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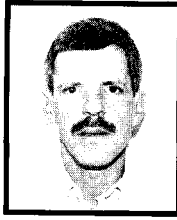
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A rational approach to the design of bracing to resist stability forces and a review of the CSA S16.1-99 proposals

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The design of bracing to resist stability forces involves a complex interaction between the bracing system and the member. Most studies have involved sophisticated computer analysis and as such the behavioural aspects have been clouded, especially when the results of such studies evolve into design methods.

This paper deals with simplified mathematical models which clearly illustrate the interaction between the stiffness and strength of the brace itself and the buckling capacity of the strut. The proposed design rules are consistent with the behaviour, have a sound theoretical basis and are simple enough to be applied directly in a design context.

INTRODUCTION

It is common practice to utilise bracing systems to increase the capacity of struts and beams by reducing the unrestrained length of members susceptible to buckling. Designers tacitly assume that points where the member is attached to the bracing system are fully restrained and determine effective lengths and resistance values for the members on that basis. All bracing systems are in fact flexible, and the interaction between the flexibility of the bracing system and the stiffness of the member is often not fully appreciated by the designer. This lack of understanding of the interactive nature of bracing systems has, perhaps, been caused by provisions in some current design codes which specify a nominal design force, commonly expressed as a percentage of the design load in the strut. The use of a simple force criterion is possibly an over-simplification of a complex problem and may have been instrumental in disguising the real behaviour of bracing systems. Under ideal conditions, that is, where the braced point undergoes no displacement, the force in the brace is zero, but this condition would require an infinitely high bracing stiffness.

In this paper, a theoretical model is derived from first principles by examining the buckling of a strut restrained by a flexible bracing system at a point midway between two fixed points. The model subsequently presented clearly illustrates the interaction between the flexural stiffness of the strut and the bracing system. The problem is also approached by considering the results obtained from applying force and stiffness criteria to the design of bracing members. It is subsequently shown that different values of stiffness are obtained from designing simple bracing members for axial tension or compression. The proposals of CSA S16.1-99 (Canadian Standards Association 1999) are used as a basis for discussing proposed rules for the design of stability bracing. Simplified rules are proposed which are formulated from considering upper bound criteria.

Proposed theoretical stiffness model

The interaction between the stiffness of a bracing member and the resistance of the member itself is complex and involves an evaluation of the likely eccentricities of the unloaded strut as well as the ratio of the actual load in the strut to the buckling resistance. The model proposed in this paper is based upon a closed form solution of the force and stiffness required to restrain an axially loaded strut at mid-height. It is assumed that the strut in its unloaded form has a small initial eccentricity at the brace, leading to deformation of the brace as the load is increased.

Consider the strut of length 2L, restrained by an elastic brace at mid-height, as shown in figure 1. The axial force in the strut equals the actual buckling resistance of the strut and the strut is therefore considered to be in a condition of neutral equilibrium.
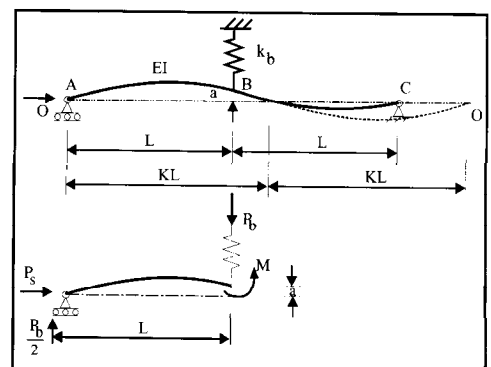

Figure 1 Theoretical model for a strut with flexible bracing

P_b = force in the brace
P_s = axial force in the strut

For the strut as shown in figure 1, the primary interaction between the stiffness of the brace and the buckling resistance of the strut may be appreciated by considering the influence of the stiffness of the brace on the position of the point of contra flexure. For a given level of load in the strut, the effective

length of the portion AB will exceed L and may be expressed as KL where K will always exceed unity. As the buckling load of the strut is unique, the portion BC will also have an effective length KL appropriate to the dotted line intersecting the revised axis OO. The force in the bracing is equilibriated by lateral reactions at points A and C. The displacement of portion AB may be expressed as:

$$y = A \sin \frac{\pi x}{KL}$$

And:

$$\frac{dy}{dx} = A \frac{\pi}{KL} \cos \frac{\pi x}{KL}$$

$$\frac{d^2 y}{dx^2} = -A \frac{\pi^2}{(KL)^2} \sin \frac{\pi x}{KL}$$

$$EI \frac{d^2 y}{dx^2} = -A \frac{\pi^2 EI}{(KL)^2} \sin \frac{\pi x}{KL}$$

$$= -A P_e \sin \frac{\pi x}{KL}$$

Where: $P_e = \frac{\pi^2 EI}{(KL)^2}$ is the critical elastic load of the strut, allowing for displacement at the brace. Note that the value of the effective length factor K is determined by the displacement of the braced point and therefore by the stiffness of the bracing system. Also note that the shift of the inflexion point caused by lateral displacement of the braced point indicates a reduction in the buckling capacity of the strut.
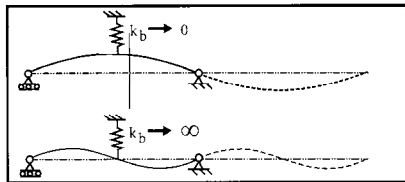

Figure 2 Limiting cases of brace stiffness

In the extreme case of a brace having a very low stiffness, the value of the effective length factor, K, would tend towards 2, indicating that the brace has no influence on the buckling resistance of the strut, as illustrated in figure 2.

The following boundary conditions apply at point B:
y=a and the bending moment in the strut is equal to :

$$\frac{P_b}{2} L - P_s a = -EI \frac{d^2 y}{dx^2} /_{x-L}$$

$$\therefore \frac{P_b}{2} L - P_s a = EIA \frac{\pi^2}{(KL)^2} \sin \frac{\pi L}{KL}$$

$$= A P_e \sin \frac{\pi}{K}$$

$$= a P_e$$

From which the force in the brace may be written as:

$$P_b = 2 \frac{a}{L} (P_e + P_s)$$

$$\therefore P_b = 2 \frac{a}{L} \left(\frac{P_s}{P_e} + 1 \right) P_e \quad (1)$$

And the stiffnes of the brace may be expressed as:

$$k_b = \frac{P_b}{a} = \frac{2 P_e}{L} \left(\frac{P_s}{P_e} + 1 \right) \quad (2)$$

The stiffness of the brace may be determined from equation (2) and is shown to be a function of the actual load in the strut and the critical elastic load, as well as the length of the strut. In general terms, equation (2) shows that the stiffer the strut, and the greater the value of axial load in the strut, the greater the required brace stiffness will be. In the case of pure elastic buckling, the maximum value of the load in the strut will equal the elastic buckling resistance of the strut, $\pi^2 EI / (KL)^2$ and the maximum value of equation (2) will be as given by equation (3)

$$k_b = \frac{4 P_e}{L}$$

$$k_b = \frac{4 \pi^2 EI}{(KL)^2 .L} \quad (3)$$

Equation (4) demonstrates clearly that the full buckling capacity of the strut can only be realised if the stiffness of the brace exceeds $\frac{4 P_e}{L}$, so achieving a value of K = 1.

Stiffness requirements of a single brace

By setting the value of the effective length factor K, equal to 1 in equation (4), the required bracing stiffness may be derived to achieve a condition where the buckling resistance of the strut is equal to that commonly assumed by the designer. This requirement would maximise the value of equation (4) which may then be expressed as equation (5)

$$k_b = \frac{4 P_e}{L} \quad (5)$$

This equation is fundamentally similar to, and illustrates the background to clause 20.2 of the CSA S 16.1-99 proposals given as equation (6)

$$K_e = \frac{\beta C_f}{\phi_r L} \left[1 + \frac{\Delta_o}{\Delta_b + \Delta_{ap}} \right] \quad (6)$$

where:

K_e = required effective stiffness of the bracing assembly
Δ_o = initial misalignment of the braced member at the point of support
Δ_b = elastic displacement of the brace caused by the initial misalignment
Δ_{ap}=displacement of the brace point due to movement of the anchor bracing point relative to adjacent brace points
β = 2, 3, 3.41, or 3.63 for 1, 2, 3, or 4 equally spaced braces, respectively
C_f = force in column, the compressed portion of a flexural member, or the

compression chord of a truss, under factored loads
L = length between brace points
ϕ_r = partial resistance factor

The similarity between equations (5) and (6) may be demonstrated by recognising that for the case under consideration, β = 2, and in order to maximise the term in brackets of equation (6), Δ_{ap} should be set equal to zero. Equations (5) and (6) are then identical, with the exception of the introduction of the capacity reduction factor, ϕ_r. The commentary to CSA S16.1-99 indicates that the value of ß should be taken as 1 when considering the ends of a braced member, and this may be appreciated by considering the theoretical model, and the basic equilibrium thereof. The sum of the reactions should equal the force in the brace, which in turn is related to the stiffness of the brace, and hence supports the requirement of a value of $\beta = 1$.

Most current bracing rules are based on the proposals of Winter (Winter 1960). The theoretical model used in this paper is similar to Winter's approach, but clearly indicates the influence of the brace stiffness on the buckling load of the strut.

Equation (4) indicates that, for the case under consideration, the brace becomes ineffective at a stiffness of one quarter of that required to be fully effective.

It is also important to note that the greater the stiffness provided in the brace, the lesser the force in the brace.

Force criteria and the design of bracing

The force in the brace is linearly related to the displacement of the brace, and multiplication of equation (5) by the displacement of the brace allows the force in the brace to be calculated.

CSA S16.1-99 also calculates the bracing force directly from the displacement of the braced point as given by equation (6). It is important to note that the force in the brace tends towards zero if the brace is infinitely stiff, and increases with the displacement of the braced point.

Design codes typically specify nominal loads for the design of bracing systems having values between 1,5% and 2,5% of the force in the strut. The fundamental problem of specifying a nominal design force without a stiffness criterion may only be appreciated by recognising that such an approach cannot ensure that the conditions required by equation (4) to achieve an effective length factor consistent to that assumed by the designer, are met.

CSA S16.1-99 specifies an iterative procedure whereby the brace stiffness is initially calculated from equation (6), considering the recommendation contained in the commentary of maximising the term in brackets, and using a value in the region of, or exceeding 2, which is

essentially the same as using equation (5), as previously discussed. A preliminary size of brace is then selected and the force in the brace is calculated using an assumed displacement of the braced point. The displacement of the braced point is then re-calculated using the actual stiffness and the first iteration value of the force in the brace. The procedure is repeated until force and displacement criteria are simultaneously satisfied.

This procedure may be regarded as too tedious and time consuming, especially when considering the relative cost of bracing as a proportion of the overall cost of the structure.

Force criteria traditionally contained in design codes stipulate nominal design forces varying between 1,5% and 2,5% of the force in the strut. Using equation (5) and multiplying the stiffness by the displacement, such values are seen to be consistent with displacements of the braced point of between L/267 to L/160. These values are, of course total values and relate to the final position of the braced points relative to adjacent supports, and are greater than the initial misalignment.

Design of bracing members to force and stiffness criteria

A fundamental issue which should be resolved is, whether both criteria, stiffness and strength, need to be satisfied within a design context. To answer this question, it is necessary to consider the problem from a different angle, viz the actual performance of the bracing element. Consider the very simple case of a tie installed perpendicular to the strut to act as a brace.
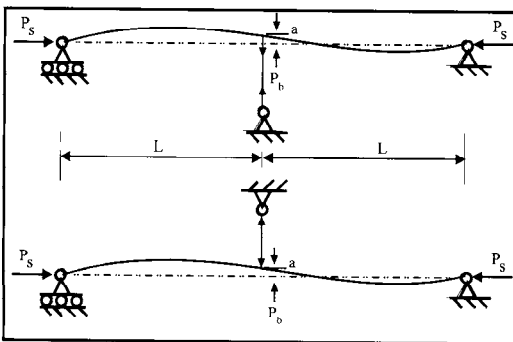

Figure 3 Simple bracing system using tie or strut braces

If the sizing of the tie is performed on the basis of a force criterion the following minimum size for the element is obtained:

$$P_b = A_b \cdot f_y$$

Where:
A_b = area of brace
f_y = yield strength of brace

By applying a criterion of minimum stiffness, the following result is obtained:

$$k_b = \frac{E_b A_b}{L_b}$$

Where:
L_b = length of brace
E_b = modulus of elasticity of the brace
The two results may be related by dividing the first equation by the displacement of the brace, a and then equating the result to the second equation, then:

$$\frac{A_b f_y}{a} = \frac{E_b A_b}{L_b}$$

Which may be written as:

$$\frac{a}{L_b} = \frac{f_y}{E_b} \qquad (7)$$

For structural steel the ratio of yield stress to elastic modulus lies between ,001 and ,002.

The maximum deformation per unit length of a brace designed as an *axially loaded tie*, simultaneously satisfying both the force and the stiffness criteria, therefore lies between 0,001 and 0,002, and is shown to be a function of the yield stress of the material, as well as the value of the modulus of elasticity.

The situation is more complicated when the brace is in compression, the minimum strength of the brace can then be expressed as:

$$P_b = \frac{\pi^2 E_b I b \alpha}{(L_b)^2} \qquad \text{where the factor } \alpha \text{ is}$$

used to convert the elastic critical load to an inelastic buckling load of the brace. The stiffness of a brace in compression is however, greatly reduced if the brace has an initial curvature.

In order to simultaneously satisfy the stiffness and strength criteria, the following relationship is obtained:

$$\frac{\pi^2 E_b I_b \alpha}{a(L_b)^2} = \frac{E_b A_b}{L_b}$$

$$\therefore \frac{\pi^2 \alpha}{(\lambda)^2} = \frac{a}{L_b} \qquad (8)$$

Equation (8) illustrates that the more slender the strut, the greater the deformation of the brace per unit length. A slenderness limit commonly adopted in design codes is $\lambda = 180$.

At this value of slenderness the elastic critical load is almost equal to the inelastic load and the value of α may be conveniently taken as equal to one. The deformation limit of a slender strut used as a brace, simultaneously satisfying the proposed force and stiffness criteria, is then given by:

$$\frac{a}{L_b} = 0,000346$$

The following significant conclusions may be drawn from considering the results of applying force and stiffness criteria to the design of simple, axially loaded bracing members:

- The stiffness of a tie designed to satisfy force criteria only is significantly less than that obtained for a bracing member designed for axial compression.

- Bracing members designed as ties using simple force criteria will not necessarily possess the required stiffness.

- Bracing members acting as struts, designed to stiffness criteria only, will not necessarily possess sufficient buckling resistance.

Upper bound limits in design using stiffness and force criteria

Proposed upper bound limit stiffness criterion

In defining an upper bound solution used for design purposes, it is important to consider not only elastic behaviour but the influence of yielding.

Consider the stiffness criterion as given by equation (5):

$$k_b = \frac{4P_e}{L} \qquad (5)$$

Equation (5) was derived by considering elastic buckling only. In real struts the buckling load and therefore required stiffness is reduced by inelastic behaviour. As the slenderness of the strut is reduced, the value of E in the expression $\frac{\pi^2 I}{L^2}$ will be reduced from the elastic value to the strain-hardening value of the material. The upper limit of P_e is determined by the squash load of the strut. The increase in the value of the squash load caused by strain-hardening is small and may be assumed to be covered by the introduction of a partial resistance factor, which will be discussed subsequently.

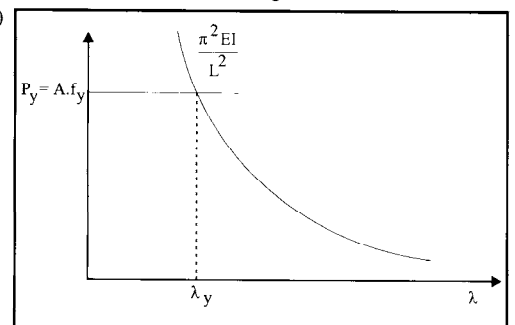Consider the hypothetical case of a strut with no residual stress or imperfections, as shown in figure 4:

Figure 4 Resistance curve for a column with no residual stress or imperfections

For the strut shown in figure 4, the maximum value of bracing stiffness required will coincide with a slenderness ratio producing the maximum value of axial force in the strut, and this will occur at the point of intersection of the critical elastic load with the squash load of the strut.

At slenderness ratios lower than this value, yielding will reduce the stiffness of the strut and consequently the required brace stiffness. The upper limit of equation (4) may therefore be expressed as:

$$k_b = \frac{4Af_y}{L^2}$$

The proposed upper bound stiffness criterion is therefore formulated as follows:

For $P_e < Afy$: $\quad k_b = \frac{4P_e}{L}$

For $P_e > Afy$: $\quad k_b = \frac{4Af_y}{L}$

Proposed upper bound limit force criterion

The force in the brace is a function of the final displacement of the brace, which in turn is dependent on the initial misalignment.

As the design bracing force is dependent on the initial misalignment of the braced point, it is useful to consider the requirements of some design codes regarding the misalignment of column restraints. BS 5400 (BS 5400 Part 3 1982) specifies a misalignment of the sum of two adjacent un-braced lengths divided by 500 and BS 5950 (BS 5950 Part 1 1985) as L/600. The draft Canadian Code CSA 1-99 (Canadian Standards Association 1999) recommends an initial misalignment of L/1000. Only the Eurocode specifies a final misalignment of L/250 to be considered in the design of bracing, used in conjunction with an initial imperfection of L/500, indicating a modal amplification factor of 2.

The final displacement of the brace is a function of the initial misalignment or sweep and the ratio of the load in the strut to the elastic buckling load of the strut. The final displacement of the brace may also be determined by an iterative process which is basically similar to the procedure suggested by the CSA S16.1-99.

Stanway et al (1992) and Winter (1960) have considered the equilibrium of simple bar link systems as shown in figure 5:
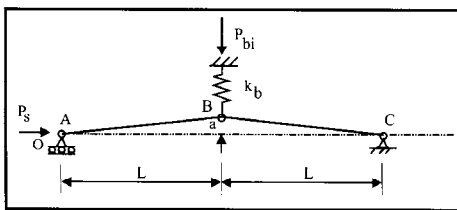

Figure 5 Equivalent bar link system representing braced strut

Considering basic equilibrium of the spring and the applied load of the system shown in figure 5, the force in the brace is given as:

$$P_{bi} = 2P_s \frac{a_{ini}}{L} \tag{9}$$

The bracing force is given in terms of the bracing stiffness as:

$$P_b = k_b a_{fin} \tag{10}$$

The bar link system shown in figure 5 constitutes a non-linear problem where, for any small initial displacement, a_i, the final force in the brace may be written as follows:

Table 1 Comparison of force and stiffness criteria for three different sections use as struts, having the same axial resistance and the same distance between braced points

Distance between braced points	Section	Factored member resistance	Ultimate braced strength of column	Elastic buckling load (P_e)	Minimum bracing stiffness (eq 5)
4 000 mm	120 x 120 x 8 L	107 kN	504 kN	130 kN	130 kN/mm
4 000 mm	IPE 180	104 kN	513 kN	125 kN	125 kN/mm
4 000 mm	102 x 2,8 CHS	108 kN	235 kN	132 kN	132 kN/mm

$$P_b = \frac{2P_s}{L}(a_i + \delta_1 + \delta_2 + \delta_3 + \ldots \ldots \delta_n)$$

For the bar link system to be stable, the second order displacements should decrease in magnitude, therefore:

$$\delta_1 < a_i$$

$$\delta_2 < \delta_1$$

$$\ldots \ldots$$

$$\delta_n < \delta_{n-1}$$

This requirement is clearly met if the stiffness of the brace is greater than $2P_s/L$.

If the stiffness of the brace, however, is provided in terms of equation (5), the brace stiffness will be double that required for stability of the bar link system. The second order displacements will therefore be halved in each iteration. The final displacement of the braced point may therefore be expressed in terms of a series as given by equation (11):

$$a_{fin} = a_i(1 + \sum_{n=1}^{n \to \infty} \frac{1}{2^n}) \tag{11}$$

The series $\sum_{n=1}^{n \to \infty} \frac{1}{2^n}$ can be shown to have a convergence value of 1.

In more general terms, the amplification factor for the system may be expressed as:

$$a_{fin} = a_i(1 + \sum_{n=1}^{n \to \infty} \frac{1}{x^n}) \tag{12}$$

Where:

$$x = \frac{k_b}{2P_e/L}$$

Provided that the minimum stiffness requirements of equation (5) are met, the final displacement of the braced point will therefore be equal to double the initial displacement, and the force in the brace may be expressed as:

$$P_b = 4P_s \frac{a_i}{L}$$

Examples using the proposed bracing design criteria

The above proposals are best illustrated by considering some examples of using the proposed criteria. Three common structural sections will be used, a circular hollow section, a hot-rolled IPE-section and an equal-legged angle. The force and stiffness criteria are reviewed by comparing suitable bracing elements designed as

struts or ties, providing central restraint to an axially loaded strut, as shown in figure 2. (See table 1 above.)

The calculation of the design force in the bracing and the influence of modal amplification is now illustrated using the circular section.

An iterative procedure is followed. The initial force in the bracing is calculated using a minimum value of brace stiffness as required by equation (5) and the initial misalignment of the braced point. The force in the strut was taken as the factored resistance of the strut. The bracing force is then progressively adjusted using the total displacement of the braced point. Satisfactory convergence is obtained after approximately nine iterations. This procedure is similar to that proposed in CSA 1-99. The results are shown in table 2.

Table 2 Bracing force and final displacement for an initial misalignment: L/200 - minimum brace stiffness: 0,132 kN/mm, initial displacement of braced point: 20 mm

Iteration	Bracing force (kN)	Displacement of braced point (mm)	Total displacement (mm)
1	1,32	10	30
2	1,98	15	35
3	2,31	17,5	37,5
4	2,475	18,75	38,75
5	2,5575	19,375	39,375
6	2,598	19,6875	39,6875
7	2,619	19,84375	39,843
8	2,6297	19,9219	39,9219
9	2,6348	19,961	39,961
	1,996% of P_b		L/100

The results are similar to using equation (11) to calculate the modal amplification. The final misalignment equals double the initial misalignment, because the brace stiffness is double the minimum value given by equation (9), and the design bracing force equals 2% of the force in the strut.

SUMMARY OF BRACING DESIGN PROPOSALS
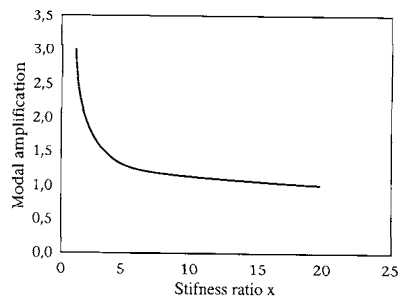
Minimum stiffness of braced point:

$$k_b > \frac{4P_e}{L} \; or \; \frac{4Af_y}{L}$$

whichever is the lesser.

Minimum design force in bracing:

$$P_b = \frac{2P_s}{L} a_i \left(1 + \sum_{n=1}^{n \to \infty} \frac{1}{x^n} \right) \quad \text{where} \quad x = \frac{k_b}{2P_e/L}$$

The progression of the modal amplification factor as a function of the stiffness ratio x is shown in figure 6:


Figure 6 Values of modal amplification factor (ratio of initial misalignment to final displacement) as a function of the stiffness ratio of the brace

In a common design situation, the designer may assume the modal amplification factor to equal 2, provided that the stiffness of the brace exceeds the minimum required by equation (5).

COMPLEX BRACING SYSTEMS

Bracing systems commonly consist of a number of members acting in compression and tension to form a lattice truss system. As a general rule, all such systems can be reduced to a system of equivalent springs acting in series as shown in figure 7:
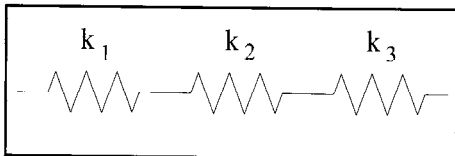
Figure 7 Equivalent spring system for complex bracing system

The effective spring stiffness for such systems may be obtained by using the series addition rule, ie

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots$$

In practice, however, the designer will commonly opt to use a simple frame analysis program and determine the effec-

tive stiffness of a system by applying a unit load at the appropriate position and calculating the corresponding deflection.

EVALUATION OF STIFFNESS OF BRACING SYSTEMS

Excepting the cases where very simple bracing systems such as axially loaded links connected directly to rigid mediums are employed, frame analysis software offers the most convenient method of evaluating the stiffness of bracing systems. By applying unit loads at the appropriate positions, the deflections obtained from such analysis directly reflect the stiffness of the bracing system at such positions.

Where bolted connections are used to connect bracing members, slip in such connections will obviously reduce the effective stiffness of the system. The complexities involved in quantifying the influence of slip in the connections is probably not warranted, but it is recommended that the theoretical stiffness of the system be reduced by some 20% where the bracing system consists of members with bolted end connections. The above comment would only apply to connections utilising bolts in bearing.

Conclusion

A rational approach to the design of bracing elements to provide stability to compression elements has been proposed. By specifying both force and stiffness criteria in the design of such members or systems, the following benefits are achieved:

- The stiffness criterion ensures that bracing systems possess adequate stiffness to prevent a reduction in the buckling capacity of the primary members caused by large deflections of the braced points. This criterion is of particular importance when bracing elements are subjected to tensile loads only.

- The force criterion ensures that compression members in the bracing system are adequately designed to prevent buckling of the bracing members themselves.

- By specifying force and stiffness criteria as well as a method of calculating modal amplification effects, the iterative procedure required by the Canadian Code is not required.

- The benefit of providing additional stiffness over and above the minimum specified by equation (5) is reflected in the reduced design force in the bracing, as proposed in the upper bound force criterion.

- The designer can evaluate the design bracing force for any initial displacement of the braced point.

- The reduction in stiffness caused by yielding in stocky struts ($P_e > Af_y$) is considered in formulating the upper bound stiffness criterion and leads to more economical bracing design at low slenderness ratios.

- Although the discussion has been limited to single brace systems, the model may be expanded to include multiple braced points using the principles outlined in CSA-S16.1-99.

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