If the equal weighted portfolio is so great, why isn’t it working in South Africa?

This paper considers the recent underperformance of the equal weighted portfolio of South African Top 40 stocks relative to the market capitalisation weighted portfolio. It highlights the impact of the increased concentration of market capitalisation weights in the Top 40, which is currently at extreme levels. Furthermore, lower levels in the benefits of diversification, through higher average correlations, has reduced the positive impact of rebalancing. Finally, the turnover in index constituents has been higher than average in recent years and this has caused a further drag on performance. The combination of these effects has had a negative impact on the equal weighted portfolio’s relative performance. A rudimentary linear model, with these factors as inputs, that highlights the importance of monitoring these drivers to improve the equal weighted portfolio’s relative performance is presented.

Keywords: equal weight portfolio; equities; stochastic portfolio theory; portfolio optimization; diversification; leakage

Introduction

It is well documented that the equal weighted stock portfolio is more efficient than the market capitalisation (cap weighted) portfolio over the long-term. See for example Plyakha et al. (2012), Bolognesi et al. (2013) and Malladi and Fabozzi (2017). However, in recent years the equal weighted portfolio in South African equities has underwhelmed, to the extent that CoreShares abandoned its exchange traded fund (ETF) tracking the FTSE/JSE Equal Weighted Top 40 Index by re-mandating the fund to a multi-factor approach[1].

In Taljaard and Maré (2019), the authors consider the use of an equal weighted index as an alternative to cap weighted indices as benchmarks. Since then the relative performance of the equal weighted portfolio has worsened significantly. This paper, as an extension, analyses the performance of the equal weighted portfolio within South African equities to understand the lacklustre performance relative to the cap weighted portfolio over recent years. Many papers on this topic, such as Plyakha et al. (2012) and Malladi and Fabozzi (2017), focus on the equal weighted portfolio's relative overweight position in smaller cap stocks and the return from rebalancing as drivers of the relative performance. While these are, of course, valid explanations, this paper takes a different approach by using applications from stochastic portfolio theory (see Fernholz, 2002; and Fernholz & Karatzas, 2009) to identify the main drivers of relative performance, namely, levels of concentration in the market capitalisation weights (cap weights), the diversification benefits, and the impact of leakage as stocks fall out of the index.

Increasing levels of concentration in cap weights would, of course, have a negative impact on a portfolio that is overweight smaller cap stocks and so these two ideas are inter-related. The act of rebalancing as a source of return is usually attributed to volatility harvesting or volatility return (Bouchey et al., 2012, Bouchey et al, 2015; and Hallerbach, 2014, for example). The model used in this paper, however, reframes this as a diversification return, whereby rebalancing is done to achieve a higher level of diversification similar to Booth and Fama (1992).

The South African equity market has a long history of battling with high levels of concentration. This is well documented, for example, in Kruger and Van Rensburg (2008) and Raubenheimer (2010), and led to the introduction of the FTSE/JSE Shareholder Weighted Top 40 Index (SWIX40) and the FTSE/JSE Capped Top 40 Index (CAPI40) as an attempt to reduce the levels of concentration. While the absolute level of concentration does not necessarily detract from the equal weighted portfolio's relative performance, a growing level of concentration will lead to the equal weighted portfolio selling stocks as they continue to appreciate relative to the overall market.

Unlike cap weighted portfolios, the equal weighted portfolio's rebalancing function involves selling stocks that have outperformed the average stock and buying those that have underperformed. In the short-term this would require some level of mean reversion in stocks to outperform the cap weighted portfolio. More importantly if momentum is particularly positive in stocks with higher cap weights then this will exacerbate the underperformance.

The second main contributor to the relative performance of the equal weighted portfolio is the level of diversification. While the benefits of diversification are always positive, its contribution may be hampered by either low volatilities within stocks (thereby negating the need for diversification) and/or a high correlation between stocks (reducing the impact of diversifying).

Finally, the impact of forming portfolios on a subset of the market, such as in the case of the FTSE/JSE Top 40 Index (Top 40), is considered. This term is called leakage and can be thought of as the net impact of the equal weighted portfolio having to sell holdings in stocks as they exit the Top 40 index.

Much of the current literature, such as Bouchey et al. (2012, 2015) and Hallerbach (2014), has focused on the role of rebalancing or volatility harvesting on portfolio growth rates or, such as Plyakha et al (2012), by decomposing the equal weighted portfolio's return into various factors such as market, size, and value. Stochastic portfolio theory, on the other hand, derives a robust general framework for any systematically constructed portfolio relative to the cap weighted index and allows us to decompose its long-term return relative to the cap weighted market portfolio. This paper adds to the literature by using these robust theoretical concepts to form a complete view of the empirical performance of the equal weighted portfolio in South African equities. Furthermore, the theory lends itself to an attribution of the relative performance of the equal weighted performance in South Africa to understand how the main contributors to relative return have behaved in recent years. The importance of monitoring these factors in relative performance is further illustrated through a rudimentary linear regression model to select between the cap- and equal weighted portfolios.

This paper is organised as follows: Section 2 provides a brief introduction to stochastic portfolio theory and develops the intuition behind some of the important expressions in stochastic portfolio theory, which are used in the following sections, Section 3 focuses on the empirical analysis of the equal weighted portfolio formed on the Top 40 constituents, and Section 4 presents a rudimentary model to improve relative performance before concluding remarks are discussed in the final section.

The model

This section provides the necessary expressions for the analyses conducted in the next section and is based on stochastic portfolio theory. An intuitive explanation is provided here, together with any related mathematical expressions, although, the reader is referred to Fernholz (1999a, 1999b, 2002) and Fernholz and Karatzas (2009) for a rigorous treatment of stochastic portfolio theory.

A portfolio of stocks

Consider two stocks, stocks A and B, that are negatively correlated. In each period, a stock has a 50% chance of a 25% gain, and a 50% chance of a 20% loss. Stock A, for example, may have a 25% gain in one period, which would mean stock B has a 20% loss in the same period (and vice versa). The return in each period is independent from any other period.

Consider an equal weight portfolio between the two which is rebalanced each period. The geometric growth rate of each stock is 0% and yet the expected growth rate of the equal weighted portfolio would be 2.5% in each period.

This return is a function of the volatility of the two stocks and the correlation structure between them. In Bouchey et al. (2012, 2015) and Hallerbach (2014), this is referred to as volatility harvesting or volatility return. In stochastic portfolio theory, this return is referred to as the portfolio excess growth rate and represents the return attributable to diversification in the portfolio. The idea of diversification directly impacting the return of a portfolio is echoed by Booth and Fama (1992) and Cuthbertson et al. (2016).

Mathematically, consider a stock i with stock price represented by X_i and assume that the price process follows the familiar geometric Brownian motion model

$$dX_i(t) = \alpha_i(t)dt + \sigma_i dW_i(t),\ t \in [0, \infty),$$

where α_i and σ_i represent the stock's arithmetic growth rate and volatility, respectively. W_i is a Brownian motion process. The geometric growth rate, γ_i, is related to the arithmetic growth rate, $\alpha_i(t)$, by, $\alpha_i(t) = \gamma_i(t) + \frac{1}{2}\sigma_i^2(t)$.

Consider now a portfolio with weights given by $\pi_i(t)$ for each stock i at time t, where portfolio weights are non-negative. The portfolio's wealth process is denoted by $Z_\pi(t)$. The change in portfolio wealth for each period would be a weighted sum of the individual stock's return:

$$\frac{dZ_\pi(t)}{Z_\pi(t)} = \sum_{i=1}^{n} \pi_i(t)\frac{dX_i(t)}{X_i(t)} \tag{1}$$

The equation for $dX_i(t)$ can be substituted into the above equation, and with some simplification, the following result for the log portfolio wealth process is obtained,

$$d\log Z_\pi(t) = \gamma_\pi(t)dt + \sum_{i=1}^{n} \pi_i(t)\sigma_i(t)dW_i(t). \tag{2}$$

This has a similar form to the individual stock price process, however, the portfolio growth rate γ_π has an interesting form,

$$\gamma_\pi(t) = \sum_{i=1}^{n} \pi_i(t)\gamma_i(t) + \gamma_\pi^*(t). \quad (3)$$

That is, the portfolio growth rate is a function of the weighted individual stock growth rates and an additional term called the portfolio excess growth rate, γ_π^*.

The portfolio excess growth rate is, in turn, a function of the individual stock volatilities and covariances,

$$\gamma_\pi^*(t) = \frac{1}{2}\left(\sum_{i=1}^{n} \pi_i(t)\sigma_i^2(t) - \sum_{i,j=1}^{n} \pi_i(t)\pi_j(t)\sigma_{ij}(t)\right). \quad (4)$$

Intuitively, equation (4) can be thought of as a measure of diversification since it is the difference between the average stock volatility and the resulting portfolio's volatility. Although $\gamma_\pi^*(t) \geq 0$, its absolute level will vary depending on the individual stock volatilities and their correlations. If the correlation between stocks is small, then $\gamma_\pi^*(t)$ would be larger leading to a much higher contribution to the overall portfolio growth rate. Conversely, if correlations are high then there is less diversification and $\gamma_\pi^*(t)$ is lower. Similarly, should individual stock volatilities be low, then $\gamma_\pi^*(t)$ would also be low as there would be less scope for a reduction in portfolio volatility.

Returning to the example above, since the individual stock growth rates are zero, $(1.25)(0.8) - 1 = 0$, the portfolio growth rate is equal to the excess growth rate,

$$\gamma_\pi^* = \frac{1}{2}\left(\pi_1(1-\pi_1)\sigma_1^2 + \pi_2(1-\pi_2)\sigma_2^2 - 2\sigma_{1,2}\pi_1\pi_2\right),$$

where

$$\pi_1 = \pi_2 = 0.5$$

$$\sigma_1^2 = \sigma_2^2 = -\sigma_{1,2} = 0.5.$$

This yields a portfolio growth rate of

$$\gamma_\pi = \gamma_\pi^* = 0.025$$

as expected.

Note that by inserting equation (3) into equation (2) and rearranging terms we get

$$d\log Z_{\pi}(t) = \sum_{i=1}^{n} \pi_i(t) d\log X_i(t) + \gamma_{\pi}^{*}(t)dt. \qquad (5)$$

This form of equation (2) will become useful in the next section.

Relative portfolio returns

Although stand-alone portfolio performance is important to consider, in practice most portfolio managers must consider portfolio performance relative to some benchmark. The most often used benchmark for portfolio managers in equities is the cap weighted portfolio or index. In this portfolio stocks are weighted according to their relative market capitalisation, which is denoted by $\mu_i(t)$ and represents the cap weight for stock i at time t. That is,

$$\mu_i(t) = \frac{X_i(t)}{X_1(t)+\cdots+X_n(t)}\,, \qquad (6)$$

where $X_i(t)$ represents the market capitalisation of stock i at time t.

Let Z_{μ} denote the value process of the cap weighted portfolio with weights given by equation (6). Equation (6) can now be written as

$$\mu_i(t) = \frac{X_i(t)}{Z_{\mu}(t)}. \qquad (7)$$

Although, this portfolio process follows the same structure in equation (5), the difference is that its weights change given each stock’s relative performance over a period.

Consider the example above again. Of course, each period in the example is random and independent from the previous period, however, consider an example where the returns for each of the stocks alternates from one period to the next. If in period 1, for example, stock A has a return of -20% and stock B returns 25%, then in period 2 stock A would return 25% and stock B would return -20%.

The equal weighted portfolio would be unaffected, as at each period the portfolio would be rebalanced. The cap weighted portfolio, however, would become concentrated in the stock which had outperformed in the prior period only for that stock to underperform in the next.

Stock A, in this example, would have a cap weight of 39% after period 1, which would revert to 50% after period 2, in which it outperforms stock B. In relative terms, the equal weight outperforms the cap weighted portfolio (by about 5% in this example), and it does so for two reasons:

- The cap weighted portfolio becomes more concentrated in a single stock (stock B in this example) after the first period only for that stock to then underperform the other. In other words, the cap weights become concentrated in period 1 and then less concentrated in period 2 and this directly impacts the cap weighted portfolio's return relative to the equal weighted portfolio.
- The cap weighted portfolio has a lower excess growth rate than that of the equal weighted portfolio since it is more concentrated in a single stock.

The simple example above demonstrates how movements in the cap weights affects the relative return of the equal weighted portfolio. In the example, the cap weight of each stock mean reverts, and this benefits the equal weighted portfolios relative return. The reverse can of course occur as well; the cap weights could exhibit momentum and the stocks with larger cap weights could continue to outperform the average stock leading to further concentration in the cap weighted portfolio. This would have the reverse effect on the equal weighted portfolio and create a drag on relative performance.

This drag on relative performance can be expressed mathematically using stochastic portfolio theory. Consider equation (5) and subtract $d\log Z_{\mu}(t)$ on each side

$$d\log Z_\pi(t) - d\log Z_\mu(t) = \sum_{i=1}^{n} \pi_i(t) d\log X_i(t) - d\log Z_\mu(t) + \gamma_\pi^*(t)dt.$$

Noting that the weights, $\pi_i(t)$ sum to one, this can be rewritten as

$$d\log\left(\frac{Z_\pi(t)}{Z_\mu(t)}\right) = \sum_{i=1}^{n} \pi_i(t) d\log\left(\frac{X_i(t)}{Z_\mu(t)}\right) + \gamma_\pi^*(t)dt. \qquad (8)$$

Equation (8) is true for any two portfolios, however, in the case of the cap weighted portfolio, and using equation (7), equation (8) can be simplified into

$$d\log\left(\frac{Z_\pi(t)}{Z_\mu(t)}\right) = \sum_{i=1}^{n} \pi_i(t) d\log\mu_i(t) + \gamma_\pi^*(t)dt \qquad (9)$$

That is, the equal weighted portfolio's return relative to the cap weighted portfolio is equal to a term involving the change in the log cap weights and the portfolio's excess growth rate. Equation (9) holds for any portfolio with weights $\pi(t)$, although the focus here is on the equal weighted portfolio.

When considering the equal weighted portfolio, in particular, the first term in equation (9) can be rewritten as,

$$\begin{aligned} \sum_{i=1}^{n} \pi_i(t) d\log\mu_i(t) &= \frac{1}{n}\sum_{i=1}^{n} d\log\mu_i(t) \\ &= \frac{1}{n} d\log\left(\prod_{i=1}^{n} \mu_i(t)\right) \\ &= d\log\left(\prod_{i=1}^{n} \mu_i(t)\right)^{1/n} \end{aligned}$$

That is, when considering the equal weighted portfolio relative to the cap weighted portfolio, the first term becomes the change in the log of the geometric mean of the cap weights. The log of the geometric mean will be negative, given that the cap weights are less than one. Therefore, as the geometric mean increases, the log of the geometric mean decreases. That is, as the cap weights become more concentrated, the log of the geometric mean will decline (become more negative) and cause a drag on the equal weighted portfolio's relative performance, through equation (9). In summary then, the relative return of the equal weighted portfolio is a function of the change in the geometric mean of cap weights and the excess growth rates.

An important assumption in stochastic portfolio theory is that no one stock can dominate the market or, alternatively, the maximum cap weight for a stock is bounded. Under this assumption the geometric mean of cap weights, and the term containing it in equation (9), is bounded and over the long term its contribution to the relative performance of the equal weighted portfolio should be negligible. In the context of equation (9), this implies that that over the long-term the equal weighted portfolio is likely to outperform the cap weighted portfolio since the excess growth rate is greater than, or equal to, zero and the geometric mean term is bounded. This confirms, theoretically at least, the findings of empirical studies into equal weighted portfolio performance such as Plyakha et al. (2012), Bolognesi et al. (2013), and Malladi and Fabozzi (2017).

Although the geometric mean term is bounded over the long-term, in the short-term, this geometric mean term can be a significant driver of relative performance, dominating the excess growth rate term in equation (9).

In stochastic portfolio theory the geometric mean of cap weights is called the portfolio generating function of the equal weighted portfolio and is usually denoted as $S(t)$ where

$$\mathrm{S(t)} = \left(\prod_{i=1}^{n} \mu_i(t)\right)^{1/n}. \quad (10)$$

The portfolio generating function, as the name implies, is a function which can be used to generate the weights of a portfolio and has a key role to play in the performance of the resulting portfolio relative to a cap weighted portfolio. The reader is referred to Fernholz (1999b, 2002) for a complete understanding of portfolio generating functions.

To conclude, and for completeness, we include the difference in dividends between the portfolios as the difference in the two dividends rates, denoted by δ_π and δ_μ for the equal weighted and cap weighted portfolios, respectively.

The model of the equal weighted portfolio's relative return to the cap weighted portfolio, equation (9), can now be written as

$$d\log\left(\frac{Z_\pi(t)}{Z_\mu(t)}\right) = d\log\big(S(t)\big) + \int_0^t \big(\delta_\pi(s) - \delta_\mu(s)\big)\, ds + \gamma_\pi^*(t)dt. \qquad (11)$$

Leakage

Completing the model of the equal weighted portfolio's relative return requires the consideration of one more component. This component is called leakage and is a direct result of forming market indices on a subset of the overall market.

Equation (11) assumes that both the equal weighted and cap weighted portfolios are formed over the entire equity market. This is unlikely to be the case in practice, however. The most common equity benchmark in South Africa, for example, is the Top 40 index, which is formed on the top 40 stocks in the SA equity market. The total number of stocks in the SA equity market, however, is far greater, with around 150 stocks in the JSE All-Share index that represent 99% of the total market value of the equity market.

Consider the example of the two stocks again. In the previous section, the equal weighted portfolio outperformed the cap weighted portfolio because the concentration in cap weights was bounded and because the returns mean reverted. That is, the equal weight was able to outperform because it held an equal share in the stock that outperformed in each period.

In that example there are two stocks in the market. Consider, however, if there were three stocks and an index is formed on the top two stocks. Stock A returns -20% as before but is now ejected from the top two stock index and replaced by stock C. The equal weighted portfolio no longer has any exposure to stock B's recovery in the next period as it must sell its holdings in this stock and rotate into stock C.

The same effect occurs in an index like the Top 40 and, as stocks move in and out of the index at each rebalance, the equal weighted portfolio finds itself selling stocks that may later recover (in which case it would have no exposure). In terms of equation (11), although the geometric mean term (or portfolio generating function) is bounded over the entire market, the equal weighted portfolio may feel the brunt of an increase in concentration in cap weights but may not fully benefit from this term decreasing as concentration in cap weights declines.

The cap weighted portfolio experiences the same effect as it must sell positions in any stocks that leave the index. The difference is that the weight of these stocks leaving is much lower and so the net effect on the equal weighted portfolio's relative return is negative if there is an increase in the turnover of the constituents in the index.

In this paper we make use of the suggestion in Fernholz (2002) to estimate the net impact of leakage on a portfolio in each period by computing the difference between:

- The portfolio generating function in equation (10) assuming the constituents are unchanged but using the end of the period cap weights, and
- The portfolio generating function using the updated constituents at the end of the period and the cap weights at the end of the period.

Intuitively, this is a measurement of the impact that changes in the constituents has had on the portfolio generating function. The exact calculation for leakage requires the use of local times and the reader is referred to Fernholz (2002) for a complete description.

Denoting the net impact of leakage as $dL_{\pi/\mu}(t)$, equation (11) can now be extended to

$$d\log\left(\frac{Z_\pi(t)}{Z_\mu(t)}\right) = d\log\big(S(t)\big) + \int_0^t \big(\delta_\pi(s) - \delta_\mu(s)\big)\,ds + \gamma_\pi^*(t)dt \; + \; dL_{\pi/\mu}(t). \qquad (12)$$

In summary, the equal weighted portfolio's return relative to the cap weighted portfolio is a function of:

- The changes in the concentration of the cap weights (the portfolio generating function),
- the difference in dividends,
- the equal weighted portfolio's excess growth rate, and
- the net impact of leakage.

Empirical performance of the equal weighted portfolio in SA stocks

In the previous section the relative performance of the equal weighted portfolio was explained through the combination of four components: the change in the portfolio generating function $S(\mu(t))$, representing the concentration of weights in the cap weighted portfolio, the difference in dividends, the net impact of leakage and the excess growth rate $\gamma_\pi^*(t)$ representing the benefits of diversification.

This section focuses on the empirical performance of an equal weighted portfolio of the Top 40 stocks relative to the cap weighted portfolio since 2002 (when the index was first launched). The cap weights are constructed using market capitalisation data directly and, although the constituents mirror the official Top 40 index at all times, the weights in the cap weighted portfolio considered here will likely differ from the Top 40 weights given different rebalancing frequencies and various rules implemented in the Top 40 index that are not considered here.

The portfolios are rebalanced monthly using the Top 40 constituents as at the beginning of the month and include transaction costs of 15 basis points (bps). Data is sourced from Bloomberg and dividends and corporate actions are assumed to be paid in cash. This cash is then used by the portfolio at the next rebalance date. Cash balances between rebalance dates are assumed to earn no interest.

Relative performance and its drivers

Figure 1 depicts the cumulative performance on a log scale since 2002. While the equal weighted portfolio did outperform the cap weighted portfolio in the first portion of our sample period, the cap weighted portfolio appears to have overtaken the equal weighted portfolio from 2018 in total return over the full period.

[INSERT FIGURE 1 ABOUT HERE]

This is also evident from Figure 2, which shows the relative performance of the equal weighted portfolio. The equal weighted portfolio's relative performance begins to decline in 2013 and as at May 2020, the equal weighted portfolio has underperformed the cap weighted portfolio by almost 20% according to these figures. Note that these results include assumed transaction costs of 15bps.

[INSERT FIGURE 2 ABOUT HERE]

Table 1 shows the relative returns of the equal and cap weighted portfolio since 2003 on a two -year period basis. This further highlights the poor performance of the equal weighted portfolio since 2011. In fact, the equal weighted portfolio has not outperformed the cap weighted portfolio over a two-year period since 2011, registering its worst relative performance since 2019.

According to the model in the previous section, the source of this underperformance should be evident by dissecting the relative performance into the cap weighted concentration, $S(\mu(t))$, the equal weighted portfolio's diversification benefits, $\gamma_\pi^*(t)$, dividends, and the net impact of leakage.

Figure 3 depicts the log generating function of the equal weighted portfolio, $\log S(\mu(t))$, for the Top 40 stocks which is calculated using the log of the geometric mean of cap weights. This has declined significantly, indicating a growing concentration in cap weights of the Top 40 since 2002 and contributing negatively to the equal weighted portfolio's relative performance. The two periods where the log generating function increased, namely 2002 to 2005 and 2008 to 2010, coincided with significant outperformance over the cap weighted portfolio.

[INSERT FIGURE 3 ABOUT HERE]

Furthermore, the recent declines in the stock market due to the Corona virus has led to one of the worst declines in the log generating function, indicating a significant increase in concentration in cap weights within the Top 40. While concentration in the Top 40 has always been an issue in South African equities, current levels, as measured by the generating function of the equal weighted portfolio indicate concentration in cap weights has continued to increase substantially since 2010, marking extreme levels (albeit measured over a relatively short period).

Figure 4 depicts the corresponding log generating function for the S&P500 equal weighted portfolio and highlights that the increase in concentration of the Top 40 stocks, while clearly more extreme, is not completely unique to the South African stock market.

[INSERT FIGURE 4 ABOUT HERE]

Another measure for concentration in cap weights is the level of entropy; a concept used in information theory and introduced by Shannon (1948). Entropy has also been used as a measure of diversity in stock markets, in Fernholz (1999a) for example, and is analogous to the portfolio generating function in terms of concentration. Entropy for a portfolio such as the cap weighted portfolio with weights $\mu_i(t)$ for stock i is defined as

$$S_E(\mu(t)) = -\sum_{i=1}^{n} \mu_i(t)\log\mu_i(t)$$

Figure 5 displays the level of entropy for the S&P500 and Top 40, normalised to the starting period level of entropy for each index, respectively. As noted above, the increase in concentration in the Top 40 is not unique and the S&P500 has experienced the same phenomenon. However, the S&P500's level of entropy does not appear as extreme as the Top 40 as would be expected given the difference in number of constituents.

[INSERT FIGURE 5 ABOUT HERE]

The second component of the equal weighted portfolio's relative performance, per equation (1), is the benefit of diversification or $\gamma_\pi^*(t)$. This is estimated here using the rolling three-month, historical, correlation matrix. Figure 6 depicts the resulting excess growth rate for the equal weighted portfolio of Top 40 stocks.

[INSERT FIGURE 6 ABOUT HERE]

While the excess growth rate has increased significantly in 2020, it has averaged a lower level since 2010 than that of 2002 to 2008, reaching a low point in 2012. Figure 7 highlights that the spikes in the excess growth rate are mostly due to spikes in volatilities. The underperformance between 2012 and 2015 also makes sense as during this period average volatility was extraordinarily low. Relatively higher levels of correlation during this period also appear to have contributed to lowering the benefits of diversification. It does appear, however, that previous spikes in volatilities have led to some outperformance by the equal weighted portfolio.

[INSERT FIGURE 7 ABOUT HERE]

The final component considered in this section is the net impact of leakage on the equal weighted portfolio. Recall that leakage is the net impact of constituents falling out of the index, or subset of the market being tracked, on the equal weighted portfolio. Since the equal weighted portfolio has a higher weight assigned to these stocks than the cap weighted portfolio, leakage is almost always a net drag on relative returns.

Figure 8 shows the cumulative impact of leakage on the equal weighted portfolio since the start of 2003, together with the rolling 12 month change in constituents. The total cumulative impact of leakage on the equal weighted portfolio's relative return since 2003 is -44.4% or about -3.3% per annum on average. Leakage was muted during the period 2009 to 2016 when changes in Top 40 constituents were somewhat lower than on average. The impact of leakage has, however, accelerated again as changes in Top 40 membership have increased over the past few years and has created a drag of 17.8% on the equal weighted portfolio's relative return since 2016.

[INSERT FIGURE 8 ABOUT HERE]

Attribution of relative returns

In this section we combine all the individual drivers into a complete view of the relative performance of the equal weighted portfolio since 2002. Figure 9 shows the cumulative contribution from each factor since 2003. The excess growth rate contributes a consistently positive return, as expected, with a return of about 90% since 2003. Interestingly, leakage has contributed a more negative return than the portfolio generating function, with each contributing -44% and -34%, respectively. This highlights the importance of monitoring changes in the index composition when considering performance relative to the cap weighted portfolio.

[INSERT FIGURE 9 ABOUT HERE]

There appears to be some residual component when comparing these theoretical returns to the actual returns that, although, generally stable over the period considered, could highlight errors in estimation either of the excess growth rate (given the need to estimate the covariance matrix) or the impact of leakage.

The contribution of transaction costs and dividends is relatively small over this sample period.

[INSERT TABLE 2 ABOUT HERE]

Table 2 extends the analysis of relative returns in Table 1 by decomposing the relative return into the various drivers. Figure 10 displays this information visually. Although, the contribution of the excess growth rate is always positive, its size does vary with a large contribution in 2007 to 2009 of 18%, declining to between 5% and 7% thereafter.

In general, when the equal weighted portfolio has underperformed it has been due to large declines in the portfolio generating function (an increase in cap weight concentration) such as in 2005 to 2007, 2013 to 2015 and 2019 to 2020 (although, this last period represents only a few months). Notably, underperformance in 2017 to 2019, is largely attributable to leakage as the number of changes in index compositions increased (Figure 8). While in 2007 to 2009, although, the equal weighted portfolio's relative performance was moderately positive, it was dragged down by a significant impact from leakage.

[INSERT FIGURE 10 ABOUT HERE]

Impact of Naspers and Prosus

A common topic of discussion in the South Africa equity market over the last five years has been the dominance Naspers (NPN) and Prosus (PRX), which was spun off from

NPN in 2019, in cap weighted indices. These two stocks together, have accounted for at least 25% to 30% of the Top 40 index over the past few years. The portfolio generating function can be used to understand the size of the impact of NPN and PRX on the relative performance of an index like the equal weighted portfolio, which is significantly underweight compared to these two stocks. In this analysis, NPN and PRX are excluded initially from the calculation of the portfolio generating function, with their weights distributed proportionately to the remaining stocks. This adjusted portfolio generating function is compared to the true value to get a sense of the impact these two stocks have had.

These figures are displayed in Table 3 over two periods: 2005 to 2015 and 2016 to 2020. The difference is negligible at 3.6% for the period 2005 to 2015, however, in 2016 the cap weight of NPN's starts to increase and by 31 March 2020 its impact (together with PRX) is about 22%. The portfolio generating function has contributed a -23% return on the equal weighted portfolio since 2016 (see Table 2), highlighting the significant role that NPNs and PRX have had.

[INSERT TABLE 3 ABOUT HERE]

Improving the performance of the equal weighted portfolio

The previous section highlighted the various ways in which the components of equation (12) appear to have contributed to the relative underperformance of the equal weighted portfolio in South African equities. This section sets out to use the portfolio generating function, excess growth rate, and net leakage to actively select between the cap weighted and equal weighted portfolio given that these appear to be the largest drivers in empirical performance.

This is done by using a rudimentary linear regression model to forecast the next month's relative performance using the average monthly change in the log portfolio generating function, $\log S(\mu(t))$ over the past three months and the latest estimate of the net leakage impact, $dL_{\pi/\mu}(t)$, plus the most recent estimate of the excess growth rate, $\gamma_{\pi}^{*}(t)$. This is to mirror the structure of equation (12), albeit by using the past three-month average in the portfolio generating function and the latest estimate of net leakage as an estimate for the future values of both. The excess growth rate is estimated using volatilities and correlations over the prior three months. The model is trained on the prior three years' data.

Mathematically this would be expressed as

$$y(t+1) = \beta_0 + \beta_1 X(t) \qquad t = 3, \dots, n$$

Where $y(t+1)$ is the next month's relative return of the equal weighted portfolio, $X(t)$ is given by

$$X(t) = \frac{1}{12}\gamma_{\pi}^{*}(t) + \frac{1}{3}\sum_{i=t-2}^{t} d\log S(\mu(i)) + dL_{\pi/\mu}(t),$$

And where $d\log S(\mu(i))$ is the one-month change in the log of the portfolio generating function and $dL_{\pi/\mu}(t)$ is the latest one-month net leakage impact for the equal weighted portfolio at time t. The coefficients β_0 and β_1 are estimated using ordinary least squares.

As in the previous section's analysis, transaction costs of 15bps are included for all portfolios. As a result, the model only switches from equal weights to cap weights (or vice versa) if the relative return is predicted to, at least, offset these transaction costs.

[INSERT FIGURE 8 ABOUT HERE]

Figure 8 shows the cumulative performance of all three portfolios, including the model portfolio. It appears as if the model portfolio matches the equal weighted portfolio's outperformance in the earlier part of the period before outperforming the equal weighted portfolio more recently. Figure 9, which depicts performance relative to the cap weighted portfolio, highlights how during this period, the model portfolio switches into the cap weighted portfolio and has remained in the cap weighted portfolio since 2016. The model portfolio spends much of its time using equal weights and only switches on eight occasions to cap weights. The time spent in total using equal weights is approximately 60%. We should expect the equal weighted portfolio to outperform over the long-term and it should be expected that the model portfolio spends most of the time using equal weights.

[INSERT FIGURE 9 ABOUT HERE]

This simple model illustrates how monitoring the levels of concentration, benefits of diversification and net impact of leakage in South African stocks can substantially improve the performance of an equal weighted portfolio approach. Table 4 presents the performance and risk metrics for the three portfolios. Not only does the model portfolio outperform on a risk-adjusted basis but it does so with a much lower relative drawdown and a much-improved information ratio.

Table 4 also shows the p-values of both the information ratio and the Sharpe ratio. These p-values reflect the statistical significance of the difference between the equal weighted and optimal weighted portfolio using the method in Ledoit and Wolf (2008). This method uses a studentised circular bootstrap approach to construct a confidence interval at a given significance level. The hypothesis test is a two-sided test with the null hypothesis that the difference in the Sharpe, or information ratio, is zero. It is worth bearing in mind that the model portfolio spends 60% of its time using equal weights and the bootstrap approach would, therefore, cover many areas where the model portfolio is exactly equal to the equal weighted portfolio.

[INSERT TABLE 4 ABOUT HERE]

Table 5 further illustrates the improvement by highlighting the first and second half of the sample period's returns for each portfolio. While the equal weighted portfolio outperforms in the first half of the period, its underperformance in the second half weighs significantly on the overall returns. The model portfolio, on the other hand, achieves most of the equal weighted portfolio's outperformance in the first half and limits the relative drawdown in the second half by switching to the cap weighted portfolio.

[INSERT TABLE 5 ABOUT HERE]

Conclusion

This paper analysed the recent underperformance of the equal weighted Top 40 portfolio of South African stocks. Although, it has been widely shown that the equal weighted portfolio appears to outperform the cap weighted portfolio over the long-term, the South African equal weighted portfolio has disappointed, underperforming by almost 20% since 2002 including transaction costs.

Stochastic portfolio theory provides a robust theoretical framework for understanding this underperformance by considering the drivers of the relative performance, namely, the level of concentration in cap weights (portfolio generating function), the level of diversification benefits (the excess growth rate), and the impact of leakage (as stocks move in and out of the index). Section 3 considered the empirical evidence, highlighting that the level of concentration in the Top 40 has increased significantly since 2002 and is currently at extreme levels.

Furthermore, while the excess growth rate may have offset this headwind to relative performance, lower levels of volatility and higher average correlations since

2012 have led to low levels in the diversification benefit. This has coincided with the significant underperformance of the equal weighted portfolio relative to the cap weighted portfolio.

Finally, the impact of net leakage is often not considered and yet in South Africa has been a drag of about 3% on the equal weighted portfolio's relative return since 2002. The net impact of leakage is related to the changes in the index's constituents given that the equal weighted portfolio will hold a higher weight in stocks being replaced. Many smart beta strategies result in portfolios with higher weights allocated to stocks with lower cap weights and their returns can, therefore, be impacted by net leakage in a very similar manner to the equal weighted portfolio. These strategies may be expected to outperform the cap weighted portfolio over the long-term, however, if they are constructed on indices that have high turnover in constituents, their long-term performance is likely to be much lower as a result of leakage.

The importance of monitoring the level of concentration, excess growth rate of the equal weighted portfolio, and net leakage was demonstrated using a rudimentary linear model. This model, using these metrics as inputs, improved the performance of the equal weighted portfolio by switching to cap weights when optimal to do so. This reduced the relative drawdowns and improved risk-adjusted performance.

While this paper has focused on the equal weighted portfolio, some of these results can be extended to other smart beta strategies. Strategies that deviate from the cap weights by allocating more weight to smaller stocks are likely to be impacted by changes in the concentration of cap weights which could impact short-term performance significantly even if these strategies are expected to outperform over the long-term.

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Notes

1. https://senspdf.jse.co.za/documents/SENS_20190218_S411132.pdf
2. 1 January 2019 to April 2020
3. Compounded annual growth rate
4. One-month JIBAR used as risk-free rate
5. P-values for both Sharpe and information ratios are shown in brackets
6. Maximum relative return drawdown

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Table 1. Performance of equal weighted and cap weighted portfolios on a rolling two-year basis since 2003.

Period	Equal Weighted (%)	Cap Weighted (%)	Relative (%)
2003 - 2005	133.6	107.1	12.8
2005 - 2007	129.8	151.9	-8.8
2007 - 2009	20.8	23.3	-2.1
2009 - 2011	71.7	64.0	4.7
2011 - 2013	57.0	60.1	-1.9
2013 - 2015	28.0	52.6	-16.1
2015 - 2017	18.6	30.1	-8.9
2017 - 2019	5.2	16.0	-9.3
2019 - 2020[2]	-26.6	-1.1	-25.8

Table 2. Decomposition of relative performance of the equal weighted portfolio on a rolling two-year basis since 2003.

Period	Excess growth rate (%)	Port. Generating function (%)	Leakage (%)	Divid-ends (%)	Costs (%)	Residual (%)	Total (%)
2003 - 2005	10.5	5.4	-5.5	0.8	-0.1	-1.2	10.1
2005 - 2007	7.1	-6.5	-5.1	0.8	-0.1	-3.4	-7.1
2007 - 2009	17.9	2.1	-18.7	1.5	-0.1	-2.8	3.5
2009 - 2011	4.3	-8.9	-0.4	0.4	-0.1	8.0	3.4
2011 - 2013	4.7	-4.5	0.7	-0.2	-0.1	-4.1	-3.5
2013 - 2015	5.7	-9.6	0.7	-0.2	-0.1	-6.1	-9.5
2015 - 2017	7.0	-2.5	-9.4	0.7	-0.1	3.6	-0.0
2017 - 2019	7.8	3.6	-16.2	0.6	-0.1	-4.6	-6.3
2019 - 2020	1.8	-24.2	-0.2	-0.0	-0.0	1.7	-21.2

Table3. Equal weighted portfolio generating function with all stocks and excluding Naspers (NPN) and Prosus (PRX).

Period	All Stocks (%)	Excluding NPN, PRX (%)	Difference (%)
2005 - 2015	-30.5	-27.9	-3.6
2016 - 2020	-24.0	-2.7	-21.9

Table 4. Performance and risk metrics for the equal weighted, cap weighted, and model portfolios. Includes 15bps of transaction costs.

Portfolio	CAGR[3] (%)	Volatility (%)	Sharpe ratio[4,5]	Information ratio	Drawdown[6] (%)
Model	15.2	19.2	0.45 (0.28)	0.25 (0.10)	11.2
Equal weighted	12.4	19.2	0.32	-0.17	46.1
Cap Weighted	13.9	20.2	0.38		

Table 5. Performance of the equal weighted, cap weighted, and model portfolios over the first and second half of sample period. Includes 15bps of transaction costs.

Portfolio	2002 - 2011		2011 - 2020	
	CAGR	Volatility	CAGR	Volatility
Model	19.7%	21.2%	10.2%	16.7%
Equal weighted	19.7%	20.1%	4.6%	18.0%
Cap weighted	17.5%	23.1%	9.8%	16.3%

Figure 1. Cumulative log performance of the equal weighted and cap weighted Top 40 portfolios since 2002.

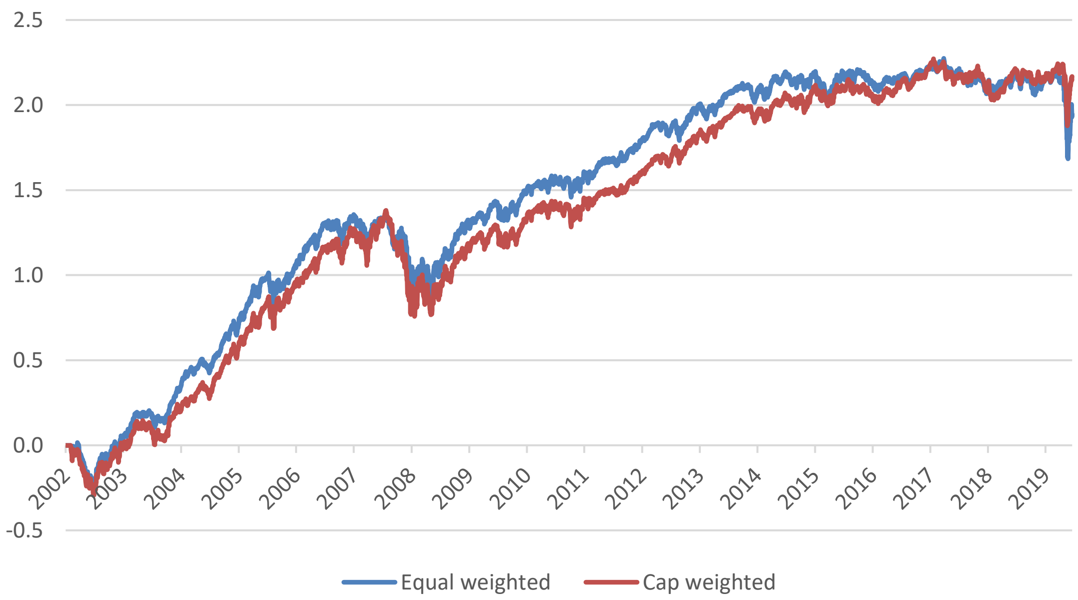

Figure 2. Relative cumulative performance of the equal weighted portfolio relative to the cap weighted portfolio.

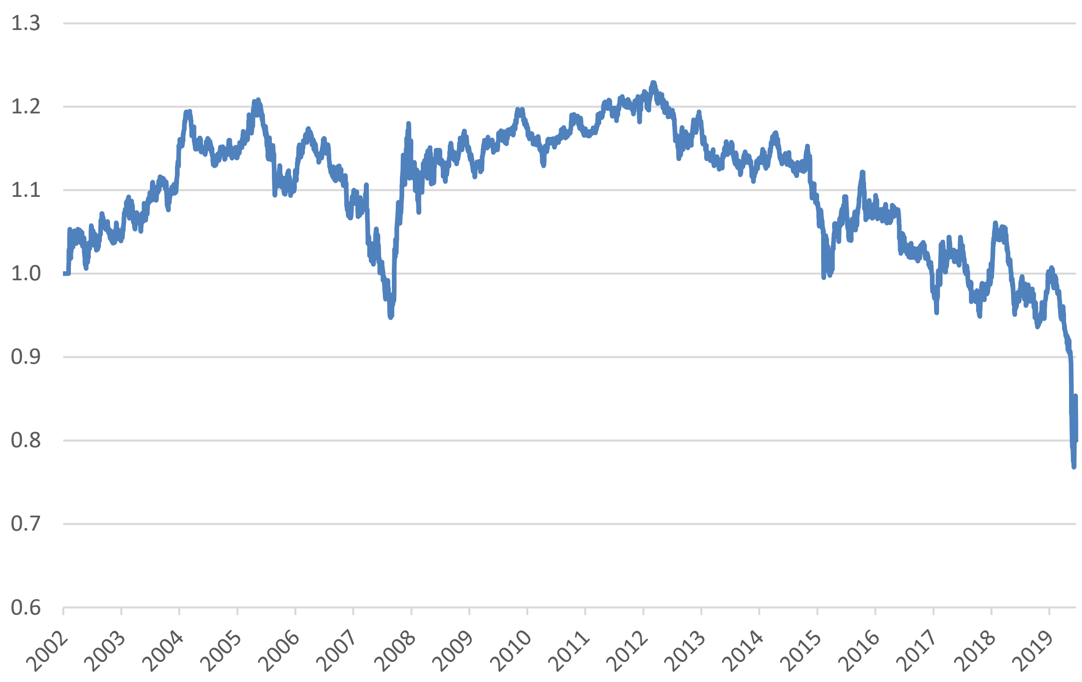

Figure 3. Log generating function, $\log S(\mu(t))$, of the equal weighted portfolio.

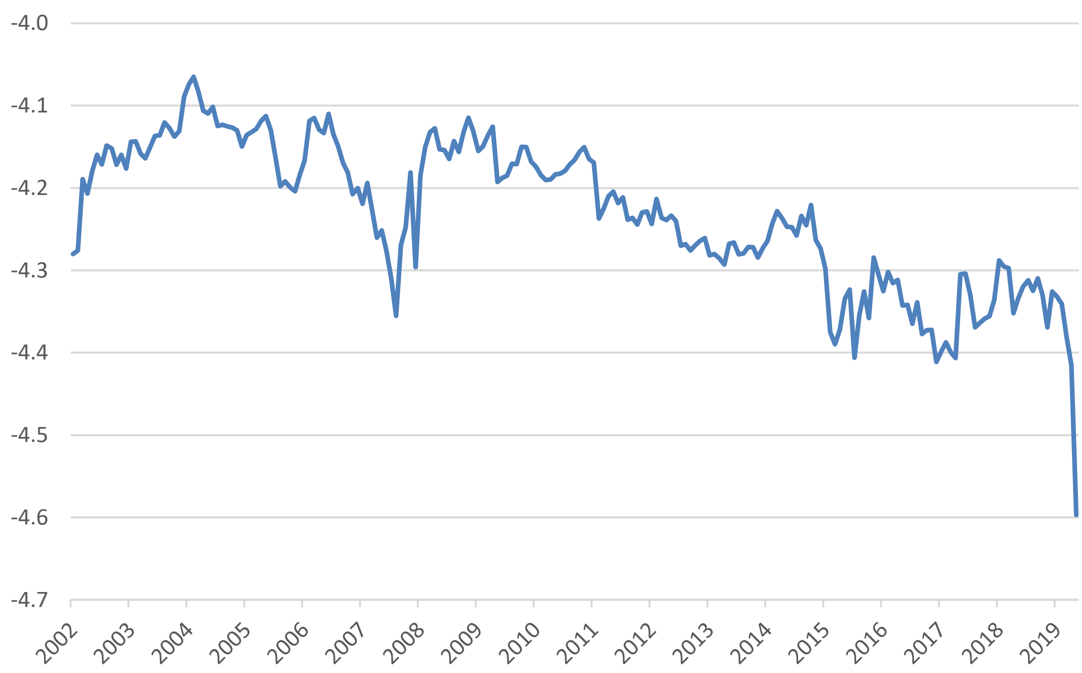

Figure 4. Log generating function for an equal weighted portfolio of S&P500 stocks.

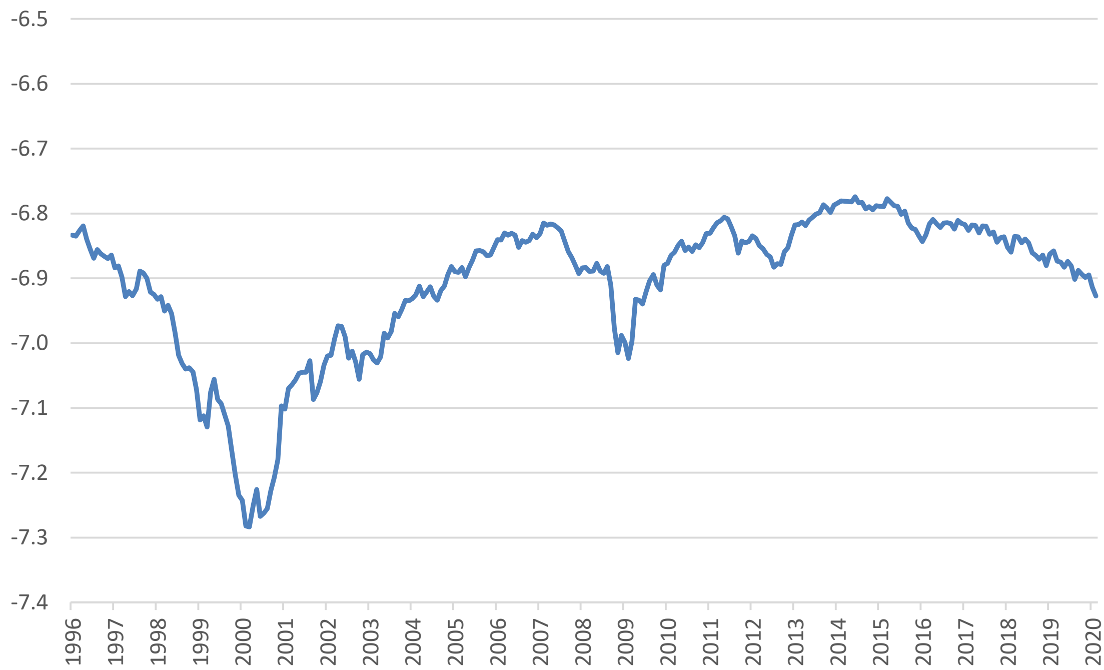

Figure 5. Normalised level of entropy for the Top 40 and S&P500 stocks.

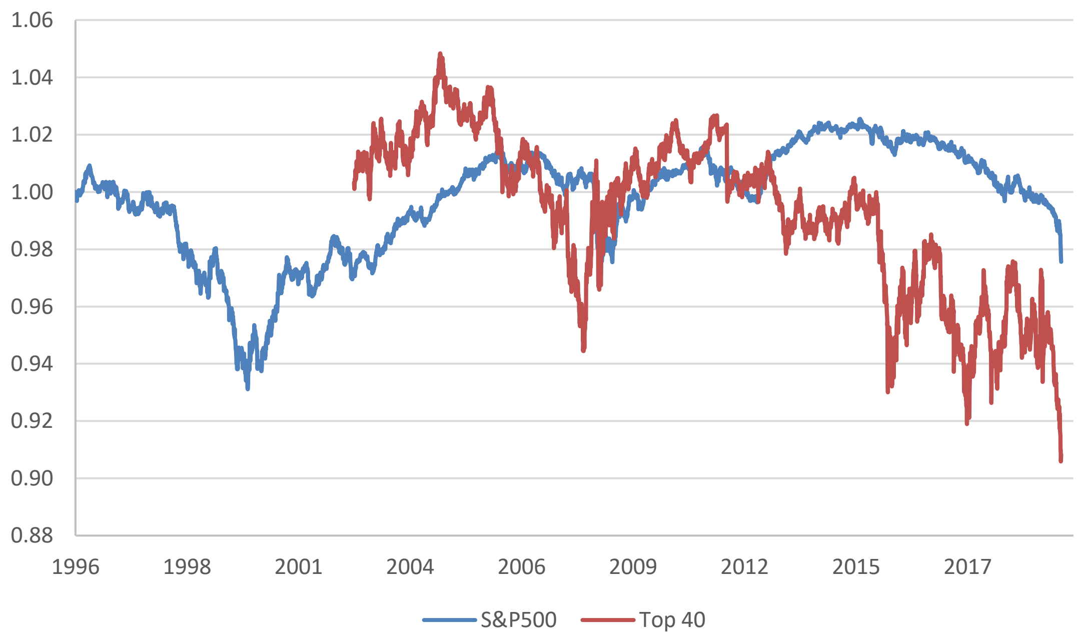

Figure 6. Excess growth rate for the equal weighted portfolio of Top 40 stocks.

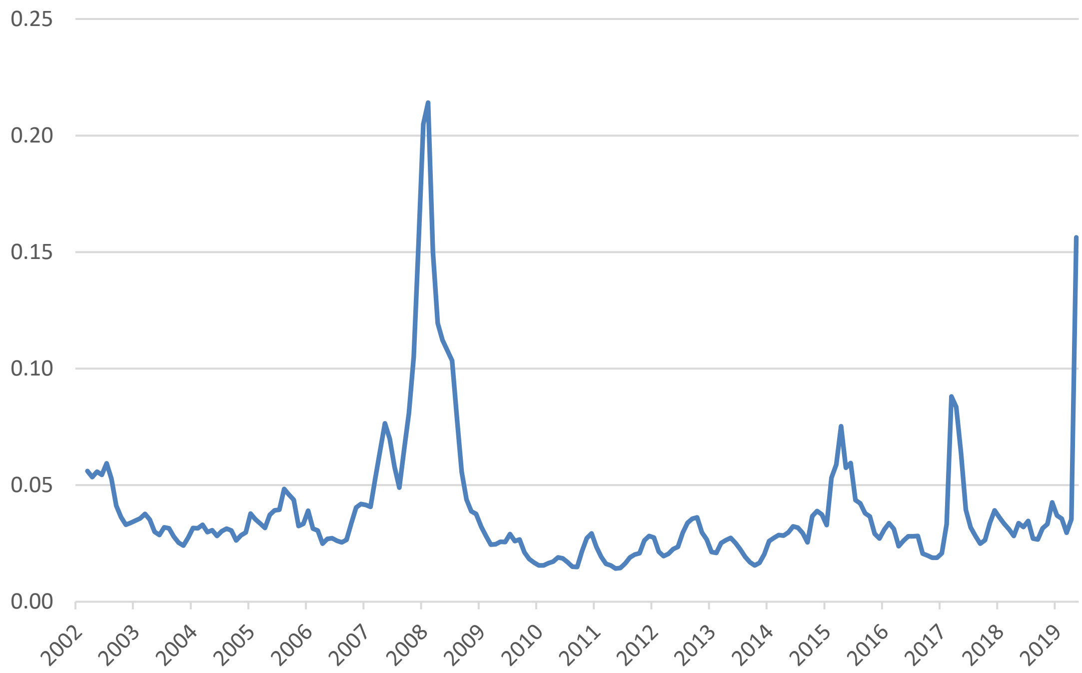

Figure 7. Average volatility and correlations for the Top 40 stocks.

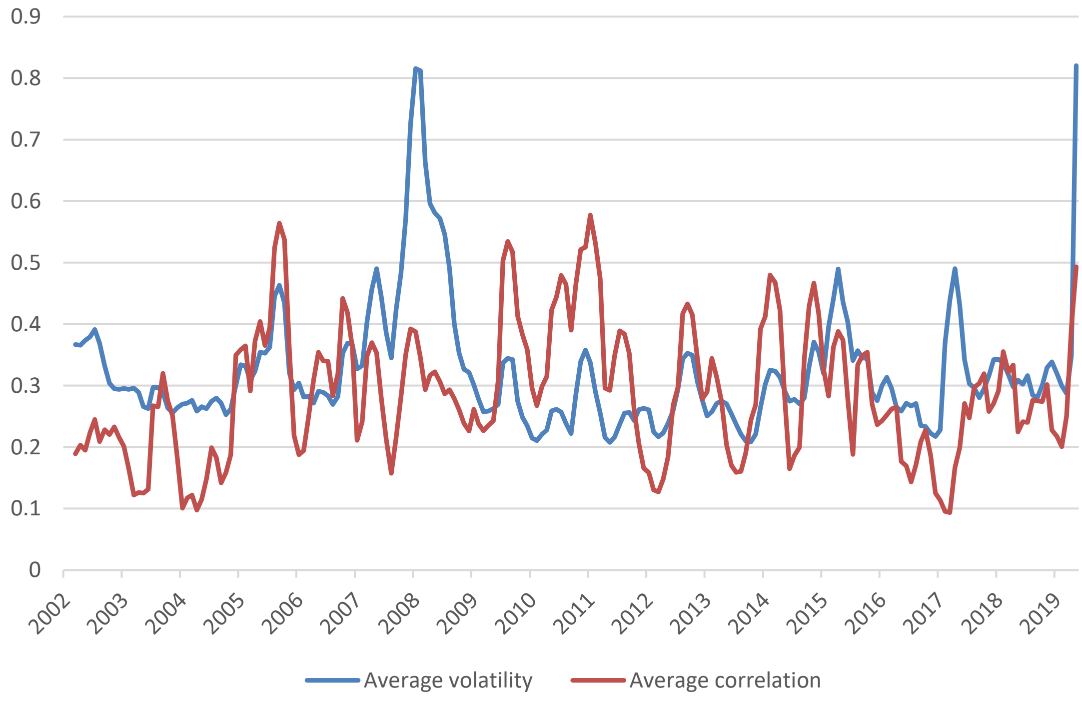

Figure 8. Cumulative return from net leakage in the equal weighted portfolio, overlayed with rolling 12-month change in constituents.

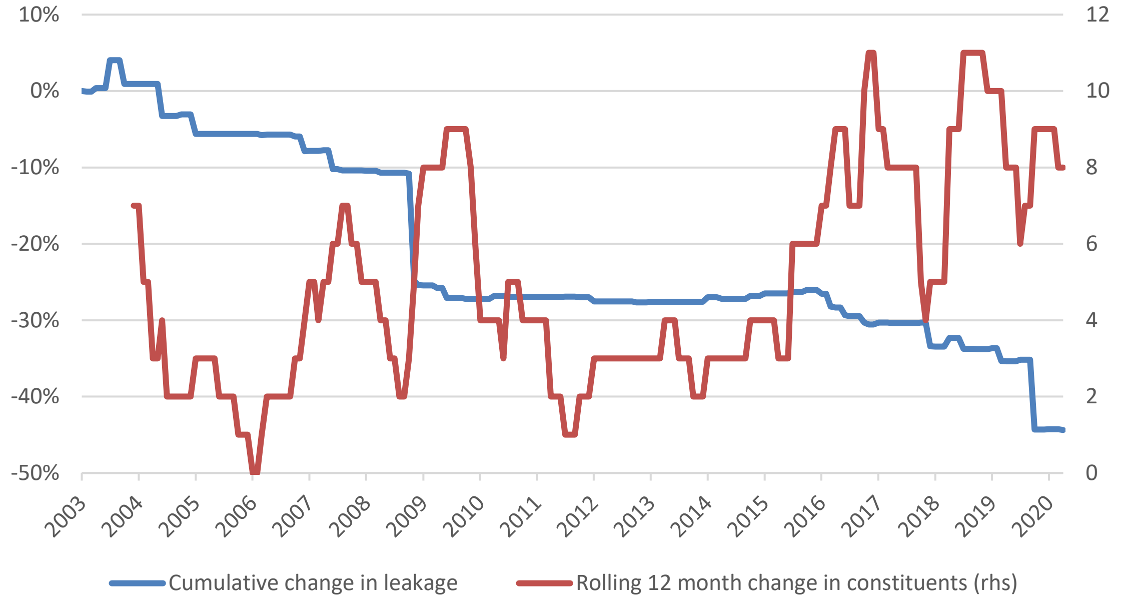

Figure 9. Cumulative contribution of drivers in the relative returns of the equal weighted portfolio.

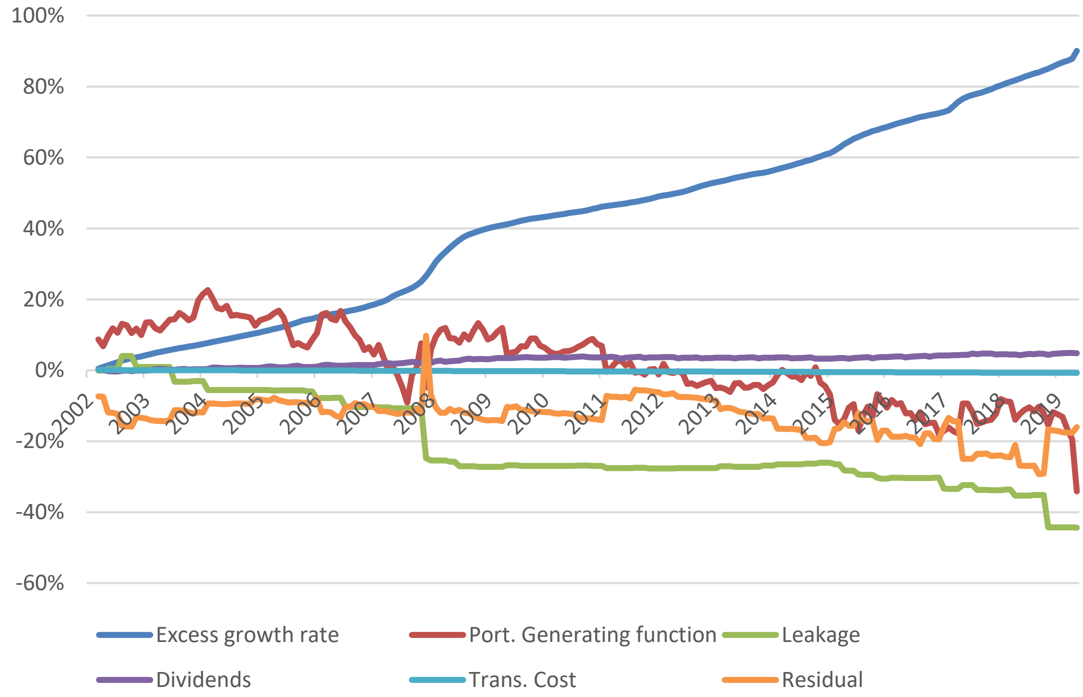

Figure 10. Decomposition of relative performance of the equal weighted portfolio on a rolling two-year basis since 2003.

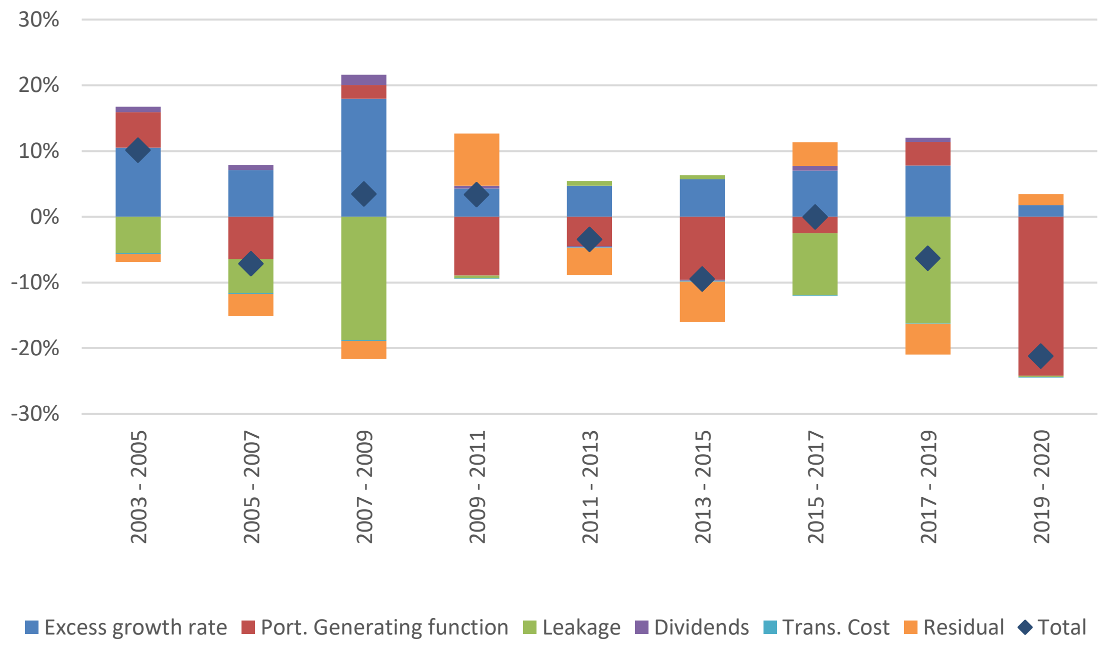

Figure 10. Log cumulative performance for the cap weighted, equal weighted and model portfolios.

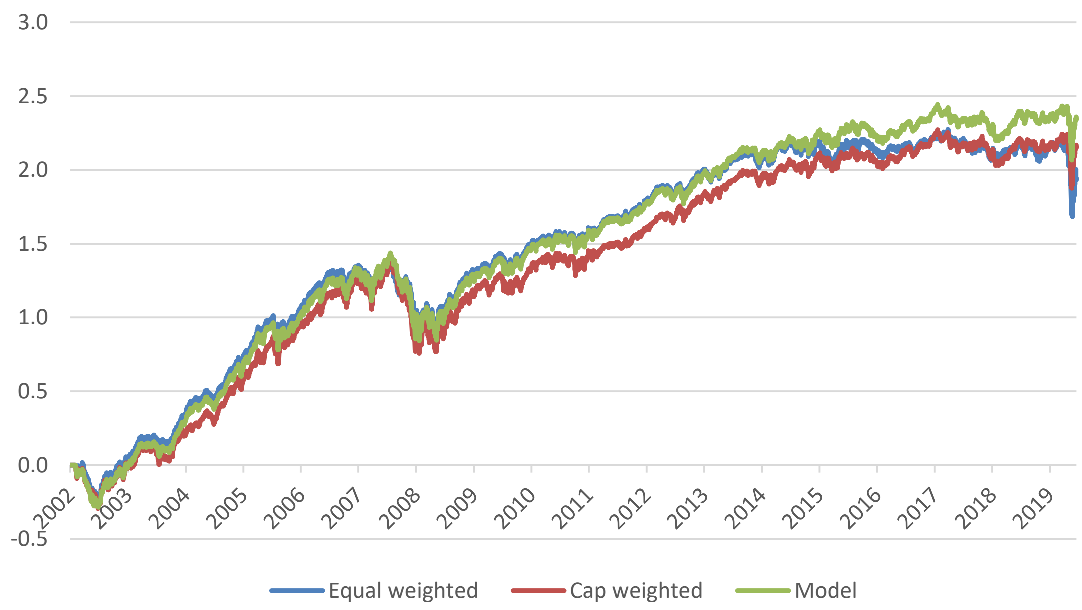

Figure 11. Relative performance of the equal weighted and model portfolios.