

W M G Burdzik and N W Dekker

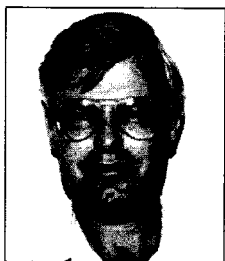
# Bracing of timber roofs

## Synopsis

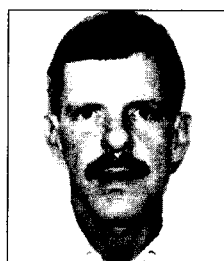
Problems attributed to the bracing of timber roofs have necessitated a fresh look at the bracing criterion given in the timber design code, SABS 0163 (1994). The difference between bracing provided for stability of the overall structure and bracing that is used to reduce the effective length of compression members is discussed. The differences between bracing at discrete intervals and bracing by means of a continuous elastic diaphragm are shown. Revised design rules are proposed, which, if implemented, should solve the problems currently being experienced with timber roofs.

## Samevatting

Probleme waarvoor die verspanning van houtdakke die skuld gegee word, het genoodsaak dat opnuut gekyk word na die verspanningskriteria wat in die houtontwerpkode SABS 0163 (1994) gegee word. Die verskil tussen die verspanning wat stabiliteit aan die hele struktuur verskaf en die verspanning wat gebruik word om die effektiewe lengte van 'n drukdeel te verander, word bespreek. Die verskille tussen die verspanning wat die drukdeel op diskrete intervalle lateraal steun en verspanning deur middel van 'n kontinue diafragma word uiteengesit. Gewysigde reëls word voorgestel wat, indien hulle geïmplementeer sou word, die probleme wat huidiglik met houtdakke ondervind word, sou oplos.



**Walter Burdzik** received the degrees BSc Eng (Civil), BEng Hons, MEng and PhD from the University of Pretoria. He spent three years working for consulting engineers and then returned to the University of Pretoria as a senior lecturer, later being promoted to Professor of Structural Engineering. Together with Ben van Rensburg, he started the timber engineering research unit at the University of Pretoria and expanded the facilities to the extent where it is now one of the few recognized timber testing facilities in South Africa. He has been intimately involved in the committees charged with writing the timber design codes and has undertaken numerous research projects as a result of this work.



**Nick Dekker** received the degrees BSc Eng, BEng Hons and MEng from the University of Pretoria and a PhD from the University of the Witwatersrand. He has spent most of his professional career with BKS, where he was responsible for the design of a wide range of structures, including bridges, industrial and commercial buildings, shopping centres, sports stadia and process buildings. In 1996 he co-founded the practice Dekker & Gelderblom and was appointed a Professor of Structural Engineering at the University of Pretoria.

## Introduction

A marked increase in problems associated with the bracing of timber roofs has been noted in recent years, perhaps precipitated by the increase in size and span of such roofs. Failures observed range from buckling of the top chord to total collapse, the latter failure commonly associated with a complete absence of bracing. Buckling of the top chords has occurred in spite of the designs complying with SABS 0243 (1992). The problems are generally confined to tiled roofs, perhaps owing to the diaphragm action induced by roof sheeting.

In most instances the problems have manifested themselves only after a number of years. It is believed that some of the bracing may be provided by friction between the tiles, which in turn may be reduced by wind uplift and a gradual movement of the tiles induced by thermal expansion and contraction. In most cases, buckling of the compression chord has been noted on the side of the roof with the greater exposure to direct sunlight. Considering that a large number of the problem cases do in fact comply with SABS 0243 (1992), one may conclude that the current bracing provisions are inadequate or do not reflect prevailing construction conditions in South Africa.

It is necessary to distinguish clearly between two types of bracing, ie bracing required to provide a structure with overall stability (Fig 1) and bracing provided to reduce the effective length of compression or flexural members (Fig 2). The design and sizing of the former type of bracing is clearly determined by the magnitude of external forces, while the design of the latter type is more complex.

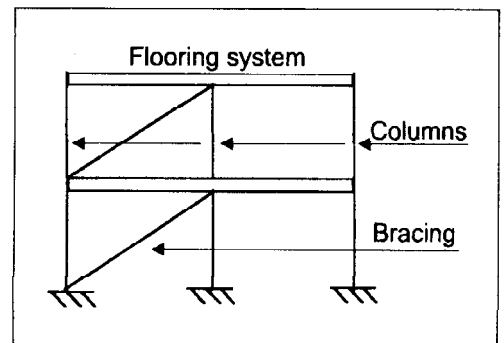


Fig 1: Bracing required for stability against horizontal forces

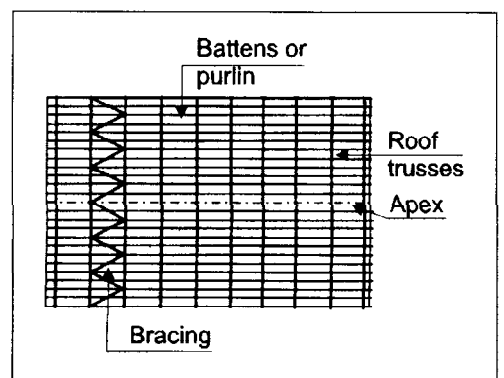


Fig 2: Bracing required to decrease effective length of compression members

Two types of bracing systems are commonly employed, a prefabricated bracing frame (Fig 3), providing the compression member with a continuous elastic support, and a single diagonal providing discrete support for the compression elements at certain intervals (Fig 4).

The interaction between the bracing system and the compression element is very complex and the efficiency of the bracing system is dependent on the stiffness of the brace relative to the member being restrained. It is therefore understandable that codes have traditionally opted for a relatively simple criterion for the design of bracing systems, the most common being a design action expressed as a force, which is a percentage of the force in the strut. It will subsequently be shown that this method will not necessarily ensure that the brace possesses adequate stiffness.

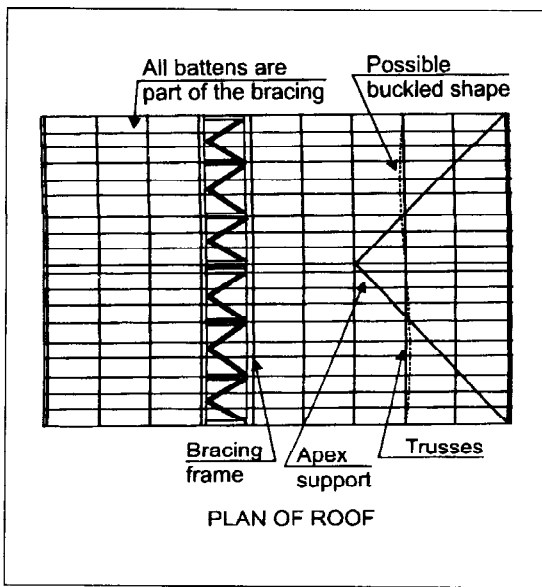


Fig 3 (left): Bracing frame that provides continuous elastic support

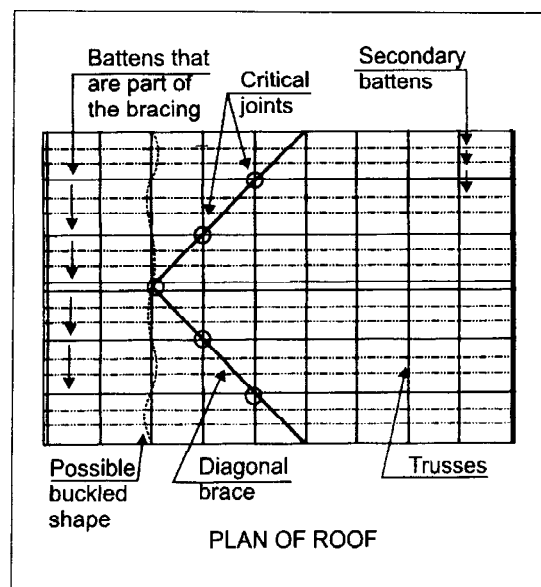


Fig 4 (right): Discrete lateral support to top chord by means of diagonal brace

### Current bracing rules

The Eurocode for timber design, EC5 (1992), stipulates the following requirements:

#### Members braced at discrete intervals (Fig 5)

The displacement of the brace itself at the point of attachment of the brace has a fundamental influence on the buckling resistance of the strut it is required to restrain. An out of straightness limit, for single compression members, of  $L/500$  for glue laminated products and  $L/300$  for all others is proposed. This requirement limits the displacement of the strut, referred to as  $d_b$ . The required spring stiffness  $C$  is given by:

$$C = k_s \pi^2 EI / a^3 \quad (1)$$

where:

- $C$  = spring stiffness
- $E$  =  $E_{0,05} f_{md} / f_{mk}$
- $E_{0,05}$  = fifth percentile modulus of elasticity
- $f_{md}$  = the design stress
- $f_{mk}$  = characteristic stress
- $k_s$  =  $2(1 + \cos\pi/m)$
- $m$  = the number of bays with a length of  $a$
- $a$  = distance between lateral supports

The design resistance  $F_d$  of the bracing is given in terms of the mean design force  $N_d$ :

$$\begin{aligned} F_d &= N_d / 50 \text{ for solid timber} \\ F_d &= N_d / 80 \text{ for glued laminated sections} \end{aligned} \quad (2)$$

It is important to note that both the strength and stiffness criteria must be satisfied. It does not suffice only to apply the force criterion as the force criterion is based on the assumption that the deflection will be limited by the stiffness of the spring.

#### Members braced by a continuous bracing system (elastic support)

For a series of parallel laterally supported members, the design force per unit length on the bracing system  $q_d$ , additional to any other forces induced by horizontal loads, is given by:

$$q_d = k_1 \frac{nN_d}{30L} \quad (3)$$

where:

- $n$  = the number of members being supported
- $N_d$  = the design axial force in the member

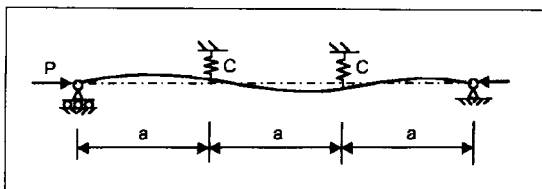


Fig 5: Axially loaded compression member with  $m - 1$  lateral supports

$L$  = span of the beam or distance from the eaves support to the apex support

$k_1$  = minimum of 1 or  $\sqrt{15}/L$

In order to satisfy the stiffness requirement, the mid-span deflection of the bracing system caused by the load  $q_d$  alone should not exceed  $\text{Span}/700$ . If  $q_d$  acts in combination with other loads, the requirement is amended to  $\text{Span}/500$ .

SABS 0163 (1994) by comparison stipulates no stiffness requirement and the design load for the bracing system is given by:

$$P_b = 0,1 n^{0,7} \frac{P_A}{N + 1} \quad (4)$$

where:

- $P_b$  = force in each lateral brace
- $n$  = number of trusses or members that are braced by the system
- $P_A$  = the average force in the compression member
- $N$  = the number of lateral restraints

The Australian steel design code, AS1250-1972, stipulates both a stiffness and a strength criterion for the bracing of the compression flanges of beams. The criterion for the strength of the bracing is given as 2,5 per cent of the axial force, which according to Nethercott (1982) is considered too conservative in many cases. A value of two per cent would be in keeping with that given in EC5 (1992). The stiffness of a brace is required to be at least  $10P/L$ , where  $L$  is the overall length and  $P$  the force in the compression flange. To account for the cumulative effect of a number of members supported by a bracing system, the Australian steel code suggests that the total force should be taken as the force induced by the seven most heavily loaded members.

### Theoretical models

#### Bracing at discrete intervals

Most bracing rules are based on the work of George Winter (1960). Winter investigated the influence of the two principal parameters, stiffness and strength, which are required to provide a compression member with effective lateral bracing. Winter considered columns with one to four lateral spring restraints as well as columns provided with continuous lateral support as would be the case for compression members that are connected by sheeting.

In the design of bracing to resist external horizontal forces, the stiffness requirement is not that important and the strength requirement will govern. In the case of bracing that is used to decrease the buckling length of compression members, ie reduce the slenderness ratio of compression members, both stiffness and strength are equally important. In considering the requirements for bracing given by SABS 0163 (1994), the lack of a requirement for the stiffness of the bracing system is significant. SABS 0163 (1994) only stipulates a provision for a nominal design force for the bracing. It may be argued that the strength criterion should also ensure that the bracing system possesses adequate stiffness. This is not necessarily the case.

For a column that is laterally supported by  $n$  number of elastic sup-

ports, the required spring constant was found by Winter (1960) to be a function of the Euler buckling strength and the distance between the lateral supports. The idealized spring constant, for an initially straight column, may be written as:

$$k_{id} = \frac{k_s P_c}{L} \quad (5)$$

where:

- $k_{id}$  = idealized spring constant for initially straight column
- $k_s$  = factor for the number of lateral supports  
=  $2(1 + \cos(\pi/m))$ , given in EC5, 1992
- $m$  = number of bays of length  $L$
- $P_c$  = Euler buckling strength
- $L$  = distance between the equally spaced lateral supports

Winter (1960) recommended that the value of the spring constant be increased for columns with an initial curvature. This increase depends on the initial deflection and the final displacement. The required spring constant,  $k_{req}$ , may then be written as:

$$k_{req} = k_{id} [(d_0/d) + 1] \quad (6)$$

where:

- $d_0$  = initial deflection due to lack of straightness (see Fig 6)
- $d$  = additional deflection after buckling

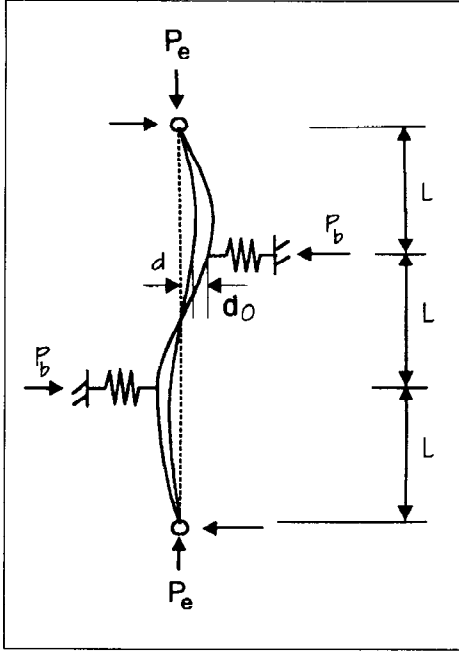


Fig 6: Buckled shape of laterally braced compression member, showing deflections and supports

The force that will be induced in the spring or brace will then equal the spring constant times the total deflection at the point of support, therefore:

$$P_b = k_{req} (d_0 + d) \quad (7)$$

Coates (1988) describes a method that may be used to determine the critical force in a member that is laterally supported at a discrete point by a single spring. The results are the same as those given by Winter (1960).

#### Bracing of members by a continuous bracing system

Winter (1960) used the theory of a compression member on an elastic foundation to determine the stiffness requirement of continuous lateral support. This method is consistent with the case of a roof that is braced by a pre-fabricated bracing frame, where every batten is fixed to the bracing frame. Owing to the close spacing of the battens and the overall stiffness of the frame, this type of bracing system is more representative of a continuous lateral restraint than restraints at discrete points. The theoretical value of the modulus of the elastic support medium,  $\beta_{act}$ , can be obtained from the following formulae:

$$\frac{\beta_{id} L^2}{P_c} = \pi^2 \left( \frac{P_{cr}}{P_c} - 1 \right) \text{ for } 0 < \frac{\beta_{id} L^2}{P_c} \leq 30 \quad (8)$$

$$\frac{\beta_{id} L^2}{P_c} = \frac{\pi^2}{4} \left( \frac{P_{cr}}{P_c} - 0,6 \right)^2 \text{ for } \frac{\beta_{id} L^2}{P_c} \geq 30 \quad (9)$$

Similar to columns having an initial curvature that are supported at discrete intervals, continuously supported columns with an initial curvature will require a greater value of the modulus of the elastic support modulus.

$$\beta_{req} = \beta_{id} (d_0 + d) \quad (10)$$

If the actual elastic modulus,  $\beta_{act}$ , is greater than the required modulus, the force induced in the bracing will be:

$$\omega_{req} = d_0 \frac{\beta_{id}}{1 - (\beta_{id} / \beta_{act})} \quad (11)$$

Timoshenko et al (1961) used a slightly different approach to determine the required stiffness of the bracing, ie the elastic support modulus,  $\beta$ , for compression elements that are braced by a continuous system. An energy method is used to determine the critical load,  $P_{cr}$ , which is given by:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \left( m^2 + \frac{\beta L^4}{m^2 \pi^4 EI} \right) \quad (12)$$

where:

- $m$  = number of half sine waves that form when the compression member buckles
- $L$  = length of the compression member
- $E$  = modulus of elasticity
- $I$  = second moment of area perpendicular to the plane of buckling
- $\beta$  = modulus of the elastic foundation

In all cases Eqn 12 can be represented in the form:

$$P_{cr} = \frac{\pi^2 EI}{l^2} \quad (13)$$

where  $l$  equals a reduced length reflecting the influence of the elastic support.

A series of values for  $l/L$  are given in the accompanying table.

#### Reduced length $l$ for a compression member on an elastic foundation

$\beta l^4 / (16 EI)$	0	1	3	5	10	15	20
$l/L$	1	0,927	0,819	0,741	0,615	0,537	0,483
$\beta l^4 / (16 EI)$	30	40	50	75	100	200	300
$l/L$	0,437	0,421	0,406	0,376	0,351	0,286	0,263
$\beta l^4 / (16 EI)$	500	700	1 000	1 500	2 000	3 000	4 000
$l/L$	0,235	0,214	0,195	0,179	0,165	0,149	0,140

Eqn 12 can be modified and expressed in terms of the Euler buckling load,  $P_c$ , and the critical load applied to the member,  $P_{cr}$ . The equation for  $P_{cr}$  as a proportion of  $P_c$  is given by:

$$\frac{P_{cr}}{P_c} = \left( m^2 + \frac{\beta_{id} L^2}{m^2 \pi^2 P_c} \right) \quad (14)$$

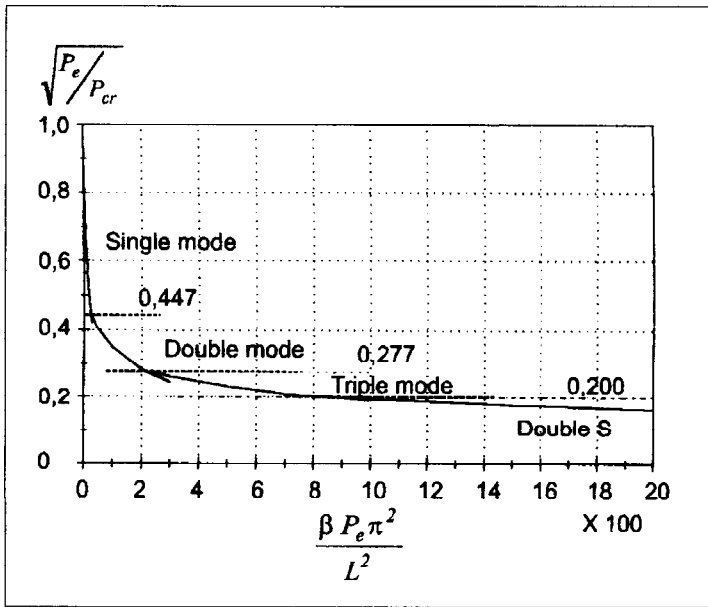
where  $m$  = number of half sine waves and  $\beta_{id}$  = the ideal modulus of the bracing.

The critical values of  $l/L$ , where the buckled shape changes from a half sine wave,  $m = 1$ , to a full, one-and-a-half and two full sine waves are 0,447, 0,277 and 0,200 respectively (see Graph 1). The value of  $l/L$  is given by  $\sqrt{P_c / P_{cr}}$ . If the mode shape  $m$  is known, the ideal  $\beta_{id}$  value can be calculated. The ideal  $\beta_{id}$  is given by:

$$\beta = \frac{m^2 \pi^2 P_c}{L^2} \left( \frac{P_{cr}}{P_c} - m^2 \right) \quad (15)$$

The theoretical elastic support modulus must be increased by the factor of  $(l + d_0/d)$  to allow for initial curvature and a partial factor of safety of at least 2,22 should be applied.

The force in the bracing can be calculated by multiplying the stiffness by the theoretical deflection. If the initial curvature is known and the additional deflection is given, the load in the bracing is the stiffness multiplied by the sum total deflection. In the case of multiple members sup-



Graph 1: The effect of  $\beta$ , the stiffness modulus of the lateral support, on the buckled shape of a compression member

ported by a single bracing system, it is unlikely that all contributing members will have similar initial curvatures, thereby causing a cumulative effect in terms of forces on the bracing. It is therefore presumed that the individual contribution of the members will decrease with an increase in the number of members supported by the same bracing system (SABS 0163, 1994).

#### Comparison of theoretical formulae with code requirements

The theoretical models clearly indicate the fundamental importance of the stiffness of the bracing system, as opposed to a criterion based solely on a nominal design force. It is therefore of vital importance that a new set of bracing rules has both stiffness and strength criteria. Bracing rules must, furthermore, also consider the different types of bracing that can be used, namely bracing at discrete intervals and continuous braces.

#### Proposed bracing rules

##### Members braced at discrete points

For members that are braced at discrete points along the length, we propose that the required stiffness of the bracing be as suggested by Winter (1960), namely:

$$k_{id} = \frac{k_s P}{L} \quad (16)$$

where:

$$k_s = \text{factor for number of lateral supports} = 2(1 + \cos(\pi/m)) \quad (17)$$

$P$  = compressive force in the member due to dead load only

$L$  = distance between lateral supports

$$k_{req} = k_{id}(1 + \delta_0/\delta) \quad (18)$$

If an initial curvature, with average amplitude for all the trusses in the braced system,  $\delta_0$ , equal to  $L/500$  is assumed and a final additional deflection,  $\delta$ , of  $L/500$  is possible, then the required stiffness should be at least equal to 2,0 times the ideal.

A partial factor of safety of at least 2,0 should be applied to the theoretical value. The required stiffness is then equal to:

$$k_{req} = 4,0 k_{id}$$

For the specific case of a single lateral brace the required stiffness is then:

$$k_{single} = 8,0 P/L \quad (19)$$

For the case of multiple lateral supports the values of  $\cos(\pi/m)$  in Eqn 17 will tend towards 1 and the required stiffness at each support will be given by:

$$k_{many} = 16,0 P/L \quad (20)$$

The force in the support may be obtained by multiplication of the stiff-

ness by the deflection. If a final additional deflection,  $\delta$ , of  $L/500$  is assumed, then the force in the lateral support is equal to:

$$F = k_{req} \delta \quad (21)$$

For a single central lateral support the force in the brace is 1,6 per cent of the axial force in the member and for multiple lateral supports 3,2 per cent of the axial force. The latter requirement would apply to the case where a number of members are supported by the same lateral support. Where a single member is laterally braced, the requirement should be more severe as the initial curvature could be as high as  $L/200$ . The value of 3,2 per cent of the axial force caused by dead load only is similar to that required by the Australian code, ie 2,5 per cent of total load, but is significantly more than the value given by SABS 0163, ie 10 per cent divided by the number of lateral supports. The value recommended in this paper is higher to allow for realistic values of initial curvatures in the members.

##### Member supported by an elastic bracing frame

For members that are braced by a continuous bracing system, the proposals are based on the method described by Timoshenko et al (1961).

$$\text{For } \sqrt{\frac{P_c}{P_{cr}}} \leq 0,447 \quad \beta_{id} = \frac{\pi^2 P_c}{L^2} \left( \frac{P_{cr}}{P_c} - 1 \right) \quad (22)$$

$$0,447 < \sqrt{\frac{P_c}{P_{cr}}} \leq 0,277 \quad \beta_{id} = \frac{4\pi^2 P_c}{L^2} \left( \frac{P_{cr}}{P_c} - 4 \right) \quad (23)$$

$$0,277 < \sqrt{\frac{P_c}{P_{cr}}} \leq 0,200 \quad \beta_{id} = \frac{9\pi^2 P_c}{L^2} \left( \frac{P_{cr}}{P_c} - 9 \right) \quad (24)$$

$$0,200 < \sqrt{\frac{P_c}{P_{cr}}} \quad \beta_{id} = \frac{16\pi^2 P_c}{L^2} \left( \frac{P_{cr}}{P_c} - 16 \right) \quad (25)$$

The required stiffness will be greater than the theoretical stiffness and is given by:

$$\beta_{req} = \beta_{id} \left( 1 + \frac{\delta_0}{\delta} \right) \quad (26)$$

If an initial curvature, with maximum amplitude  $\delta_0$  equal to  $L/300$  is assumed and a final additional deflection  $\delta$  of  $L/500$  is possible, then the stiffness required should be equal to at least 2,667 times the ideal.

If a partial load factor of 2,22 is then applied to this value, the required lateral stiffness of the bracing system would be equal to 5,921 times the theoretical stiffness.

$$\beta_{req} = 5,921 \beta_{id}$$

The nominal force induced in the bracing system will be equal to the required stiffness multiplied by the additional deflection of  $L/500$ .

$$q = 5,921 \beta_{id} L/500 * 0,637 = 3,772 \beta_{id} L/500 \quad (27)$$

The factor of 0,637 reflects the buckled shape, which is assumed to be a half sine wave. In the case of a buckled shape in the form of a full sine wave, the bracing system will be subjected to load reversal along its length, which will reduce the total load on the system.

#### Summary of bracing requirements

The following bracing criteria are proposed for timber structures and could be modified slightly for the bracing of steel structures.

##### Compression members braced at discrete intervals

Stiffness:

$$k_{req} = \frac{4k_s P}{L} \quad (28)$$

where:

$$k_s = 2(1 + \cos(\pi/m))$$

$m$  = number of equal bays of length  $L$  between apex and eaves

$P$  = axial load in compression member, at the lateral support, owing to dead load only

Force in lateral support,  $P_l$ : For many lateral supports to a compression mem-

ber,  $P_b$  = three per cent of the average axial load  $P$  in the compression member.

*Compression members continuously braced by a membrane or bracing frame*

Required stiffness modulus,  $\beta_{req}$ :

$$\beta_{req} = \frac{5,921 m^2 \pi^2 P_e}{L^2} \left( \frac{P_{cr}}{P_e} - m^2 \right) \quad (29)$$

where:

$\beta_{req}$  = required stiffness modulus

$$m = 1 \text{ for } \sqrt{\frac{P_e}{P_{cr}}} \leq 0,447 \text{ (buckling in half sine wave)}$$

$$= 2 \text{ for } 0,447 < \sqrt{\frac{P_e}{P_{cr}}} \leq 0,227 \text{ (buckling in full sine wave)}$$

$$= 3 \text{ for } 0,227 < \sqrt{\frac{P_e}{P_{cr}}} \leq 0,200 \text{ (buckling in one and a half sine wave)}$$

$$= 4 \text{ for } 0,200 < \sqrt{\frac{P_e}{P_{cr}}} \text{ (buckling in a double sine wave)}$$

$P_{cr}$  = axial load in member due to dead load alone

$P_e$  = Euler buckling load

$$= \frac{\pi^2 E_{0,05} I}{L^2} \quad (30)$$

$L$  = length of beam or distance between eaves support and apex support of truss

$E_{0,05}$  = fifth percentile modulus of elasticity

*Nominal design load,  $q$  (based on a single half wave buckle):*

$$q = \frac{0,06 P}{L} \quad (31)$$

where:

$P$  = axial force in member due to dead load alone

$L$  = length of beam or distance from eaves support to apex support of truss

If the buckled shape of the compression member assumes a full wave as opposed to a half sine wave, the moments that are induced in the bracing will be reduced. It is therefore conservative to assume a half sine wave buckle and to base the force on that buckled shape.

#### Acknowledgements

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