

# Quality of Service Provisioning through Resource Optimisation in Heterogeneous Cognitive Radio Sensor Networks

Babatunde S. Awoyemi, Bodhaswar T. Maharaj

*University of Pretoria; awoyemibabatunde@gmail.com, sunil.maharaj@up.ac.za*

---

## Abstract

Recently, cognitive radio sensor networks (CRSN) have evolved as a result of the introduction of cognitive capabilities to conventional wireless sensor networks. In most CRSN designs, secondary users and/or sensor nodes are permitted, under certain constraints, to use the limited resources of a primary network. One major challenge with CRSN is how to optimally appropriate and use the limited resources available in driving their communication demands. To overcome this challenge, in this paper, we develop a resource allocation (RA) model that is capable of achieving a target quality of service (QoS) demand for the heterogeneous CRSN, despite the huge resource constraints imposed on the network. The RA problem developed is a complex optimisation problem. We analyse and solve the complex RA problem using the optimisation approaches of integer linear programming, Lagrangian duality and by a heuristic. We then study the performance of the RA model for the different solution approaches investigated. The results obtained are used to establish the optimality-complexity trade-off, which is a critical criterion for QoS decision-making in practical CRSN applications.

### *Keywords:*

Cognitive radio sensor networks, heterogeneous networks, optimisation, resource allocation, wireless sensor networks.

---

## 1. Introduction

The quest for highly reliable and very productive wireless communication models and designs has continued to increase exponentially, so much that

keeping up remains a challenge for telecommunication stakeholders. In meeting this daunting challenge, researchers, both in academic field and the industry, now have the huge task of investigating and employing next-generation (xG) wireless network prototypes with the capability of achieving outstanding results, especially in the aspects of data rates, throughput, latency, speed, etc. that such networks can realise [1]. Some of the emerging technologies being developed are the fifth generation communication, internet-of-things networks, massive multiple input multiple output networks, autonomous vehicles and artificial intelligence-based technologies.

As new technologies emerge to meet present-day and future wireless communication expectations, a major challenge, however, is that the newly evolving wireless technologies usually require large bandwidths and substantial spectrum space for efficient service provisioning [2]. Regrettably, the frequency spectrum is no longer an ubiquitous communication resource. As it stands, there is already an ongoing problem of spectrum scarcity or unavailability for wireless communication. The problem of spectrum scarcity is one of the major barriers to achieving the desired results for xG communications, since it significantly undermines the effectiveness and productivity of most emerging technologies. In response, some new technologies are being advanced to help overcome the obstacle of spectrum scarcity for xG systems. One such technology is cognitive radio networks (CRN) [3].

The CRN promises to mitigate the limiting problem of spectrum scarcity, and has therefore continued to gain attention as one of the ideal candidate technologies for emerging xG wireless communication networks. In typical CRN designs, the same spectrum space is assigned to and used by both primary users (PUs) and secondary users (SUs), either simultaneously or in an opportunistic manner, under certain predefined conditions (or constraints). A very good example of such constraints is the amount of interference that the PUs can accommodate while the SUs transmit their data. For the CRN to be effective, the SUs have to transmit below the interference limits of the PU network while they communicate [4]. At the same time, the SUs must seek to utilise their very limited resources optimally to realise the best productivity for their network. To achieve this, appropriate resource allocation (RA) models to help carry out fair and efficient distribution of limited resources, so as to achieve utmost results for the CRN, are necessary [5]. Already, several useful RA models for the CRN are being developed and applied.

Just as new and/or improved RA models for the CRN are being developed, the CRN is also finding application in many other emerging wireless

network prototypes. One such recent application of the CRN is in wireless sensor networks (WSN). The intermix of WSN and CRN has resulted in a relatively new xG wireless technology called cognitive radio sensor networks (CRSN) [6]. The CRSN design is equipped with adaptability and flexibility in spectrum allocation, which potentially improves spectrum utilisation above that in conventional WSN. Hence, in the CRSN, some benefits are derived that may not always be achievable in conventional WSN designs. Such benefits include the ability to use multiple channels simultaneously by each user to fit in with a number of spectrum regulation demands, user adaptability to help reduce the amount of power consumed during transmission, the use of channels in an opportunistic manner during bursty traffic, and much more [7].

The CRSN paradigm is still an evolving technology. As it develops, a critical aspect of its evolution is the development of RA models and solutions for practical CRSN applications. In this paper, we develop and analyse an appropriate RA model for the heterogeneous CRSN. In the model, three aspects of heterogeneity are considered for the CRSN, namely a heterogeneous network, heterogeneous channels and heterogeneous users (or user demands). The incorporation of heterogeneity into the RA model for the CRSN gives a more practical representation of the network, albeit exacerbating the complexity of the RA problem. We then investigate three very useful solution approaches to the RA problem developed. From the approaches investigated, both optimal and suboptimal solutions are realised for the RA problem. The results obtained are used to establish the optimality-complexity trade-off, which is a critical consideration in quality of service (QoS) decision-making for practical CRSN applications.

A summary of the important contributions in this paper are as follows:

- An appropriate RA model that incorporates the concept of heterogeneity into the CRSN is designed. This gives a more realistic representation of the CRSN.
- An investigation into practical optimisation approaches for solving the RA problem developed for heterogeneous CRSN is carried out.
- The various optimisation solution approaches investigated are compared to establish an optimality-complexity trade-off, which gives an important basis for QoS decision-making in practical CRSN applications.

The remaining parts of this paper are organised thus: Section 2 gives a review of recent works on the CRSN, Section 3 presents the system model for RA in heterogeneous CRSN, Section 4 shows how the RA problem for heterogeneous CRSN is successfully reformulated as an integer linear programming (ILP) problem and then solved using classical optimisation, Section 5 describes the use of the concept of Lagrangian duality for solving the RA problem for heterogeneous CRSN, Section 6 describes the use of an appropriate heuristic for solving the RA problem for heterogeneous CRSN, Section 7 provides and compares some important results from the solution approaches investigated, and Section 8 gives the concluding remarks.

## 2. Review of Relevant Literature

The problems of RA in modern wireless communication networks have generally been described as optimisation problems [5]. Investigating techniques for solving RA problems in newly emerging wireless networks is an ongoing research focus. Several approaches are being developed to address these RA problems. A comprehensive review of RA problems and solution approaches in CRN has been carried out in [8], while an equally impressive review of RA problems and solutions for CRSN has also been provided in [9]. In this section, we discuss some of the most recent works on RA for the CRSN.

The authors in [10] employed the simultaneous wireless information and power transfer technique for resource sharing in CRSN. The model used the concepts of cooperative relaying, energy harvesting, wireless power transfer and power splitting in realising a low-cost CRSN for remote monitoring applications. Exact expressions for the outage probability and throughput were derived for the CRSN, making the results very useful and most likely transferable. Other work that has considered the use of wireless power transfer in RA for the CRSN is reported in [11]. In the model, both the SU and PU in the CRSN are wirelessly powered, while the SU rate is maximised using a dual optimisation method.

In [12], the authors employed the techniques of spectrum leasing and wireless energy harvesting for achieving RA in CRSN. The RA problem is developed as a joint subchannel, power and leasing time optimisation problem and solved suboptimally after splitting it into three sub-problems, namely subchannel allocation, power allocation and time allocation. The authors in [13] and [14] employed the technique of deep reinforcement learning in

developing RA solutions for the CRSN. One important advantage of employing deep learning architecture is that the network devices can learn from their environment, and adjust their parameters, thereby achieving very high efficiency in the use of network resources. The results presented show improvement in energy conservation, PU protection and decision on the best choice of spectrum and power in the CRSN.

The authors in [6] developed a two-level heterogeneous CRSN and investigated an RA solution for the network. The two aspects of heterogeneity that were considered are the heterogeneous radio environment and heterogeneous traffic. The RA scheme employed for the heterogeneous CRSN developed is called the improved pliable cognitive medium access protocol. Furthermore, a scheduling and queueing-based approach is used in determining the average delay and throughput performances of the CRSN. Results presented show improvement in the performance measures of interest over similar or comparative models for the CRSN.

From the RA problems and solution approaches reviewed above (and other similar approaches for the CRSN), it is clear that more than one approach may be employed for solving RA problems, with each approach having its own uniqueness, benefits, limitations and/or challenges. The implication of this is that employing a single approach to address RA problems in practical CRSN applications may not be in the best interest of the CRSN. This is because single-approach-based solutions may be inadequate in providing the desired result, especially when comparative analyses are required for some important decision-making, which could be in terms of QoS, cost, return on investment, etc.

In practical CRSN applications, there are some RA solution approaches that are capable of providing timeous solutions, although such solutions may be significantly suboptimal. Meanwhile, there may be cases where achieving optimal solutions would be quite critical, almost non-negotiable, for the type of applications or demands for which the network is to be employed. In contrast, even though the goal for most networks is usually to achieve optimal solutions, there may be other practical cases and/or applications of CRSN where suboptimal solutions that are sufficiently close to optimal would be good enough, especially if such solutions are obtained with some significant savings in the amount of time, resources and/or computational demands required in obtaining them. These considerations are important in QoS decision-making for practical networks, as the RA model for heterogeneous CRSN developed in this paper shows.

### 3. System Model

In this section, we describe and analyse the system model for the heterogeneous CRSN. This helps to establish the basis for the application of the different RA solution approaches investigated and analysed in this paper.

#### 3.1. Model Description

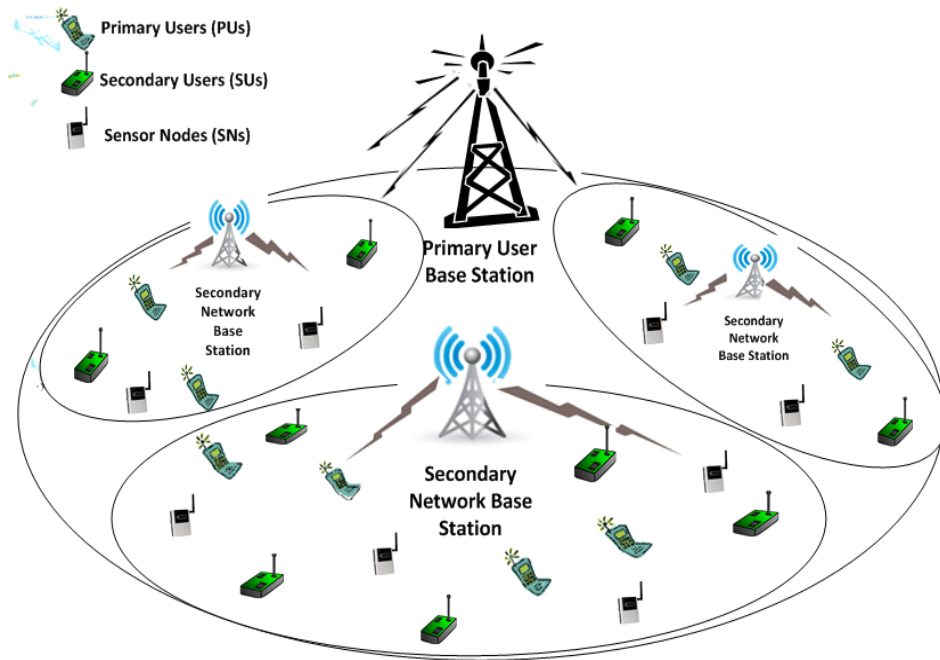


Figure 1: The system model for underlay heterogeneous CRSN

The system model developed in this paper is an underlay, heterogeneous CRSN model. The model is presented in Figure 1. The underlay architecture is employed because it makes it possible for all the network subchannels to be available for use by the secondary network, as long as the secondary network devices transmit their data within the interference threshold permitted by the PUs. The model can, however, be easily studied for the overlay CRSN. This can be achieved by simply including the probabilities of miss-detection and false alarm in the permissible interference constraint.

In the CRSN model developed, three heterogeneous considerations are incorporated, namely network heterogeneity, channel heterogeneity and user heterogeneity. Network heterogeneity is incorporated by designing the primary network to run differently from but simultaneously with the secondary network. This allows each network to use different configurations and/or parameters to drive its operations. Channel heterogeneity is incorporated in the design by the use of the orthogonal frequency division multiple access (OFDMA) technique. Then, user heterogeneity is incorporated in the design by having both SUs and sensor nodes (SNs) working alongside in the secondary network of the CRSN.

Furthermore, in the CRSN system model being studied, the primary network covers a large geographical area made up of a number of PUs. The activities of the PUs are coordinated by a PU base station. Within this geographical area, a number of smaller secondary networks are permitted to work independently but simultaneously with the primary network. A secondary network base station (SNBS) coordinates the activities of the devices of the secondary network. The SNs in the CRSN are similar to the SUs in that they contain cognitive radio units that enable them to act like the SUs. Thus, the SNs are able to dynamically adapt their communication parameters such as the subchannels, modulation options, transmission power, etc. to meet their communication demands. However, the SNs in the secondary networks are different from the SUs in that they have the inherent limitations of other conventional SNs, such as limitation in memory capacity, processing capability, transmission power, etc.

### *3.2. Model Analysis*

In the design, there are  $L$  PUs (indexed by  $l$ ) and  $K$  secondary network devices (indexed by  $k$ ) in the CRSN model. The secondary network devices are divided into  $K_1$  SUs and  $K_2 = (K - K_1)$  SNs. There are  $N$  OFDMA subchannels (indexed by  $n$ ) in the space of each SNBS. Furthermore, the  $N$  subchannels are divided into  $N_1$  and  $N_2$ , where  $N_1$  represents the subchannels allocated to the SUs (that is, the subchannels allocated to  $K_1$ ) and  $N_2$  represent the subchannels allocated to the SNs (that is, the subchannels allocated to  $K_2$ ). In all,  $N = N_1 + N_2$ . Rate weights  $w_1$  for SUs and  $w_2$  for SNs are associated with the respective secondary network types.

The decision on which subchannels are to be allocated to each SU or SN is made by the SNBS. This decision is passed to the SUs and SNs through a

distinct control channel. We assume that the SUs/SNs and the SNBS communicate perfectly over the control channel. A slow fading model is used for modelling the environment of the communication network. The modulation scheme for a subchannel determines the data rate  $c$  transmitted on that subchannel. We have employed four modulation schemes for the secondary network in our model. These modulation schemes are the binary phase shift keying (BPSK), quadrature amplitude modulation (4-QAM), 16-QAM and 64-QAM. The bit rates that can be transmitted by the four modulation schemes employed are  $c = 1, 2, 4$  and 6 bits per OFDMA symbol respectively. However, there is an important difference between allocations for the SNs and for the SUs. Since the SNs are very limited in computational power and memory space, assigning high modulation schemes to them is undesirable. Therefore, to avoid high modulation schemes being assigned to the SNs, the modulation schemes assigned to them are restricted to the BPSK and 4-QAM.

Furthermore, a parameter  $\rho$  is used to define the bit error rate (BER) requirement at the receiver end of the secondary network. In a subchannel, for a given value of  $\rho$ , the modulation scheme that is applied will determine the minimum power  $P_r(c, \rho)$  required to achieve  $\rho$  [15]. Therefore, it is easy to calculate the minimum power required to achieve  $\rho$  for the four modulation schemes under consideration. For BPSK modulation, the minimum power is given as  $P_{BPSK}(1, \rho) = Z_\phi [\text{erfc}^{-1}(2\rho)]^2$ . Similarly, the minimum power for the M-ary QAM schemes is given as  $P_{M-QAM}(c, \rho) = \frac{2(2^c-1)Z_\phi}{3} [\text{erfc}^{-1}(\frac{c\rho\sqrt{2^c}}{2(\sqrt{2^c-1})})]^2$ . In the minimum power equations,  $\text{erfc}(x) = (\frac{1}{\sqrt{2\pi}}) \int_x^\infty e^{-\frac{t^2}{2}} dt$  is the complementary error function,  $\pi = (22/7)$ , and  $Z_\phi$  is the noise power spectral density. The value of  $Z_\phi$  remains constant for all subchannels in the network.

Generally, the transmission power of the secondary network devices will increase in a non-linear manner when the number of bits on the subchannels of those devices increases, for a particular  $\rho$  value. We represent the power gain matrix between a PU and the SNBS as  $H^p \in R^{L \times N}$ , meaning that the subchannel power gain between the PU  $l$  and the SNBS at any subchannel  $n$  is  $H_{l,n}^p$ . We then represent the subchannel power gain matrix between any SU or SN and the SNBS as  $H^s \in R^{K \times N}$ , meaning that the power gain between the  $k$ th SU or SN and the SNBS at the  $n$ th subchannel is  $H_{k,n}^s$ . Hence, to transmit  $c_{k,n}$  bits over the  $n$ th subchannel to the  $k$ th SU or SN, and with a BER threshold  $\rho$ , transmission power  $P_{k,n}(c_{k,n}, \rho)$  is required.  $P_{k,n}(c_{k,n}, \rho)$  is



given as [15];

$$P_{k,n}(c_{k,n}, \rho) = \frac{P(c_{k,n}, \rho)}{H_{k,n}^s}. \quad (1)$$

Since the SUs can be assigned any modulation scheme to transmit, they can be treated as users having a minimum rate requirement. On the other hand, since the SNs are assigned to transmit with the lower modulation schemes, they are invariably best-effort users, meaning that the resources available have to be shared among all the SNs using a fair proportionality factor. With the SUs having a minimum rate requirement, we denote  $R_k$  as the minimum data rate required by SU  $k$  (more broadly, each secondary network device  $i$  has a data rate  $R_i$  assigned to it). Also, with the SNs being best-effort users, each SN has a value  $\gamma_k$  as the normalised proportional fairness factor assigned to it.

Furthermore, we represent the maximum power available on the  $n$ th subchannel by  $\Phi_n = \sum_{k=1}^K P_{k,n}$ , while we represent the transmission power of the  $k$ th SU or SN over that subchannel  $n$  by  $P_{k,n}$ . We represent the interference channel gain between the SNBS and the  $l$ th PU on subchannel  $n$  by  $H_{l,n}^p$ , while we represent the permissible interference power to the PU  $l$  from all the SUs and SNs combined by  $\varepsilon_l$ . Finally, we represent the maximum transmission power of the SNBS by  $P_{\max}$ . The formulation of the RA optimisation problem for heterogeneous CRSN becomes;

$$\max y = \left( \sum_{n=1}^{N_1} \sum_{k=1}^{K_1} w_1 c_{k,n} + \sum_{n=(N_1+1)}^N \sum_{k=(K_1+1)}^K w_2 c_{k,n} \right), \quad (2)$$

$$c_{k,n} \in \{0, 1, 2, 4, 6\}$$

subject to

$$\sum_{n=1}^N c_{k,n} \geq R_k; \quad k = 1, 2, \dots, K_1 \quad (3)$$

$$\frac{R_k}{\sum_{i=(K_1+1)}^K R_i} = \gamma_k; \quad k = K_1 + 1, K_1 + 2, \dots, K \quad (4)$$

$$\left( \sum_{n=1}^{N_1} \sum_{k=1}^{K_1} P_{k,n} + \sum_{n=(N_1+1)}^N \sum_{k=(K_1+1)}^K P_{k,n} \right) \leq P_{\max} \quad (5)$$

$$\left( \sum_{n=1}^{N_1} \Phi_n H_{l,n}^p + \sum_{n=(N_1+1)}^N \Phi_n H_{l,n}^p \right) \leq \varepsilon_l; \quad l = 1, 2, \dots, L \quad (6)$$

$$c_{k,n} = 0 \text{ if } c_{k',n} \neq 0, \quad \forall k' \neq k; \quad k = 1, 2, \dots, K \quad (7)$$

$$c_{k,n} \in \{0, 1, 2\} \quad \forall n \in N_2. \quad (8)$$

The throughput or total data rate that the secondary network achieves (that is, the combined data rate of all the SUs and SNs) is captured in the objective function of Equation (2). The constraint in Equation (3) is the constraint employed to meet the SUs' QoS demand. Thus, Equation (3) establishes that for the model to be feasible, the minimum data rate required to achieve the QoS demand for each SU must always be provided. In Equation (4), a fair distribution of the data rates to the SNs is achieved. This is done by the use of  $\gamma_k$  (a proportional fairness factor) to appropriately allocate data rates to all available SNs. Equation (5) represents the power constraint. The constraint simply means that when the transmission power of all SUs and SNs is added, the total must still be less than or equal to the maximum power of the SNBS. Equation (6) is the interference constraint. The constraint shows that for each PU in the network, when the total interference from all surrounding secondary devices (SUs and SNs) is combined, the total interference must still not exceed the threshold interference limit that the PU can accommodate. Equation (7) establishes the OFDMA requirement of mutual exclusivity in subchannel allocation for the SUs and SNs. The constraint means that a subchannel, once allocated to a secondary network device, cannot be allocated to any other secondary network device within the network in that same time frame. The constraint in Equation (8) explains that only the BPSK and 4-QAM modulation schemes can be assigned to the SNs whenever they have data to transmit (0 data rate is allocated to the SNs when they do not have any data to transmit). Equation (4) can be equivalently rewritten as;

$$R_1 : R_2 : \dots : R_{K_2} = \tilde{\gamma}_1 : \tilde{\gamma}_2 : \dots : \tilde{\gamma}_{K_2}, \quad k = K_1 + 1, K_1 + 2, \dots, K, \quad (9)$$

with  $\tilde{\gamma}_k$  being the product of  $\gamma_k$  and  $\sum_{i=(K_1+1)}^K R_i$ .

The RA problem formulation for heterogeneous CRSN given in Equations (2) - (8) is an NP-hard problem. This is easily established by closely examining the power constraint in Equation (5). Generally, NP-hard optimisation problems are quite difficult to solve. We investigate three different

but very important approaches to solve the RA problem for heterogeneous CRSN. The solutions are then compared and used to establish an important basis for QoS decision-making in practical CRSN applications.

#### 4. Solution through Integer Linear Programming Reformulation

Despite the fact that it is a complex NP-hard problem, an important contribution of this paper is that we are able to obtain optimal solutions for the RA problem for heterogeneous CRSN. In this section, we present the ILP solution approach as a good approach for optimally solving the RA problem for heterogeneous CRSN. Indeed, the approach is able to achieve optimal results for the complex NP-hard RA problem because the particular reformulation process employed does not in any way alter the scope of the original RA problem.

In the ILP approach, the original RA problem for heterogeneous CRSN is cleverly reformulated as an ILP problem while capturing all the details in the original problem. This is achieved by studying the structure of the problem. The idea used in achieving the reformulation is not entirely new, as it has been explored in some previous works on RA for the CRN, such as [16] and [15]. The idea is only now expanded in this paper to capture the intricacies of the heterogeneous CRSN. From a close look at the structure of the RA problem for heterogeneous CRSN developed in Equations (2) - (8), two distinct points stand out. The first clear observation is that the bits (0,1,2,4 or 6) assigned to the subchannels in the secondary network are strictly integer values. The other important observation is that in allocating bits to the SUs and SNs, each subchannel either has one or more bits assigned to it for data transmission or does not have any bit assigned to it, which is invariably a binary decision.

The above-mentioned observations are jointly considered and employed in carrying out the reformulation of the RA problem for heterogeneous CRSN in Equations (2) - (8) to an ILP. The reformulated ILP problem is much easier to solve, and any of the classical optimisation techniques for solving ILP problems can be employed for solving the reformulated problem. Since the process of achieving the ILP reformulation of the RA problem developed for heterogeneous CRSN is in many ways similar to the ILP reformulations for CRN that have already been carried out and well documented by the authors in some of their previous works, the details on the ILP reformulation are not presented in this paper. The authors encourage interested readers to

access their earlier works in [15] and [16] for a detailed explanation of the reformulation process. However, for necessary completeness, the reformulated problem is summarised in this section.

Let  $\mathbf{x}_1$  represent the bit allocation vector for the SUs and  $\mathbf{x}_2$  represent the bit allocation vector for the SNs ( $\mathbf{x}$  being the combined allocation vector for all SUs and SNs), let  $\mathbf{b}_1$  be the modulation order vector for the SUs and  $\mathbf{b}_2$  be the modulation order vector for the SNs, let  $\mathbf{B}_i \in \mathbb{Z}^{K_1 \times N K_1 C}$  be the data rate matrix for the SUs and  $\mathbf{B}_j \in \mathbb{Z}^{(K-K_1) \times N(K-K_1)C}$  be the equivalent data rate matrix for the SNs, let  $\mathbf{A} \in \{0, 1\}^{N \times N K C}$  be the total power matrix for the secondary network with  $\mathbf{p}_1$  being the power transmission vector for the SUs and  $\mathbf{p}_2$  being the equivalent power transmission vector for the SNs. The operator  $\odot$  signifies the Schur-Hadamard product (that is, the entry-wise product) of two entities. Then, the ILP reformulation of the RA problem for heterogeneous CRSN given in Equations (2) - (8) becomes:

$$y^* = \max_x [(w_1 \odot \mathbf{b}_1)^T \mathbf{x}_1 + (w_2 \odot \mathbf{b}_2)^T \mathbf{x}_2] \quad (10)$$

subject to

$$\mathbf{B}_i \mathbf{x}_1 \geq R_k; \quad k = 1, 2, \dots, K_1 \quad (11)$$

$$\mathbf{B}_j \mathbf{x}_2 = \tilde{\gamma}_k; \quad k = K_1 + 1, K_1 + 2, \dots, K \quad (12)$$

$$[(\mathbf{p}_1)^T \mathbf{x}_1 + (\mathbf{p}_2)^T \mathbf{x}_2] \leq P_{\max} \quad (13)$$

$$(\mathbf{H}^p[\mathbf{A}(\mathbf{p}_1 \odot \mathbf{x}_1)] + \mathbf{H}^p[\mathbf{A}(\mathbf{p}_2 \odot \mathbf{x}_2)]) \leq \boldsymbol{\varepsilon}_l \quad (14)$$

$$\mathbf{0}_N \leq \mathbf{A} \mathbf{x} \leq \mathbf{1}_N \quad (15)$$

$$\mathbf{x} \in \{0, 1\}, \quad \mathbf{b}_2 \in \{0, 1, 2\}, \quad \{w_1, w_2\} \in \mathbb{R}^+. \quad (16)$$

The newly reformulated RA problem in Equations (10) - (16) is a combinatorial linear optimisation problem. A number of standard solution techniques for solving combinatorial optimisation problems have been developed already. For the reformulated RA problem for heterogeneous CRSN presented in Equations (10) - (16), the well-established branch-and-bound (BnB) optimisation technique, embedded with the implicit enumeration tool, is used for solving the RA problem. This optimisation technique is employed because of its advantage of being able to identify the most viable branch for optimal solutions quite quickly [15].

## 5. Solution through Lagrangian Duality

Indeed, the ILP analysis of the RA problem for heterogeneous CRSN presented in the previous section is capable of achieving optimal solutions. However, the high level of computational complexity would make its implementation almost impracticable, especially when the network is of a large. Therefore, it is necessary to develop and employ other useful approaches that reduce the computational demand for the network. This is another important contribution of this paper. In this section, the Lagrangian duality approach is employed as a computationally less demanding approach to solving the complex RA problem of heterogeneous CRSN.

In the Lagrangian duality approach, the major constraints in the RA problem are first dualised. With the dualisation achieved, a new dual problem is formed. The new dual problem is then solved by applying the well-known Karush Kuhn Tucker (KKT) conditions to it [17]. To help formulate the dual problem, we take the reformulated ILP problem in Equations (10) - (16) as the primal problem. Then, the dual problem is obtained by minimising the dual function over the dual variables (this helps to obtain the highest value of the upper bound for the problem). The Lagrangian dual problem is developed in this section.

First, we write the Lagrangian function as follows;

$$\begin{aligned}
L(x, \eta, \delta, \tau, \varrho) = & ((w_1 \odot \mathbf{b}_1)^T \mathbf{x}_1 + (w_2 \odot \mathbf{b}_2)^T \mathbf{x}_2) \\
& + \eta^T (\mathbf{B}_i \mathbf{x}_1 - \mathbf{R}_k) + \delta^T (\mathbf{B}_j \mathbf{x}_2 - \boldsymbol{\gamma}_k) + \tau^T ((\mathbf{p}_1)^T \mathbf{x}_1 + (\mathbf{p}_2)^T \mathbf{x}_2 - P_{\max}) \\
& + \varrho^T (\mathbf{H}^p [\mathbf{A}(\mathbf{p}_1 \odot \mathbf{x}_1)] + \mathbf{H}^p [\mathbf{A}(\mathbf{p}_2 \odot \mathbf{x}_2)] - \boldsymbol{\varepsilon}_l),
\end{aligned} \tag{17}$$

in which case  $\eta, \delta, \tau, \varrho$  are the Lagrangian multipliers. Then, we obtain the corresponding dual function of the Lagrangian function. This is given as;

$$g(\eta, \delta, \tau, \varrho) = \max_x L(x, \eta, \delta, \tau, \varrho). \tag{18}$$

This means that,

$$\begin{aligned}
g(\eta, \delta, \tau, \varrho) = & \max_x \left[ ((w_1 \odot \mathbf{b}_1)^T \mathbf{x}_1 + (w_2 \odot \mathbf{b}_2)^T \mathbf{x}_2) \right. \\
& + \eta^T (\mathbf{B}_i \mathbf{x}_1 - \mathbf{R}_k) + \delta^T (\mathbf{B}_j \mathbf{x}_2 - \boldsymbol{\gamma}_k) + \tau^T ((\mathbf{p}_1)^T \mathbf{x}_1 + (\mathbf{p}_2)^T \mathbf{x}_2 - P_{\max}) \\
& \left. + \varrho^T (\mathbf{H}^p [\mathbf{A}(\mathbf{p}_1 \odot \mathbf{x}_1)] + \mathbf{H}^p [\mathbf{A}(\mathbf{p}_2 \odot \mathbf{x}_2)] - \boldsymbol{\varepsilon}_l) \right].
\end{aligned} \tag{19}$$

Factorising, it becomes;

$$\begin{aligned}
g(\eta, \delta, \tau, \varrho) &= -\mathbf{R}_k^T \eta - \gamma_k^T \delta - P_{\max}^T \tau - \boldsymbol{\varepsilon}_l^T \varrho \\
&+ \max_x \left[ (w_1 \odot \mathbf{b}_1)^T \mathbf{x}_1 + (w_2 \odot \mathbf{b}_2)^T \mathbf{x}_2 + \mathbf{B}_i^T \eta \mathbf{x}_1 \right. \\
&\left. + \mathbf{B}_j^T \delta \mathbf{x}_2 + \mathbf{p}_1 \tau \mathbf{x}_1 + \mathbf{p}_2 \tau \mathbf{x}_2 + \left( \mathbf{H}^p [\mathbf{A}(\mathbf{p}_1 \odot \mathbf{x}_1)] + \mathbf{H}^p [\mathbf{A}(\mathbf{p}_2 \odot \mathbf{x}_2)] \right)^T \varrho \right].
\end{aligned}$$

Therefore, the dual problem  $d^*$  is obtained as follows;

$$d^* = \min_{\eta, \delta, \tau, \varrho} g(\eta, \delta, \tau, \varrho) = \min_{\eta, \delta, \tau, \varrho} \max_x L(\mathbf{x}, \eta, \delta, \tau, \varrho).$$

That is,

$$d^* = \min_{\eta, \delta, \tau, \varrho} -\mathbf{R}_k^T \eta - \gamma_k^T \delta - P_{\max}^T \tau - \boldsymbol{\varepsilon}_l^T \varrho \quad (20)$$

subject to,

$$\eta, \delta, \tau, \varrho \geq 0 \quad (21)$$

$$(w_1 \odot \mathbf{b}_1)^T + (w_2 \odot \mathbf{b}_2)^T + \mathbf{B}_i^T \eta + \mathbf{B}_j^T \delta + \mathbf{p}_1 \tau + \mathbf{p}_2 \tau + \mathbf{H}^p \mathbf{p}_1 \mathbf{A} \varrho + \mathbf{H}^p \mathbf{p}_2 \mathbf{A} \varrho = 0 \quad (22)$$

$$\mathbf{b}_2 \in \{0, 1, 2\}. \quad (23)$$

The dual optimisation problem of the RA for heterogeneous CRSN given in Equations (20) - (23) is easily solved using the KKT conditions. Furthermore, it provides a good upper bound for the reformulated ILP problem in Section 4.

## 6. Solution through Fast BnB-based Heuristic

In this section, we employ a heuristic as a very useful third approach to solving the RA problem developed for heterogeneous CRSN. The heuristic employed is quite fast, and it is based on the BnB solution initially investigated in Section 4. It is common knowledge that, more often than not, solutions obtained using heuristics turn out to be suboptimal. Yet, in the RA problems for xG networks, such as for the heterogeneous CRSN being considered, heuristic solutions are still very useful, especially in cases where QoS expectations and optimality-complexity trade-off are critical criteria in the network design.

The heuristic algorithm developed in this paper seeks to circumvent the negative impact of one of the constraints in the heterogeneous CRSN design.

The constraint is that the decision variable  $\mathbf{x}$  must always take binary integer values of either 0 or 1. This constraint significantly exacerbates the complexity of the RA problem. Therefore, in the reformulated ILP RA problem for heterogeneous CRSN presented in Equations (10) - (16), if it is possible to integer-relax the decision variable  $\mathbf{x}$ , the complexity of the problem can be significantly reduced. Integer relaxation means that the variable  $\mathbf{x}$  is permitted to take decimal values between 0 and 1, unlike in the previous Sections 4 and 5 where the variable  $\mathbf{x}$  was strictly binary (either 0 or 1).

The new problem obtained by integer-relaxing  $\mathbf{x}$  is solved using the optimisation technique employed in Section 4. Right from the first iteration, it is obvious that the decision variable will take the value of 1 for some subchannels. For all those subchannels in which  $\mathbf{x}$  is 1 at the first iteration, the allocation is taken to be already determined. After this first iteration, a very simple algorithm is used to allocate appropriate data rates to all the remaining unallocated subchannels (that is, the subchannels in which the value of  $\mathbf{x}$  gave decimal numbers). The heuristic algorithm used in this paper evolves from the work in [18]. The algorithm is employed because it is fast and BnB-based. The algorithm is presented in Table 1.

## 7. Discussion of Results

In the paper, we have developed an important RA model for heterogeneous CRSN and have investigated and employed three different but useful approaches to solving the resulting complex NP-hard RA problem. In this section, we now present some numerical and simulation results for the RA problem. Further, by comparing the different solution approaches, we are able to establish the optimality-complexity trade-off, which is an important basis for QoS decision-making in heterogeneous CRSN.

In the simulation, a MATLAB-based optimisation solver called YALMIP, developed in [19], is used to carry out the optimisation process. To model the heterogeneous CRSN, we set up an OFDMA-based network with the number of subchannels  $N = 64$ . Random multipath fading is used as the fading model. The total number of PUs in the system  $L = 4$ . The SUs  $K_1 = 2$ , with each having a minimum rate requirement of 64 bits per symbol. The remaining data are allocated to the SNs  $K_2 = 2$  in a proportionally distributed manner using a fairness factor. In the setup, the small number of devices used is intended to make the interpretation of the results easy to understand and follow. The small network design also means that network

Table 1: BnB-based heuristic for RA in heterogeneous CRSN

Steps	Algorithm for fast suboptimal subchannel allocation
1	Relax the strict integer constraint in the reformulated ILP RA problem for heterogeneous CRSN presented in Equations (9) - (16). To achieve this, change the constraint $\mathbf{x} \in [0, 1]$ in Equation (16) to $0 \leq \mathbf{x} \leq 1$ . This means that $\mathbf{x}$ can now be any value between 0 and 1, decimal values inclusive.
2	The integer-relaxed problem is solved using the BnB method to obtain a solution for $\mathbf{x}$ .
3	The results from step (2) are used to identify and separate the subchannels that turn out $\mathbf{x} = 1$ in the solution. The number of subchannels that turn out $\mathbf{x} = 1$ in step (2) is represented by $M$ . We make $\Omega_M$ a set indexing all subchannels in $M$ , with $\Omega'_M$ being the set indexing the remaining subchannels that do not appear in $M$ .
4	All subchannels in $\Omega_M$ are allocated based on the positions of the 1s in $\mathbf{x}_N^n (n \in \Omega_M)$ . Clearly then, only the subchannels in $\Omega'_M$ remain to be allocated. The network's complexity is reduced very much after this step.
5	The remaining subchannels (that is, the subchannels in $\Omega'_M$ ), are now allocated as follows: initialise $m = M$ and $TheBest = \emptyset$ (that is, an empty set); use the BnB technique to solve the resulting problem to obtain the solution or $TheBest$ .
6	After solving, the subchannels in $\Omega'_M$ are allocated according to the positions of 1s in $TheBest$ and the algorithm is stopped. Also, if $TheBest = \emptyset$ , the message 'FAIL' is indicated and the algorithm is stopped.

complexity is kept minimal as much as possible. However, the RA model developed is quite scalable, and the solution approaches investigated can be employed for a much larger network.

Another important parameter for the simulation is the interference. When the PUs transmit, they interfere with the SUs and SNs. Such interference is seen as noise at the SUs and SNs and a constant power spectral density of 0.01 mW/subchannel is used to capture that interference. The BER  $\rho = 0.01$  is used for all SUs and SNs. The simulation is carried out using 100 random channel pairs of  $H^s$  and  $H^p$ . The rate weights  $w_1 = w_2 = 1$  are used for both



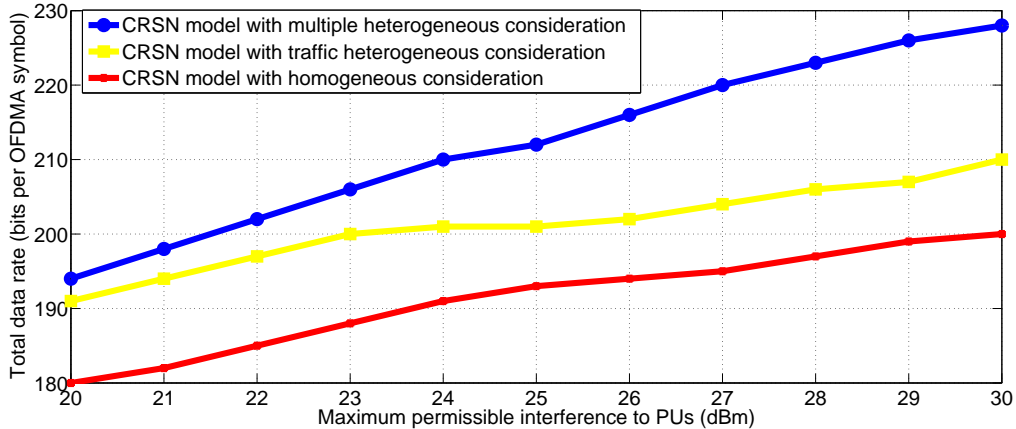


Figure 2: A plot of the comparative total data rates for three different models, namely a homogeneous CRSN model, a traffic-based heterogeneous CRSN model and the multiple-based heterogeneous CRSN model developed in this paper. The maximum SNBS power is set as 30 dBm.

secondary network categories.

For the purpose of validation, we compare the results obtained in this paper with the results from two similar RA models for the CRSN. Figure 2 provides such a comparison. We have chosen the results from the work in [20], which is a homogeneous CRSN design, and the results from the work in [21], which is a traffic-based heterogeneous CRSN design. The difference between our model and the models in the comparative works is clear. Unlike in [20], our model is not homogeneous but heterogeneous. Further, unlike in [21], our model incorporates not only traffic (or user demand) heterogeneity but also channel and network heterogeneity in its CRSN design. We compare the results for the total data rate or throughput that the network achieves, since it is a common performance measure investigated in all three models. To make the results comparable, we used equivalent parameters for the models being compared to derive similar total data rate results, which are then compared to the results from our model. In comparing our model with others, for fairness, we use the results for the ILP approach because it gives an optimal solution for the heterogeneous CRSN, and compare these with the best results obtained in the other models. The results show that our ILP solution outperforms the two comparative models in terms of throughput. This is because of the various heterogeneous concepts incorporated in the design,

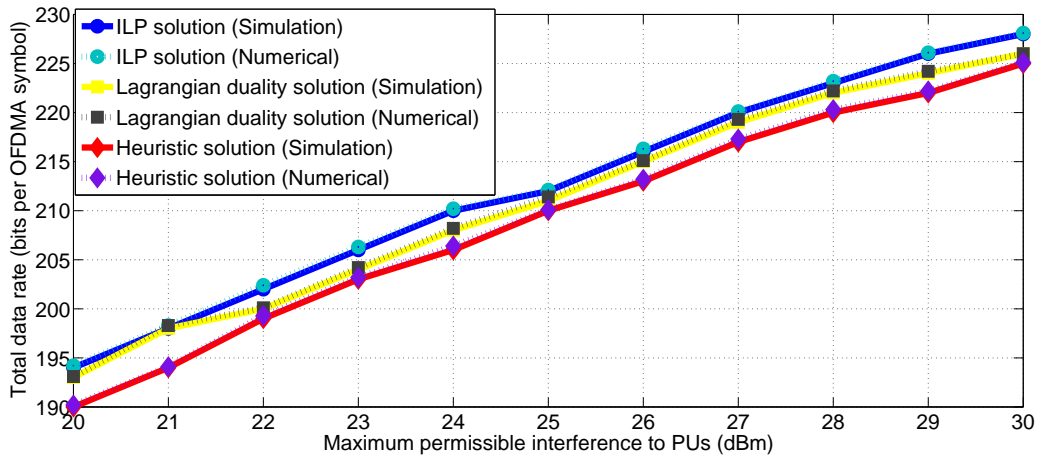


Figure 3: A plot of the total data rate versus the maximum permissible interference by the PUs for both numerical and simulation results. The maximum SNBS power is set as 30 dBm. The results for the ILP, Lagrangian duality and by heuristic are compared.

and because of the optimal solution realised by successfully reformulating the RA problem and solving optimally using the ILP approach.

Figure 3 presents the results of the throughput or total data rate of the secondary network (both SUs and SNs combined) when the three solution approaches investigated in this paper are compared. In addition, results from both the numerical analysis and the simulation are presented. In the graph, the interference threshold of the PUs was gradually increased from 20 to 30 dBm, at a constant SNBS power of 30 dBm. The important observation from this plot is that all three approaches employed provide similar solution patterns, which stands as a strong validation of the results. The solutions are also similar to the results from comparative CRN and CRSN models, such as in [15, 22], which helps to validate the results further.

Importantly, in Figure 3, by carrying out a comparative evaluation of the results for the three solution approaches investigated, we can easily provide an important basis for QoS decision-making in the heterogeneous CRSN. In terms of data transmission (that is, considering the throughput or total data rate achieved by the network), the results in Figure 3 show that the ILP approach outperforms the Lagrangian duality approach, which itself outperforms the heuristic approach. The implication of this result is that in certain CRSN scenarios where optimality is the critical QoS consideration, classical optimisation solution approaches such as the ILP, which achieves optimal

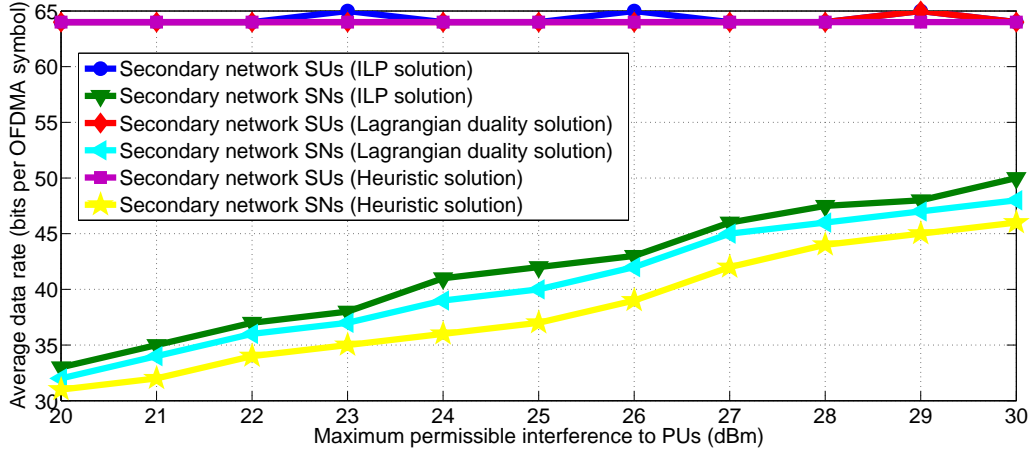


Figure 4: A plot of the average data rate versus the maximum permissible interference by PUs. The maximum SNBS power is set as 30 dBm. The results for the ILP, Lagrangian duality and by heuristic are compared.

or almost optimal solutions, are the preferred solution approaches for such CRSN applications.

In Figure 4, the results of the average data rates achieved by the different types of devices in the secondary network are presented. The results compare the three solution approaches developed in this paper. The interference threshold of the PUs was gradually increased from 20 to 30 dBm. A constant SNBS power of 30 dBm was assumed. It should be observed that in the results obtained, the QoS demands of the SUs were consistently met by providing the minimum rate requirement for SUs. The SNs only share the remaining resources after the SUs' requirements have been satisfied. However, as the amount of resources available for transmission is gradually increased, the performance of the SNs improves while the SUs' performance remains fairly constant, signifying that the SNs' capabilities can be improved by increasing available resources for them. The results for the average data rate are in line with the total data rate results presented in Figure 3. Therefore, the interpretations and implications are similar. The ILP approach outperformed the Lagrangian duality and heuristic approaches, again signifying that when optimality is the critical QoS consideration, the ILP approach is the preferred solution method for heterogeneous CRSN applications. However, in applications where optimality is not a crucial demand, other solution approaches such as the Lagrangian duality or the heuristic presented in this

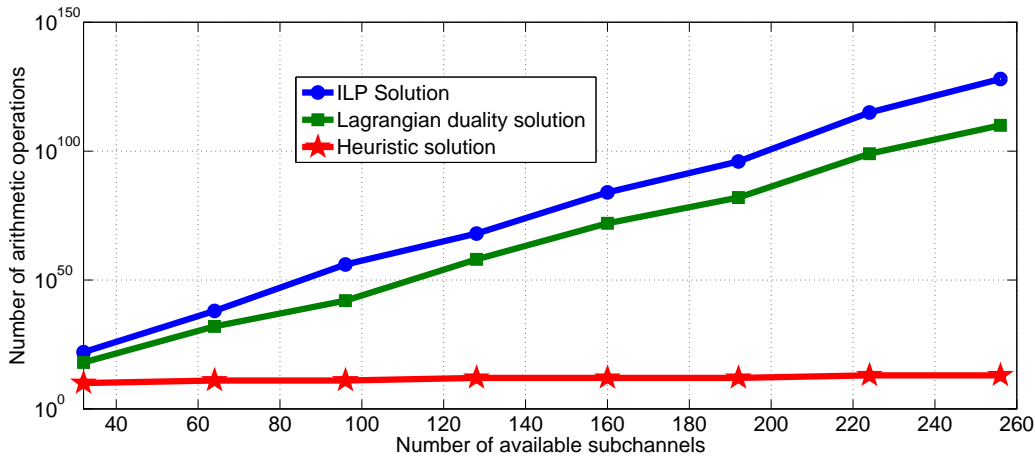


Figure 5: A comparison of the different solution approaches in terms of the complexity of the various solution models investigated. The computational demand is plotted against an increasing number of subchannels.

paper may be favourably considered.

Figure 5 compares the computational implications of the three RA solution approaches investigated for the heterogeneous CRSN. For the ILP and the Lagrangian duality approaches, the number of arithmetic operations performed before arriving at solutions is used to calculate the computational complexity of the model investigated. The total computational complexity of the heuristic is calculated by summing the number of operations carried out in the two parts of the algorithm (the first part being the solution of the relaxed problem and the second part being the allocation of the remaining subchannels). The results presented show that the computational complexity of the heuristic is significantly less than those of the ILP and the Lagrangian duality, especially as the network gets larger. From the results, it can be deduced that in heterogeneous CRSN scenarios where network complexity and/or the amount of time and resources spent to arrive at solutions are the most critical QoS considerations (that is, the time taken and the level of network complexity involved are more important than obtaining optimal solutions), developing heuristics for solving such RA problems is the preferred solution approach.

From all the results presented, the importance of the Lagrangian duality approach can be deduced. In terms of performance, this approach stands between the approaches of ILP (which gives solutions that are optimal or

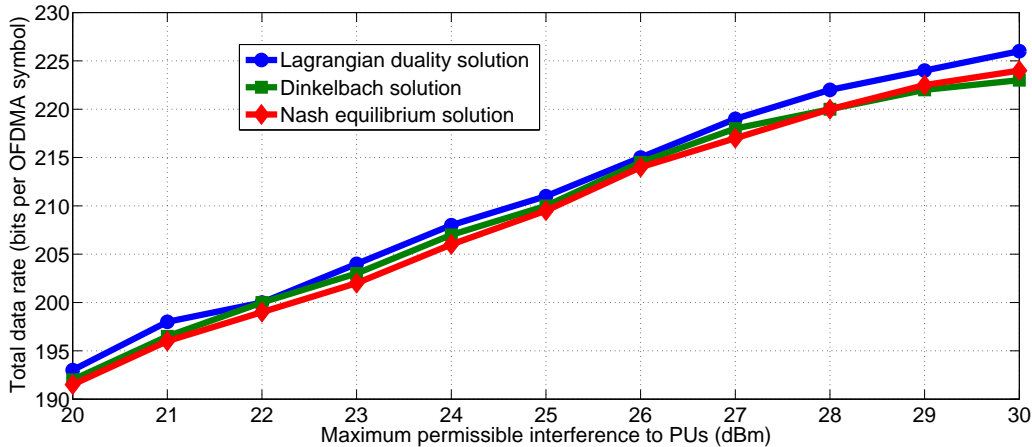


Figure 6: A comparison of different suboptimal solution approaches for solving RA problems in the CRSN. The Lagrangian duality approach employed in this paper is compared with the Dinkelbach approach used in [23] and the Nash equilibrium approach used in [24].

closest to optimal) and heuristic (which always gives suboptimal solutions). While it is not as optimal as the ILP, the Lagrangian duality approach is also not as computationally demanding as the ILP. Again, while it is closer to optimal than the heuristic, the Lagrangian duality approach is also more computationally complex than the heuristic. Therefore, the Lagrangian duality (or any other approach that may be employed) is good for consideration in heterogeneous CRSN scenarios where results are supposed to be near optimal, while at the same time not being too computationally demanding. Some other non-heuristic approaches that have been used in recent works on the CRSN are the Dinkelbach approach used in [23] and the Nash equilibrium approach used in [24]. Figure 6 compares the Lagrangian duality solution employed in this paper with the suboptimal solutions achieved in [23] and [24]. The solutions are quite similar once the same parameters are employed for simulating the network.

## 8. Conclusion

In this paper, a practical and very realistic RA model for heterogeneous CRSN has been developed and analysed. The inclusion of heterogeneity in the CRSN model exacerbates the complexity of the RA problem formulation. By studying the structure of the problem, a number of approaches that solves

the RA problem for heterogeneous CRSN are investigated. In the first solution approach, an ILP reformulation of the original problem is carried out. We then use the well-developed BnB optimisation technique with the implicit enumeration tool to solve the reformulated ILP problem. Optimal solutions are obtained using this approach. However, we argue that the high computational complexity requirement could make it an improbable approach for practical considerations, especially when the network is substantial in size. A second approach that reduces the computational complexity demand is then investigated. The second approach investigated is the Lagrangian duality approach. The approach uses the fact that, by dualising the ILP problem, the complex constraint in the ILP formulation is easily eliminated. The resulting dualised problem is solved using the well-established KKT conditions. In the third approach, a heuristic is developed to solve the RA problem. The heuristic relaxes the integer constraint, thereby making the complexity significantly less. Solutions from all three approaches are compared in terms of optimality and computational complexity, and the importance of each solution approach is established. The results are shown to be quite useful for QoS decision-making in practical heterogeneous CRSN applications.

### **Acknowledgements**

Financial support was provided by the SENTECH Chair in BWMC at the University of Pretoria. The authors deeply appreciate the invaluable contributions of Prof. Attahiru S. Alfa to this research work.

### **Abbreviations**

The following abbreviations are used in this paper:

BER	Bit error rate
BnB	Branch and bound
BPSK	Binary phase shift keying
CRN	Cognitive radio networks
CRSN	Cognitive radio sensor networks
ILP	Integer linear programming
KKT	Karuch-Khun-Tucker
OFDMA	Orthogonal frequency division multiple access
PU, PUs	Primary user, primary users
QAM	Quadrature amplitude modulation
QoS	Quality of service
RA	Resource allocation
SN, SNs	Sensor node, sensor nodes
SNBS	Sensor node base station
SU, SUs	Secondary user, secondary users
WSN	Wireless sensor networks

## References

- [1] B. S. Awoyemi, A. S. Alfa, B. T. Maharaj, Network restoration in wireless sensor networks for next-generation applications, *IEEE Sensors Journal* 19 (18) (2019) 8352–8363. doi:10.1109/JSEN.2019.2917998.
- [2] B. S. Awoyemi, A. S. Alfa, B. T. Maharaj, Resource optimisation in 5G and Internet-of-Things networking, *Wireless Personal Communications* 111 (4) (2020) 2671–2702. doi:10.1007/s11277-019-07010-9.
- [3] S. Haykin, P. Setoodeh, Cognitive radio networks: The spectrum supply chain paradigm, *Cognitive Communications and Networking, IEEE Transactions on* 1 (1) (2015) 3–28. doi:10.1109/TCCN.2015.2488627.
- [4] S. D. Okegbile, B. T. Maharaj, A. S. Alfa, Interference characterization in underlay cognitive networks with intra-network and inter-network dependence, *IEEE Transactions on Mobile Computing* (2020) 1–10.
- [5] B. Awoyemi, B. Maharaj, A. Alfa, Optimal resource allocation solutions for heterogeneous cognitive radio networks, *Digital Communications and Networks* 3 (2) (2017) 129 – 139. doi:https://doi.org/10.1016/j.dcan.2016.11.003.  
URL <http://www.sciencedirect.com/science/article/pii/S2352864816301043>

- [6] M. Al-Medhwahi, F. Hashim, B. M. Ali, A. Sali, A. Alkholidi, Resource allocation in heterogeneous cognitive radio sensor networks, *International Journal of Distributed Sensor Networks* 15 (7) (2019) 1550147719851944. arXiv:<https://doi.org/10.1177/1550147719851944>, doi:10.1177/1550147719851944.  
URL <https://doi.org/10.1177/1550147719851944>
- [7] O. B. Akan, O. B. Karli, O. Ergul, Cognitive radio sensor networks, *IEEE Network* 23 (4) (2009) 34–40. doi:10.1109/MNET.2009.5191144.
- [8] B. S. Awoyemi, B. T. J. Maharaj, A. S. Alfa, Solving resource allocation problems in cognitive radio networks: a survey, *EURASIP Journal on Wireless Communications and Networking* 2016 (1) (2016) 176. doi:10.1186/s13638-016-0673-6.  
URL <https://doi.org/10.1186/s13638-016-0673-6>
- [9] A. Ahmad, S. Ahmad, M. H. Rehmani, N. U. Hassan, A survey on radio resource allocation in cognitive radio sensor networks, *IEEE Communications Surveys Tutorials* 17 (2) (2015) 888–917. doi:10.1109/COMST.2015.2401597.
- [10] D. S. Gurjar, H. H. Nguyen, P. Pattanayak, Performance of wireless powered cognitive radio sensor networks with nonlinear energy harvester, *IEEE Sensors Letters* 3 (8) (2019) 1–4.
- [11] D. Xu, Q. Li, Resource allocation in cognitive wireless powered communication networks with wirelessly powered secondary users and primary users, *Science China Information Sciences* 62 (2) (2019) 29303.
- [12] Z. Liu, M. Zhao, Y. Yuan, X. Guan, Subchannel and resource allocation in cognitive radio sensor network with wireless energy harvesting, *Computer Networks* 167 (2020) 107028.
- [13] C. Wu, Y. Wang, Z. Yin, Energy-efficiency opportunistic spectrum allocation in cognitive wireless sensor network, *EURASIP Journal on Wireless Communications and Networking* 2018 (1) (2018) 1–14.
- [14] M. Liu, T. Song, J. Hu, J. Yang, G. Gui, Deep learning-inspired message passing algorithm for efficient resource allocation in cognitive radio networks, *IEEE Transactions on Vehicular Technology* 68 (1) (2019) 641–653.



- [15] B. S. Awoyemi, B. T. Maharaj, A. S. Alfa, Resource allocation for heterogeneous cognitive radio networks, in: Proc. IEEE WCNC, 2015, pp. 1759–1763. doi:10.1109/WCNC.2015.7127734.
- [16] B. S. Awoyemi, B. T. Maharaj, A. S. Alfa, Resource allocation in heterogeneous cooperative cognitive radio networks, International Journal of Communication Systems 30 (11) (2017) e3247, e3247 dac.3247. arXiv:<https://onlinelibrary.wiley.com/doi/pdf/10.1002/dac.3247>, doi:10.1002/dac.3247. URL <https://onlinelibrary.wiley.com/doi/abs/10.1002/dac.3247>
- [17] P. Pedregal, Introduction to Optimization, Texts in Applied Mathematics, Springer-Verlag, New York, USA, 2004.
- [18] Z. Mao, X. Wang, Efficient optimal and suboptimal radio resource allocation in OFDMA system, Wireless Communications, IEEE Transactions on 7 (2) (2008) 440–445. doi:10.1109/TWC.2008.060546.
- [19] J. Lofberg, YALMIP: a toolbox for modeling and optimization in MATLAB, in: Computer Aided Control Systems Design, 2004 IEEE International Symposium on, 2004, pp. 284–289. doi:10.1109/CACSD.2004.1393890.
- [20] J. Huang, X. Zeng, X. Jian, X. Tan, Q. Zhang, Opportunistic capacity-based resource allocation for chunk-based multi-carrier cognitive radio sensor networks, Sensors 17 (1) (2017). doi:10.3390/s17010175. URL <https://www.mdpi.com/1424-8220/17/1/175>
- [21] G. A. Shah, V. C. Gungor, O. B. Akan, A cross-layer design for qos support in cognitive radio sensor networks for smart grid applications, in: 2012 IEEE International Conference on Communications (ICC), 2012, pp. 1378–1382.
- [22] B. S. Awoyemi, B. T. Maharaj, A. S. Alfa, QoS provisioning in heterogeneous cognitive radio networks through dynamic resource allocation, in: Proc. IEEE AFRICON, 2015, pp. 1–6. doi:10.1109/AFRCON.2015.7331941.
- [23] M. Naeem, K. Illanko, A. Karmokar, A. Anpalagan, M. Jaseemuddin, Energy-efficient cognitive radio sensor networks: Parametric

and convex transformations, *Sensors* 13 (8) (2013) 11032–11050.  
doi:10.3390/s130811032.  
URL <https://www.mdpi.com/1424-8220/13/8/11032>

- [24] H. Xu, H. Gao, C. Zhou, R. Duan, X. Zhou, Resource allocation in cognitive radio wireless sensor networks with energy harvesting, *Sensors* (Basel, Switzerland) 19 (23) (November 2019). doi:10.3390/s19235115.  
URL <https://europepmc.org/articles/PMC6928818>