Strategic Location Modelling for Reaction Vehicles of the Private Security Industry in South Africa

by

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Executive Summary

Title: Strategic Location Modelling for Reaction Vehicles of the Private Security Industry in South Africa

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Since the early 1960s location problems have been used throughout various industries and in various countries. During recent years the field of location problems has become increasingly popular due to the fact that it is applicable in real life situations – especially in emergency services such as hospital, police station and ambulance locations to name a few. Despite the fact that location problems are so widely used with great success, it is still not being used to full potential in industries where it can have a major impact. One of these industries is the private security industry in South Africa. This dissertation addresses various mathematical models that can assist the management of privately owned security companies to determine strategic locations for their reaction vehicles, these locations will increase both resource utilization and improve the level of service they provide to customers. These models are used in different scenarios to see how the models adapt to input changes.

Keywords: Private security industry, South Africa, covering problems, location problems, reaction vehicles.

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List of Acronyms

RV	Reaction Vehicle
SAPS	South African Police Service
SAIDSA	South African Intruder Detection Services Association
PSIRA	Private Security Industry Authority Association
SASA	Security Association of South Africa
VESA	Vehicle Security Association of South Africa
SCP	Set Covering Problem
MCLP	Maximal Covering Location Problem
BCLP	Backup Covering Location Problem
LSCP	Location Set Covering Problem
PIPS	Polygon Intersection Point Set

Chapter 1

Introduction

1.1 Crime in South Africa

South Africa is one of the countries in the world with the highest number of burglaries per capita, which leads to an increase in the number of South Africans that make use of private security companies to protect themselves and their assets (Irish: 1999). South Africa ranks tenth in the world with approximately 8.9 burglaries per 1000 people (see figure 1) (Rapid Intelligence: 2005). This is the highest number of burglaries per capita in Africa. Although there are certain areas with higher crime rates than others, the problem with crime is the inability to predict when or where it will occur. One can only take steps to help prevent burglaries (Weber: 2012) or respond to it when it occurs. One way to respond to crime is to have an efficient alarm system that is installed and maintained by a registered security company that provides armed response or reaction vehicle (RV) services.

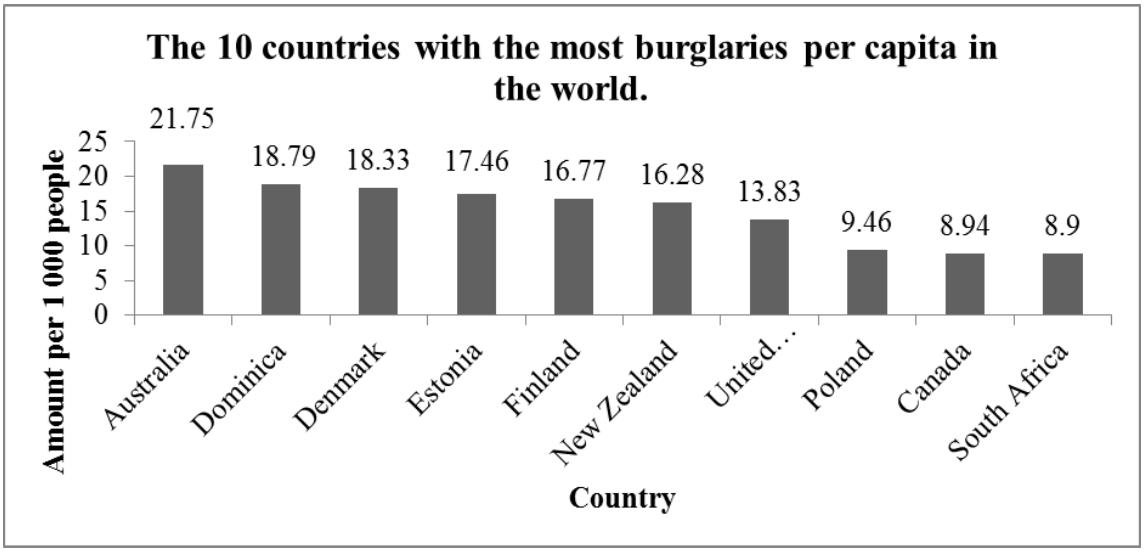

Figure 1 Burglaries per Capita

1.2 Private Security of South Africa

The private security industry of South Africa is a very competitive environment (Irish: 1999). According to the Private Security Industry Regulatory Authority's (PSIRA) annual report for the period 2009/2010 the registered active security businesses have increased from 6 392 to 7 459 (nTier Software Services: 2007) in one year, which is also confirmed by Irish (1999) that stated: "The private security industry is one of the rapidly growing economic sectors in South Africa".

There are various reasons for the rapid growth in the private security industry, some of these reasons are (Irish: 1999):

- Scaling down and withdrawal of police from some of its functions.
- The perception that the police are unable to protect the public.
- Insurance companies insist on having increased security.

Due to the large number of security businesses and the competitiveness of the industry, it is essential to continuously improve the methods and processes of the business, as well as to comply with industry standards and be registered with the relevant associations.

It is important to differentiate between the South African Police Service (SAPS) and the private security industry – the objective of the SAPS is to protect the public at large, whereas the private security industry has a profit motive and is only accountable to its clients (Irish: 1999).

The private security industry consists of various components, these components can be seen as the various services provided by private security companies, and these components are (Irish: 1999):

- *Guarding*: This is guarding of fixed assets, for example buildings, shopping malls and schools.
- *Armed response*: This is a fleet of response vehicles that responds when a service is required, this can be due to a burglary, fire or medical emergencies to name but a few (Manic Creations: 2013).
- *Cash-in-transit*: This is transport of cash between various companies for example shops and banks.
- *Electronic hardware*: This is the installation of alarms and other security devices.
- *Investigation and risk management*: This is the field of private investigators and risk consultants. Certain companies also provide investigators that operate as debt collectors and tracing agents.

This dissertation, only focuses on the armed response component.

According to South African Intruder Detection Services Association (SAIDSA) (nTier Software Services: 2007) an RV is a vehicle that is solely dedicated to the purpose of responding to clients in a predetermined area.

The only requirements for RVs with regard to performance stated by SAIDSA are:

- A minimum of 2 vehicles must be fully equipped, manned and available for 24-hours per day with a minimum of one fully-equipped backup vehicle.
- The areas allocated to each vehicle must be predetermined and clearly marked on a map, which must be maintained for inspection.
- A log must be maintained for all cases where the reaction time exceeds 15 minutes. If 10% of the occurrences in a specific area exceed 15 minutes, the situation must be reviewed and required steps must be taken to comply with the requirements of a 15 minute maximum reaction time.

The guidelines by SAIDSA also clearly state that "there can be no guarantee that a reaction service will arrive at a site within a specific time period" (Rudolph: 2011). This is understandable since one cannot exactly predict where burglaries or crimes will occur and companies therefore must take strategic actions to minimize the response time when service is required.

Currently, the management of RVs are done by managers of security companies without using any specific tools or techniques.

1.3 Rationale for the Research

1.3.1 Managerial Implications

The private security industry of South Africa is unique due to the context in which it was developed, this industry in the US and Europe was developed without any input from the state – this was not the case in South Africa (Irish: 1999). Very little formal research has been done on the techniques and procedures used in South Africa.

Contracts with large private security companies often become very expensive, which leads to cancelation of contracts even though they still need the service. This leads to an increase in small companies with lower tariffs, unfortunately these tariffs are usually achieved by cutting running costs to a minimum – this will directly affect the quality of service provided (Irish: 1999). The research done will help companies to use their resources more effectively and help to determine the achievable service levels.

Various location models have been developed to help with decision making when locating facilities, some of these criteria used are cost and distance from demand points (Farahani et al: 2012). In this dissertation these models will be adapted to fulfil the needs of private security companies of South Africa.

1.3.2 Questions to be addressed

The preliminary research indicates that various questions with regard to reaction vehicles still need to be answered. The following research questions need answers:

- Does the demand for reaction vehicles follow any trends?
- Do certain areas have a higher demand than others?
- Is there a relationship between the number of customers in an area and the demand for reaction vehicles?
- Does the time of day have an influence on demand for reaction vehicles?
- Does the time of day have an influence on the response time of vehicles?
- How should reaction vehicle locations change according to the time of day?
- How do company policies influence reaction vehicle locations?
- What is the current service level of reaction vehicles?
- What should be done to increase the service level of reaction vehicles?
- What is the relevance of queueing theory to reaction vehicle operation?
- How should the workforce of a private security company be allocated/ scheduled to improve service level or to achieve a certain service level?
- Can models be developed and implemented to solve reaction vehicle location problems?

All these problems should be addressed in such a fashion that they would be applicable to all private security companies in South Africa (those that provide armed response or reaction vehicle services).

1.3.3 Research Objectives

By addressing the research questions above it becomes clear that more research needs to be done on reaction vehicles – especially in the South African environment. To address the research problem, models will be developed to assist managers of security companies with the positioning of RVs. From these models managers will further be able to predict how many vehicles would be required during specific time periods as well as the service level they will be able to achieve.

The models will be developed in such a way that they can be used by any privately owned security company in South Africa (those that need to comply with the governing bodies mentioned in this document) by entering the company's own input data such as area of operation, response times, customer locations and number of call-outs.

The proposed models will assist companies to improve the utilization of their resources, to respond to demand as rapidly as possible and to comply with industry standards and regulations.

1.3.4 Importance of the Research Problem

According to the annual statistics of the South African Police Service (2012) the number of burglaries and robberies at residential premises decreased over the past few years, but with 16 766 robberies and 245 531 burglaries in 2011/2012, the numbers remain some of the highest in the world.

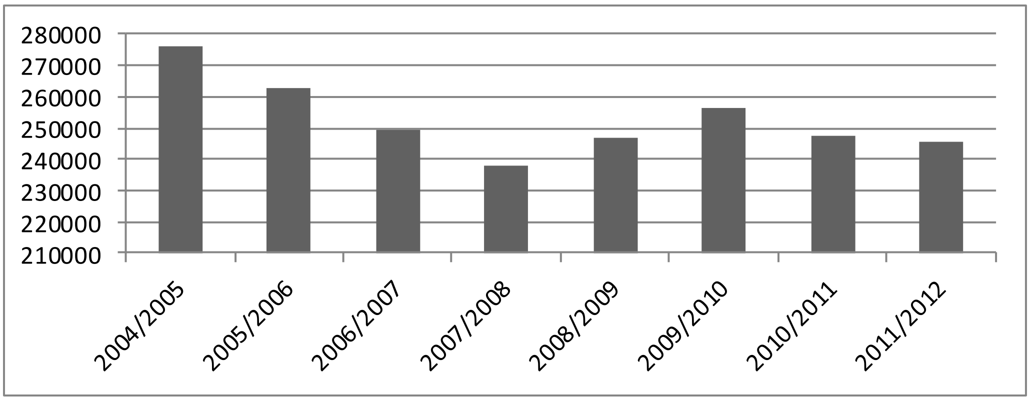

Figure 2 Burglaries in South Africa

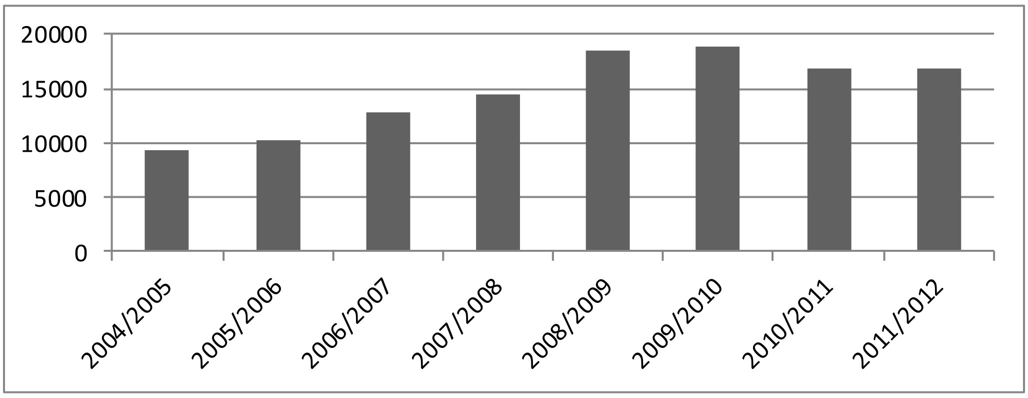

Figure 3 Robberies in South Africa

A research of crime survey was conducted by Statistics South Africa (Stats SA) (2012) on private households in all nine provinces in South Africa. The research showed that 59.3% of all households perceive housebreaking/burglary to be one of the most common types of crime and 35.2% of the households believed that it increased over recent years. Studies further showed that the crime most feared by households was housebreaking/burglary (57,4%).

According to the study approximately half of the households took physical measures to protect their homes (these include burglar bars, electric fencing and alarm systems). Most incidents occur during the night, afternoon hours or early morning hours.

The research conducted showed that only 60% of housebreaking/burglary events were reported to the police. Some people who did not report the crime to the police did report the crime to a private security company. This shows that South African citizens feel that the South African police cannot help them. This statement is supported by the fact that 20% of all the households feel that more money should be spent on law enforcement.

People feel that they have the need to get support and protection with the assistance of security services. This research will help privately owned security companies to increase the level of service they provide to their customers and give them the necessary guidelines required to increase their service levels.

1.3.5 Limitations and Assumptions of the Study

Although statistics are available on past occurrences of crime, one remains unable to accurately predict when and where it will take place. Only limited research will be done on how to prevent crime, focus will be placed on responding to the point where the crime took place as soon as possible and the utilization of resources.

The models developed during the research will only be applicable within the context of the private security industry in South Africa and the requirements set out by SAIDSA (South African Intruder Detection Services Association) will be taken into consideration.

1.4 Concluding Remarks and Scope of Work

This research comprises a comprehensive literature review of location models that are currently used by various industries. The aim of this research is to develop location models for RVs in the context of the private security industry of South Africa. The literature review is undertaken to analyse previous location models. The gathered information will be used as a framework to develop generic location models that can be used by private security industry of South Africa.

Further research was carried out to understand how RVs of the private security industry operate and what the guidelines set out by governing bodies are. Data was also analysed to determince how the number of customers and number of call-outs influence the predetermined locations of RVs.

In summary, RVs are used to respond to a crime or burglary when it does occur (this has nothing to do with crime prevention). Privately owned security companies need to make strategic decisions when determining the locations of RVs. Several location models for RVs are studied in this dissertation:

Chapter 1 is introductory in nature and gives a brief description of the importance of RVs in the South African context in responding to crime.

The literature review discusses various location models that are currently used in different industries. This in discussed in Chapter 2.

In Chapter 3, a generic location model is developed, executed and interpreted for companies that operate in a fairly small area. This model determines which RVs should respond to which call-outs/customers at street level.

In Chapter 4, models are developed, executed and interpreted where companies locate all their vehicles at a single location. The model uses queueing theory to determine how many RVs should be used by the privately owned security company.

When companies operate in a larger area it is often not possible to determine the response area up to street level, in these instances the RV would be located to respond to a specific area.

In Chapter 5 multi-objective optimization models are developed, executed and interpreted for scenarios where the number of available RVs is finite or infinite. This chapter also takes queueing theory into consideration when locating RVs.

In Chapter 6, the concluding remarks of the dissertation are presented.

The following flow chart presents a systematic way of the thesis.

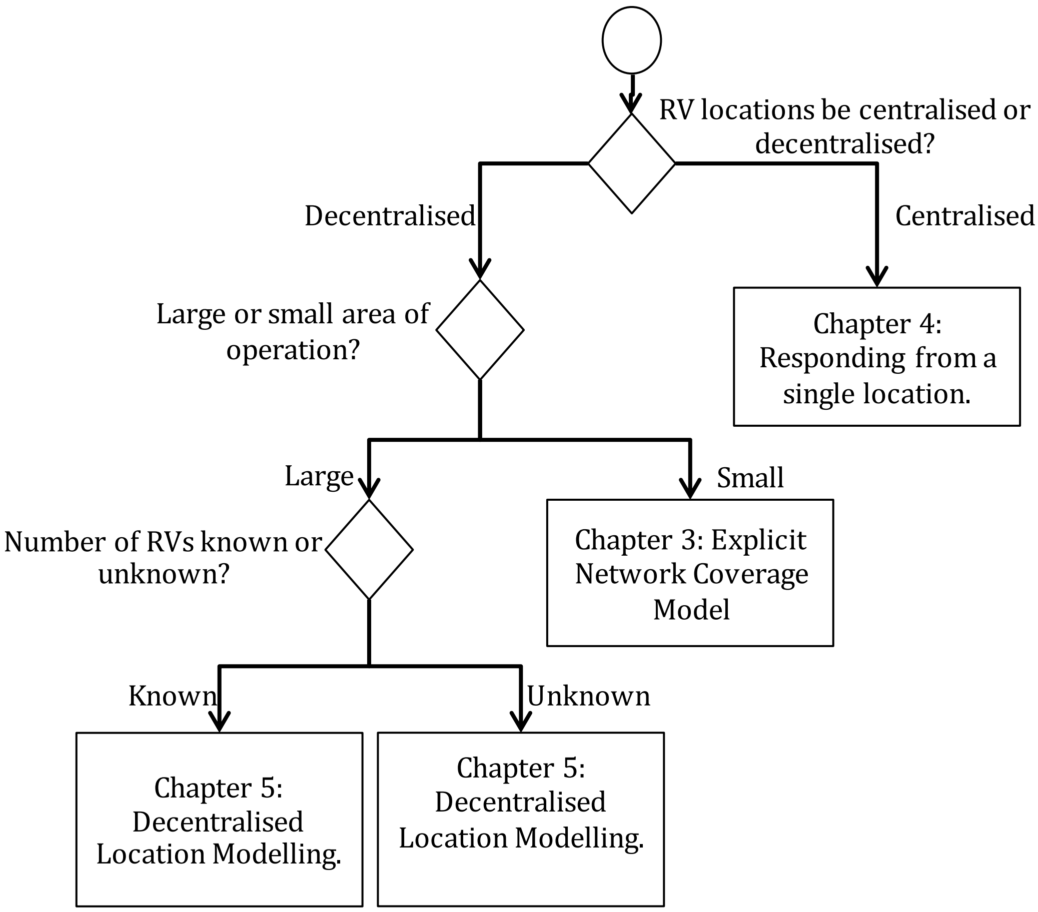

Figure 4 Document Flow

Chapter 2

Literature Review

2.1 **Introduction**

This chapter focuses on providing an overview of location models that is currently used in the various industries. Section 2.2 gives an overview on the present-state-of-art of covering models. Section 2.3 discusses the difference between explicit and implicit location models. Section 2.4 discusses the Location Set Covering Problem (LSCP), section 2.5 the Maximal Covering Location Problem (MCLP) and section 2.6 the Backup Covering Location Problem (BCLP). Section 2.7 concludes the chapter.

2.2 **Covering Models: Present-state-of-art**

Location problems are not a new research field and much in depth research has been done over the past years on this topic (Farahani et al: 2011). It is believed that mathematical location modelling can identify the "optimal" location for facilities when realistic objectives are identified by some quantified measure (Church et al: 1974). This research field is attractive due to the fact that it is very applicable in real-life situations, especially for services and emergency facilities (Faharani et al: 2011) such as emergency medical services, ambulances and fire response stations (Murray et al: 2010). If good decisions are made when locating facilities that provide goods and services it will reduce the operational costs (Murray et al: 2010) and improve service.

The first location problems was developed by Hakimi (1964), where the purpose of his research was to find optimum locations for switching centres on a communication network and police stations on a highway system. The optimum locations were determined by allocating weights to both the vertices and branches in the networks and locating facilities closer to the vertices or branches with the greatest weights.

Different covering problems have different goals and constraints. In certain location problems a customer is serviced by at least one facility within a critical distance (this distance also relates to the time it takes to reach the customer from the facility). In most covering problems a customer receives services by facilities depending on the distance between the customer and the facility (Farahani et al: 2011).

According to Schilling et al. (1993) covering models can be divided into two categories namely Set Covering Problems (SCPs) and Maximal Covering Location Problems (MCLPs). Murray et al.

(2010) suggests that a third type of deterministic problem should be added, known as the backup covering location problem (BCLP). These models have to do with (Murray et al: 2010):

- completely covering all the demand by using the minimum number of facilities,
- covering as much demand as possible by using only a limited number of facilities, and
- increasing the likelihood of facility availability for service through the provision of backup coverage by other facilities.

2.3 Implicit and Explicit models

When a demand area is presented by a space instead of a point the solution of locations problems becomes more complicated (Murray et al: 2010). This is explained in figure 5 below where it can be seen that the demand area is not completely covered by one of the two facilities. Both of the facilities partially cover the area. This causes errors in the solutions of the problems by locating too many or too few facilities and overestimating or underestimating demand served. To address these errors, the implicit and explicit covering problems were developed.

Implicit and explicit models work in practice, because it is necessary to locate facilities in a complimentary fashion (Murray et al: 2010).

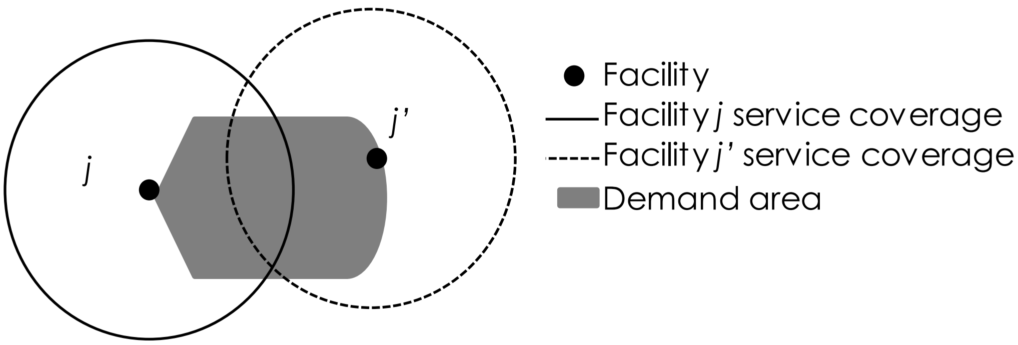

Figure 5 Area Coverage

Implicit coverage is the concept that these relationships may be modelled without tracking combinations of facilities providing coverage to an area (Murray et al: 2010). Implicit coverage takes into consideration that an accepted level of coverage is reached if a certain percentage (user defined) is covered by the facility or combination of facilities. In figure 6 below the area is approximately 50% covered (using implicit covering).

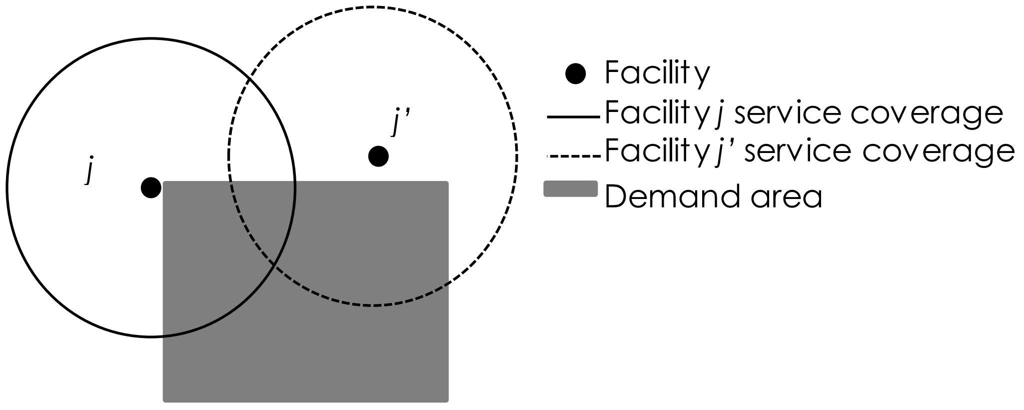

Figure 6 Implicit Coverage

Explicit coverage is exactly accounting for coverage provided to a demand area by a specific set of facilities (Murray et al: 2010). By using explicit coverage all combinations of potential coverage must be taken into consideration. The demand area will be completely covered when explicit coverage is used, see the figure below.

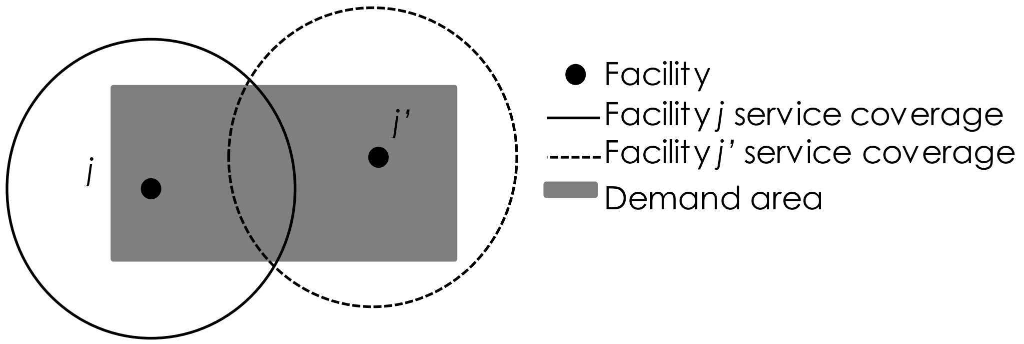

Figure 7 Explicit Coverage

The main difference between implicit coverage and explicit coverage is that by using implicit coverage the level of coverage is estimated, but by using explicit coverage an exact level of coverage is used.

2.4 Location Set Covering Problem (LSCP)

2.4.1 Background

The location set covering problem attempts to minimize the cost associated with facilities (minimize the number of facilities located) and their locations, while still providing a specified level of coverage (Farahani et al: 2012)(Murray et al: 2010) – the specified level of coverage is achieved if the response distance or time is within the upper limit determined by the user (Toregas et al: 1971). By doing this the maximum time or distance that seperates the facility and the demand point is the crucial parameter (Toregas et al: 1971).

The mathematical formulation of the LSCP is as follows:

$i \triangleq demand\ nodes$

$j \triangleq facility$

$x_j \triangleq \begin{cases} 1\ if\ a\ facility\ should\ be\ located\ at\ node\ j \\ 0\ otherwise \end{cases}$

$S \triangleq the\ maximum\ servicing\ distance$

$c_j \triangleq cost\ of\ locating\ facility\ at\ node\ j$

$a_{ij} \triangleq \begin{cases} 1\ if\ the\ distance\ between\ node\ i\ and\ j\ is\ smaller\ than\ S \\ 0\ otherwise \end{cases}$

$$Minimize\ \sum_{j=1}^{n} c_j x_j \tag{2.1}$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1 \qquad \forall\, i \in \{1..m\} \tag{2.2}$$

$$x_j \in \{0;1\} \qquad \forall\, j \in \{1..n\} \tag{2.3}$$

The objective function (2.1) minimizes the cost of locating facilities. If the cost of locating a facility at a node is either zero or identical for each node, the set covering problem becomes the Minimum Cardinality Set Covering Problem (MCSCP) or the Unicost Set Covering Problem (USCP). In these instances the model will minimize the number of located facilities. Constraint (2.2) ensures that all of the demand nodes are covered by at least one facility. Constraint (2.3) is a binary constraint.

The Set Covering Problem has some problems that are addressed in other adaptions of the model, one of the problems is capacity constraints that are for example not taken into consideration, the model does not take the number of demands serviced by each facility or the area size (if the areas do not overlap) into consideration. This means that some facilities can cover a very small area or very little demand.

2.4.2 Location Set Covering Problem (LSCP) Implicit and Explicit

2.4.2.1 Implicit-Model

The notation of the model is as follows:

$i \triangleq the\ index\ of\ demand\ areas$

$j \triangleq the\ index\ of\ facilities$

$k \triangleq the\ index\ of\ coverage\ levels$

$x_j \triangleq \begin{cases} 1\ if\ a\ facility\ should\ be\ located\ at\ point\ j \\ 0\ otherwise \end{cases}$

$Y_{ik} \triangleq \begin{cases} 1\ if\ area\ i\ is\ covered\ at\ level\ k \\ 0\ otherwise \end{cases}$

$\beta_k \triangleq the\ minimum\ required\ percentage\ at\ level\ k$

$\alpha_k \triangleq the\ minimum\ number\ of\ required\ facilities\ for\ covering\ completely\ at\ the\ k^{th}\ level$

$\Omega_{ik} \triangleq the\ set\ of\ potential\ facilities\ cover\ area\ i\ at\ least\ \beta_k$

The LSCP-Implicit mathematical model is as follows:

$$Minimize\ z = \sum_j x_j \tag{2.4}$$

Subject to:

$$\sum_{j \in \Omega_{ik}} x_j \geq \alpha_k Y_{ik} \qquad \forall\, i, k \tag{2.5}$$

$$\sum_k Y_{ik} = 1 \qquad \forall\, i \tag{2.6}$$

$$Y_{ik} \in \{0; 1\} \qquad \forall\, i, k \tag{2.7}$$

$$x_j \in \{0; 1\} \qquad \forall\, j \tag{2.8}$$

The objective function (2.4) minimizes the number of facilities that must be located. Constraint (2.5) states that for completely covering demand area at level k, α_k facilities must be located. Constraint (2.6) ensures the existence of the coverage at level k. Constraints (2.7) and (2.8) are binary constraints.

2.4.2.2 Explicit-Model

The notation of the model is as follows:

$i \triangleq the\ index\ of\ demand\ areas$

$j \triangleq potential\ locations\ for\ facilities$

$k \triangleq the\ index\ of\ coverage\ levels$

$l \triangleq$ *the index of facility configuration*

$\Psi_{ik} \triangleq$ *the set of k facility configurations completely covering area i*

$\Delta_{ikl} \triangleq$ *the set of k facilities in* l^{th} *configuration which covers area i completely*

$$x_j \triangleq \begin{cases} 1 \text{ if a facility should be located at point } j \\ 0 \text{ otherwise} \end{cases}$$

$$Z_{ikl} \triangleq \begin{cases} 1 \text{ if area } i \text{ is covered by configuration } l \text{ at level } k \\ 0 \text{ otherwise} \end{cases}$$

The LSCP-Explicit mathematical model is as follows:

$$Minimize\ z = \sum_j x_j \qquad (2.9)$$

Subject to:

$$\sum_k \sum_{l \in \Psi_{ik}} Z_{ikl} = 1 \qquad \forall\, i \qquad (2.10)$$

$$x_j \geq Z_{ikl} \qquad \forall\, i, k, l \in \Psi_{ik},\ j \in \Delta_{ikl} \qquad (2.11)$$

$$Z_{ikl} \in \{0; 1\} \qquad \forall\, i, k, l \in \Psi_{ik} \qquad (2.12)$$

$$x_j \in \{0; 1\} \qquad \forall\, j \qquad (2.13)$$

The objective function (2.9) minimizes the number of facilities that must be located. According to constraint (2.10) a k-facility configuration must be chosen to completely cover a demand area. Constraint (2.11) indicates that a configuration can be assumed only when the required facilities are located. Constraints (2.12) and (2.13) are both binary constraints on the decision variables.

2.4.3 Present-State-of-Art: Location Set Covering Problem

Year	Author/s	Method and Approach	Contribution and/or Comments
1971	Toregas, Swain, ReVelle, Bergman	• Location set covering problem. • Minimum Cardinality Set Covering Problem (MCSCP). • Unicost set covering problem. • Integer programming problem.	• Use to site emergency service facilities.
1976	ReVelle, Toregas, Falkson	• Applications of the set covering problem. • Arc covering formulation. • The problem of ambulance location.	• Adapt set covering problem to apply to new situations.
1979	Church, Meadows	• Location modelling utilizing maximum service distance criteria.	• Facilities can be placed anywhere on node. • In optimal solution, the facilities are located on network intersect point set (NIPS). • Improves coverage by placing facilities along the arcs of the network.
1988	Current, Storbeck	• Capacitated covering models	• Assumes facilities in location covering models to this date are uncapacitated. • Places capacities on the facilities.
1992	Current, O'Kelly	• Application of location set covering problem.	• Determines locations of emergency warning sirens. • Analyse cost implications of various policies.
1994	Marianov, Revelle	• Queueing probabilistic location set covering	• Takes probability that server is available into

		problem. • Apply queueing theory to availability constraint.	consideration.
2002	Beraldi, Ruszczynski	• Probabilistic Set-Covering Problem	• Uses probabilistically efficient points of binary vectors. • Develop enumeration methods. • Branch-and-Bound algorithms.
2006	Rajagopalan, Saydam, Xiao	• Multi-period set covering location model for dynamic redeployment of ambulances. • Validated using simulation model. • Use Tabu search algorithm.	• Address the issue of demand fluctuations throughout different time periods. • Determine minimum number of ambulances and locations for each time cluster. • Determine probability that ambulance is busy. • Experimental datasets used.
2008	Eiselt, Marianov	• Gradual location set covering with service quality.	• Replaces covered/not covered with gradual covering. • Includes quality of service criterion. • Compares size and features of models.
2008	Saxena, Goyal, Lejeune	• Mixed Integer Programming formulations for the probabilistic set covering problem.	• Introduce the concepts of p-inefficiency and polarity cuts. • Uses test-bed of almost 10 000 probabilistic instances. • Procedure is faster than existing approaches.
2010	Murray, Tong, Kim	• Implicit covering location problem.	• Uses GIS.

		• Explicit covering location problem.	
2013	Drezner, Drezner	• Multiple facility location problem with gradual coverage. • Heuristic algorithms. • Tabu search algorithm.	• Includes partial coverage. • Positioning of cell phone towers. • Demand point covered as much as possible and no demand points with low coverage.

Table 1 Location Set Covering Problem Literature

2.5 Maximal Covering Location Problem (MCLP)

2.5.1 Background

Users of location models recognized that it is often too costly to service all demand nodes since it may be impractical or budgetary constraints will not allow it (Church et al: 1974). When a company locates four facilities which cover 90% of the demand and five facilities cover 100% of the demand, it is often too costly to locate an extra facility for the 10% increase in coverage (Church et al: 1974). For this reason the MCLP was developed. The MCLP locates a specified number of facilities while maximizing the total demand covered (Murray et al: 2010).

The mathematical model is as follows (Farahani et al: 2011):

$i \triangleq demand\ nodes$

$j \triangleq potential\ facility\ locations$

$h_i \triangleq demand\ at\ node\ i$

$S \triangleq the\ desired\ time\ from\ the\ facility\ to\ the\ server$

$P \triangleq the\ number\ of\ facilities\ to\ locate$

$$\alpha_{ij} \triangleq \begin{cases} 1\ if\ the\ distance\ between\ i\ and\ j\ is\ smaller\ than\ S \\ 0\ otherwise \end{cases}$$

$$x_j \triangleq \begin{cases} 1\ if\ facility\ must\ be\ located\ at\ node\ j \\ 0\ otherwise \end{cases}$$

$$z_i \triangleq \begin{cases} 1\ if\ node\ i\ is\ covered \\ 0\ otherwise \end{cases}$$

$$Maximize\ h_i \times z_i \qquad (2.14)$$

Subject to:

$$z_i \le \sum_j \alpha_{ij} \times x_j \qquad \forall\, i \qquad (2.15)$$

$$\sum_j x_j \le P \qquad (2.16)$$

$$z_i \in \{0; 1\} \qquad \forall\, i \qquad (2.17)$$

$$x_j \in \{0; 1\} \qquad \forall\, j \qquad (2.18)$$

The objective function (2.14) maximizes the demand covered by the facilities. Constraint (2.15) is the relation between the coverage and location variables. It states that demand node j is covered if at least one facility at one of the potential sites are able to cover node j, is located. Constraint (2.16) limits the number of facilities that must be located. Constraints (2.17) and (2.18) are both binary constraints on the decision variables.

2.5.2 Maximum Covering Location Problem (MCLP) Implicit and Explicit

2.5.2.1 Implicit-Model

Before using the Implicit MCLP model, a value of β_k must be chosen, where β_k is the minimum acceptable coverage percentage at level k. For RVs the entire demand area must be covered, thus it would be 100%.

$i \triangleq$ *demand areas*

$j \triangleq$ *potential locations for facilities*

$k \triangleq$ *index of coverage levels* $(1,2,3,\ldots,K)$

$\Omega_{ik} \triangleq$ *set of potential facilities j partially covering area i at least* β_k

$$X_j \triangleq \begin{cases} 1 \text{ if a facility is located at location } j \\ 0 \text{ otherwise} \end{cases}$$

$\alpha_k \triangleq$ *minimum number of facilities needed for complete coverage at level k*

$$Y_{ik} \triangleq \begin{cases} 1 \text{ if area } i \text{ is covered at level } k \\ 0 \text{ otherwise} \end{cases}$$

$p \triangleq$ *the number of facilities to locate*

$$Z_i \triangleq \begin{cases} 1 \text{ if area } i \text{ is suitably covered by a sited facility} \\ 0 \text{ otherwise} \end{cases}$$

$g_i \triangleq$ *demand for service in area i*

$$Maximize\ z = \sum_i g_i \times Z_i \tag{2.19}$$

Subject to:

$$\sum_{j\in\Omega_{ik}} X_j \geq \alpha_k Y_{ik} \qquad \forall i,k \tag{2.20}$$

$$\sum_j X_j = p \tag{2.21}$$

$$\sum_k Y_{ik} = Z_i \qquad \forall i \tag{2.22}$$

$$X_j = \{0,1\} \qquad \forall j \tag{2.23}$$

$Y_{ik} = \{0,1\}$ $\forall i, k$ (2.24)

$Z_i = \{0,1\}$ $\forall i$ (2.25)

The objective function (2.19) maximizes the total demand covered. Constraint (2.20) links facility siting decisions to the coverage of demand area i. Constraint (2.21) puts a limit on the number of facilities that must be located. Constraint (2.22) tracks whether coverage has been provided at some level k. Constraints (2.23) to (2.25) impose binary integer restrictions on the decision variables.

2.5.2.2 Explicit-Model

$i \triangleq$ *index of demand areas*

$j \triangleq$ *index of potential facility sites*

$k \triangleq$ *index of coverage levels* $(1,2,3,\dots,K)$

$l \triangleq$ *index of facility configurations*

$\Psi'_{ik} \triangleq$ *set of k facility configurations partially or completely covering area i*

$g_i \triangleq$ *demand for service in area i*

$c_{ikl} \triangleq$ *fraction of area i covered by configuration l containing k facilities*

$$Z_{ikl} \triangleq \begin{cases} 1 \text{ if area } i \text{ is suitably covered at level } k \text{ by configuration } l \\ 0 \text{ otherwise} \end{cases}$$

$$X_j \triangleq \begin{cases} 1 \text{ if a facility is sited at location } j \\ 0 \text{ otherwise} \end{cases}$$

$p \triangleq$ *number of facilities to site*

$\Delta'_{ikl} \triangleq$ *set of k facilities in configuration l that partially or completely cover area i*

$$Maximize \sum_i \sum_k \sum_{l \in \Psi'_{ik}} g_i c_{ikl} Z_{ikl} \qquad (2.26)$$

Subject to:

$\sum_k \sum_{l \in \Psi'_{ik}} Z_{ikl} \leq 1$ $\forall i$ (2.27)

$\sum_j X_j = p$ (2.28)

$X_j \geq Z_{ikl}$ $\forall i, k, l \in \Psi'_{ik}, j \in \Delta'_{ikl}$ (2.29)

$$X_j = \{0,1\} \qquad \forall j \qquad (2.30)$$

$$Z_{ikl} = \{0,1\} \qquad \forall i, k, l \in \Psi'_{ik} \qquad (2.31)$$

The objective function (2.26) is to maximise the total demand covered. Constraint (2.27) stipulates that at most one-level configuration combination can account for the coverage of demand area i. Constraint (2.28) puts a limit on the number of facilities that must be located. Constraint (2.29) limits coverage to facilities that have been located. Constraints (2.30) and (2.31) are binary constraints on the decision variables.

A problem associated with the MCLP Explicit model is the number of constraints and variables generated since it is considerably more than for the other models (Murray et al: 2010).

2.5.3 Present-State-of-Art: Maximal Covering Location Problem

Year	Author/s	Method and Approach	Contribution and/or Comments
1974	Church, Revelle	• The maximal covering location problem. • Greedy Adding (GA) Algorithm. • Greedy Adding with Substitution (GAS) Algorithm. • Linear programming.	• Use for emergency facilities such as fire stations and ambulance dispatching stations. • Considers cost-effectiveness. • Resources insufficient to provide complete coverage.
1979	Church, Meadows	• Location modelling utilizing maximum service distance criteria.	• Facilities can be placed anywhere on node. • In optimal solution, the facilities are located on network intersect point set (NIPS). • Improves coverage by placing facilities along the arcs of the network.
1984	Church	• Planar Maximal Covering Location Problem	• Potential sites are not on a network. • Uses both Euclidean and Rectilinear distances.
1988	Current, Storbeck	• Capacitated covering models	• Assumes facilities in location covering models to this date are uncapacitated. • Places capacities on the facilities.
1989	ReVelle, Hogan	• Maximum Availability Location Problem. • Zero-one linear programming problem.	• Maximizes the population that will find a server available within a time standard with certain reliability. • Introduces concept of backup covering. • Solved on a medium-sized transportation

			network.
1992	Current, O'Kelly	• Application of location set covering problem.	• Determines locations of emergency warning sirens. • Analyse cost implications of various policies.
1995	Marianov, ReVell	• Queueing maximal availability location problem. • Probabilistic version of the MCLP.	• Adds randomness into availability of servers. • Probabilities that servers are busy are independent. • Limited number of emergency vehicles can respond within a certain time with certain reliability.
2001	Espejo, Galvao, Boffey	• Dual-based heuristics for a hierarchical covering location problem. • Heuristic approach.	• A 2-level hierarchical extension of MCLP. • Effective method for solution developed. • Reduces knapsack problem.
2002	Berman, Krass	• Generalized MCLP.	• Models partial coverage. • Level of coverage is a non-linear step function of distance to nearest facility. • Used for retail facilities.
2004	Karasakal, Karasakal	• Maximal covering location model with partial coverage.	• Considers partial coverage when siting facilities. • Level of service drops as distance to facility increases.
2009	Tong, Murray, Xiao	• Genetic Algorithm for Coverage Maximization.	• Use GIS. • Explores the geographical structure of the

			problem.
2010	Berman, Drezner, Krass	• MCLP with mixed weights.	• Takes into consideration that you don't want to cover some demand points (for example you don't want to site a retail store in an area with a high crime rate).
2010	Alexandris, Giannikos	• Integer programming model for partial coverage.	• Uses Geographic Information System (GIS). • Demand points are spatial objects. • Model provides larger coverage than traditional models.
2010	Murray, Tong, Kim	• Implicit covering location problem. • Explicit covering location problem.	• Uses GIS.

Table 2 Maximal Covering Location Problem Literature

2.6 Backup Covering Location Problem (BCLP)

2.6.1 Background

A problem that arises with both the LSCP and the MCLP is whether a facility providing service to a specific demand area is available. For example what will a fire station do if a vehicle is responding to a call for service and a second call for service is received (Murray et al: 2010)? For this reason it might be beneficial to let an area be covered by more than one facility. This is also a more realistic approach since areas in which a facility provides service can overlap.

In defining the objectives we let:

$$i \triangleq demand\,nodes$$

$$j \triangleq potential\,facility\,locations$$

$$h_i \triangleq demand\,at\,node\,i$$

$$z_i \triangleq \begin{cases} 1\; if\; node\; i\; is\; covered \\ 0\; otherwise \end{cases}$$

$$u_i \triangleq \begin{cases} 1\; if\; area\; i\; is\; covered\; by\; two\; or\; more\; sited\; facilities \\ 0\; otherwise \end{cases}$$

$$x_j \triangleq \begin{cases} 1\; if\; facility\; must\; be\; located\; at\; node\; j \\ 0\; otherwise \end{cases}$$

$$P \triangleq the\; number\; of\; facilities\; to\; locate$$

$$Maximize\; \sum_i h_i z_i \tag{2.32}$$

$$Maximize\; \sum_i h_i u_i \tag{2.33}$$

Subject to:

$$\sum_j x_j \geq z_i + u_i \qquad \forall\, i \tag{2.34}$$

$$\sum_j x_j \leq P \tag{2.35}$$

$$z_i \geq u_i \qquad \forall\, i \tag{2.36}$$

$$x_j \in \{0; 1\} \qquad \forall\, j \tag{2.37}$$

$$z_i \in \{0; 1\} \qquad \forall\, i \tag{2.38}$$

$$u_i \in \{0; 1\} \qquad \forall\, i \tag{2.39}$$

The first objective (2.32) of BCLP is to maximise the total demand covered, the second objective (2.33) is to maximise the total demand that receives coverage from two or more facilities. Constraint (2.34) links facility siting decisions to the primary and secondary coverage of demand area i. Constraint (2.35) states that a fixed number of facilities must be located. Constraint (2.36) states that an area must first receive primary coverage before secondary coverage can be received. Constraints (2.37) to (2.39) are binary constraints on the decision variables.

An issue that arises with the BCLP is the fact that it needs multi-objective approaches to be solved. This makes the solving of BCLP more challenging and complicated (Murray et al: 2010).

2.6.2 Backup Covering Location Problem (BCLP) Implicit and Explicit

2.6.2.1 Implicit-Model

$i \triangleq$ *index of demand areas*

$j \triangleq$ *index of potential facility sites*

$k \triangleq$ *index of coverage levels* $(1,2,3,\dots,K)$

$g_i \triangleq$ *demand for service in area i*

$Z_i \triangleq \begin{cases} 1 \text{ if area } i \text{ is suitably covered by a sited facility} \\ 0 \text{ otherwise} \end{cases}$

$V_i \triangleq \begin{cases} 1 \text{ if area } i \text{ is provided suitably secondary coverage} \\ 0 \text{ otherwise} \end{cases}$

$\Omega_{ik} \triangleq$ *set of potential facilities j partially covering area i at least* β_k

$X_j \triangleq \begin{cases} 1 \text{ if a facility is sited at location } j \\ 0 \text{ otherwise} \end{cases}$

$\alpha_k \triangleq$ *minimum number of facilities needed to complete coverage at level k*

$\widehat{U}_{ik} \triangleq \begin{cases} 1 \text{ if area } i \text{ has secondary coverage at level } k \\ 0 \text{ otherwise} \end{cases}$

$p \triangleq$ *number of facilities to site*

$Y_{ik} \triangleq \begin{cases} 1 \text{ if area } i \text{ is covered at level } k \\ 0 \text{ otherwise} \end{cases}$

$$Maximize \sum_i g_i Z_i \qquad (2.40)$$

$$Maximize \sum_i g_i V_i \qquad (2.41)$$

Subject to:

$$\sum_{j \in \Omega_{ik}} X_j \geq \alpha_k Y_{ik} + \alpha_k \widehat{U}_{ik} \qquad \forall i, k \qquad (2.42)$$

$$\sum_j X_j = p \qquad (2.43)$$

$$\sum_k Y_{ik} = Z_i \qquad \forall i \qquad (2.44)$$

$$\sum_k \widehat{U}_{ik} = V_i \qquad \forall i \qquad (2.45)$$

$$Z_i \geq V_i \qquad \forall i \qquad (2.46)$$

$$X_j = \{0,1\} \qquad \forall j \qquad (2.47)$$

$$Y_{ik} = \{0,1\} \qquad \forall i, k \qquad (2.48)$$

$$\widehat{U}_{ik} = \{0,1\} \qquad \forall i, k \qquad (2.49)$$

$$Z_i = \{0,1\} \qquad \forall i \qquad (2.50)$$

$$V_i = \{0,1\} \qquad \forall i \qquad (2.51)$$

The first objective function (2.40) is to maximize the total demand that receives primary coverage and the second objective function (2.41) is to maximize the total demand that receives secondary coverage. Constraint (2.42) links facility siting decisions to primary and secondary coverage of demand area i. Constraint (2.43) sets a limit on the number of facilities that must be sited. Constraint (2.44) determines whether primary coverage is provided at some level k. Constraint (2.45) determines whether secondary coverage is provided at some level k. Constraint (2.46) states that secondary coverage can only be obtained if primary coverage is already provided. Constraints (2.47) to (2.51) are binary constraints on the decision variables.

2.6.2.2 Explicit-Model

$i \triangleq$ *index of demand areas*

$j \triangleq$ *index of potential facility sites*

$k \triangleq$ *index of coverage levels* $(1,2,3, \ldots, K)$

$l \triangleq$ *index of facility configurations*

$\Psi'_{ik} \triangleq$ *set of k facility configurations partially or completely covering area i*

$g_i \triangleq$ *demand of service in area i*

$c_{ikl} \triangleq$ *fraction of area i covered by configuration l containing k facilities*

$Z_{ikl} \triangleq \begin{cases} 1\ if\ area\ i\ is\ suitably\ covered\ at\ level\ k\ by\ configuration\ l \\ 0\ otherwise \end{cases}$

$o_{ikl} \triangleq\ fraction\ of\ area\ i\ receiving\ secondary\ coverage\ by\ configuration\ l$

$containing\ k\ facilities$

$X_j \triangleq \begin{cases} 1\ if\ facility\ is\ sited\ at\ location\ j \\ 0\ otherwise \end{cases}$

$p \triangleq\ number\ of\ facilities\ to\ site$

$\Delta'_{ikl} \triangleq\ set\ of\ k\ facilities\ in\ configuration\ l\ that\ partially\ or$

$completely\ cover\ area\ i$

$$Maximise\ \sum_i \sum_k \sum_{l\in\Psi'_{ik}} g_i c_{ikl} Z_{ikl} \quad (2.52)$$

$$Maximise\ \sum_i \sum_k \sum_{l\in\Psi'_{ik}} g_i o_{ikl} Z_{ikl} \quad (2.53)$$

Subject to:

$$\sum_k \sum_{l\in\Psi'_{ik}} Z_{ikl} \leq 1 \qquad \forall i \quad (2.54)$$

$$\sum_j X_j = p \quad (2.55)$$

$$X_j \geq Z_{ikl} \qquad \forall i, k, l \in \Psi'_{ik}, j \in \Delta'_{ikl} \quad (2.56)$$

$$X_j = \{0,1\} \qquad \forall j \quad (2.57)$$

$$Z_{ikl} = \{0,1\} \qquad \forall i, k, l \in \Psi'_{ik} \quad (2.58)$$

The first objective function (2.52) is to maximise the demand that receives primary coverage, the second objective function (2.53) maximises the demand that receives secondary coverage. Constraint (2.54) stipulates that at most one level configuration combination can account for the coverage of demand area i. Constraint (2.55) sets a limit on the number of facilities that must be located. Constraint (2.56) states that coverage can only be provided by sites where facilities have been located. Constraints (2.57) and (2.58) are binary constraints on the decision variables.

The BCLP-Explicit and the MCLP-Explicit models have the same constraints, the only difference is that the BCLP-Explicit model provides additional secondary coverage (Murray et al: 2010).

2.7 Conclusion

From the literature it can be see that numerous location models have been developed and used to solve problems faced by various industries. The key issue is to be able to identify which of the location models would deliver the best results for the locations of RVs.

All three location problems discussed in this literature review are linked and make the following assumptions: demand is known and service coverage is certain (Murray et al: 2010). By using these models as a framework, other characteristics can also be taken into consideration for example capacity issues, facility availability and uncertainty (Murray et al: 2010) to adapt the model to the specific situation.

This chapter provides the building blocks of the dissertation to develop a generic approach to model and solve location problems associated with RVs that are used by the private security industry of South Africa. These models are addressed in the subsequent chapters.

Chapter 3

Explicit Network Coverage Model

3.1 Introduction

The aim of this model is to assist security companies (that operate in a small area) to improve the service they provide to their customers by responding to customers as rapidly as possible when service is required. The model uses the company's data as inputs to determine the best locations for reaction vehicles, taking operational constraints into consideration. The model also shows in which area each vehicle must operate (see figure 8 below).

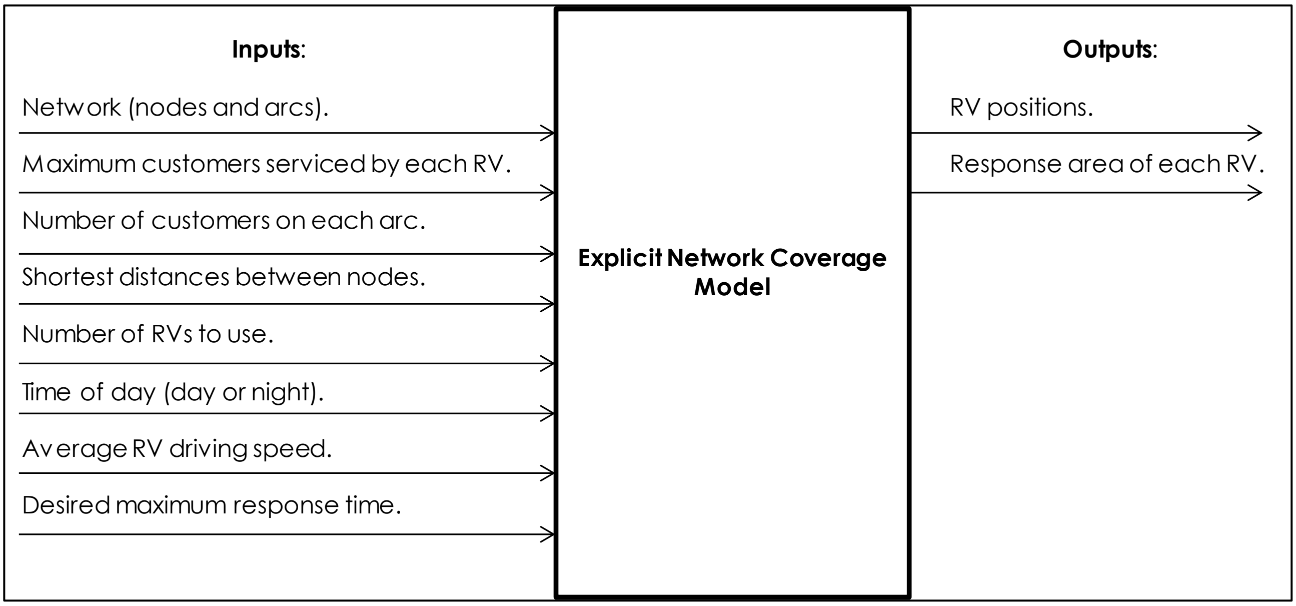

Figure 8 Explicit Network Coverage Model (Inputs and Outputs)

This model should primarily be used in the following situations:

- When a company wishes to allocate a fixed number of vehicles to a small area with a high crime rate.
- When a company has a very small customer base.
- When a company operates in a very small area.

To fully comprehend this model it is important for the user to understand that demand in this industry consists of both of the following:

- Customer: A physical entity that pays a monthly fee to a security company to monitor their alarm system.

- Call-Out: This is the action that takes place when an alarm is triggered and a RV must respond to the customer. It is possible for a customer to not have any call-outs during a specific time period.

Throughout this chapter three different datasets will be used to demonstrate how to obtain the inputs for the model, how to execute and program the model and how to interpret the results generated by the explicit network coverage model (see appendix A for the various datasets).

3.2 Mathematical Model

Define the sets of indices I, J, K ∈ {1, 2,..., n}, where n is the number of nodes in the network.

$$X_k \triangleq \begin{cases} 1 \text{ if node } k \text{ should be a fixed location for RVs, where } k \in K \\ 0 \text{ otherwise} \end{cases}$$

$$S_{kij} \triangleq \begin{cases} 1 \text{ if the arc between nodes } i \text{ and } j \text{ is serviced by RVs located at node } k, \\ \text{where } i \in I, j \in J \text{ and } k \in K \\ 0 \text{ otherwise} \end{cases}$$

$$N_{ij} \triangleq \begin{cases} 1 \text{ if nodes } i \text{ and } j \text{ are linked with an arc, where } i \in I \text{ and } j \in J, \forall\, i < j \\ 0 \text{ otherwise} \end{cases}$$

$$T \triangleq \text{the given maximum number of customers that can be serviced by an RV}$$

$$W_{ij} \triangleq \begin{cases} \text{the weight allocated to arc } ij \text{ if weight is allocated to the arc,} \\ \text{where } i \in I \text{ andand } j \in J \\ 1 \text{ otherwise} \end{cases}$$

$$C_{ij} \triangleq \text{the number of customers situated on arc } ij, \text{where } i \in I \text{ and } j \in J$$

$$AS \triangleq \text{the given average RV driving speed } \left(\frac{km}{h}\right)$$

$$DR \triangleq \text{the given maximum response time (minutes)}$$

$$D_{ki} \triangleq \text{the shortest distance from node } k \text{ to node } i, \text{where } i \in I \text{ and } k \in K$$

$$D_{kj} \triangleq \text{the shortest distance from node } k \text{ to node } j, \text{where } j \in J \text{ and } k \in K$$

$$R \triangleq \text{the given number of RVs to use}$$

$$D \triangleq \begin{cases} 1 \text{ if the time of day is day} \\ 2 \text{ if the time of day is night} \end{cases}$$

The formulation is given as:

$$\text{Minimize } \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} W_{ij} \times \frac{D_{ki} + D_{kj}}{2} \times S_{kij} \qquad (3.1)$$

Subject to:

$$\sum_{k \in K} X_k = \left\lceil \frac{R}{D} \right\rceil \qquad (3.2)$$

$$0 \leq \left[\left(\sum_{k \in K} S_{kij}\right) - 1\right] + M_1 \times y_{ij} \qquad \forall i \in I, j \in J, \ j > i \qquad (3.3)$$

$$N_{ij} \leq M_2\left(1 - y_{ij}\right) \qquad \forall i \in I, j \in J, j > i \qquad (3.4)$$

$$y_{ij} = \{0,1\} \qquad \forall i \in I, j \in J, j > i \qquad (3.5)$$

$$\sum_{k \in K} S_{kij} \leq 1 \qquad \forall i \in I, j \in J \qquad (3.6)$$

$$S_{kij} - X_k \leq 0 \qquad \forall i \in I, j \in J, k \in K \qquad (3.7)$$

$$\sum_{i \in I} \sum_{j \in J} C_{ij} \times S_{kij} \leq T \times D \qquad \forall k \in K \qquad (3.8)$$

$$\frac{D_{ki} + D_{kj}}{2} \times S_{kij} \leq AS \times \frac{1000}{60} \times DR \qquad \forall i \in I, j \in J, k \in K \qquad (3.9)$$

$$X_k \in \{0,1\} \qquad \forall k \in K \qquad (3.10)$$

$$S_{kij} \in \{0,1\} \qquad \forall i \in I, j \in J, k \in K \qquad (3.11)$$

Explaining the objectives:

The objective (3.1) stated above is to reduce the average time/distance travelled from the fixed location to the customer where the service is required. The weight is fixed for each arc, this means the model will allocate RVs closer to arcs with a higher weight. Constraint (3.2) states that only a certain number of positions for RVs must be determined. Constraints (3.3), (3.4) and (3.5) state that if the nodes are connected by an arc they must be serviced by at least one RV, it is only necessary to execute for j>i, since arc ij and arc ji refer to the same arc. Constraint (3.6) ensures that only one vehicle is servicing each street. Constraint (3.7) allows assignment only to sites to which RVs have been allocated. Constraint (3.8) ensures that each RV can only service the maximum predetermined number of customers. Constraint (3.9) places a limit on the maximum distance from the RV's location to the demand point where service is required, taking into consideration the desired maximum response time and the average driving speed of RVs. Constraints (3.10) and (3.11) are binary constraints.

3.3 Model Assumptions

- Deterministic model.
- The locations of RVs are fixed (the location do not change with time).
- The locations of a fixed number of RVs must be determined.
- A customer/demand point is covered if the RV is within an acceptable distance.

- RVs service the arcs within the network.
- The positions of the RVs are on the nodes of the network.
- Each arc will only be serviced from one location, this means that areas serviced by the RVs will not overlap.
- During the night two RVs will be located together for safety purposes, if an uneven number of RVs are used management must decide which of the RVs should be grouped together and which should be located singly. If all RVs should be grouped an additional RV must be used or one less should be used.
- The number of customers and number of call-outs on the arc carries the same weight, thus 10 customers on the arc would carry the same weight as 10 call-outs on the arc.

3.4 Model Execution

The following diagram has been compiled to assist the user in the use of the mathematical model (see figure 9 below):

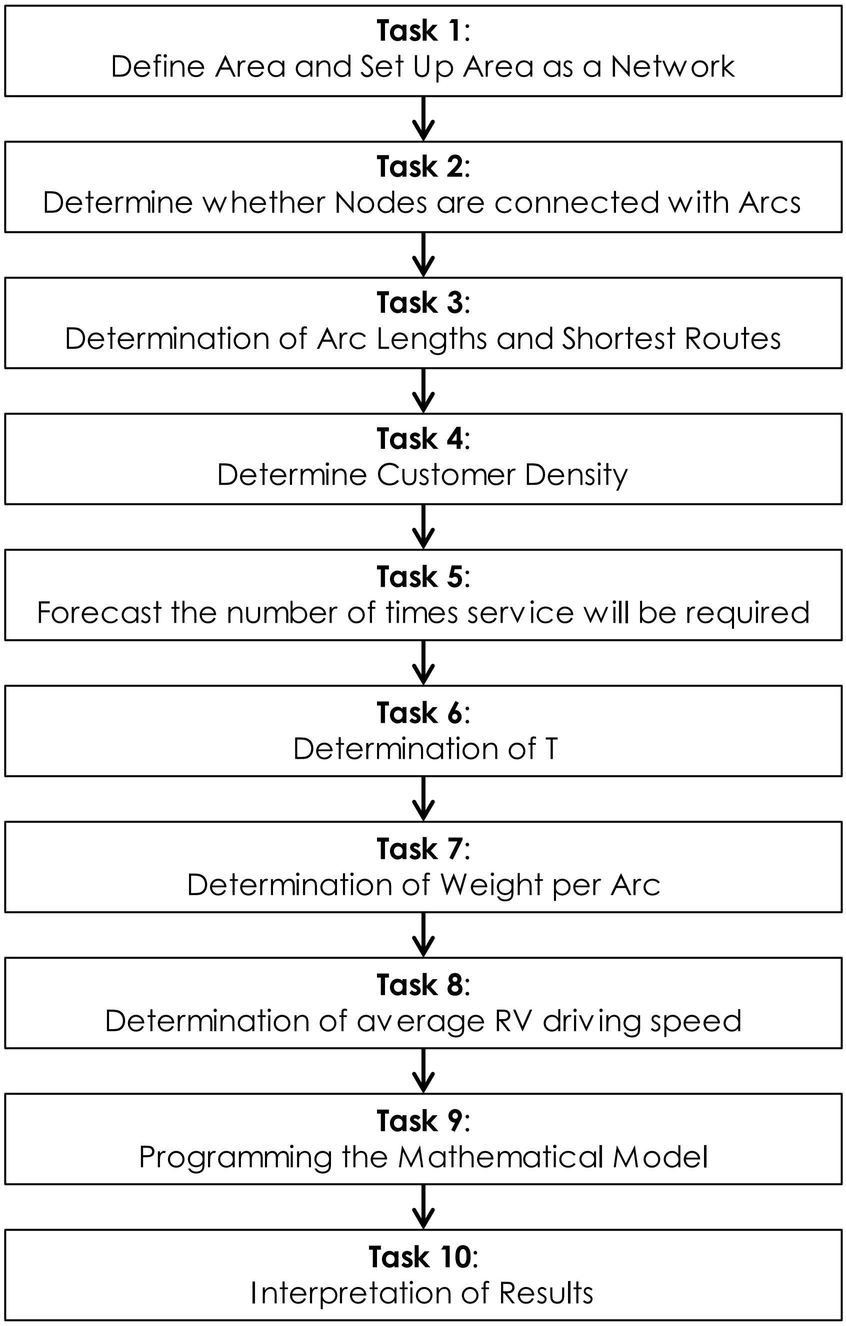

Figure 9 Model Execution Tasks

Task 1: Define Area and Set Up Area as a Network

It is the responsibility of management to determine the area for which the model will be used (the operating area). When making the decision with regard to area size, management must bear in mind the capability of the available linear programming software and the hardware capability. Not all of the available linear programming solver softwares and versions allow the same number of constraints and variables.

A map of the area should be obtained for reference of specific positions, areas and streets. It is also required by SAIDSA that the company must have a map displayed showing the locations of RVs and their operating areas at all times. For an area to be used in a mathematical model, it must be set up as a network consisting of nodes and arcs. When the intersections (nodes) on the map are connected via a road it is designated as an arc. Each node on the map must be labelled with a number to be used as reference points.

Figure 10 below shows the area that will be used in the model and figure 11 shows the area where the nodes have already been labelled (the same area will be used for all three scenarios). Randgate, a suburb of Randfontein, will be used as the area to which the mathematical model will be applied. The area is bordered by the red lines in the figures below.

Figure 10 Map Extract

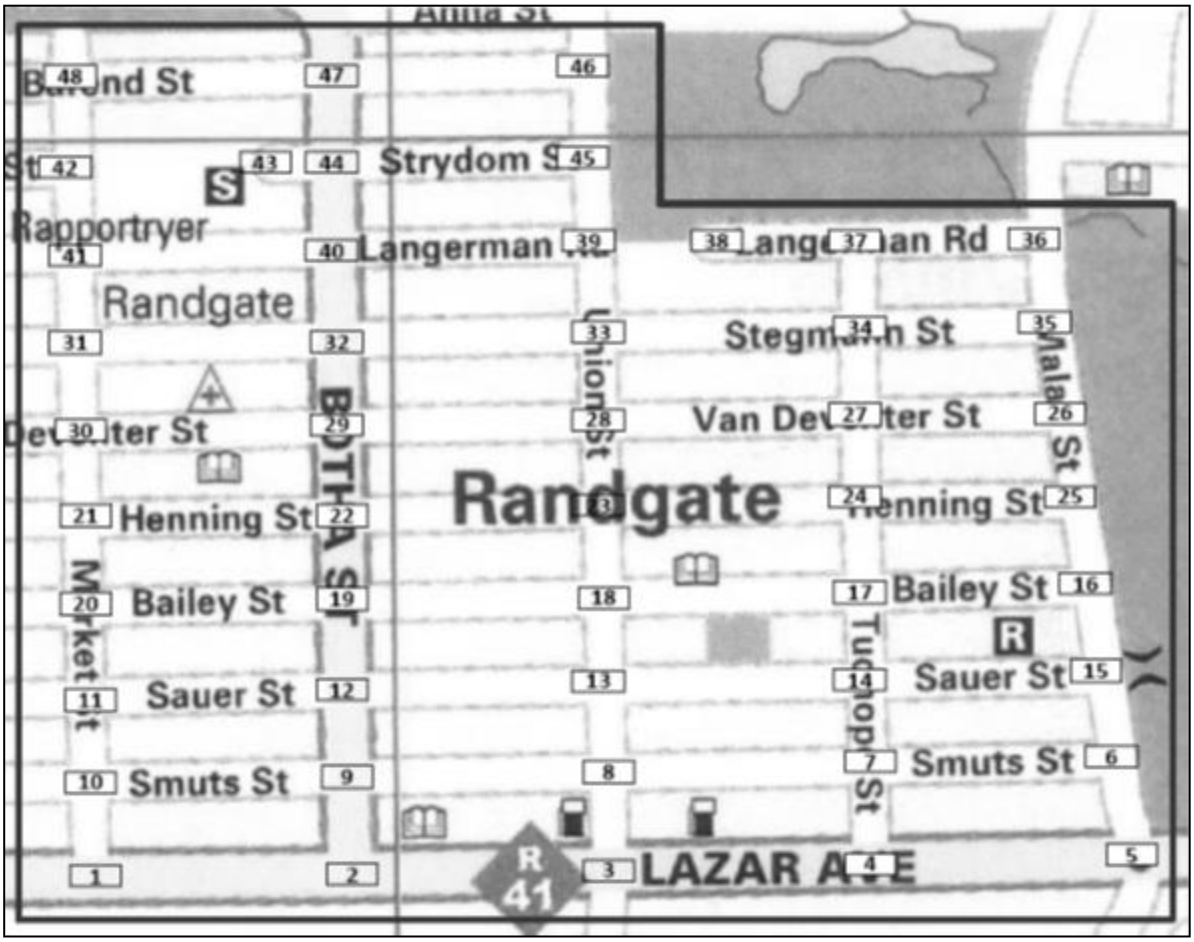

Figure 11 Defining the Area

Task 2: Determine whether Nodes are connected with Arcs

When two nodes in the network are connected with an arc it must be labelled in the matrix (N_{ij}) with an 1 otherwise it must be labelled with a 0. Only the portion above the diagonal of the matrix needs to be completed.

All values below the diagonal of the matrix must be zero, when the value of N_{ij} is 1 the mathematical model will make sure that arc *ij* is serviced. Arc *ij* refers to the same street section as arc *ji*, thus only arc *ij* or arc *ji* need to be serviced.

The table below shows an extract of the completed matrix– note that all of the values below the diagonal are zero (for the complete matrix see appendix A).

	1	2	3	4	5	6	7	8	9	10
1	**0**	1	0	0	0	0	0	0	0	1
2	0	**0**	1	0	0	0	0	0	1	0
3	0	0	**0**	1	0	0	0	1	0	0
4	0	0	0	**0**	1	0	1	0	0	0
5	0	0	0	0	**0**	1	0	0	0	0
6	0	0	0	0	0	**0**	1	0	0	0
7	0	0	0	0	0	0	**0**	1	0	0
8	0	0	0	0	0	0	0	**0**	1	0
9	0	0	0	0	0	0	0	0	**0**	1
10	0	0	0	0	0	0	0	0	0	**0**

Table 3 Matrix Nij

Task 3: Determination of Arc Lengths and Shortest Routes

In order to determine the travelling distances of the RVs and the distance from the fixed location to the point where service is required (customer), the length of the arcs in the network is required.

Floyd's Algorithm is an algorithm that is frequently used to find the shortest path between all of the nodes within a network. It differs from Dijkstra's algorithm in the sense that when using Dijkstra's algorithm one must specify a node to which the shortest distance must be determined in advance. Floyd's algorithm determines the shortest path between all of the nodes in the network at once, so you do not need to specify any nodes in advance when using the algorithm.

Floyd's algorithm deals with a network n nodes. The network is viewed as a square matrix. This means it has exactly the same number of rows and columns. Entry (i, j) of the matrix gives the distance d_{ij} from node i to node j. This entry in the matrix is finite if i and j are connected directly, otherwise the entry is set as infinity.

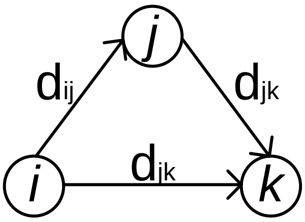

Figure 12 Floyd's Algorithm

Floyd's algorithm is based on the following logic: if three nodes i, j and k are given (figure 12 above) with the distances displayed on the three arcs. It would be shorter to reach k from i passing through j if $d_{ij} + d_{jk} < d_{ik}$. If this is the case it would be better to replace the direct route from node i to node k with the indirect path going through node j. This principle is applied to the whole network in a systematic fashion.

Steps for the algorithm as described by Kasana et al. (2004):

Step 0: Define the starting distance matrix D^0 as given subsequently. The diagonal elements given are marked with a very large number (10^{15}) to indicate that they are blocked. Set $k = 1$.

Step k: Define row k and column k as pivot row and pivot column. Apply the triangle operation to each entry d_{ij} in D^{k-1}, for all i and j if the condition $d_{ik} + d_{kj} < d_{ij} \quad (i \neq k, j \neq k, i \neq j)$ is satisfied, the following change needs to be made:

- Construct D^k by replacing d_{ij} in D_{k-1} with $d_{ik} + d_{kj}$;

Set $k = k + 1$, and repeat step k until no changes are given by triangle operation.

Thus, after n steps, one may determine the shortest distance between nodes i and j from matrix D^n by using of the following rule:

- From D^n, d_{ij} gives the shortest distance between nodes i and j.

All arc lengths must be captured in a matrix format, this matrix will not be used in the mathematical model but is necessary for calculating the shortest distance between the various nodes. The matrix will be symmetrical around the diagonal as long as there are no one-way streets within the area and have a size of n×n, where n is the number of nodes present in the network (a macros has been developed in Microsoft Excel, using Visual Basic to automatically complete the section of the matrix below the diagonal, see Appendix A). When two nodes are not connected by an arc, the cell must contain a very large value, this is necessary for the execution of Floyd's Algorithm (a value of 10^{15} is sufficient). This matrix is used as the input to Floyd's Algorithm, to determine the shortest distance.

The table below shows the lengths of the various arcs, this is only an extract of the matrix for the complete matrix see appendix A. The length of the arcs will be the same for all the datasets, since the same area is used for all datasets.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	**0**	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150
2	250	**0**	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15
3	1E+15	250	**0**	250	1E+15	1E+15	1E+15	150	1E+15	1E+15
4	1E+15	1E+15	250	**0**	250	1E+15	150	1E+15	1E+15	1E+15
5	1E+15	1E+15	1E+15	250	**0**	150	1E+15	1E+15	1E+15	1E+15
6	1E+15	1E+15	1E+15	1E+15	150	**0**	250	1E+15	1E+15	1E+15
7	1E+15	1E+15	1E+15	150	1E+15	250	**0**	250	1E+15	1E+15
8	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	**0**	250	1E+15
9	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	**0**	250
10	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	**0**

Table 4 Matrix dij

To determine the shortest route from one node in the network to another node in the network, Floyd's Algorithm must be used. Floyd's algorithm can be effectively used since it determines the distances between all nodes in the network as proven by Kasana & Kumar (2004).

A Macro was developed in Microsoft Excel with the use of Visual Basic to solve Floyd's Algorithm, see appendix C.

The output of the Floyd's Algorithm is used to compile matrix D_{ij}, which will be used as an input to the mathematical model. An extract of matrix D_{ij} can be viewed in table 5 below (for a complete matrix, refer to appendix A). Matrix D_{ij} is the same for all of the datasets, since the same area is used for all of the scenarios.

	1	2	3	4	5	6	7	8	9	10
1	0	250	500	750	1000	1150	900	650	400	150
2	250	0	250	500	750	900	650	400	150	400
3	500	250	0	250	500	650	400	150	400	650
4	750	500	250	0	250	400	150	400	650	900
5	1000	750	500	250	0	150	400	650	900	1150
6	1150	900	650	400	150	0	250	500	750	1000
7	900	650	400	150	400	250	0	250	500	750
8	650	400	150	400	650	500	250	0	250	500
9	400	150	400	650	900	750	500	250	0	250
10	150	400	650	900	1150	1000	750	500	250	0

Table 5 Matrix Dij

Task 4: Determination of Customer Density

The customer density refers to the number of customers living in each street. There are various ways to determine this number - the company can either obtain the data from a database if the function is available or it can be determined by using a complete customer list that contains information about customer addresses.

A list of streets together with the number of customers situated in each street is given in table 6 below. The customers per street data will be the same for all of the datasets, because the same area is used in all instances.

Street Name	Customers Per Street
Bailey Street	30
Barend Street	7
Botha Street	21
Henning Street	3
Langerman Street	15

Malan Street	16
Market Street	22
Sauer Street	0
Smuts Street	0
Stegmann Street	12
Strydom Street	17
Tudhope Street	19
Union Street	25
Van Deventer Street	29

Table 6 Customer Density

Task 5: Forecast the number of times service will be required

For the company to strategically position their vehicles in areas, they must attempt to predict/forecast where the service will be required. It would be advantageous for a company to position vehicles closer to either (i) an area where more customers are situated and/or (ii) where more call-outs or burglaries are likely to occur (this mathematical model takes both of these factors into account, by calculating a weight for each arc). Although less than 10% of all call-outs are actual burglaries (IMC Reaction:2013) companies cannot distinguish between call-outs and actual burglaries since they are obliged by law to respond to any situation (alarm triggered).

To forecast the number of times a service will be required, historical data first needs to be captured. According to SAIDSA, all private security companies in South Africa must keep a record of the information when a service is delivered. This information includes how frequently a vehicle responds to a specific area or to a specific customer. Some companies store this data on an area-level (for example 50 responses in Pretoria and 90 responses in Johannesburg), where other may store it at street-level (for example 10 responses in Church Street and 13 responses on Pretorius Street). To obtain better results from the mathematical model, it would be best for the company to capture the information at street-level, but processing the area-level information can also be used effectively.

The number of times service was required for the previous periods must be obtained - if more periods are used, the results of the forecasting will be more accurate. A minimum of three periods will be sufficient for the purposes of this model.

The user must first attempt to forecast using linear regression. If this is insufficient forecasting must be done by using moving averages. For the complete process on how to perform forecasting, refer to appendix E.

The data needs to be processed further if it was captured on an area-level. For the model to work efficiently all data needs to be at a street-level. The following formula must be used to transform the data from area-level to street-level:

$$Fs = \frac{FaCs}{Ca}$$

Fs : Forecast number of call-outs at street-level.

Fa : Forecast number of call-outs at an area-level.

Ca : The number of customers in the area.

Cs : The number of customers in the street.

In the scenarios used, all of the data was captured at a street-level, thus further processing is not needed.

The table below shows the forecast number of call-outs for each street, this data is the same for all the datasets since the same area is used for all instances.

Street Name	Historical Data		
	Number of Times Service was required during month X		
	1	2	3
Bailey Street	1199	1154	1357
Barend Street	886	860	964
Botha Street	209	217	268
Henning Street	484	112	151
Langerman Street	997	868	763
Malan Street	728	1374	1472
Market Street	113	447	201
Sauer Street	20	20	20
Smuts Street	20	20	20
Stegmann Street	434	647	960
Strydom Street	992	991	994
Tudhope Street	764	1432	858
Union Street	333	260	173
Van Deventer Street	85	92	26

Table 7 Historical Data (Call-Outs)

Street Name	Linear Regression								
	Calculations		Slope	Y Intercept	Forecast Value for Next Period	Linear Regression Error			
	SSXY	SSX	b1	b0	Y4	SSR	SSE	SST	r^2
Bailey Street	158	2	79	1078.666667	1395	12482	10250.66667	22732.66667	0.549077686
Barend Street	78	2	39	825.3333333	981	3042	2816.666667	5858.666667	0.519230769
Botha Street	59	2	29.5	172.3333333	290	1740.5	308.1666667	2048.666667	0.849576961
Henning Street	-333	2	-166.5	582	-84	55444.5	28153.5	83598	0.663227589
Langerman Street	-234	2	-117	1110	642	27378	96	27474	0.996505787
Malan Street	744	2	372	447.3333333	1935	276768	50050.66667	326818.6667	0.846854933
Market Street	88	2	44	165.6666667	342	3872	56066.66667	59938.66667	0.064599368
Sauer Street	0	2	0	20	20	0	0	0	#DIV/0!
Smuts Street	0	2	0	20	20	0	0	0	#DIV/0!
Stegmann Street	526	2	263	154.3333333	1206	138338	1666.666667	140004.6667	0.988095635
Strydom Street	2	2	1	990.3333333	994	2	2.666666667	4.666666667	0.428571429
Tudhope Street	94	2	47	924	1112	4418	257094	261512	0.016894062
Union Street	-160	2	-80	415.3333333	95	12800	32.66666667	12832.66667	0.997454413
Van Deventer Street	-59	2	-29.5	126.6666667	9	1740.5	888.1666667	2628.666667	0.662122749

Table 8 Call-Outs Linear Regression

Street Name	Moving Averages	Forecast Value to be Used
	Forecast Value for Next Period	
	Y4	
Bailey Street	1236.666667	1237
Barend Street	903.3333333	903
Botha Street	231.3333333	290
Henning Street	249	249
Langerman Street	876	642
Malan Street	1191.333333	1935
Market Street	253.6666667	254
Sauer Street	20	0
Smuts Street	20	0
Stegmann Street	680.3333333	1206
Strydom Street	992.3333333	992
Tudhope Street	1018	1018
Union Street	255.3333333	95
Van Deventer Street	67.66666667	68

Table 9 Forecast Call-Outs

Task 6: Determination of T

T is the maximum number of clients that can be serviced by each RV. No legislation or standard currently exists that can be used to determine the maximum number of customers that each vehicle is required to service. Companies have their own unique policies that are compiled by their management. Such policies may contain information regarding the number of customers that are serviced by each individual vehicle. The datasets have different values for T, these can be seen below:

Dataset 1:	100
Dataset 2:	100
Dataset 3:	100
Dataset 4:	50

Task 7: Determination of Weight per Arc

Many streets cover long distances (some extending over multiple areas). It may therefore be possible for an RV to only service parts of a street. This is why each individual street must first be divided into several sections. Being a network model, this can easily be achieved since a street will consist of multiple arcs. The following formula can be used as a guideline to determine the number of arcs per street:

$A = N - 1$

A : The number of arcs in the street.

N : The number of nodes in the street.

The weight, in both the single facility location model and the multiple facility location model, is the number of trips between the facility and a specific point (Francis, White: 1974) (Love, Morris, Wesolowsky: 1988). In the location model for RVs, the number of trips between the fixed location and the point where service is required are unknown and a weight must be calculated by taking into account the forecast number of occurrences where service will be required and the customer density.

The following formulae must be used to determine the weight of each arc:

$$Wa = \left(\left(\frac{F_s}{F_T} \times 100\right) + \left(\frac{C_s}{C_T} \times 100\right)\right) \div A$$

Wa : The weight per arc.

C_T : The total number of customers in the area.

Cs : The number of customers in the street.

F_T : The total number of call-outs forecast for area.

Fs : The number of call-outs forecast per street.

A : The number of arcs in the street.

The formula above assumes that the weight for both number of customers and number of call-outs are the same. This means if the specific arc has 10% of the customers and 10% of the call-outs the two factors will carry the same weight. See table 10 below.

Wa	F_s	F_T	C_s	C_T	A
7.78	60	200	50	300	6
7.78	50	300	60	200	6

Table 10 Weight Calculations

If a company should decide that the number of customers must carry a higher or lower weight, with regard to the number of call-outs, the multiplying factor should be adjusted accordingly (larger number for more weight, lower number for less weight).

If an arc has zero value, then it should be changed to one, the zero value suggests that an arc either does not have any customers or if it has customers that no call-outs have been triggered during the time period.

This data should be used to compile matrices C_{ij} and W_{ij} that are used as inputs in the mathematical model. These matrices are the same for all of the datasets, the complete matrices can be seen in appendix A.

Task 8: Determination of average RV driving speed

*T*he mathematical model calculates the maximum distance that a vehicle is allowed to travel to the point where service is required. According to the standard specified by SAIDSA, all call-outs must be serviced within 15 minutes. The management of the company can decide what the maximum allowed response time of their vehicles to the point where service is required should be.

To determine the maximum allowed travel distance the user must initially determine the average speed that RVs travel at. Due to the geography and street layout differing from area to area, vehicles don't necessarily achieve an average speed that is similar to the speed limit. Private security companies keep record of the average response times and the average distance to the location where service is required for each of their vehicles. The historical data of the RVs has been captured in and the average driving speed of RVs has been calculated. The average driving speed for the various datasets can be seen below:

Dataset 1: 65 km/h

Dataset 2: 65 km/h

Dataset 3: 40 km/h

Dataset 4: 40 km/h

Task 9: Programming the Mathematical Model

Appropriate linear programming software needs to be used to effectively solve the model. When choosing the software to be used, one must bear in mind the number of variables and constraints that will be used. Different software has different limits on the number of variables and constraints.

If the mathematical model is executed and the user is unable to find a solution, it means that an insufficient number of RVs are used and the model is incapable of adhering to all constraints. Additional RVs should be used to solve such a problem.

Task 10: Interpretation of Results

After the model has been solved by using the appropriate linear solving software, it remains the responsibility of management to interpret the results. The model has been programmed and solved in Lingo (see appendix C for the Lingo code).

When the value of X_k is equal to one, it means that intersection *k* should be a fixed location for an RV. Sometimes it is physically impossible for a vehicle to be positioned at the exact calculated location, thus management must determine a position as close to the intersection or node as possible that will suffice.

When the value of S_{kij} is equal to one, the RVs located at node *k* must service the arc between nodes *i* and *j*.

The following figures show the results that have been obtained after the models have been executed.

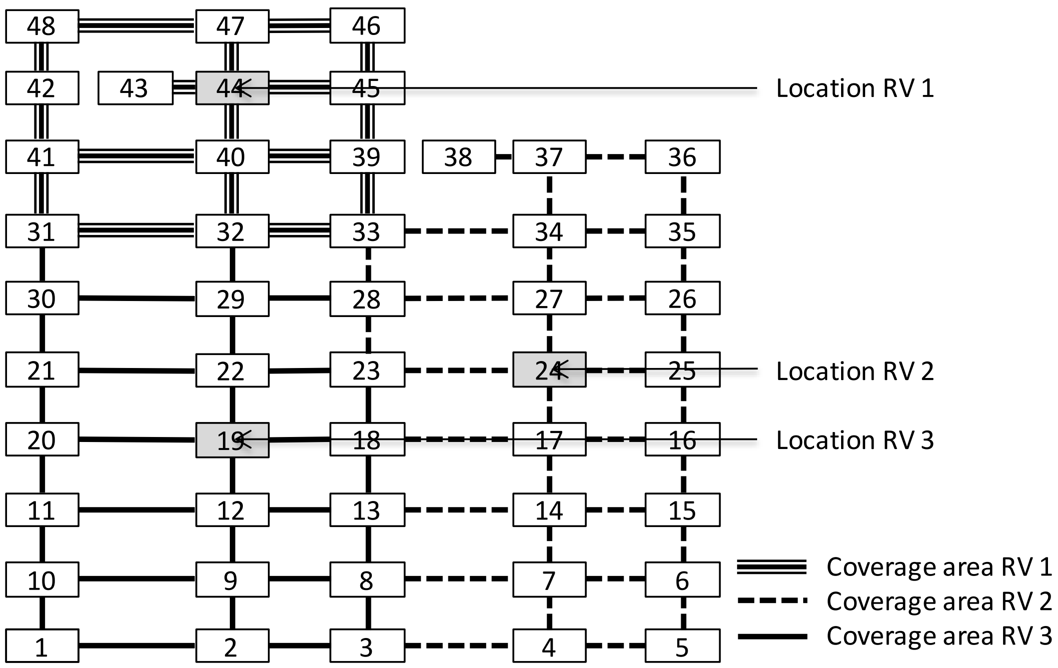

Figure 13 Results Scenario 1

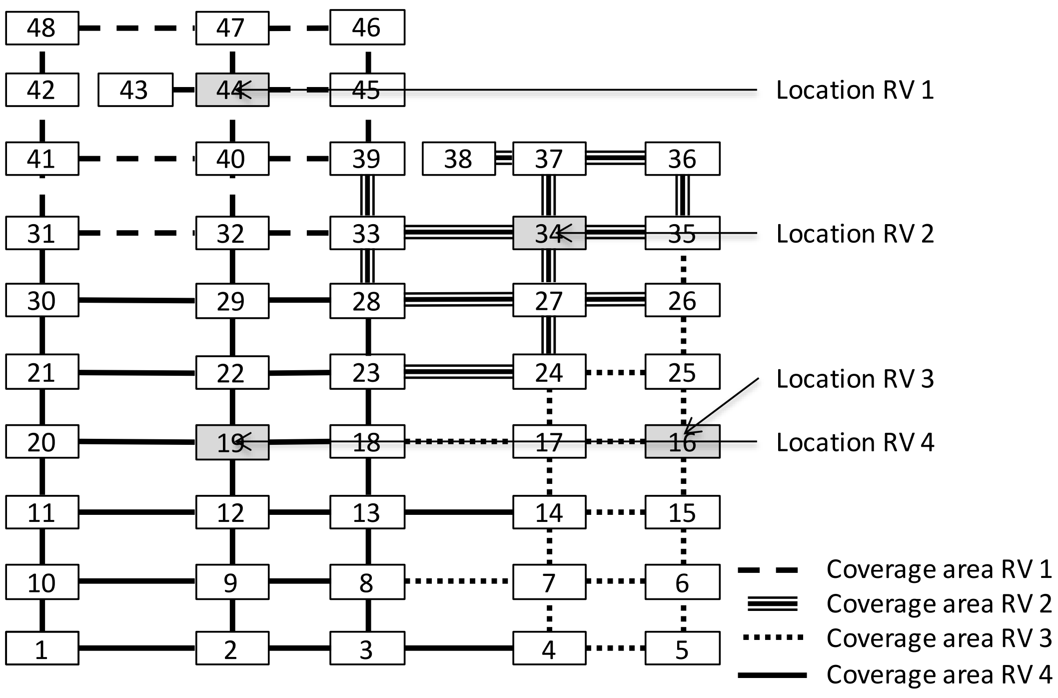

Figure 14 Results Scenario 2

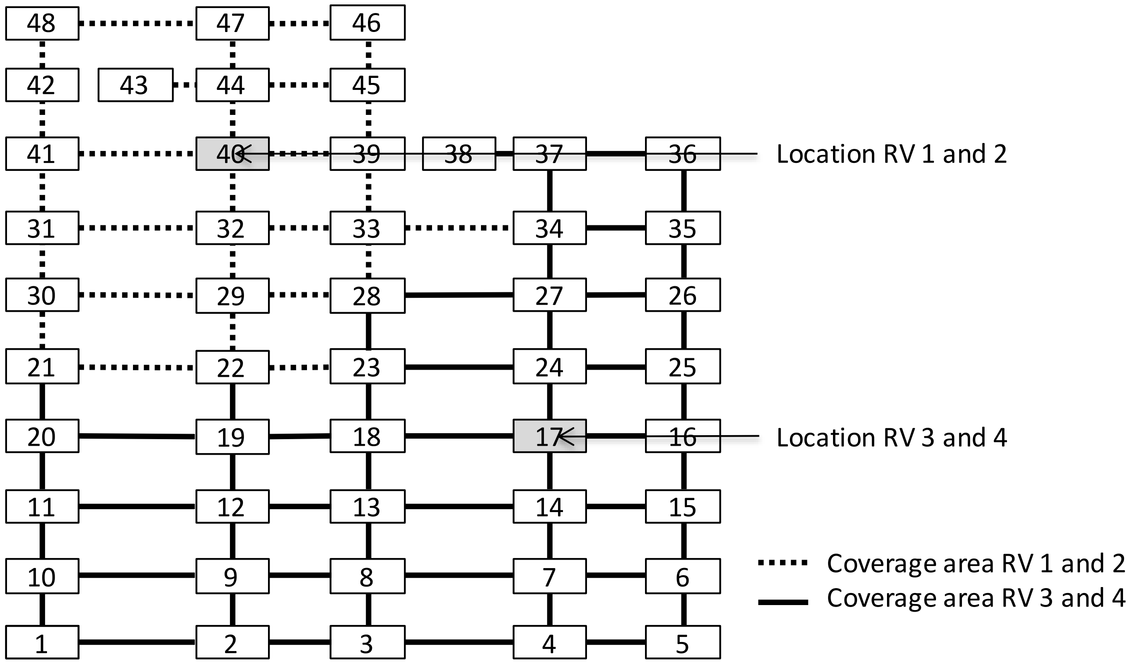

Figure 15 Results Scenario 3

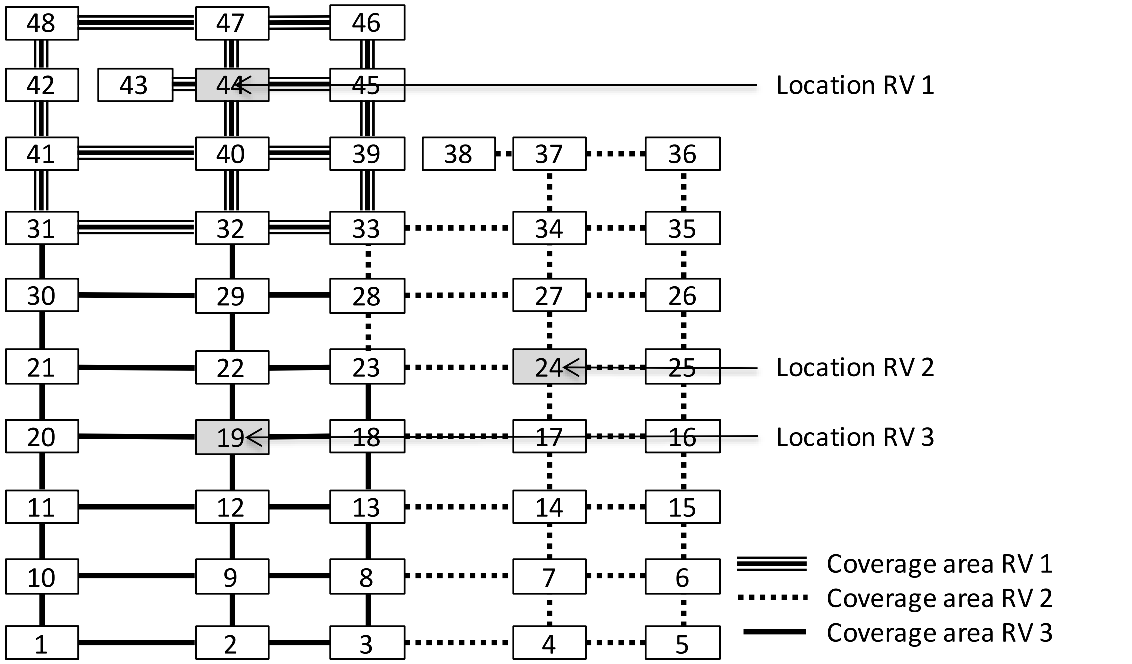

Figure 16 Results Scenario 4

The results generated by the mathematical model can be seen in figures 13 to 16 above.

Figure 13 shows the results of dataset 1. In dataset 1 the locations for three RVs are determined, each RV is allowed to serve up to 100 customers. The time of day used is day. The desired response time is 1.5 minutes and the average driving speed of the RVs has been calculated as 65km/h.

Figure 14 shows the results of dataset 2. In dataset 2 the locations for four RVs are determined, each RV is allowed to serve up to 100 customers. The time of day used is day – this means RVs are parked individually (no grouping is done). The desired response time is 1.5 minutes and the average driving speed of the RVs has been calculated as 65km/h.

Figure 15 shows the results of dataset 3. In dataset 3 the locations for four RVs are determined, each RV is allowed to serve up to 100 customers. The time of day used is night – this means RVs are parked together in groups of two. The desired response time is 1.5 minutes and the average driving speed of the RVs has been calculated as 65km/h.

Figure 16 shows the results of dataset 4. In dataset 4 the locations for six RVs are determined, each RV is allowed to serve up to 50 customers. The time of day used is night – this means RVs are parked together in groups of two. The desired response time is 1.5 minutes and the average driving speed of the RVs has been calculated as 40km/h.

According to the data used, Malan Street (5-6-15-16-25-26-35-36) and Bailey Street (16-17-18-19-20) had the highest number of burglaries, but Bailey Street and Van Deventer Street (26-27-28-29-30) had the highest number of customers. The weight calculated for the model takes the combination of both number of burglaries and number of customers into consideration. The streets with the highest calculated weights were: Bailey Street, Barend Street and Stegmann Street (31-32-33-34-35).

In all four executions of the model, the results placed the RVs very close to the streets with the highest weights, if the RV was not placed directly on one of the nodes of the street.

3.5 Conclusion

In this chapter an explicit network coverage model is presented, explained and executed. This model should be used by smaller security companies that wish to site RVs and allocate specific streets to the various RVs.

The model takes various operational factors and constraints into consideration i.e. weight allocated to streets, the average driving speed of the RVs, the maximum desired response time, the number of RVs available and the time of the day (day or night).

The weight allocated to the various arcs (street sections) takes into consideration both the number of customers and the number of call-outs.

The chapter also has a list of tasks which the user needs to perform to collect the appropriate inputs and get it in the correct format to be used in model execution.

Chapter 4

Responding from a Single Location

4.1 Introduction

In this chapter the response of RVs from a single location (or facility) will be discussed. From literature it can be seen that using a single facility has numerous advantages (Dineshbakshi: 2013):

- Reduced coordination problems within the organisation, because everything is managed from one place.
- There is uniformity within the organisation, everyone needs to adhere to the same policies and plans.
- Centralisation organisations are best suited where resources and information has to move swiftly, especially in emergencies.
- Duplication of functions and facilities is minimised which in turn reduces costs.

If a company is just starting it would be beneficial for them to have one facility and as the company grows, they might consider decentralising their resources.

This chapter can be divided into two phases as shown in the figure below. The first phase is to determine where the centralised facility location should be located, this centralised location can be determined by either using only the customers' locations (Model 1a) or using both the expected number of call-outs and the customers' locations (Model 1b). A company that is only starting will typically not know the number of call-outs that they can expect, since this is dependent on factors of the area in which the company operates for example: crime rate, number of customers and income level of customers.

The second phase is to determine how many RVs should be allocated to this facility. This number can change with time (higher number of call-outs during the evening) or can be fixed throughout the day where the arrival rate is constant throughout the day. The data currently captured does not differentiate between night and day, but it is advised that companies should consider this, research by Stats SA (2012) showed that burglaries occur more frequently during the night. To determine the number of RVs required both the service rate of RVs and the arrival rate of call-outs should be taken into account.

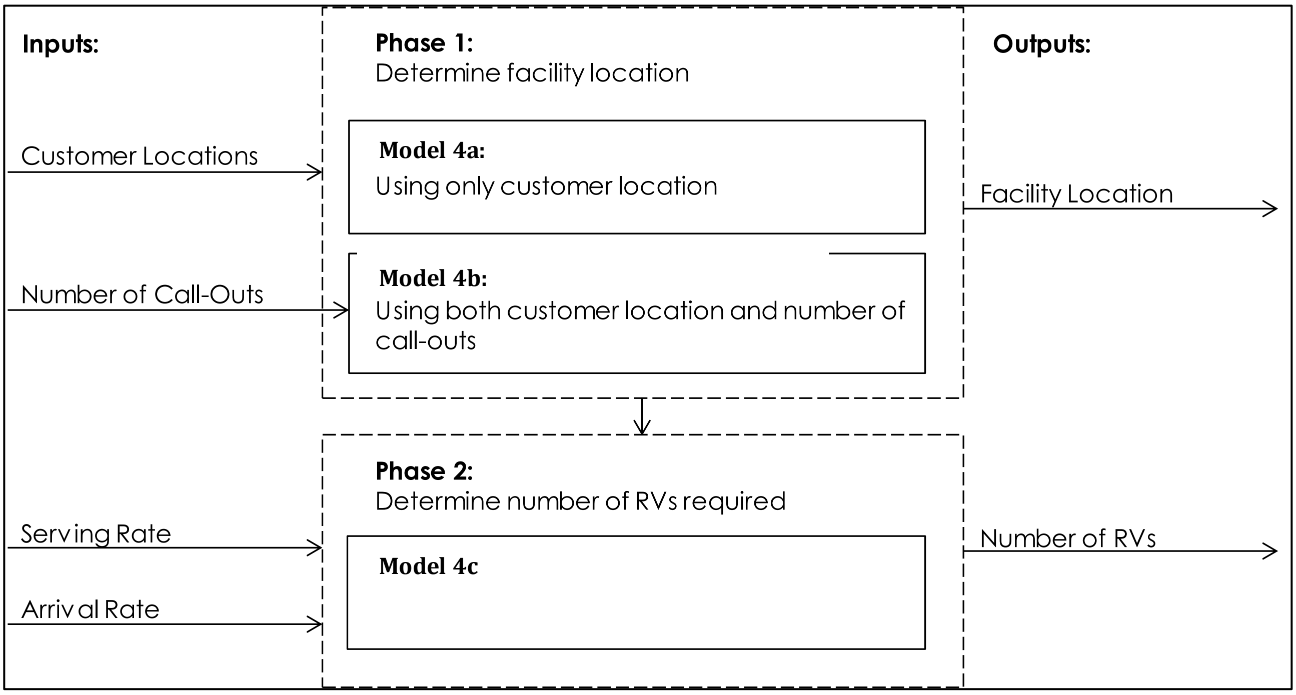

Figure 17 Single Location Model (Inputs and Outputs)

The location problems taken into consideration are all single-facility location problems, because, as the name suggests, all of them are used to determine a good location for a single facility taking specific constraints into consideration. In these problems if one needs to determine the distance from the facility to the customer or demand point, there are various methods used in the literature to determine the distance from the facility to the customer (Tompkins et al: 2010)for example:

- Rectilinear distance: For rectilinear distance measurement, distances are measured along paths that are orthogonal (or perpendicular) to each other.
- Straight-line distance: In straight line distance measurement, distances are measured along the straight line between two points.
- Chebyshew distance: In Chebyshew distance measurement, the distance is the greater one of the horizontal and vertical distance travelled.
- Actual distance: In actual distance measurement, distances are measured along the actual path between two points.

For measuring the distances travelled by RVs, using actual distances are the most accurate. Unfortunately this would be impractical since complicated models must be developed for each area, which will be too time consuming and costly. Rectilinear distance measurement would be sufficient for the use of the models in this section, because most of the streets in South Africa are perpendicular to one another or very close thereto, as can be seen in the figures below.

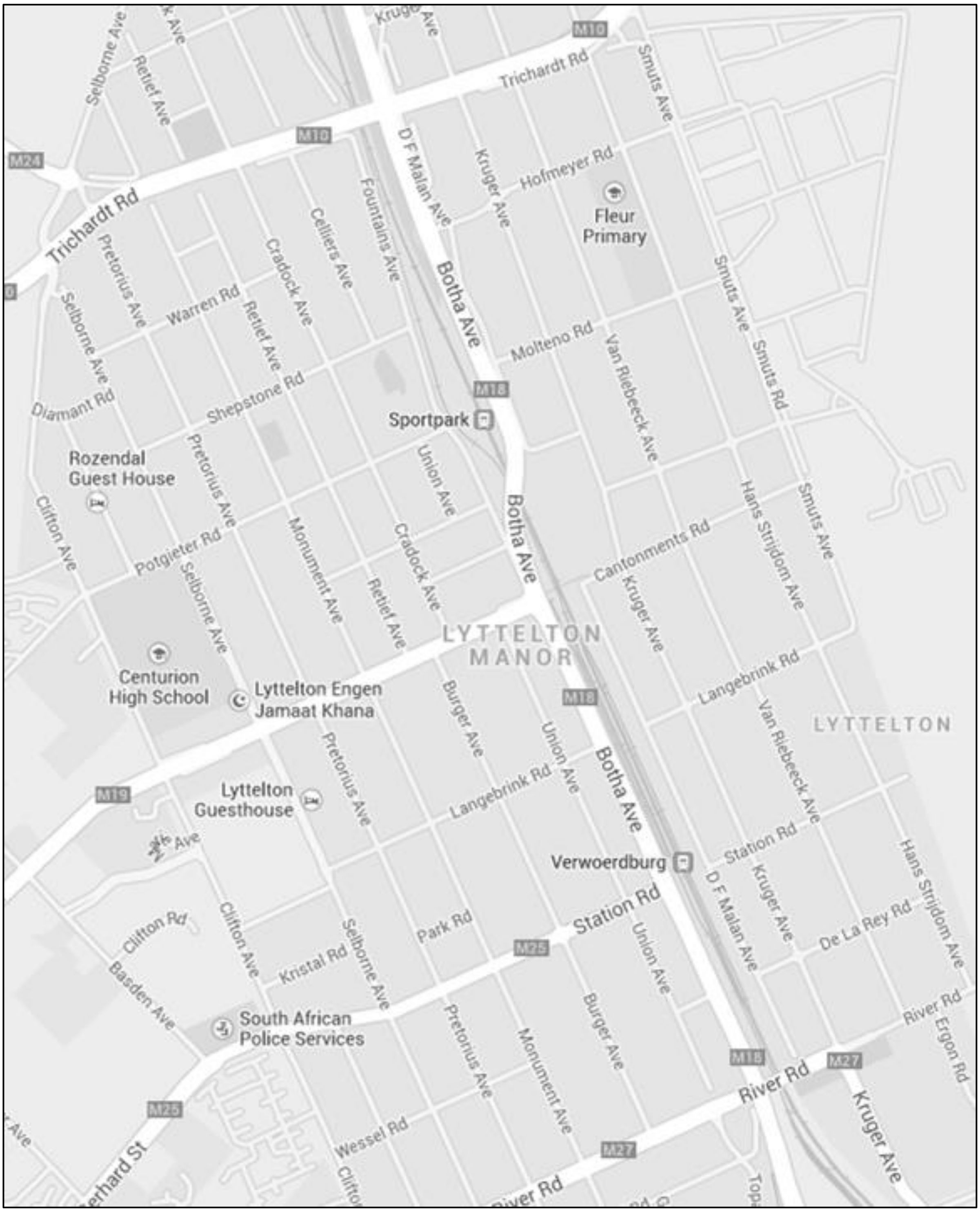

Figure 18 Street Layout South Africa

The models in this section can be used by any privately owned security company and different results would be obtained by each, since the models use each company's unique data independently.

Two single location facility models have been used in this section: these are the minisum location problem and the minimax location problem.

The aim of the minisum location problem is to minimize the weighted distances between the facility and the customers' locations. The aim of the minimax location problem on the other hand is to minimize the distance between the facility and the furthest customer.

When taking these problems into consideration both of them have their advantages and disadvantages, using the minisum location problem (which takes demand and customer location into consideration) would improve the average customer service, but this may cause some customers to be very far away from the facility. This means that poor service would be provided to them.

When a location of a facility needs to be determined and only the customer's locations are taken into consideration it would be better to use the minimax location model, because for a small company just starting their business operations it would be beneficial to be able to respond to each of their clients within the predetermined time. If a customer is too far away from the facility, the RV would not be able to reach the customer within the minimum allowed time.

Throughout this section four datasets will be used to show how the models work (see appendix B).

4.2 Mathematical Model

4.2.1 Using only customer location (Model 4a)

Mathematical formulation for the minimax location problem (Tompkins et al: 2010):

Define the sets of indices $I \in \{1,2,\ldots,n\}$, where n is the number of customers in the area under investigation.

$$x \triangleq the\ x - coordinate\ of\ the\ single\ facility$$

$$y \triangleq the\ y - coordinate\ of\ the\ single\ facility$$

$$a_i \triangleq the\ x - coordinate\ of\ customer\ i, where\ i \in I$$

$$b_i \triangleq the\ y - coordinate\ of\ customer\ i, where\ i \in I$$

The aim of this model is to minimize the distance from the facility to the furthest customer, the formulation is given as:

$$min\, z = max(|x - a_i| + |y - b_i| + g_i) \tag{4.1}$$

To obtain the minimax solution we let:

$$c_1 = minimum\,(a_i + b_i - g_i) \tag{4.2}$$

$$c_2 = maximum\,(a_i + b_i + g_i) \tag{4.3}$$

$$c_3 = minimum\,(-a_i + b_i - g_i) \tag{4.4}$$

$$c_4 = maximum\,(-\,a_i + \ b_i \ + \ g_i) \qquad (4.5)$$

$$c_5 = maximum\,(c_2 \ - \ c_1, c_4 \ - \ c_3) \qquad (4.6)$$

After the values above have been determined the optimal solution would be any point on a line connecting the following points:

$$(x_1^*, y_1^*) = 0.5(c_1 - c_3, c_1 + c_3 + c_5) \qquad (4.7)$$

$$(x_2^*, y_2^*) = 0.5(c_2 - c_4, c_2 + c_4 - c_5) \qquad (4.8)$$

From these equations the maximum distance from the facility to the customers can be determined by:

$$Maximum\,Distance = \ c_5 \div 2 \qquad (4.9)$$

If the maximum distance is further than the distance a RV can travel in the predetermined response time, it is the responsibility of management to address the issue, although it is recommended that a different model (see chapter 5) should be used and the RVs be decentralised.

4.2.2 Using both customer location and number of call-outs (Model 1b)

In the mathematical model, discussed in the previous section one does not take the expected number of call-outs into consideration. In this section the expected number of call-outs are takin into consideration when determining the location of a new facility.

This model will be used by companies that wish to centralise their resources. Both the locations of customers and the expected number of call-outs or the historical call-out data are available.

Define a sets of indices I ∈ {1,2,...,n}, where n is the number of customers in the area.

$x \ \triangleq \ the\ x\ coordinate\ of\ the\ new\ facility$

$y \ \triangleq \ the\ y\ coordinate\ of\ the\ new\ facility$

$a_i \ \triangleq \ the\ given\ x\ coordinate\ of\ customer\ i\ in\ the\ area, where\ i \ \in I$

$b_i \ \triangleq \ the\ given\ y\ coordinate\ of\ customer\ i\ in\ the\ area\ where\ i \ \in I$

$w_i \ \triangleq the\ given\ number\ of\ call-outs\ expected\ at\ customer\ i, where\ i \ \in I$

$d_i \triangleq the\ calculated\ distance\ between\ the\ facility\ location\ and\ customer\ i, i \ \in I$

$$min\,z \ = \ \textstyle\sum_{i \in I} w_i \times d_i \qquad (4.10)$$

$$d_i = |x - a_i| + |y - b_i| \qquad \forall i \in I \qquad (4.11)$$

Explaining the objectives:

The aim (4.10) of this mathematical model is to minimize the average distance from the single facility to the customer where service is required. Equation (4.11) is a calculation to determine the distance between the facility and all of the customers, by using the rectilinear distance method.

4.2.3 Number of RVs to be used (Queueing Theory)

The input of the process is called the arrival process, the entities arriving at the process are the customers (Winston: 2004). In the models used in this section it will be assumed that only one arrival can occur at a given instant – this is a realistic assumption when working with RVs, since multiple arrivals are impossible (an example of a multiple arrival is a group of people arriving at a restaurant). The arrival process of RVs are unaffected by the number of customers currently in the system, a probability distribution will be determined that governs the time between entity arrivals.

To describe the output process of a queueing system, one should specify a probability distribution (service time distribution) that describes the customers' service time. The service time distribution is independent of the number of customers presently in the system, this means a server will not work faster or slower as the number of customers in the system increases or decreases (Winston: 2004). In the queueing system of RVs, the servers (RVs) are in parallel, this means the customer only needs to be serviced by one server (RV) to complete the service (the servers are independent of one another).

The queueing discipline used by RVs is the First Come First Served (FCFS) discipline. This means that customers will be served in the order that they arrive in the system. For the purpose of RVs it will be assumed that both the arrival rate and service rate follow an exponential distribution.

The model can be written as M/M/s/GD/∞ / ∞ and the calculations will be done using the following formulae:

λ : rate at which quantity arrives at system.

μ : rate at which quantity is serviced in system.

s : number of servers in the system.

$$\rho = \lambda / s\mu \tag{4.12}$$

For $\rho \geq 1$, no steady state exists. For $\rho < 1$,

$$\pi_0 = \frac{1}{\sum_{i=0}^{i=s-1} \frac{(s\rho)^i}{i!} + \frac{(s\rho)^s}{s!(1-\rho)}} \tag{4.13}$$

$$\pi_j = \frac{(s\rho)^j \pi_0}{j!} \qquad (j=1,2,\ldots,s) \tag{4.14}$$

$$\pi_j = \frac{(s\rho)^j \pi_0}{s! s^{j-1}} \qquad (j=1, s+1, s+2,\ldots) \tag{4.15}$$

$$P(j \geq s) = \frac{(s\rho)^s \pi_0}{s!(1-\rho)} \tag{4.16}$$

$L_q = \frac{P(j \geq s)\rho}{1-\rho}$: average number of customers waiting in line. (4.17)

$W_q = \frac{P(j \geq s)}{s\mu - \lambda}$: average time a customer spends in line. (4.18)

$L_s = \frac{\lambda}{\mu}$: average number of customers in service. (4.19)

$W_s = \frac{1}{\mu}$: average time a customer spends in service. (4.20)

$L = L_q + \frac{\lambda}{\mu}$: average number of customers present in the queueing system. (4.21)

$W = \frac{L}{\lambda}$: average time customer spends in system. (4.22)

4.3 Model Assumptions

- Arrival rate of call-outs follow an exponential distribution.
- Service rate of RVs follow an exponential distribution.
- Historical/ expected number of call-outs are known when determining the number of RVs to use.
- The arrival rate is constant throughout the day – according to research most crimes are committed during the evening, this should theoretically increase the arrival rate of call-outs during the evening.
- The service rate is constant throughout the day – service rate can vary throughout the day due to external factors such as traffic.
- There is no resource capacity constraint, any number of RVs can be used at any time.

4.4 Model Execution

The following diagram has been compiled to assist the user with the use of the mathematical model (see figure 19 below):

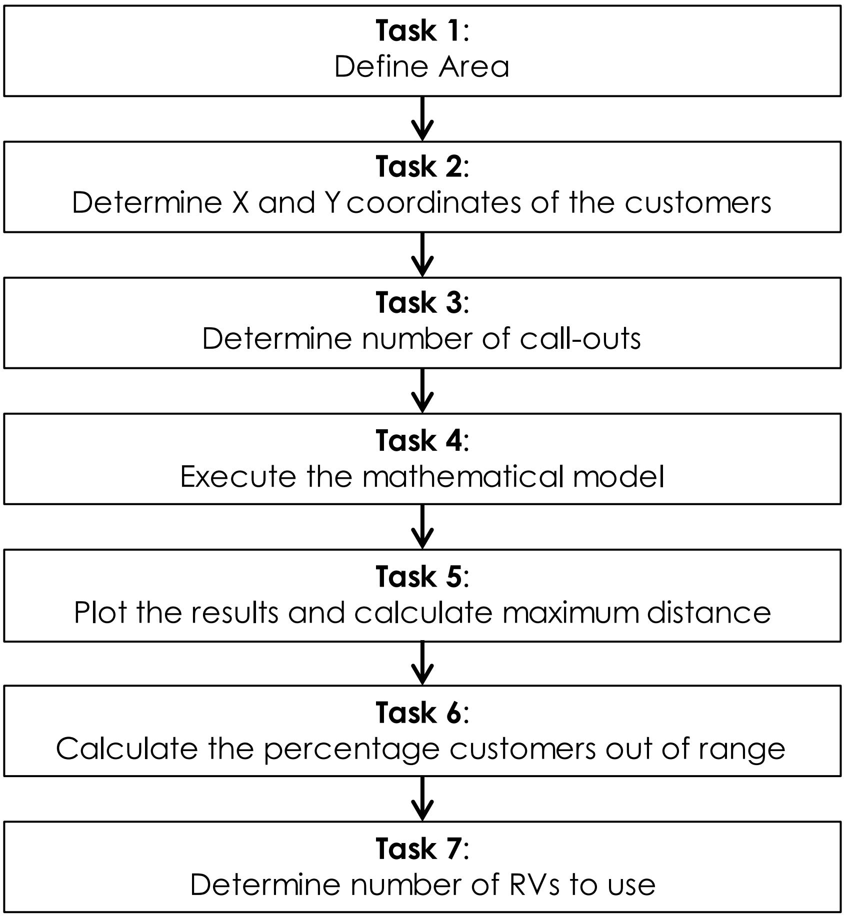

Figure 19 Single Location Model Execution

Task 1: Define Area

It is the responsibility of management to determine the area for which the model will be used (the operating area). The areas for the different datasets are given in the table below:

Dataset:	**Area Size:**	**Width (X):**	**Length (Y):**
Dataset 1	581km^2	31.20 km	18.62 km
Dataset 2	491km^2	41.37 km	11.87 km
Dataset 3	125 km^2	15.90 km	7.86 km
Dataset 4	15 km^2	6.60 km	2.24 km

Table 11 Area Sizes

Task 2: Determine X and Y coordinates of the customers

The models in this section require the X and Y coordinates of all the customers in the area used. It is advised to capture the coordinates in a spread sheet format (Microsoft Excel for example) to increase the ease of use. The table below is a small extract of the X and Y coordinates of one of the datasets used. To view the X and Y coordinates of the customers of all the datasets, refer to appendix B.

Customer Number	Location	
	X	y
1	1.935176	0.677712
2	5.177669	0.031462
3	5.083995	1.828095
4	4.738574	0.101793
5	2.767024	1.93612
6	5.28286	0.838389
7	6.428833	1.955931
8	0.375837	2.131248
9	2.432775	1.178049
10	5.126687	0.120089

Table 12 Customer Location Coordinates

Task 3: Determine number of call-outs

If management wishes to use both the customers' locations and the number of times each customer has requested a service (Model 4b) then the number of call-outs need to be captured. The number of call-outs can be forecast (see appendix E or the data of the previous time period be used.

If the number of call-outs is unavailable (new company for example) or management does not wish to take it into consideration then this task can be excluded and the next task can be performed.

Task 4: Execute the mathematical model

In this section management needs to execute the mathematical model, if the management only use the locations of the customers then the model in section 4.2.1 needs to be executed and if both the number of call-outs and customer locations need to be used then the model in section 4.2.2 needs to be executed.

Both the models have been executed for the various datasets, the datasets can be viewed in appendix B.

Task 5: Plot the results and calculate maximum distance.

In this task, the output of the model needs to be plotted on a graph or map to see where the RVs (or facility) should be positioned. The maximum distance from the location to the customers also need to be determined to help decide which of the RVs will be able to respond to the customers in the desired response time.

Figures 20 to 23 below show the location of the customers and the centralised facility when only customer locations are considered (Model 4a).

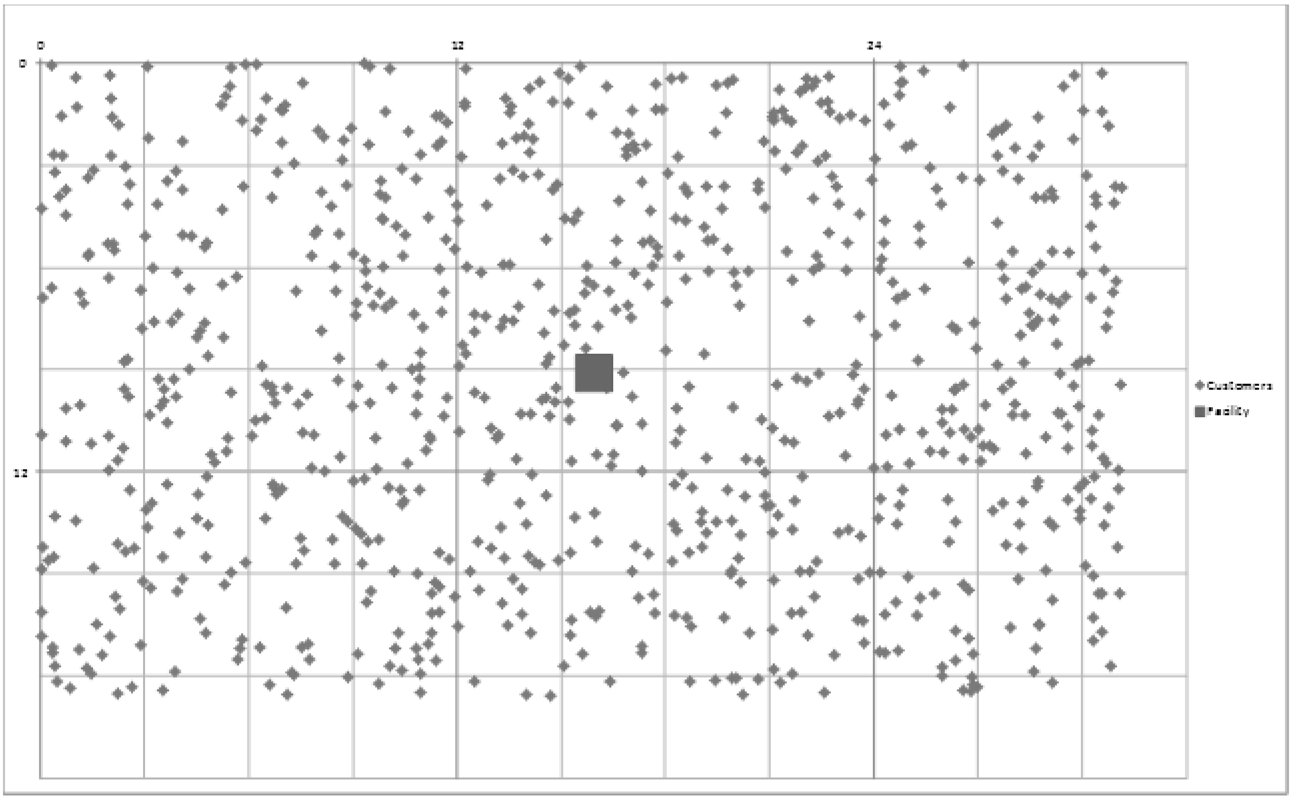

Figure 20 Results Dataset 1 (locations only)

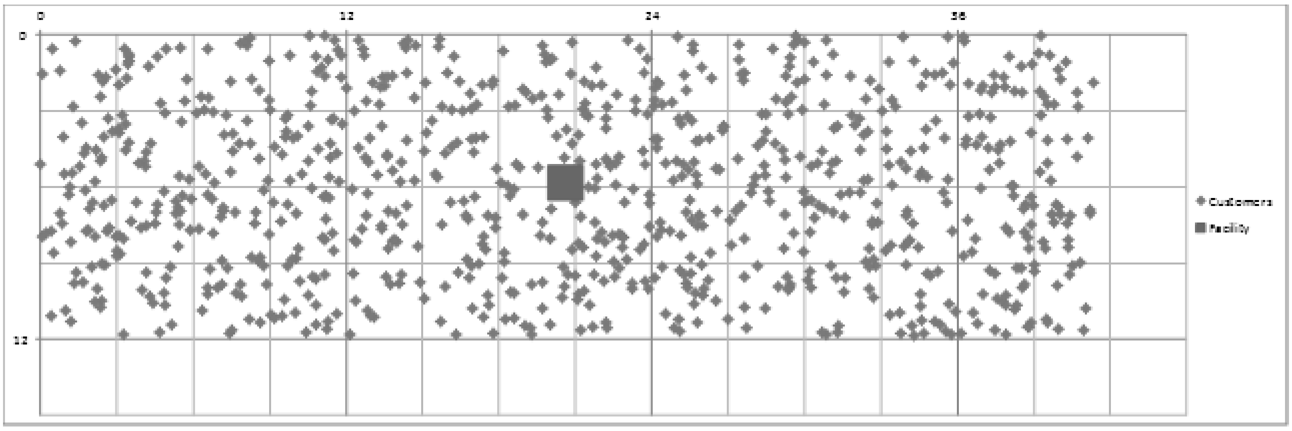

Figure 21 Results Dataset 2 (locations only)

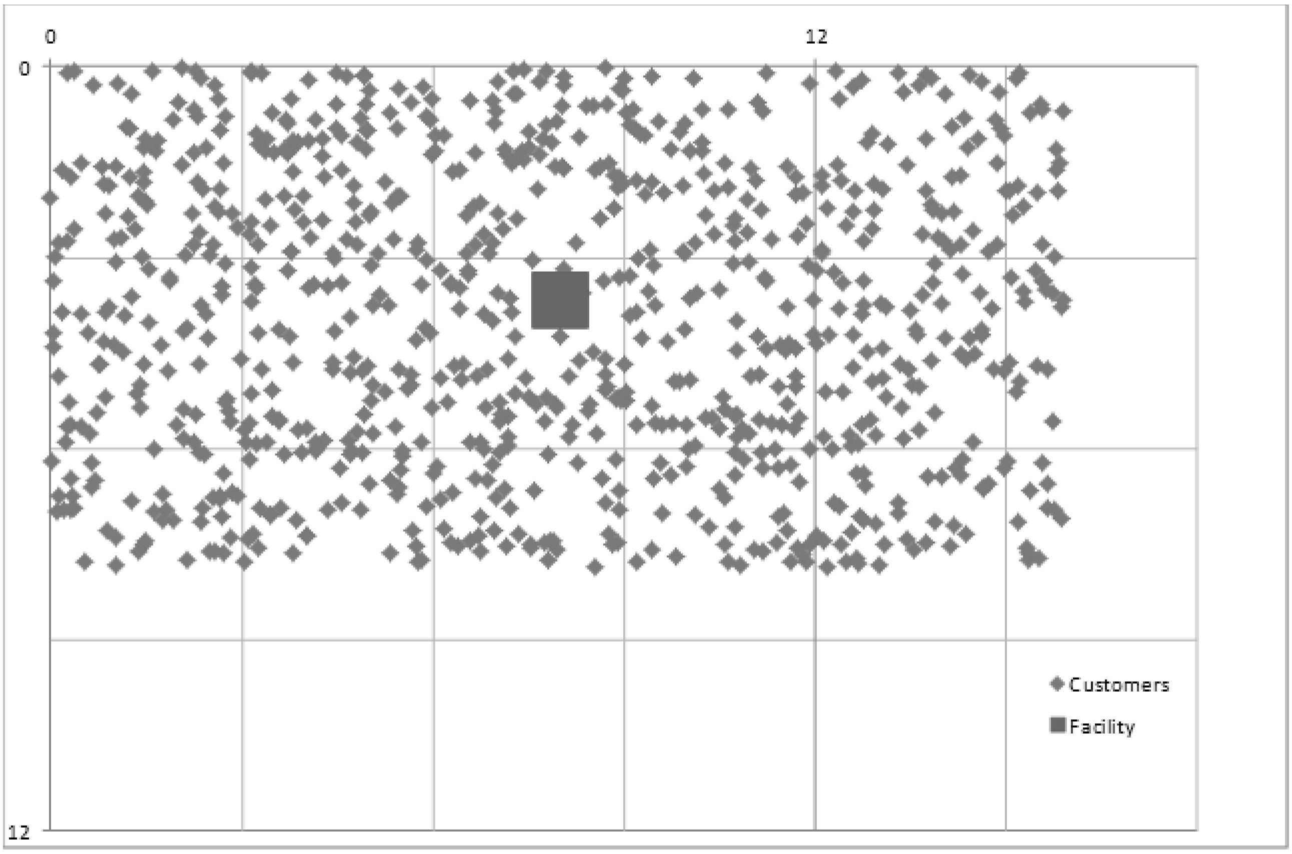

Figure 22 Results Dataset 3 (locations only)

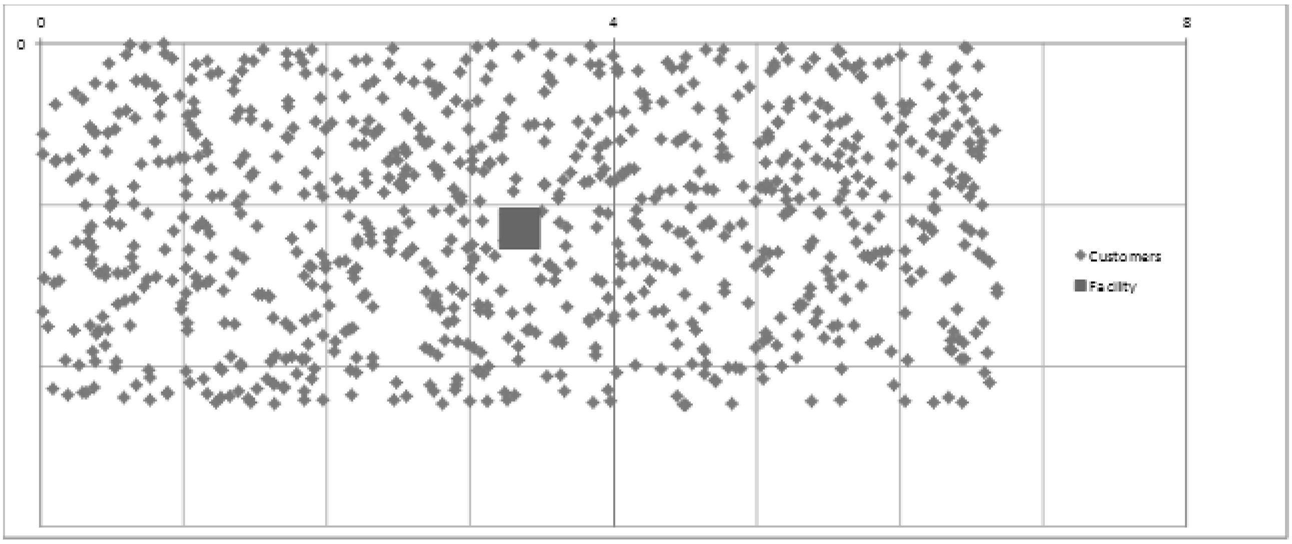

Figure 23 Results Dataset 4 (locations only)

The maximum distance from the centralised facility to the furthest customer needs to be determined, these values are in the table below:

Dataset:	**Maximum distance:**
Dataset 1	24.7 km
Dataset 2	26.27 km
Dataset 3	11.57 km
Dataset 4	4.25 km

Table 13 Maximum Distances

Figures 24 to 27 below show the location of the customers and the centralised facility when both the locations of the customers and the expected number of call-outs are used as inputs (Model 4b).

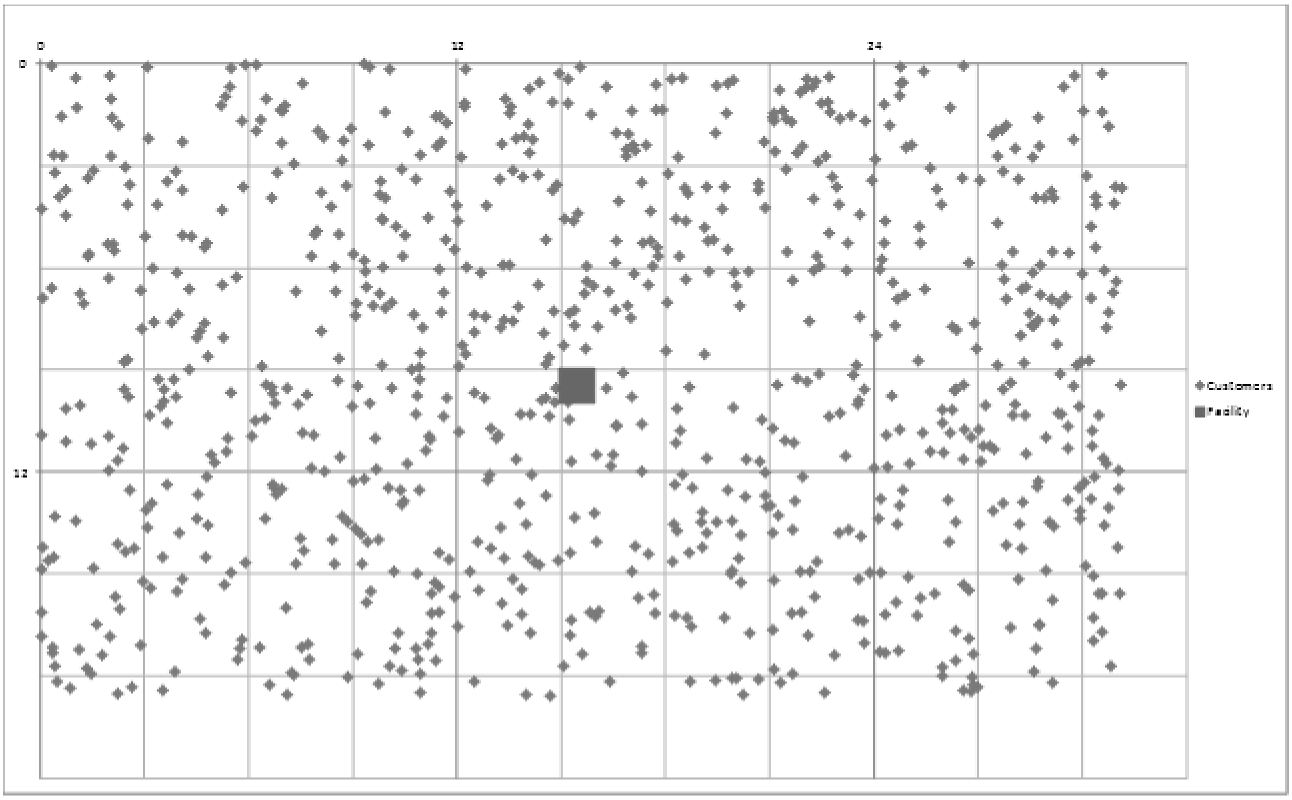

Figure 24 Results Dataset 1 (call-outs and customer location)

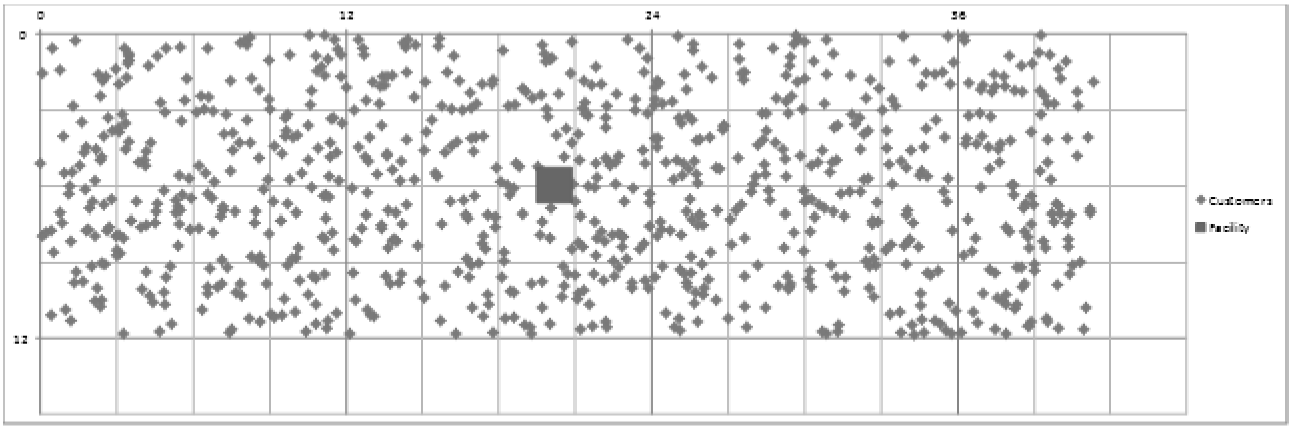

Figure 25 Results Dataset 2 (call-outs and customer location)

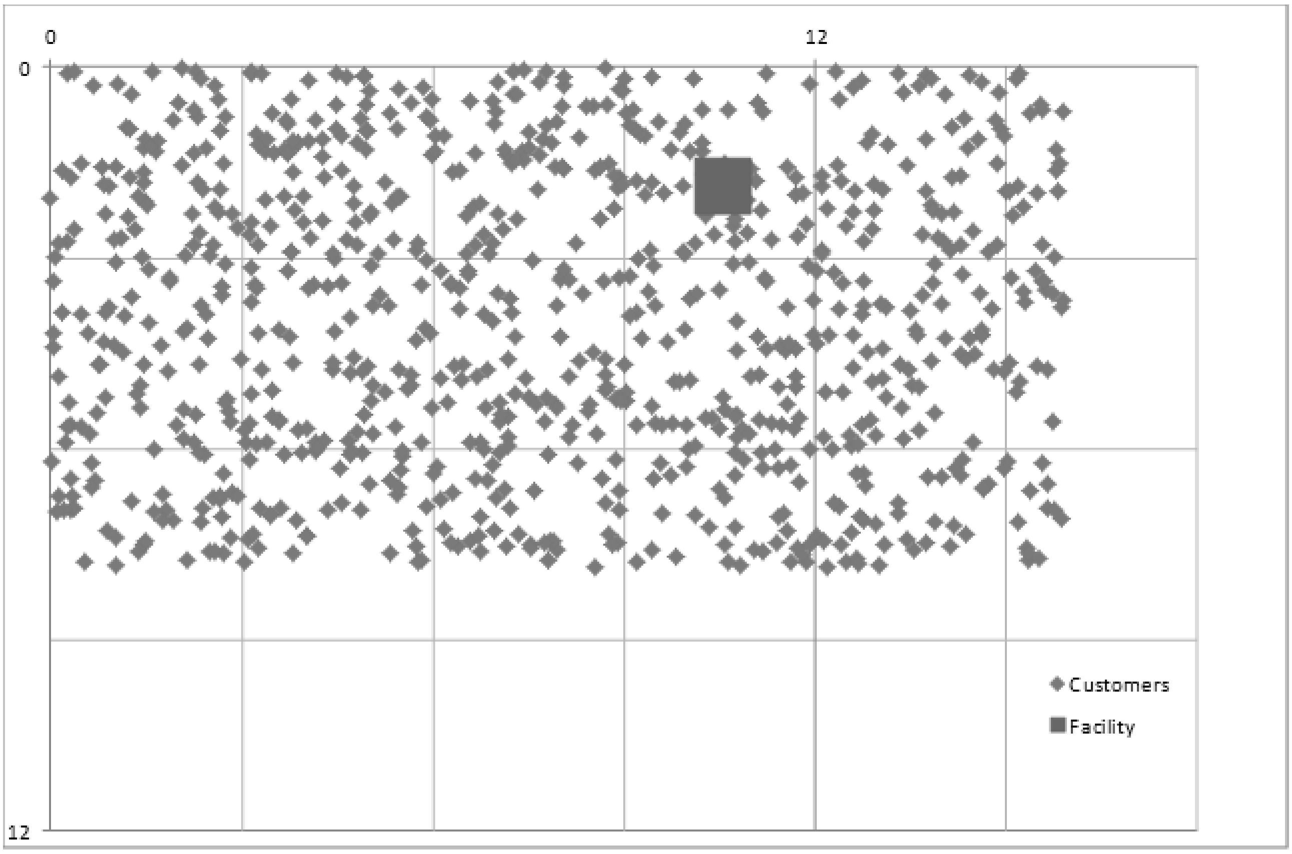

Figure 26 Results Dataset 3 (call-outs and customer location)

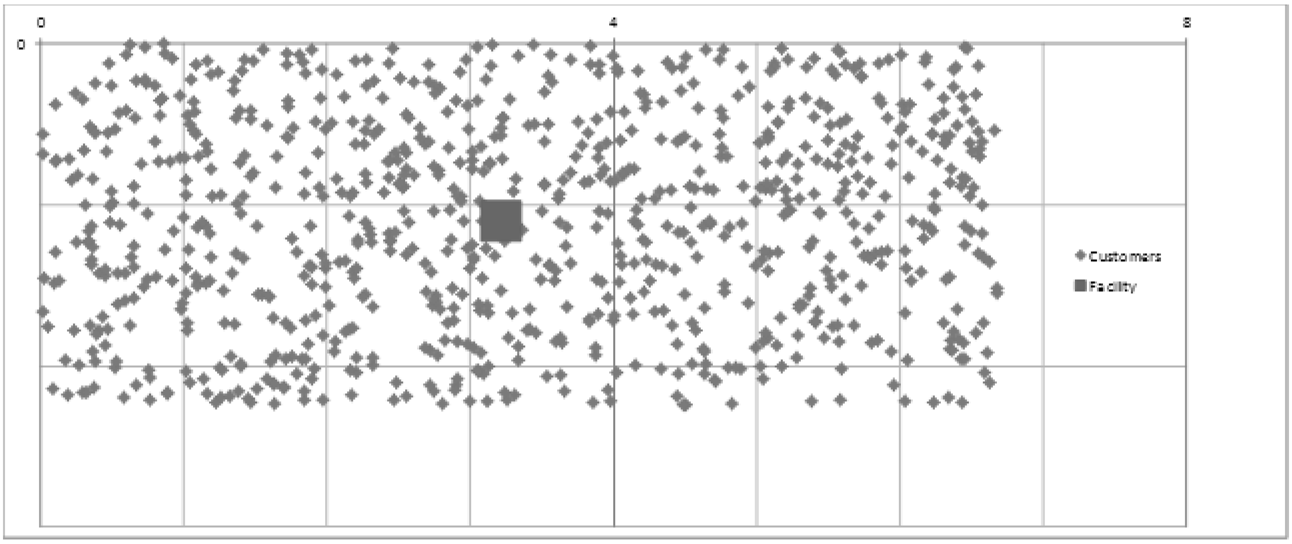

Figure 27 Results Dataset 4 (call-outs and customer location)

The maximum distance from the facility to the furthest customer need to be determined, these values are given in the table below:

Dataset:	Maximum distance:
Dataset 1	24.87 km
Dataset 2	26.75 km
Dataset 3	15.95 km
Dataset 4	4.42 km

Table 14 Maximum Distances

From the results it can be seen that models 4a and 4b locate the centralised facility very close to each other, the number of call-outs expected at the customers does not play a significant role when determining a facility location as expected. A problem identified with model 4b is that the distance from the facility to the furthest customer increases in every instance, independent of the dataset use. When looking at dataset 3, when model 4a was used all the customers were within the desired 15km. When using model 4b the maximum distance is over 15 km, this means that the RVs will not be able to reach the customer in 15 minutes. From these findings it is recommended that model 4a should be used when determining the location of a centralised facility.

If the maximum distance determined is further than the distance an RV is able to travel in the desired response time it is a good indication that the locations of RVs should be decentralised. SAIDSA states that the response time should be less than 15 minutes, if a vehicle travels at an average speed of 60km/h, it will only be possible to respond to customers closer than 15km from the RV's location. In the next task, the percentage of customers out of range will be determined, this will further support management's decision to centralise or decentralise their RVs.

Task 6: Calculate the percentage customers out of range

In this section the percentage of customers out of the desired response range should be determined This task will further support management's decision to centralise or decentralise the locations or RVs. A good example of where this task is important is with the use of dataset 3 when using the model that takes both the customers location and expected number of call-outs. The maximum distance calculated for this dataset is 15.95 km, this means that the customer at this location is just out of range. The task in this section will help determine how many customers are out of range. If it is only one or two customers, management may still wish to use a centralised location for their RVs. The percentage of customers further than 15km (assumes average driving speed is 60km/h) from the facility is given in the tables below.

When the model only takes customer locations into consideration (Model 4a):

Dataset:	**Percentage customers out of range:**
Dataset 1	34.09%
Dataset 2	79.35%
Dataset 3	0%
Dataset 4	0%

Table 15 Percentage Customers out of Range (only location)

When the model takes both customer locations and expected number of call-outs into consideration (Model 4b):

Dataset:	**Percentage customers out of range:**
Dataset 1	34.02%
Dataset 2	40.12%
Dataset 3	0.44%
Dataset 4	0%

Table 16 Percentage Customers out of Range (call-outs and customer locations)

Task 7: Determine number of RVs to use

In this task the number of RVs to be allocate to the facility will be calculated. It is advised that the queueing theory should only be applied when the percentage of customers out of range is 0%, if the percentage of customers that is out of range is greater than 0, the RVs should be placed at decentralised locations, refer to chapter 3 or chapter 5.

To determine the number of RVs to use, the queueing theory discussed in section 4.2.3 will be used.

The mathematical model is given below:

$s \triangleq$ *the number of RVs to use*

$W \triangleq$ *average time a customer spends in the system (in hours)*

DT ≜ the given desired response time (in hours)

$L \triangleq$ *average number of customers present in the queuing system*

$\lambda \triangleq$ *average number of arrivals entering the system (in customers per hour)*

$\mu \triangleq$ *the average service rate by the RVs (in customers per hours)*

$L_q \triangleq$ *the average number of customers waiting in line*

$P(j \geq s) \triangleq$ *the probability that all servers are busy*

$\rho \triangleq$ *service rate to arrival rate ratio*

The mathematical formulation is given as:

$$Minimize\ s \tag{4.23}$$

$$W \leq DT \tag{4.24}$$

$$W = \frac{L}{\lambda} \tag{4.25}$$

$$L = L_q + \frac{\lambda}{\mu} \tag{4.26}$$

$$L_q = \frac{P(j \geq s)\rho}{1-\rho} \tag{4.27}$$

$$\rho = \frac{\lambda}{s\mu} \tag{4.28}$$

Explaining the objectives:

The aim of this model (4.23) is to minimize the number of RVs used, while still providing an acceptable level of service to the customers. (4.24) The average time a customer spends in the system should be less or equal to the desired time determined by management. (4.25),(4.26),(4.27) and (4.28) are all calculations used to calculate the average time a customer spends in the system when using queueing theory.

Because the number of customers that is out of range for both dataset 1 and 2 is greater than zero, it will not be used with queueing theory, it will only be used in the models where the locations of RVs are decentralised (Chapter 5). RVs will physically not be able to respond to most of the customers within 15 minutes.

In task 5 it was seen that Model 4a and Model 4b deliver similar results, in some instance Model 4b locates a facility that is too far for the RVs to reach in 15 minutes, for this reason only the outputs of Model 1a will be used in this section. Datasets 3 and 4 were used as inputs to these models. The models were solved using Lingo, the Lingo code can be viewed in Appendix C.

Dataset 3 Inputs and Results:

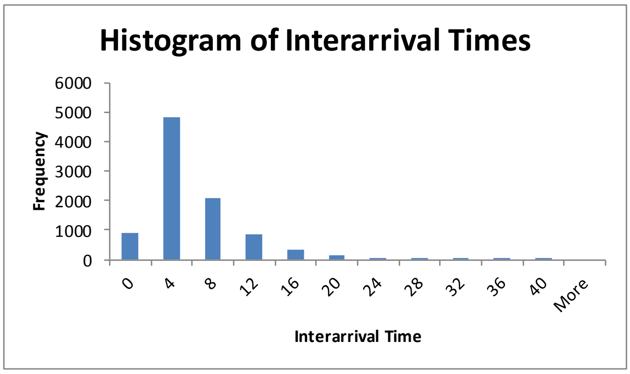

Figure 28 Inter-arrival Times

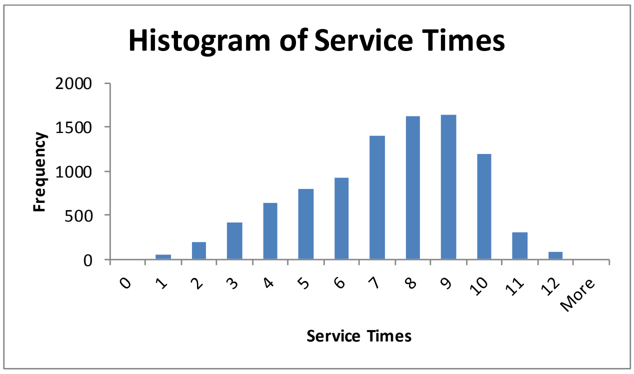

Figure 29 Service Times

Dataset:	**Arrival Rate:**	**Service Rate:**
Dataset 3	12.85 call-outs per hour	8.81 call-outs per hour

Table 17 Arrival Rate and Service Rate

Output:	**Desired Time: 15 min**	**Desired Time: 10 min**
Number of RVs	$1.977 \approx 2$	$2.39 \approx 3$
Average time in system	15 min	10 min
Average number of call-outs in system	3.21 call-outs	2.14 call-outs
Average number of call-outs	1.75 call-outs	0.68 call-outs

waiting in line		

Table 18 Model Outputs

Dataset 4 Inputs and Results:

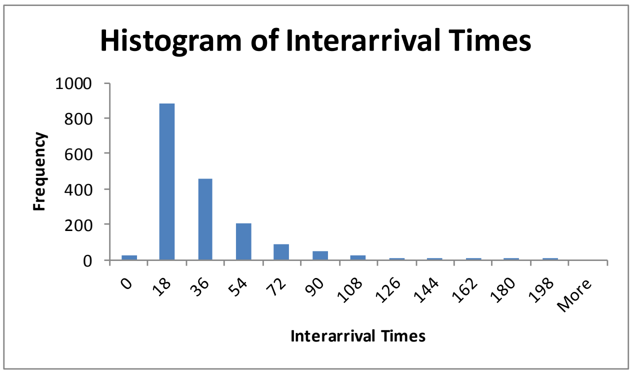

Figure 30 Inter-arrival Times

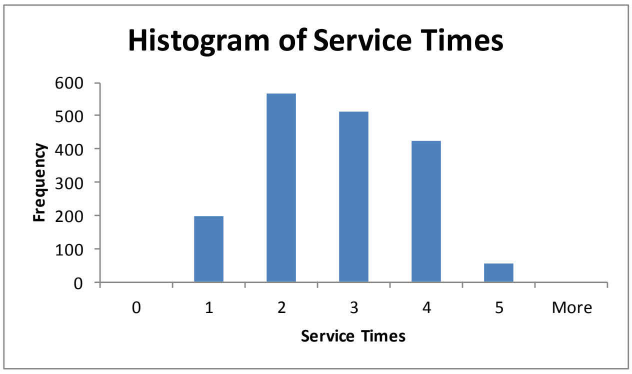

Figure 31 Service Times

Dataset:	Arrival Rate:	Service Rate:
Dataset 4	12.85 call-outs per hour	8.81 call-outs per hour

Table 19 Arrival Rate and Service Rate

Output:	Desired Time: 15 min	Desired Time: 10 min
Number of RVs	$0.24 \approx 1$	$0.32 \approx 1$
Average time in system	15 min	10 min
Average number of call-outs in system	0.61 call-outs	0.41 call-outs
Average number of call-outs waiting in line	0.52 call-outs	0.32 call-outs

Table 20 Model Outputs

4.5 Conclusion

This chapter discusses the response of RVs from a single location. The models will typically be used by security companies that operate in a fairly small area or are just starting their operations.

The chapter is divided into two sections – firstly determining the centralised location and secondly determining the number of RVs to use (using queueing theory). When determining the centralised location the user can either use the locations of the customers or a combination of both customers' locations and expected demand. A criterion is also discussed to determine whether a centralised location can be sufficiently used or if decentralised locations should be considered.

The tasks required to gather data and execute the models are also explained and used to execute various scenarios.

The results shows that RVs used for Dataset 1 and Dataset 2 should rather be decentralised as 34% of the customers of Dataset 1 and 79% of the customers in Dataset 2 won't be serviced within the desired time. In Dataset 3 and Dataset 4 the number of customers too far from the centralised facility to be serviced within the desired time are negligible, thus the facility can be centralised. Dataset 3 requires two RVs to respond to customers within 15 minutes and three RVs to respond to customers within 10 minutes. Dataset 4 requires only one RV to respond to customers within 10 minutes.

Chapter 5

Decentralised Location Modelling with Queueing Theory

5.1 Introduction

In this chapter the response of RVs from multiple locations (or facilities) will be discussed. From literature it can be seen that using multiple facilities has numerous advantages (Langley et al.: 2008):

- As the number of RVs increases, the number of customers that can be serviced by the security company increases.
- The operational area of the company increases.
- The response time to customers is decreased, if facilities are located at the correct strategic sites.
- The number of customers and potential customers that is covered increases.

When a company is initially starting it would be beneficial for them to have one facility and as the company grows, they might consider decentralising their resources – the main reason for security companies to let their RVs respond from multiple locations will be to reduce the response time to the location where a crime or call-out has occurred and to increase the area that is covered.

This chapter is divided into two sections as can be seen in the figure below. In the first section the LSCP will be used but it will be adapted to develop Model 5a, a model that takes the various operational constraints security companies face into consideration. Security companies need to service demand and cover all of their customers. Although these two factors (demand and number of customers) are often directly proportional to each other it can happen that an area with a low number of customers has a high number of call outs and vice versa. This model will inform the management of security companies of how many RVs are required and where they should be located in the operational area to respond to demand within the given limited time period.

In the second section the MCLP problem will be taken into consideration to develop Model 5b. In this model the number of RVs that will be used is a constant constraint, and they will be located to provide the best level of service. This model should be used when the number of RVs the security company has available are insufficient to provide complete coverage to all the customers (output given by Model 5a). It is not desirable for security companies to use this model but often due to external forces (vacation leave, sick leave and strikes) it becomes necessary to use this model. The output of this model will inform management of security companies on where to locate the RVs when resources (RVs) are limited to provide the best level of service. In many cases it is too costly to add another resource when the increase in service level is small due to the increase in number of resources, this model can also be used to track the demand and number of customers covered versus the number of RVs used.

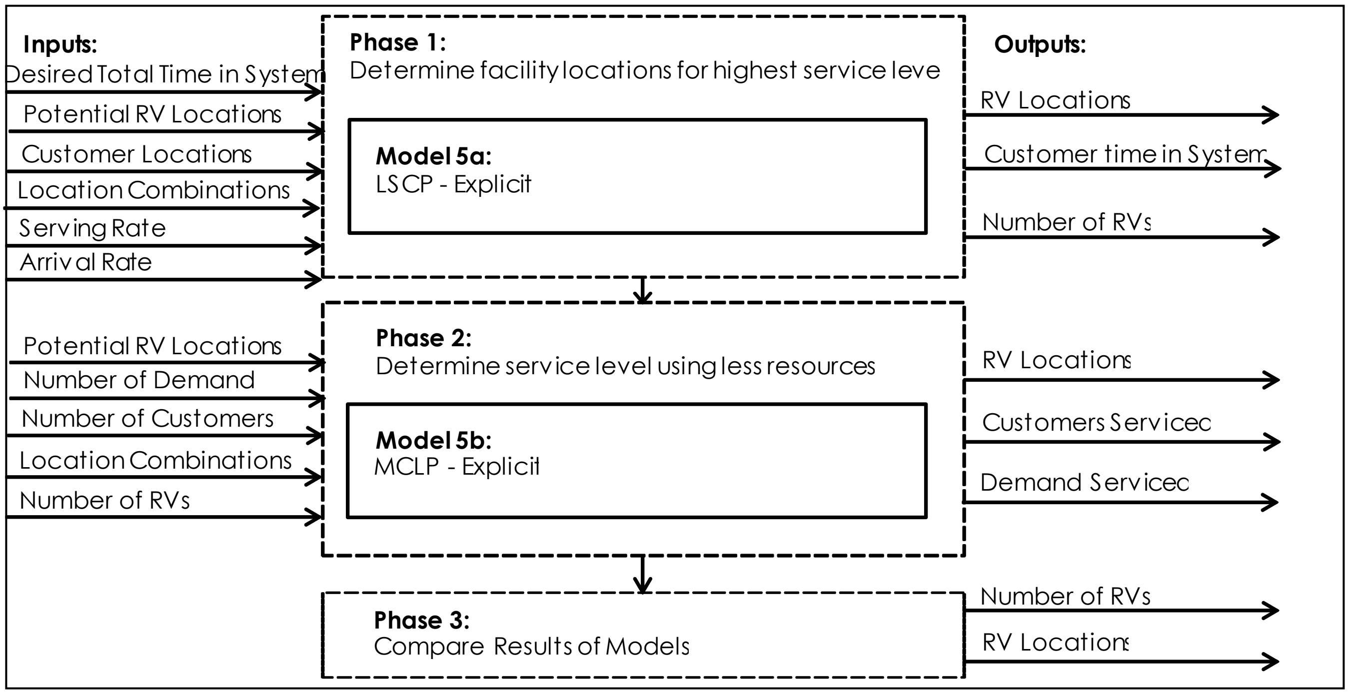

Figure 32 Multiple Location Models (Inputs and Outputs)

The location problems taken into consideration are the Location Set Covering Problem (LSCP), Maximal Covering Location Problem (MCLP) and the Backup Covering Location Problem (BCLP).

In this chapter the explicit derivatives of the models will only be used. It is essential for these models to be explicit because according to the regulations set out by SAIDSA (Rudolph: 2011), all customers must be covered and the locations of all RVs must be displayed within the company – this means that no areas are allowed to be partially covered, which is done by traditional coverage and location models.

Throughout this section four datasets will be used to show how the models work. For a comparison and explanation of the various datasets used see Appendix B.

5.2 Mathematical Model

5.2.1 Explicit Location set covering problem with queueing theory

In traditional LSC problems only the locations of the customers are taken into consideration when determining the facilities' locations. This is not the case for RV companies. The RV companies must locate the RVs in such a way that:

- all customers are within an acceptable distance from at least one RV, and
- the RVs must respond to the customer within 15 minutes, to achieve this queueing theory need to be taken into consideration when companies determine the locations of their RVs.

To achieve these objectives the following mathematical model needs to be used:

In defining the objectives let:

$\bar{I} \in \{1..n\}, where\ n\ is\ the\ given\ number\ of\ potential\ RV\ locations\ in\ the\ operational\ area.$

$\bar{L} \in \{1..n\}, where\ n\ is\ a\ given\ list\ of\ all\ possible\ combinations\ of\ potential\ RV\ locations.$

$\bar{K} \in \{1..n\}, where\ n\ is\ the\ given\ number\ of\ sub-areas\ in\ the\ operational\ area.$

$X_i \triangleq the\ number\ of\ RVs\ that\ should\ be\ located\ at\ site\ i, where\ i \in \bar{I}$

$Y_{li} \triangleq \begin{cases} 1\ if\ permutation\ l\ has\ a\ vehicle\ sited\ at\ location\ i, where\ l \in \bar{L}, i \in \bar{I} \\ 0\ otherwise \end{cases}$

$P_l \triangleq the\ number\ of\ time\ combination\ l\ must\ be\ used, where\ l \in \bar{L}$

$M_{lk} \begin{cases} 1\ if\ combination\ completely\ covers\ area\ k, where\ l \in \bar{L}\ and\ k \in \bar{K} \\ 0\ otherwise \end{cases}$

$s_k \triangleq the\ number\ of\ RVs\ that\ should\ cover\ area\ k, where\ \forall k \in \bar{K}$

$W_k \triangleq average\ time\ a\ customer\ from\ area\ k\ spends\ in\ the\ system\ (in\ hours), where\ \forall k \in \bar{K}$

$DT \triangleq the\ given\ desired\ total\ time\ in\ system\ (in\ hours)$

$L_k \triangleq average\ number\ of\ customers\ present\ in\ the\ queuing\ system\ of\ area\ k, where\ \forall k \in \bar{K}$

$AR_k \triangleq the\ given\ average\ number\ of\ arrivals\ entering\ the\ system\ (in\ customers\ per\ hour)\ in\ area\ k, where\ \forall k \in \bar{K}$

$SR_k \triangleq the\ average\ service\ rate\ by\ the\ RVs\ (in\ customers\ per\ hours) in\ area\ k, where\ \forall k \in \bar{K}$

$LQ_k \triangleq the\ given\ the\ average\ number\ of\ customers\ waiting\ in\ line\ in\ area\ k,$

$where\ \forall k \in \bar{K}$

$P(j \geq s)_k \triangleq$ $the\ probability\ that\ all\ servers\ servicing\ are\ k\ are\ busy, where\ \forall k \in \bar{K}$

$R_k \triangleq$ $service\ rate\ to\ arrival\ rate\ ratio\ for\ area\ k, where\ \forall k \in \bar{K}$

The formulation is then given as:

$$Min\ Z = \sum_i X_i \quad (5.1)$$

Subject to:

$X_i \geq Y_{li} \times P_l$		(5.2)
$\sum_l P_l \times M_{lk} = s_k \forall k \in \bar{K}$		(5.3)
$X_i \geq 0$	$\forall\ i \in \bar{I}$	(5.4)
$P_l \geq 0\ \forall$	$l \in \bar{L}$	(5.5)
$W_k \leq DT$	$\forall k \in \bar{K}$	(5.6)
$W_k = \frac{L_k}{AR_k}$	$\forall k \in \bar{K}$	(5.7)
$L_k = LQ_k + \frac{AR_k}{SR_k}$	$\forall k \in \bar{K}$	(5.8)
$LQ_k = \frac{P(j \geq s)_k R_k}{1 - R_k}$	$\forall k \in \bar{K}$	(5.9)
$R_k = \frac{AR_k}{s \times SR_k}$	$\forall k \in \bar{K}$	(5.10)
$Y_{li} \in \{0,1\}$	$\forall i \in \bar{I}, \forall\ l \in \bar{L}$	(5.11)
$M_{lk} \in \{0,1\}$	$\forall\ l \in \bar{L}, \forall k \in \bar{K}$	(5.12)

Explaining the objectives:

The objective (5.1) of this model is to minimize the number of RVs required to achieve the desired operational performance. Constraint (5.2) states that if a specific combination of RV locations is used that RVs must be sited at all the locations contained within the combination. Constraint (5.3) determines how many RVs should cover the specific area, taking queueing theory into consideration. Constraints (5.4) and (5.5) states that both the siting and combination variable should have a value greater than zero. (5.6)The average time a customer spends in the system should be less or equal to the desired time determined by management. Constraints (5.7),(5.8),(5.9) and (5.10) are all calculations used to calculate the average time a customer spends in the system when using queueing theory. Constraints (5.11) and (5.12) are binary constraints.

5.2.2 Maximum coverage location problem with multiple objectives

In defining the objectives let:

$\bar{I} \in \{1..n\}, where\ n\ is\ the\ given\ number\ of\ potential\ RV\ locations\ in\ the\ operational\ area.$

$\bar{L} \in \{1..n\}, where\ n\ is\ a\ given\ list\ of\ all\ possible\ combinations\ of\ potential$
$RV\ locations.$

$\bar{K} \in \{1..n\}, where\ n\ is\ the\ given\ number\ of\ sub-areas\ in\ the\ operational\ area.$

$X_i \triangleq \begin{cases} 1\ if\ an\ RV\ is\ to\ be\ sited\ at\ location\ i, where\ i \in \bar{I}. \\ 0\ otherwise \end{cases}$

$Y_{li} \triangleq \begin{cases} 1\ if\ combination\ l\ has\ a\ vehicle\ site\ at\ location\ i, where\ i \in \bar{I}\ and\ l \in \bar{L}. \\ 0\ otherwise \end{cases}$

$P_l \triangleq$ $The\ number\ of\ time\ combination\ l\ must\ be\ used, where\ l \in \bar{L}$

$M_{lk} \triangleq \begin{cases} 1\ if\ combination\ l\ completely\ covers\ area\ k, where\ l \in \bar{L}\ and\ k \in \bar{K}. \\ 0\ otherwise \end{cases}$

$C_{lk} \triangleq \begin{cases} 1\ if\ combination\ l\ should\ be\ used\ to\ cover\ area\ k,\ where\ l \in \bar{L}\ and\ k \in \bar{K}. \\ 0\ otherwise \end{cases}$

$d_k \triangleq$ *The given demand of area k, where* $k \in \bar{K}$.

$n_k \triangleq$ *The given number of customers in area k, where* $k \in \bar{K}$.

$R \triangleq$ *The given number of RVs to be sited.*

$$max\, Z_1 = \sum_l \sum_k C_{lk} \times n_k \qquad (5.13)$$

$$max\, Z_2 = \sum_l \sum_k C_{lk} \times d_k \qquad (5.14)$$

$$\sum_i X_i = R \qquad (5.15)$$

$$\sum_l C_{lk} \leq 1 \qquad \forall k \in \bar{K} \qquad (5.16)$$

$$C_{lk} \leq M_{lk} \times P_l \qquad \forall l \in \bar{L}, k \in \bar{K} \qquad (5.17)$$

$$X_i = \sum_l Y_{li} \times P_l \qquad \forall i \in \bar{I} \qquad (5.18)$$

$$P_l \in \{0,1\} \qquad \forall l \in \bar{L} \qquad (5.19)$$

$$C_{lk} \in \{0,1\} \qquad \forall l \in \bar{L}, k \in \bar{K} \qquad (5.20)$$

The multiple objectives of this model is to (5.13) maximise the number of customers covered by RVs and to (5.14) maximise the demand that is covered by RVs. Constraint (5.15) states that the number of sited RVs should be equal to the number of RVs given as an input. (5.16) Each one of the sub-areas should be covered by a maximum of one combination of RVs. (5.17) A sub-area can only be covered by a combination if the locations within the combination cover the area completely. (5.18) The number of RVs sited at each location should be equal to the number of RVs in the combination at the specific location, if the combination should be used. Constraints (5.19) and (5.20) are binary constraints.

5.3 Model Assumptions

Model 5a (LSCP-Explicit)

- The inter arrival rate is exponential.
- The service rate is exponential.
- Arrival rate and service rate is constant over time, it doesn't change throughout the day.
- All of the sub-areas contain some customers and should be covered.

- There is no constraint on the number of resources (RVs) available.

Model 5b (MCLP-Explicit)

- There is a constraint on the number of resources (RVs) available.
- The time the customer spends in the system is not a priority, the priority is to cover as many customers and demand as possible.

5.4 Model Execution

The following diagram has been compiled to assist the user with the use of the mathematical models (see figure 33 below):

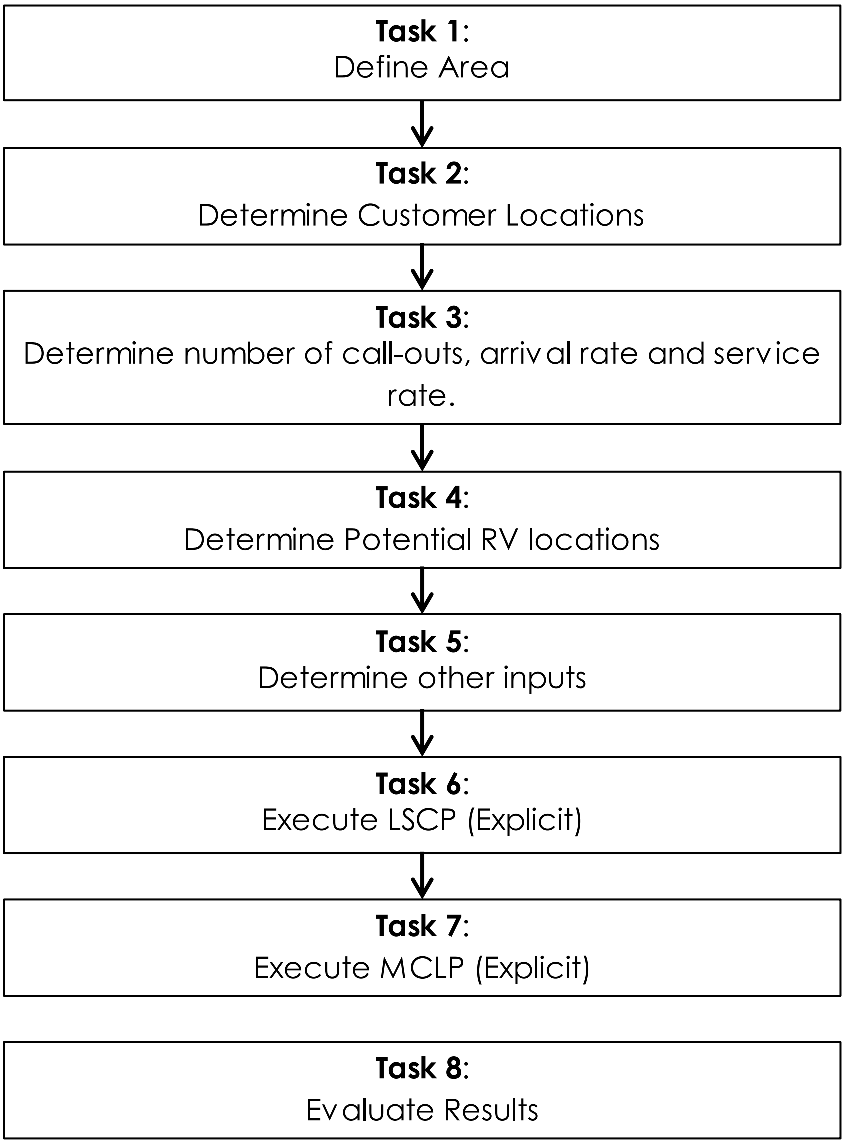

Figure 33 Multiple Location Models (Execution)

Task 1: Define Area

The area that should be used in the model should be determined by the managers of the company, although the boundaries of the area should actually be determined by the spread of the customer locations. The area should now be divided into clusters (referred to as sub-areas throughout this chapter). The sizes of the clusters can vary, but to reduce computational difficulty it is suggested that the same size should be used throughout. In the datasets used the areas are divided into sub-areas of 3km ×3km. See the figures below:

1	2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33
34	35	36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65	66
67	68	69	70	71	72	73	74	75	76	77

Figure 34 Dataset 1 Sub-Areas

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75

Figure 35 Dataset 2 Sub-Areas

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

Figure 36 Dataset 3 Sub-Areas

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24

Figure 37 Dataset 4 Sub-Areas

Task 2: Determine Customer Locations

The coordinates of the customers must be used to determine in which of the sub-areas the customer is situated (see chapter 4, section 4.4 for coordinates). The total number of customers in each of the sub-areas should be calculated and used to compile matrix n_k that is used in the MCLP model. At this stage it is important to check that all customers are contained in the sub-areas.

	Dataset 1	Dataset 2	Dataset 3	Dataset 4
k	n_k	n_k	n_k	n_k
1	73	50	289	47
2	62	70	327	59
3	69	57	291	66
4	56	48	288	58
5	101	66	282	51
6	80	70	87	72
7	62	66	80	47
8	78	59	86	0
9	62	56	93	52
10	71	78	75	53
11	25	58	76	59
12	66	62	23	53
13	63	60	48	51

14	66	52	39	54
15	62	0	46	38
16	71	56	53	0
17	70	63	48	13
18	75	60	17	26
19	65	63	0	13
20	64	49	0	15
21	61	74	0	13
22	28	62	0	7
23	64	60	0	6
24	67	76	0	0
25	52	59		
26	65	58		
27	82	70		
28	65	67		
29	68	40		
30	55	0		
31	67	73		
32	82	55		
33	30	58		
34	75	52		
35	80	66		
36	79	72		
37	67	68		
38	61	70		
39	61	64		
40	64	60		
41	65	65		
42	68	59		
43	84	67		
44	25	56		
45	63	0		
46	86	54		
47	76	40		
48	65	58		
49	71	66		
50	67	64		

51	74	62		
52	65	65		
53	64	64		
54	74	81		
55	27	57		
56	70	60		
57	83	54		
58	70	65		
59	77	46		
60	62	0		
61	65	0		
62	57	0		
63	59	0		
64	76	0		
65	57	0		
66	35	0		
67	10	0		
68	9	0		
69	15	0		
70	17	0		
71	18	0		
72	22	0		
73	21	0		
74	10	0		
75	24	0		
76	16			
77	13			

Table 21 Matrix N_k All Datasets

Task 3: Determine number of call-outs and arrival rate

Historical data should be used to determine the number of call-outs of each customer and this is used to compile matrix d_k that is used in the MCLP. The inter arrival times of the call-outs should also be determined at a sub-area level to determine the arrival rate, the arrival rate is used to compile matrix AR_k that is used as an input to the LSCP. See the table below for matrices d_k , SR_k and AR_k for the various datasets.

	Dataset 1			Dataset 2			Dataset 3			Dataset 4		
k	AR_k	SR_k	d_k	AR_k	SR_k	d_k	AR_k	SR_k	d_k	AR_k	SR_k	d_k
1	1	6	146	1	6	400	1	6	578	1	6	47
2	1	6	372	1	6	210	10	6	6540	1	6	177
3	1	6	276	1	6	57	2	6	873	1	6	66
4	1	6	112	1	6	144	1	6	0	1	6	174
5	1	6	303	1	6	198	2	6	846	1	6	204
6	1	6	160	1	6	350	1	6	174	1	6	0
7	1	6	620	1	6	132	1	6	160	1	6	141
8	1	6	312	1	6	118	1	6	172	1	6	0
9	1	6	372	1	6	56	1	6	93	1	6	156
10	1	6	639	1	6	312	1	6	375	1	6	106
11	1	6	0	1	6	232	2	6	1140	1	6	236
12	1	6	660	1	6	124	1	6	69	1	6	106
13	1	6	63	1	6	60	1	6	144	1	6	204
14	1	6	396	1	6	104	1	6	0	1	6	270
15	1	6	186	1	6	0	1	6	184	1	6	76
16	1	6	710	1	6	280	1	6	106	1	6	0
17	1	6	70	1	6	378	1	6	0	1	6	13
18	1	6	300	1	6	180	1	6	17	1	6	130
19	1	6	130	1	6	252	1	6	0	1	6	26
20	1	6	576	1	6	294	1	6	0	1	6	45
21	1	6	122	1	6	222	1	6	0	1	6	26
22	1	6	168	1	6	310	1	6	0	1	6	14
23	1	6	576	1	6	240	1	6	0	1	6	30
24	1	6	469	1	6	380	1	6	0	1	6	0
25	1	6	364	1	6	236						
26	1	6	390	1	6	58						
27	2	6	738	1	6	210						
28	1	6	455	1	6	0						
29	1	6	476	1	6	160						
30	1	6	330	1	6	0						
31	1	6	201	1	6	219						
32	2	6	738	1	6	110						
33	1	6	150	1	6	116						
34	1	6	225	1	6	104						
35	1	6	0	2	6	792						

36	1	6	158	1	6	360						
37	1	6	670	1	6	68						
38	1	6	366	1	6	210						
39	1	6	488	1	6	384						
40	1	6	512	1	6	180						
41	1	6	260	1	6	195						
42	1	6	204	1	6	177						
43	1	6	588	1	6	268						
44	1	6	125	1	6	168						
45	1	6	378	1	6	0						
46	2	6	774	1	6	54						
47	1	6	380	1	6	400						
48	1	6	0	1	6	232						
49	1	6	497	1	6	132						
50	1	6	536	1	6	384						
51	1	6	666	1	6	310						
52	1	6	65	1	6	650						
53	1	6	192	1	6	256						
54	1	6	74	1	6	81						
55	1	6	135	1	6	228						
56	1	6	420	1	6	120						
57	1	6	0	1	6	270						
58	1	6	630	1	6	65						
59	1	6	0	1	6	230						
60	1	6	372	1	6	0						
61	1	6	130	1	6	0						
62	1	6	171	1	6	0						
63	1	6	177	1	6	0						
64	1	6	76	1	6	0						
65	1	6	285	1	6	0						
66	1	6	315	1	6	0						
67	1	6	10	1	6	0						
68	1	6	0	1	6	0						
69	1	6	60	1	6	0						
70	1	6	0	1	6	0						
71	1	6	162	1	6	0						
72	1	6	198	1	6	0						

73	1	6	168	1	6	0						
74	1	6	30	1	6	0						
75	1	6	168	1	6	0						
76	1	6	64									
77	1	6	13									

Table 22 Matrices AR_k, SR_k and d_k for all Datasets

The service rate for all of the datasets is 6 customers per hour (this means the RV will respond to the customer in 10 minutes) and is aligned with the distance used to determine potential RV sites (Task 4) and with the SAIDSA guidelines.

Task 4: Determine potential RV locations

Since the RVs can be sited anywhere within the area it is necessary to determine potential locations that can be used to site RVs. Murray and Tong (2007) showed that the best potential sites for allocating facilities in a continuous space can be determined by using the Polygon Intersection Point Set (PIPS) approach. The difference between discrete and continuous approaches is: in discrete approaches the locations of potential facilities are known in advance whereas facilities in a continuous space can be located anywhere within that space (Murray & Tong: 2007).

The locations of RVs can be anywhere in a specific area because no infrastructure is required, for this reason the space for RVs can be seen as a continuous space.

The PIPS approach uses the following steps (Murray & Tong: 2007):

- Identify spatial demand objects (points, lines and/or polygons) in need of coverage.

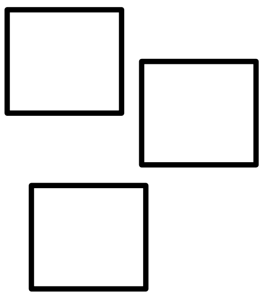

Figure 38 Potential Facility Location

- Extract object vertices as potential facility locations.

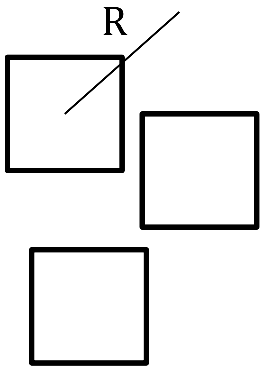

Figure 39 Potential Facility Location

- Derive covering boundaries (areas) for each demand object.

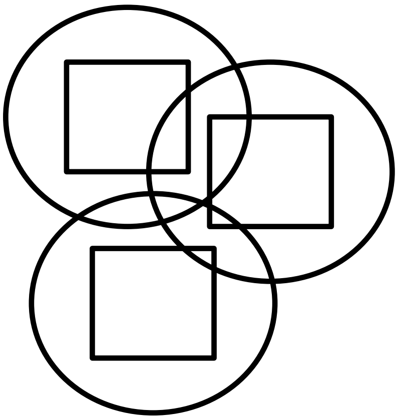

Figure 40 Potential Facility Location

- Identify the intersection points of covering boundaries as potential facility locations.

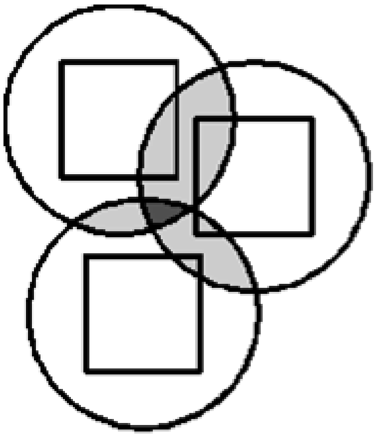

Figure 41 Potential Facility Location

- (Optional) Remove dominated critical locations.

Step 1 of the PIPS is very straightforward, it only requires the determination and identification of demand areas. Step 2 requires the user to determine the acceptable distance between the facility and the demand point in order to deliver an acceptable level of service. Step 3 and step 4 are the most important steps since they help with the actual location determination. Step 3 determines the location of a facility to service each area. Step 4 identifies whether these areas overlap; the more areas that overlap the better the location for facilities. The light grey areas in figure 41 would be good potential locations but the dark grey would be even better.

Step 1 of the PIPS approach was already completed in Task 1, when the area used within the models was defined. To determine the acceptable distance between the RV and the customer it is assumed that the desired response time is 10 minutes and that the vehicles travel at an average speed of 60km/h. The following mathematical model and figure was used to determine the minimum distance.

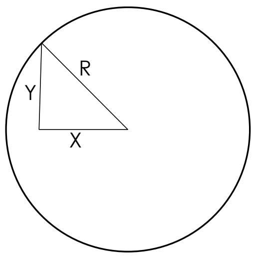

Figure 42 Pythagorean Theorem

$$minZ = R \quad (5.21)$$

$$X^2 + Y^2 = R^2 \quad (5.22)$$

$$X + Y = 15 \quad (5.23)$$

The (5.21) objective is to minimise the traveling radius using the constraints (5.22), (5.23) of the Pythagorean Theorem, assuming the roads used to travel are perpendicular to one another.

The figures below, shows how PIPS approach was used to determine potential RV locations for the various datasets:

- **Dataset 1**

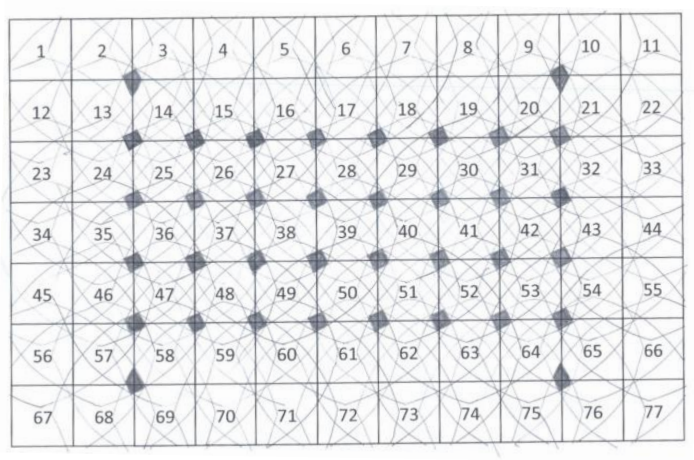

Figure 43 Dataset 1 PIPS Results

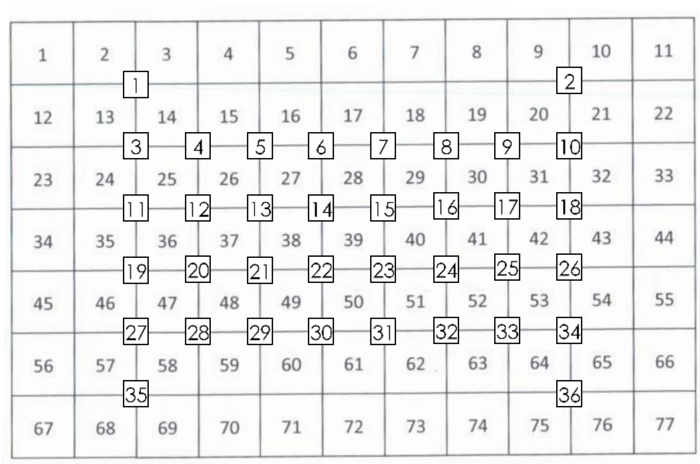

Figure 44 Dataset 1 Potential RV Sites

- **Dataset** 2

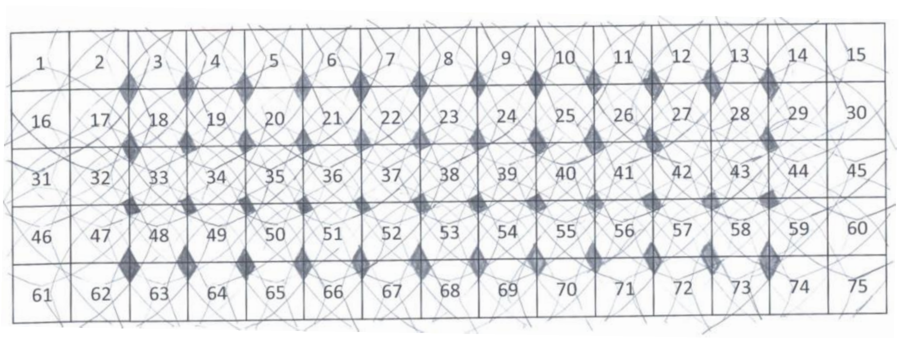

Figure 45 Dataset 2 PIPS Results

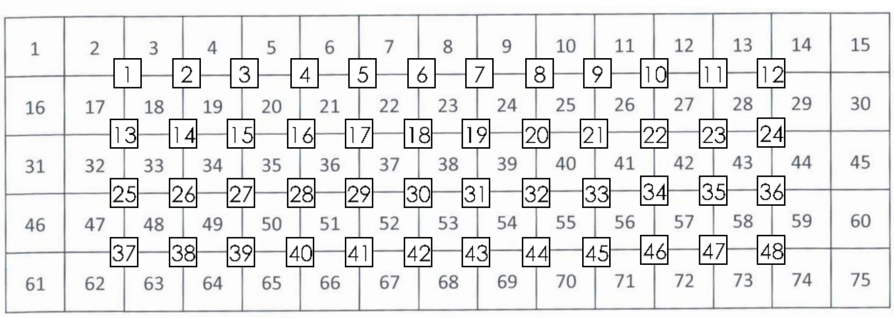

Figure 46 Dataset 2 Potential RV Sites

- **Dataset 3**

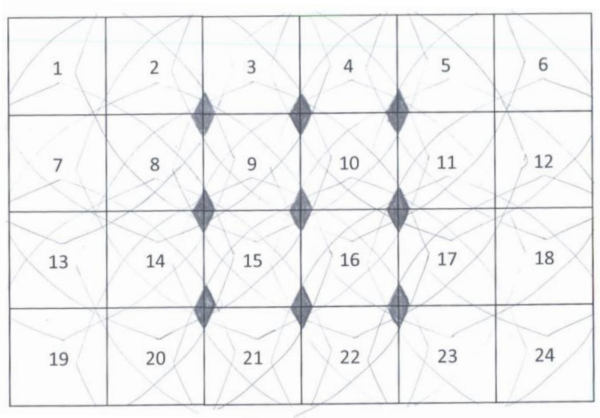

Figure 47 Dataset 3 PIPS Results

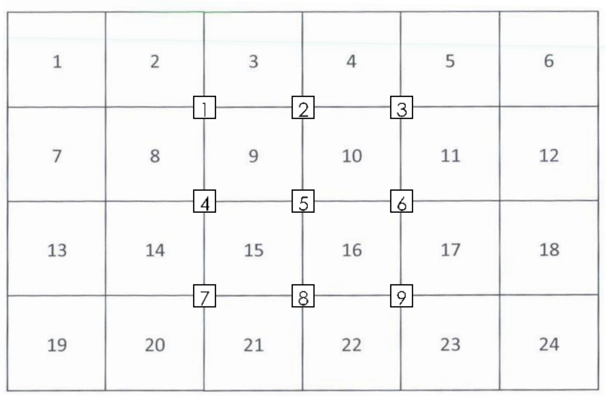

Figure 48 Dataset 3 Potential RV Sites

- **Dataset 4**

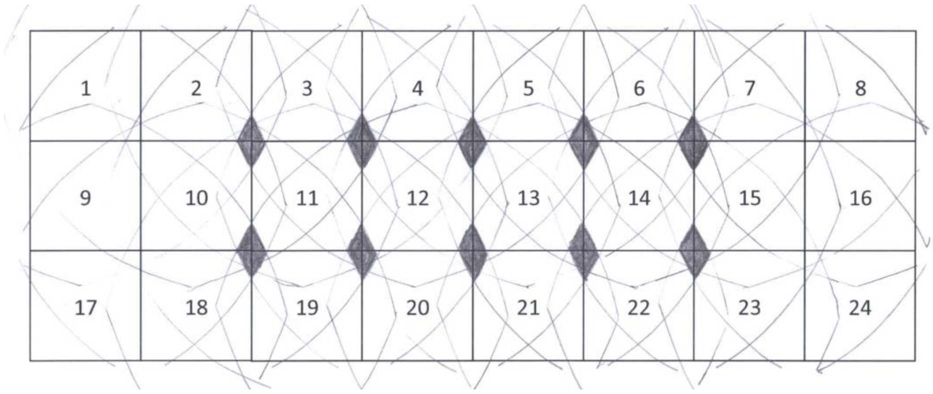

Figure 49 Dataset 4 PIPS Results

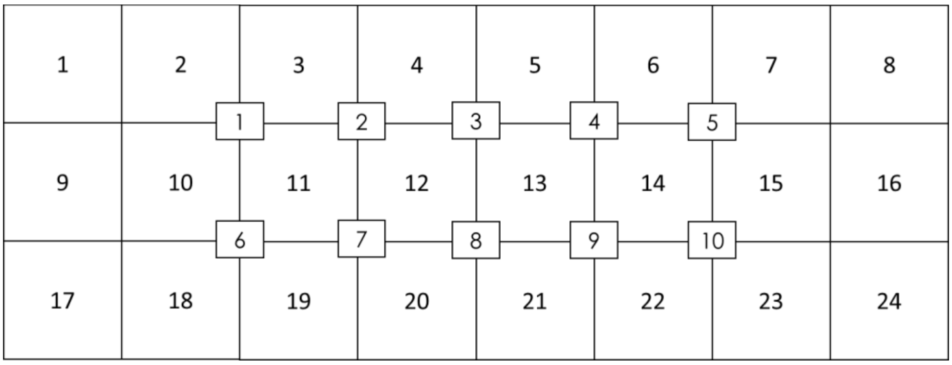

Figure 50 Dataset 4 Potential RV Sites

Although the PIPS approach is very effective for determining potential RV locations, it still gives a large number of potential RV locations that can be manually reduced. The only thing the user must bear in mind is that the sites used must be able to cover the entire area. The figures below show the reduced number of potential RV locations for both Dataset 1 and Dataset 2.

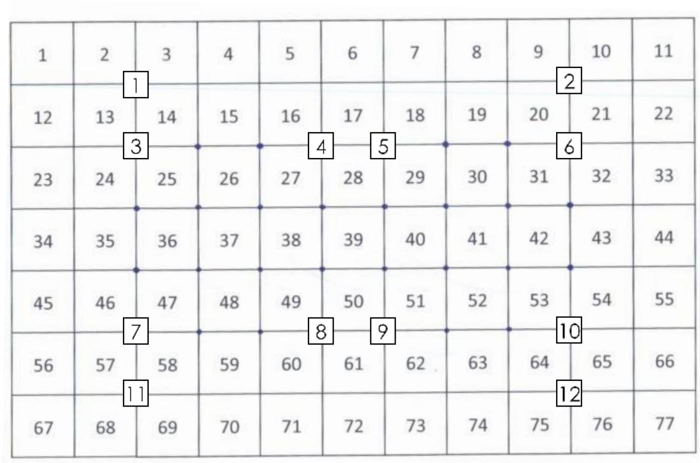

Figure 51 Dataset 1 Reduced RV Sites

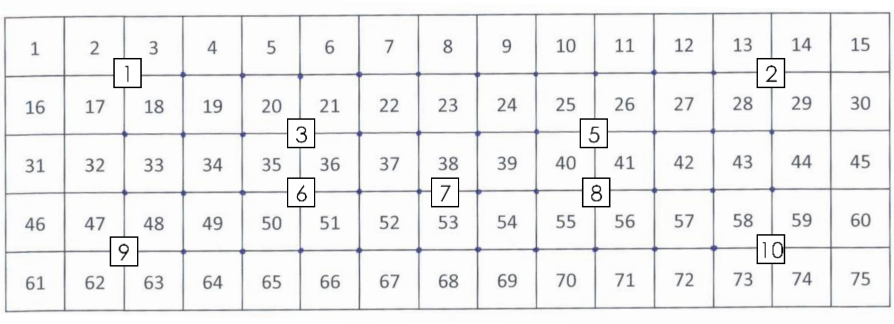

Figure 52 Dataset 2 Reduced RV Sites

Task 5: Determine other inputs

DT: This is the maximum time the user wishes the customer to be in the system, when working with RVs it is important to remember that the time in system consists of: the time the customer waits in the queue, the time the RVs drive to the customer's location and the time the security official takes to inspect the premises. In all of the executions a time of 15 minutes was used. DT is only an input in the LSCP model.

M_{lk}: This matrix is the input that determines whether a combination completely covers a specific area, this is used in the model to track which areas are covered. M_{lk} is used in both the LSCP and the MCLP models.

Y_{li}: This matrix is use to determine whether the specific combination has a RV sited at a specific location. Y_{li} is used as an input in both the LSCP and the MCLP models.

Due to the size of matrices M_{lk} and Y_{li} they are not included in this document.

Task 6: Execute the LSCP (Explicit)

The LSCP (Explicit) model can now be solved by using linear programming software. Lingo 14.0 was used and all of the models were solved in less than 10 minutes. The following results were obtained:

- **Dataset 1**

Location	Location Used	Number of RVs at Location
1	Yes	1
2	Yes	1
3	Yes	1
4	Yes	1
5	Yes	1
6	Yes	1
7	Yes	1

8	Yes	1
9	Yes	1
10	Yes	1
11	Yes	1
12	Yes	1

Table 23 Dataset 1 Output

Sub-Area	Arrival rate (Customers per Hour)	Service Rate (Customers per Hour)	Number of Servers	Average Time of Customer in System (Hours)
1	1	6	1	0.2
2	1	6	2	0.1678322
3	1	6	2	0.1678322
4	1	6	1	0.2
5	1	6	1	0.2
6	1	6	1	0.2
7	1	6	1	0.2
8	1	6	1	0.2
9	1	6	1	0.2
10	1	6	1	0.2
11	1	6	1	0.2
12	1	6	2	0.1678322
13	1	6	2	0.1678322
14	1	6	2	0.1678322
15	1	6	2	0.1678322
16	1	6	1	0.2
17	1	6	1	0.2
18	1	6	1	0.2
19	1	6	1	0.2
20	1	6	1	0.2
21	1	6	1	0.2
22	1	6	1	0.2
23	1	6	1	0.2
24	1	6	2	0.1678322
25	1	6	2	0.1678322

26	1	6	1	0.2
27	2	6	1	0.25
28	1	6	1	0.2
29	1	6	1	0.2
30	1	6	1	0.2
31	1	6	1	0.2
32	2	6	1	0.25
33	1	6	1	0.2
34	1	6	1	0.2
35	1	6	1	0.2
36	1	6	1	0.2
37	1	6	1	0.2
38	1	6	1	0.2
39	1	6	1	0.2
40	1	6	1	0.2
41	1	6	1	0.2
42	1	6	1	0.2
43	1	6	1	0.2
44	1	6	1	0.2
45	1	6	1	0.2
46	2	6	1	0.25
47	1	6	1	0.2
48	1	6	1	0.2
49	1	6	1	0.2
50	1	6	1	0.2
51	1	6	1	0.2
52	1	6	1	0.2
53	1	6	1	0.2
54	1	6	1	0.2
55	1	6	1	0.2
56	1	6	1	0.2
57	1	6	1	0.2
58	1	6	1	0.2
59	1	6	1	0.2
60	1	6	1	0.2
61	1	6	1	0.2
62	1	6	1	0.2

63	1	6	1	0.2
64	1	6	1	0.2
65	1	6	1	0.2
66	1	6	1	0.2
67	1	6	1	0.2
68	1	6	1	0.2
69	1	6	1	0.2
70	1	6	1	0.2
71	1	6	1	0.2
72	1	6	1	0.2
73	1	6	1	0.2
74	1	6	1	0.2
75	1	6	1	0.2
76	1	6	1	0.2
77	1	6	1	0.2

Table 24 Dataset 1 Queueing Theory

- **Dataset 2**

Location	Location Used	Number of RVs at Location
1	Yes	1
2	Yes	1
3	Yes	1
4	Yes	1
5	Yes	1
6	Yes	1
7	Yes	1
8	Yes	1
9	Yes	1
10	Yes	1

Table 25 Dataset 2 Output

Sub-Area	Arrival rate (Customers per Hour)	Service Rate (Customers per Hour)	Number of Servers	Average Time of Customer in System (Hours)
1	1	6	1	0.2
2	1	6	1	0.2
3	1	6	1	0.2
4	1	6	1	0.2
5	1	6	1	0.2
6	1	6	1	0.2
7	1	6	1	0.2
8	1	6	1	0.2
9	1	6	1	0.2
10	1	6	1	0.2
11	1	6	1	0.2
12	1	6	1	0.2
13	1	6	1	0.2
14	1	6	1	0.2
15	1	6	1	0.2
16	1	6	1	0.2
17	1	6	1	0.2
18	1	6	1	0.2
19	1	6	1	0.2
20	1	6	1	0.2
21	1	6	1	0.2
22	1	6	1	0.2
23	1	6	1	0.2
24	1	6	1	0.2
25	1	6	1	0.2
26	1	6	1	0.2
27	1	6	1	0.2
28	1	6	1	0.2
29	1	6	1	0.2
30	1	6	1	0.2
31	1	6	1	0.2
32	1	6	1	0.2

33	1	6	1	0.2
34	1	6	1	0.2
35	2	6	1	0.25
36	1	6	1	0.2
37	1	6	1	0.2
38	1	6	1	0.2
39	1	6	1	0.2
40	1	6	1	0.2
41	1	6	1	0.2
42	1	6	1	0.2
43	1	6	1	0.2
44	1	6	1	0.2
45	1	6	1	0.2
46	1	6	1	0.2
47	1	6	1	0.2
48	1	6	1	0.2
49	1	6	1	0.2
50	1	6	1	0.2
51	1	6	1	0.2
52	1	6	1	0.2
53	1	6	1	0.2
54	1	6	1	0.2
55	1	6	1	0.2
56	1	6	1	0.2
57	1	6	1	0.2
58	1	6	1	0.2
59	1	6	1	0.2
60	1	6	1	0.2
61	1	6	1	0.2
62	1	6	1	0.2
63	1	6	1	0.2
64	1	6	1	0.2
65	1	6	1	0.2
66	1	6	1	0.2
67	1	6	1	0.2
68	1	6	1	0.2
69	1	6	1	0.2

70	1	6	1	0.2
71	1	6	1	0.2
72	1	6	1	0.2
73	1	6	1	0.2
74	1	6	1	0.2
75	1	6	1	0.2

Table 26 Dataset 2 Queueing Theory

- **Dataset 3**

Location	Location Used	Number of RVs at Location
1	Yes	1
2	Yes	2
3	Yes	1
4	No	0
5	No	0
6	No	0
7	Yes	1
8	No	0
9	Yes	1

Table 27 Dataset 3 Output

Sub-Area	Arrival rate (Customers per Hour)	Service Rate (Customers per Hour)	Number of Servers	Average Time of Customer in System (Hours)
1	1	6	1	0.2
2	10	6	3	0.2041367
3	2	6	3	0.1669776
4	1	6	3	0.1667073
5	2	6	2	0.1714286
6	1	6	1	0.2
7	1	6	1	0.2
8	1	6	3	0.1667073
9	1	6	3	0.1667073
10	1	6	3	0.1667073
11	2	6	2	0.1714286

12	1	6	1	0.2
13	1	6	1	0.2
14	1	6	2	0.1678322
15	1	6	3	0.1667073
16	1	6	2	0.1678322
17	1	6	2	0.1678322
18	1	6	1	0.2
19	1	6	1	0.2
20	1	6	1	0.2
21	1	6	2	0.1678322
22	1	6	2	0.1678322
23	1	6	1	0.2
24	1	6	1	0.2

Table 28 Dataset 3 Queueing Theory

- **Dataset 4**

Location	Location Used	Number of RVs at Location
1	Yes	1
2	Yes	1
3	No	0
4	No	0
5	Yes	1
6	Yes	1
7	No	0
8	No	0
9	No	0
10	Yes	1

Table 29 Dataset 4 Output

Sub-Area	Arrival rate (Customers per Hour)	Service Rate (Customers per Hour)	Number of Servers	Average Time of Customer in System (Hours)
1	1	6	1	0.2
2	1	6	1	0.2
3	1	6	1	0.2

4	1	6	1	0.2
5	1	6	2	0.1678322
6	1	6	1	0.2
7	1	6	1	0.2
8	1	6	1	0.2
9	1	6	1	0.2
10	1	6	1	0.2
11	1	6	1	0.2
12	1	6	1	0.2
13	1	6	2	0.1678322
14	1	6	1	0.2
15	1	6	1	0.2
16	1	6	1	0.2
17	1	6	1	0.2
18	1	6	1	0.2
19	1	6	1	0.2
20	1	6	1	0.2
21	1	6	1	0.2
22	1	6	1	0.2
23	1	6	1	0.2
24	1	6	1	0.2

Table 30 Dataset 4 Queueing Theory

From the results in the tables above it can be seen that the minimum expected time a customer will spend is 0.17 hours (10.2 minutes) and the maximum time is 0.25 hours (15 minutes).

Datasets 1 and 2 have RVs located at all of the potential sites, this is also an indication that the PIPS approach is a good technique for determining potential RV sites. Datasets 3 and 4 do not have RVs at all of the potential sites, this means that the areas covered by the sites overlap. At some locations there are more than 1 RV sited, this is due to the service rate and arrival rate of call-outs in the sub-areas covered by the RVs from the specific location. More RVs are required to reduce the time a customer spends in the system.

Task 8: Execute the MCLP (Explicit)

To solve the MCLP (Explicit) it is advised that a preemptive optimisation approach should be used. It is the decision of the user to determine whether it is more important to cover a large number of customers or cover a large demand (number of call-outs). This decision should be aligned with the strategic goals of the company.

Preempting Z_1 (Number of Customers)

$$max\, Z_1{}^* = \sum_l \sum_k C_{lk} \times n_k$$

Subject to

$\sum_i X_i = R$

$\sum_l C_{lk} \leq 1 \quad \forall k \in \bar{K}$

$C_{lk} \leq M_{lk} \times P_l \quad \forall l \in \bar{L}, k \in \bar{K}$

$X_i = \sum_l Y_{li} \times P_l \quad \forall i \in \bar{I}$

$P_l \in \{0,1\} \quad \forall l \in \bar{L}$

$C_{lk} \in \{0,1\} \quad \forall l \in \bar{L}, k \in \bar{K}$

$$max\, Z_2{}^* = \sum_l \sum_k C_{lk} \times d_k$$

Subject to:

$\sum_l \sum_k C_{lk} \times n_k \geq Z_1{}^*$

$\sum_i X_i = R$

$\sum_l C_{lk} \leq 1 \quad \forall k \in \bar{K}$

$C_{lk} \leq M_{lk} \times P_l \quad \forall l \in \bar{L}, k \in \bar{K}$

$X_i = \sum_l Y_{li} \times P_l \quad \forall i \in \bar{I}$

$P_l \in \{0,1\} \quad \forall l \in \bar{L}$

$C_{lk} \in \{0,1\} \quad \forall l \in \bar{L}, k \in \bar{K}$

Preempting Z_2 (Demand Covered)

$$max\, Z_2{}^* = \sum_l \sum_k C_{lk} \times d_k$$

Subject to

$\sum_i X_i = R$

$\sum_l C_{lk} \leq 1 \quad \forall k \in \bar{K}$

$C_{lk} \leq M_{lk} \times P_l \quad \forall l \in \bar{L}, k \in \bar{K}$

$X_i = \sum_l Y_{li} \times P_l \quad \forall i \in \bar{I}$

$P_l \in \{0,1\} \quad \forall l \in \bar{L}$

$C_{lk} \in \{0,1\} \quad \forall l \in \bar{L}, k \in \bar{K}$

$$max\, Z_1{}^* = \sum_l \sum_k C_{lk} \times n_k$$

Subject to

$\sum_l \sum_k C_{lk} \times d_k \geq Z_2{}^*$

$\sum_i X_i = R$

$\sum_l C_{lk} \leq 1 \quad \forall k \in \bar{K}$

$$C_{lk} \leq M_{lk} \times P_l \qquad \forall l \in \bar{L}, k \in \bar{K}$$
$$X_i = \sum_l Y_{li} \times P_l \qquad \forall i \in \bar{I}$$
$$P_l \in \{0,1\} \qquad \forall l \in \bar{L}$$
$$C_{lk} \in \{0,1\} \qquad \forall l \in \bar{L}, k \in \bar{K}$$

The MCLP (Explicit) model can now be solved using the appropriate linear programming software, Lingo 14.0 was used and all of the models were solved in less than 20 minutes.

- **Dataset 1**

Customer Preemptive						
Number of RVs:	2	4	6	8	10	12
RV Locations:	4,7	4,6,7,9	2,3,5,7,8,10	1,2,3,4,7,9,10	1,2,3,4,5,6,7,9,10,11	1,2,3,4,5,6,7,8,9,10,11,12
Demand Covered:	8112	15878	21265	21832	22117	22322
Customers Covered:	1701	3062	4118	4351	4433	4474
Demand Preemptive						
Number of RVs:	2	4	6	8	10	12
RV Locations:	3,5	3,5,6,8	2,3,5,7,8,10	2,3,4,5,6,7,9,10	1,2,3,4,5,6,7,8,9,10	1,2,3,4,5,6,7,8,9,10,11,10
Demand Covered:	8899	16221	21265	21849	22269	22322
Customers Covered:	1613	2868	4118	4277	4424	4474

Table 31 Dataset 1: Model 5b Results

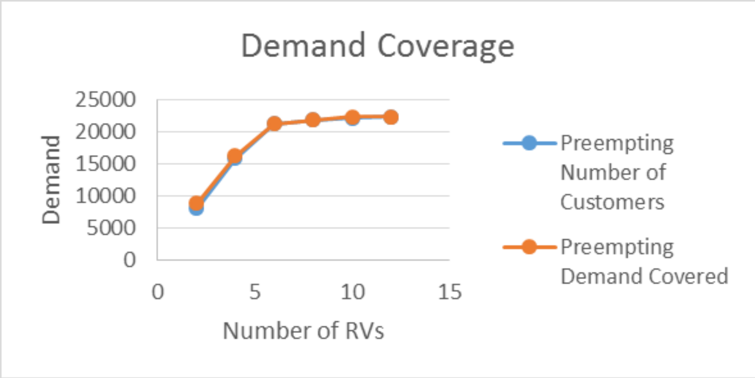

Figure 53 Dataset 1 Demand Coverage

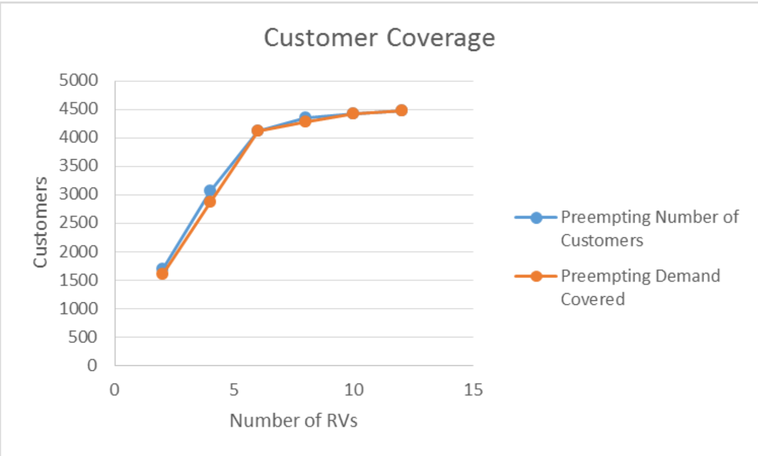

Figure 54 Dataset 1 Customer Coverage

- **Dataset 2**

Customer Preemptive					
Number of RVs:	2	4	6	8	10
RV Locations:	3,5	1,3,4,5	1,2,3,4,5,9	1,2,3,4,5,7,9,10	1,2,3,4,5,6,7,8,9,10
Demand Covered:	5976	9974	11895	12460	12460
Customers Covered:	1456	2570	3265	3430	3430
Demand Preemptive					
Number of RVs:	2	4	6	8	10
RV Locations:	3,4	1,3,4,5	1,3,4,5,9,10	1,2,3,4,5,7,9,10	1,2,3,4,5,6,7,8,9,10
Demand Covered:	6070	9974	12012	12460	12460
Customers Covered:	1348	2570	3149	3430	3430

Table 32 Dataset 2: Model 5b Results

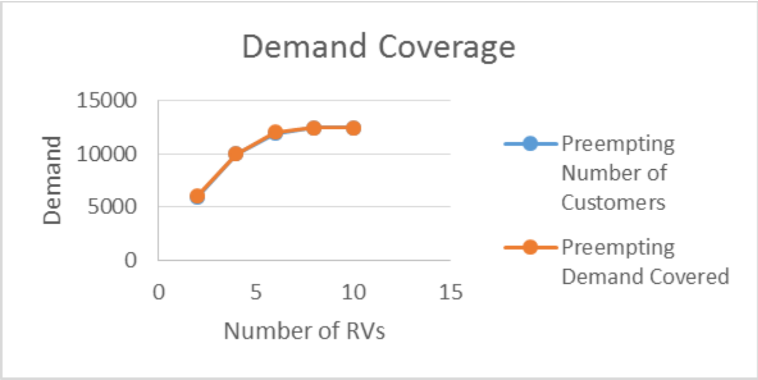

Figure 55 Dataset 2 Demand Coverage

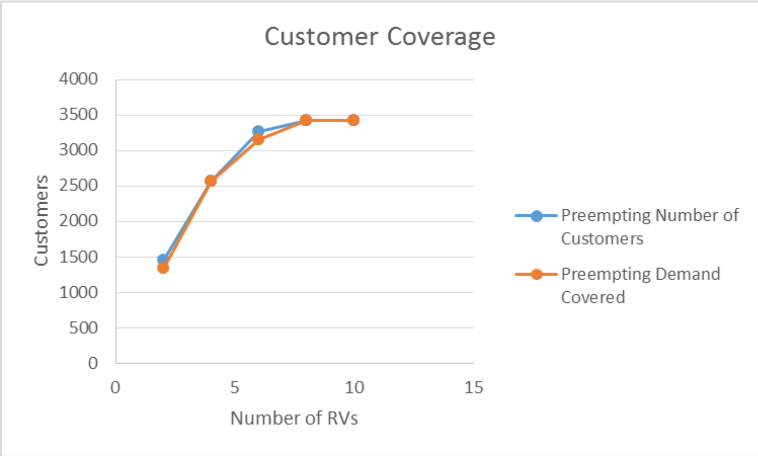

Figure 56 Dataset 2 Customer Coverage

- **Dataset 3**

Customer Preemptive						
Number of RVs:	1	2	3	4	5	6
RV Locations:	2	1,3	1,3,7	1,3,4,6	1,3,4,6,8	1,2,3,4,5,6
Demand Covered:	10329	11310	11454	11471	11471	11471
Customers Covered:	1617	2183	2231	2248	2248	2248
Demand Preemptive						
Number of RVs:	1	2	3	4	5	6
RV Locations:	2	1,3	1,3,7	1,3,4,6	1,3,4,6,8	1,2,3,4,5,6
Demand Covered:	10329	11310	11454	11471	11471	11471
Customers Covered:	1617	2183	2231	2248	2248	2248

Table 33 Dataset 3: Model 5b Results

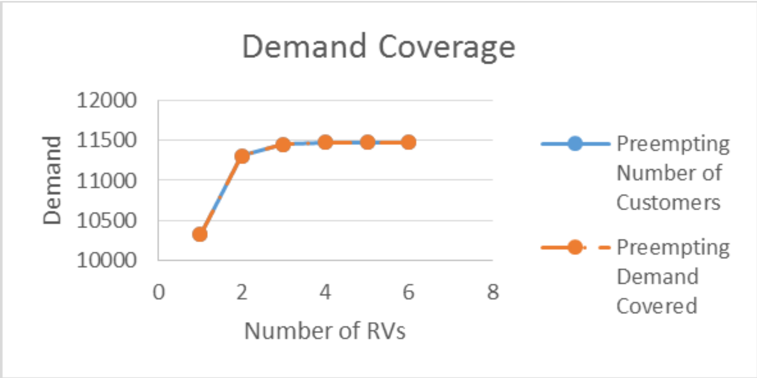

Figure 57 Dataset 3 Demand Coverage

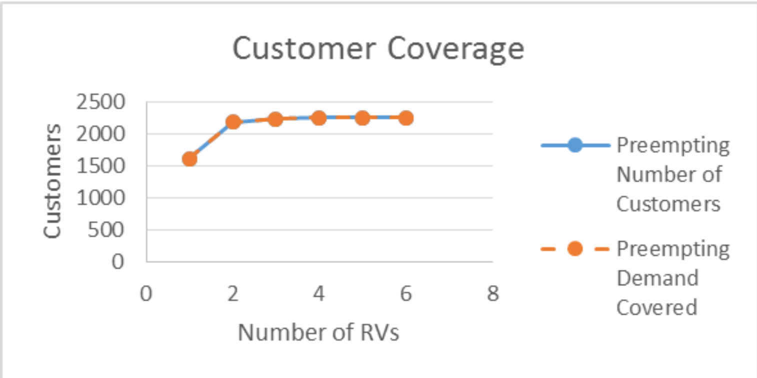

Figure 58 Dataset 3 Customer Coverage

- **Dataset 4**

Customer Preemptive					
Number of RVs:	1	2	3	4	5
RV Locations:	2	1,9	1,2,9	1,2,6,9	1,2,3,5,6
Demand Covered:	1224	2189	2234	2247	2247
Customers Covered:	486	825	840	853	853
Demand Preemptive					
Number of RVs:	1	2	3	4	5
RV Locations:	2	1,9	1,2,9	1,2,6,9	1,2,3,5,6
Demand Covered:	1344	2189	2234	2247	2247
Customers Covered:	478	825	840	853	853

Table 34 Dataset 4: Model 5b Results

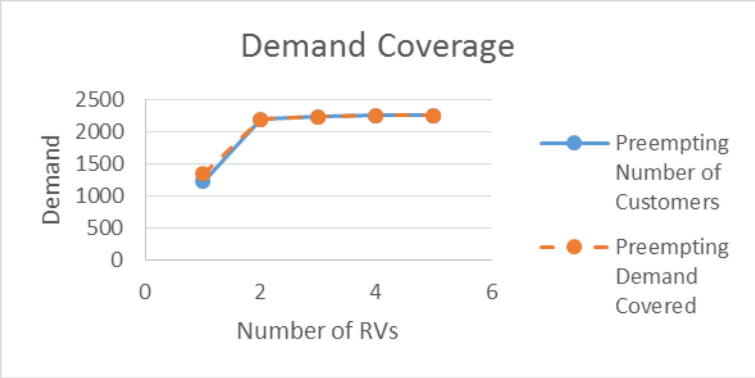

Figure 59 Dataset 4 Demand Coverage

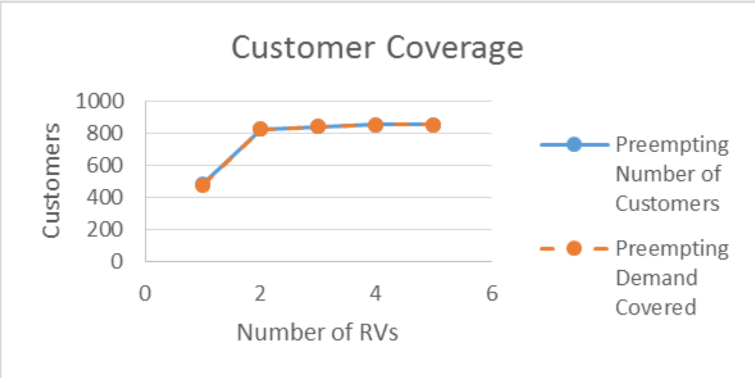

Figure 60 Dataset 4 Customer Coverage

From the results above it can be seen that the level of service provided increases as the number of RVs increase. This model is helpful to determine how many resources are needed to achieve sufficient level of coverage – from the graphs it can be seen that there is a strong increase in coverage when few RVs are used, but later the increase is very low that shows redundant resources.

There can further be seen that there is a strong correlation between the number of customers and demand covered by RVs. As the number of customers covered increases the number of demand covered also increases.

Task 9: Evaluate the Results

From the results it can be seen that the LSCP (Explicit) model always suggests a higher number of RVs to be used, but the MCLP (Explicit) shows that high levels of coverage can still be obtained when fewer RVs are used. It will not always be feasible for a company to have a high number of resources, the lower number of RVs can still cover many customers and demand, but this will increase the time a customer spends in the system. A long time in the system will lower customer service levels and customer satisfaction.

The following figures show the recommendation for RV locations according to both the LSCP (Explicit) models and the MCLP (Explicit) model:

5.5 Conclusions

This chapter discusses the siting of RVs at multiple (decentralised) locations. The models will typically be used by larger security companies or companies that have customers in a large area (where it will take too long for RVs to respond from a single location).

The chapter has two models: the first one uses both location problems and queueing theory to determine where to site RVs and how many to site assuming that there is an unlimited number of resources available. The second model assumes that there is a limit on the number of RVs the company has available and sites RVs at locations to cover the most demand or customers depending on the option preferred by management.

The results by the first model show how many RVs are required by a security company to service all customers within a specified time. The results also shows the estimated time a customer will wait to get serviced for each of the different areas used in the model.

The results obtained by the second model show that there is often a strong correlation between the demand and the number of customers covered. The only real difference can be seen in the results of Dataset 1 and Dataset 2. When four RVs are used in Dataset 1 and 6 RVs are used in Dataset 2, it can be seen that more customers are covered when the number of customers covered are pre-empted.

These models should be used in conjunction to make informed decisions, because it is often too costly to add additional RVs for a small increase in the service level provided to the customers.

The chapter also includes a stepwise procedure to use and interpret the models. Four different scenarios were used to execute the models to see how the models adapt to different situations.

Chapter 6

Conclusions

In recent years there has been an increase in the use of covering models since it is applicable to real life situations, especially in siting emergency service facilities such as ambulance, hospital and police station locations. In many of these applications the models have been used with great success. Although these models are successful there are still numerous industries and countries that do not use them although they can have a major impact on the service level provided to customers and resource utilization. One of such industries is the private security industry of South Africa. Although management of these companies put effort into the siting of response vehicles, there is currently no formal method or technique used to justify these decisions. Another factor that makes the siting of response vehicle so important is that the private security industry of South Africa is continuously growing and very competitive.

This dissertation was motivated by the need of privately owned security companies of South Africa to develop and implement location models to improve the service level they provide to their customers. By using these models and techniques they can also improve their resource utilization which will have a cost benefit for the companies. In addition, there exists a need to improve the services of security companies in South Africa due to the high crime rate, South Africans also feel that they need extra protection for themselves and their belongings and that the South African Police Department cannot provide the service they require. A further motivation is triggered from an industrial engineering perpective, a number of the smaller industries and privately owned companies in South Africa are still unaware of the analytical techniques used by industrial engineers and how it can improve their companies by reducing costs and improving service. The objective of the dissertation was meant to identify how location models are currently being used in the various industries, especially the emergency services industries. The next step was to change and adapt these models so that they can be used by companies of the private security industry of South Africa to site reaction vehicles.

The focus of this dissertation was mainly to develop mathematical location models for three different scenarios: siting reaction vehicles in a small area where the streets and intersections within the area can be seen as nodes and arcs of a network, locating a site for a facility if the company wishes to site all of the vehicles at a single location and to determine how many reaction vehicles there should be at this site and last of all a model that companies can be used when they wish to site their vehicles at different locations. In general these models will assist companies to determine the number of reaction vehicle required and where these vehicles should be positioned to give customers the best service level under operational constraints.

The research gathered illustrates how location problems have changed and improved throughout the years to adapt to different needs and constraints of the various industries. These models have been changed to be used within the private security industry in South Africa. The newly developed models takes additional constraints into consideration such as time of day, number of available resources and queueing theory.

The models address the concerns such as resource availability and response time. From a decision making perspective these models will assist the management of security companies to determine the number of vehicles required and where they should be sited. This will ultimately lead to better service levels and lower costs that will give the company a competitive advantage in a very competitive environment.

All of the models discussed in the dissertation are accompanied with a list of tasks that should be done to correctly and effectively use them. This makes the use of the models generic so that any security company in South Africa may use it with ease.

In conclusion, one can never determine when and where a crime may take place, the only thing that can be done is to attempt to prevent crime or respond to it when it does occur. The dissertation focuses on minimizing the time it takes for a reaction vehicles to get to the place where service is required. By using these models privately owned security companies in South Africa can greatly increase the service level they provide to customers without occurring unnecessary expenses on additional resources.

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Appendix: A

Explicit Network Coverage Model Inputs

1 Matrix N_{ij}

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 35 Matrix Nij (Part 1)

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0

40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 36 Matrix Nij (Part 2)

2 Distances between nodes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
2	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
3	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
4	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
5	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
6	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
7	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
8	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
9	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
10	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
11	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
12	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
13	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15

32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15

33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15

Table 37 Distance Matrix (Part 1)

18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	26
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	27
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	28
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	29
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	30
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	31
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	32
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	33
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	34
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	35
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	36
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	37
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	38
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	39
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	40
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	41
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	42
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	43
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	44
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	45
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	46
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	47
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	48
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	49
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	50

	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19
	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15

38	39	40	41	42	43	44	45	46	47	48	49	50
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
0	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	0	250	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	250	0	250	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	150	0	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	0	50	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	150	1E+15	1E+15	50	0	250	1E+15	150	1E+15	1E+15	1E+15
1E+15	150	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	250	0	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0

Table 38 Distance Matrix (Part 2)

3 Floyd's Algorithm: Output

	1	2	3	4
1	0	250	500	750
2	250	0	250	500
3	500	250	0	250
4	750	500	250	0
5	1000	750	500	250
6	1150	900	650	400
7	900	650	400	150
8	650	400	150	400
9	400	150	400	650
10	150	400	650	900
11	300	550	800	1050
12	550	300	550	800
13	800	550	300	550
14	1050	800	550	300
15	1300	1050	800	550
16	1450	1200	950	700
17	1200	950	700	450
18	950	700	450	700
19	700	450	700	950
20	450	700	950	1200
21	600	850	1100	1350
22	850	600	850	1100
23	1100	850	600	850
24	1350	1100	850	600
25	1600	1350	1100	850

23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5
1100	850	600	450	700	950	1200	1450	1300	1050	800	550	300	150	400	650	900	1150	1000
850	600	850	700	450	700	950	1200	1050	800	550	300	550	400	150	400	650	900	750
600	850	1100	950	700	450	700	950	800	550	300	550	800	650	400	150	400	650	500
850	1100	1350	1200	950	700	450	700	550	300	550	800	1050	900	650	400	150	400	250
1100	1350	1600	1450	1200	950	700	450	300	550	800	1050	1300	1150	900	650	400	150	0
950	1200	1450	1300	1050	800	550	300	150	400	650	900	1150	1000	750	500	250	0	150
700	950	1200	1050	800	550	300	550	400	150	400	650	900	750	500	250	0	250	400
450	700	950	800	550	300	550	800	650	400	150	400	650	500	250	0	250	500	650
700	450	700	550	300	550	800	1050	900	650	400	150	400	250	0	250	500	750	900
950	700	450	300	550	800	1050	1300	1150	900	650	400	150	0	250	500	750	1000	1150
800	550	300	150	400	650	900	1150	1000	750	500	250	0	150	400	650	900	1150	1300
550	300	550	400	150	400	650	900	750	500	250	0	250	400	150	400	650	900	1050
300	550	800	650	400	150	400	650	500	250	0	250	500	650	400	150	400	650	800
550	800	1050	900	650	400	150	400	250	0	250	500	750	900	650	400	150	400	550
800	1050	1300	1150	900	650	400	150	0	250	500	750	1000	1150	900	650	400	150	300
650	900	1150	1000	750	500	250	0	150	400	650	900	1150	1300	1050	800	550	300	450
400	650	900	750	500	250	0	250	400	150	400	650	900	1050	800	550	300	550	700
150	400	650	500	250	0	250	500	650	400	150	400	650	800	550	300	550	800	950
400	150	400	250	0	250	500	750	900	650	400	150	400	550	300	550	800	1050	1200
650	400	150	0	250	500	750	1000	1150	900	650	400	150	300	550	800	1050	1300	1450
500	250	0	150	400	650	900	1150	1300	1050	800	550	300	450	700	950	1200	1450	1600
250	0	250	400	150	400	650	900	1050	800	550	300	550	700	450	700	950	1200	1350
0	250	500	650	400	150	400	650	800	550	300	550	800	950	700	450	700	950	1100
250	500	750	900	650	400	150	400	550	300	550	800	1050	1200	950	700	450	700	850
500	750	1000	1150	900	650	400	150	300	550	800	1050	1300	1450	1200	950	700	450	600

42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24
1200	1050	1300	1550	1950	1800	2050	1900	1650	1400	1150	900	750	1000	1250	1500	1750	1600	1350
1450	1300	1050	1300	1700	1550	1800	1650	1400	1150	900	1150	1000	750	1000	1250	1500	1350	1100
1700	1550	1300	1050	1450	1300	1550	1400	1150	900	1150	1400	1250	1000	750	1000	1250	1100	850
1950	1800	1550	1300	1200	1050	1300	1150	900	1150	1400	1650	1500	1250	1000	750	1000	850	600
2200	2050	1800	1550	1450	1300	1050	900	1150	1400	1650	1900	1750	1500	1250	1000	750	600	850
2050	1900	1650	1400	1300	1150	900	750	1000	1250	1500	1750	1600	1350	1100	850	600	450	700
1800	1650	1400	1150	1050	900	1150	1000	750	1000	1250	1500	1350	1100	850	600		700	450
1550	1400	1150	900	1300	1150	1400	1250	1000	750	1000	1250	1100	850	600	850	1100	950	700
1300	1150	900	1150	1550	1400	1650	1500	1250	1000	750	1000	850	600	850	1100	1350	1200	950
1050	900	1150	1400	1800	1650	1900	1750	1500	1250	1000	750	600	850	1100	1350	1600	1450	1200
900	750	1000	1250	1650	1500	1750	1600	1350	1100	850	600	450	700	950	1200	1450	1300	1050
1150	1000	750	1000	1400	1250	1500	1350	1100	850	600	850	700	450	700	950	1200	1050	800
1400	1250	1000	750	1150	1000	1250	1100	850	600	850	1100	950	700	450	700	950	800	550
1650	1500	1250	1000	900	750	1000	850	600	850	1100	1350	1200	950	700	450	700	550	300
1900	1750	1500	1250	1150	1000	750	600	850	1100	1350	1600	1450	1200	950	700	450	300	550
1750	1600	1350	1100	1000	850	600	450	700	950	1200	1450	1300	1050	800	550	300	150	400
1500	1350	1100	850	750	600	850	700	450	700	950	1200	1050	800	550	300	550	400	150
1250	1100	850	600	1000	850	1100	950	700	450	700	950	800	550	300	550	800	650	400
1000	850	600	850	1250	1100	1350	1200	950	700	450	700	550	300	550	800	1050	900	650
750	600	850	1100	1500	1350	1600	1450	1200	950	700	450	300	550	800	1050	1300	1150	900
600	450	700	950	1350	1200	1450	1300	1050	800	550	300	150	400	650	900	1150	1000	750
850	700	450	700	1100	950	1200	1050	800	550	300	550	400	150	400	650	900	750	500
1100	950	700	450	850	700	950	800	550	300	550	800	650	400	150	400	650	500	250
1350	1200	950	700	600	450	700	550	300	550	800	1050	900	650	400	150	400	250	0
1600	1450	1200	950	850	700	450	300	550	800	1050	1300	1150	900	650	400	150	0	250

11	10	9	8	7	6	5	4	3	2	1	
1450	1600	1350	1100	850	600	750	1000	1250	1500	1750	26
1200	1350	1100	850	600	850	1000	750	1000	1250	1500	27
950	1100	850	600	850	1100	1250	1000	750	1000	1250	28
700	850	600	850	1100	1350	1500	1250	1000	750	1000	29
450	600	850	1100	1350	1600	1750	1500	1250	1000	750	30
600	750	1000	1250	1500	1750	1900	1650	1400	1150	900	31
850	1000	750	1000	1250	1500	1650	1400	1150	900	1150	32
1100	1250	1000	750	1000	1250	1400	1150	900	1150	1400	33
1350	1500	1250	1000	750	1000	1150	900	1150	1400	1650	34
1600	1750	1500	1250	1000	750	900	1150	1400	1650	1900	35
1750	1900	1650	1400	1150	900	1050	1300	1550	1800	2050	36
1500	1650	1400	1150	900	1150	1300	1050	1300	1550	1800	37
1650	1800	1550	1300	1050	1300	1450	1200	1450	1700	1950	38
1250	1400	1150	900	1150	1400	1550	1300	1050	1300	1550	39
1000	1150	900	1150	1400	1650	1800	1550	1300	1050	1300	40
750	900	1150	1400	1650	1900	2050	1800	1550	1300	1050	41
900	1050	1300	1550	1800	2050	2200	1950	1700	1450	1200	42
1200	1350	1100	1350	1600	1850	2000	1750	1500	1250	1500	43
1150	1300	1050	1300	1550	1800	1950	1700	1450	1200	1450	44
1400	1550	1300	1050	1300	1550	1700	1450	1200	1450	1700	45
1550	1700	1450	1200	1450	1700	1850	1600	1350	1600	1850	46
1300	1450	1200	1450	1700	1950	2100	1850	1600	1350	1600	47
1050	1200	1450	1700	1950	2200	2350	2100	1850	1600	1350	48
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	49
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	50

50	49	48	47	46	45	44	43
1E+15	1E+15	1350	1600	1850	1700	1450	1500
1E+15	1E+15	1600	1350	1600	1450	1200	1250
1E+15	1E+15	1850	1600	1350	1200	1450	1500
1E+15	1E+15	2100	1850	1600	1450	1700	1750
1E+15	1E+15	2350	2100	1850	1700	1950	2000
1E+15	1E+15	2200	1950	1700	1550	1800	1850
1E+15	1E+15	1950	1700	1450	1300	1550	1600
1E+15	1E+15	1700	1450	1200	1050	1300	1350
1E+15	1E+15	1450	1200	1450	1300	1050	1100
1E+15	1E+15	1200	1450	1700	1550	1300	1350
1E+15	1E+15	1050	1300	1550	1400	1150	1200
1E+15	1E+15	1300	1050	1300	1150	900	950
1E+15	1E+15	1550	1300	1050	900	1150	1200
1E+15	1E+15	1800	1550	1300	1150	1400	1450
1E+15	1E+15	2050	1800	1550	1400	1650	1700
1E+15	1E+15	1900	1650	1400	1250	1500	1550
1E+15	1E+15	1650	1400	1150	1000	1250	1300
1E+15	1E+15	1400	1150	900	750	1000	1050
1E+15	1E+15	1150	900	1150	1000	750	800
1E+15	1E+15	900	1150	1400	1250	1000	1050
1E+15	1E+15	750	1000	1250	1100	850	900
1E+15	1E+15	1000	750	1000	850	600	650
1E+15	1E+15	1250	1000	750	600	850	900
1E+15	1E+15	1500	1250	1000	850	1100	1150
1E+15	1E+15	1750	1500	1250	1100	1350	1400

Table 39 Floyd's Algorithm (Part 1)

33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12
650	900	1150	1000	750	500	250	0	150	400	650	900	1150	1300	1050	800	550	300	450	700	950	1200
400	650	900	750	500	250	0	250	400	150	400	650	900	1050	800	550	300	550	700	450	700	950
150	400	650	500	250	0	250	500	650	400	150	400	650	800	550	300	550	800	950	700	450	700
400	150	400	250	0	250	500	750	900	650	400	150	400	550	300	550	800	1050	1200	950	700	450
650	400	150	0	250	500	750	1000	1150	900	650	400	150	300	550	800	1050	1300	1450	1200	950	700
500	250	0	150	400	650	900	1150	1300	1050	800	550	300	450	700	950	1200	1450	1600	1350	1100	850
250	0	250	400	150	400	650	900	1050	800	550	300	550	700	450	700	950	1200	1350	1100	850	600
0	250	500	650	400	150	400	650	800	550	300	550	800	950	700	450	700	950	1100	850	600	850
250	500	750	900	650	400	150	400	550	300	550	800	1050	1200	950	700	450	700	850	600	850	1100
500	750	1000	1150	900	650	400	150	300	550	800	1050	1300	1450	1200	950	700	450	600	850	1100	1350
650	900	1150	1300	1050	800	550	300	450	700	950	1200	1450	1600	1350	1100	850	600	750	1000	1250	1500
400	650	900	1050	800	550	300	550	700	450	700	950	1200	1350	1100	850	600	850	1000	750	1000	1250
550	800	1050	1200	950	700	450	700	850	600	850	1100	1350	1500	1250	1000	750	1000	1150	900	1150	1400
150	400	650	800	550	300	550	800	950	700	450	700	950	1100	850	600	850	1100	1250	1000	750	1000
400	150	400	550	300	550	800	1050	1200	950	700	450	700	850	600	850	1100	1350	1500	1250	1000	750
650	400	150	300	550	800	1050	1300	1450	1200	950	700	450	600	850	1100	1350	1600	1750	1500	1250	1000
800	550	300	450	700	950	1200	1450	1600	1350	1100	850	600	750	1000	1250	1500	1750	1900	1650	1400	1150
600	350	600	750	500	750	1000	1250	1400	1150	900	650	900	1050	800	1050	1300	1550	1700	1450	1200	950
550	300	550	700	450	700	950	1200	1350	1100	850	600	850	1000	750	1000	1250	1500	1650	1400	1150	900
300	550	800	950	700	450	700	950	1100	850	600	850	1100	1250	1000	750	1000	1250	1400	1150	900	1150
450	700	950	1100	850	600	850	1100	1250	1000	750	1000	1250	1400	1150	900	1150	1400	1550	1300	1050	1300
700	450	700	850	600	850	1100	1350	1500	1250	1000	750	1000	1150	900	1150	1400	1650	1800	1550	1300	1050
950	700	450	600	850	1100	1350	1600	1750	1500	1250	1000	750	900	1150	1400	1650	1900	2050	1800	1550	1300
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15

	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
	400	150	300	550	700	800	1050	1300	1450	1250	1200	950	1100	1350	1600	1E+15	1E+15
	150	400	550	300	450	550	800	1050	1200	1000	950	700	850	1100	1350	1E+15	1E+15
	400	650	800	550	700	300	550	800	950	750	700	450	600	850	1100	1E+15	1E+15
	650	900	1050	800	950	550	300	550	700	500	450	700	850	600	850	1E+15	1E+15
	900	1150	1300	1050	1200	800	550	300	450	750	700	950	1100	850	600	1E+15	1E+15
	750	1000	1150	900	1050	650	400	150	300	600	550	800	950	700	450	1E+15	1E+15
	500	750	900	650	800	400	150	400	550	350	300	550	700	450	700	1E+15	1E+15
	250	500	650	400	550	150	400	650	800	600	550	300	450	700	950	1E+15	1E+15
	0	250	400	150	300	400	650	900	1050	850	800	550	700	950	1200	1E+15	1E+15
	250	0	150	400	550	650	900	1150	1300	1100	1050	800	950	1200	1450	1E+15	1E+15
	400	150	0	250	400	800	1050	1300	1450	1250	1200	950	1100	1350	1600	1E+15	1E+15
	150	400	250	0	150	550	800	1050	1200	1000	950	700	850	1100	1350	1E+15	1E+15
	300	550	400	150	0	700	950	1200	1350	1150	1100	850	1000	1250	1500	1E+15	1E+15
	400	650	800	550	700	0	250	500	650	450	400	150	300	550	800	1E+15	1E+15
	650	900	1050	800	950	250	0	250	400	200	150	400	550	300	550	1E+15	1E+15
	900	1150	1300	1050	1200	500	250	0	150	450	400	650	800	550	300	1E+15	1E+15
	1050	1300	1450	1200	1350	650	400	150	0	600	550	800	650	400	150	1E+15	1E+15
	850	1100	1250	1000	1150	450	200	450	600	0	50	300	450	200	450	1E+15	1E+15
	800	1050	1200	950	1100	400	150	400	550	50	0	250	400	150	400	1E+15	1E+15
	550	800	950	700	850	150	400	650	800	300	250	0	150	400	650	1E+15	1E+15
	700	950	1100	850	1000	300	550	800	650	450	400	150	0	250	500	1E+15	1E+15
	950	1200	1350	1100	1250	550	300	550	400	200	150	400	250	0	250	1E+15	1E+15
	1200	1450	1600	1350	1500	800	550	300	150	450	400	650	500	250	0	1E+15	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0	1E+15
	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0

Table 40 Floyd's Algorithm (Part 2)

4 Matrix C_{ij}

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	1	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	2	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
7	0	0	0	3	0	1	0	1	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

9	0	2	0	0	0	0	0	1	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
10	2	0	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0
12	0	0	0	0	0	0	0	0	2	0	1	0	1	0	0	0	0	0	2	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	3	0	0	0	0	0	1	0	1	0	3	0	0	0	0	0	0	0	0
15	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	8	0	0	0	0	0	0	0	2
17	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	8	0	8	0	0	0	0	0	3	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	8	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	8	0	8	0	2	0	0	0
20	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	8	0	2	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0	1	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	1	0	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 41 Matrix Cij (Part 1)

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	7	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	7	0	7	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	7	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	7	0	7	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	7	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	2	0	3	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
32	0	0	0	2	0	3	0	3	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	3	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	3	0	0	0	0	0	3	0	3	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0
35	2	0	0	0	0	0	0	0	3	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	2	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	3	0	4	0	4	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	2	0	0	0	0	0	0	4	0	4	0	0	2	0	0	0	0	0	0
41	0	0	0	0	0	2	0	0	0	0	0	0	0	0	4	0	2	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	2	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	9	0	9	0	2	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	4	0	4	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	4	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 42 Matrix Cij (Part 2)

5 Matrix W_{ij}

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	1	0	0	0	0	0	0	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	1	0	2.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	4.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.9	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.2	0	0	0	0	0	0	0	0.1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.9	0	0	0	0	0	0.1	0	0.1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0.1	0	0.1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	0.1	0	0.1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	0.1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	0	0	0	0	0	0	0.1	0	1.4	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	0	0	0	0	0.1	0	0.1	0	1.4	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0.1	0	0.1	0	0	0	0.1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.9	0	0.1	0	0.1	0	0	0	0	0	2.9	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.2	0	0.1	0	0	0	0	0	0	0	4.2
0	0	0	0	0	0	0	0	0	0	0	4.2	0	0	0	0	0	0	0	6.9	0	4.2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	2.9	0	0	0	0	0	6.9	0	6.9	0	2.9	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0	6.9	0	6.9	0	0	0	0.1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	6.9	0	6.9	0	0	0	0	0	1.4	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	6.9	0	0	0	0	0	0	0	1.4	0	0	0	0	0
0	0	0	0	0	0	1.4	0	0	0	0	0	0	0	1	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1.4	0	0	0	0	0	1	0	1	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0.1	0	0	0	1	0	1	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2.9	0	1	0	1	0	0	0	0	0	2.9	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	4.2	0	1	0	0	0	0	0	0	0	4.2	0	0	0	0	0	0	0	0	0	0

37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 43 Matrix Wij (Part 1)

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
0	0	0	0	0	0	0	0	0	0	0	4.2	0	0	0	0	0	0	0	3.5	0	4.2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	2.9	0	0	0	0	0	3.5	0	3.5	0	2.9	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0	3.5	0	3.5	0	0	0	0.1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	3.5	0	3.5	0	0	0	0	0	1.4	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	3.5	0	0	0	0	0	0	0	1.4	0	0	0	0	0
0	0	0	0	0	1.4	0	0	0	0	0	0	0	0	4.8	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1.4	0	0	0	0	0	0	4.8	0	4.8	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.1	0	0	0	0	4.8	0	4.8	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2.9	0	4.8	0	4.8	0	0	0	0	0	2.9	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	4.2	0	4.8	0	0	0	0	0	0	0	4.2	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	3.5	0	4.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	3.5	0	3.5	0	2.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	3.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0.1	0	0	0	0	3.5	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1.4	0	0	3.5	0	3.5	0	0	0	0	0	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1.4	0	3.5	0	0	0	0	0	0	0	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	9.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	9.5	0	9.5	0	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	9.5	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6.7	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	6.7	**0**	6.7	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.4	0	0	0	0	6.7	**0**	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**

Table 44 Matrix Wij (Part 2)

Appendix B: Datasets/Scenarios

1 Introduction

For this research project numerous private security companies have been approached for support and inputs. Although a number of insight regarding RV requirements and operations were obtained, the author was unable to obtain any data regarding customer locations and call-out history due to security reasons.

Due to this factor it was decided to develop different realistic scenarios that could be used as the inputs for the models. How the models adapt to the input changes was interpreted..

The following factors change in the various scenarios:

- The number of customers.
- The size of the operational area.
- The average number of call-outs per customer.
- The number of areas with an exceptionally high crime rate.
- The locations of the customers.
- The number of clusters in the area (clusters are all $9km^2$).
- The geography of the area (roads and population density).
- Ratio of number of customers versus number of call-outs.

In this Appendix each of the scenarios is as follows:

2 Scenario 1

In Scenario 1 both the location and number of call-outs of the customers are randomly distributed, there is no trend in customer density nor in the locations of the call-outs.

The area covers an area of $581km^2$ and is divided into square clusters of $9km^2$, as can be seen in the figures below:

1	2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33
34	35	36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65	66
67	68	69	70	71	72	73	74	75	76	77

Figure 61 Clusters Scenario 1

The area has 4474 customers, at a customer density of approximately 8 per square kilometre. The X and Y coordinates for all the customers were captured, but due to the size the table is not included in the report, the locations are displayed in the figure below.

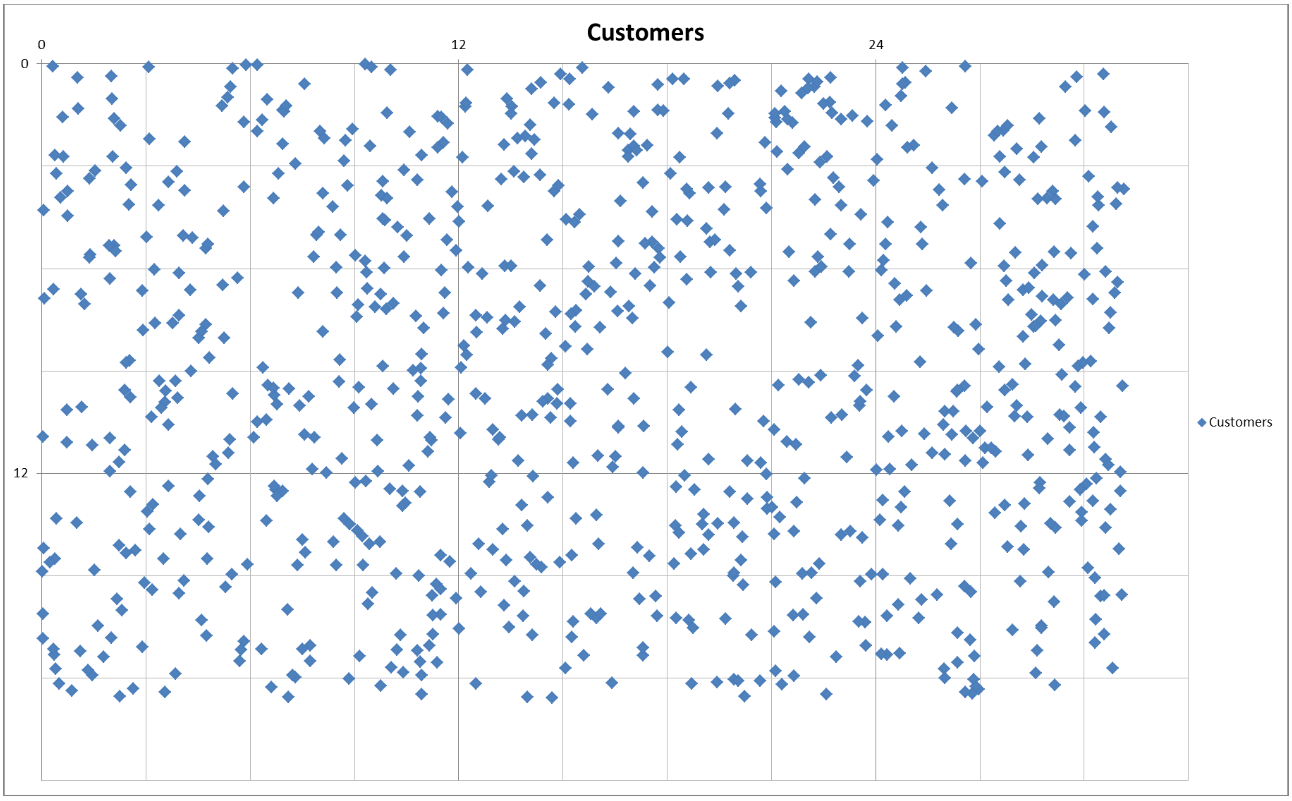

Figure 62 Customers Scenario 1

After the customer locations were determined, calculations were done to determine how many customers there were in each of the clusters and how many call-outs were made in each of the clusters. The data was then used to generate the following graph.

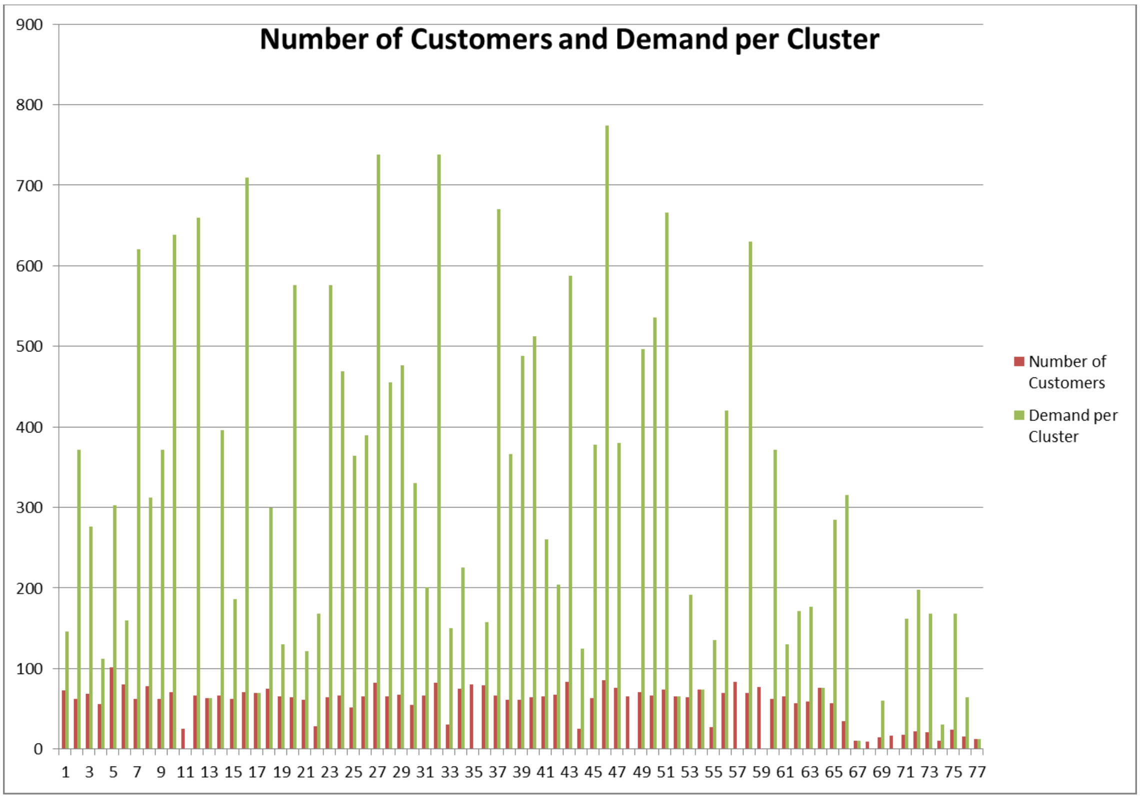

Figure 63 Customers and Demand per Cluster (Scenario 1)

3 Scenario 2

In Scenario 2 the location of the customers are randomly distributed and there is no trend in customer locations. Although the locations are randomly distributed, there are certain areas with a higher crime rate than others as can be seen in the figures and diagrams below.

The area covers an area of 491km^2 and is divided into square clusters of 9km^2, as can be seen in the figure below.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75

Figure 64 Clusters Scenario 2

The area has 3430 customers, at a customer density of approximately 7 per square kilometre. The X and Y coordinates for all the customers were captured, but due to the size the table is not included in the report, the locations are displayed in the figure below.

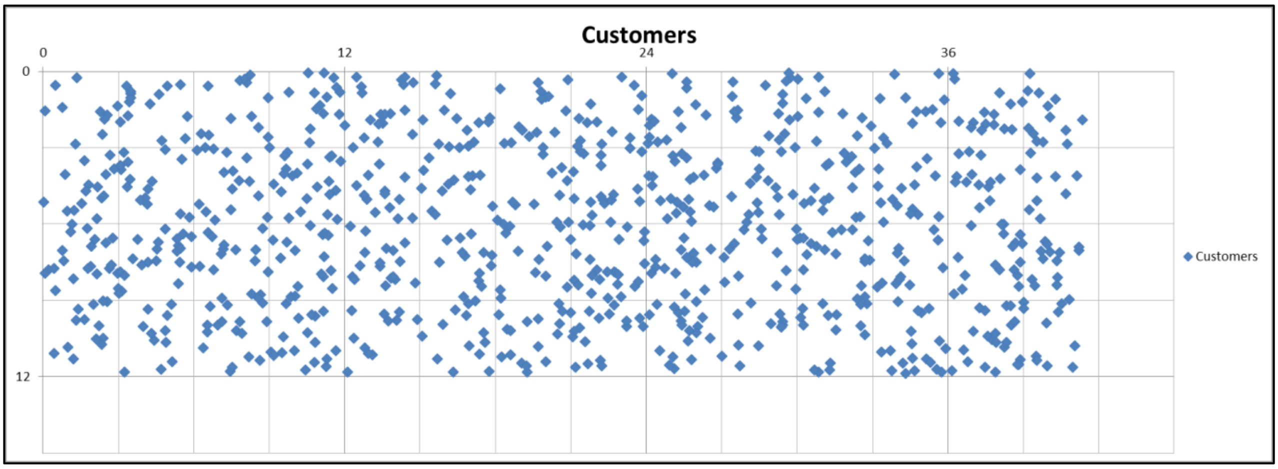

Figure 65 Customers Scenario 2

After the customer locations were determined, calculations were done to determine how many customers were in each of the clusters and how many call-outs were made in each of the clusters. The data was then used to generate the following graph.

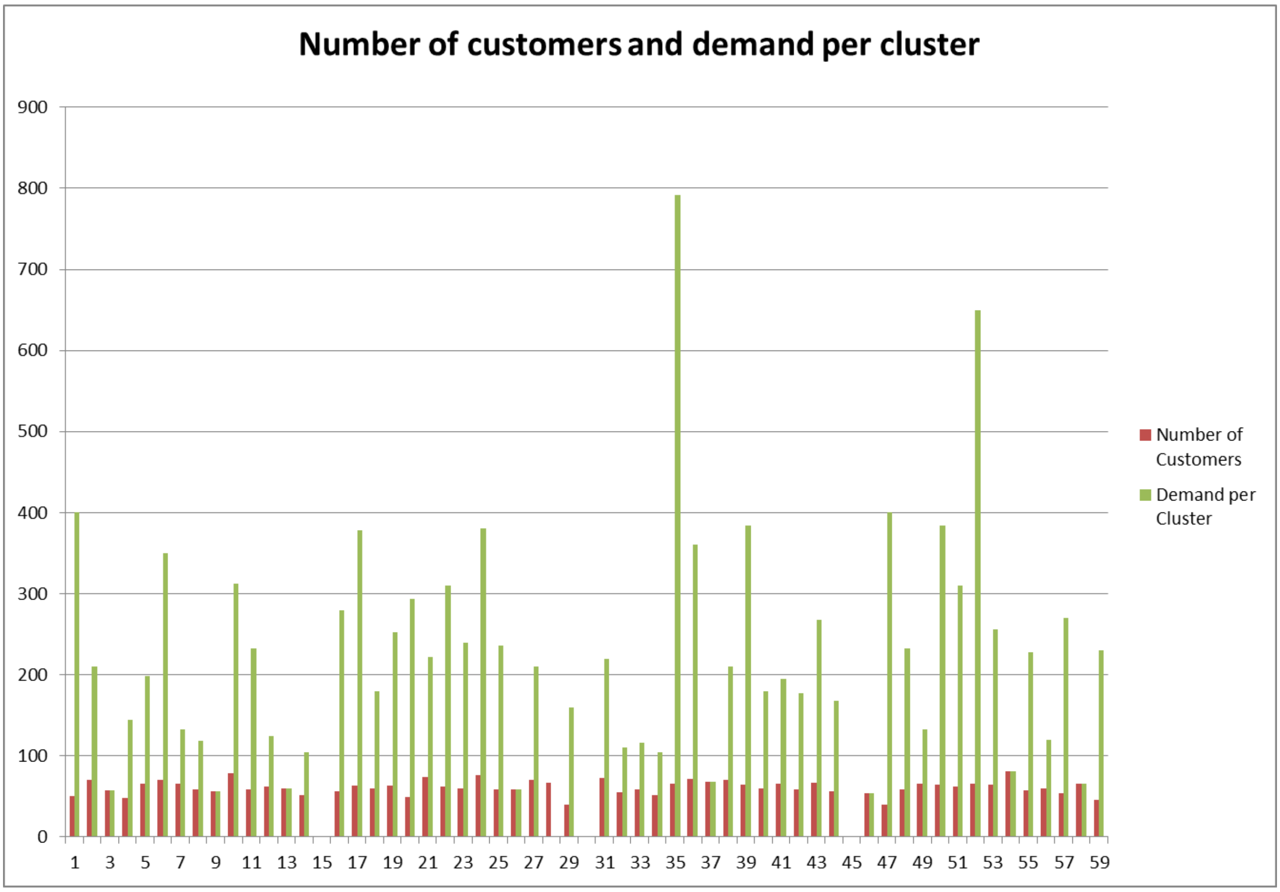

Figure 66 Customers and Demand per Cluster (Scenario 2)

4 Scenario 3

In Scenario 3 there are areas with a high number of customers and there is an area with an extremely high demand as can be seen in the table below:

Cluster Number	Number of Customers	Demand per Cluster
1	289	578
2	327	6540
3	291	873
4	288	0
5	282	846
6	87	174
7	80	160
8	86	172
9	93	93
10	75	375
11	76	1140
12	23	69
13	48	144
14	39	0
15	46	184
16	53	106
17	48	0
18	17	17
19	0	0
20	0	0
21	0	0
22	0	0
23	0	0
24	0	0

Table 45 Cluster Information Scenario 3

The area covers an area of 125 km^2 and is divided into square clusters of $9km^2$ as can be seen in the figures below.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24

Figure 67 Clusters Scenario 3

The area has 2248 customers, at a customer density of approximately 18 per square kilometre. The X and Y coordinates for all the customers were captured, but due to the size the table is not included in the report, the locations are displayed in the figure below.

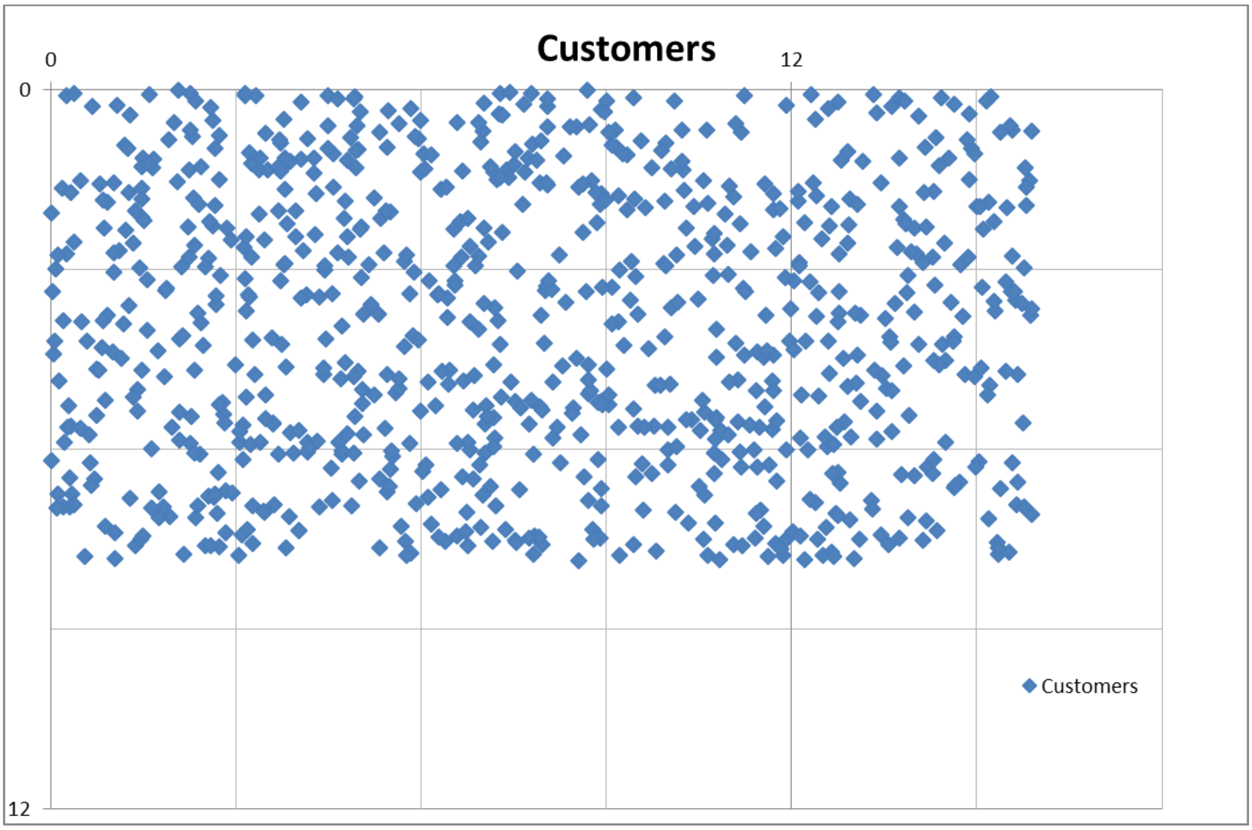

Figure 68 Customers Scenario 3

After the customer locations were determined, calculations were done to determine how many customers there are in each of the clusters and how many call-outs were made in each of the clusters. The data was then used to generate the following graph.

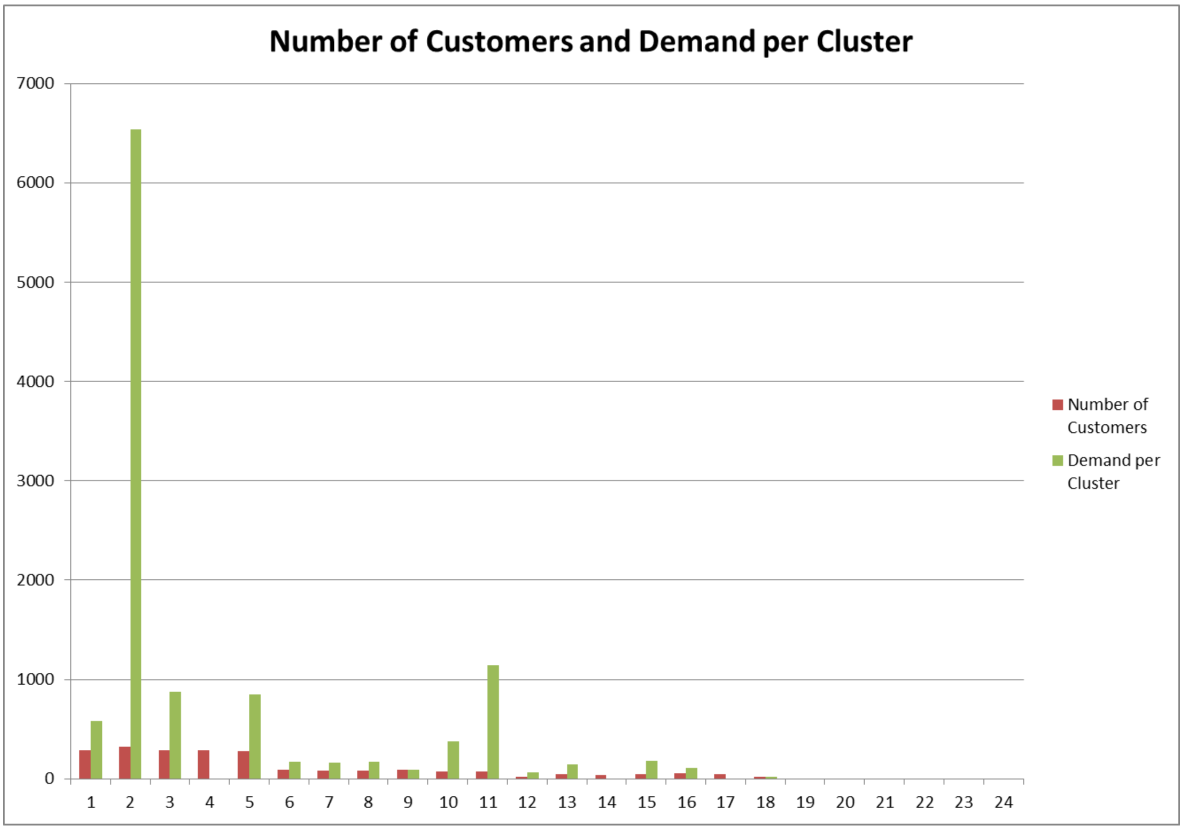

Figure 69 Customers and Demand per Cluster (Scenario 3)

5 Scenario 4

In Scenario 4 the both the location and number of call-outs of the customers are randomly distributed, there is no trend in customer density nor in the locations of the call-outs. The difference in this scenario is that the operational area is a lot smaller than those in the other scenarios, although this operational area has a large number of customers.

Cluster Number	**Number of Customers**	**Demand per Cluster**
1	47	47
2	59	177
3	66	66
4	58	174
5	51	204
6	72	0

7	47	141
8	0	0
9	52	156
10	53	106
11	59	236
12	53	106
13	51	204
14	54	270
15	38	76
16	0	0
17	13	13
18	26	130
19	13	26
20	15	45
21	13	26
22	7	14
23	6	30
24	0	0

Table 46 Cluster Information Scenario 4

The area covers an area of 15 km^2 and is divided into square clusters of 9km^2, as can be seen in the figures below.

1	2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31	32	33
34	35	36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65	66
67	68	69	70	71	72	73	74	75	76	77

Figure 70 Clusters Scenario 4

The area has 853 customers, at a customer density of approximately 57 per square kilometre. The X and Y coordinates for all the customers were captured, but due to the size the table is not included in the report, the locations are displayed in the figure below.

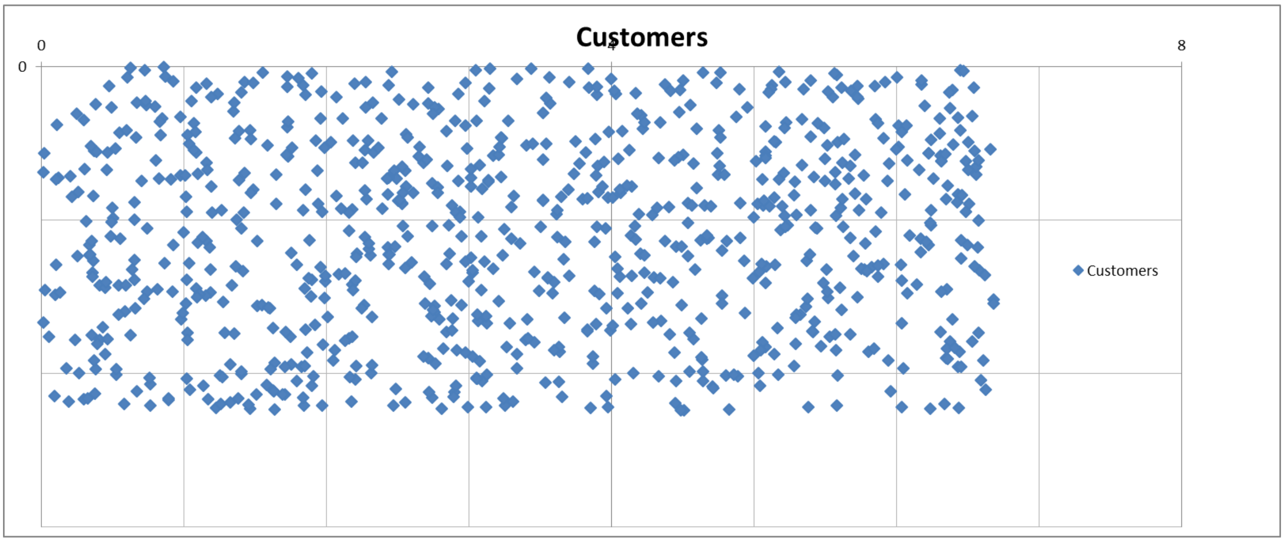

Figure 71 Customers Scenario 4

After the customer locations were determined, calculations were done to determine how many customers there are in each of the clusters and how many call-outs were made in each of the clusters. The data was then used to generate the following graph.

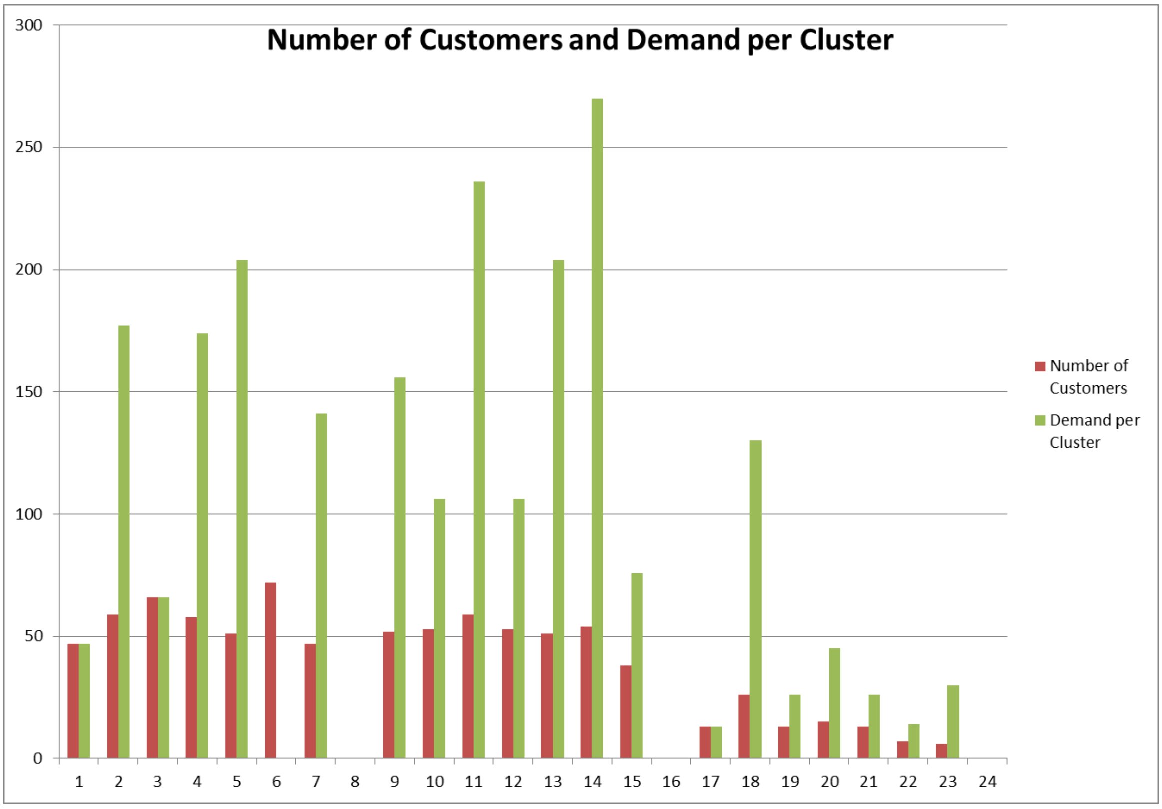

Figure 72 Customers and Demand per Cluster (Scenario 4)

Appendix C

Lingo Code

1 Explicit Network Coverage Model

```
SETS:
!Set Members;
POSITIONS/1..48/ : X;
FROM_NODE/1..48/:;
TO_NODE/1..48/:;
COMBINATIONS (POSITIONS, FROM_NODE, TO_NODE): S;
ARCS (FROM_NODE, TO_NODE): N, W, C, DI, DJ, Y;
ENDSETS

DATA:
T= Value of T;
R= The number of RVs to use;
D= Time of Day;
AS= Average Driving Speed;
DR= Desired Response Time;
N= Matrix Nij;
W= Matrix Wij;
DI= Matrix Dij;
DJ= Matrix Dij;
C= Matrix Cij;
ENDDATA

!1 OBJECTIVE FUNCTION;
MIN = @SUM(COMBINATIONS(k,i,j):W(i,j)*(DI(k,i)+DJ(k,j)/2)*S(k,i,j));

!2 ;
@SUM(POSITIONS(k):X(k))= @ROUNDUP(R/D,0);

!3-5 If statement;
```

```
@FOR (ARCS(i,j): 0<@SUM(POSITIONS(k):S(k,i,j))-1+9999*Y(i,j));
@FOR (ARCS(i,j): N(i,j)<=9999*(1-Y(i,j)));
@FOR (ARCS(i,j):@BIN(Y(i,j)));

!6;
@FOR (ARCS(i,j):@SUM(POSITIONS(k):S(k,i,j))<=1);

!7;
@FOR (COMBINATIONS(i,j,k):S(k,i,j)-X(k)<=0);

!8;
@FOR (POSITIONS(k):@SUM(ARCS(i,j):C(i,j)*S(k,i,j))<=T*D);

!9;
@FOR(COMBINATIONS(k,i,j):((DI(k,i)+DJ(k,j))/2)*S(k,i,j)<=AS*(1000/60)*DR);

!10;
@FOR (POSITIONS(k):@BIN(X(k)));

!11;
@FOR (COMBINATIONS(k,i,j):@BIN(S(k,i,j)));

END
```

2 Single Location Model using both number of call-outs and customer locations.

```
SETS:
Customers/1..4474/:w,a,b,d;
ENDSETS

DATA:
w = Expected number of call-outs;
a = X coordinate of customers;
b = Y coordinate of customers;
ENDDATA
```

```
MIN = @SUM(Customers(i):w(i)* d(i));
@FOR(Customers(i):d(i)= @abs(x-a(i))+@abs(y-b(i)));
END
```

3 Queueing Theory

```
SETS:
!Set Members;
ENDSETS

DATA:
LAM = Arrival Rate of Call-Outs;
MU =  Service Rate;
DT =  Desired Time in System;
ENDDATA

MIN = s;
W <= DT;
W = (L/LAM);
L = LQ+ (LAM/MU);
LQ = ((@PEB(LAM/MU,s))*p)/(1-p);
p = LAM/(s*MU);
END
```

4 Location Set Covering Problem (Explicit)

```
SETS:
!Set Members;
Sites/1..3/: X;  !i;
Area/1..3/: LAM, MU, DT, s, W, LS, LQ, R;
Combination/1..7/: P;  !l;
CombinationSites(Combination,Sites)  : Y;  !li;
CombinationArea(Combination,Area)  :M; !lk;
ENDSETS
```

```
DATA:
LAM = Arrival Rate;
MU = Service Rate;
DT = Desired Response Time ;
Y =;
M =;
ENDDATA


MIN = @SUM(Sites(i):X(i));
@for(Sites(i): X(i)>= @SUM(Combination(l):P(l)*Y(l,i)));
@for(Area(k):@Sum(Combination(l):P(l)*M(l,k))= S(k));
@for(Sites(i):X(i)>=0);
@For(Combination(l):P(l)>=0);
@FOR(Area(k):
W(k) <= DT;
W(k) = (LS(k)/LAM(k));
LS(k) = LQ(k)+ (LAM(k)/MU(k));
LQ(k) = ((@PEB(LAM(k)/MU(k),s(k)))*R(k))/(1-R(k));
R(k) = LAM(k)/(s(k)*MU(k));
@GIN(s(k));
);
END
```

5 Maximal Covering Location Problem (Explicit)

5.1 Z1 Pre-empting Number of Customers

```
SETS:
!Set Members;
Sites/1..12/: X;            !i;
Area/1..77/: D,N;                   !k;
Combination/1..4095/:P ;        !l;
CombinationSites(Combination,Sites) :Y ; !li;
CombinationArea(Combination,Area) : M,C; !lk;
```

```
ENDSETS
DATA:
R = ;
D = ;
Y = ;
M = ;
ENDDATA

Max = @SUM(Combination(l):@SUM(Area(k): C(l,k)*D(k))) ;
@SUM(Sites(i):X(i))=R;
@FOR(Area(k):@SUM(Combination(l):C(l,k))<=1);
@FOR(Combination(l):@FOR(Area(k):C(l,k)<=M(l,k)*P(l)));
@FOR(Sites(i):X(i)=@SUM(Combination(l):Y(l,i)*P(l)));
@FOR(Combination(l):@BIN(P(l)));
@FOR(Combination(l):@FOR(Area(k):@BIN(C(l,k))));
END
```

5.2 Z2 Pre-empting Number of Demand

```
SETS:
!Set Members;
Sites/1..12/: X;            !i;
Area/1..77/: D,N;                  !k;
Combination/1..4095/:P ;        !l;
CombinationSites(Combination,Sites) :Y ; !li;
CombinationArea(Combination,Area) : M,C; !lk;
ENDSETS

DATA:
R = ;
D = ;
N = ;
Y = ;
M =;
ENDDATA
```

```
Max = @SUM(Combination(l):@SUM(Area(k): C(l,k)*N(k)));
@SUM(Combination(l):@SUM(Area(k): C(l,k)*D(k))) >= Output from Z1 model ;
@SUM(Sites(i):X(i))=R;
@FOR(Area(k):@SUM(Combination(l):C(l,k))<=1);
@FOR(Combination(l):@FOR(Area(k):C(l,k)<=M(l,k)*P(l)));
@FOR(Sites(i):X(i)=@SUM(Combination(l):Y(l,i)*P(l)));
@FOR(Combination(l):@BIN(P(l)));
@FOR(Combination(l):@FOR(Area(k):@BIN(C(l,k))));
END
```

Appendix D:

Macros

1 Auto complete symmetrical matrix

```
Sub Macro1()
 mynum = 1
   Do Until mynum = 50
      Cells(mynum + 1, mynum + 2).Select
      Range(Selection, Selection.End(xlToRight)).Select
      Selection.Copy
      Cells(mynum + 2, mynum + 1).Select
      Selection.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= _
         False, Transpose:=True
      Application.CutCopyMode = False
      mynum = mynum + 1
   Loop
   Cells(1, 1).Select
End Sub
```

2 Floyd's Algorithm

```
Sub Macro2()
   Dim i, j, k, n
   ActiveWorkbook.Sheets("Floyd Input").Activate
   n = ActiveWorkbook.Sheets("Floyd Input").Cells(1, 16).Value

   'Clear output sheet
   For i = 1 To n
      For j = 1 To n
         Sheets("Dij").Cells(1 + i, 1 + j).Value = Null
      Next j
   Next i

   'Copy weights to output sheet
```

```
  For i = 1 To n
    For j = 1 To n
      Sheets("Dij").Cells(1 + i, 1 + j).Value = Sheets("Floyd Input").Cells(2 + i, 1 + j)
      If Sheets("Floyd Input").Cells(2 + i, 1 + j).Value = "" Then Sheets("Dij").Cells(1 + i, 1 + j).Value =
10 ^ 15
    Next j
  Next i

  'Run algorithm
  Sheets("Dij").Activate
  For k = 1 To n
    For i = 1 To n
      For j = 1 To n
        If (Sheets("Dij").Cells(1 + i, 1 + j) > (Sheets("Dij").Cells(1 + i, 1 + k) + Sheets("Dij").Cells(1 + k, 1
+ j))) Then
          Sheets("Dij").Cells(1 + i, 1 + j) = Sheets("Dij").Cells(1 + i, 1 + k) + Sheets("Dij").Cells(1 + k, 1
+ j)
        End If
      Next j
    Next i
  Next k
End Sub
```

Appendix E:

Forecasting

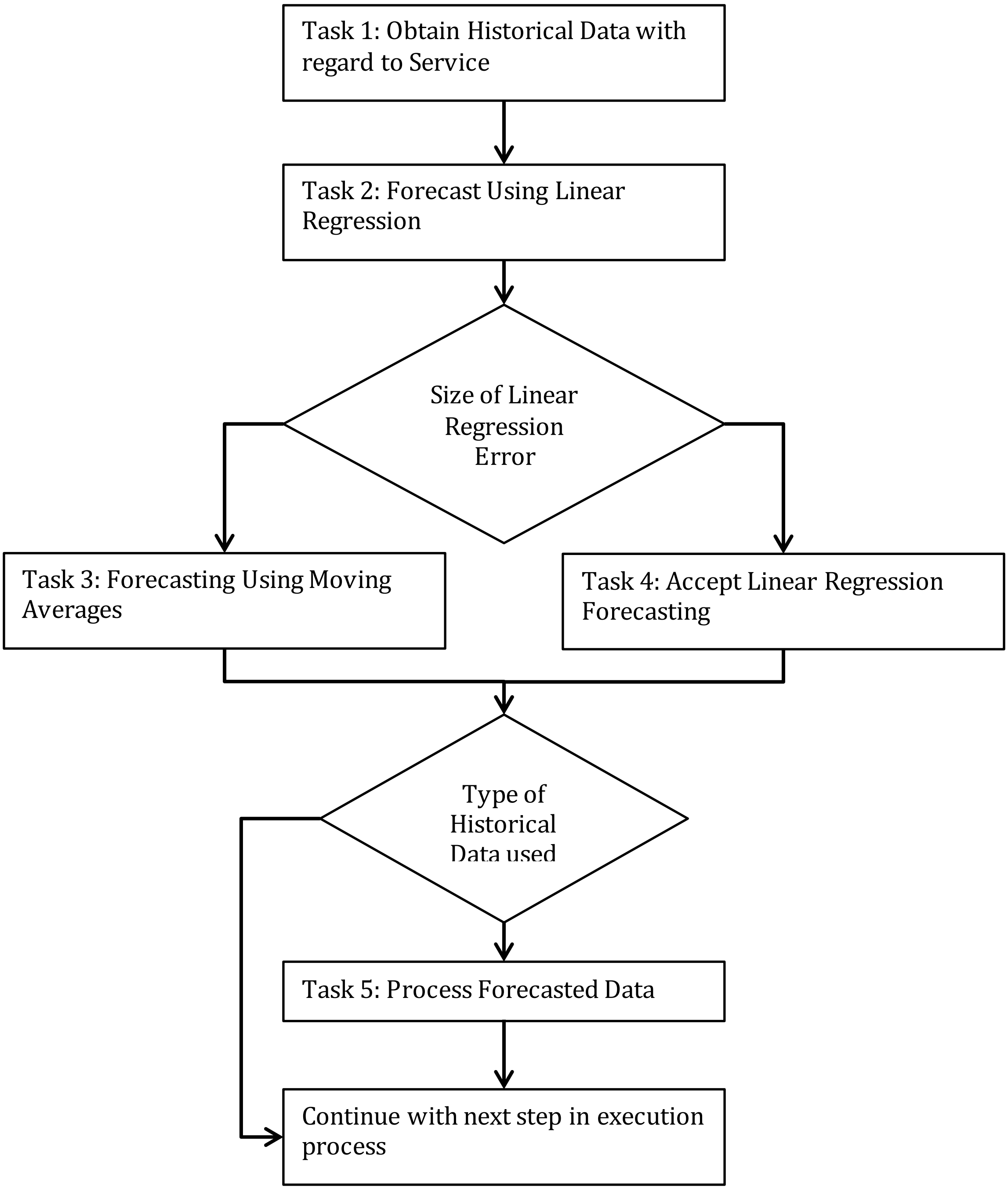

Figure 73 Forecasting Tasks

Task 1: Obtain Historical Data with regard to Service

All private security companies in South Africa keep records of the information when a service is delivered. The information includes how frequently a vehicle responds to a specific area or to a specific customer. Certain companies store this data at an area-level, where other may store it at street-level. To obtain improved results from the mathematical model, it is best for the company to record the information at street-level, but processing the area-level information it can also be used effectively.

The number of times service was required for previous periods must be obtained, if more periods are used, the results of the forecasting will be more accurate. A minimum of three periods will be sufficient for the purposes of this model.

In the below example, the frequency that service was required by customers for the previous three months has been captured in a spreadsheet, this variable will be used in steps 2 to 5 below to do forecasting. The spreadsheet extract with the data can be viewed in the table below.

Street Name	**Historical Data**		
	Number of Times Service was required during month X		
	1	2	3
Bailey Street	1199	1154	1357
Barend Street	886	860	964
Botha Street	209	217	268
Henning Street	484	112	151
Langerman Street	997	868	763
Malan Street	728	1374	1472
Market Street	113	447	201
Sauer Street	0	0	0
Smuts Street	0	0	0
Stegmann Street	434	647	960
Strydom Street	992	991	994
Tudhope Street	764	1432	858
Union Street	333	260	173
Van Deventer Street	85	92	26

Task 2: Forecast Using Linear Regression

The historical data captured in Task 1 must be used as inputs to forecast the number of times service will be required in the future period. The regression sum of squares (SSR), error sum of squares (SSE) and the total sum of squares (SST) must also be calculated. It is used to determine the ratio of the regression sum of squares to the total sum of squares (r^2).

The value of r^2 will always be less than 1. If the value of r^2 is exactly 1 it means that a perfect straight line relationship is obtained between the x and y values of the linear equation. If the value of r^2 is very small which means that the data does not form a straight line and another method of forecasting must be used.

The linear forecasting and forecasting errors for the example can be viewed in the table below.

Linear Regression								
Calculations		Slope	Y Intercept	Forecasted Value for Next	Linear Regression			
SSXY	SSX	b1	b0	Y4	SSR	SSE	SST	r^2
158	2	79	1078.666667	1395	12482	10250.66667	22732.66667	0.549077686
78	2	39	825.3333333	981	3042	2816.666667	5858.666667	0.519230769
59	2	29.5	172.3333333	290	1740.5	308.1666667	2048.666667	0.849576961
-333	2	-166.5	582	-84	55444.5	28153.5	83598	0.663227589
-234	2	-117	1110	642	27378	96	27474	0.996505787
744	2	372	447.3333333	1935	276768	50050.66667	326818.6667	0.846854933
88	2	44	165.6666667	342	3872	56066.66667	59938.66667	0.064599368
0	2	0	0	0	0	0	0	#DIV/0!
0	2	0	0	0	0	0	0	#DIV/0!
526	2	263	154.3333333	1206	138338	1666.666667	140004.6667	0.988095635
2	2	1	990.3333333	994	2	2.666666667	4.666666667	0.428571429
94	2	47	924	1112	4418	257094	261512	0.016894062
-160	2	-80	415.3333333	95	12800	32.66666667	12832.66667	0.997454413
-59	2	-29.5	126.6666667	9	1740.5	888.1666667	2628.666667	0.662122749

Task 3: Forecast Using Moving Averages

If the value of r^2 is less than 0.7, the forecast value would not be accurate enough for this model and moving averages should be used instead as a more accurate forecasting method.

Forecasting for the example by using moving averages can be viewed in the table below.

Street Names	**Forecast Value for Next Period**
Bailey Street	1236.666667
Barend Street	903.3333333
Botha Street	231.3333333
Henning Street	249

Langerman Street	876
Malan Street	1191.333333
Market Street	253.6666667
Sauer Street	0
Smuts Street	0
Stegmann Street	680.3333333
Strydom Street	992.3333333
Tudhope Street	1018
Union Street	255.3333333
Van Deventer Street	67.66666667

Task 4: Accept Linear Regression Forecasting

If r^2 is greater than 0.7, the forecast value by using linear regression would be accurate enough for this model. In the table below, all the values in the blocks that have a green background have r^2 greater than 0.7.

Street Name	Linear Regression Forecasted Value for Next Y4	Error r^2	Moving Averages Forecasted Value for Next Y4	Forecasted Value to be Used
Bailey Street	1395	0.549077686	1236.666667	1237
Barend Street	981	0.519230769	903.3333333	903
Botha Street	290	0.849576961	231.3333333	290
Henning Street	-84	0.663227589	249	249
Langerman Street	642	0.996505787	876	642
Malan Street	1935	0.846854933	1191.333333	1935
Market Street	342	0.064599368	253.6666667	254
Sauer Street	0	#DIV/0!	0	#DIV/0!
Smuts Street	0	#DIV/0!	0	#DIV/0!
Stegmann Street	1206	0.988095635	680.3333333	1206
Strydom Street	994	0.428571429	992.3333333	992
Tudhope Street	1112	0.016894062	1018	1018
Union Street	95	0.997454413	255.3333333	95
Van Deventer Street	9	0.662122749	67.66666667	68

Task 5: Process Forecast Data

The forecast data only needs to be processed further if the data that was used as inputs were captured at an area-level. For the model to perform efficiently all data needs to be at a street-level. The following formula must be used to transform the data from area-level to street-level:

$$\text{Forecasted Value at Street-Level} = \frac{\text{Forecasted Value at Area-Level}}{\text{Number of Customers in Area}} \times \text{Number of Customers in Street}$$

In the current example, all of the data was captured at a street-level, thus no further processing is required.

Formulae for linear regression:

$\hat{Y}_i =$ The predicted values of Y.

$X_i =$ The independent X values.

$Y_i =$ The dependant Y values.

$\bar{X} =$ The mean of the X values.

$\bar{Y} =$ The mean of the Y values.

$n =$ The number of observations.

Determining the slope:

$$b_1 = \frac{SSXY}{SSX}$$

$$SSX = \sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=1}^{n} X_i^2 - \frac{(\sum_{i=1}^{n} X_i)^2}{n}$$

$$SSXY = \sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^{n} X_i Y_i - \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{n}$$

Determining the Y intercept:

$$b_0 = \bar{Y} - b_1\bar{X}$$

Determining the Error:

$$r^2 = \frac{SSR}{SST}$$

$$SSE = \sum_{i=1}^{n} Y_i^2 - b_0\sum_{i=1}^{n} Y_i - b_1\sum_{i=1}^{n} X_i Y_i$$

$$SSR = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

$$SST = SSR + SSE$$

Formula for Moving Averages:

$$Y_n = \frac{Y_{n-1} + Y_{n-2} + Y_{n-3}}{3}$$