Co-digestion of water hyacinth, municipal solid waste and cow dung: a methane optimised biogas-liquid petroleum gas hybrid system

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Abstract

Fossil fuels are still the major source of energy in developing countries, howbeit expensive and environmentally unsustainable. Co-digestion substrate proportions and the respective biogas potentials for a huge number of biomaterials for anaerobic digestion are yet to be ascertained let alone optimised. This paper presents a novel methane-optimised biogas-liquid petroleum gas hybrid system concept. Herein this research, biogas is produced from the anaerobic co-digestion of water hyacinth, municipal solid waste and cow dung. A model that incorporated seasonal variations of biomass feedstocks was developed; an optimisation problem was formulated and solved using the Optimisation Interface tool (OptiTool) in combination with the Solving Constraint Integer Programs (SCIP) toolbox in Matrix Laboratory (MATLAB). The biogas production reactions are optimised in such a way that the methane component of the biogas is maximised, and the other components minimised by the integration of a model which necessitates the feed in of optimal substrate masses as per the ratios ascertained for the substrates considered thereby yielding a high quality combustible biogas product. The methane-optimised biogas is channelled towards some community gas demand and liquid petroleum gas comes in to fill the discrepancy between the methane-optimised biogas and the gas demand. Consideration of seasonality changes in the availability of substrates in the modelling and optimisation led to an increase of 174.58 % in annual biogas output. A 6.97 % annual lowest cost savings was realised in winter and 18.24 % annual highest cost savings was realised in summer from the methane-optimised biogas-liquid petroleum gas hybrid system.

Keywords: Anaerobic digestion; co-digestion; substrate blend ratios; biogas hybrid system; biogas modeling and optimisation

1. Introduction

The heating, cooling and transport sectors, which account for 80 % of global total final energy consumption, are lagging behind in view of meeting Sustainable Development Goal 7 (SDG 7) - (affordable and clean energy) and thus require accelerated action towards the renewable energy transformation [1]. One lucrative avenue towards solving the issue is venturing into biofuels such as biogas which is a form of bioenergy. Bioenergy can be regarded as the most substantial renewable energy source due to its cost-effective advantages and its great potential as an alternative to fossil fuels [2]. It is a renewable energy that is derived from biomass material which is any biological organic matter obtained from plants or animals. Bioenergy is obtained from a broad variety of resources and produced in many diverse routes [3].

Biomass energy sources include but are not limited to terrestrial plants, aquatic plants, timber processing residues, municipal solid wastes, animal dung, sewage sludge, agricultural crop residues and forestry residues. These different types of biomass have to be linked to the various energy flows and conversions in order to meet both renewable energy needs and solve waste management challenges [4]. Bioenergy is one of the most versatile among other renewable energies since it can be made available in solid, liquid and/or gaseous forms [5]. Biogas is one such bioenergy source in the form of a gaseous biofuel. In contrast with other biofuels, biogas production is flexible to different substrates on condition that they are biodegradable. Biogas is produced by the process of anaerobic digestion of biodegradable organic matter. Anaerobic digestion is the breakdown of biomass materials with the aid of bacteria in the absence of oxygen producing a mixture of gases [6]. Production of biogas through the anaerobic digestion process is an environmental friendly process utilizing the increasing amounts of organic wastes produced [7]. This technology reduces greenhouse gas emissions and as such a sustainable form of energy, biogas, a biofuel is obtained [8].

Rozy et al., [9], experimentally investigated the effect of varying physicochemical parameters on biogas production from water hyacinth (WH) in combination with cow dung and obtained enhanced yield parameters. They however, emphasised the need to enhance and optimise methane generation from WH and other such substrates. In anaerobic co-digestion combinations, it is of paramount importance to know the mass ratio of each substrate to be fed in the blend mixture so as to achieve the highest

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possible proportion of methane in the output biogas. In as much as WH is a nuisance to waterways and sources, municipal solid waste (MSW) and cow dung (CD) are as well pausing some detrimental effects to the environment. Anaerobic co-digestion of these bio-materials among others leads to increased biogas yields when compared to mono-digestion of the same due to enhanced bio-degradability, bio-accessibility, and bio-availability among other synergisms in the process reactions [10].

Biogas production can be enhanced by utilisation of high-methane potential substrates, enzymes and microbial addition, optimisation of process conditions and parameters, co-digestion of various substrates, pre-treatment of the feed material and separating the digestion process into phases (multi-stage digestion) [11]. Dependable anaerobic co-digestion modelling is essential to clearly forecast the consequence of blending substrates in a reactor and do away with possible undesirable outcomes from blending combinations established on arbitrary and/or heuristic conclusions. To optimise is to determine the maximum or minimum values of a specified function that is subject to certain constraints [1]. Hagos et al., [12], highlighted that process optimisation and improvement of biogas production still needs more investigations to be done and that the use of modelling and simulation ways can lead to realisation of substantial enhancement of biogas yields. Diverse optimisation approaches are established in literature in a bid to obtain the best reaction conditions, best reaction parameters and best substrate ratios for different feed stocks so as to enhance and optimise the biogas production process. Sreekrishnan et al., [13], also notes that use of additives, recycling of slurry and slurry filtrate, variation of operational parameters like temperature, hydraulic retention time (HRT) and particle size of the substrate and use of fixed film/biofilters are some of the techniques for enhancing biogas production.

The conventional method of optimisation of anaerobic digestion comprise of laboratory batch experiments varying reaction conditions and parameters as well as co-digestion of varied feed stocks to evaluate the digestibility and biogas potential of different substrates. Co-digestion of varied substrates has shown that an improved biogas production potential can be realised as compared to mono-digestion of single substrates [14]. Artificial Neural Networks (ANNs) and Genetic Algorithms (GA) are some of the modern tools that are used to solve complex problems which cannot be unravelled by conventional solutions [15]. Linear programming approaches [16], response surface methodologies [17], as well as simplex-centroid mixture design and central composite design [18], are some of the optimisation approaches which have been applied in anaerobic digestion.

There has been a considerable increase in demand for energy in developing countries like Zimbabwe while the supply and/or generation capacity is lagging behind [19]. As a result consumers are shifting to alternative renewable energy options and also to other available fossil derived and imported fuels such as Liquid Petroleum Gas (LPG). The availability of non-renewable forms of energy such as LPG, derived from fossil fuels will continue to decrease while at the same time their costs will continue to increase [20]. The interchangeability of fuels has to be compared in terms of the Wobbe Index (WI) when considering shifting from one fuel to the other. The Wobbe Index (WI) is an indicator of the interchangeabiity of fuel gasses. It is the key pointer to the replacement of one fuel with another and is very useful in comparing the burning efficiency of fuel gases [21].

This research deduced from previous works/studies that solar PV-Biogas hybrid systems have been developed and optimised to ease the energy demand mainly being fostered by inadequate conventional energy supplies. Nawaz et al. [22], carried out a feasibility study on a solar photovoltaic-biogas hybrid system and also did an optimisation of the same. Kwok et al. [23], investigated the hybridisation of solar, wind and biogas in a bid to optimise energy generation from these renewable energy sources. In some instances the solar PV-Biogas systems have been tied with the grid to minimise energy costs and at the same time ensuring consistent supply of energy at all times [24]. It was also however, realised that not much has been reported and/or researched with respect to integrating optimised biogas systems and LPG for heating, lighting and power purposes. According to the authors' literature survey, no research was found to report on optimised biogas-LPG hybrid systems. In order to meet the growing energy demands and to do away with waste disposal problems, the production of biogas and the respective optimisation of its bio-methane major constituent is of uttermost importance in addition to hybridisation of the energy supply alternatives such as LPG [25].

In the authors' previous paper [26], a model for determining biogas production potential from water hyacinth (WH), municipal solid watse (MSW), and cow dung (CD) was presented and an optimisation of the co-digestion mixing ratios of these substrates was carried out in a bid to obtain the highest possible amount of biogas from the co-digestion mixture. The same substrates are used in this present work. However, the model developed and being reported in its own novel way in this current paper differs with the previous one in the following ways:

- Seasonal variations of the substrates are taken into consideration in the modelling and optimisation.

- Methane is maximised whilst carbon dioxide, ammonia, and hydrogen sulphide are minimised to obtain more methane in the biogas mixture and thus improving the quality of the biogas produced.

- The enhanced biogas produced is hybridised with Liquid Petroleum Gas (LPG) to supply some gas demand

It is hereby being emphasised that according to the authors' knowledge and research investigations, no previous studies are reported to have looked at the effect of substrate/feedstock seasonal variations on co-digestion and at the same time incorporate
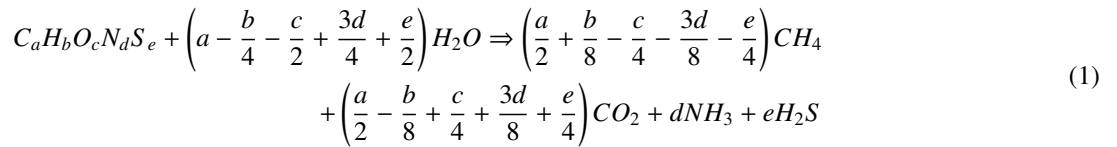
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[1]https://www.dictionary.com

the same in modelling and optimisations. As such, this current research is unique and innovative in that regard and the findings are one of their own kind, contributing immensely to the anaerobic digestion research niche. The purpose and contribution of this current work is the development of a model which facilitates the attainment of high quality biogas constituted of a high proportion of methane while at the same time taking into consideration the seasonality changes of the substrates. Consequently, the resultant co-digestion substrate blending ratios vary for each month and so does the biogas yield unlike in the previous work where a single average blend ratio and an annual average biogas output was obtained. The high quality optimised biogas produced is channeled towards the gas requirements of a community in a hybrid system with Liquid Petroleum Gas (LPG) where LPG meets the rest of the demand not met by the biogas. This work contributes to the reduction of reliance on imported energy and adds great value by supplying a high quality bio-methane gas thereby substituting a great proportion of LPG consequently reducing import costs as well as minimising environmental pollution. The model and the method used herein this study are general enough for use in many countries, and can be applied with many other varied biomass resources.

Section 2 of this paper gives the modelling and optimisation materials and methods, section 3 gives a case study, section 4 gives the results & discussion and section 5 concludes the paper. The algorithm is given as the appendix.
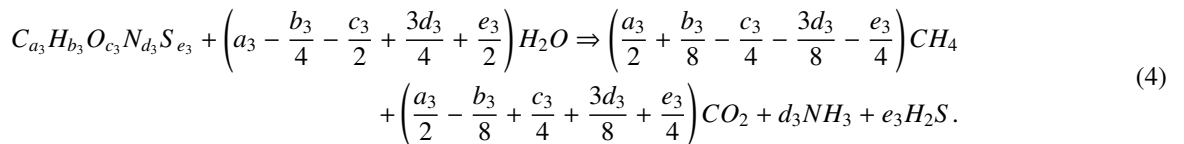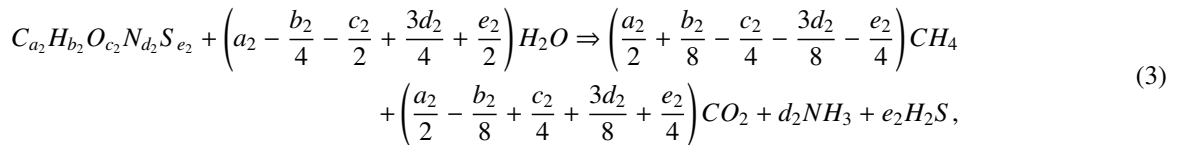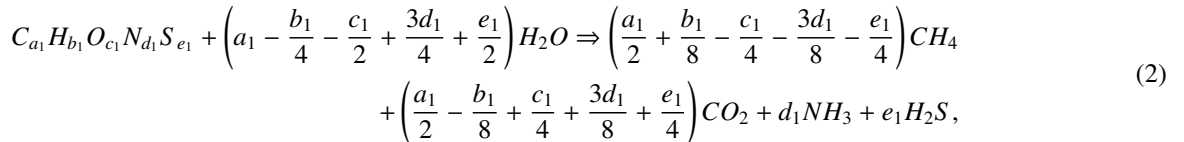
2. Modelling and Optimisation

2.1. Problem formulation

The Buswell & Mueller modified equation [27] shown in (1) is herein taken as the biogas production reaction equation.

$$C_aH_bO_cN_dS_e + \left(a - \frac{b}{4} - \frac{c}{2} + \frac{3d}{4} + \frac{e}{2}\right)H_2O \Rightarrow \left(\frac{a}{2} + \frac{b}{8} - \frac{c}{4} - \frac{3d}{8} - \frac{e}{4}\right)CH_4$$
$$+ \left(\frac{a}{2} - \frac{b}{8} + \frac{c}{4} + \frac{3d}{8} + \frac{e}{4}\right)CO_2 + dNH_3 + eH_2S \tag{1}$$

where the constants a, b, c, d and e in $C_aH_bO_cN_dS_e$ are obtained from the ultimate analysis of each of the elements devided by the relative atomic mass (Ar) of each of the elements as depicted in Appendix A

For the three materials under co-digestion in this study equations (2), (3) and (4) are formulated to represent the biogas generation reactions from WH, MSW and CD respectively [26].

$$C_{a_1}H_{b_1}O_{c_1}N_{d_1}S_{e_1} + \left(a_1 - \frac{b_1}{4} - \frac{c_1}{2} + \frac{3d_1}{4} + \frac{e_1}{2}\right)H_2O \Rightarrow \left(\frac{a_1}{2} + \frac{b_1}{8} - \frac{c_1}{4} - \frac{3d_1}{8} - \frac{e_1}{4}\right)CH_4$$
$$+ \left(\frac{a_1}{2} - \frac{b_1}{8} + \frac{c_1}{4} + \frac{3d_1}{8} + \frac{e_1}{4}\right)CO_2 + d_1NH_3 + e_1H_2S, \tag{2}$$

$$C_{a_2}H_{b_2}O_{c_2}N_{d_2}S_{e_2} + \left(a_2 - \frac{b_2}{4} - \frac{c_2}{2} + \frac{3d_2}{4} + \frac{e_2}{2}\right)H_2O \Rightarrow \left(\frac{a_2}{2} + \frac{b_2}{8} - \frac{c_2}{4} - \frac{3d_2}{8} - \frac{e_2}{4}\right)CH_4$$
$$+ \left(\frac{a_2}{2} - \frac{b_2}{8} + \frac{c_2}{4} + \frac{3d_2}{8} + \frac{e_2}{4}\right)CO_2 + d_2NH_3 + e_2H_2S, \tag{3}$$

$$C_{a_3}H_{b_3}O_{c_3}N_{d_3}S_{e_3} + \left(a_3 - \frac{b_3}{4} - \frac{c_3}{2} + \frac{3d_3}{4} + \frac{e_3}{2}\right)H_2O \Rightarrow \left(\frac{a_3}{2} + \frac{b_3}{8} - \frac{c_3}{4} - \frac{3d_3}{8} - \frac{e_3}{4}\right)CH_4$$
$$+ \left(\frac{a_3}{2} - \frac{b_3}{8} + \frac{c_3}{4} + \frac{3d_3}{8} + \frac{e_3}{4}\right)CO_2 + d_3NH_3 + e_3H_2S. \tag{4}$$

A MATLAB toolbox, the Optimisation Interface (OPTI) [28] was used to construct the optimisation problem and the Solving Constraint Integer Programs (SCIP) solver was applied to solve the formulated optimisation problem. The objective is to improve the quality of biogas produced by maximising methane while at the same time minimising carbon dioxide, ammonia and hydrogen sulphide. The optimised biogas is then integrated with LPG in a hybrid system for supplying gas to the community. The objective function and the constraints are inputed into the optimisation model which then gives the number of moles for WH, MSW and CD respectively for each month which are then used for computing the monthly Stoichiometric masses of each of the co-digestion materials to be fed into the digester to obtain the optimum methane.

2.2. Objective function

The objective function is expressed as

$$\sum_{j=1}^{N}\sum_{i=1}^{3} G_i(x_{i,j}) \tag{5}$$

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where j is time in months from January through to December, N is number of months which is equal to 12. i is the substrate material index. $i = 1$ for substrate material WH, $i = 2$ indicates substrate material MSW, and $i = 3$ indicates substrate material CD. $x_{i,j}$ are the number of moles of substrate material i in the jth month. $G_i(x_{i,j})$ is the monthly biogas produced from material i in month j. For a particular month j, the total biogas (G_{tot}) produced is expressed as:

$$G_{tot,j} = G_1(x_{1,j}) + G_2(x_{2,j}) + G_3(x_{3,j}), \tag{6}$$

where

$$G_1(x_{1,j}) = (22.4 \times 10^{-3}) \times \left(\frac{CO_{2_{1,j}} + NH_{3_{1,j}} + H_2S_{1,j} - CH_{4_{1,j}}}{Mr_{WH}} \right), \tag{7}$$

$$G_2(x_{2,j}) = (22.4 \times 10^{-3}) \times \left(\frac{CO_{2_{2,j}} + NH_{3_{2,j}} + H_2S_{2,j} - CH_{4_{2,j}}}{Mr_{MSW}} \right), \tag{8}$$

$$G_3(x_{3,j}) = (22.4 \times 10^{-3}) \times \left(\frac{CO_{2_{3,j}} + NH_{3_{3,j}} + H_2S_{3,j} - CH_{4_{3,j}}}{Mr_{CD}} \right). \tag{9}$$

In equations (7), (8 and (9); $CO_{2_{\{1,2,3\},j}}$, $NH_{3_{\{1,2,3\},j}}$, $H_2S_{\{1,2,3\},j}$ and $CH_{4_{\{1,2,3\},j}}$ are the number of moles of carbon dioxide, ammonia, hydrogen sulphide and methane for WH, MSW and CD respectively and Appendix B shows how to determine these moles. Mr_{WH}, Mr_{MSW} and Mr_{CD} are the relative molecular masses of WH, MSW and CD respectively and these are as denoted in Appendix B as well. These relative molecular masses are assumed to be constant and as such are not affected by seasonal variations.

2.3. Constraints

The objective function is subject to the constraints of carbon to nitrogen ratio ($C : N$) as shown in inequalities (10), the reactor volume (V_R) constraint as shown in equation (11) and gas demand satisfaction constraint as stated in equation (21).

$$(C : N)^{min} \leqslant \left(\frac{a_1 + a_2 + a_3}{d_1 + d_2 + d_3} \right) \sum_{i=1}^{3} x_{i,j} \leqslant (C : N)^{max}, \tag{10}$$

where a_1, a_2 and a_3 are the WH, MSW and CD carbon ultimate compositions respectively; and d_1, d_2, and d_3 are the WH, MSW and CD nitrogen ultimate compositions respectively.

$$\left[\left(V_{WH,j} + \left(\frac{(22.4 \times 10^{-3}) \times H_2O_{1,j}}{Mr_{WH}} \times R_{H_2O_{(1,j)}} \right) \right) x_{1,j} + \left(V_{MSW,j} + \left(\frac{(22.4 \times 10^{-3}) \times H_2O_{2,j}}{Mr_{MSW}} \times R_{H_2O_{(2,j)}} \right) \right) x_{2,j} \right.$$
$$\left. + \left(V_{CD,j} + \left(\frac{(22.4 \times 10^{-3}) \times H_2O_{3,j}}{Mr_{CD}} \times R_{H_2O_{(3,j)}} \right) \right) x_{3,j} \right] = V_R, \tag{11}$$

where

$$H_2O_{1,j} \ (moles) = \left(a_1 - \frac{b_1}{4} - \frac{c_1}{2} + \frac{3d_1}{4} + \frac{e_1}{2} \right) x_{1,j}, \tag{12}$$

$$H_2O_{2,j} \ (moles) = \left(a_2 - \frac{b_2}{4} - \frac{c_2}{2} + \frac{3d_2}{4} + \frac{e_2}{2} \right) x_{2,j}, \tag{13}$$

$$H_2O_{3,j} \ (moles) = \left(a_3 - \frac{b_3}{4} - \frac{c_3}{2} + \frac{3d_3}{4} + \frac{e_3}{2} \right) x_{3,j}, \tag{14}$$

$$V_{WH,j} \ (volume \ of \ WH \ in \ month \ j) = \frac{m_{WH,j}}{\rho_{WH}}, \tag{15}$$

$$m_{WH,j} = Mr_{WH} \times n_{WH,j}, \tag{16}$$

ρ_{WH} is the density of water hyacinth.

$$V_{MSW,j} \ (volume \ of \ MSW \ in \ month \ j) = \frac{m_{MSW,j}}{\rho_{MSW}}, \tag{17}$$

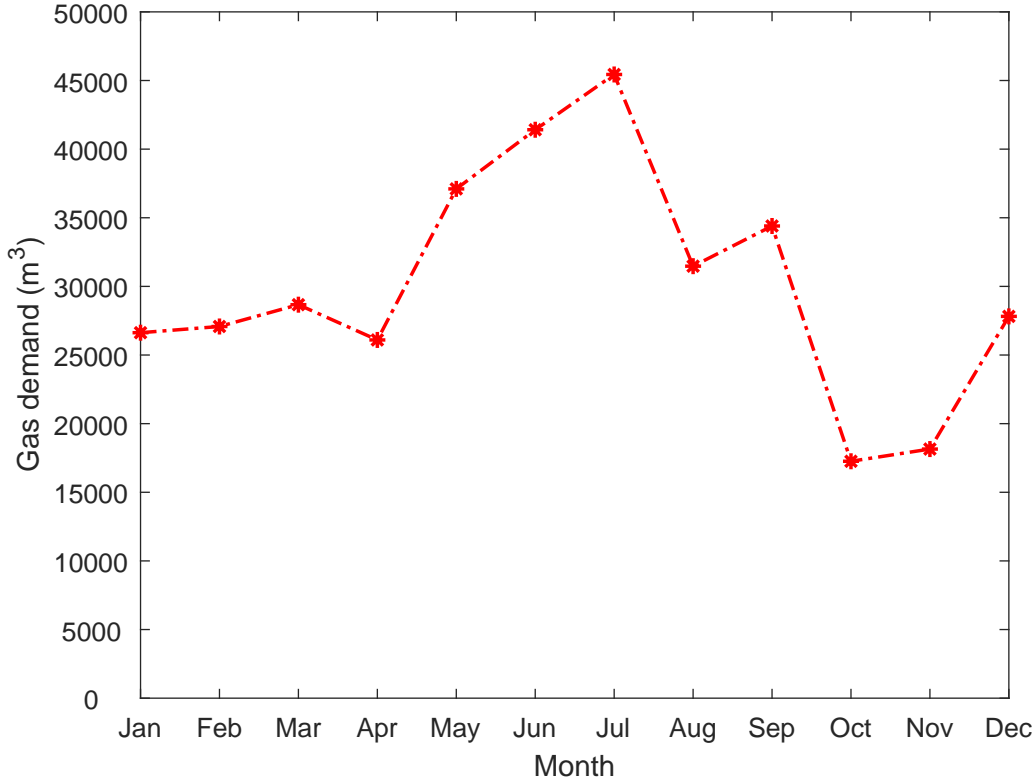$$m_{MSW,j} = Mr_{MSW} \times n_{MSW,j}, \tag{18}$$

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Figure 1: Monthly gas demand

ρ_{MSW} is the density of municipal solid waste.

$$V_{CD,j} \; (volume \; of \; CD \; in \; month \; j) = \frac{m_{CD,j}}{\rho_{CD}}, \tag{19}$$

$$m_{CD,j} = Mr_{CD} \times n_{CD,j}, \tag{20}$$

ρ_{CD} is the density of cow dung. $R_{H_2O_{(1,j)}}$, $R_{H_2O_{(2,j)}}$ and $R_{H_2O_{(3,j)}}$ are the ratios or the proportions of water to be added to the WH, MSW and CD substrates respectively in order to attain the required total solids content and V_R is the reactor volume in m^3.

$$\left[(22.4 \times 10^{-3}) \times \left(\left(\frac{CO_{2_{1,j}} + NH_{3_{1,j}} + H_2S_{1,j} - CH_{4_{1,j}}}{Mr_{WH}} \right) x_{1,j} + \left(\frac{CO_{2_{2,j}} + NH_{3_{2,j}} + H_2S_{2,j} - CH_{4_{2,j}}}{Mr_{MSW}} \right) x_{2,j} \right. \right.$$
$$\left. \left. + \left(\frac{CO_{2_{3,j}} + NH_{3_{3,j}} + H_2S_{3,j} - CH_{4_{3,j}}}{Mr_{CD}} \right) x_{3,j} \right) \right] \tag{21}$$
$$+ LPG_j = \text{Gas demand}_j,$$

where LPG is imported energy that balances up the demand not met by the biogas; bio-methane in this case and the gas demand is depicted as monthly consumption as shown in Fig. 1. WI for LPG is around 85 MJ/m^3 and that of methane from biogas is 36 MJ/m^3 [29]. This shows that LPG and methane from biogas cannot be directly interchanged. According to Ananthakrishnan et al., [30], 1 m^3 of biogas is equivalent to 0.45 kg of LPG. As such when substituting LPG with biogas this factor was taken into consideration.

Inequalities (22), (23) and (24) show the the lower and upper bounds constraints for WH, MSW and CD respectively.

$$V_{WH,j}^{min} \leqslant V_{WH,j} \leqslant V_{WH,j}^{max}, \tag{22}$$

$$V_{MSW}^{min} \leqslant V_{MSW,j} \leqslant V_{MSW}^{max}, \tag{23}$$

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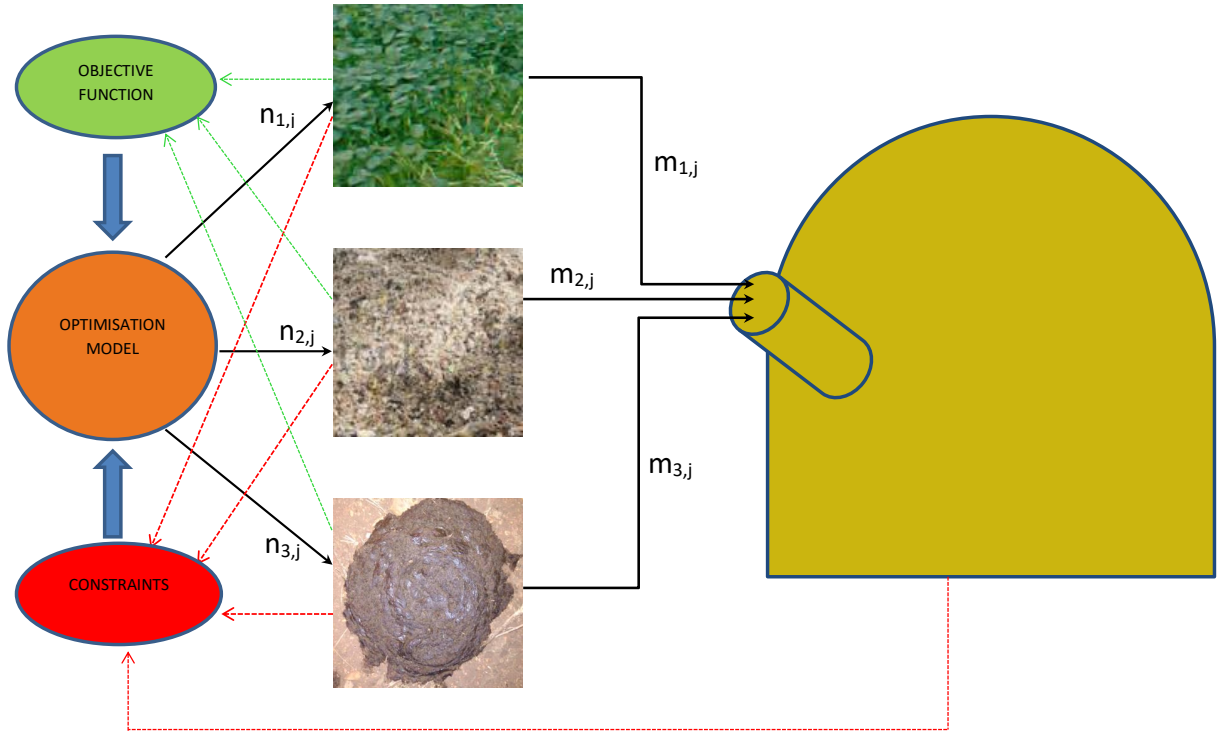
Figure 2: Model layout diagram

Table 1: Case study data values

Parameters	C	H	O	N	S	ρ (kgm^{-3})	Source
WH	33.13	4.35	29.71	1.66	0.37	85.00	[31]
MSW	48.00	6.40	37.60	2.60	0.40	217.50	[32]
CD	45.32	5.87	27.38	5.16	0.45	400.00[a]	[33]

[a]https://www.aqua-calc.com/page/density-table/substance/manure

$$V_{CD}^{min} \leqslant V_{CD,j} \leqslant V_{CD}^{max}. \tag{24}$$

The modelling and optimisation is summarised in Figure 2. The function to be optimised and its respective constraints are fed into the optimisation model. The optimisation model inturn gives the respective optimal number of moles $x_{i,j}$ ($x_{1,j}$, $x_{2,j}$ and $x_{3,j}$) for WH, MSW and CD respectively. In Figure 2, $n_{1,j} \triangleq x_{1,j}$, $n_{2,j} \triangleq x_{2,j}$ and $n_{3,j} \triangleq x_{3,j}$. These Stoichiometric moles obtainable from the optimisation model are then used in computations of the respective optimal substrate mass blend ratios m_1, m_2 and m_3 for WH, MSW and CD to be fed to the digester/reactor, where $m_{i,j} = x_{i,j} \times Mr_i$. The detailed algorithm is given in Appendix C.

3. Case study

The WH substrate is obtained from Lake Chivero in Harare - Zimbabwe. MSW is obtained from Norton, an urban town in Zimbabwe. Cow dung is obtained from cattle in the Norton part of Chegutu district. Fig. 3 gives the monthly (seasonal) available substrate resources for WH, MSW and CD. Table 1 gives the Case study data values used in this research.
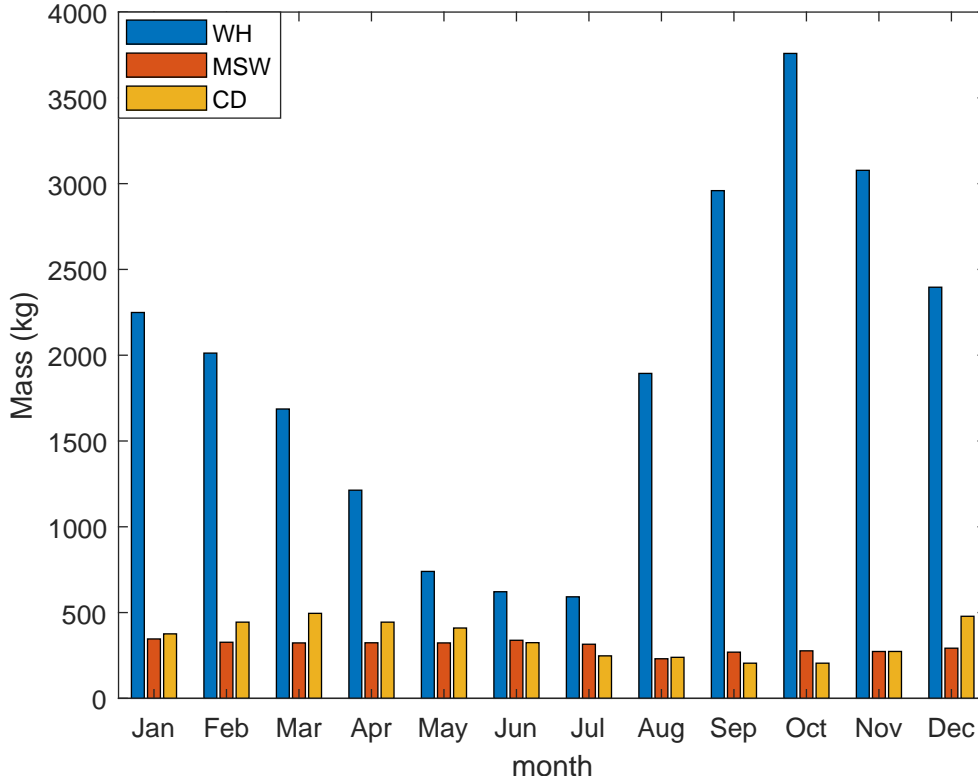
Figure 3: Substrate monthly resources

$$
lb = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{4N \times 1} ,
\tag{25}
$$

where $N = 12$. The lower bounds (lb) are as shown in equation (25) and the upper bounds (ub) are as shown in Table 2. These lower and upper bounds are congruent to constraint inequalities (22), (23) and (24).

4. Results and discussion

The SCIP solver in conjunction with 'Spatial Branch and Bound using IPOPT and SoPlex' algorithm gave the global sub-optimal mole ratios for the co-digestion of water hyacinth, municipal solid waste and cow dung as shown in Table 3. Using the results in Table 3 and applying the Stoichiometric relationship; *mass = number of moles * molar mass* [34], substrate mass blending ratios presented in Table 4 are arrived at. The mass blending ratios in Table 4 translate to optimal percentage substrate mass blend ratios shown in Table 5 for the co-digestion of water hyacinth, municipal solid waste and cow dung. Fig. 4 which is derived from Table 5 shows a graphical presentation of the optimal percentage substrate mass blend ratios which maximises the methane component in the output biogas yield.

Fig. 5 shows the monthly optimised biogas production. Summation of the monthly biogas potential yields gives an annual total of 38 465.68 m^3. This is an increase by 174.58 % when compared to an annual average of 14 008.8 m^3 from the previous study [26], which did not take into account seasonality changes in co-odigestion substrate availability. The results of this study agrees with Lovrak et al. [35], who highlighted that there is a great need to consider seasonalities when evaluating the biogas potential of lignocellulosic agricultural wastes in a study in which they proposed a GIS-based technique for assessing the spatial distribution of biogas generation capacity while considering seasonality variations in feedstock production. Shukla et al. [36] also reported getting the highest biogas yields in summer months in a study in which they investigated the effect of seasonal variation on biogas production from different food wastes. This present study further highlights that seasonality changes have to be considered not

Table 2: Upper bound *(ub)* limits

Month	WH (*moles*)	MSW (*moles*)	CD (*moles*)	LPG (*moles*)
January	374 911.10	45 463.84	62 661.13	50 000
February	335 446.77	42 938.07	74 054.06	50 000
March	281 143.86	42 432.92	82 598.76	50 000
April	202 256.25	42 533.95	74 054.06	50 000
May	123 327.60	42 432.92	68 357.60	50 000
June	103 592.28	44 453.53	54 116.43	50 000
July	98 660.82	41 422.61	41 299.38	50 000
August	315 714.61	30 309.23	39 875.26	50 000
September	493 304.08	35 360.76	34 178.80	50 000
October	626 611.42	36 371.07	34 178.80	50 000
November	513 036.24	35 865.92	45 571.73	50 000
December	399 464.23	38 391.69	79 750.53	50 000

Table 3: Monthly mole ratios

Month	WH (*moles*)	MSW (*moles*)	CD (*moles*)	LPG (*moles*)
January	161 744.86	45 463.84	0.00	29773.56
February	164 747.61	42 938.07	0.00	30339.76
March	165 348.16	42 432.92	0.00	31950.29
April	165 228.05	42 533.95	0.00	29377.11
May	123 327.6	42 432.92	24 748.64	39807.65
June	103 592.28	44 453.53	34 957.24	43788.13
July	98 660.82	41 422.61	39 983.92	47833.00
August	179 761.36	30 309.23	0.00	35299.08
September	173 755.86	35 360.76	0.00	38000.12
October	172 554.76	36 371.07	0.00	20814.42
November	173 155.31	35 865.92	0.00	21726.42
December	170 152.56	38 391.69	0.00	31279.86

Table 4: Monthly co-digestion masses

Month	WH (kg)	MSW (kg)	CD (kg)	Total mass (kg)
January	970.47	345.53	0.00	1 315.99
February	988.49	326.33	0.00	1 314.81
March	992.09	322.49	0.00	1 314.58
April	991.37	323.26	0.00	1 314.63
May	739.97	322.49	148.49	1 210.95
June	621.55	337.85	209.74	1 169.14
July	591.96	314.81	239.90	1 146.68
August	1 078.57	230.35	0.00	1 308.92
September	1 042.54	268.74	0.00	1 311.28
October	1 035.33	276.42	0.00	1 311.75
November	1 038.93	272.58	0.00	1 311.51
December	1 020.92	291.78	0.00	1 312.69

Table 5: Monthly co-digestion percentage blend ratios

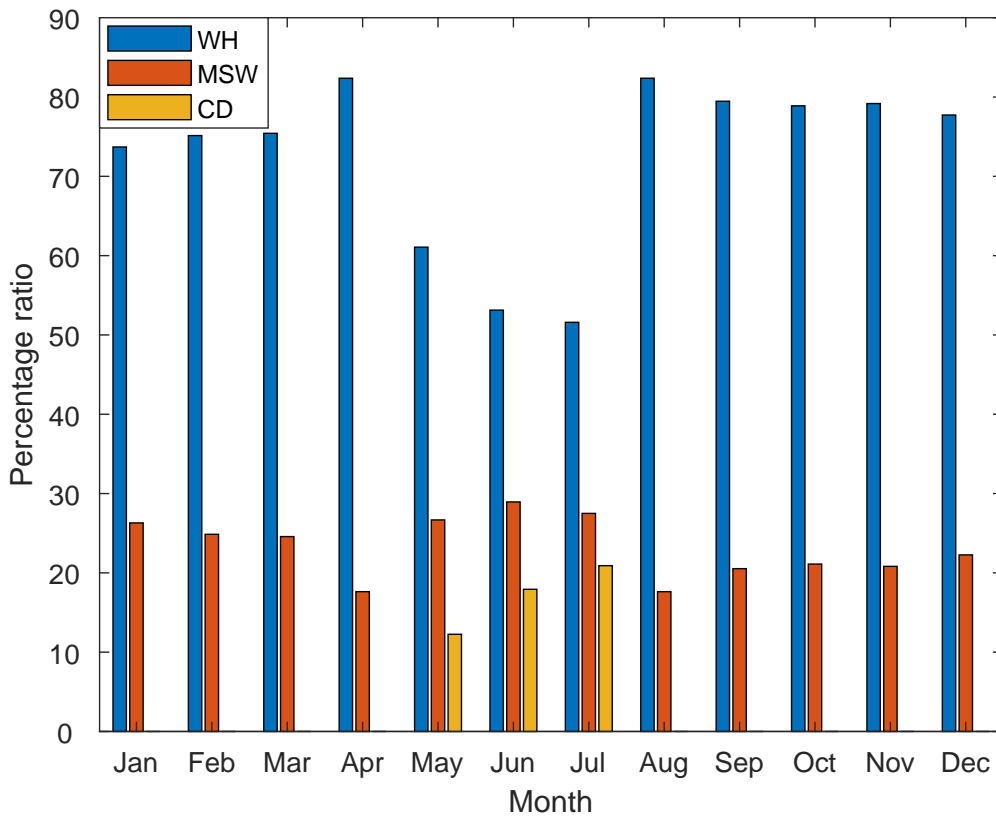| Month	% ratio ($WH : MSW : CD$)
January	73.74 : 26.26 : 0.00
February	75.18 : 24.82 : 0.00
March	75.47 : 24.53 : 0.00
April	75.41 : 24.59 : 0.00
May	61.11 : 26.63 : 12.26
June	53.16 : 28.90 : 17.94
July	51.62 : 27.45 : 20.92
August	82.40 : 17.60 : 0.00
September	79.51 : 20.49 : 0.00
October	78.93 : 21.07 : 0.00
November	79.22 : 20.78 : 0.00
December	77.77 : 22.23 : 0.00

Figure 4: Combined substrates monthly substrate feed ratios

only for lignocellulosic agricultural wastes but for all biomass feedstocks especially when anaerobic co-digestion is the biomass-to-energy conversion route being applied. This implies that storage arrangements have to be put in place for accummulating the feedstocks in times of plenty for later use in times when the resources are insufficient and/or when demand is very high.

Biogas production is higher in summer months than in winter months owing to the overally higher co-digestion substrate quantities in the summer season and the opposite is true for the winter season. This trend is also attributed to by the high light intensities in the summer months which facilitate enhanced photosynthesis consequently generating more sugars which are a key component in the biochemical reactions of biogas production. The findings of this study agrees with D'Este et al. [37], who also reported higher methane yields in summer months in a study which focussed on seasonal and spatial variations of algae as a potential biomass feedstock for biogas production. In this present study, the optimised biogas generated is channeled to feed part of the community's gas demand and as such the amount of LPG imports are reduced. The reduction in LPG quantities implies that there will be reduced carbon emissions since biogas is from renewable sources and is regarded as carbon neutral. Fig. 6 shows the resultant LPG gas needed to satisfy the demand in the biogas-LPG gas hybrid system.

The demand profile (as seen in Fig. 1) shows highest gas consupmption in winter and lowest gas consumption in summer. This is a typical demand profile analogous to the electricity demand profile which shows similar trends for the winter and summer seasons [38]. The optimised biogas is more of bio-methane since it constitutes of maximised methane (CH_4) component in the biogas and minimised carbon dioxide (CO_2), ammonia (NH_3) and hydrogen sulphide (H_2S). Fig. 7 shows the effect of optimised biogas-LPG hybridisation on gas demand. It can be deduced that methane-optimised biogas production followed by subsequent hybridisation reduces the LPG gas demand.

Table 6 shows the monthly LPG gas costs before optimisation and hybridisation as well monthly LPG gas costs after hybridisation and optimisation.

Fig. 8 shows the effect of optimisation and hybridisation on LPG costs. The monthly percentage cost savings are shown in this figure and it ranges from 6.93 % to 18.24 %.

May, June and July are characterised by the least cost savings due to the high gas demand during these winter months mainly for the purposes of heating and cooking. October and November have huge cost savings due to the lower gas demand during the summer season. Globally, only a few countries are priviledged to be endowed with oil and petroleum resources, whereas the bulky of the nations rely on importing the same to cater for their transportation, industial, agricultural and domestic fuel requirements. The results of this study are of critical importance to such a dire pillar of the economy in providing a home-grown
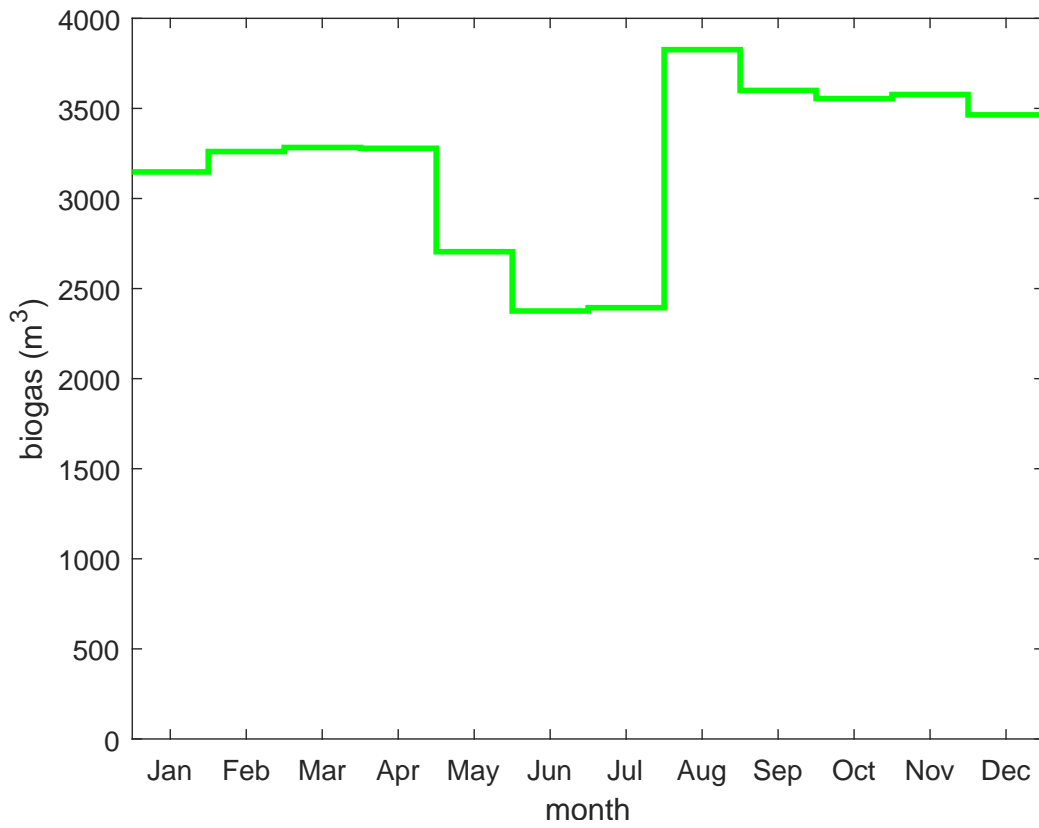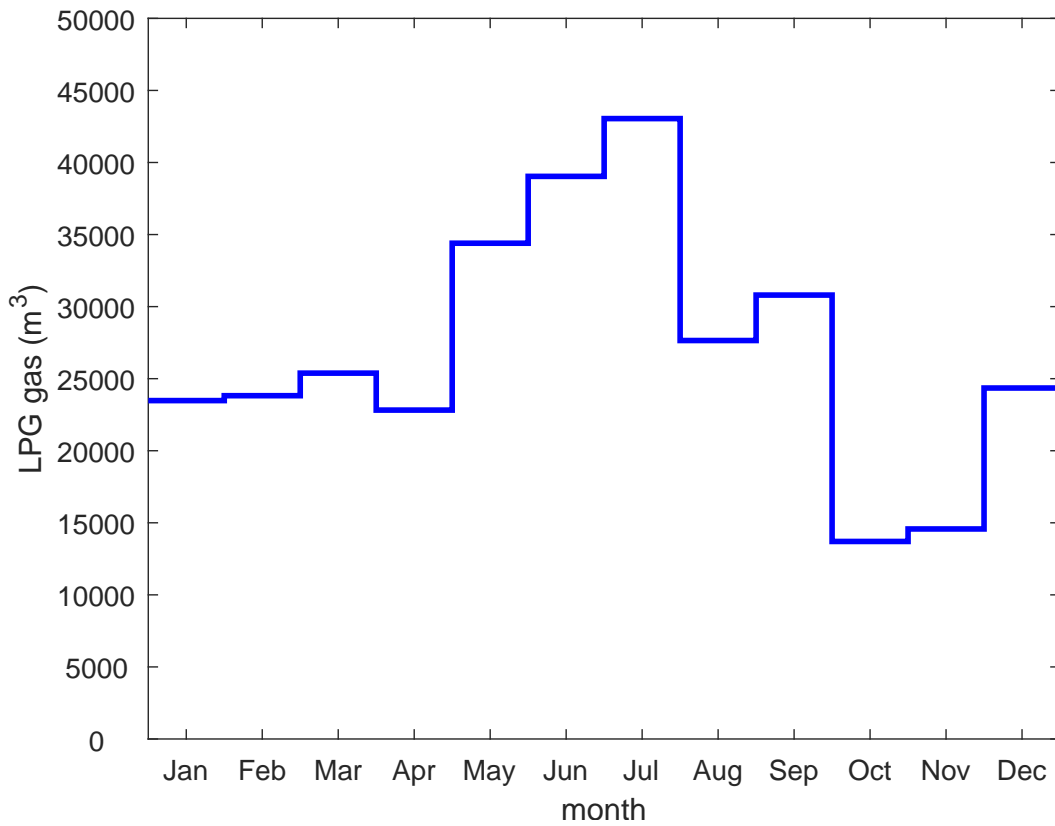
Figure 5: Optimised biogas production

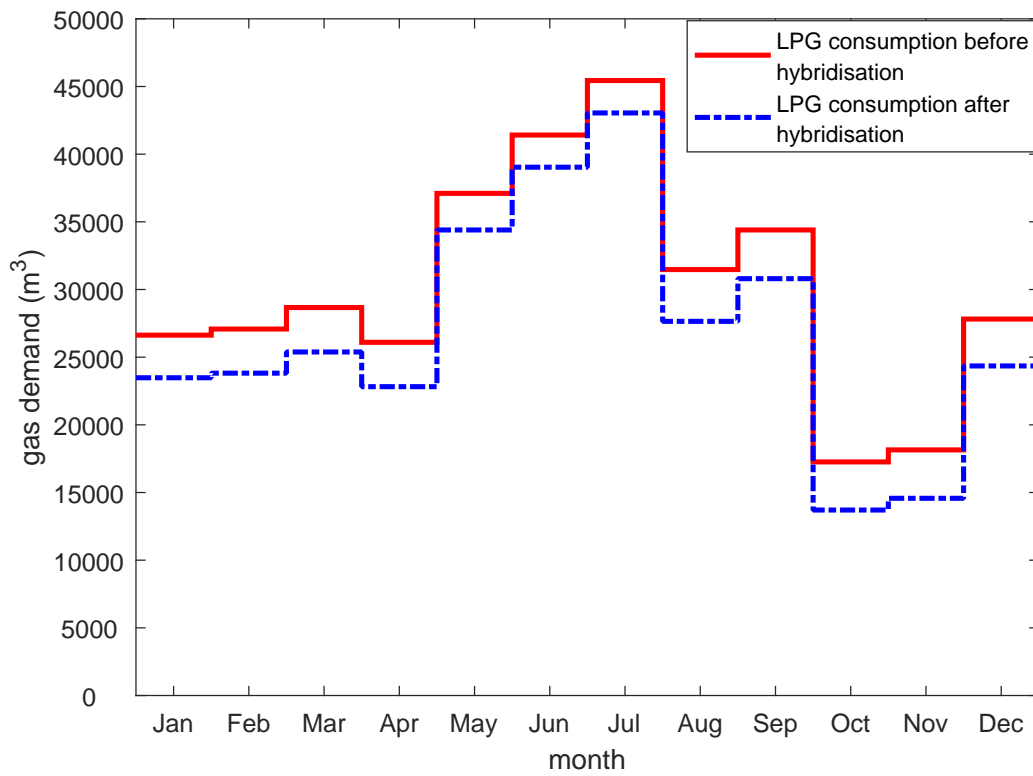Figure 6: Resultant LPG gas needed

Figure 7: Hybridisation effect on gas demand

Table 6: LPG costs before and after hybridisation

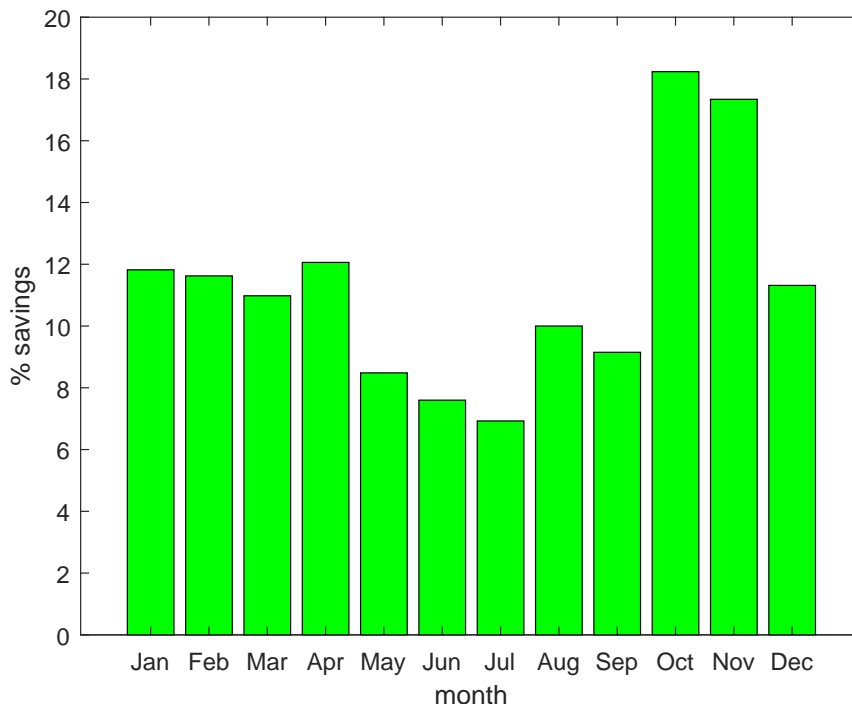| Month	LPG costs before hybridisation ($)	LPG costs after hybridisation ($)
January	5257.96	4636.43
February	5347.43	4725.90
March	5661.00	5039.47
April	5153.76	4532.23
May	7326.86	6705.33
June	8177.89	7556.36
July	8973.05	8351.51
August	6215.07	5593.54
September	6793.13	6171.60
October	3408.35	2786.82
November	3583.97	2962.44
December	5492.86	4871.33

Figure 8: Monthly cost savings

optimal solution to fuel challenges.The methane-optimised biogas will go a long way in substituting fossil derived fuels which are posing detrimental climatic effects by way of emmiting hazadous pollutants. Hybridising the optimised biogas with other conventional fuels such as LPG will guarantee continuous availability of the fuel and meeting of the demand at all times.

In-depth research on co-digestion for increasing the yield of biogas and meeting the requirements of load over the whole year have been undertaken in previous studies. However, the approaches taken to ascertaining the co-substrate blend ratios has mainly been uninformed experimental guesses whereby two or three substrates are apportioned into certain proportions and the mixture that gives the highest biogas was taken be having the optimal blend ratio. This is the optimal ratio of what has been put in place but not necessarily the optimal ratios for the substrates under investigation. More so, the modelling and optimisation strategies employed in some few studies which employed this approach did not consider the variability of the different substrate quantities across seasons of the year. This work contributes the novel aspect of optimal monthly substrate mix ratios and the incorporation of biomass co-digestion feedstock quantity seasonality changes into the modelling and optimisation of anaerobic digestion research domain.

This research is unique in that it incorporates substrate seasonal fluctuations and enhances biogas quality by maximising the principal preferred methane component of biogas while simultaneously minimising the undesired components; carbon dioxide, hydrogen sulphide, and ammonia in the modelling and optimisation. The concept of integrating the methane-optimised biogas in a hybrid system with liquid petroleum gas to supply a gas demand is another unique contribution of this study. To date, to the authors' best knowledge, the methodology and approach taken in this current work has neither been published nor reported in the previous works by any other researchers and as such is a unique contribution to the biogas fraternity. A case study is used to validate the proposed modelling and optimisation and the results show that the objective of the research is achieved and the findings are the first of their own kind.

The model developed herein this paper and its accompanying optimisation and hybridisation methodologies are not limited to the case study location, it is applicable to other geographical locations world-over having varied seasonal changes. Numerous bio-degradable biomass materials from varied sources can also be used as co-digestion substrates with this model. It is also possible to hybridise the methane-optimised biogas with any other conventional or non-conventional fuel in a bid to reach some meaningful trade-off between fuel costs, demand satisfaction and environmental consequenses.

5. Conclusion

This paper brings in the concepts of co-digestion, modelling and optimisation to the anaerobic digestion research niche and introduces the novel aspect of biomass co-digestion feedstock seasonal variations into the modelling and optimisation. Further, in the objective function, unwanted biogas components (CO_2, H_2S and NH_3) have been minimised and the major desired component (CH_4) has been maximised. The incorporation of seasonal variations and the control of biogas quality is unique to this paper.

Hybridisation of biogas with other energy sources such as solar have been investigated in previous works, however, hybridisation of biogas with other conventional fuels such as liquid petroleum gas (LPG) is still at infancy and the authors could hardly find any reported research works on this. Renewable energy hybrid systems in combination with conventional energy sources have the potential to bring about a huge difference to energy transformation in developing countries. Incorporation of modelling and optimisation in addition to hybridisation of these systems leads to enhanced energy yields, reduction in energy costs as well as improved environmental sustainability. Biogas demand is higher in winter months than in summer months due to increased heating requirements during this period. More cost savings were realised in the summer season than in the winter season in a case study as more biogas was produced in summer than in winter. This study concludes that the employment of mathematical analytic tools in combination with modelling and optimisation and the incoporation of seasonality changes in substrate availability into the modelling and optimisation of biogas production in co-digestions increases the overal biogas yields. It is hereby being emphasised that the hybridisation of the optimally generated biogas with conventional fuels such as liquid petroleum gas goes a long way in the reduction of fuel import costs and meeting of demand. The model developed herein this work can be applied with any other bio-degradable materials in co-digestion combinations and the methodology is applicable in other countries with the same or different geographical and environmental conditions.

References

[1] L. Mead, Renewables report: Heating, cooling, transport lag behind power sector in energy transformation, Tech. rep., International Institute for Sustainable Development (IISD) (2018).

[2] SANEDI, Appraisal of implementation of fossil fuel and renewable energy hybrid technologies in South Africa, Tech. rep., South African National Energy Development Institute (2017).

[3] A. Faaij, Modern biomass conversion technologies, Mitigation and Adaptation Strategies for Global Change 11 (2) (2006) 343–375.

[4] P. Lens, B. Hamelers, H. Hoitink, W. Bidlingmaier, Resource recovery and reuse in organic solid waste management, Integrated Environmental Technology Series, IWA Publishing, 2004.

[5] F. Rosillo-Calle, J. Woods, The biomass assessment handbook, 2nd Edition, Routledge, New York, 2012.

[6] M. C. Gould, Chapter 18 - Bioenergy and anaerobic digestion, in: Bioenergy, Academic Press, Boston, 2015, pp. 297–317.

[7] H. D. Matthews, N. P. Gillett, P. A. Stott, K. Zickfeld, The proportionality of global warming to cumulative carbon emissions, Nature 459 (7248) (2009) 829.

[8] J. L. Sawin, F. Sverrisson, K. Seyboth, R. Adib, H. E. Murdock, C. Lins, et al., Renewables 2016 global status report. Key findings., Tech. rep., International Atomic Energy Agency (2016).

[9] R. Rozy, R. A. Dar, U. G. Phutela, Optimization of biogas production from water hyacinth (eichhornia crassipes), Journal of Applied and Natural Science 9 (4) (2017) 2062–2067.

[10] L. M. Colla, A. C. F. Margarites, A. Decesaro, F. G. Magro, N. Kreling, A. Rempel, et al., Waste biomass and blended bioresources in biogas production, in: Improving Biogas Production, Springer, 2019, pp. 1–23.

[11] D. Divya, L. Gopinath, P. M. Christy, A review on current aspects and diverse prospects for enhancing biogas production in sustainable means, Renewable and Sustainable Energy Reviews 42 (2015) 690–699.

[12] K. Hagos, J. Zong, D. Li, C. Liu, X. Lu, Anaerobic co-digestion process for biogas production: Progress, challenges and perspectives, Renewable and Sustainable Energy Reviews 76 (2017) 1485–1496.

[13] T. Sreekrishnan, S. Kohli, V. Rana, Santosh, Y. Gupta, Enhancement of biogas production from solid substrates using different techniques–a review, Bioresource Technology 95 (1) (2004) 1–10.

[14] A. Maragkaki, I. Vasileiadis, M. Fountoulakis, A. Kyriakou, K. Lasaridi, T. Manios, Improving biogas production from anaerobic co-digestion of sewage sludge with a thermal dried mixture of food waste, cheese whey and olive mill wastewater, Waste Management 71 (2018) 644–651.

[15] H. A. Qdais, K. B. Hani, N. Shatnawi, Modeling and optimization of biogas production from a waste digester using artificial neural network and genetic algorithm, Resources, Conservation and Recycling 54 (6) (2010) 359–363.

[16] A. Gil, J. Siles, P. Márquez, M. Gutiérrez, M. Martín, Optimizing the selection of organic waste for biomethanization, Environmental Technology 40 (5) (2019) 564–575.

[17] X. Lu, W. Jin, S. Xue, X. Wang, Effects of waste sources on performance of anaerobic co-digestion of complex organic wastes: taking food waste as an example, Scientific Reports 7 (1) (2017) 15702.

[18] X. Wang, G. Yang, F. Li, Y. Feng, G. Ren, X. Han, Evaluation of two statistical methods for optimizing the feeding composition in anaerobic co-digestion: Mixture design and central composite design, Bioresource Technology 131 (2013) 172–178.

[19] C. Wolfram, O. Shelef, P. Gertler, How will energy demand develop in the developing world?, Journal of Economic Perspectives 26 (1) (2012) 119–38.

[20] S. Odngam, N. Khaewnak, T. Dolwichai, J. Srisertpol, A comparative study on gasoline, LPG and biogas affecting the dynamic responses of SI Engine, in: Future Information Technology, Springer, 2014, pp. 927–932.

[21] N. Pannucharoenwong, A. Worasaen, C. Benjapiyaporn, J. Jongpluempiti, P. Vengsungnle, Comparison of bio-methane gas Wobbe Index in different animal manure substrate, Energy Procedia 138 (2017) 273–277.

[22] M. A. Nawaz, A. Raheem, R. Shakoor, Z. Anwar, Feasibility and optimization of standalone PV-biogas hybrid distributed renewable system for rural electrifcation: A case study of a Cholistan community, Mehran University Research Journal of Engineering & Technology 38 (2) (2019) 453.

[23] C. S. Kwok, S. Krishnan, B. H. Tesfagergis, H. Y. Hsu, Feasibility study of using photovoltaic (PV), wind and biogas renewable energy system in South Australia household to maximize the energy production from renewable sources, in: 2019 Advances in Science and Engineering Technology International Conferences (ASET), IEEE, 2019, pp. 1–5.

[24] D. K. Yadav, T. Bhatti, A. Verma, Study of integrated rural electrification system using wind-biogas based hybrid system and limited grid supply system, International Journal of Renewable Energy Research 7 (1) (2017) 1–11.

[25] R. Feng, A. A. Zaidi, K. Zhang, Y. Shi, Optimisation of microwave pretreatment for biogas enhancement through anaerobic digestion of microalgal biomass, Periodica Polytechnica Chemical Engineering 63 (1) (2019) 65–72.

[26] T. Kunatsa, X. Xia, L. Zhang, Biogas potential determination and production optimisation through optimal substrate ratio feeding in co-digestion of water hyacinth, municipal solid waste, and cow dung, Biofuels (2020). `doi:10.1080/17597269.2020.1835452`.

[27] W. Boyle, Energy recovery from sanitary landfills-a review, in: The Proceedings of a Seminar Sponsored by the UN Institute for Training and Research (UNITAR) and the Ministry for Research and Technology of the Federal Republic of Germany held in Göttingen, October 1976, Microbial Energy Conversion, Elsevier, 1977, pp. 119–138.

[28] T. Achterberg, SCIP: solving constraint integer programs, Mathematical Programming Computation 1 (1) (2009) 1–41.

[29] S. Suwansri, J. Moran, P. Aggarangsi, N. Tippayawong, A. Bunkham, P. Rerkkriangkrai, Converting LPG stoves to use biomethane, Distributed Generation and Alternative Energy Journal 30 (1) (2015) 38–57.

[30] R. Ananthakrishnan, K. Sudhakar, A. Goyal, S. S. Sravan, Economic feasibility of substituting LPG with biogas for MANIT hostels, International Journal of Chemtech Research 5 (2) (2013) 891–893.

[31] ECN laboratories, Phyllis classification-Water hyacinth, Tech. rep., ECN (Netherlands). https://www.ecn.nl/phyllis2/Biomass/View/1148 (1998).

[32] R. Chandrappa, D. B. Das, Waste quantities and characteristics. Environmental Science and Engineering (Environmental Engineering)., in: Solid Waste Management, Berlin, Heidelberg, 2012, pp. 47–63. doi:https://doi.org/10.1007/978-3-642-28681-0_2.

[33] J. M. Sweeten, J. Korenberg, W. A. LePori, K. Annamalai, C. B. Parnell, Combustion of cattle feedlot manure for energy production, Energy in Agriculture 5 (1) (1986) 55–72.

[34] M. McGlashan, Amount of substance and the mole, Metrologia 31 (6) (1995) 447.

[35] A. Lovrak, T. Pukšec, N. Duić, A Geographical Information System (GIS) based approach for assessing the spatial distribution and seasonal variation of biogas production potential from agricultural residues and municipal biowaste, Applied Energy 267 (2020) 115010.

[36] P. V. Shukla, T. S. Bhalerao, S. Ingle, Effect of seasonal variation on biogas production from different food waste., Journal of Pure and Applied Microbiology 4 (1) (2010) 333–337.

[37] M. D' Este, M. Alvarado-Morales, A. Ciofalo, I. Angelidaki, Macroalgae Laminaria digitata and Saccharina latissima as potential biomasses for biogas and total phenolics production: focusing on seasonal and spatial variations of the algae, Energy & Fuels 31 (7) (2017) 7166–7175.

[38] S. Yilmaz, J. Chambers, M. Patel, Comparison of clustering approaches for domestic electricity load profile characterisation-implications for demand side management, Energy 180 (2019) 665–677.

[39] E. M. Wanjiru, S. M. Sichilalu, X. Xia, Optimal control of heat pump water heater-instantaneous shower using integrated renewable-grid energy systems, Applied Energy 201 (2017) 332–342.

Appendices

A. Biogas production reaction equation constants

$$a = \frac{Carbon\ ultimate\ mass}{Ar_C} \triangleq \frac{Carbon\ ultimate\ mass}{12.017},$$

$$b = \frac{Hydrogen\ ultimate\ mass}{Ar_H} \triangleq \frac{Hydrogen\ ultimate\ mass}{1.0079},$$

$$c = \frac{Oxygen\ ultimate\ mass}{Ar_O} \triangleq \frac{Oxygen\ ultimate\ mass}{15.999},$$

$$d = \frac{Nitrogen\ ultimate\ mass}{Ar_N} \triangleq \frac{Nitrogen\ ultimate\ mass}{14.0067},$$

$$e = \frac{Sulphur\ ultimate\ mass}{Ar_S} \triangleq \frac{Sulphur\ ultimate\ mass}{32.065}.$$

B. Fragmented objective function equations

$$CH_{4_{1,j}} = \left(\frac{a_1}{2} + \frac{b_1}{8} - \frac{c_1}{4} - \frac{3d_1}{8} - \frac{e_1}{4}\right)x_{1,j},$$

$$CO_{2_{1,j}} = \left(\frac{a_1}{2} - \frac{b_1}{8} + \frac{c_1}{4} + \frac{3d_1}{8} + \frac{e_1}{4}\right)x_{1,j},$$

$$NH_{3_{1,j}} = d_1 x_{1,j},$$

$$H_2S_{1,j} = e_1 x_{1,j},$$

$$CH_{4_{2,j}} = \left(\frac{a_2}{2} + \frac{b_2}{8} - \frac{c_2}{4} - \frac{3d_2}{8} - \frac{e_2}{4}\right)x_{2,j},$$

$$CO_{2_{2,j}} = \left(\frac{a_2}{2} - \frac{b_2}{8} + \frac{c_2}{4} + \frac{3d_2}{8} + \frac{e_2}{4}\right)x_{2,j},$$

$$NH_{3_{2,j}} = d_2 x_{2,j}$$

$$H_2S_{2,j} = e_2 x_{2,j},$$

$$CH_{4_{3,j}} = \left(\frac{a_3}{2} + \frac{b_3}{8} - \frac{c_3}{4} - \frac{3d_3}{8} - \frac{e_3}{4}\right)x_{3,j},$$

$$CO_{2_{3,j}} = \left(\frac{a_3}{2} - \frac{b_3}{8} + \frac{c_3}{4} + \frac{3d_3}{8} + \frac{e_3}{4}\right)x_{3,j},$$

$$NH_{3_{3,j}} = d_3 x_{3,j}$$

$$H_2S_{3,j} = e_3 x_{3,j},$$

$$Mr_{WH} \ (kgmol^{-1}) = a_1 * Ar_C + b_1 * Ar_H + c_1 * Ar_O + d_1 * Ar_N + e_1 * Ar_S,$$

$$Mr_{MSW} \ (kgmol^{-1}) = a_2 * Ar_C + b_2 * Ar_H + c_2 * Ar_O + d_2 * Ar_N + e_2 * Ar_S,$$

$$Mr_{CD} \ (kgmol^{-1}) = a_3 * Ar_C + b_3 * Ar_H + c_3 * Ar_O + d_3 * Ar_N + e_3 * Ar_S.$$

C. Algorithm

A linear programming optimisation approach was adopted to solve the objective function using the canonical form [39]. The mathematical formulation is as shown below.

$$\min_{x} f^T x \quad \text{such that} \quad \begin{cases} A.x \leq b, \\ A_{eq}.x = b_{eq}, \\ lb \leq x \leq ub, \end{cases}$$

where f, x, b, b_{eq}, lb and ub are vectors, and A and A_{eq} are matrices, and $f^T x$ is the objective function and the equalities and inequalities are the constraints.

$$X = \begin{bmatrix} x(1), & \cdots, & x(N), & x(N+1), & \cdots, & x(2N), & x(2N+1), & \cdots, & x(3N), & x(3N+1), & \cdots, & x(4N) \end{bmatrix}^T_{4N \times 1},$$

where $x(1), \cdots, x(N), x(N+1), \cdots, x(2N), x(2N+1), \cdots, x(3N), x(3N+1), \cdots, x(4N)$ are the number of moles of WH, MSW, CD and LPG respectively. $x(1), \cdots, x(N) \triangleq x_{1,j}, x(N+1), \cdots, x(2N) \triangleq x_{2,j}, x(2N+1), \cdots, x(3N) \triangleq x_{3,j}, x(3N+1), \cdots, x(4N) \triangleq LPG_j$

$$f^T = \begin{bmatrix} 0.0239 \times Ones\,(1,N), & -0.0162 \times Ones\,(1,N), & 0.0175 \times Ones\,(1,N), & zeros(1,N) \end{bmatrix}_{1 \times 4N},$$

$$A_1 = \begin{bmatrix} r_{1,j} & 0 & \cdots & 0 & r_{2,j} & 0 & \cdots & 0 & r_{3,j} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & r_{1,j} & \cdots & 0 & 0 & r_{2,j} & \cdots & 0 & 0 & r_{3,j} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{1,j} & 0 & 0 & \cdots & r_{2,j} & 0 & 0 & \cdots & r_{3,j} & 0 & 0 & \cdots & 0 \end{bmatrix}_{N \times 4N},$$

where

$$r_{1,j} = (10d_1 - a_1),$$

$$r_{2,j} = (10d_2 - a_2),$$

$$r_{3,j} = (10d_3 - a_3),$$

$$b_1 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{N \times 1},$$

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$$A_2 = \begin{bmatrix} r^-_{1,j} & 0 & \cdots & 0 & r^-_{2,j} & 0 & \cdots & 0 & r^-_{3,j} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & r^-_{1,j} & \cdots & 0 & 0 & r^-_{2,j} & \cdots & 0 & 0 & r^-_{3,j} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r^-_{1,j} & 0 & 0 & \cdots & r^-_{2,j} & 0 & 0 & \cdots & r^-_{3,j} & 0 & 0 & \cdots & 0 \end{bmatrix}_{N \times 4N},$$

where

$$r^-_{1,j} = (a_1 - 35d_1),$$

$$r^-_{2,j} = (a_2 - 35d_2),$$

$$r^-_{3,j} = (a_3 - 35d_3),$$

$$b_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{N \times 1},$$

$$A = [A_1; A_2]_{2N \times 4N},$$

$$b = [b_1; b_2]_{2N \times 1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{2N \times 1},$$

$$A_{eq_1} = \begin{bmatrix} \alpha_{1,j} & 0 & \cdots & 0 & \beta_{2,j} & 0 & \cdots & 0 & \gamma_{3,j} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \alpha_{1,j} & \cdots & 0 & 0 & \beta_{2,j} & \cdots & 0 & 0 & \gamma_{3,j} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_{1,j} & 0 & 0 & \cdots & \beta_{2,j} & 0 & 0 & \cdots & \gamma_{3,j} & 0 & 0 & \cdots & 0 \end{bmatrix}_{N \times 4N},$$

where $A_{eq_{1,j}}$ is the first equality constraint. $\alpha_{1,j}$, $\beta_{2,j}$ and $\gamma_{3,j}$ are the sums of volumes of water hyacinth, municipal solid waste and cow dung and the respective quantity of water to be added to each substrate as denoted below.

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$$\alpha_{1,j} = V_{WH,j} + \left(\frac{(22.4 \times 10^{-3}) \times H_2O_{1,j}}{Mr_{WH}} \times R_{H_2O_{(1,j)}} \right),$$

$$\beta_{2,j} = V_{MSW,j} + \left(\frac{(22.4 \times 10^{-3}) \times H_2O_{2,j}}{Mr_{MSW}} \times R_{H_2O_{(2,j)}} \right),$$

$$\gamma_{3,j} = V_{CD,j} + \left(\frac{(22.4 \times 10^{-3}) \times H_2O_{3,j}}{Mr_{CD}} \times R_{H_2O_{(3,j)}} \right),$$

$$b_{eq_1} = \begin{bmatrix} 150 \\ 150 \\ \vdots \\ 150 \end{bmatrix}_{N \times 1},$$

$$A_{eq_2} = \begin{bmatrix} 0.0239 \times ones\,(N,N) & -0.0162 \times ones\,(N,N) & 0.0175 \times ones\,(N,N) & eye(N,N) \end{bmatrix}_{N \times 4N},$$

$$b_{eq_2} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_N \end{bmatrix}_{N \times 1},$$

where $D_1, D_2 \ldots D_N$ are the respective gas demands for each month from January up to December.

$$A_{eq} = [A_{eq_1}; A_{eq_2}]_{2N \times 4N},$$

$$b_{eq} = [b_{eq_1}; b_{eq_2}]_{2N \times 1}.$$

The initial starting guess is denoted as

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{4N \times 1}.$$