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A rational approach to obtaining effective lengths of compression members in framed structures

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With the common and widespread use of PC-based design packages in the design of steel structures, it is of fundamental importance to revisit the basic principles contained in and sometimes omitted from common analysis packages. The basic methods of stability analysis are briefly discussed and the influence of inelastic buckling incorporated and then illustrated by using representative case studies. Common errors in the use of such design packages, and the importance of simple calibration checks are used to illustrate the problems which can lead to failures.

INTRODUCTION

Since the first appearance of computer-based analysis programs, ample warnings have been issued to maintain the use of simple manual calculations. The use of integrated design packages linked to analysis packages can further obscure errors in structural modelling. In the case of design packages involving members susceptible to failure through instability, special care should be taken to ensure that the correct boundary conditions are used in both planes. It is necessary to review some basic concepts inherent to the design of such members.

SIMPLE LATTICE STRUCTURES AND THE K-FACTOR

In-plane buckling

The K-factor or effective length factor is used to adjust the actual unrestrained length of a compression member to account for prevailing boundary conditions. Many software packages use a default effective length factor of 0,85, implying some form of rotational joint restraint by adjacent members. Some design codes also specify effective length factors for compression members in lattice trusses. BS 5400 Part 3 (2000) in table 11 specifies effective length factors for buckling in the plane of the truss as well as out of

the plane of the truss. In all cases the values given in table 11 of BS 5400 Part 3 are less or equal to 0,85. Eurocode 5 (1995) gives the effective column length for members of triangulated trusses with loading at the nodes as the bay length. Furthermore, for strength verification the calculated force must be increased by 10 %. SANS 10162:1 (2004), 15.3.1, states that: 'The effective length for buckling in the plane of the truss shall be taken as the distance between the lines of intersection of the working points of the web members and the chord. The effective length for buckling perpendicular to the plane of the truss shall be equal to the distance between the points of lateral support.' This is then a conservative solution.

Boundary conditions are not merely a function of connection details and continuity, but are influenced by the capacity of adjacent members at the node. Consider the example of a simple lattice truss with a constant section shown in figure 1.

The compression chord ABCDEFG is divided into equal portions. The basic principle that the buckling load for member ABCDEF is unique shows that an effective length factor of less than one for a particular member is consistent with an effective length factor of greater than one in the adjacent members, as shown in equation 1 below:

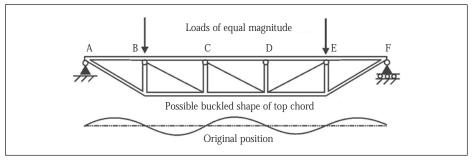


Figure 1 In-plane buckling of top chord of lattice truss – case 1

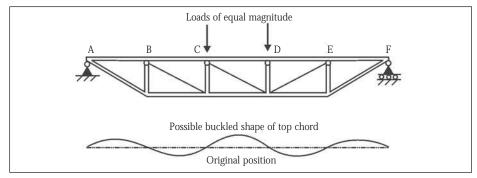


Figure 2 In-plane buckling of top chord of lattice truss - case 2

$$\frac{\pi^2 EI}{(k_{AB} L)^2 \cdot P_{AB}} = \frac{\pi^2 EI}{(k_{BC} L)^2 \cdot P_{BC}} = \frac{\pi^2 EI}{(k_{CD} L)^2 \cdot P_{CD}} = \frac{\pi^2 EI}{(k_{$$

$$\frac{\pi^2 EI}{(k_{DE} L)^2 \cdot P_{DE}} = \frac{\pi^2 EI}{(k_{EF} L)^2 \cdot P_{EF}}$$
(1)

For the loading as shown, the force in ABCDEF will be constant over the length of the truss. If the member is of constant section, equation 1 can only be satisfied if the buckling length is taken as $L = L_{AB} = L_{BC} = L_{CD} = L_{DE} = L_{EF}$ If the stiffness of the lacing elements is ignored, an effective length factor of less than one is clearly incorrect.

Now consider the same system but with a different loading combination, as shown in figure 2.

In the second case shown in figure 2, the compression force in BC, CD and DE are greater than the forces in AB and EF. $P_{BC} = P_{CD} = P_{DE} > P_{AB}, P_{EE}$. Once again, the buckling strength of the chord ABCDEF is unique. In order to still satisfy equation 1, the effective length factors for the members with the lower forces are greater than for those members with the higher forces. It should now be clear that subtle differences exist between the buckling modes of the same top chord for the two different load cases.

In the first case, the buckling lengths are equal on AB, BC etc and the only sensible choice of an effective length factor is 1,0. For the second load case shown in figure 2 above, segment BC, CD and DE are critical while segments AB and EF are sub-critical. An effective length factor of less than 1,0 is therefore justified for BC, CD and DE by virtue of the restraint action exercised by AB and EF. A rigorous analysis will show that the effective length factors of the adjacent portions AB and EF are in fact greater than 1,0, this being coupled to a smaller force in the member.

It may be argued that, in real trusses, some rotational restraint from the lacing members would always be present at the nodes and therefore the effective length factor may be reduced somewhat. This argument only holds true in the following cases:

- Some of the connecting members at a node are in tension.
- Some of the compression members at a node are sub-critical.

A sub-critical condition in adjacent members can be achieved by having the

same size member subjected to a lower compression force or a reduced distance between restraints.

If every member is optimised in a lattice structure, care should be taken in assuming effective length factors less than one.

It is significant that the Eurocodes for steel design specifically, have discarded the practice of using tabulated effective length factors in the design of both compression and flexural members. Elastic buckling loads are used as a basis of design, and such loads are commonly calculated using computer programs.

Stanway, Chapman and Dowling (1992) have discussed the influence of elastic supports at any position of the length of the strut, thereby considering the influence of unequal bay lengths and the beneficial restraint offered by adjacent sub-critical elements having a shorter buckling length. The basis of elastic buckling analysis is subsequently discussed.

BUCKLING ANALYSIS OF FRAMED AND LATTICE STRUCTURES

In-plane buckling

Most PC-based analysis packages are capable of performing buckling analysis on framed structures using beam elements, and individual members using shell elements. It is important that the user is aware of the actual process and the premises on which such analyses are based. The method of buckling analysis of a frame structure is presented in Coates, Coutie and Kong (1980), as described below.

In the case of a linear elastic analysis of a framed structure, deformation is linearly related to load, or, expressed in matrix form:

$$P = K_s \Delta$$
 (2)
Where:
 P is the force or load matrix

 Δ is the displacement matrix $K_{\mbox{\tiny c}}$ is the stiffness matrix

The terms of K_s are constant for a given structure, provided that second order effects are neglected, therefore K_s is independent of P. If, however, the influence of axial forces on member stiffness is included, K_s becomes a function of P, or $K_s = K_s(P)$.

In the case where axial loads are not neglected, equation 2 becomes non-linear, but if the axial loads are known, the deflections may be calculated.

$$\lambda P = K_s(\lambda P)\Delta \tag{3}$$

In equation 3 the term λ has been inserted as a load multiplier. As the loads are progressively increased, a state of neutral equilibrium is achieved where any deflection is possible for a given load level. This state defines instability and may be referred to as $\lambda = \lambda_{cr}$. The critical state is consistent with the matrix $K_s(\lambda P)$ becoming singular.

A test of the singularity of the matrix K_s can therefore be used as a check on stability, if it is non-singular and positive definite, the structure is stable – if it is singular the structure is on the point of collapse. The value of λ_{cr} is therefore a multiple whereby an arbitrarily chosen load can be multiplied to achieve a state of collapse. The following comments regarding the value of λ_{cr} should be clearly noted:

- $lack \lambda_{cr}$ is *not* a safety factor. Even if *P* is chosen to represent load effects at working loads, the influence of inelastic buckling is not taken into account in an elastic buckling analysis.
- In the case where buckling modes are de-coupled, for example lattice structures consisting of pin-ended members, the value of λ_{cr} applies to the member most susceptible to buckling and has no application to other members.
- If a two-dimensional analysis were to be carried out to determine λ_{cr} , the value so obtained obviously does not apply to out of plane buckling.

Inelastic buckling

The significance of an elastic buckling analysis is that the value of $\lambda_{\rm cr}P$ is the elastic buckling load of the critical member or portion of a structure, or of the structure as a whole. In order to calculate the inelastic buckling load, and therefore the factored resistance of the critical member, the following procedure should be followed:

Calculate the equivalent effective unbraced length from the relationship:

$$\lambda_{cr} P = \frac{\pi^2 EI}{(KL)^2}$$
therefore: $KL = \sqrt{\frac{\pi^2 EI}{\lambda_{cr} P}}$
or $K = \sqrt{\frac{P_e}{\lambda_{cr} \cdot P}}$ (4)

Where:

 P_{e} = Euler buckling load for compression member hinged at both ends

P = applied load

Calculate the inelastic compressive resistance C_r using the value of KL obtained from

Table 1 Results of an elastic buckling analysis performed on a simple frame

Load case and frame conditions	Axial load in top chord at buckling, P	Load factor, $\lambda_{_{\mathrm{cr}}}$	$P_{\rm e} = \frac{\pi^2 EI}{L^2}$	Effective length factor, $K = \sqrt{\frac{P_e}{\lambda_{cr} . P}}$
LC 1 fixed	16,67 kN	1,32	10,33 kN	0,69
LC 1 pinned	16,67 kN	0,62	10,33 kN	1,00
LC 2 fixed	33,3 kN	0,70	10,33 kN	0,67
LC 2 pinned	33,3 kN	0,36	10,33 kN	0,93

Table 2 Results of elastic buckling analysis on node-frame (values in parentheses indicate hypothetical k-factors of non-critical members)

P_1	P ₂	P ₁ / P _e	P ₂ /P _e	λ_{cr}	K _{AB,BC}	K _{BD}
10,28 kN	28,55 kN	1,0	1,0	1,00	1,0	1,0
10,28 kN	0 kN	1,0	0	1,36	0,85	-
0	28,55 kN	0	1,0	1,46	-	0,68
5,14 kN	28,55 kN	0,5	1,0	1,28	(0,63)	0,88
10,28 kN	14,28 kN	1,0	0,5	1,21	0,91	(0,64)

Table 3 Results of elastic buckling analysis on node-frame – out-of-plane buckling (values in parentheses indicate hypothetical k-factors of non-critical members)

\mathbf{P}_{1}	P_2	P_1/P_e	P_2/P_e	λ_{cr}	K _{AB/BC}	K _{BD}
54,97 kN	152,71 kN	1,0	1,0	1,00	1,00	1,00
54,97 kN	0 kN	1,0	0	1,19	0,917	-
0	152,71 kN	0	1,0	1,08	-	0,962
27,48 kN	152,71	0,5	1,0	1,07	(0,684)	0,967
54,97 kN	76,35 kN	1,0	0,5	1,17	0,925	(0,654)

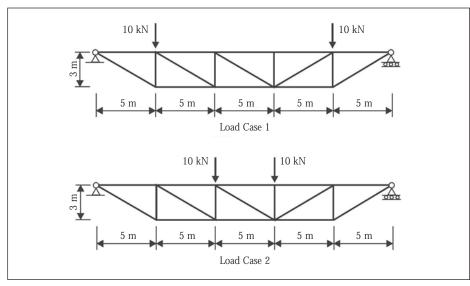


Figure 3 Frame parameters for in-plane buckling analysis

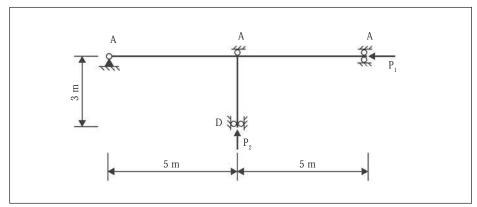


Figure 4 Node frame model

equation 4 for the appropriate member size. For example, if SANS 10162:1 (2004) is used, the appropriate slenderness ratio is

$$\frac{KL}{r}\sqrt{\frac{300}{f}}$$

from which C_r can be calculated. In order to demonstrate the principles discussed above, a commonly available PC-based analysis package, Prokon (2003), was used to calculate the effective length factors of the top chord of the frame shown in figures 1 and 2 above. The parameters used in the analysis are shown in figure 3.

It is assumed that the trusses in figure 3 are made up out of equal angle $50 \times 50 \times 6$ mm sections. The second moment of area, I, of chords, verticals and diagonal = 128×0^{-9} m⁴ and E = 206 GPa.

Two cases were considered. In the first case rigid nodes were used, that is, a frame analysis was carried out. In the second case the nodes were considered as pinned, that is, only axial forces exist in the truss members. Significant differences exist between the two cases of pinned and fixed joints for the load cases considered, and are shown in table 1.

It is important to note the following:

- Significant differences exist between the cases of fixed and pinned frame joints. In this particular case the differences were amplified by the choice of stiffness of the lacing members as being equal to the chords. In real trusses the lacing members would commonly have significantly less stiffness than the chord members.
- The effective length factor of the compression chord is not only dependent on the joint conditions, but also on the loading conditions, as is evident from the differences in the critical buckling loads between load cases 1 and 2.

It is not commonly accepted that the restraining influence of lacing members is influenced by the magnitude of the axial forces resulting from load effects, to the capacity of the members. In order to demonstrate this principle, a buckling analysis was performed on a simple sub-frame representing two bays of a chord member and one vertical element. Different combinations of axial forces relative to the buckling capacities of the members were used as shown in figure 4:

$$I_{abc} = I_{bd} = 127 \text{ x } 10^{-9} \text{ m}^4 \text{ and } E = 206 \text{ GPa}$$

For the node-frame model shown in figure 4, the critical buckling loads (Euler), for ABC and for DB using an effective length factor K=1, are 10,28 kN and 28,55 kN, respectively. The results of the analysis are shown in table 2.

The following important conclusions may be drawn from studying the results shown in table 2:

■ The degree of restraint offered to a compression member by connecting members at a node is dependent on the magnitude of the axial forces in the connecting

Table 4 Results of elastic buckling analysis on node-frame – out-of-plane buckling (values in parentheses indicate hypothetical k-factors of non-critical members)

P_1	P ₂	P ₁ /P _e	P ₂ /P _e	λ_{cr}	K _{AB/BC}	K _{BD}
116,3 kN	323,0 kN	1,0	1,0	1,03	0,985	0,985
116,3 kN	0 kN	1,0	0	1,35	0,861	-
0 kN	323,0 kN	0	1,0	1,06	-	0,971
58,15 kN	323,0 kN	0,5	1,0	1,05	(0,690)	0,976
116,3 kN	161,5 kN	1,0	0,5	1,33	0,867	(0,613)

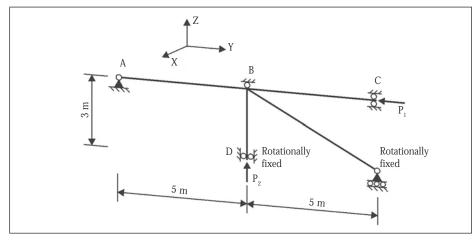


Figure 5 Node frame model for out-of-plane buckling

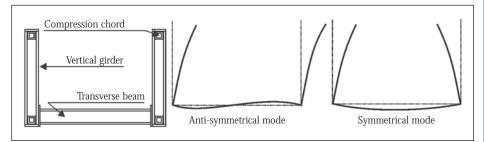


Figure 6 Cross section and buckling modes of girder with elastic lateral supports

members. If the connecting members are subjected to axial compression, the ability of such members to restrain adjacent critical members is reduced.

- If the connecting members are subjected to axial compressive forces equal to their critical buckling loads, such members offer no restraint action.
- Design nomographs expressing effective length factors of compression members in a frame as a function of the stiffness of connecting members do not account for the reduction in stiffness of connecting members themselves subjected to compressive forces.
- Tensile forces in connecting members increase their stiffness and lead to a reduction in the buckling length of compression members. The particular software package used in this analysis does not consider this effect.
- Unless a buckling analysis has been performed on a frame or truss, it is difficult to justify the use of a general effective length factor less than one.

Out-of-plane buckling behaviour

The preceding discussion and analysis have been limited to in-plane behaviour. In

considering the out-of-plane buckling behaviour of planar trusses, the behaviour is somewhat different. Rotational restraint to the compression chord can now only be provided by the torsional stiffness of the connecting members. In order to demonstrate this principle, the same node frame model previously used was modified to incorporate a diagonal member and the members were changed to angle sections. Out-of-plane restraints were provided at all joints in the X-direction.

Use an angle section - 125 x 75 x 8 mm with $I_{xx}=2,47$ x $10^{-6},\ I_{zz}=0,676$ x $10^{-6},\ J=36,7$ x 10^{-9} and E=206 GPa.

For the node-frame model shown in figure 5, the critical buckling loads (Euler), for ABC and for DB using an effective length factor K=1 are 54,97 kN and 152,71 kN, respectively. The results of the analysis are shown in table 3.

The following important conclusions may be drawn from studying the results shown in table 3:

Owing to the low torsional stiffness of angle sections, adjacent sub-critical members offer very little restraint to out-ofplane buckling. In order to demonstrate this principle, the same node frame as shown in figure 5 was re-analysed using square hollow section, $90 \times 90 \times 3.5$ mm, members. The section properties and member parameters used in this analysis were as follows:

$$I_{xx} = I_{yy} = 1,43 \times 10^{-6}, J = 2,33 \times 10^{-6}, P_e \text{ (AB)}$$

= 116,3 kN, $P_e \text{ (BD)} = 323,0 \text{ kN}$

For the case where ABC and BD are both critical, that is, the load effects equal the elastic buckling load with K=1 (Euler), the critical load factor equalled 1,03.

For the case where BD was critical but the axial load in ABC was zero, the critical load factor equalled 1,06 reflecting an effective length factor of 0,971.

It is clear that there is no justification for the use of an effective length factor of 0,85.

LATERAL BUCKLING OF GIRDERS WITH FLEXIBLE LATERAL SUPPORTS

Perhaps the case of incorrect application of effective length factors with the most serious consequences may be found where the compression chords of girders or trusses are restrained by flexible supports. This case is commonly found in the older type bridge girders where the carriageway is attached to the bottom chords and the top chords are not restrained by a dedicated separate bracing system. In this case the lateral stability of the top chord is controlled by the U-frame action of the transverse beams acting with the uprights of the girders (see figure 6).

Two buckling modes of the U-frame are shown in figure 6, a symmetrical mode where the transverse beam is in single curvature, and an anti-symmetrical mode where the transverse beam is in double curvature. The lowest buckling mode is associated with the lowest lateral restraint to the compression chord and is therefore consistent with the symmetrical case where the transverse beam is in single curvature.

The buckling resistance of the compression chord is now determined by the flexural stiffness of the U-frame comprising the transverse beam and the verticals. Designing the compression chord using a buckling length equal to the spacing between U-frames, without consideration of the lateral stiffness of the U-frame at chord level, will result in lateral buckling occurring before in-plane buckling. The use of a simple force criterion to size the U-frames will not provide sufficient stiffness to satisfy the assumption of a buckling length equal to the spacing between U-frames. Dekker and Burdzik (2000) have proposed the use of a combined force and stiffness criterion, which may be applied to this case.

This case is covered in BS5400 Part 3 (2000), clause 12.5.1, where the effective length of a compression chord that is laterally restrained by U-frame action is given by:

Table 5 Member sizes for the girder truss pedestrian bridge

Member	Element	A – mm²	Ixx - mm ⁴	Iyy – mm ⁴
Top chord	PFC 180 x 70 – web horizontal	2,68 x 10 ³	13,5 x 10 ⁶	1,27 x 10 ⁶
Bottom chord	PFC100 x 50 – web horizontal	1,29 x 10 ³	2,05 x 10 ⁶	0,320 x 10 ⁶
Diagonals	100 x 100 x 8 angle section	1,55 x 10 ³	1,45 x 10 ⁶	1,45 x 10 ⁶
Verticals and floor beams	PFC 100 x 50 – web vertical	1,29 x 10 ³	2,05 x 10 ⁶	0,320 x 10 ⁶

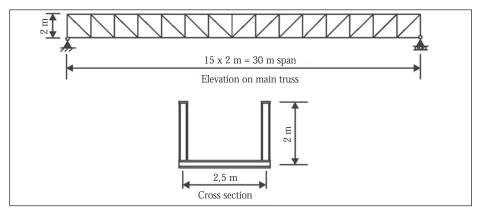


Figure 7 Typical pedestrian bridge with U-frame restraint to top chords

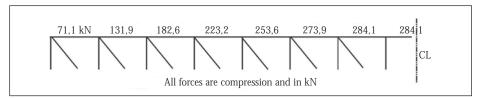


Figure 8 Compression forces in top chord of monoplanar truss

$$l_e = 1.5k_3 (EI_c a\delta)^{0.25}$$

Where:

- k_3 depends on the flexural stiffness of the vertical members of the truss and may be taken as 1,0
- I_c is second moment of area of the chord about the out-of-plane axis
- a is the distance between U-frames
- δ is the nominal lateral deflection of the Uframe under the application of a unit load, therefore representing the effective stiffness of the U-frame

For the case where the U-frame is symmetrical and cross members and verticals have a constant section over their length, this clause of BS 5400 Part 3 provides a method of calculating the required stiffness from the value of δ but the process is time consuming. In some cases the top chords of Warren-type trusses are braced horizontally but the bracing cannot be completed to the supports and a stiffened U-frame is placed at each end to allow clearance at the ends of the bridge. This case is covered in BS 5400 Part 3 in clause 12.5.3.2.

Girders using U-frame action to achieve stability of the compression chords may be conveniently analysed using buckling analysis options of PC-based software. Two methods may be used:

- Modelling the compression chord in isolation with the U-frame stiffness represented by spring supports
- Modelling the complete structure on a three-dimensional basis

The design of a simple truss used to support a pedestrian walkway was used to demonstrate this principle, as shown in figure 7.

The arrangement shown in figure 7 is typical of a system where the stiffness of the vertical elements and the transverse beams combine to prevent lateral buckling of the top chord.

The main girders were first analysed as planar structures for nodal loads of 9 kN, which reflects combined unfactored deadand live-load effects. The self-weight of the truss is additional to this. The top chord forces are given in figure 8.

These forces were used to size the members assuming lateral restraint at each node point and the preliminary member sizes given in table 5 were adopted.

The in-plane Euler buckling resistance of the top chord can be determined from the section properties and is equal to:

$$P_e = \frac{\pi^2 E \cdot I_y}{I_c^2} = 645 \text{ kN}$$

For out-of-plane buckling one would strive to obtain a similar buckling strength.

Three alternative methods used to establish the out-of-plane resistance of the top chord will now be discussed in turn.

Top chord analysed in isolation

Compression member with spring supports
Top chord modelled as a strut with rigid
vertical supports and lateral springs at
the node points and the load distribution

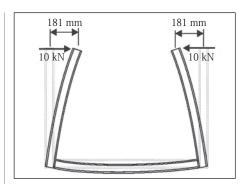


Figure 9 Equivalent lateral spring representing crossframe stiffness

as shown in figure 8 – that is, load varies between bays. The lateral spring is the lateral support afforded by the transverse frames.

The lateral spring stiffness is obtained from a plane frame analysis considering the floor beam in single curvature as shown in figure 9.

The stiffness obtained equals 10/0,181 = 55 kN/m.

The top chord may now be modelled as shown in figure 10. Care must be taken that the software is capable of performing a buckling analysis when springs are used as supports. If not, axially loaded members with equivalent stiffness may be used to replace the springs.

As a first step, the springs are replaced with rigid lateral supports in order to obtain the buckling factor for in-plane buckling. A critical buckling factor of 2,46 was obtained for this case. This value is then consistent with the in-plane buckling resistance of the selected member size and reflects an elastic buckling load of 2,46 x 284,1 kN = 698,9 kN in the critical portion of the top chord.

The lateral restraints were then replaced by springs with a stiffness of 55 kN/m. A critical buckling factor of 2,38 was obtained, reflecting an elastic buckling load of 676,2 kN. Figure 11 shows the increase in the buckling factor with an increase of the U-frame stiffness. A buckling factor in excess of 2,46 would then induce in-plane buckling rather than out of plane buckling and a further increase in U-frame stiffness would be a waste of material. A stiffness of about 60 kN/m would suffice.

Theoretical stiffness requirement for top chord buckling

Winter (1960) used the theory of a compression member on an elastic foundation to determine the stiffness requirement of continuous lateral support. The theoretical value of the modulus of the elastic support medium, β_{id} , can be obtained from the following formulae:

$$\frac{\beta_{id}L^2}{P_E} = \pi^2 \left(\frac{P_{cr}}{P_E} - 1\right) \text{ for } 0 < \frac{\beta_{id}L^2}{P_E} \le 30$$

$$\frac{\beta_{id}L^2}{P_E} = \frac{\pi^2}{4} \left(\frac{P_{cr}}{P_E} - 0.6 \right)^2 \text{ for } \frac{\beta_{id}L^2}{P_E} \ge 30$$

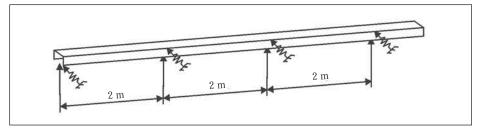


Figure 10 Top chord modelled in isolation

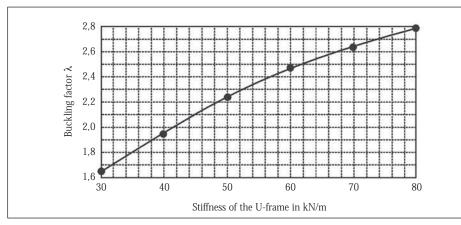


Figure 11 Increase in the buckling factor with an increase in the stiffness of the U-frame

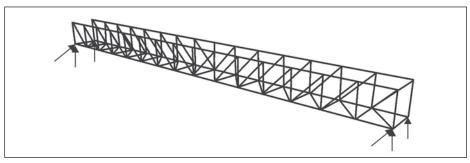


Figure 12 Space truss modelling of trusses and floor elements

Where:

 eta_{id} = the ideal stiffness of the elastic support in kN/m/m

 P_{cr} = the critical load

 P_E = the Euler buckling strength, based on the full laterally unsupported length

L = the total length of the member

This method is consistent with the case of a top chord that is braced by U-frames.

One can apply these equations to the top chord of the girder truss with U-frames as lateral support for out of plane buckling. One must, however, assume uniform axial loading to use these equations.

$$P_e = 30,5 \text{ kN}$$

 $P_{cr} = 698,9 \text{ kN}$

$$\frac{\beta_{id} L^2}{P_E} = 7.331 \le 30$$

$$\frac{\beta_{id}L^2}{P_E} = \frac{\pi^2}{4} \left(\frac{P_{cr}}{P_E} - 0.6 \right)^2 \text{ thus governs}$$

 $\beta_{\rm req}$ = 41,65 kN/m/m which at 2 m spacing would require a spring stiffness of 83,3 kN/m.

As the axial load is not uniform it would not be inconsistent for a spring with slightly less stiffness to suffice.

Full three-dimensional buckling analysis of structure

Modelling the top chord in isolation does not account for the influence of the vertical compression members and the diagonal tension members in the truss. As a third step, the entire system was modelled including both trusses as well as the transverse beams. This model is shown in figure 12.

Unfactored loads were applied to the node points and a buckling analysis was performed. The buckling factor in this case was 2,04 for symmetrical buckling. Increasing the stiffness of the U-frame to 112 kN/m by using a PFC - 120 x 55 increased the symmetrical buckling factor to a value of 3,01. The primary buckling mode changed from a symmetrical to an anti-symmetrical torsional buckling mode with the structure rotating about its longitudinal axis. The buckling factor for the anti-symmetrical buckling was 2,83, which is still greater than the required 2,48.

CONCLUSIONS

■ The use of a uniform effective length factor between 0,7 and 0,85 for the design of compression members in lattice trusses cannot be justified by rigorous analysis.

An elastic buckling analysis using PC-based software provides the most reliable method of calculating the buckling lengths of such members.

- In the absence of a buckling analysis of the whole structure, the use of an effective length factor of 1,0 is recommended for design purposes.
- An elastic buckling analysis provides the means of calculating the correct effective length factor to be used in sizing compression members using design codes. The critical load factor obtained from an elastic buckling analysis is not a safety factor.
- Any system where flexible members provide restraint against buckling is best analysed on a global basis. The approach of EC3 in omitting the use of effective length factors, and using the elastic buckling load in calculating the inelastic buckling resistance, should be encouraged.

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