# Double sampling monitoring schemes: A literature review and some future research ideas

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**Abstract**

Adapted from the acceptance sampling field, the double sampling monitoring schemes implement a two-stage strategy to decide whether the process being monitored is in-control or out-of-control. That is, a master sample is split into two separate subgroup samples, with the first subgroup sample used in the first stage and, depending on which type of double sampling method is used, either only the second or the combined first and second, subgroup sample(s) are used in the second stage. This strategy has been proven to effectively decrease the sampling effort and, at the same time, to decrease the time to detect potential out-of-control situations. For these reasons, it has received some attention in the statistical process monitoring (SPM) literature and, in this review paper, all 87 existing publications on the basic double sampling monitoring schemes and other different schemes that are integrated with the basic double sampling schemes are reviewed. The double sampling schemes are categorized and summarized so that any research gaps in the SPM literature can easily be identified. Finally, concluding remarks and some directions for future research ideas are given.

**Keywords:** Control chart, Double sampling, Monitoring scheme, Statistical process monitoring (SPM), Run-length, Phase I, Phase II.

## 1. Introduction

The double sampling monitoring strategy is one of the most powerful tools used in statistical process monitoring (SPM) to detect unexpected changes in various types of processes (such as business, health and manufacturing) as quickly as possible. One of the main purposes of any monitoring scheme is to distinguish between assignable and common causes of variation. A process that works only in the presence of common causes of variability is said to be statistically in-control (IC). When a given sample has assignable causes of variation then a process is said to be out-of-control (OOC). Double sampling monitoring schemes implement a two-stage monitoring procedure to decide whether the process being monitored is IC or OOC. The first introduction of the double sampling strategy in the SPM context was reported in Croasdale (1974). Since then, there has been about 86 additional publications on related double sampling monitoring schemes, which are outlined in Table 1 and sorted chronologically (from 1974 to 2020).

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**Table 1:** Classification of articles discussing double sampling schemes in SPM (sorted chronologically)

| | Quality characteristic | | | | Econ and/or Econ-Stat design | Parameter(s) | | Type of Data | | Process Type | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Paper** | Location | Variability | Location & Variability | Linear Profiles | | Known | Unknown | Univariate | Multivariate | I.I.D. | Serial Dependence | Type of DS scheme |
| Croasdale (1974) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Daudin et al. (1990) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Daudin (1992) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Irianto and Shinozaki (1998) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Carot et al. (2002) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS-VSI $\bar{X}$ |
| He et al. (2002) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS & NSSTS $\bar{X}$ |
| He and Grigoryan (2002) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS $S$ |
| He and Grigoryan (2003) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS $S$ |
| Hsu (2004) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS & NSSTS $\bar{X}$ |
| Khoo (2004) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS $S^2$ |
| He and Grigoryan (2005) | ✓ | | | | | ✓ | | | ✓ | ✓ | | MS $\chi^2$ |
| Grigoryan and He (2005) | | ✓ | | | | ✓ | | | ✓ | ✓ | | DS $\|\mathbf{S}\|$ |
| He and Grigoryan (2006) | | | ✓ | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$&$S$ |
| Hsu (2007) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS $S$ |
| Claro et al. (2008) | ✓ | | | | | ✓ | | ✓ | | | ✓ | NSSDS $\bar{X}$ |
| Costa and Claro (2008) | ✓ | | | | | ✓ | | ✓ | | | ✓ | NSSDS $\bar{X}$ |
| Champ and Aparisi (2008) | ✓ | | | | | ✓ | | | ✓ | ✓ | | DS $T^2$ |
| Costa and Machado (2008) | ✓ | | | | | ✓ | | | ✓ | ✓ | | DS Bivariate $T^2$ |
| Machado and Costa (2008) | | ✓ | | | | ✓ | | | ✓ | ✓ | | DS Bivariate VMAX |
| Torng et al. (2009a) | ✓ | | | | ✓ | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Torng et al. (2009b) | ✓ | | | | ✓ | ✓ | | ✓ | | | ✓ | NSSDS $\bar{X}$ |
| Torng and Lee (2009) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Lee et al. (2009) | ✓ | | | | | ✓ | | ✓ | | | ✓ | NSSDS $\bar{X}$ |
| Irianto and Juliani (2010) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Torng et al. (2010) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS-VSI $\bar{X}$ |
| Lee et al. (2010) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS $S$ |
| Costa and Machado (2011) | ✓ | | | | | ✓ | | ✓ | | | ✓ | NSSDS $\bar{X}$ |
| De Araújo Rodrigues et al. (2011) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $np$ |
| Khoo et al. (2011) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSS synthetic DS $\bar{X}$ |
| Faraz et al. (2012) | ✓ | | | | ✓ | ✓ | | ✓ | | ✓ | | DS $T^2$ |
| Lee et al. (2012a) | ✓ | | | | ✓ | ✓ | | ✓ | | ✓ | | NSSDS-VSI $\bar{X}$ |
| Lee et al. (2012b) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS-VSI $S$ |
| Khoo et al. (2013a) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Khoo et al. (2013b) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Khoo et al. (2013c) | ✓ | | | | | ✓ | | | ✓ | ✓ | | synthetic DS $T^2$ |
| Teoh et al. (2013) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Lee (2013) | | | ✓ | | | ✓ | | ✓ | | ✓ | | NSSDS-VSI $\bar{X}$&$S$ |
| Chong et al. (2014) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSS synthetic DS $np$ |
| Teoh et al. (2014a) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Teoh et al. (2014b) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $\bar{X}$ |

**Abbreviations:** I.I.D. – independent and identically distributed; NSS – Non-side sensitive; DS – Double sampling; VSI – Variable sampling interval; NSSTS – Non-side sensitive triple sampling; MS – Multiple sampling; RSS – Revised side sensitive; MSS – Modified side sensitive; VSSI – Variable sampling size and interval; SSGRDS – Side sensitive group runs double sampling; MSSGRDS – Modified side sensitive group runs double sampling; EWMA – Exponentially weighted moving average; CV – Coefficient of variation; AIB – Auxiliary information based; ME – Measurement errors.

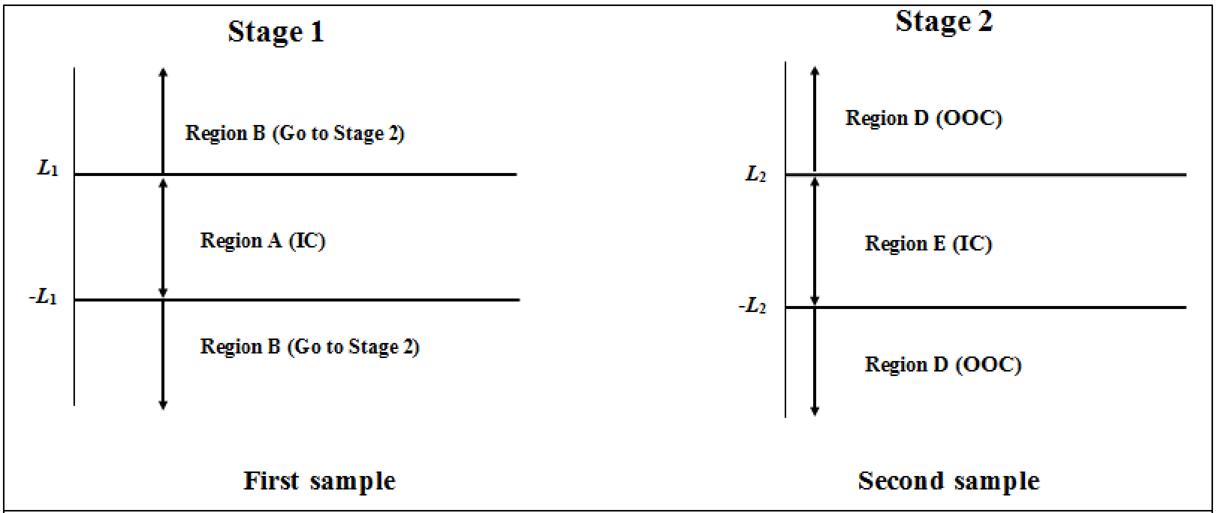
**Table 1:** (continued)

| Paper | Quality characteristic: Location | Quality characteristic: Variability | Quality characteristic: Location & Variability | Quality characteristic: Linear Profiles | Econ and/or Econ-Stat design | Parameter(s): Known | Parameter(s): Unknown | Type of Data: Univariate | Type of Data: Multivariate | Process Type: I.I.D. | Process Type: Serial Dependence | Type of DS scheme |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Costa and Machado (2015) | ✓ | | | | | ✓ | | ✓ | | ✓ | | RSS synthetic DS $\bar{X}$ |
| Noorossana et al. (2015) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS-VSSI $\bar{X}$ |
| Teoh et al. (2015) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| You et al. (2015) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSS synthetic DS $\bar{X}$ |
| Khoo et al. (2015) | ✓ | | | | | ✓ | | ✓ | | ✓ | | SSGRDS $\bar{X}$ |
| Joekes et al. (2015) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $p$ |
| Khoo et al. (2016) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Teoh et al. (2016a) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Teoh et al. (2016b) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $\bar{X}$ |
| Aghaulor and Ezekwem (2016) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSS synthetic DS $\bar{X}$ |
| Costa (2017) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS $Range$ |
| You (2017) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSS synthetic DS $\bar{X}$ |
| Lee and Khoo (2017a) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSS synthetic DS $S$ |
| Lee and Khoo (2017b) | | | ✓ | | | ✓ | | ✓ | | ✓ | | NSS synthetic DS $Max$ |
| Lee and Khoo (2017c) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS-VSI $np$ |
| Chong et al. (2017) | ✓ | | | | | ✓ | | ✓ | | ✓ | | SSGRDS $np$ |
| Castagliola et al. (2017) | | ✓ | | | | | ✓ | ✓ | | ✓ | | NSSDS $S^2$ |
| Yang and Wu (2017a) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS EWMA-sign |
| Yang and Wu (2017b) | | ✓ | | | | ✓ | | ✓ | | ✓ | | NSSDS EWMA-sign |
| Chong et al (2018) | ✓ | | | | | ✓ | | ✓ | | ✓ | | SSGRDS $\bar{X}$ |
| Haq and Khoo (2018) | ✓ | | | | | ✓ | | ✓ | | | ✓ | NSSDS $\bar{X}$ with AIB |
| You (2018) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSS synthetic DS $\bar{X}$ |
| Ng et al. (2018) | | | ✓ | | | ✓ | | ✓ | | ✓ | | NSSDS $CV$ |
| Saha et al. (2018) | ✓ | | | | | ✓ | | ✓ | | ✓ | | MSSGRDS $\bar{X}$ |
| Chu et al. (2018) | | ✓ | | | | ✓ | | | ✓ | ✓ | | DS-VSI $\|\mathbf{S}\|$ |
| Lee and Khoo (2018a) | | ✓ | | | ✓ | ✓ | | ✓ | | ✓ | | NSSDS $S$ |
| Lee and Khoo (2018b) | | ✓ | | | | ✓ | | | ✓ | ✓ | | DS-VSSI $\|\mathbf{S}\|$ |
| Khatun et al. (2018) | ✓ | | | | | ✓ | | | ✓ | ✓ | | DS-VSI $T^2$ |
| Malela-Majika and Rapoo (2019) | ✓ | | | | | ✓ | | ✓ | | ✓ | | MSS synthetic DS $\bar{X}$ |
| Malela-Majika et al. (2019) | ✓ | | | | | ✓ | | ✓ | | ✓ | | SSDS $\bar{X}$ |
| Malela-Majika (2019) | | | ✓ | | | ✓ | | ✓ | | ✓ | | MSS synthetic DS $\bar{X}$&$S^2$ |
| Lee and Khoo (2019a) | ✓ | | | | ✓ | ✓ | | ✓ | | ✓ | | NSS synthetic DS $\bar{X}$ |
| Lee and Khoo (2019b) | ✓ | | | | ✓ | ✓ | | | ✓ | ✓ | | synthetic DS $T^2$ |
| Lee and Khoo (2019c) | ✓ | | | | | | ✓ | ✓ | | ✓ | | NSSDS $np$ |
| Haq and Khoo (2019) | ✓ | | | | | ✓ | | ✓ | | | ✓ | NSS synthetic DS $\bar{X}$ with AIB |
| Rozi et al. (2019) | | | ✓ | | | ✓ | | ✓ | | ✓ | | NSSDS $CV$ |
| Lee et al. (2019) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $\bar{X}$ with ME |
| Lee et al. (2020) | | | ✓ | | | ✓ | | ✓ | | ✓ | | NSSDS $CV$ |
| Katebi and Moghadam (2020) | ✓ | | | | | ✓ | | | ✓ | ✓ | | DS-VSSI $T^2$ |
| Motsepa et al. (2020) | ✓ | | | | | | ✓ | ✓ | | ✓ | | SSDS $\bar{X}$ |
| Zhou et al. (2020) | | | ✓ | | ✓ | ✓ | | ✓ | | ✓ | | NSSDS $np_x$ |
| Umar et al. (2020a) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS-VSI $\bar{X}$ with AIB |
| Umar et al. (2020b) | ✓ | | | | | ✓ | | ✓ | | ✓ | | MSSGRDS $\bar{X}$ |
| Tuh et al. (2020) | ✓ | | | | | ✓ | | ✓ | | ✓ | | NSSDS $np$ |
| Mosquera and Aparisi (2020) | | | ✓ | | | ✓ | | ✓ | | ✓ | | NSSDS gauge-based |
| Eizi et al. (2020) | | | | ✓ | ✓ | | ✓ | | ✓ | | ✓ | DS $\hat{\beta}_1$ & DS $T^2$ |
| Tomohiro et al. (2020) | | | ✓ | | ✓ | ✓ | | ✓ | | ✓ | | NSSDS $C_{pm}$ |

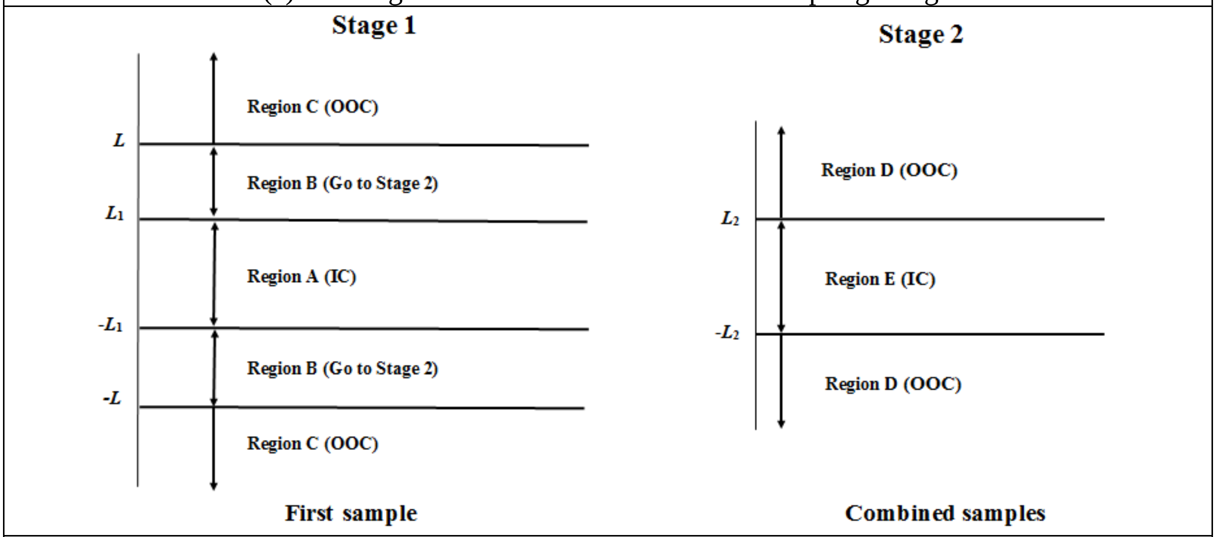
Croasdale (1974) adapted the idea of double sampling procedure from the acceptance sampling field and implemented its use in the SPM field. Croasdale (1974)'s method entails the use of a sample of size $n_1$ in stage 1 and of size $n_2$ in stage 2 to compute the corresponding charting statistic, both sub-samples from the same master sample of size $n$ $(= n_1 + n_2)$, where $n_2 > n_1$. Consequently, as an improvement to Croasdale (1974)'s method, Daudin et al. (1990) and Daudin (1992) showed that the use of the sample size $n_1$ in stage 1 and both $n_1$ and $n_2$ (i.e., $n_1 + n_2$) in stage 2 yields an even more improved performance and reduces the number of items to be inspected. In real-life applications, the investigation of the use of small samples in process monitoring (through control charts) is more important than that of the use of large sample sizes. Based on the latter, the vast majority of discussions on double sampling schemes done post-1992 were more focused on the method by Daudin (1992) rather than the original version by Croasdale (1974).

There are three main different designs of Shewhart-type *univariate* double sampling schemes charting regions, which are defined as: (i) original non-side-sensitive, (ii) improved non-side-sensitive and, (iii) side-sensitive. The first non-side-sensitive double sampling scheme is a two-stage scheme based on two *unconnected* samples (i.e. the first sample of size $n_1$ in stage 1 and the second sample of size $n_2$ in stage 2). It has only been discussed in 5 research works (see Figure 1(a) for its charting regions) – and it was first proposed in Croasdale (1974). The second non-side-sensitive double sampling scheme (by Daudin, 1992) is the most used design by almost 90% of publications on this topic; see Figure 1(b) for its charting regions – henceforth denoted by NSSDS. Unlike Croasdale (1974)'s scheme, the Daudin (1992)'s scheme is a two-stage scheme based on two *connected* samples (i.e. the first sample of size $n_1$ in stage 1 and the second combined sample of size $n_1 + n_2$ in stage 2). The third one is called the side-sensitive double sampling with its charting regions given in Figure 1(c) – henceforth denoted by SSDS – this is proposed in Malela-Majika et al. (2019). The SSDS scheme is based on two *connected* samples. It is important to note from Figure 1 that the Croasdale (1974) charting regions imply that a monitoring process never go to a state of OOC in stage 1, but it only does in stage 2. However, the charting regions in Figures 1(b) and (c) do allow for an OOC signal to take place in stage 1, making it more efficient. It is important to note that *bivariate* and *multivariate* double sampling schemes have charting that are upper one-sided only and consequently, the NSSDS or SSDS regions do not apply for this scenario.

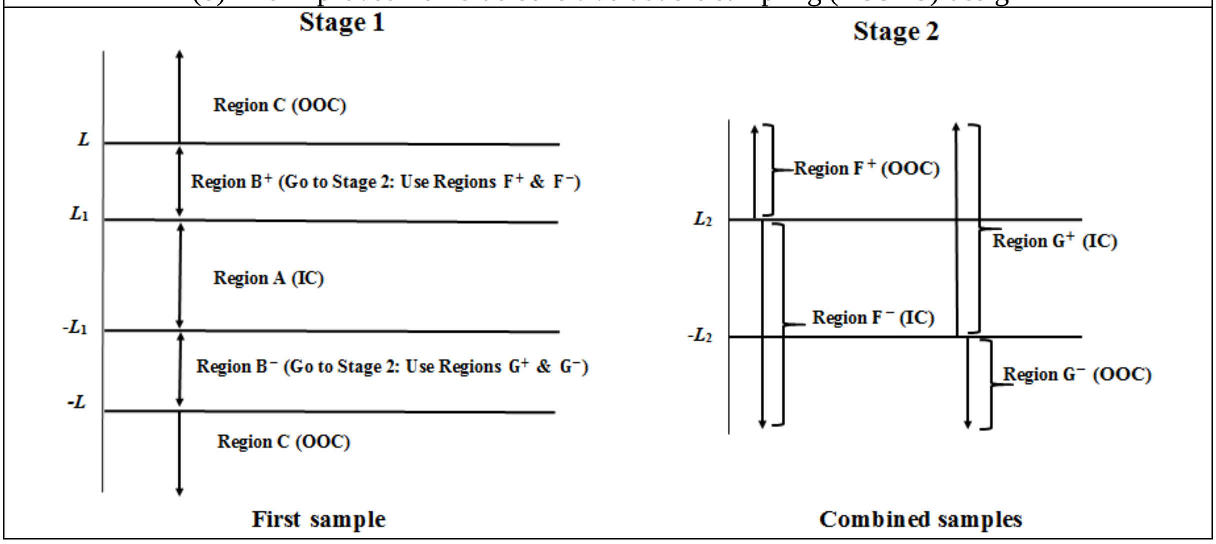
While the majority of the double sampling schemes are focused on the monitoring of the process location parameter(s), there is a variety of other different parameters that can be monitored by these schemes, e.g. the standard deviation, variance, range, coefficient of variation, linear profiles, etc. All the publications up to November 2020 that we could find in the literature are summarized in Table 1 and the corresponding journals or conference proceedings that published these ones are outlined in Table 2. Next, different authors that have made a contribution of at least two publications in this area of research are listed in Table 3 along with their respective affiliations and number of publications. It is observed from Tables 2 and 3 that *Communications in Statistics – Simulation and Computation* as well as *Quality* *and Reliability Engineering International* journals have the most publications on double sampling schemes, and that Prof M.B.C. Khoo (from Universiti Sains Malaysia, Malaysia) significantly has the most publications than any other author / researcher.



(a) The original non-side-sensitive double sampling design



(b) The improved non-side-sensitive double sampling (NSSDS) design



(c) The side-sensitive double sampling (SSDS) design

**Figure 1:** The charting regions in stages 1 and 2 of the different double sampling designs

**Table 2:** Journals / conference proceedings that published research on double sampling monitoring schemes

| Journal / Conference proceedings title | Number of publications |
|---|---|
| Communications in Statistics – Simulation and Computation | 9 |
| Quality and Reliability Engineering International | 9 |
| International Journal of Production Research | 7 |
| Communications in Statistics – Theory and Methods | 6 |
| International Journal of Production Economics | 6 |
| Computers & Industrial Engineering | 5 |
| Journal of Applied Statistics | 3 |
| European Journal of Operational Research | 2 |
| IIE Transactions | 2 |
| International Journal of Advanced Manufacturing Technology | 2 |
| Journal of Statistical Computation and Simulation | 2 |
| South African Journal of Industrial Engineering | 2 |
| Academic Journal of Science | 1 |
| Advances in Mathematics: Scientific Journal | 1 |
| COMPUSOFT, An International Journal of Advanced Computer Technology | 1 |
| European Journal of Industrial Engineering | 1 |
| Expert Systems with Applications | 1 |
| IEEE Access | 1 |
| IEEE International Conference on Control and Robotics Engineering | 1 |
| IEEE International Conference on Industrial Engineering and Engineering Management | 1 |
| International Conference on Management Engineering, Software Engineering and Service Sciences | 1 |
| International Conference on Smart Sensors and Application | 1 |
| International Journal of Applied Engineering Research | 1 |
| International Journal of Computing and Mathematics | 1 |
| International Journal of Engineering Research & Technology | 1 |
| International Journal of Industrial Engineering – Theory, Applications & Practice | 1 |
| International Journal of Production Development | 1 |
| International Journal of Pure and Applied Mathematics | 1 |
| International Journal of Quality Research | 1 |
| IOP Conference Series: Materials Science and Engineering | 1 |
| ITB Journal of Engineering Science | 1 |
| Journal of Probability and Statistics | 1 |
| Journal of Quality Measurement and Analysis | 1 |
| Journal of Quality Technology | 1 |
| Journal of Testing and Evaluation | 1 |
| Kongzhi yu Juece / Control and Decision | 1 |
| MATEC Web of Conferences | 1 |
| Pesquisa Operacional | 1 |
| PLoS ONE | 1 |
| Quality Engineering | 1 |
| Quality Technology and Quantitative Management | 1 |
| Revue de Statistique Appliquée | 1 |
| Statistical Methodology | 1 |
| Transactions of the Institute of Measurement and Control | 1 |
| **TOTAL** | **87** |

**Table 3:** Top researchers in SPM with at least two publications on double sampling schemes

| Author | Affiliation | Number of publications |
|---|---|---|
| Khoo, M.B.C. | Universiti Sains Malaysia; Malaysia | 32 |
| Lee, M.H. | Swinburne University of Technology; Malaysia | 14 |
| Teoh, W.L. | Heriot-Watt University Malaysia; Malaysia | 12 |
| Castagliola, P. | Université de Nantes & LS2N UMR CNRS 6004; France | 10 |
| Lee, P.-H. | Fujian University of technology, China | 10 |
| Torng, C.-C. | National Yunlin University of Science and Technology; Taiwan | 9 |
| Costa, A.F.B. | Sao Paulo State University; Brazil | 7 |
| Yeong, W.C. | University of Malaya; Malaysia | 7 |
| Teh, S.Y. | Universiti Sains Malaysia; Malaysia | 6 |
| He, D. | University of Illinois; USA | 6 |
| Grigoryan, A. | University of Illinois; USA | 6 |
| Chong, Z.L. | Universiti Sains Malaysia; Malaysia | 5 |
| Machado, M.A.G. | Sao Paulo State University; Brazil | 5 |
| Malela-Majika, J.-C. | University of South Africa; South Africa | 4 |
| Haq, A. | Quaid-i-Azam University, Pakistan | 3 |
| Irianto, D. | Institute of Technology Bandung; Indonesia | 3 |
| Saha, S. | International University of Business Agriculture and Technology, Bangladesh | 3 |
| Tseng, C.-C. | National Yunlin University of Science and Technology; Taiwan | 3 |
| You, H.W. | Universiti Kebangsaan Malaysia; Malaysia | 3 |
| Aparisi, F. | Universidad Politécnica de Valencia, Spain | 2 |
| Chakraborti, S. | University of Alabama; USA | 2 |
| Claro, F.A.E. | Sao Paulo State University; Brazil | 2 |
| Daudin, J.J. | UMR MIA 518 AgroParisTech/INRA; France | 2 |
| Hsu, L.F. | City University of New York; USA | 2 |
| Lau, E.M.F. | Swinburne University of Technology, Australia | 2 |
| Lee, H.C. | Universiti Sains Malaysia; Malaysia | 2 |
| Liao, H.-S. | National Yunlin University of Science and Technology; Taiwan | 2 |
| Liao, N.-Y. | National Yunlin University of Science and Technology; Taiwan | 2 |
| Motsepa, C.M. | University of South Africa; South Africa | 2 |
| Then, P.H.H. | Swinburne University of Technology, Malaysia | 2 |
| Umar, A.A. | Universiti Sains Malaysia; Malaysia | 2 |
| Wu, Z. | Nanyang Technological University; Singapore | 2 |
| Wu, S.-H. | National Chengchi University; Taiwan | 2 |
| Yang, S.-F. | National Chengchi University; Taiwan | 2 |

The rest of the review is structured as follows: The operation of the NSSDS scheme is outlined in Section 2 and the corresponding run-length properties are discussed in the Appendix. Univariate and multivariate double sampling schemes are discussed in Sections 3 and 4, respectively. A variety of monitoring schemes combined with the double sampling procedure are discussed in Section 5. Finally, in Section 6, concluding remarks and future research ideas are given.

## 2. Operation of the basic double sampling monitoring scheme

Remark: from Table 1, all the publications on double sampling schemes are based on observations sampled from an i.i.d. (independent and identically distributed) sequence of data from some underlying parametric distribution (except for Yang and Wu (2017a, b) and the seven publications that are based on serially correlated data).

Assume that $Y_{tj}$ are i.i.d. observations from a specified distribution used to calculate some quality characteristic of interest. Using these $Y_{tj}$ observations, a master sample of size $n$ is formed. Then, from the master sample, a first subgroup sample of size $n_1$ is collected at the $t^{th}$ sampling time (denoted as $Y_{1tj}$, $t = 1, 2, \ldots,$ and $j = 1, 2, \ldots, n_1$). If the standardized charting statistic based on the first sample falls on a region that requires a second stage to make a decision, then a second subgroup sample of size $n_2$ is also collected from the master sample at the $t^{th}$ sampling time (denoted as $Y_{2tj}$, $t = 1, 2, \ldots,$ and $j = 1, 2, \ldots, n_2$); where, in most publications, $n_2 \geq n_1$. Then any double sampling monitoring scheme uses these two separate sub-samples taken from the same master sample to decide whether the process is IC or OOC, and these sub-samples are used to compute the charting statistics of the two stages shown in Figure 1. Since the majority of the double sampling schemes in Table 1 are based on the univariate sample mean using the standard normal distribution, hence, for illustration purpose of the stages and the operation, we use the NSSDS $\bar{X}$ scheme when parameters are known (i.e. Case K) in Section 2.1 and unknown (i.e. Case U) in Section 2.2.

### 2.1 Case K

For Case K, the stages are implemented as discussed below.

**Stage 1:** Let $\bar{Y}_{1t} = \sum_{j=1}^{n_1} Y_{1tj}/n_1$ be the mean of the first sample of subgroup size $n_1$ at the $t^{th}$ sampling time. Hence, the standardized statistic for the first sample at the $t^{th}$ sampling time is then given by

$$Z_{1t} = \frac{\bar{Y}_{1t} - \mu_0}{\sigma_0/\sqrt{n_1}} \tag{1}$$

where $\bar{Y}_{1t} \sim N(\mu_0 + \delta\sigma_0, \frac{\sigma_0}{\sqrt{n_1}})$ and $\delta = |\mu_1 - \mu_0|/\sigma_0$ represents the magnitude of the standardized mean shift with the OOC mean $\mu_1$ ($\mu_1 = \mu_0 + \delta\sigma_0$), so that $\delta = 0$ means that the process is IC. In this case, $Z_{1t}$ follows a standard normal distribution (i.e. $Z_{1t} \sim N(0,1)$). However, when $\delta \neq 0$, the process is OOC and $Z_{1t} \sim N(\delta, 1)$.

**Stage 2:** At the $t^{th}$ sampling time of the second sample, the sample mean, i.e. $\bar{Y}_{2t} = \sum_{j=1}^{n_2} Y_{2tj}/n_2$, and the combined (or pooled) sample mean, i.e. $\bar{Y}_t = (n_1\bar{Y}_{1t} + n_2\bar{Y}_{2t})/(n_1 + n_2)$ are calculated, respectively. Hence, the standardized charting statistic for the combined samples at the $t^{th}$ sampling time is then given by

$$Z_{2t} = \frac{\bar{Y}_t - \mu_0}{\sigma_0/\sqrt{n_1 + n_2}}. \tag{2}$$

where $\bar{Y}_t \sim N(\mu_0 + \delta\sigma_0, \frac{\sigma_0}{\sqrt{n_1+n_2}})$. When the process is IC, $Z_{2t} \sim N(0, 1)$ since $\delta = 0$ and when the process is OOC, $Z_{2t} \sim N(\delta, 1)$.

### 2.2 Case U

For Case U, there is a need to first conduct a Phase I parameter estimation (see the parameter estimation reviews by Jensen et al. (2006) and Psarakis et al. (2013)), and then implement the stages in Phase II.

**Phase I parameter estimation**

Since the IC process parameters, $\mu_0$ and $\sigma_0$, are usually unknown they have to be estimated from $m$ Phase I subgroup samples, each of size $n$, i.e. $X_{ij}$, $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$. Hence, the estimated IC process parameters, $\hat{\mu}_0$ and $\hat{\sigma}_0$, are given by $\hat{\mu}_0 = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}$ and $\hat{\sigma}_0 = \sqrt{\frac{1}{m(n-1)}\sum_{i=1}^{m}\sum_{j=1}^{n}\left(X_{ij} - \bar{X}_i\right)^2}$, respectively, where $\bar{X}_i = \sum_{j=1}^{n} X_{ij}/n$.

**Phase II charting statistics and operation procedure: Stages 1 and 2**

Let $Y_{tj}$ be the Phase II observations from i.i.d. $N(\mu_1, \sigma_0)$, where $\mu_1$ is as defined in Case K. In Phase II of the NSSDS $\bar{X}$ scheme, there are two distinct standardized charting statistics in Case U (i.e. $\hat{Z}_{1t}$ and $\hat{Z}_{2t}$, shown below) used during stages 1 and 2, respectively.

**Stage 1:** Similarly, as in Case K, $\bar{Y}_{1t} = \sum_{j=1}^{n_1} Y_{1tj}/n_1$; so that

$$\hat{Z}_{1t} = \frac{\bar{Y}_{1t} - \hat{\mu}_0}{\hat{\sigma}_0/\sqrt{n_1}}. \tag{3}$$

**Stage 2:** Similarly, as in Case K, $\bar{Y}_{2t} = \sum_{j=1}^{n_2} Y_{2tj}/n_2$ and $\bar{Y}_t = (n_1\bar{Y}_{1t} + n_2\bar{Y}_{2t})/(n_1 + n_2)$, so that

$$\hat{Z}_{2t} = \frac{\bar{Y}_t - \hat{\mu}_0}{\hat{\sigma}_0/\sqrt{n_1 + n_2}}. \tag{4}$$

In essence, Equations (1) to (4) imply that there are two distinct standardized charting statistics used during stages 1 and 2 (if needed), respectively. Consequently, based on the abovementioned stages, then it follows that the Phase I and Phase II operational procedure of the NSSDS $\bar{X}$ scheme is as summarized in Figure 2. Note that when the parameters are assumed known, only the Phase II portion is relevant. Although Figure 2 is done for the NSSDS $\bar{X}$ scheme, it can easily be modified to account for the different designs outlined in Figure 1 as well as for different charting statistics, i.e. the median, standard deviation, coefficient of variation, etc.

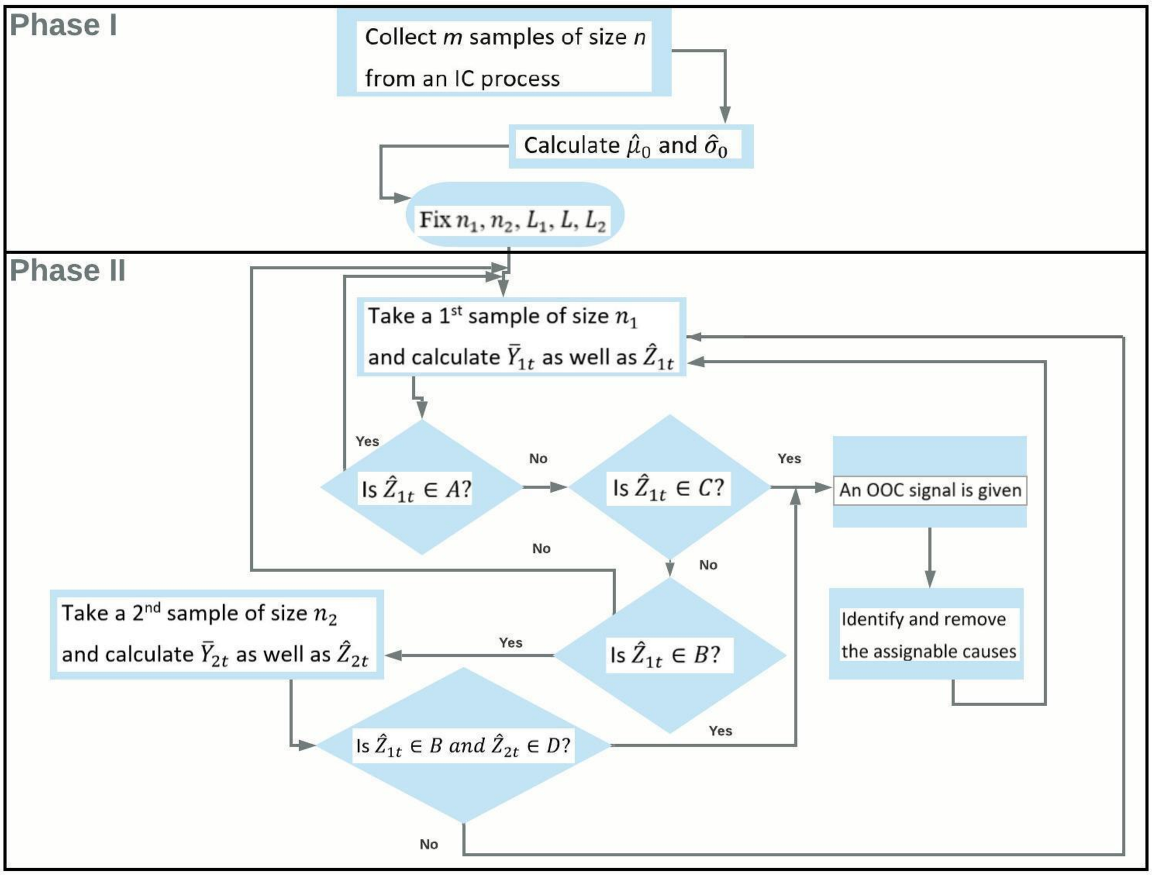



**Figure 2:** Flowchart of the operation of the NSSDS $\bar{X}$ scheme in Phase I and Phase II

## 3. Univariate double sampling monitoring schemes

In this section, the publications discussing basic double sampling schemes for location, variability and both the mean and variability simultaneously, are discussed in sub-sections 3.1, 3.2 and 3.3, respectively. Note that double sampling schemes combined with other monitoring schemes to monitor location, variability and both the mean and variability simultaneously, are discussed in Section 5.

### 3.1 Location

In an effort to design double sampling schemes for monitoring the mean, many authors have studied the same scheme; however, they have designed it by taking into account different design aspects. For instance, Irianto and Juliani (2010) outlined the following three design criterions:

(i) Minimize the expected number of sampling and inspections,
(ii) Maximize the OOC detection power (or minimizing the customer risk),
(iii) Minimize the false alarm rate (or minimizing the producer risk).

Daudin (1992)’s method (i.e. Figure 1(b)) prioritized (i) and (iii), whereas Irianto and Shinozaki (1998) used the charting regions in Figure 1(a) as these regions mainly prioritizes (ii). Irianto and Juliani (2010) used the charting regions in Figure 1(a) and formulated a model that takes into account (ii) and (iii), simultaneously. Although the latter two papers used Croasdale (1974)’s charting regions, in stage 2, they used a combined sample of size $n_1 + n_2$ instead of just $n_2$ as done in Croasdale (1974). Next, other publications discussed here, tend to ignore design criterion (iii), by keeping the false alarm rate constant and prioritizing (i) and (ii), simultaneously.

He et al. (2002) compared the performance of the NSSDS scheme to a triple sampling scheme (i.e. with 3 stages) and they observed that increasing the number of stages improves the detection ability of a monitoring scheme. However, Hsu (2004) raised some valid concerns regarding the manner in which the generic algorithm in He et al. (2002) was designed as it only took into account the *ASS* only, without using other run-length performance measures.

As outlined in Table 1, seven publications for serially dependent observations using the NSSDS $\bar{X}$ scheme. Costa and Claro (2008) used the autoregressive moving average with order (1,1), whereas Claro et al. (2008) and Costa and Machado (2011) used the first-order autoregressive model; however, Torng et al. (2009a) and Lee et al. (2009) used a correlation model proposed in Yang and Hancock (1990). Finally, Haq and Khoo (2018) designed a NSSDS $\bar{X}$ scheme based on a regression-type estimator of the process mean with an auxiliary variable under some specific conditions of correlation.

Torng and Lee (2009) studied the NSSDS $\bar{X}$ scheme using a variety of *t*- and gamma distributions with different parameters and observed that it is as good as the variable parameter $\bar{X}$ scheme; however, it turns to be much better than the basic $\bar{X}$ scheme in terms of a variety of run-length performance measures.

There have been numerous articles that have investigated the performance of the NSSDS $\bar{X}$ scheme when parameters are unknown, these are: Khoo et al. (2013a, b), Teoh et al. (2013, 2014b, 2015, 2016a, 2016b), You et al. (2015) and You (2018). That is, these latter articles studied the NSSDS scheme in Case U for a variety of design criterion and contexts. The design parameters that are obtained while the process is IC are such that the following performance metrics used in the latter articles (e.g. the unconditional average run-length ($ARL$), the unconditional median run-length ($MRL$), the unconditional expected $ARL$ ($EARL$), the unconditional average sample size ($ASS$), the unconditional average number of observations to signal ($ANOS$), etc.) are minimized when the process is in a state of OOC. Some of these Case U metrics are shown how they were derived in Appendix B. In the case of SSDS $\bar{X}$ scheme, Motsepa et al. (2020) studied the effect of parameter estimation as well as the effect of Phase I sample size on the Phase II OOC performance.

The economic and economic-statistical design of the NSSDS $\bar{X}$ scheme in Case K have been conducted in an effort to find the optimal set of parameters which minimizes the net sum of all costs involved, so that the scheme can be operated at an economically optimal level by using the classical cost model in Lorenzen and Vance (1986) as well as the sensitivity analysis. The latter was studied by Torng et al. (2009a, b) when observations are serially correlated and i.i.d., respectively.

Until more recently, all the publications on NSSDS $\bar{X}$ schemes have assumed that the observations were obtained using perfect measurements, i.e. without contaminated observations. Note that as discussed in

Maleki et al. (2017), this is hardly ever true in real life applications; hence, Lee et al. (2019) investigated the effect of measurement errors on the NSSDS scheme using a linearly covariate error model to capture the inherent measurement inaccuracy. To reduce the negative effect of measurement errors, Lee et al. (2019) used the multiple measurements sampling strategy (instead of the standard single measurement). Following a similar operational procedure as that in Figure 2, De Araújo Rodrigues et al. (2011) formulated the first double sampling scheme for attribute data called the NSSDS $np$ scheme which monitors the number of nonconforming items in a sample and it was shown to have a significantly better performance than the basic $np$ scheme in terms of the *ARL* metric. However, Tuh et al. (2020) criticized the use of the $ARL$ metric as a sole performance measure because it does not provide a clear picture about the NSSDS $np$ scheme's performance when the run-length distribution is too skewed. Thus, they designed the NSSDS $np$ scheme using the *MRL* metric as it provides a better interpretation of the run-length distribution. Joekes et al. (2015) showed that De Araújo Rodrigues et al. (2011)'s scheme is applicable in the case of large sample sizes only because the binomial-normal approximation used is valid for large sample sizes. Thus, they proposed the use of a Cornish-Fisher corrected control limits in the first stage of the NSSDS $p$ scheme so that the binomial-normal approximation is also valid for small sample sizes. More recently, Lee and Khoo (2019c) investigated the performance of the NSSDS $np$ scheme in Phase II when the process parameter is estimated from some IC historical Phase I data.

### 3.2 Variability

The first NSSDS scheme for variability was proposed in He and Grigoryan (2002), where the sample standard deviation is computed by, $S = \frac{1}{n}\sum_{j=1}^{n}\left(X_{tj} - \bar{X}\right)^2$ in each stage, accordingly, by using the operational procedure in Figure 2. Lee et al. (2010) illustrated a real-life application of the NSSDS $S$ scheme using a wire bonding process of packaging, where they showed the effectiveness of the scheme in reducing the cost as it requires fewer samples. Next, He and Grigoryan (2003) presented an improved version of the scheme in He and Grigoryan (2002) without the normality assumption for the sample standard deviation. Similar to the manner that Hsu (2004) showed that the sole use of the *ASS* without other run-length measures may, in some cases, yield misleading results; Hsu (2007) showed that He and Grigoryan (2003)'s sole use of the *ASS* is questionable because the conclusion is invalid when using other run-length properties. The economic-statistical design that minimizes cost using statistical constraints for the NSSDS $S$ scheme was studied by Lee and Khoo (2018a). A comparison analysis shows that it is more cost-efficient than the basic Shewhart $S$ scheme. Next, Khoo (2004) investigated the performance of the NSSDS scheme for monitoring the variability using the $S^2$ statistic when the underlying parameters are known and later, Castagliola et al. (2017) conducted the same study when the underlying parameters were estimated from a Phase I IC data and they also investigated the effect of Phase I sample size on the Phase II OOC performance.

Contrary, to the above publications that use either the standard deviation or the variance to monitor variability, Costa (2017) proposed an NSSDS scheme based on the sample ranges.

### 3.3 Location and variability

In the review paper by McCracken and Chakraborti (2012), the authors observed that monitoring the process mean alone would imply ignoring the changes in the process standard deviation, despite being well known that the latter can be greatly affected when the mean value gives a poor measure of central tendency. For monitoring both the mean and variability simultaneously, it is assumed that the process is OOC if either the process mean shifts from $\mu_0$ to $\mu_1 = \mu_0 \pm \delta\sigma_0$ (i.e., $|\delta| > 0$) and/or the process standard deviation shifts from $\sigma_0$ to $\sigma_1 = \gamma\sigma_0$ (i.e., $\gamma > 1$ for increase in $\sigma_0$, or $0 < \gamma < 1$ for decrease in $\sigma_0$). The process is IC if $\delta = 0$ and $\gamma = 1$. He and Grigoryan (2006) first proposed the NSSDS scheme to monitor both the mean and standard deviation simultaneously using the NSSDS $\bar{X}$ sub-scheme by Daudin (1992) and the NSSDS $S$ sub-scheme by He and Grigoryan (2002) i.e., with separate schemes for the mean and standard deviation. Later, Lee and Khoo (2017b) proposed the use of the single max-type plotting statistic (see Chen and Cheng, 1998); that is, instead of separately plotting the standardized mean or standard deviation, one needs to plot the maximum value of either the standardized mean or standard deviation at each sampling point (for stage 1, and if needed, for stage 2 also) using the upper one-sided version of the charting regions in Figure 1(b).

Since there are cases in SPM application where the process mean and standard deviation may not be constant when the process is in an IC state; however, their corresponding ratios are proportional, then Ng et al. (2018) implemented the SSDS charting regions in Figure 1(c) to monitor the coefficient of variation (CV) measuring the run-length performance with the *ANOS*; however, using samples of size $n_1$ in stage 1, and $n_2$ *only* in stage 2. Next, Rozi et al. (2019) instead implemented the NSSDS charting regions in Figure 1(b) with samples of size $n_1$ in stage 1, and the combined samples of size $n_1 + n_2$ in stage 2 also using the *ANOS*. Unaware of the publications by Rozi et al. (2019), Lee et al. (2020) also studied the design and performance of the NSSDS CV scheme.

Wu et al. (2009) proposed a scheme that monitors the mean shifts of a process by using combined attribute-variable inspection (denoted as $np_x$ scheme). Given the advantage of the $np_x$ scheme over both the attributes $np$ scheme and the variables $\bar{X}$ scheme, recently, Zhou et al. (2020) studied the statistical and economic designs of the NSSDS $np_x$ scheme to monitor the process mean and variance. In an effort to minimize cost and improve statistical performance in an uncertain environment, a robust 'weighted signal-to-noise ratio (WSNR)' approach (used to control the variance) has been incorporated in the NSSDS $np_x$ scheme's model. Consequently, using the *ARL* and expected cost, it is shown that the NSSDS $np_x$ scheme is more effective than the Wu et al. (2009)'s $np_x$ scheme.

Gauge-based monitoring schemes are generally known to be inferior to basic Shewhart schemes. In an effort to improve their performance, Mosquera and Aparisi (2020) showed that the NSSDS gauge-based scheme is more efficient than the basic $\bar{X}$, $S$ and the joint $\bar{X}$ & $S$ schemes in reducing sampling cost and in quickly detecting shifts in the mean and variability.

Process capability index ($C_{pm}$) uses the process variability and process specifications limits to determine whether the process is 'capable'; that is, it compares different processes to check whether they come up to expectation. Tomohiro et al. (2020) used the Taguchi's quality loss function to evaluate the economic and statistical efficiency of the NSSDS $C_{pm}$ scheme and showed that it has a significantly higher power and lower expected operational costs than the corresponding single sampling scheme.

**4. Multivariate double sampling monitoring schemes**

A majority of publications on double sampling control charts are based on univariate monitoring schemes, with just only 11 (out of 87) publications on multivariate schemes - see the outline on Table 1. When more than one characteristics (either i.i.d. or correlated) are to be monitored, multivariate charts must be used. If observations are $p$-variate normal random variable with mean $\boldsymbol{\mu_0}$ and variance $\boldsymbol{\Sigma}_0$, then the sequence of observations is denoted by $\{\boldsymbol{X}_{tj} = \left[X_{1tj}\ X_{2tj}\ \ldots\ X_{ptj}\right]'$: $t \geq 1$; $j = 1, 2,\ldots, n\}$. The upper one-sided DS schemes for multivariate data are based on the Hotelling's $T^2 = n(\overline{\boldsymbol{X}} - \boldsymbol{\mu_0})'\boldsymbol{\Sigma}_0^{-1}(\overline{\boldsymbol{X}} - \boldsymbol{\mu_0})$ statistic and the generalized sample variance (or equivalently, the determinant of the sample covariance matrix) $|\mathbf{S}| = \left|\frac{1}{n}\sum_{j=1}^{n}(\boldsymbol{X}_{tj} - \overline{\boldsymbol{X}})'(\boldsymbol{X}_{tj} - \overline{\boldsymbol{X}})\right|$ which are usually used to monitor the multivariate sample mean and standard deviation, respectively. The latter were first proposed by Champ and Aparisi (2008) and Grigoryan and He (2005), respectively. It is worth mentioning that He and Grigoryan (2005) presented a general case of multiple (instead of specifically double) sampling Hotelling's $\chi^2$ scheme where it was shown that as the number of sampling stages (which is equal to 2) are greater than or equal to 2, the monitoring scheme has a higher capability in detecting small shifts. Moreover, when the sampling stages are greater than 2, the Hotelling's $\chi^2$ scheme had an improved small shifts performance than the multivariate CUSUM and EWMA schemes in most situations.

Note that, unlike the univariate double sampling schemes, the multivariate ones tend to be designed as one-sided schemes. Faraz et al. (2012) conducted an intensive economic-statistical design for the optimal set of parameters for the DS $T^2$ scheme and they showed that, in most cases, it even outperforms the well-known multivariate EWMA $T^2$ scheme.

For the specific bivariate case, Costa and Machado (2008) showed that the one-sided DS $T^2$ scheme has a better performance than the basic, VSS, VSI $T^2$ schemes. Moreover, they observed that the one-sided version of Croasdale (1974)'s regions in Figure 1(a) are more favourable in terms of implementation it is known that the OOC signal can only take place in stage 2, and in some cases, it yields better OOC performance than the DS $T^2$ scheme. Next, for the bivariate sample variability, Machado and Costa (2008) proposed a DS scheme based on the VMAX statistic which can be used for monitoring a covariance matrix of a bivariate normal process, i.e. VMAX statistic utilizes the sample variances of two correlated random variables given by $\text{VMAX} = \max\{S_x^2, S_y^2\}$, where $S_x^2 = \sum_{j=1}^{n} x_j^2/n$, $S_y^2 = \sum_{j=1}^{n} y_j^2/n$ and the samples are denoted by $(x_j, y_j)$, $j = 1,2, \ldots, n$.

Profile monitoring entails the application of SPM in a process that is characterized by a relationship between two or more variables of interest. More recently, Eizi et al. (2020) studied the statistical and economic-statistical designs of the DS schemes to monitor linear profiles in Phase II. It is shown that since the *ARL*s depends only on the shift of the slope parameter ($\hat{\beta}_1$), but not on the constant parameter ($\hat{\beta}_0$); hence, the first chart is termed as the DS $\hat{\beta}_1$ scheme and the second one is termed as the DS $T^2$ scheme. When the correlation between parameters $\hat{\beta}_0$ and $\hat{\beta}_1$ is low, the DS $\hat{\beta}_1$ scheme is shown to have the smallest expected loss as compared to the DS $T^2$ scheme; otherwise, the converse is true. Moreover, it is recommended to implement the economic-statistical design rather than the statistical design when monitoring $\hat{\beta}_1$ using a DS scheme as it yields the smallest expected loss. Finally, the DS $\hat{\beta}_1$ scheme yields the smallest expected loss as compared to the single sampling $\hat{\beta}_1$ scheme.

Other multivariate double sampling schemes are discussed under the appropriate sub-sections of Section 5.

## 5. Other monitoring schemes combined with the double sampling scheme

Khoo et al. (2016) and Teoh et al. (2014a) compared the performance of the double sampling $\bar{X}$ scheme against the VSI and VSS $\bar{X}$ schemes, respectively. It was observed that the VSI schemes had a better OOC performance when moderate to large shifts are of interest using the *ATS*; that is, the NSSDS scheme has a better performance for small shifts only. Next, the NSSDS scheme has a better OOC performance than the VSS scheme when using the *ARL* and *SDRL*; however, the converse is true when using the *ASS*.

Because the purpose of integrating different monitoring schemes is to produce an improved scheme that has a better performance than the individual combined schemes, several monitoring schemes have been integrated with the basic double sampling scheme in an effort to improve its performance. Such monitoring schemes that we are aware of, so far, that have been integrated with the basic double sampling schemes are: (i) Variable sampling interval (VSI) scheme, (ii) Variable sample size and interval (VSSI) scheme, (iii) Synthetic scheme, (iv) Group-runs scheme and (v) Exponentially weighted moving average (EWMA) procedure.

### 5.1 VSI and VSSI procedure

For a better understanding of VSI and VSSI schemes, the reader is referred to the literature review by Psarakis (2015). Assume that the possible sample sizes are $n_1 < n_2$ and we define the long and short sampling intervals as $d_1$ and $d_2$, respectively, where $d_1 > d_2$. Carot et al. (2002) were the first to combine the NSSDS scheme with the VSI design using the charting regions in Figure 1(b). At each sampling point $t$, in stage 1, the sample size is fixed at $n_1$; however, the sampling interval is allowed to vary as follows

$$\begin{cases} d_2, & \text{if } Z_{1,t-1} \in \text{Region B or C} \\ d_1, & \text{if } Z_{1,t-1} \in \text{Region A.} \end{cases}$$

Later, Torng et al. (2010) studied the corresponding works with the normality assumption relaxed by using various $t$- and gamma distributions with different parameters. Note though, slightly different charting regions were used in stage 1 – see Figure 3. That is, Torng et al. (2010) defined the implementation of the sampling intervals at $Z_{1t}$ as follows

$$\begin{cases} d_2, & \text{if } Z_{1,t-1} \in \text{Region B1 or B2 or C} \\ d_1, & \text{if } Z_{1,t-1} \in \text{Region A.} \end{cases}$$

Moreover, unlike Carot et al. (2002), the charting procedure moves to stage 2 when $Z_{1t}$ falls in Region B2 in stage 1. As an improvement to Haq and Khoo (2018), Umar et al. (2020a) investigated the performance of the NSSDS scheme combined with the VSI design for monitoring the process mean with regression-type estimators under specific conditions of correlation (i.e. with auxiliary based information).

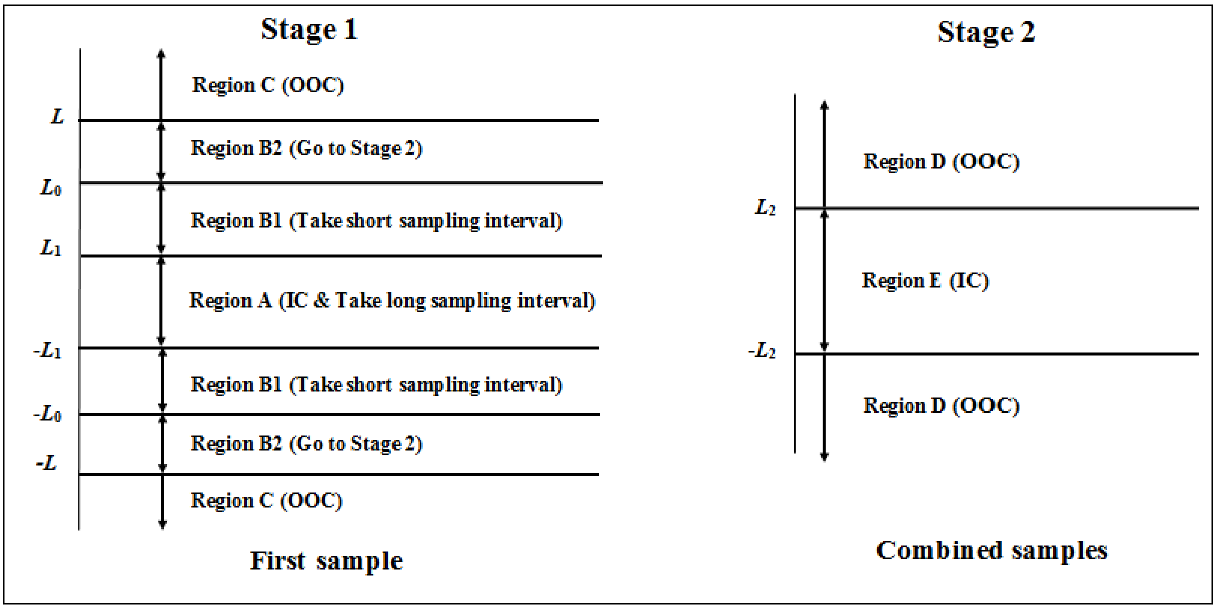



**Figure 3:** The charting regions in stages 1 and 2 of the VSI double sampling scheme

However, Noorossana et al. (2015) combined the NSSDS scheme with the VSSI design using the charting regions in Figure 3. While the NSSDS with the VSI design has $n_1$ and $n_2$ only, the double sampling with VSSI design has $n_1$, $n_2$ and $n_3$ (with $n_1 < n_2 < n_3$). Consequently, at each sampling point $t$, the sample size and sampling interval (denoted as $(n_i, d_i)$) are defined as follows

$$\begin{cases} (n_3, 0) & \text{if } Z_{1,t-1} \in \text{Region B2} \\ (n_2, d_2) & \text{if } Z_{1,t-1} \in \text{Region B1 or C} \\ (n_1, d_1) & \text{if } Z_{1,t-1} \in \text{Region A.} \end{cases}$$

That is, when a plotting statistic falls in Region B2, the charting procedure moves to stage 2 immediately (at that sampling point, i.e. the sampling interval is equal to zero) using a combined sample of size either $(n_1 + n_3)$ or $(n_2 + n_3)$ depending on which region did the previous sample (i.e. $Z_{1,t-1}$) plot on.

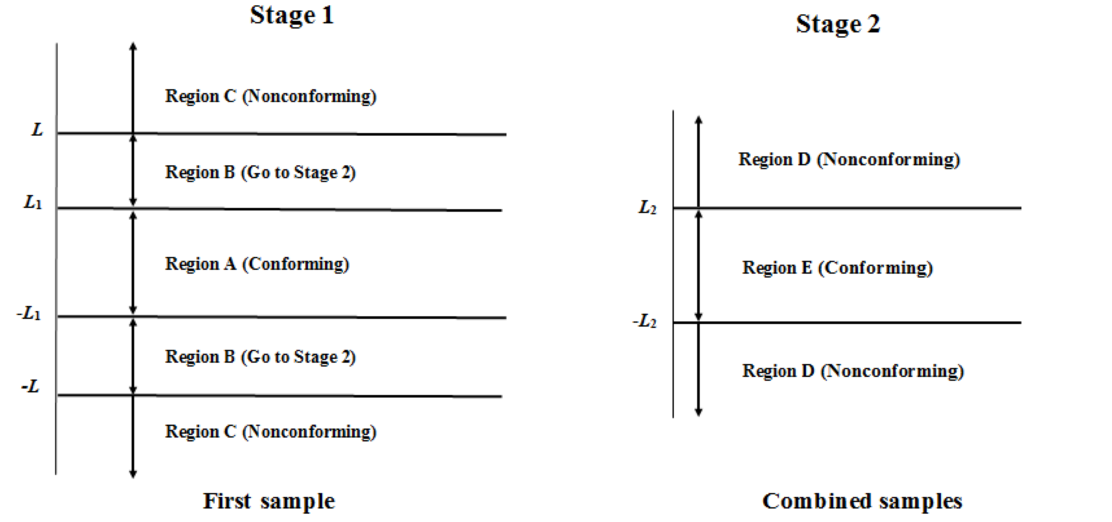
Unlike Torng et al. (2010) who used integral equations to evaluate the run-length distribution, Carot et al. (2002) and Noorossana et al. (2015) used the Markov chain approach outlined in Jensen et al. (2008) to obtain the *ATS*, average number of samples to signal (*ANSS*) and *ANOS*. Using these run-length properties, Noorossana et al. (2015) showed that the double sampling scheme with the VSSI design has a better performance than the corresponding VSI counterpart. Moreover, it performs better than all the corresponding basic Shewhart VSS, VSI and VSSI $\bar{X}$ schemes. Note that the economic design of the VSI double sampling $\bar{X}$ scheme is studied in Lee et al. (2012a).

The NSSDS $S$ scheme combined with the VSI design for monitoring the standard deviation is discussed in Lee et al. (2012b). The corresponding scheme that jointly monitors the mean and the standard deviation (i.e., which in essence incorporates the VSI design to the He and Grigoryan (2006)'s joint $\bar{X}$ and $S$ NSSDS scheme) was proposed in Lee (2013). Similarly; however, in the case of attributes data, Lee and Khoo (2017c) combined the NSSDS $np$ scheme with the VSI design.

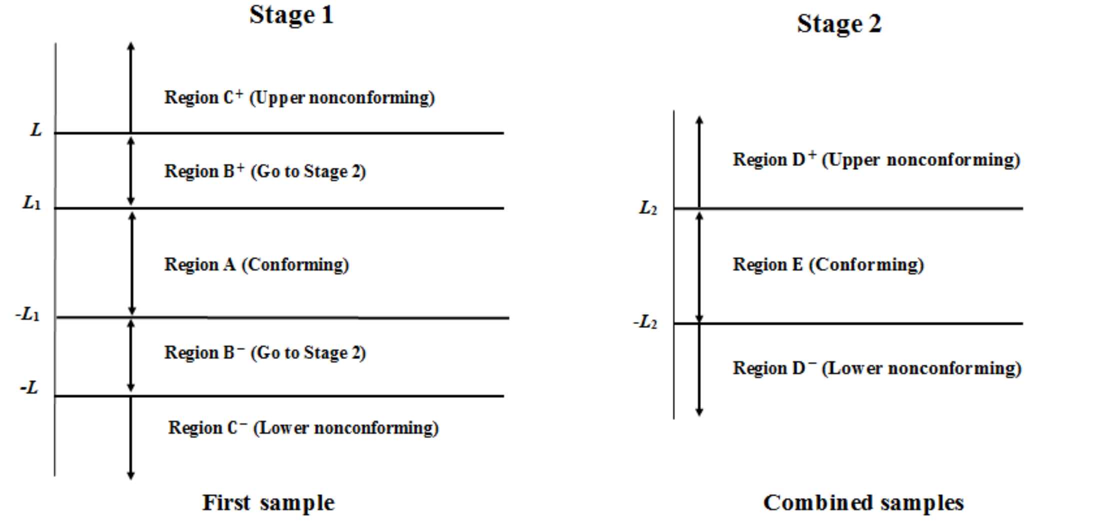
For multivariate data, Khatun et al. (2018) and Katebi and Moghadam (2020) investigated the performance of the one-sided DS $T^2$ scheme combined with the VSI and VSSI designs, respectively. In addition, the VSSI design incorporated into the one-sided DS $T^2$ scheme outperforms that of the VSI design in detecting shifts in the vector of process means. Chu et al. (2018) and Lee and Khoo (2018b) investigated the performance of the one-sided DS $|\mathbf{S}|$ scheme combined with the VSI and VSSI designs, respectively. In the latter multivariate articles, the combined schemes were shown to yield much better performance than their individual counterparts.

### 5.2 Synthetic scheme

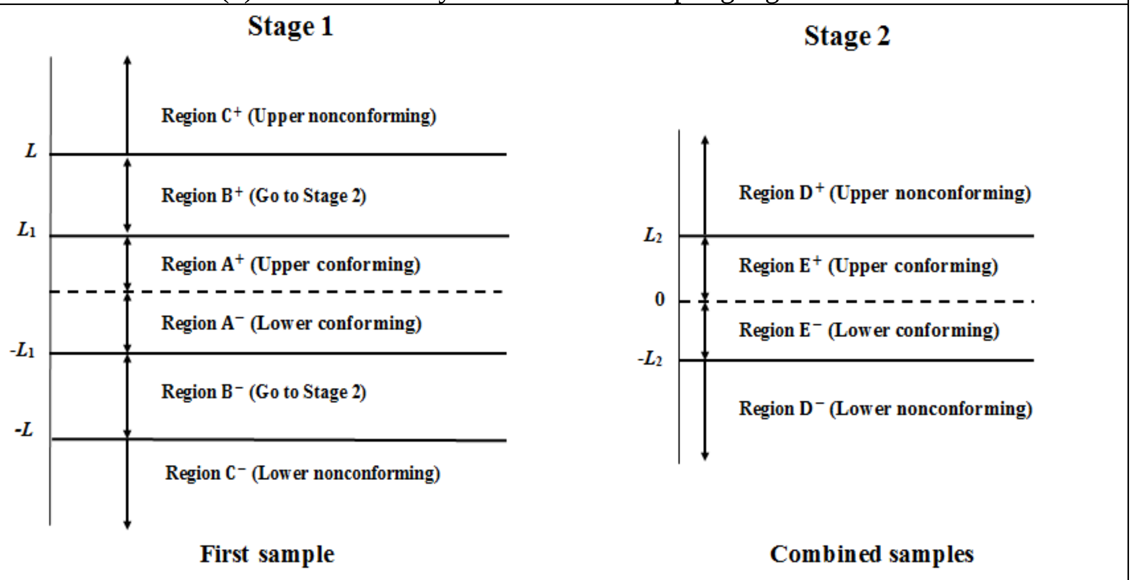
For a better understanding of synthetic schemes, the reader is referred to the literature review by Rakitzis et al. (2019). The conforming run-length (CRL) is defined as the number of samples observed between two consecutive nonconforming samples, inclusive of the nonconforming sample at the end. The main difference between a basic NSSDS scheme (in Figure 1(b)) and a non-side-sensitive (NSS) synthetic DS scheme (in Figure 4(a)) is that the latter does not issue OOC signal at the first sample point that falls on the nonconforming regions (i.e., the 'OOC regions' in Figure 1(b)). That is, the process waits until a second sample point falls on the nonconforming region and, if these two nonconforming samples are relatively close to each other (say, $CRL \leq H$), then an OOC signal is triggered. Note that $H$ is a positive integer greater than 0 and it is defined as a control limit of the CRL scheme.




(a) NSS synthetic double sampling regions




(b) SSS and RSS synthetic double sampling regions




(c) MSS synthetic double sampling regions

**Figure 4:** The charting regions in stages 1 and 2 of the synthetic double sampling scheme

Khoo et al. (2011) and Khoo et al. (2013) were respectively the first to integrate the operation of a NSS synthetic DS scheme with regions given in Figure 4(a) using the $\bar{X}$ and $T^2$ charting statistics and the CRL sub-scheme. It was observed that the integrated scheme has a significant improvement over the individual synthetic or NSSDS scheme. Next, the corresponding economic designs were studied in Lee and Khoo (2019a, b) under a variety of constraints for the univariate and multivariate NSS synthetic DS schemes, respectively. Aghaulor and Ezekwem (2016) designed the NSS synthetic DS scheme in a slightly different manner than Khoo et al. (2011); that is, they implemented an algorithm such that the samples of sizes $n_1$ and $n_2$ used in stages 1 and 2, respectively, are such that: $n_1 < n$, $n_2 < 2n$ and $n_1 + n_2 < n$.

Next, Costa and Machado (2015) realized that a side-sensitive version of the Khoo et al. (2011) scheme yields an improved performance; hence they proposed the standard side-sensitive (SSS) synthetic DS scheme using the regions in Figure 4(b). Malela-Majika and Rapoo (2019) proposed the revised and modified side-sensitive (denoted by RSS and MSS, respectively) synthetic DS schemes; and they showed that the latter two schemes outperform the other synthetic DS schemes. More recently, Motsepa et al. (2020) briefly studied the parameter estimation effect of the MSS synthetic DS $\bar{X}$ chart and showed that it has a better OOC performance than the basic SSDS $\bar{X}$ chart in Case U.

As an improvement to the NSSDS scheme for monitoring the mean with auxiliary variable (by Haq and Khoo, 2018), Haq and Khoo (2019) proceeded by combining the latter with the CRL sub-scheme to form a more effective NSS synthetic DS $\bar{X}$ chart using the regions in Figure 4(a).

Given that there was no synthetic DS scheme dedicated to simultaneously monitoring the mean and standard deviation, Malela-Majika (2019) used the regions in Figure 4(c) and the CRL sub-scheme to propose the MSS synthetic DS scheme with an OOC performance better than all its Shewhart-type competitors.

Having observed the performance of the basic NSSDS *np* scheme by De Araújo Rodrigues et al. (2011), Chong et al. (2014) investigated the corresponding NSS synthetic DS *np* scheme. For the variability case, Lee and Khoo (2017a) extended on He and Grigoryan (2002) work and proposed a NSS synthetic DS $S$ scheme.

### 5.3 Group-runs scheme

For a better understanding of group-runs schemes, the reader is referred to Gadre and Rattihalli (2007). Khoo et al. (2015) and Chong et al. (2017) proposed the side-sensitive group-runs DS (SSGRDS) scheme for the process mean and number of nonconforming items in a sample, respectively. Looking at group-runs schemes in a different way, it is a generalized version of the synthetic schemes in Section 5.2, i.e. it is similar to the CRL sub-scheme except in the decision making procedure. That is, the group-runs schemes give an OOC signal when the first CRL charting statistic is less or equal to $H$ (i.e., $CRL_1 \leq H$), or any two consecutive CRL charting statistics are both less than or equal to $H$ (i.e., $CRL_i \leq H$ and $CRL_{i+1} \leq H$, $i$=2,3,…). The SSGRDS scheme uses the charting regions in Figure 4(b) similar to those of the RSS synthetic schemes. The zero- and steady-state OOC performance of these schemes were computed using the *ANOS*. Chong et al. (2018) studied the run-length in more details by evaluating additional run-length properties for the work in Khoo et al. (2015), i.e., the median and percentile number of observations to signal (these are denoted by *MNOS* and *PNOS*, respectively).

Thereafter, Saha et al. (2018) enhanced the SSGRDS scheme by re-defining the CRL sub-scheme so that it has two limits, i.e. a warning limit (denoted by $H_1$) and a control limit (denoted by $H_2$), with $H_1 < H_2$, where $H_1$ and $H_2$ are positive integers greater than 0. This new scheme was called the modified SSGRDS (MSSGRDS). The MSSGRDS scheme gives an OOC signal when the first CRL charting statistic is less or equal to $H_2$ (i.e., $CRL_1 \leq H_2$), or any two consecutive $CRL_i \leq H_1$ and $CRL_{i+1} \leq H_2$, $i$=2,3,…). Using the *ANOS* and *EANOS*, the MSSGRDS scheme has been shown to outperform the SSGRDS scheme and a variety of other Shewhart-type competitors. Finally, Umar et al. (2020b) studied the run-length of the MSSGRDS scheme proposed by Saha et al. (2018) in more details by evaluating the *ANOS*, *MNOS* and *PNOS*.

### 5.4 EWMA procedure

Yang and Wu (2017a) used an asymmetric version of the control limits in Figure 1(b) to study the EWMA double sampling scheme based on the nonparametric sign statistic. This latter scheme was studied, and shown to yield a better performance than a variety of parametric and distribution-free schemes under the normal, double exponential, uniform, chi-square and exponential distributions. Similarly, Yang and Wu (2017b) showed that the asymmetric EWMA double sampling scheme for monitoring the variance has a better performance compared with the parametric and distribution-free schemes for monitoring variability.

## 6. Concluding remarks and future research ideas

Basic double sampling monitoring schemes are very effective in reducing sampling effort (hence a reduction in operational costs) and detect OOC situations much quicker than most Shewhart or Hotelling's competitors. Moreover, double sampling schemes combined with other procedures / schemes (e.g. VSI, VSSI, synthetic, group-runs, EWMA, gauge-based) have an even better OOC performance. Thus, this indicates that these monitoring schemes can be useful in many real life applications where the currently basic Shewhart or Hotelling's schemes are in use. Implementation tools need to be developed (e.g. using statistical software like R, Minitab, SAS, Matlab, SPSS, etc.) so that these schemes can be implemented in monitoring real-life applications.

While a majority of research work has been dedicated to NSSDS schemes, very little has been dedicated to the better performing SSDS schemes. Also, while there are a number of estimated parameter(s) research works for double sampling schemes – except the NSSDS $S^2$ scheme by Castagliola et al. (2017), these are only for the univariate process location, with none dedicated to simultaneous monitoring of the location and variability, the coefficient of variation, etc. With the exception of Eizi et al. (2020)'s work on linear profiles, no parameters unknown investigation has been done for bivariate or multivariate double sampling schemes in the case of i.i.d. or non-i.i.d. observations.

Finally, we provide below a list of some possible future research ideas that may be of interest to researchers who are interested in pursuing enhancements of double sampling schemes:

1. Majority of double sampling schemes focuses on the monitoring of process location parameter(s) for a normally distributed i.i.d. process. There are a number of pitfalls in ignoring or assuming that the corresponding standard deviation is constant or unaffected by changes in the location parameter. Thus, future research works need to focus more on monitoring both the location and variability parameters simultaneously in the case of non-normal distributions. Also, it worth monitoring the time between events using, say, the exponential distribution.
2. Double sampling schemes are mostly based on the assumption that the subgroup samples do not have either autocorrelation (within-sample correlation) or cross-correlation (between-samples correlation). However, for sequential observations, there tend to be some inherent underlying correlation within the observations – see for instance, Qiu (2019). Therefore, it is important to also focus on double sampling with more emphasis on autocorrelated observations as well as on nonparametric or distribution-free monitoring schemes.
3. With the exception of Haq and Khoo (2018, 2019), no other double sampling scheme takes into account auxiliary information. Also, there is only a single research work that takes into account measurement errors, i.e. Lee et al. (2019). Considering the importance of auxiliary information and measurement errors in real life applications; these important factors require more attention for different types of double sampling schemes.
4. With only eleven publications on multivariate schemes, there is a lot of research that need to be done based on parametric multivariate double sampling schemes as well as those for distribution-free double sampling schemes – with Qiu (2014) being the more appropriate starting point in this scenario. Moreover, for the parametric case, there is a need for a multivariate double sampling schemes based on the joint monitoring of location and variability, as well as the coefficient of variation statistic.
5. Since the combined schemes usually perform better than the individual integrated schemes, a fact that has been shown in the case of double samples with synthetic, VSI, VSSI and group-runs schemes. The latter fact needs to be tested whether it holds in the case of using double sampling approach in the case of memory-type schemes (i.e. exponentially weighted moving average (EWMA), cumulative sum (CUSUM), generally weighted moving average (GWMA), homogeneously weighted moving average (HWMA)). The only publications that have done this so far in the literature are on distribution-free method using the EWMA double sampling scheme; see Yang and Wu (2017a, b). The latter methodologies need to be adopted for the parametric scenarios and also be extended for other nonparametric scenarios. Moreover, for complex double sampling schemes, research may investigate the possibility of combining the synthetic or group-runs double sampling schemes with the VSI or VSSI designs.

6. Only a few studies on the economic and economic-statistical designs have been done in the literature. Hence, more effort needs to be done, especially when there is no assumption of i.i.d. and normality; and more importantly, when parameters are estimated.
7. With only a few studies on attributes data as well as combined attribute-variable inspection, more investigations are required in the area of double sampling schemes, specifically based on the number of nonconformities as well as high-yield processes; see the Woodall (1997) and Wu et al. (2009) as possible starting points.

In closing, the objective of this review was to give more intensive as well as more detailed background on this important branch of Shewhart-type schemes; with the hope that this will stimulate future researches on simple as well as complex double sampling schemes for monitoring a variety of quality characteristics.

**Appendix: Run-length properties of the NSSDS $\bar{X}$ scheme**

Run-length properties when the monitored observations are from a normal distribution with the underlying parameters known (i.e. Case K) and unknown (i.e. Case U) are discussed in Appendices A and B, respectively. Except for four publications, the entire double sampling schemes herein have an assumed underlying normal distribution; hence, the run-length properties are shown for this scenario. For non-normal discussions see Torng and Lee (2009), Torng et al. (2010), Yang and Wu (2017a, b).

It is worth mentioning that, for double sampling schemes, using different run-length metrics yield different outcomes; see the best examples in Hsu (2004, 2007) and Teoh et al. (2014a). Consequently, it is important to evaluate the run-length distribution of a double sampling monitoring scheme with different metrics rather than a single one.

**Appendix A: Case K**

Let $P_{0k}$ represents the probability that the process is regarded as IC at stage $k$, where $k$ = 1, 2. Then, $P_0 = P_{01} + P_{02}$ is the probability that the process is IC, where:

$$P_{01} = P(Z_{1t} \in \text{A}) = \Phi\left[L_1 + \delta\sqrt{n_1}\right] - \Phi\left[-L_1 + \delta\sqrt{n_1}\right],$$

and

$$P_{02} = P[Z_{1t} \in \text{B and } Z_{2t} \in \text{E}]$$
$$= \int_{Z_{1t}\in \text{I}^{**}} \left\{\Phi\left[cL_2 + rc\delta - z\sqrt{n_1/n_2}\right] - \Phi\left[-cL_2 + rc\delta - z\sqrt{n_1/n_2}\right]\right\} \phi(z)dz$$

where $\Phi(.)$ and $\phi(.)$ are the c.d.f. (cumulative distribution function) and p.d.f. (probability density function) of the standard normal random variable, respectively; $r^2 = n_1 + n_2$, $c = r/\sqrt{n_2}$, and $\text{I}^{**} = [-L + \delta\sqrt{n_1}, -L_1 + \delta\sqrt{n_1}) \cup (L_1 + \delta\sqrt{n_1}, L + \delta\sqrt{n_1}]$. Given that the NSSDS $\bar{X}$ scheme is a Shewhart-type one, its run-length (*RL*) follows a geometric distribution. Therefore, the c.d.f. of the *RL* distribution (denoted $F_{RL}(\ell)$) is obtained as $F_{RL}(\ell) = P(RL \leq \ell) = 1 - P_{0,}^{\ell}$ where $\ell \in \{1, 2, 3, \ldots\}$. Then, the $(100\rho)^{th}$ percentile of the *RL* distribution, $\ell_\rho$, is given by $P(RL \leq \ell_\rho - 1) \leq \rho$ and $P(RL \leq \ell_\rho) > \rho$.

Note that the most used values to evaluate the run-length distribution are $\rho$ = 0.05, 0.25, 0.50, 0.75 and 0.95, which denote the 5th, 25th, 50th (or median), 75th and 95th percentiles, respectively. Other well-known *RL* properties are the average run-length (*ARL*), standard deviation of the run-length (*SDRL*), average sample size (*ASS*) and average number of observations to signal (*ANOS*), in Case K, these are given by

$$ARL = \frac{1}{1 - P_0},$$

$$SDRL = \frac{\sqrt{P_0}}{1 - P_0},$$

$$ASS = n_1 + n_2 P_2$$

and

$$ANOS = ASS \times ARL,$$

respectively; where $P_2 = P(Z_{1t} \in B)$ is the probability of taking the second sample, and it is given by

$$P_2 = \Phi\left(L + \delta\sqrt{n_1}\right) - \Phi\left(L_1 + \delta\sqrt{n_1}\right) + \Phi\left(-L_1 + \delta\sqrt{n_1}\right) - \Phi\left(-L + \delta\sqrt{n_1}\right).$$

Since the *ANOS* depends on the *ASS* and *ARL* values, a larger *ANOS* value implies that either the monitoring scheme is inefficient and/or the cost of inspection is higher. A variety of other *RL* performance measures have been used in the literature, these include the *ATS*, *ANSS*, average number of switches (*ANSW*), standard deviation of time to signal (*SDTS*), standard deviation of number of samples to signal (*SDNSS*), standard deviation of number of switches (*SDNSW*) – most of these are thoroughly discussed in Noorossana et al. (2015).

**Appendix B: Case U**

In order to calculate the unconditional *RL* properties, we need to first derive the conditional ones, see for instance, You et al. (2018). Hence, the conditional c.d.f. of $\hat{Z}_{1t}$, given $\hat{\mu}_0$ and $\hat{\sigma}_0$ is defined as $F_{\hat{Z}_{1t}}(z|\hat{\mu}_0, \hat{\sigma}_0) = \Phi\left(\mathrm{U}\sqrt{\frac{n_1}{mn}} + Vz - \delta\sqrt{n_1}\right)$, where $U = (\hat{\mu}_0 - \mu_0)\frac{\sqrt{mn}}{\sigma_0}$ and $V = \hat{\sigma}_0/\sigma_0$. Consequently, the conditional p.d.f. of $\hat{Z}_{1t}$, is given by

$$f_{\hat{Z}_{1t}}(z|\hat{\mu}_0, \hat{\sigma}_0) = V\phi\left(\mathrm{U}\sqrt{\frac{n_1}{mn}} + Vz - \delta\sqrt{n_1}\right).$$

Since $\hat{\mu}_0 \sim N(\mu_0, \frac{\sigma_0^2}{mn})$, then $U \sim N(0,1)$ so that the p.d.f. of the random variable $U$ is simply, $f_U(u) = \phi(u)$. Next, using the fact that $V^2 = (\hat{\sigma}_0/\sigma_0)^2$ has a gamma distribution with parameters $m(n-1)/2$ and $2/[m(n-1)]$, then the p.d.f. of $V$ is defined as

$$f_v(v|m, n) = 2vf_Y\left[v^2\middle|\frac{m(n-1)}{2}, \frac{2}{m(n-1)}\right],$$

where $f_Y(.)$ is the p.d.f. of the gamma distribution with parameters $\frac{m(n-1)}{2}$ and $\frac{2}{m(n-1)}$.

Consequently, to derive the unconditional c.d.f. of the $RL$ of the proposed monitoring scheme, we need to first derive the unconditional probability of the IC process. Let $\hat{P}_{0k}$ denote the probability that the process with estimated parameters remains IC at the sampling stage $k$ (with $k = \{1, 2\}$). Then, the probability that the process is IC is given by $\hat{P}_0 = \hat{P}_{01} + \hat{P}_{02}$, where,

$$\hat{P}_{01} = \Phi\left(U\sqrt{\frac{n_1}{mn}} + VL_1 - \delta\sqrt{n_1}\right) - \Phi\left(U\sqrt{\frac{n_1}{mn}} - VL_1 - \delta\sqrt{n_1}\right)$$

and

$$\hat{P}_{02} = \int_{Z \in \mathrm{I}^{**}}^{\cdot} \hat{P}_E f_{\hat{Z}_{1t}}(z|\hat{\mu}_0, \hat{\sigma}_0) dz\,;$$

with

$$\hat{P}_E = \Phi\left[U\sqrt{\frac{n_2}{mn}} + V\left(\frac{L_2\sqrt{n_1+n_2} - z\sqrt{n_1}}{\sqrt{n_2}}\right) - \delta\sqrt{n_2}\right] - \Phi\left[U\sqrt{\frac{n_2}{mn}} - V\left(\frac{L_2\sqrt{n_1+n_2} - z\sqrt{n_1}}{\sqrt{n_2}}\right) - \delta\sqrt{n_2}\right].$$

Then, the unconditional c.d.f. of the NSSDS $\bar{X}$ scheme is given by

$$F_{RL}(\ell) = \int_{-\infty}^{+\infty}\int_{0}^{+\infty} \left(1 - \hat{P}_0^{\ell}\right) f_U(u) f_V(v)\, dv\, du,$$

where $\ell \in \{1, 2, 3, \dots, \}$. Therefore, the unconditional $ARL$ and $SDRL$ of the NSSDS $\bar{X}$ scheme are given by

$$ARL = \int_{-\infty}^{+\infty}\int_{0}^{+\infty} \left(\frac{1}{1-\hat{P}_0}\right) f_U(u) f_V(v)\, dv\, du$$

and

$$SDRL = \left[\int_{-\infty}^{+\infty}\int_{0}^{+\infty} \left(\frac{1+\hat{P}_0}{1-\hat{P}_0}\right) f_U(u) f_V(v)\, dv\, du - ARL^2\right]^{1/2}.$$

The $ASS$ is given by

$$ASS = \int_{-\infty}^{+\infty}\int_{0}^{+\infty} \left(n_1 + n_2\hat{P}_2\right) f_U(u) f_V(v)\, dv\, du,$$

where $\hat{P}_2$ is the probability of taking the second sample, which is given by $\hat{P}_2 = P\left(\hat{Z}_{1t} \in \mathrm{B} \middle| \hat{\mu}_0, \hat{\sigma}_0\right)$, or simply,

$$\hat{P}_2 = \Phi\left(U\sqrt{\frac{n_1}{mn}} + VL - \delta\sqrt{n_1}\right) - \Phi\left(U\sqrt{\frac{n_1}{mn}} + VL_1 - \delta\sqrt{n_1}\right) + \Phi\left(U\sqrt{\frac{n_1}{mn}} - VL_1 - \delta\sqrt{n_1}\right) - \Phi\left(\mathrm{U}\sqrt{\frac{n_1}{mn}} - VL - \delta\sqrt{n_1}\right).$$

Then, the $ANOS$ is given by

$$ANOS = \int_{-\infty}^{+\infty}\int_{0}^{+\infty} \left(n_1 + n_2\hat{P}_2\right)\left(\frac{1}{1-\hat{P}_0}\right) f_U(u) f_V(v)\, dv\, du.$$

Numerous authors have revealed that if a double sampling scheme is designed based on one specific size of a mean shift, it will perform poorly when the actual size of the shift is significantly different from the assumed size (see for instance You, 2017, 2018; Malela-Majika et al., 2019). Moreover, it is well-known that the *RL* distribution of a monitoring scheme is generally highly right-skewed when parameters are estimated. Also, the *ARL* is criticized because of its ineffectiveness in assessing the overall performance, especially when the aim of the study is to assess the performance of a monitoring scheme over a range of shifts. Hence, they recommended designing double sampling schemes to minimize the quality loss, which is measured by a quantity called the expected weighted run-length (*EWRL*) which is given by

$$EWRL = E[w(\delta) \times q(\delta)] = \int_{\delta_{min}}^{\delta_{max}} \left(w(\delta) \times q(\delta)\right) \times h(\delta) d\delta,$$

where $\delta$ follows some p.d.f. with a density function $h(\delta)$ and a range [$\delta_{min}$, $\delta_{max}$], where $\delta_{min}$ and $\delta_{max}$ are the lower and upper bound of the range of $\delta$, $w(\delta)$ is a weight function associated with $\delta$; and $q(\delta)$ is some specific shift run-length metric, e.g., *ARL*$(\delta)$, *ANOS*$(\delta)$, *ATS*$(\delta)$, etc. In the double sampling literature, the *EWRL* has been utilised by a number of different authors to formulate a bi-objective algorithm to obtain optimal parameter values, see for instance, Chong et al. (2014), Lee and Khoo (2017c), etc. More specifically, You (2017) used $w(\delta)$=1 to design the NSSDS scheme; while Malela-Majika et al. (2019) used $w(\delta)$=$\delta^2$ to design the SSDS scheme.

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