut-downs and saturation. There are m total flow controllers and n slave flow controllers per individual m total flow controller. The total flow controller sends the same signal to all the slave flow controllers.

The m total flow controllers are connected to the mass balance controller and prioritised by using a suitable split range philosophy ($f(M)$) (Reyes-Lúa et al., 2019).

2.2 Total Flow Linearisation and Control

The purpose of total flow control is to reject local disturbances related to the mass balance closure of numerous parallel slave flow controllers. This is often used where

10.1016/j.ifacol.2021.12.024

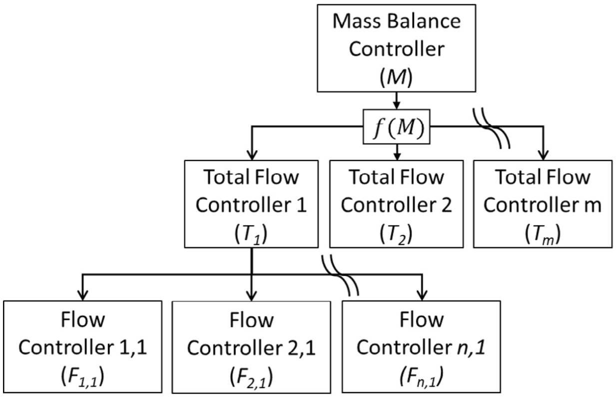

Fig. 1. Generalised structure of a mass balance controller (M), m total flow controllers (T_m), and n individual unit operation flow controllers ($F_{n,m}$) per individual total flow controller.

parallel trains of processing units supply or pull out of a common source. The other purpose of the total flow controller is to linearise the overall response of the slave controllers, i.e., the gain of M remains constant irrespective of the amount of parallel slave controllers in cascade with the total flow controllers.

One method is to use an integral-only (I-only) controller as illustrated in Fig. 2. The set-point (SP) to the total flow controller is the output (OP) received from the mass balance controller (M) through $f(M)$ as shown in Fig 1, the process value (PV) of the total flow controller is a summation of all the slave flow controller SPs pulled from the individual slave controller points and the OP of the I-only controller is sent to the flow slave controller SPs. Therefore, the I-only controller will through feedback correct the mass balance for any disturbances in the slave control loops such as process trips, removing slave loops from cascade, or other control schemes taking control of slaves through an override. Alternatively, the slave flow controller SPs can be explicitly calculated at the cost of more calculation steps and processing power.

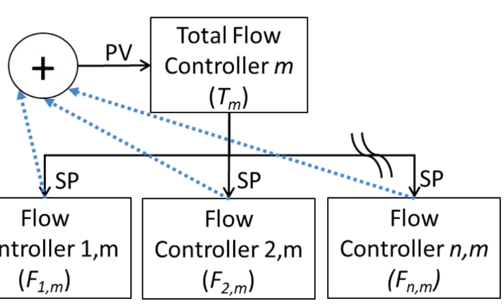

Fig. 2. Structure of a total flow controllers (T_m), and n individual unit operation flow controllers. The feedback signals from the slave flow controller set-points are shown.

2.3 Flow Biasing

A function commonly available in the baselayer is individual flow controller biasing as shown in Fig. 3. The biasing points ($B_{n,m}$) allows a manual input by a human or another controller which adds a value to the signal from the total flow controller to the parallel slave controllers. The purpose of the biasing points is to allow a specific slave flow controller to operate at a different operating region relative to the other parallel controllers. This functionality is useful in situations where there are abnormal or specific restrictions on processing equipment, the effects of which can be mitigated by the biasing point. Additionally, the biasing points can be exploited intelligently to optimise a set of parallel unit operations relative to each other based on factors such as efficiency, yield, or to avoid saturation.

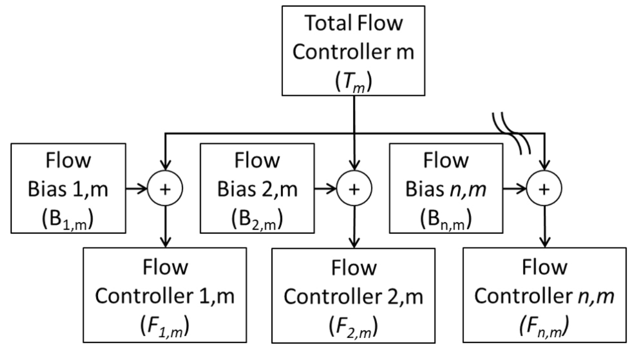

Fig. 3. Individual unit operation biasing interfaces with flow controllers.

3. DECENTRALISED MODEL PREDICTIVE CONTROL

The application of DMPC to parallel unit operations is described in this section.

3.1 Model Predictive Control

MPC applications are attractive due to the use of process models, predictive capabilities, and the implicit use of objective functions.

For MPC applications, the states ($\boldsymbol{x}$) and the model outputs ($\boldsymbol{y}$) can be discretised into $\mathcal{N}_P$ elements as,

$$\begin{aligned} \boldsymbol{x}_{k+j} &= \boldsymbol{f}(\boldsymbol{x}_{k+j-1}, \boldsymbol{u}_{k+j-1}) \quad \forall\ j\ \in\ [1, \mathcal{N}_P], \\ \boldsymbol{y}_{k+j} &= \boldsymbol{g}(\boldsymbol{x}_{k+j}, \boldsymbol{u}_{k+j}) \quad \forall\ j\ \in\ [1, \mathcal{N}_P]. \end{aligned} \tag{1}$$

The general objective function used in industrial MPCs to minimise is defined as (Qin and Badgwell, 2003),

$$\begin{aligned} \min_{\boldsymbol{u}} \Bigg[& \sum_{j=1}^{\mathcal{N}_P} \Big(||E^y_{k+j}||^2 W_Q + ||s_j||^2 W_I \Big) + \\ & \sum_{j=0}^{\mathcal{N}_C-1} \Big(||E^u_{k+j}||^2 W_R + ||\Delta \boldsymbol{u}_{k+j}||^2 W_S \Big) \Bigg], \end{aligned} \tag{2}$$

subject to,

$$\begin{aligned} \underline{\boldsymbol{y}} - s_j \le \boldsymbol{y}_{k+j} \le \bar{\boldsymbol{y}} + s_j &\quad \forall\ j\ \in\ [1, \mathcal{N}_P], \\ s_j \ge 0 &\quad \forall\ j\ \in\ [1, \mathcal{N}_P], \\ \underline{\boldsymbol{u}} \le \boldsymbol{u}_{k+j} \le \bar{\boldsymbol{u}} &\quad \forall\ j\ \in\ [0, \mathcal{N}_C - 1], \\ \Delta\underline{\boldsymbol{u}} \le \Delta\boldsymbol{u}_{k+j} \le \Delta\bar{\boldsymbol{u}} &\quad \forall\ j\ \in\ [0, \mathcal{N}_C - 1]. \end{aligned} \tag{3}$$

W_Q, W_I, W_R, and W_S are weighting matrices for the output reference error, slack variables, input reference error, and input move sizes respectively. $\mathcal{N}_P$ is the prediction horizon and $\mathcal{N}_C$ is the control horizon. E^y_{k+j} and E^u_{k+j} are deviations from the desired output and input trajectories respectively. s_j are the slack variables used to penalised output constraint violations. $\underline{\boldsymbol{y}}$ and $\bar{\boldsymbol{y}}$ are the output low and high limits, $\underline{\boldsymbol{u}}$ and $\bar{\boldsymbol{u}}$ are the input low and high limits, and $\Delta\underline{\boldsymbol{u}}$ and $\Delta\bar{\boldsymbol{u}}$ are the change in input low and high limits.

The solution of (2) is a set of proposed future $\mathcal{N}_C$ input moves,

$$\boldsymbol{u}^{\mathcal{N}_C} = [u_k, u_{k+1}, \ldots, u_{k+\mathcal{N}_C-1}]^T, \tag{4}$$

of which the first move (u_k) is executed.

3.2 Centralised Structure

For illustration purposes, a general centralised MPC (CMPC) structure to interface with the baselayer points described in Section 2 is shown in Fig. 4. Other structures are also possible. In Fig. 4 each individual flow controller $(F_{n,m})$ is used as an MV to control the mass balance variable (M). Each MV can potentially be used to optimise a specific optimisation objective per unit operation $(O_{n,m})$. Finally, there may exist i flows which can not be manipulated (D_i) and are included as DVs. The relevant variables in Fig. 4 are connected through linear models representing the dynamics of the process (ϕ).

CV\MV(DV)	$F_{1,1}$	$F_{2,1}$	$\cdots$	$F_{n,m}$	(D_i)
M	ϕ	ϕ	$\cdots$	ϕ	ϕ
$O_{1,1}$	ϕ				
$O_{2,1}$		ϕ			
$\vdots$			$\ddots$		
$O_{n,m}$				ϕ	

Fig. 4. CMPC performing mass balance, linearisation, and process optimisation functions.

The following drawbacks may result from using a centralised structure for parallel unit operation optimisation:

- The control matrix is large which can result in an undesirably large computation time and increases controller maintenance complexity.
- It is common for operations to disable the entire controller, even if it is possible to exclude specific variables, to work on a small part of the process which results in decreased controller online time.
- The optimisation objectives and control objectives interact which increases the controller tuning complexity.
- The default baselayer loops in the redundant layer are broken at a low level which increases the responsibility of the non-redundant MPC to perform robustly during abnormal situations.

3.3 Decentralised Structure and Decoupling

To improve on the possible CMPC structure shortcomings when applied to parallel unit processes, a DMPC structure can be used as shown in Fig. 5. The DMPC structure separates the mass balance control function from the individual optimisation functions. One of the best reasons for implementing decentralised control is if the systems involved have different time scales, which in turn affect the horizons of the MPCs.

The mass balance controller has the mass balance variable (M) as a CV, the total flow controller SPs (T_m) are MVs, and could potentially have i disturbances from uncontrolled flows (D_i).

There are m optimisation controllers which correspond with m total flow controllers. Each optimisation controller has a sum-of-biases CV (S_m) calculated as the sum of the biasing MVs $(B_{n,m})$,

$$S_m = \sum^n B_{n,m}. \tag{5}$$

The biasing MVs correspond with the biasing points shown in Fig. 3. The sum-of-biases CV is used to decouple the optimisation controllers from the mass balance controller by constraining the sum-of-biases to be zero.

Each n parallel processing unit per m total flow controller is assumed to have some optimisable variable $(O_{n,m})$. Examples may include saturation limits, efficiencies, and yields. Additionally, each m optimisation controller includes the corresponding total flow controller SP (T_m) as a DV. By including the total flow controller SPs as DVs, the biasing points can optimally bias the slave flows when the operating region changes due to movements in T_m.

Mass Balance Controller

CV\MV(DV)	T_1	T_2	$\cdots$	T_m	(D_i)
M	ϕ	ϕ	$\cdots$	ϕ	ϕ

Optimisation Controller *1*

CV\MV(DV)	$B_{1,1}$	$B_{2,1}$	$\cdots$	$B_{n,1}$	(T_1)
S_1	ϕ	ϕ	$\cdots$	ϕ	
$O_{1,1}$	ϕ				ϕ
$O_{2,1}$		ϕ			ϕ
$\vdots$			$\ddots$		ϕ
$O_{n,1}$				ϕ	ϕ

$\vdots$ $\vdots$

Optimisation Controller *m*

CV\MV(DV)	$B_{1,m}$	$B_{2,m}$	$\cdots$	$B_{n,m}$	(T_m)
S_m	ϕ	ϕ	$\cdots$	ϕ	
$O_{1,m}$	ϕ				ϕ
$O_{2,m}$		ϕ			ϕ
$\vdots$			$\ddots$		ϕ
$O_{n,m}$				ϕ	ϕ

Fig. 5. Generalised DMPC structure performing mass balance and process optimisation functions. The linearisation and total flow control functionality is retained in the base-layer.

The advantages of using a DMPC structure may include:

- The independent MPC controllers can be turned off separately for process requirements and abnormal conditions.
- Process and control problems can be narrowed down into smaller subsections which simplifies troubleshooting.

- The tuning objectives are clear per controller as interacting objectives are decentralised.
- The total flow controller and corresponding slave controller loops in the redundant baselayer remain unbroken thereby improving reliability and decreasing initialisation complexity.

3.4 Optimisation Strategy

When the optimisation objectives are included explicitly, they are chosen as the optimisation variables ($O_{n,m}$) and the MVs will take action to directly manipulate these optimisation objectives. This is useful in scenarios where the optimisation objective is to balance the individual unit operations such as: keeping compressors equidistant from surge lines, allowing motors to run at the same speed to minimise combined energy consumption, allowing flare valves to operate an equal distance away from opening, and keeping valves away from saturation limits.

The optimisation objectives can also be included implicitly through the use of the linear coefficient weights such as W_Q, W_I, W_R, or W_S in (2). This is useful in scenarios where a secondary optimisation objective is identified which can be influenced by the MVs. For example, if the primary objective is to ensure that parallel unit operations do not saturate, the secondary objective could be to bias more efficient unit operations higher to allow more flow to the more efficient units as long as they adhere to the primary optimisation objective, i.e., the more efficient unit operations are more loaded as long as they do not saturate.

4. INDUSTRIAL CASE STUDY

The application of DMPC to an industrial case study is described in this section.

4.1 Process Description

The industrial case study process is shown in Fig. 6. A 40 bar steam header is shown which is supplied by 8 steam boiler unit operations in parallel. The steam header supplies 6 parallel steam generation turbines and additional 40 bar consumers. The turbine dynamics are significantly faster than the boiler dynamics.

The standard baselayer built in the redundant control layer is also shown in Fig. 6. A mass balance controller (M) controls the pressure (P) which sends signals to the total boiler flow controller (T_1) and the total turbine flow controller (T_2) using a split range philosophy ($f(M)$). The boilers each have a flow controller which manipulates the steam supplied to the header per boiler ($F_n, 1$) and each turbine has a flow controller ($F_n, 2$) which manipulates the steam supplied to the turbines from the steam header. The additional 40 bar consumer flow rate (D_1) is not manipulated. The baselayer control scheme is the standard control scheme used and can be used as a suitable fall-back strategy when the advanced control schemes are turned off.

The DMPC scheme designed for, and built on the industrial case study is shown in Fig. 7. The first DMPC (C_1) manipulates the boiler and turbine total flows, the second DMPC (C_2) manipulates the biases for the individual boiler flows ($B_{n,1}$), and the third DMPC (C_3) manipulates

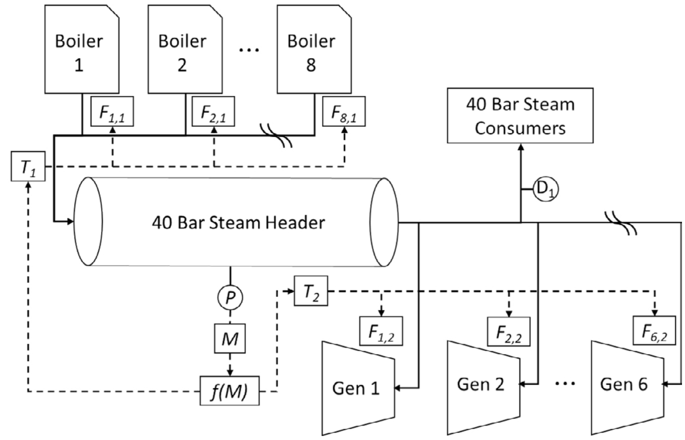

Fig. 6. Case study process diagram and corresponding base-layer control layout.

the biases for the individual turbine controllers ($B_{n,2}$). The combined DMPC structure is shown in Fig. 8. The sum-of-biases for the individual boiler and turbine flows are (S_1) and (S_2) respectively. Each boiler flow has an optimisation CV ($O_{n,1}$) and each turbine has an optimisation CV ($O_{n,2}$). The directionality of the model gains are indicated by a $+$ or $-$. The models were obtained from step-test data.

4.2 Mass Balance DMPC Design: C_1

The mass balance controller uses the 40 bar steam header pressure (P) as a CV. The MVs are the total flow controllers T_1 and T_2. The only disturbance variable is the 40 bar steam consumer flow rate (D_1).

The objective function for C_1 is,

$$\min_{T_{m\in[1,2]}} \Bigg[\sum_{j=1}^{\mathcal{N}_P} \Big(||E_j^P||^2 W_p \Big) + \sum_{j=0}^{\mathcal{N}_C - 1} \Big(||\Delta T_{m\in[1,2],j}||^2 W_{\Delta T_{m\in[1,2]}} + T_{m\in[1,2],j} W_{T_{m\in[1,2]}} \Big) \Bigg], \tag{6}$$

where E^P is the pressure deviation from the desired value with a corresponding penalty weight W_p, $\Delta T_{m\in[1,2]}$ are the total flow move sizes with corresponding penalty weights, $W_{\Delta T_{m\in[1,2]}}$. The absolute values of $T_{m\in[1,2]}$ are maximised by assigning $W_{T_{m\in[1,2]}} < 0$ to ensure that either T_1 or T_2 is at a high limit to maximise electricity production.

C_1 is subject to the following constraints,

$$\begin{aligned} \underline{P} \leq P \leq \bar{P} \quad & \forall\, j \in [1, \mathcal{N}_P], \\ \underline{T}_{m,j} \leq T_{m,j} \leq \bar{T}_{m,j} \quad & \forall\, j \in [0, \mathcal{N}_C - 1], \\ \Delta \underline{T}_{m,j} \leq \Delta T_{m,j} \leq \Delta \bar{T}_{m,j} \quad & \forall\, j \in [0, \mathcal{N}_C - 1]. \end{aligned} \tag{7}$$

4.3 Total Flow DMPC Design: C_2 and C_3

The sum-of-biases CVs (S_1 and S_2) calculated in (8) are used to decouple the mass balance controller (C_1) from the turbine and boiler optimisation controllers (C_2 and C_3).

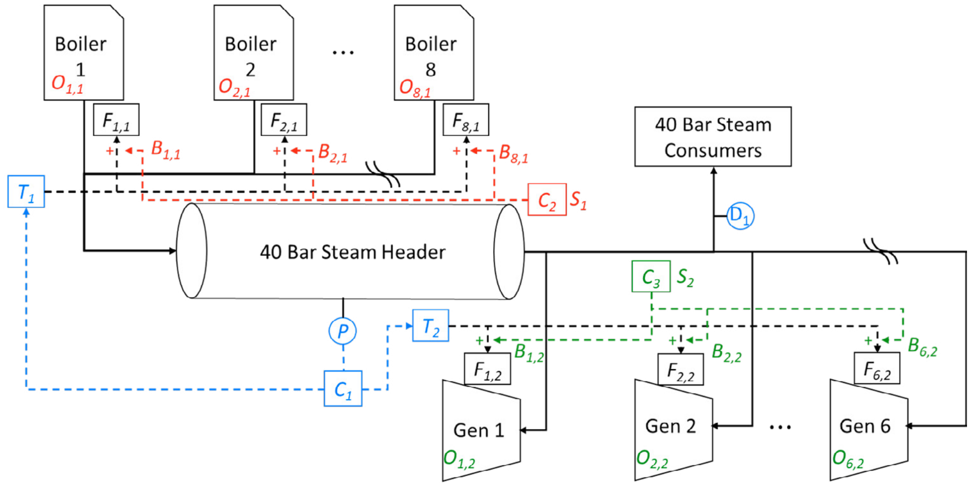

Fig. 7. Case study process diagram showing decentralised pressure control (C_1 in blue), parallel turbine optimisation (C_2 in red), and parallel boiler optimisation (C_3 in green) MPC interfaces.

Pressure Controller (C_1)

CV\MV(DV)	T_1	T_2	(D_1)
P	+	-	-

Boiler Optimisation Controller (C_2)

CV\MV(DV)	$B_{1,1}$	$B_{2,1}$	···	$B_{8,1}$	(T_1)
S_1	+	+	···	+	
$O_{1,1}$	-				-
$O_{2,1}$		-			-
⋮			⋱		-
$O_{8,1}$				-	-

Turbine Optimisation Controller (C_3)

CV\MV(DV)	$B_{1,2}$	$B_{2,2}$	···	$B_{6,2}$	(T_2)
S_2	+	+	···	+	
$O_{1,2}$	-				-
$O_{2,2}$		-			-
⋮			⋱		-
$O_{6,2}$				-	-

Fig. 8. Case Study MPC structures relating to the layout in Fig. 7.

$$S_1 = \sum_{n=1}^{n=8} B_{n,1}, \tag{8a}$$

$$S_2 = \sum_{n=1}^{n=6} B_{n,2}. \tag{8b}$$

By constraining S_1 and S_2 to zero, any move made on a given flow has to be balanced by another flow. Therefore, the biasing of individual flow rates does not interfere with the total flow rate requested by the total flow controllers (T_1 and T_2).

The optimisation is done explicitly for saturation boundaries and implicitly for a yield metric as described in Section 3.4. Each optimisation CV is equal to the remaining capacity available on a given turbine or boiler determined by the minimum between the final control element ($V_{n,m}$) capacity or steam flow ($F_{n,m}$) capacity. The bar ($\bar{}$) is used to indicate a high limit. The optimisation CVs are calculated as,

$$O_{n,m} = \min\{\bar{F}_{n,m} - F_{n,m}, (\bar{V}_{n,m} - V_{n,m})Q_{n,m}\}. \tag{9}$$

$Q_{n,m}$ is a scaling factor used to scale the final control element opening in % to the same units as the steam flow rate.

The boiler optimisation CV linear coefficients ($L_{n,1}$) are calculated at each time step by inferring the efficiency of a given boiler by dividing the amount of steam produced ($F_{n,1}$) by the amount of coal consumed ($K_{n,1}$) as,

$$L_{n,1} = \frac{F_{n,1}}{K_{n,1}} \quad n \in [1, 8]. \tag{10}$$

The turbine optimisation CV linear coefficients ($L_{n,2}$) are calculated at each time step by inferring the efficiency of a given generation turbine by dividing the amount of megawatt ($MW_{n,2}$) produced by the amount of steam consumed as,

$$L_{n,2} = \frac{MW_{n,2}}{F_{n,2}} \quad n \in [1, 6]. \tag{11}$$

The optimisation CV linear coefficients correspond with their relative optimisation CVs in the objective function of C_2 and C_3 as shown in (12a) and (12b) respectively,

$$\min_{B_{n\in[1,8],m=1}} \left[\sum_{j=1}^{\mathcal{N}_P} O_{n\in[1,8],m=1,j} L_{n\in[1,8],m=1} \right], \tag{12a}$$

$$\min_{B_{n\in[1,6],m=2}} \left[\sum_{j=1}^{\mathcal{N}_P} O_{n\in[1,6],m=2,j} L_{n\in[1,6],m=2} \right]. \tag{12b}$$

This optimisation function formulation ensures that the capacity of more efficient unit operations are minimised.

C_2 and C_3 are subject to the following constraints,

$$\begin{aligned}
0 \leq S_{m,j} \leq 0 \quad & \forall\, j \in [0, \mathcal{N}_P], \\
\underline{O}_{n,m,j} \leq O_{n,m,j} \leq \bar{O}_{n,m,j} \quad & \forall\, j \in [1, \mathcal{N}_P], \\
\underline{B}_{n,m,j} \leq B_{n,m,j} \leq \bar{B}_{n,m,j} \quad & \forall\, j \in [1, \mathcal{N}_C - 1], \\
\Delta\underline{B}_{n,m,j} \leq \Delta B_{n,m,j} \leq \Delta\bar{B}_{n,m,j} \quad & \forall\, j \in [0, \mathcal{N}_C - 1].
\end{aligned}$$

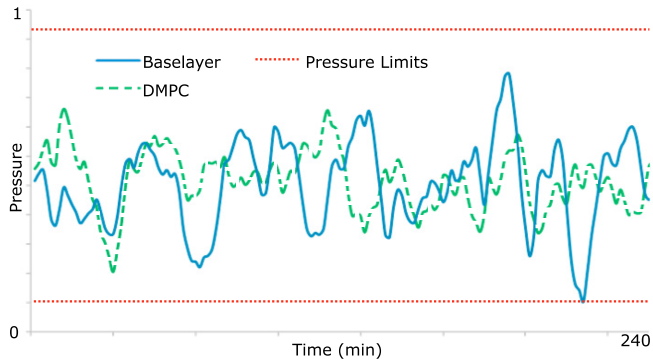

Fig. 9. C_1 and baselayer pressure control.

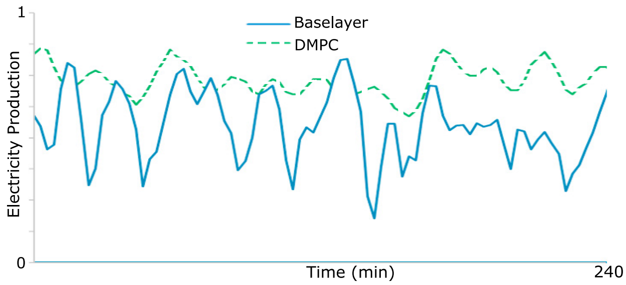

Fig. 10. C_1 and baselayer total flow control electricity production.

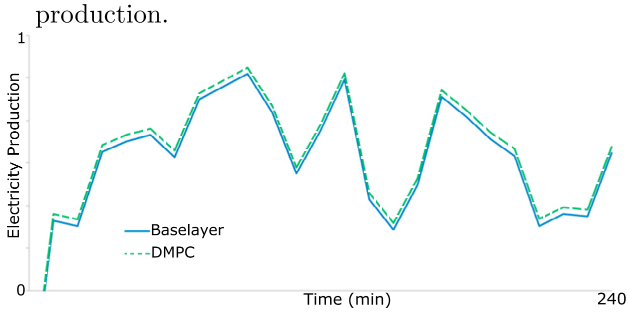

Fig. 11. C_3 and baselayer total flow control electricity production.

The total flow controllers (T_1 and T_2) used as MVs in C_1 are included as DVs in the optimisation controllers (C_2 and C_3) respectively.

4.4 Results

The case study makes use of licensed robust model predictive control technology (RMPCT) (Qin and Badgwell, 2003), a Honeywell product, to execute the controllers.

During the case study test period there was an excess of coal to be consumed. Therefore, C_2 was not in use which is made possible due to the decentralised structure. Fig. 9 shows the pressure response when using the baselayer and C_1, Fig. 10 shows the additional electricity produced due to C_1 and Fig. 11 shows the additional electricity produced due to C_3 relative to the baselayer control. For commercial reasons, the absolute pressure and electricity values are scaled to be between 0 and 1.

The DMPC controllers were commissioned on the industrial process and the data collected for a post-benefit audit. C_1 was compared to the baselayer control by comparing the electricity produced by the turbines when the process operated in similar operating regions. C_3 was compared to the baselayer control by calculating what the electricity production would be if all flow biases were zero using (11).

Table 1. Ratio of electricity production for DMPC relative to baselayer control for the time period shown in Fig. 9 to Fig. 11.

Controller	DMPC/Baselayer
C_1	1.023
C_3	1.005

From Fig. 9 it can be seen that the DMPC and baselayer control adequately controls the pressure within limits. Table 1 provides the ratio of average electricity produced between the DMPC controllers and the baselayer control. It can be seen that C_1 increases the average electricity production by 2.3% and that C_3 increases the average electricity production by 0.5% over the baselayer control which is significant for this unit. The C_3 increase of 0.5% is heavily dependent on the biasing MV limits ($\underline{B}_{n,m}$ and $\bar{B}_{n,m}$).

5. CONCLUSION

This paper provides a generalised baselayer and DMPC framework which can be applied to parallel unit operations for optimisation and control. A DMPC structure may improved online time, reduced tuning complexity, and provide easier maintenance. A DMPC scheme is applied on an industrial case study and compared with baselayer control as a baseline. Future work may investigate if any performance differences exist between DMPC and CMPC.

REFERENCES

Christofides, P.D., Scattolini, R., de la Pena, D.M., and Liu, J. (2013). Distributed model predictive control: A tutorial review and future research directions. *Computers & Chemical Engineering*, 51, 21–41.

Paparella, F., Domínguez, L., Cortinovis, A., Mercangöz, M., Pareschi, D., and Bittanti, S. (2013). Load sharing optimization of parallel compressors. In *2013 European Control Conference (ECC)*, 4059–4064. doi: 10.23919/ECC.2013.6669697.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7), 733–764.

Reyes-Lúa, A., Zotică, C., Forsman, K., and Skogestad, S. (2019). Systematic design of split range controllers. *IFAC-PapersOnLine*, 52(1), 898–903.

Viljoen, J.H., Muller, C.J., and Craig, I.K. (2020). Hybrid nonlinear model predictive control of a cooling water network. *Control Engineering Practice*, 97, 104319.

Wu, P., Lai, Z., Wu, D., and Wang, L. (2015). Optimization research of parallel pump system for improving energy efficiency. *Journal of Water Resources Planning and Management*, 141(8), 04014094.

Xenos, D.P., Cicciotti, M., Kopanos, G.M., Bouaswaig, A.E., Kahrs, O., Martinez-Botas, R., and Thornhill, N.F. (2015). Optimization of a network of compressors in parallel: Real time optimization (RTO) of compressors in chemical plants – an industrial case study. *Applied Energy*, 144, 51–63.