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New timber column design equations – new formulations and modulus of elasticity values

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Recent bending and compression tests on currently produced SA pine and low-density* Eucalyptus saligna *have shown that the fifth percentile modulus of elasticity, MOE, is lower than the values published in SABS 0163 (1994). These tests have shown that the fifth percentile values should be used when calculating the stability of timber columns and beams. Current editions of SABS 0163 (1994), however, use the mean modulus of elasticity to calculate column strengths. The author presents data and graphs to support his findings and proposes new sets of equations for column strength that look similar to the steel design code equations found in SABS 0162-1(1993). He also suggests reduced fifth percentile modulus of elasticity values to be used for the calculation of compression member strength. Graphs are presented that show the good fit of the new proposed equations with the current accepted equations.

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INTRODUCTION

In South Africa the allowable stress design of timber columns has been based on the Perry-Robertson formula. For the allowable stress design the modulus of elasticity is divided by 2,22, when calculating the effective Euler buckling strength. It must be remembered that the modulus of elasticity given in the codes is a mean value and that all stress values are based on a fifth percentile value divided by 2,22, that is, values are based on the characteristic strength. When calculating the buckling strength of bending members, the modulus of elasticity is divided by 3,0. This in effect means that the beam buckling strength is based on the characteristic modulus of elasticity and that the column buckling strength is based on a higher value.

If one looks at modulus of elasticity values given in EC5: Part 1 (1992) or BS 5268: Part 2 (1988), two values are given for every grade of timber, namely a minimum and a mean. The minimum reflects the fifth percentile value and is used when designing isolated members such as the deflection of single beams, rather than a beam in a structural system. The lower value is also used when calculating the strength of compression members. This lack of a fifth percentile value is a shortcoming in the South African timber design codes, SABS 0163 – 1 and 2 (1994), and should be addressed.

The South African limit states code is calibrated to the allowable stress code and would have all the errors and omissions of the allowable stress code. In an attempt to establish whether the strength of timber columns was being overestimated, a series of tests were undertaken. The results of these tests and the test procedure are given in a paper by Burdzik (2000). A number of interesting facts emerged from these tests, the most important being that the fifth percentile modulus of elasticity is substantially lower than the values in the codes. It was evident that new design equations were called for. The author felt that if these equations could be given in the same format as the steel design equation or in a more generic format, it would make it easier for designers to also work in timber.

BENDING MODULUS OF ELASTICITY

A short summary of the test results is given in this paper to illustrate the shortcomings of the present design codes.

Test specimens

Two grades of timber were investigated, namely grades 5 and 7. Two samples of grade 5 were obtained, one being specifically chosen with a low density and the other with the normal spread in density. The higher density grade 5 timber and the grade 7 timber were mechanically graded at the sawmill. All specimens, 50 of grade 7 and 100 of grade 5, were measured and weighed to ascertain the density. Moisture content was measured with a Wagner moisture meter. Moistures varied between 7,5% and 12%. This is the normal spread of equilibrium moisture content that one will find in South Africa. As the objective of the investigation was to determine whether the published modulus of elasticity of members was too high or too low, it was felt that more accurate moisture content measurements were unnecessary.

Test method

The specimens were cut to a length of 1 902 mm and tested for bending MOE on flat over a span of 1 778 mm. Although third of span loading is specified in SABS 1122 (1994), central point loading was applied to determine the modulus of elasticity. It was felt that the shear component of the deflection would be negligible as the specimens were being tested on flat (small load, large bending deflection). It was also felt that central point loading would give a slightly lower value for the modulus of elasticity as the

shear deflection component reduces the apparent modulus. Deflection was measured in the middle of the span. SABS 1122 (1994) requires that the span be 18 times the thickness, that is, 648 mm, when determining the modulus of elasticity and modulus of rupture. These specimens were tested over a longer length to determine the average MOE of the full specimen and not just part of the specimen.

Additional tests were undertaken on *Eucalyptus saligna* and these were tested strictly in accordance with the standard method, namely third of span loading with a span of 18 x depth.

Test results

The distribution of the higher-density grades 5 and 7 timber is shown in figures 1 and 2 respectively and the lower-density grade 5 in figure 3.

The summary of the bending test results is given in table 1.

These values are very similar to the values obtained by the South African Bureau of Standards (SABS), who have done a nation wide survey on SA pine. The SABS values are based on samples of 200 specimens from a typical sawmill in each of the forest areas of South Africa. The author suspects that the high fifth percentile value for grade 7 is as a result of the grade 7 having a large percentage of grade 10 timber specimens. The additional tests that were done on low-grade saligna also showed a fifth percentile value for modulus of elasticity that is lower than the values given in SABS 0163 (1994).

COMPRESSION TESTS

Test setup

The specimens used to determine the modulus of elasticity were cut to an initial length of 1 902 mm. To ascertain the strength increase with a decrease in slenderness, it was hoped that each specimen could be tested and then shortened from a slenderness value, L/b, of 52 to a slenderness of about 21 in four steps, namely lengths of 1 902, 1 616, 1 236 and 745. Lateral buckling deflections would be limited so as not to damage the compression fibres.

The load cells used to monitor the applied force were calibrated in a Budenberg, 50 kN hydraulic load cell calibrator. The Budenberg uses dead weights and difference in hydraulic cylinder diameters to increase the load at the load cell. The cylinders rotate to eliminate friction between the cylinder walls. The linear variable displacement transducers (LVDTs) had a measurement range of 20 mm and these were calibrated by using a digital dial gauge with an accuracy of 0,001 mm. All load deflection curves were monitored by means of a HBM Spider 8

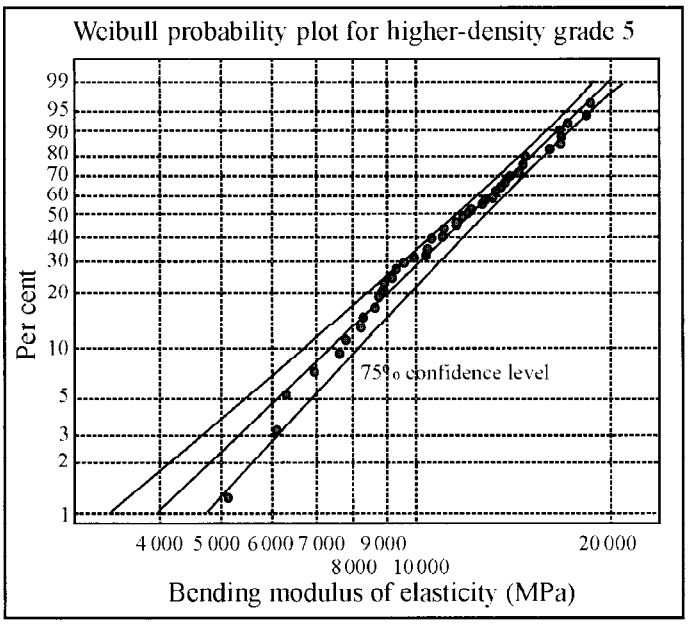


Figure 1 Distribution of the modulus of elasticity of the higher-density grade 5 timber

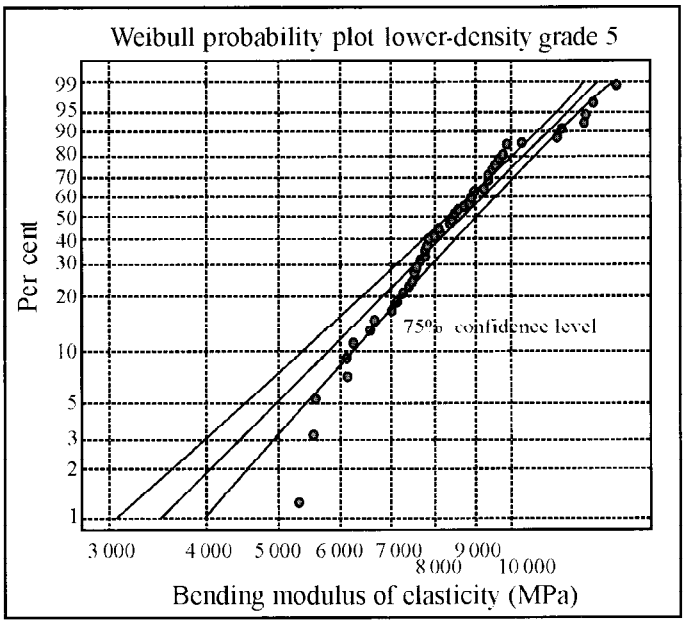


Figure 3 Distribution of the modulus of elasticity of the lower-density grade 5 timber

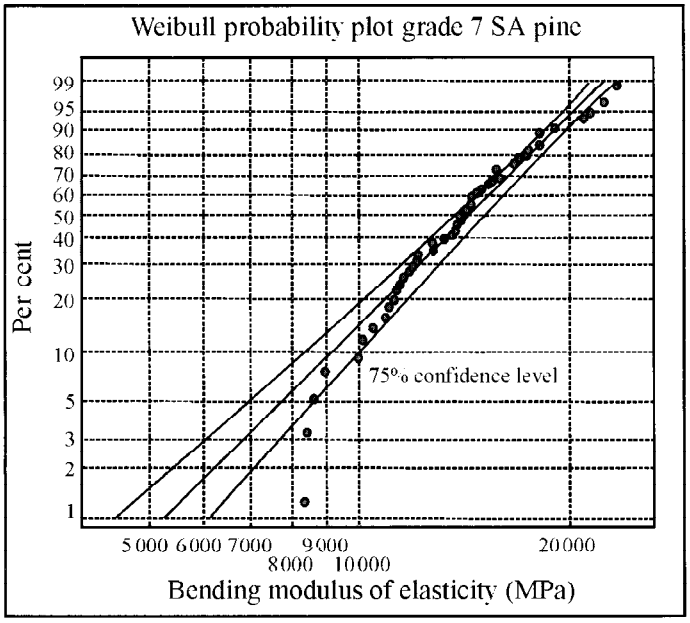


Figure 2 Distribution of the modulus of elasticity of the grade 7 timber

Table 1 Bending modulus of elasticity for SA pine and Eucalyptus from tests done at the University of Pretoria

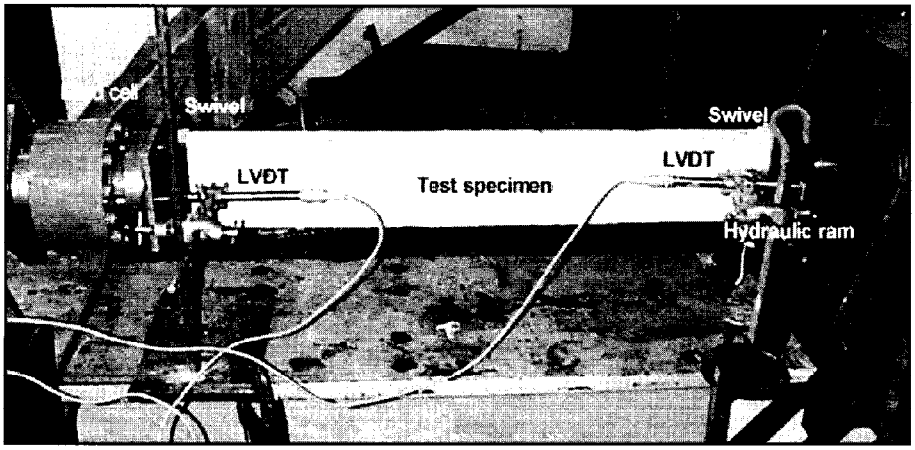
Test specimen grade	Average MOE (MPa)	5% MOE (MPa)
Low-density grade 5	8 601	5 825
Average-density grade 5	12 015	6 578
All grade 5 specimens	10 300	6 100
Average-density grade 7	14 215	8 492
Low-density Eucalyptus	11 506	8 358

data logger and recorded on a computer.

The compression was applied in a horizontal test bed, as shown in photograph 1, with the specimen's weakest axis vertical to the test bed. This ensured that lateral buckling would occur and that the self-weight of the members did not affect the initial curvature or the buckling load. Prior to and after testing, the initial curvature was measured at the centre to ensure that no permanent damage had been caused during the loading cycle. It was found that the way in which the boards were stacked caused bigger changes in the initial curvature than did the loading. Loading was applied until lateral deflections of approximately length/30 were obtained. By that stage the specimens looked as if buckling had taken place.

Three different-shaped load-deflection curves were obtained from the compression tests. The first shape, figure 4, is of a strut with a pre-curvature (initial bow). The second buckling curve, figure 5, is a typical Euler buckling of a straight strut, and the last, figure 6, of either a very straight strut or a strut that seems to have a bias towards buckling in double curvature, but then snaps through into single curvature. A bias towards double curvature could be as a result of imperfections in the member, such as knots or even a slight bow in both directions, which cannot easily be seen. The member will always snap from double curvature to single curvature, which is the state of lowest potential energy, unless the central position is held in place by an external force. The theoretical buckling strengths in figures 4, 5 and 6 are based on the bending modulus of elasticity found during the bending tests.

Only the very straight members or those that seemed to have a bias towards double curvature managed to reach the Euler buckling load. What was also apparent was that the safe load on a timber column could be governed not by the ultimate load but rather by a deflection criterion. Figure 7 shows a theoretical analysis of a member, S4, with MOE = 6 116 MPa and with an initial curvature. The graph shows how the increase in bow increases rapidly long before the buckling load is reached. With a lateral deflection of L/30, that is 54 mm, P is about 0,83 of P Euler. If this is translated into a stress, for a grade 5 with modulus of elasticity of 6 116 MPa, the characteristic stress would be 2,07 MPa.



Photograph 1 Loading of compression members

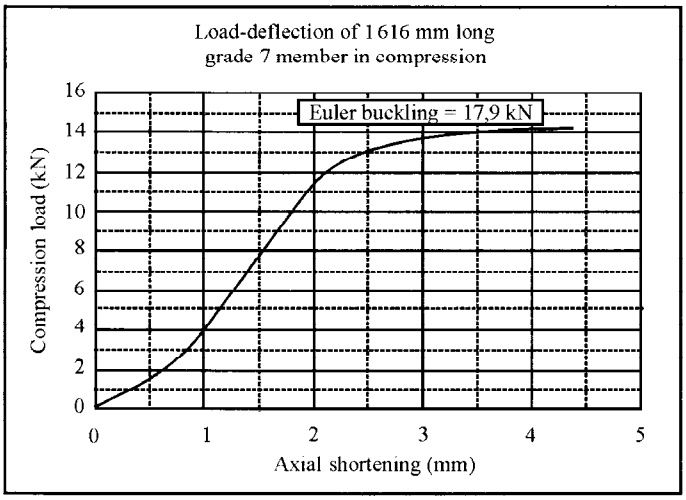


Figure 4 Compression strength of member with pre-curvature

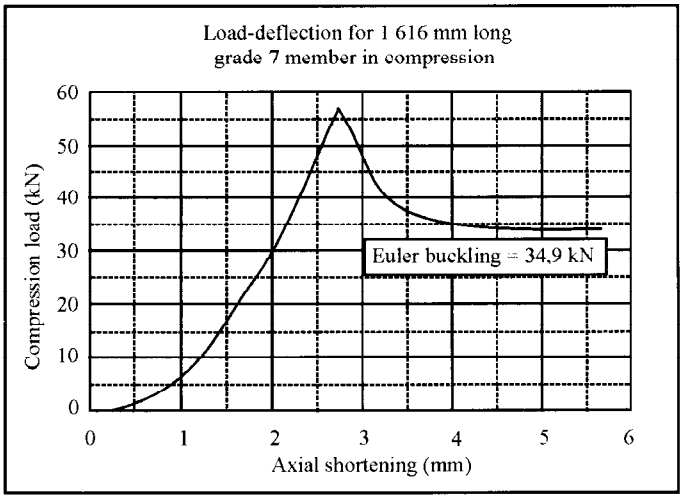


Figure 6 Compression strength of a straight member or member that has a bias to double curvature

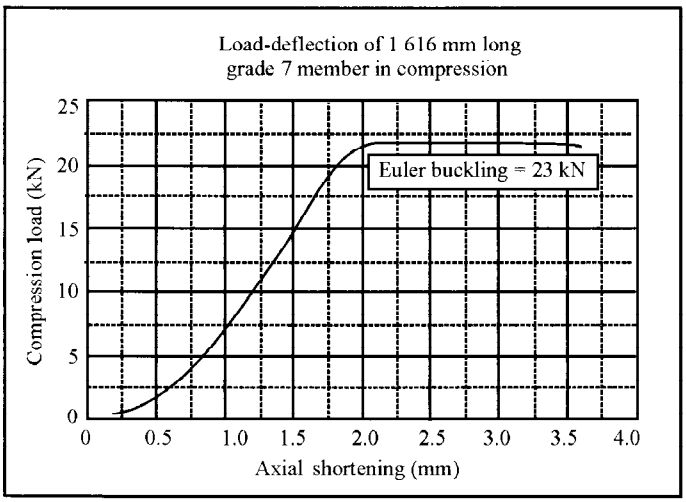


Figure 5 Compression strength of straight member or member with very little pre-curvature

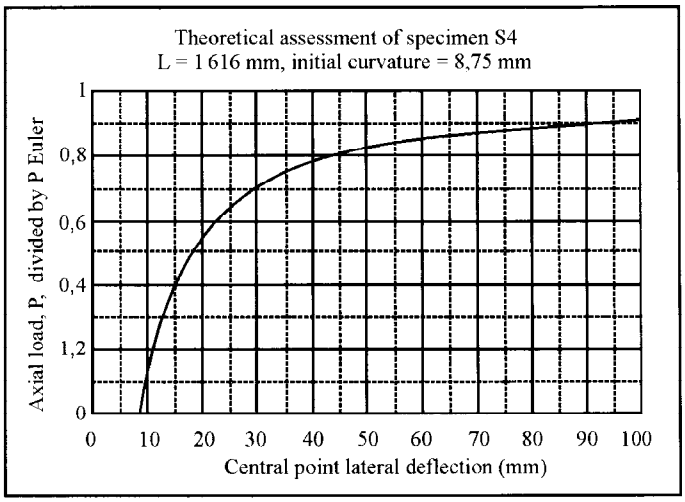


Figure 7 Theoretical evaluation of a compression member, S4, MOE = 6 116 MPa, with pre-curvature

Bear in mind that SABS 1783:1997 allows an initial central deflection of L/100. For specimen S4 this would translate into a value of 16 mm. This is excessive and should only be seen as the absolute maximum and not as a fifth percentile value. A more realistic value for a fifth percentile would be 8 mm/1 000 mm, which, for the above specimen, would translate into a value of 13 mm. For compression members that have a pre-curvature, the strength could be governed by serviceability considerations rather than strength.

NEW FORMULATION OF DESIGN EQUATION

The South African steel design code has defined a slenderness ratio whereby the yield stress is divided by the Euler buckling stress. They have also managed to reduce the five equations that defined column buckling to one equation in terms of the slenderness and a constant, which depends on the residual stresses. It was hoped that these equations could be used to describe timber column behaviour and also reduce the three equations that describe the buckling strength to two equations, one for squashing strength and one for buckling. The author realises that some minor sacrifices in strength assumptions may be required so that the design of columns could be simplified.

Limit-states formulation of compression strength

The strength of a column can be written as the lesser of the following:

$$C_r = \phi \cdot A_g \cdot \frac{f_c}{\gamma_{m1} \cdot \gamma_{m2} \cdot \gamma_{m3} \cdot \gamma_{m4} \cdot \gamma_{m5}} \quad (1)$$

where

C_r = the compressive resistance, that is the column squash load
ϕ = capacity reduction factor that takes the strength distribution of the material into account
A_g = gross area
f_c = characteristic compressive stress, material failure
γ_{mn} = modification factors

Table 2 Proposed fifth percentile MOE to be used in column equations

Grade	5% MOE (MPa)	5% MOE from mean (MPa)	Avg MOE (MPa)	f_{cy} (MPa)
5	6 100	5 780	7 800	18,0
7	7 730	7 110	9 600	22,8
10	8 880	8 890	12 000	26,2

$$C_r = \phi \cdot \beta_b \cdot A_g \cdot \frac{f_{cy}}{\gamma_{m1} \cdot \gamma_{m2} \cdot \gamma_{m3} \cdot \gamma_{m4} \cdot \gamma_{m5}} \quad (2)$$

where

β_b = buckling factor
f_{cy} = theoretical characteristic compressive yield stress, slenderness tending to 0

The buckling factor, β_b is given by:

$$\beta_b = \left(1 + \lambda^{2n}\right)^{-1/n} \quad (3)$$

with n = 1,8 and

$$\lambda = \frac{K \cdot L}{r} \cdot \sqrt{\frac{f_{cy}}{\pi^2 \cdot E_5}} \quad (4)$$

where

E_5 = fifth percentile modulus of elasticity
$K{\cdot}L$ = effective length
R = radius of gyration, $\sqrt{\frac{I}{A}}$

Alternatively the equation can be written in terms of the width of the member, b, with:

$$\lambda = \frac{\sqrt{12} \cdot K \cdot L}{b} \cdot \sqrt{\frac{f_{cy}}{\pi^2 \cdot E_5}} \quad (5)$$

If one applies this formula to the member shown in the theoretical graph, figure 7, the slenderness is 2,689 with a buckling factor of 0,136 with a resultant stress of 2,45 MPa. Compare this to the value given in SABS 0163:1, which gives a characteristic stress of 3,15 MPa, based on a MOE of 7 800 MPa.

Similar equations may be used for the allowable stress design, which will lead to simplification of column stress equations.

Allowable stress design formulation

The allowable stress for columns is the lesser of the following:

$$f_b = p_c \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot k_5 \quad (6)$$

where

p_c = grade compressive stress, squashing stress
k_n = modification factors

$$f_b = \beta_b \cdot p_{cy} \cdot k_1 \cdot k_2 \cdot k_3 \cdot k_4 \cdot k_5 \quad (7)$$

where

p_{cy} = theoretical grade compressive stress for slenderness tending to 0
$\beta_b = \left(1 + \lambda^{2n}\right)^{-1/n}$ with n = 1,8

$$\lambda = \frac{K \cdot L}{r} \cdot \sqrt{\frac{2.22 \cdot p_{cy}}{\pi^2 \cdot E_5}} \quad (8)$$

The difference between the ratios of the f_{cy}/E or p_{cy}/E values could be taken into account by varying the value of n in the buckling factor equation. Before complicating the equations and the method it is perhaps wise to see whether the difference is really significant. It may be possible to have a constant ratio between modulus of elasticity and the compressive stress without sacrificing too much of the strength. It must also be remembered that at best code equations are nothing more than curves that are fitted to some test data. These equations should not be viewed as cast in stone. If the compressive stress is well defined or well tested, it is perhaps better to adjust the modulus of elasticity. Grade 5 timber is the most popular timber in the truss industry and perhaps this should serve as the base ratio for modulus of elasticity to compressive stress. Alternatively, the mean modulus of elasticity could be divided by 1,35 to get an approximate fifth percentile value. Table 2 gives the proposed stress and MOE values; the second column is estimated from the measured fifth percentile of grade 5 and the third column from dividing mean value by 1,35. Graph 8 compares the proposed buckling factor with the presently accepted values. Graph 9 compares the proposed buckling factor with the present values, reduced to allow for the reduced modulus of elasticity.

CONCLUSION

Tests done at the University of Pretoria and by the South African Bureau of Standards have shown that the fifth per-

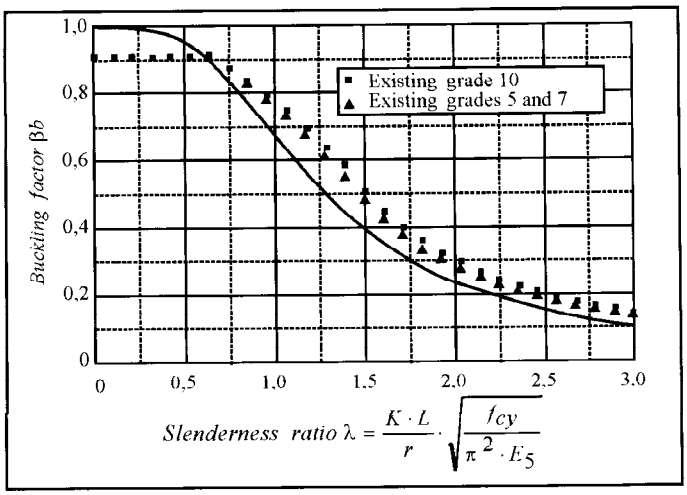


Figure 8 New buckling curve versus old curves with too high modulus of elasticity, limit-states equations

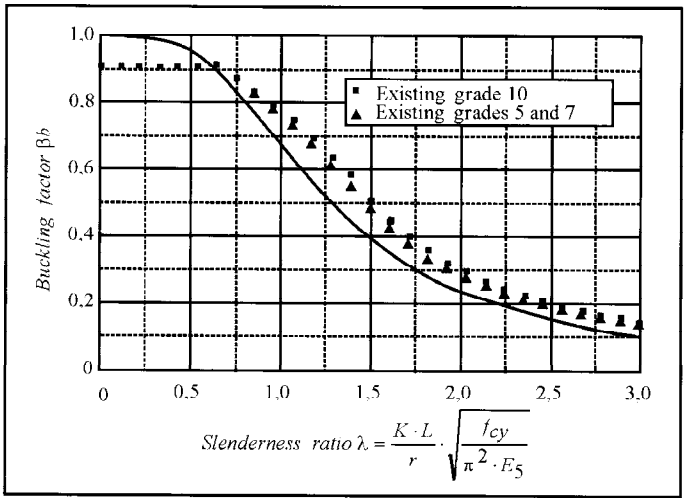


Figure 9 New buckling curve versus old buckling curve with corrected modulus of elasticity, limit-states equations

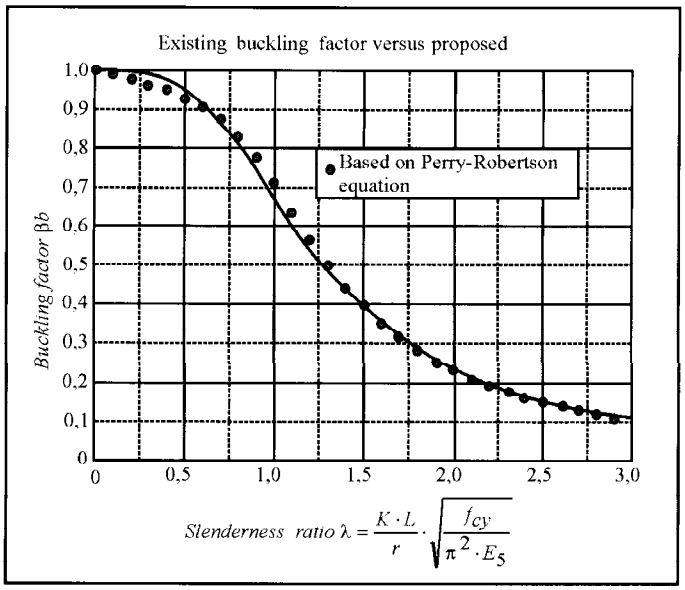


Figure 10 New proposed buckling curve versus Perry-Robertson equation with modulus of elasticity reduced to fifth percentile value, allowable stress

centile modulus of elasticity in bending is less than the values published in SABS 0163 (1994). It is thus imperative that the modulus of elasticity be reduced to reflect the true fifth percentile value. If this is done it makes sense to change the formulation of the buckling strength equation of compression members to a format that is very similar to that of the steel code.

At best, the design equations are a fit to some data from tests not necessarily done in South Africa. The fact that the proposed curve does not fall exactly on top of the existing curves should not be a deterrent to accepting a formulation of the compression member design curve. The advantage of the new curve is that it looks the same as the steel design curve and can be adjusted to fit new test data by changing the value of n in the equation for the buckling factor. SABS 0163 'The structural use of timber Parts 1 & 2' should be amended to reflect the findings of the research.

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