TECHNICAL PAPER

JOURNAL OF THE SOUTH AFRICAN INSTITUTION OF CIVIL ENGINEERING

Vol 48 No 4, 2006, Pages 14–20, Paper 610

Effective stiffness of nailed multiple-ply timber members

Walter Burdzik

WALTER BURDZIK is a professor in the Department of Civil and Biosystems Engineering at the University of Pretoria. He received his degrees from the University of Pretoria and has been involved in timber research and structural timber design for the last 25 years. Walter runs one of the few recognised timber-testing facilities in South Africa and is often involved in queries surrounding the importing of structural timber and wood-based products. He is also involved in the all the SANS committees charged with writing the South African timber design codes and specifications that have to do with wood based structural products. His fields of interest include steel design.

Contact details:
Department of Civil and Biosystems Engineering,
University of Pretoria
T 012-420-2746
walter.burdzik@up.ac.za

Keywords: Composite, flexible connectors, nails, spring model, finite element analysis

In timber roofs, multiple-ply trusses are placed next to each other and the web members are held together with nails. This paper presents various methods, among which is a simple spring model, whereby the effective stiffness of these composite web members may be calculated, based on the stiffness of the connectors and the individual web members. The spring model can be used for any number of connectors and members as long as the members are loaded within the elastic range of both the members and the connectors. Results from a limited number of finite element simulations show that this method has merit. It makes it possible to determine the bending and axial stiffness to an acceptable degree of accuracy. The paper also addresses the calculation of the stresses in the individual members.

INTRODUCTION

Reference books such as Gere (2001) give methods of calculating the stiffness and stresses in composite members. They generally state that the elements are securely fastened to one another. 'Securely fastened' would imply that there is enough shear capacity in the connectors to transfer the load equally to each ply. Gere does not specify that the connection must be rigid and no account is taken of the deformations that occur when the shear connectors transfer the load. This deformation would reduce the section stiffness and must therefore be taken into account. Loss of stiffness will also reduce the section modulus and strength.

The loss in stiffness is important when calculating the axial resistance of the compression chords of multiple-ply timber trusses. The strength of slender members depends on the effective radius of gyration, which in turn depends on the effective member geometry. Steel reinforced timber beams where the steel is fixed to the timber by means of flexible connectors would be subject to the same conditions.

Sound prediction models can help to restrict the number of tests that are required to verify the model. A spring model, as illustrated in this paper, gives one the ability to ascertain how the stiffness changes with changes in connector spacing and stiffness. Furthermore, the method makes it possible to limit the work to the testing of connector stiffness. Connector stiffness may then be used to predict the behaviour of the composite member.

As strength is an ultimate condition and because some connectors portray ductile behaviour under ultimate loading, a spring model for composite members must, of necessity, be restricted to elastic behaviour. The spring model may have an application, where the members remain elastic, but the connectors show ductile inelastic behaviour.

THEORETICAL MODEL

Prediction of the effective stiffness of composite elements

The effective flexural stiffness of composite members is a function of the stiffness of the connecting medium that transfers the shear and the elements that are being connected to form the composite member. This concept may be illustrated by considering two extreme cases, a lower limit where the stiffness of the connecting medium is too low to limit slip at the interface and an upper limit where the connecting medium has a relative stiffness that effectively prevents any slip at the interface. In the first case, the stiffness of the composite member would tend to the sum of the individual stiffnesses. In the second case, the combined stiffness of the system would approach the theoretical stiffness of a fully composite member. The stiffness of the connecting medium and of the individual elements is significant when predicting the deflection and bending stresses in composite flexural members and the slenderness and resistance of axially loaded members.

Proposed theoretical model

Any prediction model should, of necessity, reflect the interaction between the principal parameters. In the case of rigid shear connection, where no slip occurs at the interface, the deformation at the interface of the members will be the same and the stiffness can be described as a series of springs in parallel. The flexural stiffness of a simple composite member may be expressed in terms of the individual member bending stiffness together with an axial stiffness component multiplied by the square of the distance away from the neutral axis. This second term is often referred to as the Steiner term and the combined stiffness can be written as follows:

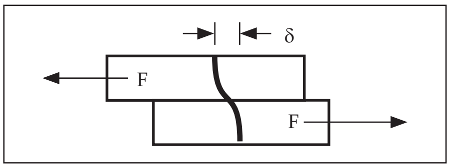

Figure 1 Load-slip that defines the stiffness of a nailed connection

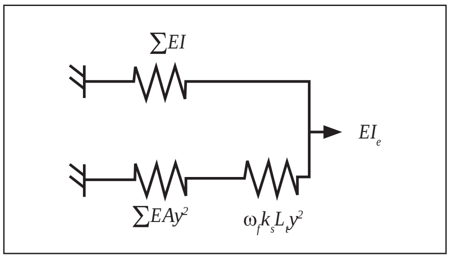

Figure 2 Spring model for predicting the effective stiffness of a composite member

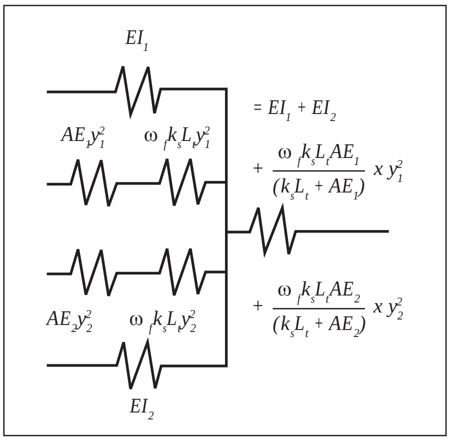

Figure 3 Expanded two-element spring model. In this case both axial stiffnesses are transformed

$$EI_e = \sum_{i=1}^{n} (E_i I_i + E_i A_i y_i^2) \qquad (1)$$

Where

EI_e is the effective stiffness of the composite member

E_iI_i is the individual stiffness of the i^{th} member

n is the number of elements

A_i is the area of the i^{th} member

y_i is the distance from the centre of the member to the neutral axis of the composite

$A_i y_i^2$ is the second moment of area of i^{th} the element about the neutral axis

In the case where the stiffness of the connectors is low compared to the axial stiffness of the individual elements, most of the composite action will disappear and each element will tend to bend about its own neutral axis. The contribution of the $E_i A_i y_i^2$ term of the equation will disappear and the effective flexural stiffness will tend towards the sum of the flexural stiffness of the individual elements.

In the case of members that are connected by flexible connectors, the axial force in the member and the spring is the same and the stiffness may then be modelled by springs in series. The radius of curvature of the individual members will be the same and the bending stiffness may be described

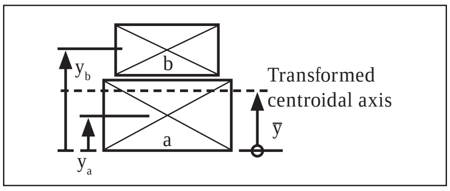

Figure 4 Two-member composite section

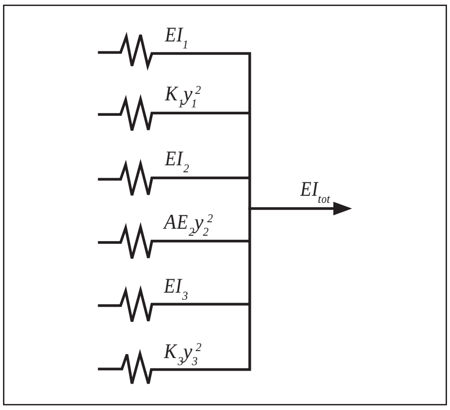

Figure 5 Equivalent spring system for three-member composite

by springs in parallel. The definition of the shear connector stiffness can be seen in figure 1. The stiffness, k_s, of the spring is given by:

$$k_s = \frac{F}{\delta}$$

Composite action may thus be represented by a number of springs acting in series and parallel, representing the stiffness of the individual elements and the connectors. The model is shown in figure 2.

E = modulus of elasticity

EI_e = effective stiffness

k_s = spring constant

L_t = transfer length over which connectors are placed, usually length divided by 2

A = Area of member that is connected by the spring

ω_f = modification term that takes the force distribution in the elements over the length L_t into account

A similar equation for composite members with flexible connectors is given in Eurocode 5 (1995) and can be applied to for up to three members in bending. The basic principles are difficult to follow thus the reason in this paper that basic principles are emphasised. Inadequacies in the equations are then easily identifiable.

The model shown in figure 3 represents a composite system consisting of two elements connected with flexible shear connectors. The same principle may be used for the case of composite members consisting of three or more elements. The combined stiffness of the system is reflected as the sum of the flexural stiffness in parallel with the sum of the Steiner term in series with the connector stiffness. In effect, the individual cross-sectional axial stiffness, AE, is transformed into an equivalent axial stiffness by the springs. The greater the spring stiffness the more the transformed section tends towards the full section and visa versa.

Using the simple two-element model shown in figure 3, the effective stiffness EI_e may be expressed as equation 2:

$$EI_e = EI_1 + EI_2 + \frac{\omega_f k_s L_t AE_1 y_1^2}{(\omega_f k_s L_t + AE_1)} + \frac{\omega_f k_s L_t AE_2 y_2^2}{(\omega_f k_s L_t + AE_2)} \qquad (2)$$

This can be written as follows:

$$EI_e = EI_1 + EI_2 + K_1 y_1^2 + K_2 y_2^2 \qquad (3)$$

Where

AE = the axial stiffness of material above or below the connector

L_t = the shear transfer length, normally the clear span/2 for simply supported beams

k_s = the connector stiffness of all connections in the transfer length

EI_i = modulus of elasticity of the i^{th} member

K_i = the effective axial stiffness of the i^{th} member in series with the spring

y_i = distance from the transformed section centroidal axis to the centre of the individual member

The distance to the neutral axis now depends on the combined stiffness of the spring and the stiffness of the members. Axial stiffness of a member is give by $\frac{AE}{L}$ and the stiffness of the spring by k_s. As the members are in series the combined stiff ness is give by $\frac{\omega_f k_s L_t AE}{\omega_f k_s L_t + AE}$. If one calls the individual stiffnesses K_i, it is relatively easy to calculate the distance to the transformed section centroidal axis. This can be calculated as follows:

$$\bar{y} = \frac{K_a y_a + K_b y_b}{K_a + K_b} \qquad (4)$$

Equation 2 clearly illustrates the influence of the shear connection stiffness on the effective flexural stiffness of the composite member. In order to obtain some appreciation of the sensitivity of equation 2 to the ratio between spring stiffness and element stiffness, a simple composite member consisting of two rectangular sections of dimensions b by $h/2$, with constant E, may be considered. Substituting the appropriate values into equation 2, will yield the following expression for the effective stiffness:

$$EI_e = \frac{Ebh^3}{48} + \frac{Ebh^3}{16}\left[\frac{\omega_f k_s L_t}{\omega_f k_s L_t + AE}\right] \qquad (5)$$

As k_sL_t tends towards 0, the second term tends towards 0 with the resultant EI_e being the same as two unconnected members lying on top of one another. As k_sL_t

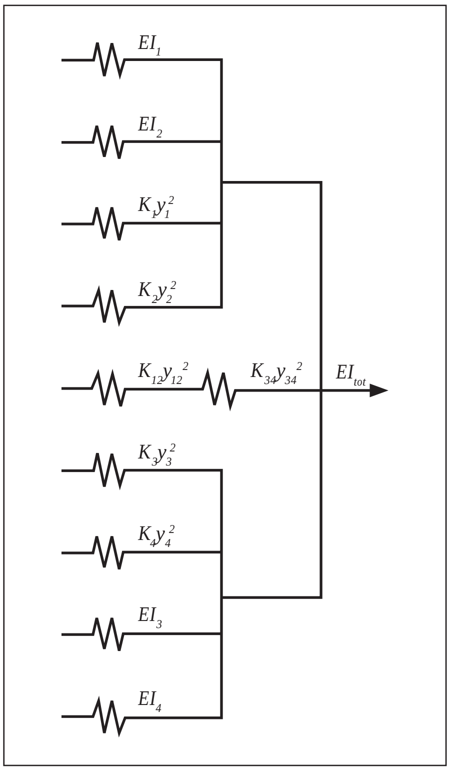

Figure 6 Equivalent spring system for four-member composite

tends towards infinity the expession in the brackets of the second term will tend towards 1 with the effective EI_e tending towards two members that are rigidly connected, that is,

$$EI_e = \frac{Ebh^3}{12}$$

This spring model can be expanded to three- and four-member composite sections, with the three member being represented by figure 5 and a four member by figure 6.

NAIL STIFFNESS

So as not to do unnecessary testing, it was decided to use the Eurocode 5 (1995) nail-stiffness formulation, Section 7, Serviceability limit-states. The nail stiffness is written in terms of the diameter of the nail and the density of the timber. This formulation takes creep of the joint into account and can be seen as a long duration loaded stiffness. For nails without pre-drilling, k_s is given by:

$$k_s = \frac{\rho_m^{1,5} d^{0,8}}{30}$$

Where
k_s = nail stiffness in kN/m or N/mm
d = diameter of the nail in mm
ρ_m = the density of the timber in kg/m^3
The South African timber truss industry generally uses two nail lengths and diameters which are in accordance with SANS 1082(2002), namely 75 mm long by 3,5 mm diameter and 100 mm long by 4 mm diameter. The density of the timber used lies between 360 kg/m^3 and 500 kg/m^3. A density of 450 kg/m^3 was assumed in this paper. The long-term stiffness of the nails can be taken as 866 kN/m and 965 kN/m for the 75 mm and 100 mm long nails respectively.

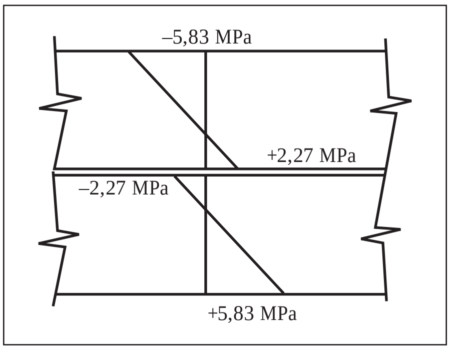

Figure 7 Stress distribution for two-ply member example with full shear transfer

Sample calculations

Two-ply member

Assume that two 36 x 111 mm grade 5 members, spanning 1,5 m, are nailed together with 75 mm nails spaced so that full shear is transferred. A central point load of 1,2 kN is applied to the beam so that a bending stress of 4,69 MPa is induced in the outer fibres of the full theoretical solid section. A 3,5 mm diameter nail can transfer 0,26 kN in shear and as the shear flow is 12,5 N/mm the spacing of the nails will be about 20 mm for full shear transfer. The bending moment = 0,45 kN.m. The modulus of elasticity of the timber is given as 7 800 MPa in SANS 10163 (2004). The transfer length is span/2, that is, 750 mm, and there will be 38 nails in the transfer length, L_t. ω_f is the factor for the force transfer. The force transfer is assumed to be triangular and ω_f will, in that case, be 0,5. The nail stiffness is k_s = 866 kN/m. As the members are going to move in opposite directions and one is working on the interface, the nail stiffness doubles to 1 732 kN/m.

AE_1 = 31,17 MN
AE_2 = 31,17 MN
EI_1 = 3,366 kN.m^2
EI_1 = 3,366 kN.m^2

y_a = 54 mm
y_b = 18 mm

$\omega_f k_s L_t$ = 0,5 x 38 x 1 732 x 0,75
= 24,681 MN

The transformed axial stiffness of the members is given by K_1 and K_2:

$$K_1 = \frac{\omega_f k_s L_t \times AE}{\omega_f k_s L_t + AE} = \frac{24{,}681 \times 31{,}17}{24{,}681 + 31{,}17} = 13{,}774 \text{ MN}$$

$$K_2 = \frac{\omega_f k_s L_t \times AE}{\omega_f k_s L_t + AE} = \frac{24{,}681 \times 31{,}17}{24{,}681 + 31{,}17} = 13{,}774 \text{ MN}$$

$$\bar{y} = \frac{K_1 y_a + K_2 y_b}{K_1 + K_2} = \frac{13{,}774 \times 54 + 13{,}774 \times 18}{13{,}774 + 13\,774} = 36 \text{ mm}$$

The effective stiffness of the composite member, EI_e is:

$$EI_e = EI_1 + EI_2 + K_1 y_1^2 + K_2 y_2^2$$

$$EI_e = 2 \times 3{,}366 + 13{,}774 \times 10^3 \times (0{,}054 - 0{,}036)^2 + 13{,}774 \times 10^3 \times (0{,}018 - 0{,}036)^2$$

$$EI_e = 15{,}66 \text{ kN.m}^2$$

A finite element analysis, which will be discussed later, gives a stiffness of 15,19 kN.m^2.

The bending moment carried by the top member is then:

$$\frac{M \times (EI_1 + K_1 y_1^2)}{EI_e} = \frac{0{,}45 \times (3{,}366 + 13{,}774 \times 10^3 \times 0{,}018^2)}{15{,}66}$$

M_{top} = 0,225 kN.m. The moment carried in bending = $\frac{M_{top} \times EI_1}{EI_1 + K_1 y_1^2}$ = 0,097 kN.m.

The moment carried in axial force M_{axial} = 0,225 – 0,097 = 0,128 kN.m. The axial force, $P = \frac{M_{axial}}{y_1} = 7{,}111$ kN.

The stress in the top member:

$$\sigma = \frac{-P}{A} \pm \frac{M}{Z} = \frac{-7{,}111 \times 10^3}{36 \times 111} \pm \frac{0{,}097 \times 10^6 \times 6}{111 \times 36^2} = -1{,}780 \pm 4{,}046 \text{ MPa}$$

Even though one has designed for full shear transfer, the bending stress in the top member will be –5,83 MPa at the top and +2,27 MPa at the bottom (compression negative, tension positive). This stress distribution is shown in figure 7.

The effective stiffness of the composite section, EI_e = 15,66 kN.m^2, that is connected by means of flexible connectors is substantially lower than that of a full 111 mm x 72 mm section, which is equal to 26,93 kN.m^2.

The highest stiffness that the nails can achieve in the long term is 15,66 kN.m^2, which is 58 % of that of the full section. For short term loading the stiffness of the nail is in the region of 1 443 kN/m and the effective EI will in that case be equal to 18,21 kN.m², which still falls short of the full section stiffness.

An alternative method of calculating the effective EI of a two-ply member, as in the example above is as follows.

Add the spring to the one member only and transform the stiffness of the member accordingly. The nail has a stiffness of 866 kN/m.

AE_1 = 31,17 MN
AE_2 = 31,17 MN
EI_1 = 3,366 kN.m²
EI_1 = 3,366 kN.m²

y_a = 54 mm
y_b = 18 mm

$\omega_f k_s L_t$ = 0,5 x 38 x 866 x 0,75
= 12,340 MN

$$K_1 = \frac{\omega_f k_s L_t \times AE}{\omega_f k_s L_t + AE} = \frac{12,340 \times 31,17}{12,340 + 31,17} = 8,8405 \text{ MN}$$

$$\bar{y} = \frac{K_1 y_a + AE_2 y_b}{K_1 + K_2} = \frac{8,8405 \times 54 + 31,17 \times 18}{8,8405 + 31,17} = 25,95 \text{ mm}$$

$$EI_e = EI_1 + EI_2 + K_1 y_1^2 + K_2 y_2^2$$

$$EI_e = 2 \times 3,366 + 8,8405 \times 10^3 \times (0,054 - 0,02595)^2 + 31,17 \times 10^3 \times (0,018 - 0,02595)^2$$

EI_e = 15,66 kN.m²

Three-ply member

Assume that three 36 mm x 111 mm grade 5 members, spanning 1,8 m, are nailed together with 75 mm nails spaced at 150 mm apart on the one side and at 75 mm apart on the other. The transfer length is 0,9 m and there will be six nails in the transfer length, L_t, and 12 on the other. ω_f is the factor for the force transfer and is equal to 0,5. As the top member and the top of the central member move in the same direction, the nail stiffness k_s remains equal to 866 kN/m.

AE_1 = 31,17 MN
AE_2 = 31,17 MN
AE_3 = 31,17 MN
EI_1 = 3,366 kN.m²
EI_2 = 3,366 kN.m²
EI_3 = 3,366 kN.m²

y_a = 90 mm,
y_b = 54 mm,
y_c = 18 mm

$\omega_f k_s L_{t1}$ = 0,5 x 6 x 866 x 0,90
= 2,338 MN

$\omega_f k_s L_{t2}$ = 0,5 x 12 x 866 x 0,90
= 4,676 MN

$$K_1 = \frac{\omega_f k_s L_{t1} \times AE}{\omega_f k_s L_{t1} + AE} = \frac{2,338 \times 31,17}{2,338 + 31,17} = 2,175 \text{ MN}$$

$$K_3 = \frac{\omega_f k_s L_{t2} \times AE}{\omega_f k_s L_{t2} + AE} = \frac{4,676 \times 31,17}{4,676 + 31,17} = 4,066 \text{ MN}$$

$$\bar{y} = \frac{K_1 y_a + AE_2 y_b + K_3 y_c}{K_1 + AE_2 + K_3} = \frac{2,175 \times 90 + 31,17 \times 54 + 4,066 \times 18}{2,175 + 31,17 + 4,066} = 52,18 \text{ mm}$$

The effective stiffness of the composite member EI_e is:

$$EI_e = EI_1 + EI_2 + EI_3 + K_1 y_1^2 + AE_2 \times y_2^2 + K_3 y_3^2$$

$$EI_e = 3 \times 3,366 + 2,175 \times 10^3 \times (0,090 - 0,0522)^2 + 31,17 \times 10^3 \times (0,054 - 0,0522)^2 + 4,066 \times 10^3 \times (0,018 - 0,0522)^2$$

EI_e = 18,06 kN.m²

A finite element analysis, which will be discussed later, gives a stiffness of 17,53 kN.m².

EI of the equivalent solid section = 90,89 kN.m² which is five times the stiffness of the nailed-together section.

Four-ply member

When analysing four-ply members one should first analyse the outer members as two connected plies and then analyse the connection of these two members to give the result for the four-ply member.

The current method of connecting multiple-ply trusses together is by means of 100 mm nails and the following will be used to illustrate the method. Assume four 36 x 111 mm members that are nailed together with 100 mm long nails and which will span 3,0 m.

The first two are nailed together and the nails clinched. The next ply is added and the nail would then penetrate all three members. The last ply is added and the nails would penetrate the top three plies. If the nails are spaced at 150 mm, then between plies 1 and 2 and 2 and 3, the spacing is effectively 75 mm and between 3 and 4, 150 mm.

The nail stiffness k_s = 965 kN/m.ω_f is the factor for the force transfer and is equal to 0,5. The transfer length is 1,500 m and there will be 20 and 10 nails when spaced at 75 mm and 150 mm respectively.

AE_1 = 31,17 MN
AE_2 = 31,17 MN
AE_3 = 31,17 MN
AE_4 = 31,17 MN
EI_1 = 3,366 kN.m²
EI_2 = 3,366 kN.m²
EI_3 = 3,366 kN.m²
EI_4 = 3,366 kN.m²

$\omega_f k_s L_{t12}$ = 0,5 x 20 x 965 x 1,5
= 14,475 MN

$\omega_f k_s L_{t23}$ = 0,5 x 20 x 965 x 1,5
= 14,475 MN

$\omega_f k_s L_{t34}$ = 0,5 x 10 x 965 x 1,5
= 7,237 MN

Members 1 and 2 combined:

$$K_1 = \frac{\omega_f k_s L_{t12} \times AE}{\omega_f k_s L_{t12} + AE} = \frac{14,475 \times 31,17}{14,475 + 31,17} = 9,885 \text{ MN}$$

$$K_2 = \frac{\omega_f k_s L_{t12} \times AE}{\omega_f k_s L_{t12} + AE} = \frac{14,475 \times 31,17}{14,475 + 31,17} = 9,885 \text{ MN}$$

$$\bar{y} = \frac{K_1 y_a + K_2 y_b}{K_1 + K_2} = \frac{9,885 \times 126 + 9,885 \times 90}{9,885 + 9,885} = 108 \text{ mm}$$

$$EI_{12} = EI_1 + EI_2 + K_1 y_1^2 + K_2 y_2^2$$

$$EI_{12} = 2 \times 3,366 + 9,885 \times 10^3 \times (0,126 - 0,108)^2 + 9,885 \times 10^3 \times (0,90 - 0,108)^2$$

EI_{12} = 13,137 kN.m² (about the neutral axis of 1 and 2 combined)

Members 3 and 4 combined:

$$K_3 = \frac{\omega_f k_s L_{t34} \times AE}{\omega_f k_s L_{t34} + AE} = \frac{7,237 \times 31,17}{7,237 + 31,17} = 5,873 \text{ MN}$$

$$K_4 = \frac{\omega_f k_s L_{t34} \times AE}{\omega_f k_s L_{t34} + AE} = \frac{7,237 \times 31,17}{7,237 + 31,17} = 5,873 \text{ MN}$$

$$\bar{y} = \frac{K_3 y_a + K_4 y_b}{K_3 + K_4} = \frac{9,885 \times 54 + 9,885 \times 18}{9,885 + 9,885} = 36 \text{ mm}$$

Table 1 Comparison between the various methods for a two-ply web member

Model	Effective stiffness of two-ply member, kN.m²							
	1,5 m length		1,8 m length		2,4 m length		3,0 m length	
	Nails at 150 mm	Nails at 300 mm	Nails at 150 mm	Nails at 300 mm	Nails at 150 mm	Nails at 300 mm	Nails at 150 mm	Nails at 300 mm
Spring	8,64	7,73	9,37	8,14	10,99	9,11	12,67	10,22
Shell	8,63	7,87	9,25	8,25	10,64	9,10	12,11	10,07
Beam	8,51	7,75	9,15	8,17	10,55	9,03	12,02	9,99

Table 2 Individual nail stiffness values

Specimen	Stiffness kN/m
1	1 721,48
2	985,06
3	1 183,49
4	1 768,04
5	1 740,89
6	1 517,54
8	1 764,09
9	1 049,13
11	1 658,47
13	787,80
15	1 174,73
16	1 319,82
17	556,27
18	1 395,59
19	1 853,12
20	1 243,07
Average	1 357,41
Fifth percentile	729,92

$$EI_{34} = EI_3 + EI_4 + K_3 y_3^2 + K_4 y_4^2$$

$$EI_{34} = 2 \times 3,366 + 5,873 \times 10^3 \times (0,054 - 0,036)^2 + 5,873 \times 10^3 \times (0,018 - 0,036)^2$$

$$EI_{34} = 10,54 \text{ kN.m}^2$$

The two sections are now connected by the spring 2 to 3. The transformed axial stiffness of the members 1 and 2 is $K_1 + K_2$ = (9,885 + 9,885) MN = 19,770 MN. The transformed axial stiffness of the members 3 and 4 is $K_3 + K_4$ = (5,873 + 5,873) MN = 11,746 kN.

Because of the spring between members 2 and 3 the axial stiffness of the combined member 1,2 is transformed once again. Remember that the two sections move in opposite directions as in the case of the two-ply member, so the stiffness of the connection should be doubled:

$$K_{12} = \frac{\omega_f k_s L_{t23} \times (K_1 + K_2)}{\omega_f k_s L_{t23} + (K_1 + K_2)} = \frac{28,950 \times 19,770}{28,950 + 19,770} = 11,747 \text{ MN}$$

$$K_{34} = \frac{\omega_f k_s L_{t23} \times (K_3 + K_4)}{\omega_f k_s L_{t23} + (K_3 + K_4)} = \frac{28,950 \times 11,746}{28,950 + 11746} = 8,356 \text{ MN}$$

Table 3 Individual modulus of elasticity for test specimens

Specimen	MOE (MPa)
1	16 256
2	13 963
3	8 081
4	13 839
5	17 871
6	12 912
7	4 820
8	11 923
9	9 286
10	6 484
11	7 824
12	12 908
13	11 139
14	5 960
15	9 259
16	11 894
17	6 733
18	7 693
19	8 274
20	17 295
Mean	10 721
Standard deviation	3 865

$$\bar{y} = \frac{K_{12} y_a + K_{34} y_b}{K_{12} + K_{34}} = \frac{11\,747 \times 108, +8\,356 \times 36}{11,747 + 8,356} = 78,07$$

The effective stiffness of the composite member, $EI_{tot,}$ is:

$$EI_{tot} = EI_{12} + EI_{34} + K_{12} y_{12}^2 + K_{34} y_{34}^2$$

$$EI_{tot} = 13,137 + 10,538 + 11,747 \times 10^3 \times (0,108 - 0,07807)^2 + 8,356 \times 10^3 \times (0,036 - 0,07807)^2$$

$$EI_{tot} = 48,99 \text{ kN.m}^2$$

The finite element method gives a stiffness of 45,84 kN.m², which differs from the spring model value by 7 %.

Compare this to the effective stiffness if no nails are present and when a rigid connection is assumed.

$EI_{no\ nails}$ = 4 x 3,366
= 13,46 kN.m²

$EI_{rigid\ connection}$ = 215,44 kN.m²

The highest section stiffness that one could expect with the currently proposed method of nailing multiple-ply trusses together is 20 % of the full section stiffness.

Structural software verification of spring model

Prokon is a suite of structural engineering programs that is available in South Africa at a reasonable price and has shown to give reliable solutions when used to solve structural systems. The possible methods of verifying the spring model using Prokon were considered:

- Solid timber elements connected by means of springs with the same stiffness as the connectors
- Timber shell elements connected by means of springs with the same stiffness as the connectors
- Timber beam elements connected by beams with the same stiffness as the connectors

Of the three available methods the solid elements proved unacceptable as they gave incorrect stiffness for a single member in bending. For instance, the solid elements overestimated the stiffness of a 1,5 m long, 36 mm x 111 mm grade 5 plank, subjected to central point loading by 36 %. This may be due to the author using the incorrect dimensional ratios for the elements. Beam and shell elements, however, gave the correct stiffness for single boards and it was decided to use the shell elements as a method of verifying the spring model, as the input is acceptably simple and the results show how the individual plies move relative to one another. The shell elements may also be used to illustrate the distribution of the stresses in the individual plies.

When using solid beam elements to determine the stiffness of a composite member, the stiffness of the nail can be modelled by using an equivalent beam element, which is fixed at both ends, with a bending stiffness that is calculated as follows:

$\frac{12EI}{L^3} \cdot \delta = k \cdot \delta$ so that the equivalent beam stiffness $EI = \frac{k \cdot L^3}{12}$

Where L is the distance between the centrelines of the members, that is, 36 mm in this case.

Table 4 Stiffness of combined members that have been joined by means of nails

Specimen combination	Measured *EI* kN. m^2	Theoretical *EI* (kN.m^2) with average nail stiffness, that is, 1 357 kN/m	Theoretical *EI* (kN.m^2) with high nail stiffness, that is, 1 853 kN/m	With rigid connection kN.m^2
1 & 16	22,34	17,89	19,70	32,2
2 & 9	20,90	15,62	17,09	26,3
3 & 13	14,50	13,73	14,95	22,0
4 & 20	23,53	19,25	21,26	39,6
5 & 6	21,92	19,00	20,96	35,2
11 & 18	12,80	11,87	12,86	18,1
7 & 17	9,57	9,42	10,07	13,6
8 & 19	16,99	14,18	15,47	25,0
15 & 14	12,97	11,50	12,41	17,1
10 & 12	19,47	13,42	14,54	20,8

Table 1 was drawn up assuming 111 mm x 36 mm grade 5 members with 75 mm nails and compares the spring model stiffness with the shell element and the beam element values for a two ply member.

Similar tables can be obtained for three- and four-ply members with the maximum difference in stiffness between the spring model and the finite element shell analysis being in the region of 7 %.

Comparison with spaced column design in Ozelton and Baird (1982)

Ozelton and Baird (1982) describe a method whereby a nailed spaced column may be designed. For a nailed column without spacer blocks the effective width is give as 2*B* and the effective length must be multiplied by a factor of 1,8. The force that is transferred by the nails must be a minimum of 1,5*P/n*, where *n* is the number of plies. For a 1,5 m long two-ply 36 mm x 111 mm grade 5 member that is to carry the maximum load, 1,8 x *L/b* = 37,5 with a permissible stress of 1,89 MPa and a resultant load of 15,104 kN. The force to be carried by

the nails is = $\frac{1{,}5 \times 15{,}104}{2} = 11{,}328$ kN. The number of nails required = 51 with a spacing of 30 mm. The effective stiffness *EI* = 8,31 kN.m^2 and the effective *b* = 48 mm.

The spring model gives an effective *EI* = 13,65 kN.m^2 which is equal to an effective *b* = 57 mm. The *Le/b* ratio is then 1 500/57 = 26,3. The Euler buckling force is 59,9 kN or if one calculates the allowable force, effective *b* = 57 mm, *Le/b* = 26,3 with an allowable stress of 3,57 MPa. The allowable force is = 28,53 kN.

A buckling analysis using beam elements gives a buckling load of 59,6 kN, which translates into an effective *EI* = 13,59 kN.m^2 with an effective *b* of 57 mm. It appears that the method in Ozelton and Baird (1982) may be conservative in the design approach of nailed together members.

EXPERIMENTAL VERIFICATION OF THE NAIL STIFFNESS AND THE COMPOSITE MODEL STIFFNESS PREDICTION

Nail stiffness tests

Test setup

Twenty specimens, using 75 mm long nails, were tested in shear using timber with a similar density profile to the timber used in the bending tests. Load versus deflection curves were obtained and the stiffness of the nail was determined from the linear portion of the curve. Table 3 gives the individual test results, which can then be compared to the Eurocode 5 (1995) values. The Eurocode 5 (1995) values are long-duration values and if the average value in table 3 is divided by 1,5 one obtains a value very similar to the 866 kN/m given in the Eurocode 5 (1995).

High stiffness values were found in the more dense timber and one must bear this in mind when comparing test values to theoretical values.

Individual board stiffness

Test setup

Twenty grade 5 SA pine 75 mm x 36 mm x 2 500 mm long members, with an estimated mean modulus of elasticity of 7 800 MPa, were used. These were measured for density and modulus of elasticity.

Modulus of elasticity was determined by applying static loads. All members were tested for bending on flat. A pre-load was applied to eliminate any twisting of the timber and to ensure that the supports were properly seated. The load was increased and the deflection measured at the centre. All members were loaded and unloaded five times and the average of the five measurements was used to determine the modulus of elasticity.

The following values were obtained for the modulus of elasticity of each specimen. These values were used to determine the stiffness of a combined section. Note that the mean modulus of elasticity is significantly higher than the estimated value for grade 5 SA pine.

It is recognised that the shear deflection plays a small part in determining the stiffness of the member. The loss in apparent stiffness as a result of the shear deflection is small when members are tested on flat and is insignificant when one looks at the lack of repeatability when measuring the MOE of a given specimen.

Composite member tests

Test setup

The single members were combined by nailing them together with 75 mm nails. These were placed in a row along the centre of the members. It was estimated that the nails have a short duration strength of 500 N and the timber a bending strength of 5,1 MPa.

To calculate the number of nails, the combined profile was estimated to be able to carry a point load that would induce a stress of 5,1 MPa. Total point load on the bending member is equal to 615 N. The shear force is equal to 308 N. Shear flow = $q = \frac{1{,}5\,V}{h}$ = 6,406 N/mm. Spacing of the nails, $s = \frac{\text{Strength of the nail}}{q} = \frac{500}{6{,}406}$ = 78,1 mm

Nails were placed at 80 mm to simplify the placing. Members were subjected to central point loading and the modulus of elasticity was determined in the same way as for the single specimen. As was expected, the apparent modulus of elasticity was less than would be obtained if the two single members were joined by means of a rigid connection. The spring model was used to predict the stiffness of the combined member and the average nail stiffness as well as the maximum nail stiffness was used in the prediction model. Table 4 gives a summary of the test values and the predicted values.

The theoretical stiffness of members joined with infinitely stiff connectors was calculated using the stiffness of the individual members.

The tests have been limited to determining nail stiffness and the stiffness of a composite bending member comprising two equally sized elements connected by nails, firstly to see how the nail stiffness compares to the nail stiffness values given in the Eurocode 5 (1995) and secondly to verify the prediction model.

CONCLUSION

Results from the theoretical spring model and the finite element model compare well. The test values and predicted values in table 4 show a larger difference. This could be attributed to the friction at the interface of the boards which would increase the stiffness of the composite member. Under long-term loading the friction would reduce as a result of timber shrinkage and expansion. The movement at the interface would reduce the effective stiffness.

The stiffness predictions show that it is extremely dangerous to assume full section stiffness when flexible connectors are used to obtain composite action. Not only can the stiffness be much less than assumed, but the stress in the individual elements can also be higher than assumed. Even when the design caters for full shear transfer, the stiffness of nailed composite members, in the long term, may be as low as 60 % of the solid section stiffness. Further research and testing is required to determine whether composite compression members lose as much stiffness as a result of slip between the elements when compared to flexural members.

The method described in Ozelton and Baird (1982), although giving conservative stiffness values, does not give the option of reducing the number of nails as one has to design the nails for 75 % of the axial load. The spring model and the finite element beam model show that it is possible to use less nails to obtain a similar stiffness compared to that given in Ozelton and Baird (1982).

The author suspects that the spring model may be conservative when estimating the stiffness of composite compression members. However, it is possible to use the spring model to determine a conservative effective slenderness of compression members that have been nailed together using the standard nailing pattern used by the truss industry in South Africa. This would certainly ensure that the slenderness is not underestimated and that the designs should be safer.

REFERENCES

European Standard EN 1995-1-1, Euro Code 5. Design of timber structures, Part 1.1: General Rules, General rules and rules for buildings, Technical Committee CEN/TC250.

Gere, J M 2001. *Mechanics of materials*. 5th ed. Pacific Grove, Calif: Brooks Cole.

Ozelton, E C & Baird, J A 1982. *Timber designers' manual*. London: Granada.

SANS 10163-1:2003. The structural use of timber, Part 1: limit-states design. Pretoria: South African Bureau of Standards.

SANS 10163–2:2001. The structural use of timber, Part 2: allowable stress design. Pretoria: South African Bureau of Standards.

SANS 1082: 2002. Mild steel nails. Pretoria: South African Bureau of Standards.