Analysis of timber girder trusses for eccentric loading

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Recent timber trussed roof failures in South Africa, especially in Gauteng, of large-span trusses have necessitated a rethink about the analysis of timber trusses in general, but especially primary girder trusses, which support major loads from secondary trusses. The failures include roofs that collapsed (these having been reported in the press) to roofs where the bottom chord of multiple-ply girder trusses had rotated to such an extent that nail plates on the outer ply failed. These roofs were repaired before collapse could occur. Some of these failures are still under investigation and to divulge their location would be inappropriate. The author has identified eccentric loading as one of the possible reasons for some of the failures. In this paper he investigates all the possible factors that may influence the strength and stiffness of the trusses and he uses these to analyse three different span of girder truss. The author shows that by ignoring the eccentric loading the plate force may be underestimated by a factor in excess of 5. He shows a simple way of determining member and plate forces and suggests that these should be used when girder trusses are designed. This may then reduce the number of failures and should lead to timber roof structures that are more robust and more capable of accommodating erection errors.

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Keywords: multi-ply, trusses, nailed, torsional, stiffness, three-dimensional

INTRODUCTION

Finite element methods – in particular the matrix-stiffness method – have made it easy for timber truss designers to analyse the many trusses that go into making up the roof of any building. The layout is drawn with great efficiency and the truss configuration generated with very little further input from the designer. Girder trusses, jack trusses and any form of truss can be analysed with greater efficiency than ever before. Not only does the program analyse each truss and choose the optimum layout and size and grade of member, but it also draws up the cutting bill and manufacturing sheet, and calculates the connector plate size. Load paths are calculated, so that every truss may be allocated the correct load. The computer programmer may have an inkling of the limitations of the program, but the general user of the software has no idea as to whether the design has catered for all the basic principles of structural analysis as well as all the requirements of the timber design code of practice.

Although the author has no doubt that the analysis of single in-plane loaded trusses may be correct, he has his reservations about the analysis of multiple-ply girder trusses. Figure 1 shows a plan and section of a possible positioning or use of a multiple-ply girder truss.

The loads from the incoming trusses are applied in the plane of the girder truss. The fact is ignored that, in a four-ply girder truss, this load may be 95 mm away from the plane. Straps, called anti-torsion straps, are used to tie the incoming trusses to the girder truss. Prior to the strapping, the girder truss bottom chord deflected laterally so much that the incoming trusses fell off their supports. The strap has been added to prevent this from happening and it is believed that they would also minimise the torsion being applied to the bottom chord. Torsion may be

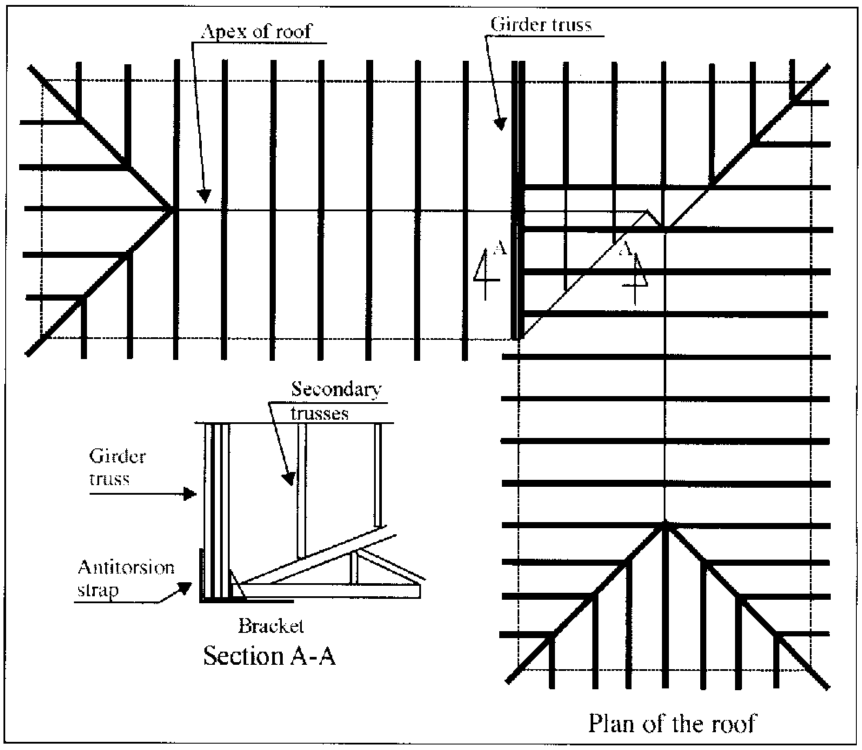

***Figure 1** Layout of roof with multiple-girder truss*

minimised if the incoming trusses butt up tightly against the girder truss and no relative rotation of the bottom chord is allowed.

The author has identified the eccentric loading as a possible cause for concern. He believes that a combination of cumulative errors may increase the probability of failure to an unacceptable level. It is important to investigate the effect of the nailing pattern on the torsional and bending stiffness of the members, as both of these will influence the forces that the individual plies may have to carry.

CODE REQUIREMENTS

No mention is made in the South African codes SANS 10163:1 and SANS 10163:2 of girder trusses that are eccentrically loaded. The British code, BS 5268-3:1998, has a requirement for girder trusses that are loaded eccentrically. The effect is taken up by a modification factor K_e for joints in eccentrically loaded components subject to tension perpendicular to grain.

At joints where a net tension force exists perpendicular to the grain direction – except those where the fastener bite is within 10 mm of the chord depth – the following condition should be satisfied in the chord member:

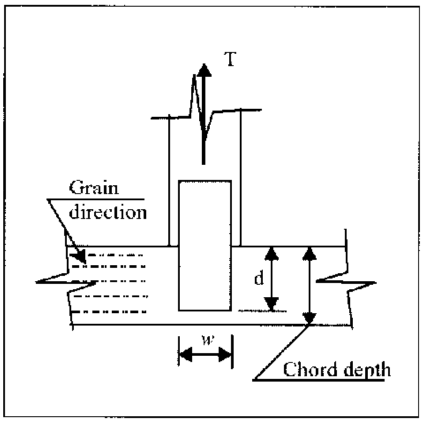

Figure 2 Plate with loading perpendicular to the grain

$$\sigma_{t,90,adm} \geq K_e \left(\frac{0,06T}{w+16d} \right)$$

Where:

$\sigma_{t,90,adm}$ = permissible tension stress perpendicular to the grain
T = the net direct tension force at the joint interface in the direction perpendicular to the grain
K_e = tension force enhancement factor.
w = is length of plate or gusset
d = fastener bite

Table 11 from BS 5268-3:1998 is given in table 1.

Table 1 Force modification factor (Ke), BS 5268-3:1998, for joints in eccentrically loaded components subject to tension perpendicular to the grain

Number of plies	Eccentricity factor (K_e)
1	1,00
2	1,33
3	2,00
4	3,00

Principal trussed rafter, ie a 'girder' should have at least two plies

An additional clause to this table states: 'Restraint systems may be used to ameliorate the effects of torsion induced by out-of-plane eccentric loading, provided their adequacy is verified by laboratory tests. For the purpose of calculating forces in plates or gussets on joints loaded eccentrically out-of-plane, the resultant tensile force determined from the above method should be increased by a further 10 % to allow for stress concentration effects.'

MULTIPLE-PLY GIRDER TRUSS ASSEMBLY

The trusses are delivered on site as single-ply trusses and stacked on flat. If a flat stretch of ground or concrete slab is available, the multiple-ply trusses are assembled in the following way. The first truss is placed on the ground and the second is then slid onto it. All the truss members are then supposed to be nailed together in accordance with SANS 10243, 1992. Any further plies are then nailed onto this assembly one at a time. The assembled girder is then lifted into position, the brackets are bolted and nailed, and finally bolts are added at each of the nodes. Figure 3 shows how the nailing and bolting is supposed to be done.

In cases where no flat assembly area is available, the trusses are placed in their positions and nailed and bolted together up in the air. The webs are flexible and can easily be broken while the nails are being hammered into place. Small plates may also become slightly dislodged, as the forces from the hammering are vertical to the direction of real strength. The temptation to leave out some of the nails increases with the difficulty of placing the nails. One can thus expect to find webs and chords with fewer than the required number of nails. The inner trusses may, in some cases, not even be nailed together; a fact which cannot be verified by the engineer inspecting the roof structure.

The importance of the nailing and bolting only becomes obvious when the out-of-plane forces and displacements are taken into account. The compression member stiffness and therefore the strength depend largely on how well the members are nailed together. Out-of-plane bending moments that apply torsion to the top and bottom chords will also apply out-of-plane bending moments to the web members.

EFFECT OF NAILING ON TORSIONAL RIGIDITY OF WEB AND CHORD MEMBERS

Introduction

Single SA pine members have been investigated by Burdzik and Nkwera 2003 for torsional rigidity, but nowhere have tests been done to investigate the torsional rigidity of SA pine members that are connected by means of flexible connectors. In the USA, laminated members are used to construct short-span bridges. The laminated beams are placed side-by-side and high-tensile tendons are installed perpendicular to the direction of the span. The tendons are stressed so that plate action can be achieved. Davalos *et al* 1996 have determined that when laminated members are joined together by means of stressing cables the torsional rigidity increases with an increase in the tendon force. In the limit the torsional rigidity would tend towards that of a fully glued section.

In the South African multiple-ply girder truss context the nails are placed 25 mm in from the edge of the timber members. As no information was available to give guidance with respect to calculating the torsional rigidity of members combined in

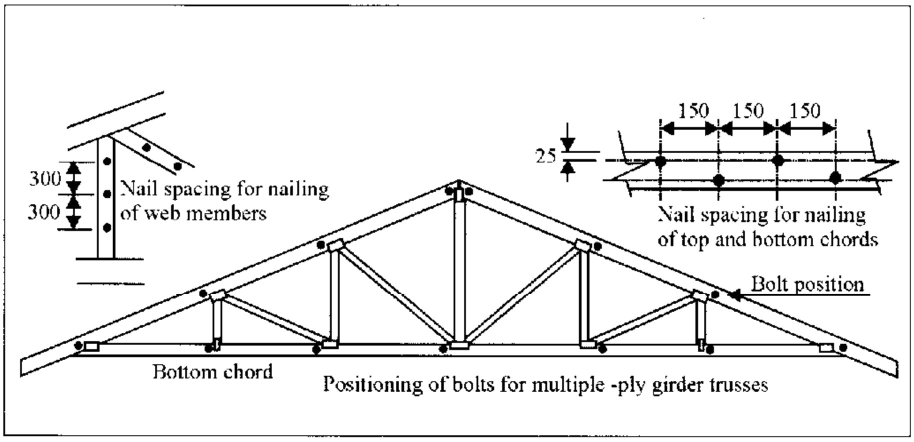

Figure 3 The bolting and nailing pattern for multiple-ply girder trusses (per SABS 0243)

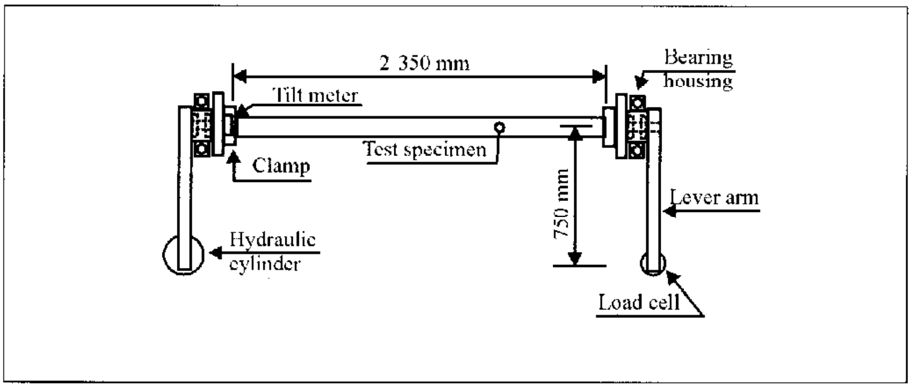

Figure 4 Test setup to determine the torsional rigidity of single and double members

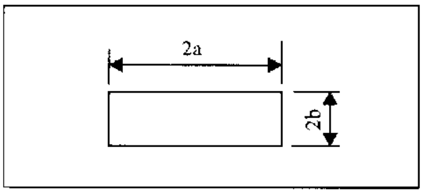

Figure 5 Dimensions to be used in equation 1

accordance with SANS 10243, 1992, a series of tests was undertaken to establish the torsional rigidity of boards nailed together with 75 mm nails at the spacing suggested in SANS 10243, 2003.

Test specimens

Forty specimens of 112 x 36 mm SA pine with a length of 2,4 m were obtained from a merchant in Pretoria. The cross-sectional dimensions were measured accurately so that the torsional section modulus, J, could be calculated using the Prandl membrane theory.

Test procedure

Two clamps were placed at the distance of 2 350 mm apart and fitted with bearings that allowed rotation through 360 degrees in either direction. A metal arm with a length of 750 mm was attached to each clamp so that torque could be applied to the specimens. One of the arms rested on a calibrated 200 kg load cell and the other arm was attached to a hydraulic cylinder. The load cell was placed so that the lever arm would remain constant. The angle of twist was measured, by means of a calibrated tilt meter, at the side where the torsion was applied. An initial load was applied to take out possible movement at the supports; this was then taken as the zero reading. The load was then increased to a maximum that would not cause damage to the timber. Both the load cell and the tilt meter were connected to a Spider 8 data-capturing device. A computer was used to save the data in ASCII format for further analytical use. A diagrammatic presentation of the setting is given in figure 4.

The torsional rigidity of the single members was determined so that matching pairs could be found. The two members of a matching pair would have a similar rigidity. The matching pairs were then nailed together with the nailing pattern as given in SANS 10243, 2000. These were then tested in a similar fashion to those of the single members.

Test results

The torsional section modulus, J, was calculated using the Prandl membrane theory for a single 36 x 111 and is equal to 1,37 x 10^6 mm^4. The torsional sectional modulus is very sensitive to the accuracy with which the width is measured. Alternatively, J may be calculated using the equation given in Roark 1965:

$$J = a \cdot b^3 \left[\frac{16}{3} - 3{,}36 \cdot \frac{b}{a} \left(1 - \frac{b^4}{12 \cdot a^4} \right) \right] \quad (1)$$

Where the dimensions are given in figure 5:

If one uses equation 1 the torsional section modulus, J = 1,374 x 10^6 mm^4.

The shear modulus, G, for the individual specimens is given in table 2, as well as the way in which the specimens were combined. A fixed torsional moment, T, was applied to the specimens and the angle of rotation, θ, was measured. This was used in equation 2 to calculate the shear modulus.

$$G = \frac{T \cdot L}{\theta \cdot J} \quad (2)$$

If the sections could be combined perfectly the torsion modulus, J, would be = 8,65 x 10^6 mm^4. Table 3 give the measured value of the torsional rigidity, GJ, as well as the theoretical combined value and the value of the two members just added together.

No combined effect could be discerned. The combined members behaved as if they were two individual members with no connection between them. The difference between the measured rigidity and the sum of the individual member rigidity can be ascribed to errors in measuring the cross-sectional dimensions and in measuring the angle of twist. The angle of

Table 2 Individual shear modulus, G, as well as the way in which members were combined

Specimens	G in MPa of first specimen	G in MPa of second specimen	GJ of the first specimen N.mm²	GJ of the second specimen N.mm²
34 & 32	500,6	538,7	6,87741E+08	7,40052E+08
5 & 13	556,3	569,7	7,64259E+08	7,82751E+08
33 & 1	588,5	605,9	8,08520E+08	8,32479E+08
7 & 37	623,5	612,4	8,56672E+08	8,41406E+08
16 & 17	628,7	632,0	8,63783E+08	8,68253E+08
36 & 8	646,7	642,5	8,88461E+08	8,82686E+08
18 & 2	649,9	651,9	8,92883E+08	8,95679E+08
6 & 29	662,5	655,3	9,10144E+08	9,00253E+08
35 & 9	665,0	673,8	9,13577E+08	9,25674E+08
21 & 40	693,1	704,8	9,52205E+08	9,68282E+08
27 & 12	758,3	711,8	1,04188E+09	9,77950E+08
11 & 26	776,2	765,6	1,06639E+09	1,05178E+09
38 & 39	785,8	801,6	1,07953E+09	1,10126E+09
10 & 15	806,5	804,3	1,10798E+09	1,10498E+09
19 & 22	834,9	860,4	1,14701E+09	1,18208E+09
25 & 23	866,8	866,7	1,19086E+09	1,19076E+09
3 & 14	904,5	904,8	1,24272E+09	1,24304E+09
31 & 4	937,9	952,5	1,28856E+09	1,30868E+09
24 & 28	1 001,5	1 044,8	1,37600E+09	1,43548E+09
20 & 30	1 055,4	1 194,9	1,45004E+09	1,64163E+09

Table 3 Measured and theoretical torsional rigidity of combined members

Specimen	Measured GJ in N.mm²	Two single members added together	Theoretical idealised combined section
34 & 32	1,4783E+09	1,4278E+09	4,2868E+09
5 & 13	1,5939E+09	1,5470E+09	4,6448E+09
33 & 1	1,6625E+09	1,6410E+09	4,9270E+09
7 & 37	1,5378E+09	1,6981E+09	5,0983E+09
16 & 17	1,8813E+09	1,7320E+09	5,2003E+09
36 & 8	1,7265E+09	1,7711E+09	5,3177E+09
18 & 2	1,9213E+09	1,7886E+09	5,3700E+09
6 & 29	1,7347E+09	1,8104E+09	5,4356E+09
35 & 9	1,7157E+09	1,8393E+09	5,5222E+09
21 & 40	1,9521E+09	1,9205E+09	5,7661E+09
27 & 12	2,0121E+09	2,0198E+09	6,0644E+09
11 & 26	2,0119E+09	2,1182E+09	6,3596E+09
38 & 39	2,3012E+09	2,1808E+09	6,5477E+09
10 & 15	2,0593E+09	2,2130E+09	6,6443E+09
19 & 22	2,2718E+09	2,3291E+09	6,9929E+09
25 & 23	2,3353E+09	2,3816E+09	7,1506E+09
3 & 14	2,3593E+09	2,4858E+09	7,4633E+09
31 & 4	2,3484E+09	2,5972E+09	7,7980E+09
24 & 28	2,7061E+09	2,8115E+09	8,4412E+09
20 & 30	3,0128E+09	3,0917E+09	9,2825E+09

twist is based on a regression of the data.

The average shear modulus, G, may be assumed to be between MOE/13 and MOE/16 (Burdzik & Nkwera 2003; Ozelton & Baird 1982) if the torsional rigidity is required in a calculation. Bear in mind that if the software one uses calculates the shear modulus using the modulus of elasticity and the Poisson ratio, the incorrect stiffness will be obtained. The torsion constant, J, will have to be adjusted to allow for the difference between the actual G and the computed value of G.

WEB AND CHORD MEMBERS THAT ARE NAILED TOGETHER

Introduction

It is generally accepted that when members are nailed together, slip has to occur on the interface of the individual trusses before the nails will transfer the shear. In the case of multiple-ply girder trusses, the forces that we are interested in will be out of plane, in other words, moments about the X-axis if we assume that the girder truss lies in the XY-plane. In general the trusses are analysed for moments about the Z-axis, that is, the axis that is vertical to the plane of the truss.

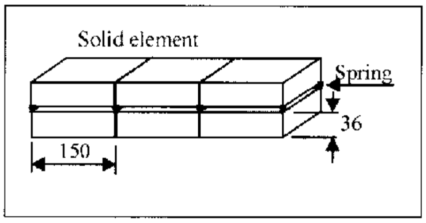

Figure 6 Finite element analysis of the members connected by means of springs

Our interest lies in determining the stiffness of the members about the weaker axis of the combined members. In order to do this one has to know what the stiffness of the nailed connection is and the type of loading that is applied to the member.

Stiffness of nailed connections

The stiffness of a nailed connection is given by the following equation (Eurocode 5 1995; Timber Engineering Step 1):

$$K = \rho_k^{1,5} \cdot \frac{d^{0,8}}{25}$$

Table 4 Loads and section properties used in the analysis of the trusses

	Single-ply	Two-ply	Four-ply
Span	6 m	9 m	12 m
Pitch of truss	25 °	25 °	25 °
Top chord	36 x 111 mm (Grade 5)	36 x 149 mm (Grade 5)	36 x 149 mm (Grade 5)
Bottom chord	36 x 225 mm (Grade 5)	36 x 149 mm (Grade 7)	36 x 225 mm (Grade 7)
Webs	36 x 111 mm (Grade 5)	36 x 73 mm (Grade 5)	36 x 73 mm (Grade 5)
Spacing of incoming trusses	750 mm	750 mm	750 mm
Weight of tiles and battens	55 kg/m²	55 kg/m²	55 kg/m²
Ceilings	10 kg/m²	10 kg/m²	10 kg/m²
Self-weight of incoming trusses	10 kg/m²	10 kg/m²	10 kg/m²
Eccentricity of bracket load	28 mm	72 mm	92 mm
Modulus of elasticity Grade 5	7 600 MPa	7 600 MPa	7 600 MPa
Shear modulus	585 MPa	585 MPa	585 MPa

Table 5 Equivalent torsional constant, J, and equivalent second moment of area about 'minor axis'

Section	J (mm⁴)	Zt (mm³)	Equivalent J (mm⁴)	Eqivalent Iyy (mm⁴)
36 x 73	$0,7843 \times 10^6$	23×10^3	$0,1449 \times 10^6$	$0,2838 \times 10^6$
2 of 36 x 73	$1,5686 \times 10^6$	46×10^3	$0,2898 \times 10^6$	$0,9082 \times 10^6$
3 of 36 x 73	$2,3529 \times 10^6$	69×10^3	$0,4348 \times 10^6$	$3,0653 \times 10^6$
4 of 36 x 73	$3,1373 \times 10^6$	92×10^3	$0,5797 \times 10^6$	$7,2659 \times 10^6$
36 x 111	$1,3739 \times 10^6$	38×10^3	$0,2539 \times 10^6$	$0,4316 \times 10^6$
2 of 36 x 111	$2,7477 \times 10^6$	76×10^3	$0,5077 \times 10^6$	1,3810 x 10v
3 of 36 x 111	$4,1216 \times 10^6$	114×10^3	$0,7616 \times 10^6$	$4,6609 \times 10^6$
4 of 36 x 111	$5,4955 \times 10^6$	152×10^3	$1,0154 \times 10^6$	$11,0481 \times 10^6$
36 x 149	$1,9647 \times 10^6$	$53,7 \times 10^3$	$0,3630 \times 10^6$	$0,9269 \times 10^6$
2 of 36 x 149	$3,9293 \times 10^6$	$107,4 \times 10^3$	$0,7260 \times 10^6$	$1,8538 \times 10^6$
3 of 36 x 149	$5,8939 \times 10^6$	$161,1 \times 10^3$	$1,0890 \times 10^6$	$6,2566 \times 10^6$
4 of 36 x 149	$7,8585 \times 10^6$	$214,8 \times 10^3$	$1,4521 \times 10^6$	$14,8304 \times 10^6$
36 x 225	$3,1465 \times 10^6$	$85,5 \times 10^3$	$0,5814 \times 10^6$	$0,8748 \times 10^6$
2 of 36 x 225	$6,2930 \times 10^6$	171×10^3	$1,1628 \times 10^6$	$2,7994 \times 10^6$
3 of 36 x 225	$9,4395 \times 10^6$	$256,5 \times 10^3$	$1,7442 \times 10^6$	$9,4478 \times 10^6$
4 of 36 x 225	$12,5860 \times 10^6$	342×10^3	$2,3256 \times 10^6$	$22,3949 \times 10^6$

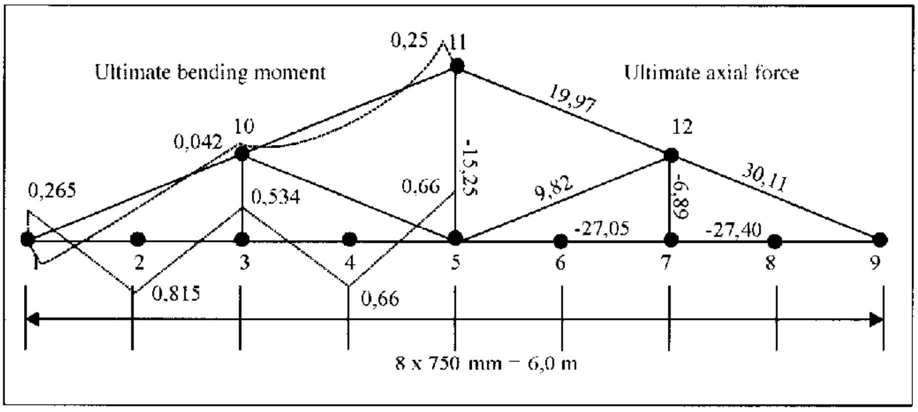

Figure 7 Ultimate bending moments and axial forces in a single-ply girder truss – point loads at positions of truss hangers. Tensile forces are negative and compressive forces are positive

Where: ρ_k = the density of the timber in kg/m^3
d = diameter of the nail in mm
K = the spring stiffness of the nail

Assuming a typical lower value for the density of the timber, that is, 400 kg/m^3, and a diameter of the nail of 3,2 mm, a spring stiffness 811 N/mm results. The author found that the fifth percentile stiffness of a limited number of nailed joints was about 730 N/mm, with an average of 1 350 N/mm. However, as more than one nail would be used in an actual truss, it makes more sense to use the average value. The average stiffness can now be used to calculate the stiffness of composite nailed-together 36 x 111 mm elements.

Analysis of nailed together sections

The stiffness of any bolts was ignored as bolts are usually fitted into oversized holes and big displacement would be required before the bolt's stiffness would take effect. As this is a theoretical evaluation, the author assumed that the sections that are nailed together would all have a similar modulus of elasticity. In reality the stiffness may vary quite considerably. However, the effect of varying modulus of elasticity is outside the scope of this investigation and would never be taken into account in the design of trusses. Assuming that the truss lies in the XY-plane, web members would be subjected to moments about the X-axis as a result of the eccentric loads from the brackets.

Solid elements were used in the finite element analysis of the multiple-ply truss members. The Grade 5 timber solid elements were connected by means of linear springs on the interface between the layers (see fig 6). This may seem like an oversimplified approach, but if one remembers that the timber has great variability in stiffness and density, a rough general estimate of the stiffness is about the best that one can hope for.

An analysis of a double 1,5 m long section that is nailed together, using the pattern as shown in figure 3 and the nail stiffness of 1 350 N/mm, shows that the bending stiffness of the double member will be in the region of 40 % of theoretical idealised member, that is, where depth is taken as double that of the single member. A 600 mm long double and quadruple member gave stiffness values of between 40 % for the double member and 25 % for the quadruple member. The 40 % of the theoretical stiffness is very similar to values that were obtained by Bosch (2003).

ANALYSIS OF MULTIPLE-PLY TRUSSES

Assumptions

Assume that the trusses lie in the XY-plane. The truss layout and analyses were simplified so that the results could be used to illustrate the dangers of ignoring the out-of-plane forces. Members were assumed to have the actual dimensions for moments about the Z-axis and were modified for moments about the X- and Y-axes. Stiffness about the weaker axis was reduced to 40 % of the theoretical value and the torsional stiffness was taken to be the sum of the individual values. Three different spans were considered. The incoming trusses were assigned the same span as the multiple-ply truss. The author realises that this may only cover a limited number of cases.

Loading on trusses

Table 4 sets out the truss spans, loads and pitch.

Table 5 sets out the equivalent torsional constant and second moment of area about the weaker axis for single and combined sections.

Results of analysis of trusses – single-ply 6 m span

Figure 7 gives the ultimate bending moments and axial forces in a single-ply truss, where it has been assumed that the connector plates are able to transfer bending moment. The truss lies in the XY-plane and the loads from incoming trusses have been applied at nodes 2 to 8. Light truss hangers, that are nailed or bolted to the bottom chord, will carry the incoming trusses. All forces are in kilo-Newtons and bending moments in kilo-Newton metres.

Discussion of results – single-ply

Torsion and shear in bottom chord

The maximum ultimate torsion moment in the bottom chord is 0,05 kN.m with a shear force of 1,96 kN. This translates into a shear stress of 0,58 MPa for the torsion and 0,36 MPa for the shear force. Total shear stress = 0,94 MPa. The resistance shear stress = 0,68 x 1,6 = 1,09 MPa.

Bending about the weaker axis, web members

The tensile resistance of a 36 x 73 and a 36 x 111 Grade 5 member are 13,30 kN and 20,23 kN respectively. Member 3-10 could then be sized 36 x 73 and member 5-11 as a 36 x 111. The plate size for member 3-10 would be designed for a force of 6,89 kN or 3,44 kN/side, and for member 5-11 a force of 15,25 kN or 7,62 kN/side.

Members 3-10 and 5-11 have a moment about the X-axis of 0,20 kN.m and 0,12 kN.m at 3 and 5 respectively. If one assumes that the bending moment is transferred by the plates only, the ultimate force in the plates of 3-10 as a result of the bending moment = 5,56 kN (0,2 kN.m/0,036 m). The maximum tension force would then be 5,56 + 6,89/2 = 9,0 kN. The actual force in the plate is then 9,0/3,44 = 2,62 times as large as the design force if torsion is ignored.

Assume that member 3-10 is 36 x 73 and the interaction formula of SANS 10163:1 must be applied.

M_u = 0,2 kN.m and T_u = 6,89 kN

$$M_r = 0{,}68 \cdot Z_e \cdot \frac{f_b}{\gamma_{m1} \cdots \gamma_{m5}} \approx 0{,}137\ kN.m$$

$$T_r = 0{,}68 \cdot A \cdot \frac{f_t}{\gamma_{m1} \cdots \gamma_{m5}} = 13{,}30\ kN$$

Interaction equation:

$$\frac{T_u}{T_r} + \frac{M_u}{M_r} = \frac{6{,}89}{13{,}30} + \frac{0{,}2}{0{,}137} = 1{,}98$$

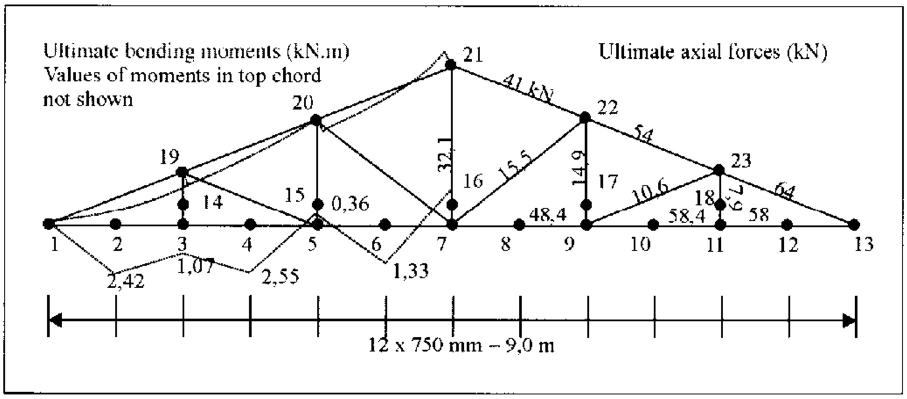

Figure 8 Ultimate bending moments and axial forces in two-ply girder truss – point loads at positions of truss hangers. Tensile forces are negative and compressive forces are positive

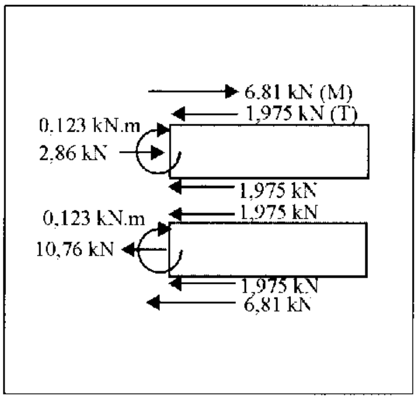

Figure 10 Distribution of forces and moments in the plies of a two-ply girder truss, loads applied to bottom chord only

Results of analysis of trusses – two-ply 9 m span

Figure 8 gives the ultimate bending moments and axial forces in a two-ply truss. Loads from incoming trusses have been applied at nodes 2, 4, 6, 8, 10, 12, 14, 15 to 18. It has been assumed, that where there is a vertical web, the hangers will be bolted to the web. The bottom chord is of a Grade 7 timber.

Torsion and shear in bottom chord

The maximum ultimate torsion moment in the bottom chord is 0,14 kN.m with a shear force of 1,29 kN. This translates into a shear stress of 0,82 MPa for the torsion and 0,12 MPa for the shear force. Total shear stress = 0,94 MPa. The resistance shear stress = 0,68 x 1,6 = 1,09 MPa.

Bending about the weaker axis, web members

The tensile resistance of a double 36 x 73 and 36 x 111 Grade 5 members are 26,6 kN and 40,45 kN respectively. Members 3-14-19 and 5-15-20 could then be sized 2 x 36 x 73 and member 7-16-21 as a 2 x 36 x 111. The plate size for member 3-14-19 would be designed for a force of 7,8 kN or 1,95 kN/plate, and for member 5-15-20 a force of 14,9 kN or

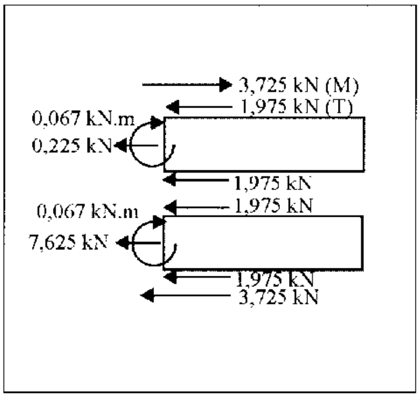

Figure 9 Distribution of forces and moments in the plies of a two-ply girder truss, some of the loads applied to bottom chord and the rest to the web members

3,73 kN/plate.

Members 3-14-19 and 5-15-20 have a moment about the X-axis of 0,27 kN.m and 0,30 kN.m at 3 and 5 respectively and 0,27 kN.m and 0,30 kN.m at 14 and 15 respectively. The plates must transfer the bending moment at 3 and 5, whereas the timber must carry the moments at 14 and 15. If one assumes that the bending moment is transferred by the plates only, the ultimate force in the plates of 3-14-19 as a result of the bending moment = 3,75 kN (0,27 x 0,036/(2 x 0,0362). The maximum tension force in the plates of 3-14-19 would then be 3,75 + 7,9/4 = 5,725 kN. The actual force in the plate is then 5,725/1,95 = 2,94 times as large as the design force if torsion is ignored.

Assume that member 3-14-19 is double 36 x 73 and this must be checked for bending about the minor axis. The interaction formula of SANS 10163:1 must be used. Figure 9 shows how statics may be used to obtain an equivalent load on each member.

M_u = 0,067 kN.m and T_u = 7,625 kN

Assuming a 36 x 73 web member the interaction formula will give the following:

$$M_r = 0{,}68 \cdot Z_e \cdot \frac{f_b}{\gamma_{m1} \cdots \gamma_{m5}} \approx 0{,}137\ kN.m$$

$$T_r = 0{,}68 \cdot A \cdot \frac{f_t}{\gamma_{m1} \cdots \gamma_{m5}} = 13{,}30\ kN$$

Interaction equation:

$$\frac{T_u}{T_r} + \frac{M_u}{M_r} = \frac{7{,}625}{13{,}30} + \frac{0{,}067}{0{,}137} = 1{,}062$$

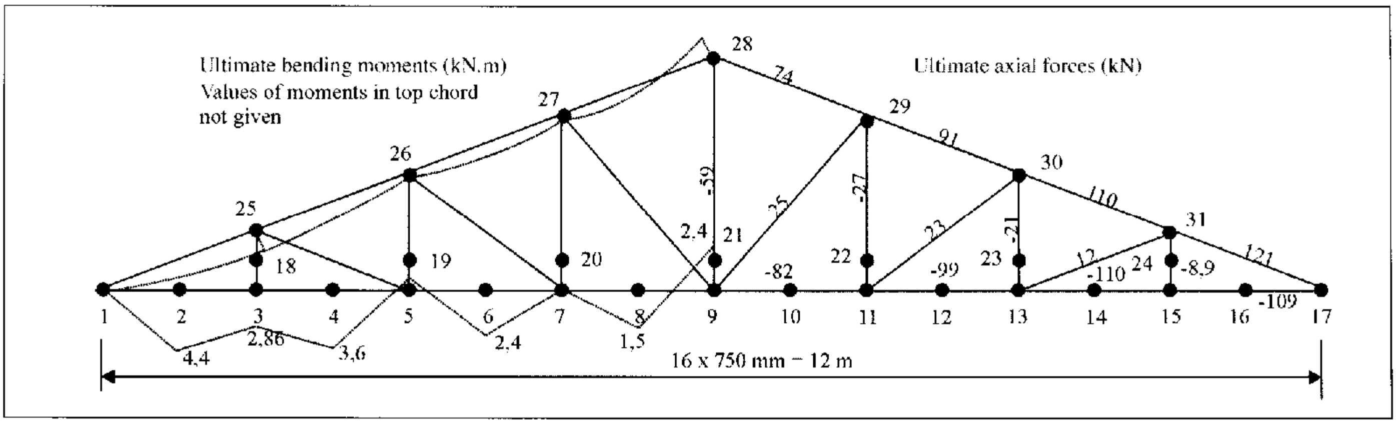

Figure 11 Ultimate loads on a 12 m span, four-ply girder truss

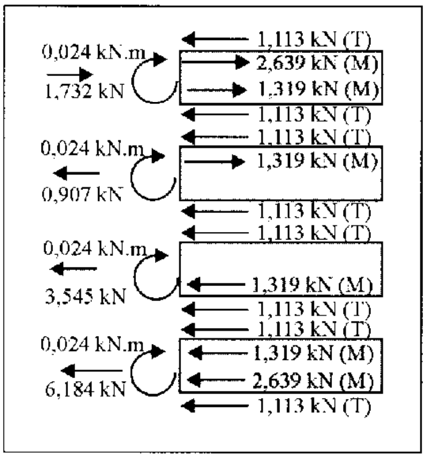

Figure 12 Distribution of forces and moments in the plies of a four-ply girder truss

Brackets connected to bottom chord only

Where the brackets are only bolted to the bottom chord and not to any vertical web members, the moments about the X-axis in the vertical webs 3-14-19, 5-15-20 and 7-16-21 increase to 0,49 kN.m, 0,37 kN.m and 0,24 kN.m respectively.

The force in the nail plates at 3, 5 and 7 would now increase to 6,81 kN, 5,14 kN and 3,33 kN respectively. Maximum tension would then be in nail plate at 3 with a value

$$T_u = 5{,}09 + \frac{8{,}9}{8} = 6{,}203\ kN$$

The force in the plate is then 4,5 times as large as the design force when the moment about the X-axis is ignored. The equivalent loads on the web members can be seen in figure 10.

The interaction equation for the bottom member:

$$\frac{T_u}{T_r} + \frac{M_u}{M_r} = \frac{10{,}76}{13{,}30} + \frac{0{,}123}{0{,}137} = 1{,}707$$

Results of analysis of trusses – four-ply 12 m span

Figure 11 gives the ultimate bending moments and axial forces in a four-ply girder truss. Loads from incoming trusses have been applied at nodes 2, 4, 6, 8, 10, 12, 14, 15, 16, 18, 19, 20, 21, 22, 23 and 24. The bottom chord is of Grade 7 timber.

Torsion and shear in bottom chord

The maximum ultimate torsion moment in the bottom chord is 0,29 kN.m with a shear force of 1,14 kN. This translates into a shear stress of 0,85 MPa for the torsion and 0,05 MPa for the shear force. Total shear stress = 0,90 MPa. The resistance shear stress = 0,68 x 1,6 = 1,09 MPa.

Table 6 Comparison of load increases for various numbers of plies in an eccentrically loaded girder truss

Number of plies	Increase in load on plate	K_e – BS 5268-3
1	2,62	1,00
2	4,5	1,33
4	5,6	3,00

Table 7 Distribution factors for the torsional moment

Member	Stiffness	Distribution factor
3-14-19	1,431	0,300
5-19	0,6043	0,127
5-15-20	0,7148	0,150
7-20	0,4877	0,103
7-16-21	0,4766	0,10
7-22	0,4877	0,103
9-17-22	0,7148	0,150
9-23	0,6043	0,127
11-18-23	1,431	0,300
Sum	4,7682	

Bending about the weaker axis, web members

The tensile resistance of a quadruple 36 x 73 and 36 x 111 Grade 5 members are 53,2 kN and 80,9 kN respectively. Members 3-18-25, 5-19-26 and 7-20-27 could then be sized 4 x 36 x 73 and member 9-21-28 as a 4 x 36 x 111. The plate size for member 7-20-27 would be designed for a force of 27 kN or 3,375 kN/plate.

Members 3-18-25, 5-19-26 and 7-20-27 have a moment about the X-axis of 0,57 kN.m, 0,42 kN.m and 0,27 at 3, 5 and 7 respectively and 0,78 kN.m, 0,80 kN.m and 0,72 kN.m at 18, 19 and 20 respectively. The plates must transfer the bending moment at 3 and 5, whereas the timber must carry the moments at 18 and 19. If one assumes that the bending moment is transferred by the plates only, the ultimate force in the plates of 3-18 as a result of the bending moment = 2,639 kN (0,57 x 0,072/(2 x $0{,}072^2$ + 4 x $0{,}036^2$). The maximum tension force would then be 2,639 + 8,9/8 = 3,139 kN.

Assume that member 3-18-25 is quadruple 36 x 73 and this must be checked for bending about the minor axis. The interaction formula of SANS 10163:1 must be used. Figure 12 shows how statics may be used to obtain an equivalent load on each member.

M_u = 0,024 kN.m and T_u = 6,184 kN

Assuming a 36 x 73 web member the interaction formula will give the following:

$$M_r = 0{,}68 \cdot Z_e \cdot \frac{f_b}{\gamma_{m1} \cdots \gamma_{m5}} \approx 0{,}137\ kN.m$$

$$T_r = 0{,}68 \cdot A \cdot \frac{f_t}{\gamma_{m1} \cdots \gamma_{m5}} = 13{,}30\ kN$$

Interaction equation:

$$\frac{T_u}{T_r} + \frac{M_u}{M_r} = \frac{6{,}184}{13{,}30} + \frac{0{,}024}{0{,}137} = 0{,}64$$

Brackets connected to bottom chord only

Where the brackets are only bolted to the bottom chord and not to any vertical web members, the moments about the X-axis in the vertical webs 3-18-25, 5-19-20 and 7-20-27 increase to 1,10 kN.m, 0,92 kN.m and 0,75 kN.m respectively.

The force in the nail plates at 3, 5 and 7 would now increase to 5,09 kN, 4,26 kN and 3,47 kN respectively. Maximum tension would then be in nail plate at 3 with a value

$$T_u = 5{,}09 + \frac{8{,}9}{8} = 6{,}203\ kN$$

The force in the plate is then 5,6 times as large as the design force when the moment about the X-axis is ignored.

The equivalent load and moment on the outer web member are M_u = 0,045 kN.m and T_u = 9,862 kN. The interaction equation:

$$\frac{T_u}{T_r} + \frac{M_u}{M_r} = \frac{9{,}862}{13{,}30} + \frac{0{,}045}{0{,}137} = 1{,}07$$

CONCLUSION

Table 6 shows how the force in the nail plate closest to the loaded side is affected by the eccentricity of the load. For comparison the BS 5268-3:1998 values for the force modification factor are included.

It appears as if the British code was based on connections that could transfer moments and that the torsional moments were reduced. The author cannot find any justification for this assumption under South African conditions, where the eccentricity may be even larger than has been assumed in this analysis. The analysis is not so complicated that designers should be afforded the easy option of using values given in a table. The author is of the opinion that even if the torsional moment can be removed, the forces in the multiple-ply trusses will not be equally distributed among the plies as the load transfer is from the bracket, to the first ply and then through the nails and bolts to the further plies. Slip has to occur before load can be transferred and as soon as slip occurs the truss with the greater deflection will also have the greater load.

A simple way of calculating the moment about the X-axis that is induced by the eccentric moment is to take the total torsion moment between the webs and to distribute this according to the web stiffness.

For instance: take the torsional moment that has been applied to the bottom chord of the 9 m two-ply truss. Each bracket will induce an ultimate moment of 0,225 kN.m. There are 8 brackets between the web members 3 to 11, thus the total moment = 1,8 kN.m. Stiffness of each member = $\frac{3 \cdot EI}{L}$ but if one assumes that all the members have the same EI value, the stiffness can be reduced to $\frac{1}{L}$.

Member stiffness and distribution factors are given in table 7.

Member 3-14-19 would then have a moment about the X-axis of $0,3 \times 1,8$ kN.m = 0,54 kN.m and member 5-15-20 = 0,27 kN.m. Compare this to the 0,47 kN.m and 0,37 kN.m obtained from the computer analysis. Although the values are different, it still gives one a fairly good indication of what is happening to the forces in the truss.

The author would like to see that the Institute for Timber Construction keeps a register of all roof failures and that a report by an independent investigator forms part of this register. This will enable design code writers to include the necessary clauses in the codes to ensure that timber structures have the same degree of safety as other structures.

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