The use of personal response systems to renegotiate the didactical contract in tertiary mathematics education

by

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DECLARATION

I, the undersigned, declare that the thesis which I hereby submit for the degree Philosophiae Doctor at the University of Pretoria, is my own, independent work and has not previously been submitted by me for a degree at this or any other tertiary institution.

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Abstract

Challenges experienced by first year students transitioning from secondary to tertiary mathematics education are examined through the lens of the didactical contract or agreement between the lecturer and students that is founded on beliefs about mutual obligations. First year students' fundamental beliefs about the nature of mathematics and mathematics teaching/learning must be challenged to negotiate a new didactical contract at tertiary level. Through pedagogy aimed at self-directed learning personal response systems (PRS) are periodically used to make students aware of their own learning and their responsibility for learning. A Likert scale questionnaire is administered at the beginning of the students' first year to gauge their beliefs about mathematics and mathematics teaching/learning and again at the end of the first semester (or term) to observe possible changes in beliefs and hence the didactical contract. The intervention consists of PRS sessions or so called Time-out sessions, regularly incorporated into the traditional transmission mode lecture to create a student-centred learning environment, aimed at influencing students' beliefs about the centredness of a mathematics classroom, mathematics learning and the responsibility for their learning. Questionnaire data is quantified and compared for the two surveys. There is evidence of a shift towards students taking ownership of their learning and a renegotiation of the didactical contract. Qualitative data generated by focus group interviews confirms the role of the PRS sessions in student beliefs.

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Chapter 1: Setting the Stage

1.1 Introduction

In a mathematics classroom various influences contribute to set the stage for teaching and learning. Just as a director would interpret a script, the teacher interprets the content of the mathematics curriculum based on their understanding and beliefs about mathematics and mathematics teaching/learning. Like each actor's interpretation of the script is based on their beliefs about mathematics and the teaching/learning of the subject, their prior knowledge will also play a critical role. The performance of these various role players in the classroom will therefore be the result of an intricate interplay of interpretations based on their prior knowledge, beliefs and attitudes.

The analogy of a stage, director and actors is used to illustrate an interpretation of didactics in mathematics as defined by Brousseau (Artigue, 2009; Balachef, 1990; D'Amore, 2008; Herbst and Kilpatrick, 1999). The didactical contract refers to the agreement about roles and responsibilities between the participants in a teaching/learning event (or didactical situation, as defined by Brousseau). The contract can be clear and/or tacit, and encompasses both the teacher's spoken and/or unspoken expectations about the student's role as well as the student's expectations of their teacher's role (D'Amore, 2008; Hourigan and O'Donoghue, 2007; Yoon, Kensington-Miller, Sneddon and Bartholomew, 2011). This "reciprocal obligation" (Pepin, 2014:653) becomes more complex at tertiary level because of the number of participants in large classrooms, but also the diverse mathematical abilities that students bring to the classroom (Biggs, 1999), especially at first year level.

1.2 Background

In recent years lecturers in the Department of Mathematics and Applied Mathematics at the University of Pretoria have had to adapt to the challenge of teaching large undergraduate classes. In the first semester of 2015 approximately 900 first year students (which increased to 1100 students in 2016) were enrolled for an applied calculus module, which meant that lecture groups varied between 70 and 400 students.

Active learning is a teaching approach that aims to introduce learning activities designed to encourage students to participate and reason (Bonwell and Eison, 1991). From an active learning perspective, successful teaching and learning can only be accomplished if students are involved in their own learning (Prince, 2004). The premise that the use of technology can help create an engaging environment in large classes led to the introduction of clickers in the applied calculus module in the first semesters of 2015 and 2016. After being exposed to the function that clickers or personal response systems can fulfil in an applied calculus classroom, their potential value in terms of the didactical contract emerged as a possible research question.

The challenge of teaching another first year calculus module to 126 first year education students in the second semester of 2015 reinforced the notion that students have definite expectations about the role of the lecturer. This notion was further strengthened when I taught the same calculus module to a new group of education students in the second semester of the following year, 2016. The students appeared to expect the lecturer to clarify mathematical methods rather than their mathematical understanding, which would lighten the burden of learning. The two different groups of second semester students have had similar first semester experiences in tertiary mathematics teaching/learning - all education students are taught on a satellite campus exclusively reserved for education students - having been taught by the same lecturer during the first semester. I realised that there was a mismatch between my expectations as lecturer and the expectations of the students, which impeded teaching and learning and needed to be addressed.

Both groups' experience of tertiary mathematics had been in the form of a pre-calculus module which focused on revising concepts from secondary mathematics education. As such, the students have not yet been confronted with the challenges inherent to a first year mathematics module. They were suddenly expected to handle complicated expressions, work with definitions of mathematical concepts and follow proofs. While I attempted to redirect their attention away from procedures towards conception, their main focus was on the "how" of solving a problem and they expected me, the lecturer, to expound the solution step by step and in the process facilitate easy learning to minimise their effort.

I realised that our expectations differed because there was a mismatch of beliefs about what mathematics and mathematics teaching and learning involves. While my beliefs about the teaching and learning of mathematics were rooted in the importance of a thorough understanding of mathematics and mathematical concepts, I realised that the students' beliefs about teaching/learning were based on their secondary education experience of teaching/learning, since their experience of tertiary mathematics was limited. I perceived students' beliefs of mathematics to be more about procedural fluency instead of conceptual understanding, and that their idea of an effective teaching/learning environment was to be coached or trained in mathematics without being challenged. This realisation led me to explore evidence about the transition from secondary to tertiary mathematics.

Researchers from various countries confirm that there appears to be a widening gap between secondary and tertiary mathematics education (Brandell, Hemmi and Thunberg, 2008; Clark and Lovric, 2009). Hourigan and O' Donoghue (2007) examine the nature of student-teacher interactions in secondary mathematics education in Ireland to explain the phenomenon of first year students failing to transition successfully from secondary to tertiary mathematics education. They conclude that the didactical contract in secondary mathematics education in Ireland is teacher-centred with preparation for the examination being the primary concern of teaching and learning. As a result, students are passive and expect the teacher to simplify the task of learning to the extent that procedures must be clearly outlined and demonstrated, new content must be revealed in relation to its relevance for the examination and its worth explained in terms of marks.

Benadé (2013) investigates the secondary-tertiary mathematics transition at a South African university and finds that students tend to view the lecturer as the source of all knowledge and learning as a transfer process. Most importantly, for them mathematics appears to be about procedural fluency. Yoon et al. (2011) investigate the didactical contract in the traditional mathematics classroom at tertiary level and find that students learn or rather memorise content outside the classroom and perceive good teaching to be the lecturer's ability to break down and model procedures with clarity.

Pepin (2014) uses the concept of the didactical contract to examine a student's transition from secondary to tertiary mathematics. She finds that at secondary level the responsibility for learning lies mainly with the teacher, whereas at tertiary level the responsibility becomes the student's. According to Pepin (2014), there is a definite change or break in the contract from secondary to tertiary level. She finds that the contract needs to be distinguished at tertiary level to help students transition successfully. Students should be supported to identify with the new contract so that the new expectations become their own. Yoon et al. (2011) mention that by including interactive learning activities in a teaching approach and focusing on conceptual rather than procedural understanding, a new didactical contract can be negotiated. The responsibility of successfully implementing student-centred activities without threatening students' comfort levels lies with the lecturer (Yoon et al., 2011). According to Selden (2005), mathematics learning at secondary level is surface learning, but at tertiary level deep conceptual learning is expected of students. Biggs (1999) advocates that a student-centred teaching approach promotes conceptual understanding in students and that good teaching is meant to lead students towards using cognitive processes at a higher level than those used in surface learning.

Dangel and Wang (2008) find that personal response systems can be used to move away from surface learning in order to nurture deep learning. They provide a framework for using personal response systems to help students use higher level cognitive processes and note that, based on a comprehensive pedagogy, these personal response systems can promote deeper learning. Dangel and Wang (2008) note that personal response systems have potential in valuing students' diverse abilities and Beatty, Leonard, Gerace and Dufresne (2006) note that proper questions can be used to record and consecutively address students' prior knowledge and beliefs. Kay and LeSage (2009) mention that the benefits of personal response systems in classrooms include student engagement, while maintaining their anonymity. Anonymity allows students to have an opinion, without being judged by peers (Kay and LeSage, 2009). Another benefit is in the form of the class discussion inspired by the use of personal response systems. Incorporating discussion after introducing a question or questions creates the opportunity for students to review and adjust their initial response or opinion (Lozanovski, Haeusler and Tobin, 2011). They mention that even though anonymity allows a student to have an opinion, the true value of the use of personal response systems in the mathematics classroom lies, based on the inherent nature of mathematics, in convincing students of the correct response and/or redirecting opinion.

1.3 Purpose Statement

The realisation of the potential value of the use of personal response systems in guiding students towards self-directed learning and closing the gap between secondary and tertiary mathematics education led to the aim of the study: to analyse students' expectations about mathematics teaching and learning at tertiary level, to distinguish and define the didactical contract at tertiary level and to use personal response systems to negotiate the didactical contract at tertiary level. The following objectives are subject to the main aim of the study: To explore the use of personal response systems based on an active learning pedagogy and to improve mathematics learning by:

- emphasising the student's responsibility for learning
- challenging and restructuring the student's beliefs about mathematics and the teaching and learning of mathematics
- encouraging a transition from surface learning to deep learning
- encouraging a transition from procedural to conceptual understanding; and
- directing students to accept responsibility for their own mathematics learning.

1.4 Research Question

The central aim of the research is to explore the potential value of personal response systems to guide students to accept responsibility for and be actively engaged in their own learning. The study aims to encourage students to accept responsibility by using personal response systems to challenge and reconstruct students' beliefs about mathematics and mathematics teaching/learning and encouraging conceptual understanding and deep learning of mathematics.

In an attempt to achieve the goal of utilising personal response systems to renegotiate the didactical contract in the mathematics classroom the following research questions are formulated:

How can personal response systems be utilised to renegotiate the didactical contract in the mathematics classroom through influencing student beliefs about

1. the centredness of the classroom
2. mathematics learning; and
3. the responsibility for their learning?

The merit of the study is situated in examining the use of personal response systems in the mathematics classroom through the lens of the didactical contract and exploring ways to incorporate personal response systems into a large mathematics classroom in order to renegotiate the didactical contract at first year level.

Balachef (1990) defines a *problématique* of research on mathematics education to include research questions that are connected to the mathematical meaning of students' conduct in the mathematics classroom. Balachef (1990) highlights aspects of Brousseau's theory of didactical situations (TDS) and mentions that learning is the result of a well-planned didactical process and according to Herbst and Kilpatrick (1999), the didactical contract allows the participants to navigate the process. In Chapter 3 Brousseau's theory of didactical situations (TDS) and didactical design will be discussed in order to establish the theoretical framework of the study.

1.5 Research Approach

The research approach employed a mixed methods case study where findings from qualitative and quantitative data are triangulated to answer the research question (Creswell, 2014).

1.5.1 The intervention. According to Balachef (1990), mathematical problems provide a trigger to challenge students' pre-conceptions and to enable development (Balachef, 1990). Since it is not possible to directly observe student learning, student behaviour and mathematical mistakes are the most meaningful sources for information about learning (Balachef, 1990). For the current study the essential mathematical content and potential student mistakes will be used to design personal response system (PRS) questions, based on principles from literature and personal experience. The questions will be in the form of so-called Time-out sessions from the traditional transmission style lecture. During these sessions the students will have the opportunity to respond to PRS questions, without correct answers being revealed. After being allowed time to have discussions with peers, the students will have the opportunity to respond to the same questions for a second time, followed by a reveal of correct answers, but no in-depth discussions. Figure 1.1 is included to elaborate terminology used when referring to personal response systems. For the purpose of my study a multiple-choice question is designed with one or more correct answers and a number of distractors. In class the student votes are recorded by means of a receiver and displayed in the form of a bar chart of student responses. In this slide a timer is included, which can be activated to count down towards the closing of student votes.

During these sessions the lecturer will encourage interactive learning inside and individual learning outside the classroom , hence creating a flipped classroom as defined by Cronhjort, Filipsson and Weurlander (2018). The lecturer will restrict dialogue with students to Socratic dialogue (Brogt, 2007), where questions and answers are utilised towards stimulating cognitive processes, and students are motivated to consult after the lecture, in order to elaborate on solutions to correct answers. The PRS questions will be aimed at encouraging conceptual understanding and emulating deeper learning of mathematical concepts. The pedagogy behind the sessions will be aimed at challenging students' prior beliefs about mathematics and mathematics teaching and learning and motivating students to take responsibility for their own learning.

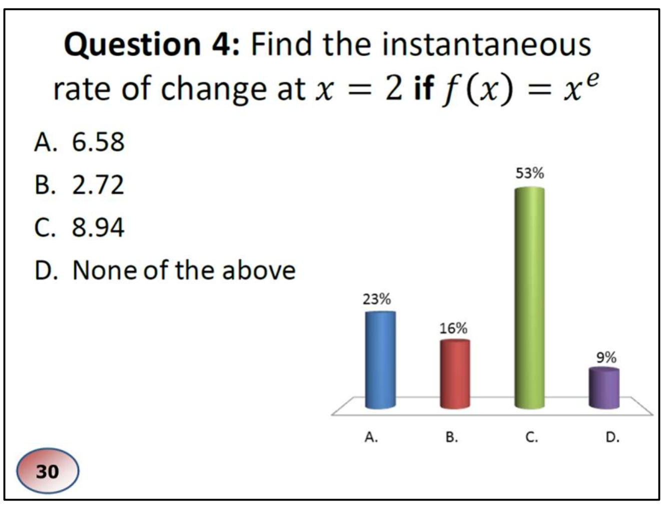


Figure 1.1: A typical PRS question with bar chart of student responses

1.5.2 Data collection. A Likert-scale questionnaire is administered at the beginning of the semester (Survey 1) to gauge students' existing beliefs about mathematics and mathematics teaching/learning. The same questionnaire is administered at the end of the semester (Survey 2) to observe potential changes in mentioned beliefs, post intervention. Quantitative methods are used to compare the questionnaire data for Surveys 1 and 2. To strengthen the research findings and allow for triangulation, focus group interviews with students selected through stratified random sampling to constitute the qualitative aspect of the study.

1.5.3 Research site. As an employee in the Department of Mathematics and Applied Mathematics at the University of Pretoria I find it convenient to conduct my research at the university. I am one of four lecturers involved in presenting a first year applied calculus module for biological sciences students in the first semester of their first year.

During the semester (one half of the year) the students write two semester tests (term tests) that contribute 70% to their semester mark (term mark) and smaller class tests and assignments that contribute 30% to their semester mark. The end of semester examination constitutes 40% of a student's final mark and the semester mark 60% of the final mark.

1.5.4 Population and sampling. The target population for the study was the 1300 students enrolled for WTW 134, a first year mathematics service module presented to biological science students in the first semester of 2018 at the University of Pretoria. The students constituted four lecture groups that were taught by four different lecturers. As a result, the approximately 598 students in my lecture group were the only students that could be involved in the study. For this reason, convenience sampling (Maree, 2012a), where the participants of the study were conveniently available to the researcher, was employed. No further sampling methods were employed for the purpose of the intervention.

The questionnaire was voluntarily completed at the beginning (Survey 1) and end of the semester (Survey 2) by 59 students from my lecture group. For the focus group interviews stratified random sampling (Maree, 2012a) was employed. Students' performance in the first of two semester tests provided the basis for categorising students into three strata. The formulation of the three strata is explained in Chapter 3, but it should be mentioned that students from the three strata groups were invited to voluntarily participate in the focus group interviews. The interviews were conducted in two focus groups on two consecutive days, the first consisting of five students and the second eight students.

1.5.5 Data analysis. For the quantitative data, three indexes or quantitative values are calculated for every student in the intervention that completed the questionnaire at the beginning (Survey 1) and end of the semester (Survey 2), i.e. the Centredness (C), Mathematics learning (M) and Responsibility Index (R). The three indexes are calculated for each of the three categories of questions in the questionnaire aimed at addressing each one of the three research questions. Also, the indexes are calculated for both data sets obtained through respectively Survey 1 and Survey 2 and compared by means of descriptive and inferential statistical analysis. For the qualitative data, codes are allocated to topics central to the research questions and themes identified by means of open coding.

1.5.6 Challenges of research approach. As part of the intervention, a Time-out session is incorporated during only six of approximately 52 lectures, due to time constraints. Another challenge of the abovementioned research design is that I am fulfilling the dual role of teacher and researcher. This challenge is overcome by allowing colleagues to conduct focus group interviews. The use of Socratic dialogue during interventions allows for restricted interaction between the researcher and students at the implementation phase (see Chapter 4), because as researcher I am obliged to deliberately manage my bias in order to be as objective as possible (Maykut and Morehouse, 1994).

Kislenko (2011) mentions that using a Likert-scale questionnaire to assess the domain of affect has its limitations, but the triangulation of qualitative (interviews) and quantitative methods (survey) will provide the researcher with opportunities for an in-depth analysis of results.

1.5.7 Ethical issues. The questionnaire includes an introductory section, informing students that completion of the questionnaire is voluntary. Also, the anonymity of the students participating in the study is maintained when reporting about the study. Ethical clearance was obtained from the relevant faculty (Faculty of Natural and Agricultural Sciences) of the University of Pretoria reference number EC180212-174 (Annexure A).

1.6 Overview

In Chapter 2 the fundamental literature is reviewed and in Chapter 3 the theoretical framework is revealed. Chapter 4 summarises the research methodology, Chapter 5 the research results and Chapter 6 concludes the study. The following has to be mentioned as influences on the design and interpretation of the study:

Brousseau uses the metaphor of teacher as actor in his theory of didactical situations (Herbst and Kilpatrick, 1999). From the perspective of didactical design, mathematical problems take centre stage and learning interventions should be designed accordingly to encourage learning (Artigue, 2009). The didactical situation becomes the actor's script and performance is based on the interplay between the teacher's and students' understanding and beliefs about mathematics and mathematics teaching/learning (Herbst and Kilpatrick, 1999). The role of the teacher remains fundamental and necessary, but is restricted. According to Herbst and Kilpatrick (1999), the didactical contract should remain implicit and should not be used as a means to direct teaching/learning, but as a mechanism that the researcher can use to study and interpret the practice of teaching/learning.

In the following chapters the potential value of using personal response systems to encourage learning through active engagement with mathematics problems will be analysed through the lens of the didactical contract in a mathematics classroom.

Chapter 2: Literature Review

The aim of the research directed my focus towards the essential literature addressing concepts such as the didactical contract; didactical design and its underlying theory; the transition from secondary to tertiary education, beliefs about mathematics and mathematics teaching and learning and closing the gap between secondary and tertiary education. Since personal response systems were used in the context of a flipped classroom, literature on active learning, pedagogies for the use of personal response systems in teaching mathematics and the flipped classroom teaching model are reviewed. The conceptual framework (Table 3.1) of the study is used as a framework to analyse the literature.

2.1 The Didactical Contract

In Chapter 1 the didactical contract was defined as an agreement which encompasses lecturers' expectations of students and vice versa, as well as expectations about teaching and learning and the responsibility for learning (D'Amour, 2008; Pepin, 2014). The concept of the didactical contract was defined by Brousseau and also referred to by researchers of mathematics teaching/learning (Kensington-Miller et al., 2011; Herbst and Kilpatrick; 1999, Pepin, 2014; Yoon et al., 2011). The didactical contract was formulated as part of Brousseau's theory of didactical situations (Balachef, 1990), aimed at identifying conditions for experiments in the didactics of mathematics (Margolinas and Drijvers, 2015). The theory of didactical situations (TDS) and the constructs of didactical engineering and didactical design will now be highlighted.

2.1.1 Theory of didactical situations, didactical engineering and design. Cottrill (2003) distinguishes between theories of learning and epistemological frameworks for mathematics education research, and notes that the latter should serve for investigating aspects of learning. He lists the frameworks of researchers at the forefront of research in mathematics education and includes the work of Balachef. According to Cottrill (2003:5), the framework is based on the idea that mathematical knowledge develops from problem situations and proposes that students be assisted to overcome "epistemological obstacles" in order to construct or reconstruct knowledge.

According to Balachef (1990), the *problématique* should include research questions exploring the significance of students' behaviour in the mathematics classroom. Balachef (1990) references Brousseau's theory of didactical situations (TDS) and dictates that student learning can be directed through the crafting of the didactical process.

The theory of didactical situations was formulated by Brousseau as a result of the modern mathematics curricular reform implemented in France in the 1970s (Margolinas and Drijvers, 2015) and researchers' attempts to assist teachers in implementing the reform. As a result, the Institute for Research in the Teaching of Mathematics (Institut de Recherches sur l'Enseignement des Mathématiques or IREMs) was established to set up research in mathematics education as one of their primary goals. According to Margolinas and Drijvers (2015), Brousseau was influential in the early development of IREMs and continued to initiate the research centre, the COREM (Center for Observation and Research on Mathematics Teaching). The COREM consisted of a school where teachers taught two-thirds of the time in order to be otherwise involved with research. For Brousseau the objective was not to observe the teachers, but to observe students' behaviour within teaching/learning situations (Margolinas and Drijvers, 2015).

Three central characteristics of TDS are identified by Grønbæk, Misfeldt and Winsløw, (2009):

The first characteristic is that the situation for learning is the principal object of TDS and learning is viewed as an "adaptation to the situation" (Artigue, 2009, Grønbæk et al., 2009). Prominence is given to the epistemological meaning behind knowledge and the features of the learning environment, or so-called *milieu*. In this respect Artigue (2009) mentions the goal of attempting to ensure a meaningful *adidactical* adaptation on the part of the students, or adaptation despite the didactics where learning is based on the mathematical problem at stake and not on the direction or support of the teacher or the didactical contract.

The second characteristic is the intricate interplay between the teacher, the student(s) and the situation for learning (Grønbæk et al., 2009). The teacher's role is limited to encouraging students to engage with the milieu, a transfer of responsibility for learning (*devolution*) and emphasising knowledge to be retained (*institutionalisation*). Grønbæk et al. (2009:88) identify four phases through which didactical situations created by the teacher are transformed into *adidactical* situations. The first phase is the "*devolution of adidactical situations of action*", where students actively explore the *milieu*. The second phase, the "*devolution of adidactical situations of formulation*" is where students are encouraged to express their observations of interactions with the *milieu*. The third phase, called *"situations of validation",* is where students attempt to validate statements generated through observations. The fourth phase is the phase of "*situations of institutionalisation"*, in which the knowledge to be retained is accentuated by the teacher. They mention that different roles are assigned to the participants in every phase, whether students, the teacher or at tertiary level, the lecturer.

The third characteristic according to Grønbæk et al. (2009), is to compare the teacher's role to that of a mathematician. The mathematician institutionalises knowledge discovered in situation or context, whereas the teacher utilises situations dictated by the epistemological meaning of the knowledge to facilitate the acquisition of the knowledge by the students, or the institutionalisation of the knowledge.

According to Artigue (2014), the concept of didactical engineering emerged from the work of Brousseau and the IREMs. The term was coined to describe didactic work that is similar to that of an engineer, where scientific knowledge and methods are the catalysts, but the engineer is challenged by the intricacy of the subject to adopt a more practical approach. Margolinas and Drijvers (2015) explain that the hypothesis at the centre of an experiment of didactical engineering is the prospect to teach in *adidactic* situations, where the student carries the responsibility for what emerges from the situation and the teacher partially transfers the facilitating role to the *milieu.*

Artigue (2009) notes that TDS and didactical engineering were instrumental in giving didactical design its fundamental role. She defines didactical design as the theory-based design of planned teaching interventions aimed at improving student learning. From the perspective of didactical design, mathematical problems are the source of mathematical meaning and the role of the teacher in the actual learning process is defined accordingly (Artigue, 2009). In the words of Artigue (2009:7) "didactical design includes all types of controlled intervention research into processes of planning, delivering and evaluating concrete mathematics education". Balachef (1990) mentions that the control and design of a didactical process or didactical design, lies at the heart of Brousseau's theory of didactical situations or TDS.

Balachef (1990) notes that research questions focused on exploring students' actions in the mathematics classroom are based on the constructivist and epistemological view of learning. The constructivist view implies that students construct their own knowledge and the epistemological view that mathematical problems are the source of mathematical knowledge (Balachef, 1990). Based on these two premises he continues to explain that a student's learning develops when they take ownership and accept and reconstruct mathematical problems as their own. To quote Balachef (1990:259): "A problem is a problem for a student only if she or he takes responsibility for the validity of the solution." In other words, in order to ensure meaningful learning, a transfer of responsibility for the truth from the teacher to the student or "devolution of responsibility" has to take effect.

According to Balachef (1990), two limitations impose on a student's personal process of devolution: the social context of mathematical knowledge and the existence of the mathematics classroom as a community. Since students learn in the social setting of the mathematics classroom, social interactions have to be navigated in order to obtain uniformity in the knowledge constructed by students. It is the responsibility of the teacher to navigate these social interactions by controlling and designing the didactical process (Balachef, 1990), governed by the rules imposed by the didactical contract.

Balachef (1990) notes that mathematical problems are fundamental in the learning process because they initiate learning and change. Since it is not possible to directly observe student learning, student behaviour has to be observed in order to gather information about learning. According to Balachef (1990), the mistakes made by students are the most observable symptoms of their conceptions and the meaning behind these mistakes is significant in observing student learning. Student mistakes must be identified not only to avoid future mistakes, but to understand and address the misconceptions that underlie the mistakes.

As mentioned before, Brousseau uses the metaphor of an actor on the stage to explain the role of the teacher (Herbst and Kilpatrick, 1999). The teacher improvises the script and reacts to the other actors' performances (Herbst and Kilpatrick, 1999). In the words of Brousseau, as quoted by Herbst and Kilpatrick (1999:8): "if [the teacher] produces her mathematical questions and answers, she deprives the student of the possibility of acting". Hence the teacher must restrict her performance to using the performance (answers or questions) provided by students and integrating them into the process of acting (or learning) (Herbst and Kilpatrick, 1999).

According to Herbst and Kilpatrick (1999), the didactical contract is not a contract in the real sense of the word, because the didactical contract is always implicit. The moment that the rules that govern the relations between students and teacher are made explicit, the contract is broken (Herbst and Kilpatrick, 1999). The contract allows for continuous negotiation of responsibility, but should not be viewed as a technical instrument for performing the practice of teaching/learning. It must be viewed as a technical instrument that enables the researcher to study and interpret the practice of teaching/learning. Herbst and Kilpatrick (1999) mention that an oversimplification and misinterpretation of the theoretical notion of didactical contract caused some to believe that through practical refinement, a teacher can create a meaningful didactical contract. This is not the case if the contract is understood to represent the abstract notion of "reciprocal obligation" (Pepin, 2014:653), necessary to regulate teaching/learning interventions.

The didactical contract affords a model or theoretical object of study (Herbst and Kilpatrick, 1999). The theoretical concept is not the didactical contract itself, but the theoretical process of finding the contract. Observation(s) of learning should attempt to "model and explain this process" (Herbst and Kilpatrick, 1999).

2.1.2 Research methodology of didactical engineering. Didactical engineering is a research methodology where the method of validation is not based on the contrast between an experimental and control group, but on an internal validation process. This internal validation process consists of five phases: the preliminary phase, the design and *a-priori* analysis phase, the intervention phase, *a-posteriori* analysis phase and the validation phase. Artigue (2009) distinguishes the five phases as essential because of the theoretical basis of the research approach.

Artigue especially emphasises the design and *a-priori* analysis phase and the validation phase of the research methodology of didactical engineering, where validation is based on a comparison between the *a-priori* and *a-posteriori* analysis (Artigue, 2009). González-Martín, Bloch, Durand-Gerrier and Maschietto (2014) note that during the preliminary analysis phase the researcher identifies so-called *didactic variables* or epistemological, cognitive and didactical factors that can influence the students' learning or adaptation to the situation, to arrive at an *adidactic* situation. In the *a-priori* analysis the use of these factors in visualising the *adidactic* situation are intricately planned, and in the design of the intervention emphasis is placed on addressing students' conceptions and/or possible misconceptions in an organised manner (González-Martín et al., 2014). For the validation phase the *a-priori* analysis forms the basis of the analysis of data that constitutes the *a-posteriori* analysis (Margolinas and Drijvers, 2015). According to Artigue (2009), the validation phase can include the use of pre-and post-tests, surveys and interviews. The phases of didactical engineering are briefly mentioned in Chapter 4, where the research methodology of the current study is discussed.

2.1.3 University didactical contract. According to Grønbæk et al. (2009), the model for a didactical situation employing four phases, as mentioned earlier, is not to be interpreted as the rule for all learning environments. They mention that university lectures are mainly situations where the *institutionalisation* of knowledge occur, where students are merely informed of the validity of the knowledge in the mathematical domain. In most lectures, knowledge is first institutionalised and then *devolution* takes the form of problems and exercises delegated by the lecturer to the students, who must take responsibility to formulate and validate their solutions. Grønbæk et al. (2009:90) refer to the "Lecture-Problems-Class Model (LPC for short)" and mention that although the four phases of a didactical situation are important, they should not essentially appear in the sequence mentioned in Section 2.1.1. They stress the importance of students learning at tertiary level to access knowledge directly, like an academic, without the presence of methodically organised learning situations. They elaborate the didactical contract for the LPC model and note that the lecturer is obliged to explain the theory with clarity to ensure that the problems and exercises will contribute towards the students' understanding and eventually assist them to pass the module. The students are obliged to actively participate in the lectures and attempt the given problems and exercises. Regarding problems and exercises the lecturer has to highlight and institutionalise valid solutions and facilitate student learning.

Personal response systems are used in my study to vary the LPC model of teaching, by introducing student-centred learning activities so that the didactical contract in tertiary mathematics education can be negotiated and students supported to transition successfully from secondary to tertiary mathematics education. For this purpose literature on active learning and pedagogies for the use of personal response systems in teaching mathematics is reviewed in Section 2.4. González-Martín et al. (2014) observe that because the difficulty level of mathematics increases considerably when transitioning from secondary to tertiary mathematics education, the epistemological lens of TDS can assist in the design of teaching/learning activities. They illuminate the suitability of the didactical contract for analysing students' learning at tertiary level. In the next section I differentiate between the didactical contract in tertiary mathematics education and secondary mathematics education.

2.2 From Secondary to Tertiary Learning

In Chapter 1 the transition from secondary to tertiary mathematics education was briefly highlighted. Researchers (Benadé, 2013; Brandell et al., 2008; Clark and Lovric, 2009; Hourigan and O' Donoghue, 2007; Pepin, 2014; Selden, 2005) acknowledge the existence of a gap between secondary and tertiary mathematics education, which implies challenges for most students in adapting their learning at tertiary level. Brandell et al. (2008) refer to the "widening gap in Sweden between secondary and tertiary level mathematics education" and Luk (2005) acknowledges its existence, but also mentions that "different places may have different transition points for the same mathematical gap". He, from the perspective of universities in Hong Kong, identifies four factors that exacerbate the transition from secondary to tertiary mathematics education.

Hourigan and O'Donoghue (2007:462) present an Irish perspective to the transition from secondary to tertiary mathematics education, which they refer to as the "mathematics problem", and attempt to identify factors that might cause students to be under-prepared for tertiary education. They describe the agreement or didactical contract between the participants in a mathematics classroom in secondary education to be teacher-centred. The main focus in classrooms at secondary level is examination preparation and from the student's perspective the teacher must provide an easy way out, by identifying content important for the examination, simplifying tasks and refraining from challenging students. The teacher must be compliant and positive, even if students are not attempting to reach their full potential (Hourigan and O'Donoghue, 2007). From the perspective of the teacher, the students are passive and experience difficulty in thinking for themselves. Hourigan and O'Donoghue (2007) mention that in order for the contract to be reformulated, the teacher's notion of student support needs to be converted into a paradigm that focuses on students' cognitive development and not just "protecting their comfort levels". At the same time, students need to realise that proper understanding requires effort on their part (Hourigan and O'Donoghue, 2007:474).

Yoon et al. (2011) explore the didactical contract in tertiary mathematics education and after examining the didactical contract in large undergraduate mathematics classrooms, find that students believe learning should take place outside the classroom, and dislike active learning concepts. Yoon et al. (2011) confirm an observation made by Hourigan and O'Donoghue (2007) that students expect the teacher to explain procedures clearly and a good teacher is mainly characterised by this ability.

Pepin (2014) finds that the transition from secondary to tertiary mathematics education is characterised by an inconsistency in the didactical contract, which holds implications for students' learning strategies. The learning environment changes from nurturing to challenging as mathematics involves more than just procedures; it requires deeper understanding and reasoning and new teaching styles necessitate adaptive learning strategies. She mentions that the most difficult challenge for students is to learn how to learn mathematics without merely imitating the actions of the lecturer. Lecturers expect students to emulate their mathematical reasoning skills, without directing students towards acquiring the necessary skills (Pepin, 2014).

In short, the student remains responsible for the transition from secondary to tertiary education, but educators can assist by creating environments for supporting the transition (Clark and Lovric, 2009) and renegotiating the didactical contract (Yoon et al., 2011).

Brandell et al. (2008), in examining the transition from a Swedish perspective, notice a mismatch between new knowledge and pre-existing knowledge, but also a mismatch of beliefs about mathematics and mathematics learning.

2.2.1 Beliefs about mathematics, mathematics teaching and learning. Brodie's (2016) explanation of the decolonisation of mathematics implies that students' mathematical beliefs and backgrounds should be considered when contemplating teaching and learning. She mentions that "students believe that mathematics is a set of procedures to be followed" and ability that only the truly gifted possess.

Benadé (2013) investigates the gap between secondary and tertiary mathematics by comparing students' beliefs about mathematics and mathematics teaching and learning with those of their lecturers. She references Ernest (1988) in stating that a person's beliefs about the teaching and learning of mathematics are grounded in their beliefs about the nature of mathematics and that these opinions are not changed easily.

Benadé (2013) distinguishes between three views of school mathematics as identified by Ernest (1988): the instrumentalist, the Platonist and the dynamic view (Benadé, 2013). According to the instrumentalist view (Benadé, 2013:69), mathematics is regarded as "a bag of tools; made up of unrelated facts, rules and skills, rules without reason". Benadé (2013:69) further elaborates that "(the learner must be) in possession of rules and be able to use it; (the) teacher demonstrates, explains in a way that learners understand; learners listen and respond to the teacher; (the) ability to get to the correct answer is evidence of understanding".

From the Platonist point of view (Benadé, 2013:68) knowledge is received passively by learners, the teacher is the expert, "learners follow instructions" and it is "the task of the teacher to see that learners master the subject." From both the instrumentalist and the Platonist viewpoint, the classroom is teacher-centred and the responsibility for learning lies with the teacher. The instrumentalist view is also characterised by an emphasis on procedural fluency.

According to the dynamic view, learning is a process of development, learners "construct concepts", teaching/learning is learner-centred and "the teacher is facilitator and stimulator of learning" (Benadé, 2013:68). Benadé (2013:75) further finds that the instrumentalist view of mathematics prevails when considering students' beliefs about mathematics.

As mentioned earlier, a teacher's view of the nature of mathematics impacts on the model of teaching and learning enacted in the teacher's classroom (Ernest, 1988). As a result, the teacher's beliefs about mathematics and mathematics teaching/learning influence learners' beliefs about mathematics and mathematics teaching/learning.

Benadé (2013) conducts her investigation at a South African university and notes that even though the school education system ostensibly adopted a learner-centred approach after 1994 (based on a constructivist view of learning), teachers' beliefs about mathematics have not necessarily changed and therefore most mathematics classrooms at secondary level remain traditional, or teacher-centred. She distinguishes between the intended and the implemented curriculum, mentioning that a gap exists between the "formal approved guidelines" and what actually happens in the classroom (Benadé, 2013:20).

Benadé (2013) asserts that secondary school teachers do not teach for conceptual understanding, but hold an instrumentalist view of mathematics (a collection of disconnected rules and procedures to be utilised) and teach accordingly. To quote Benadé (2013:19): "With a focus on good results in the Grade 12 examinations, teachers tend to coach learners procedurally with little or no conceptual understanding." Based on their experience of school mathematics, first year students see mathematics as a means to an end and underestimate the importance of proper understanding (Benadé, 2013). They hold the lecturer responsible for their mathematics learning, they expect to learn during lectures and are not willing to struggle in order to understand. At tertiary level, lecturers expect students to be responsible for their own learning (Pepin, 2014) and to be able to apply knowledge in varying contexts (Benadé, 2013).

According to Beatty and Gerace (2009), if lecturers want to change students' classroom behaviour, students' fundamental beliefs should be addressed, since beliefs govern attitudes. To quote Lozanovski et al. (2011:230): "In any subject like mathematics where there is a true answer, the aim is for all students to conform to that opinion finally."

2.2.2 Changing students' beliefs: the didactical contract perspective. Kislenko (2005) theorises about utilising didactical situations to change students' beliefs about mathematics. She mentions that according to Brousseau's theory of didactical situations, knowledge and beliefs are constructed through the student's attempts to resolve conflicts they experience in didactical situations. Kislenko (2005) reasons, based on this premise, that student errors have value in the learning process, since proper understanding includes understanding why a certain erroneous path is regarded as such. In the words of Kislenko (2005:88): "mistakes do not show what one does not know but they show what one knows". She introduces the idea of intentionally designing games (or didactical situations) aimed at creating conflict within the student. Targeting particular student beliefs in the design creates opportunities for change to occur. One of her examples involves revealing an erroneous statement and the reasoning that led to the statement, and asking students to explain why the result is wrong. She also notes the importance of peer influence on student beliefs and suggests utilising this influence towards learning. She uses the didactical engineering research design to study students' beliefs and attitudes in mathematics in Norway and Estonia. She mentions that her study does not fully support the research design at the methodological level, since her focus is on observing and not changing beliefs and attitudes.

2.3 Renegotiating the Didactical Contract

According to researchers (Benadé, 2013; Brandell et al., 2008; Clark and Lovric, 2009; Hourigan and O' Donoghue, 2007; Pepin, 2014; Selden, 2005), a mismatch exists between students' and lecturers' beliefs about mathematics and mathematics teaching/learning upon entering university. Students' beliefs are based on their experience of secondary mathematics education: they value procedural fluency and underestimate the importance of conceptual understanding (Benadé, 2013). They hold the lecturer responsible for their mathematics learning and are more comfortable with teacher-centred mathematics classrooms.

2.3.1 From teacher-centred to student-centred. Large classes have become a reality in South-African university classrooms over recent years (Baragués, Morais, Manterola and Guisasola, 2011; Strasser, 2010) and the student population represents a range of diverse abilities (Biggs, 1999). The challenge for teachers is to level the playing field or provide all students with equal opportunities to learn (Biggs, 1999).

Benadé (2013) explains that in a teacher-centred classroom the teacher holds all knowledge, and learning is a process of transferral along a route determined by the teacher. According to Benadé (2013), secondary mathematics education is teacher-centred, despite the fact that the school education system advocates learner-centred education.

According to researchers (Biggs, 1999; Bransford, Brown and Cocking, 2000), an effective learning environment should be learner-centred, characterised by teacher-facilitated learning activity on the part of the learner. Bransford et al. (2000) explain a learner-centred learning environment to be an environment where close consideration is paid to the knowledge, skills and beliefs that learners bring to the classroom.

From a constructivist point of view, students come to university with pre-existing knowledge and beliefs that influence their learning (Bransford et al., 2000; Clark and Lovric, 2009; Lantz, 2010). They build or construct their understanding of new information upon their understanding of previous knowledge, and the construct of new knowledge in combination with old knowledge characterises understanding and learning (Lantz, 2010). The construction should be guided by good teaching in a student-centred teaching environment (Biggs, 1999; Clark and Lovric, 2009).

Clark and Lovric (2009) liken the educational transition from secondary to tertiary mathematics as a rite of passage or a sequence of events, viewing it as a process of adapting to a new life experience, where old customs need to be abandoned in order to adopt new responsibilities and face new challenges. They include "cognitive conflict" and "conceptual change" in their theoretical framework (Clark and Lovric, 2009:756), in which cognitive conflict is described as the mismatch between new and pre-existing knowledge, which might lead to misconceptions. Bransford et al. (2000) advocate the design of student-centred learning activities, where students are encouraged to engage in cognitive conflict on purpose, to reconceptualise conceptual structures in order to promote learning. To engage the students in cognitive conflict, the learning activities are selected or designed around 'known misconceptions" (Bransford et al. 2000:134) or misconceptions that the lecturer are aware of.

2.3.2 Focus on conceptual understanding, not only procedural fluency. Mathematics is about integrated understanding or "relational understanding" and not just about applying rules and procedures or instrumental understanding (Skemp, 1976). Constructivism is a theory about the construction of learners' knowledge, and Van De

Walle, Lovin, Karp and Bay-Williams (2013) focuses on the construction of learners' mathematical knowledge. Van de Walle et al. (2013) argues that mathematical knowledge attained by memorisation is mathematics that is understood instrumentally, and not necessarily relational understanding. The term instrumental understanding as coined by Skemp (1976) is the application of "rules without reason". Benadé (2013) interprets Skemp's definition of instrumental understanding as the knowledge of isolated rules and procedures, and the ability to use these rules and procedures without understanding why they work. On the other hand, relational understanding is to know what to do and understand why (Skemp, 1976).

Van de Walle et al. (2013) places instrumental and relational understanding on opposite ends of the continuum of understanding (see Figure 2.1). According to Van de Walle et al. (2013), relational understanding is a meaningful and useful construct of interconnected concepts and procedures and good teaching is to teach for complete or relational understanding. According to Van de Walle et al. (2013), relational understanding helps with learning new concepts and procedures: "an idea fully understood in mathematics is easily extended when a new idea is learned". He mentions that if students are not taught to make connections, the only other option is to memorise separate rules and procedures.

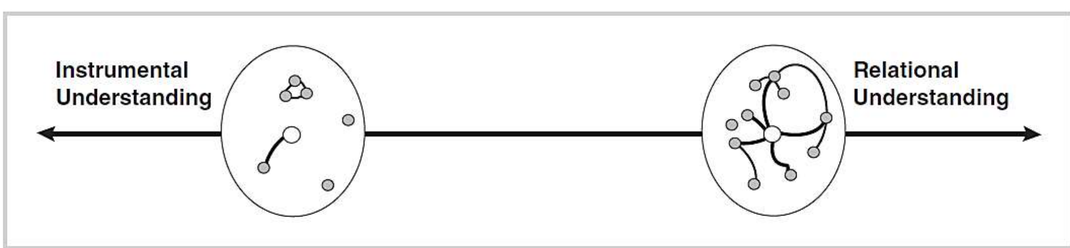


Figure 2.1: The continuum of understanding (Van de Walle et al., 2013:25)

In the document *Adding it up,* Kilpatrick, Swafford and Findell (2001:5) describe mathematical ability or skill as five interrelated abilities or skills, or in their words, "five interwoven strands of mathematical proficiency": conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and a productive disposition. They describe conceptual understanding as the understanding of mathematical concepts and procedural fluency as the skill of performing procedures efficiently and appropriately.

Conceptual understanding and procedural fluency need to be separated when attempting to understand students' learning of mathematics. Usiskin (2012) analyses the meaning of mathematical understanding or conceptual understanding, and refers to the phenomenon familiar to educators where students have the ability to "do mathematics" without having a proper understanding of mathematics (Usiskin, 2012:2). According to Usiskin (2012), conceptual understanding and procedural fluency can be mastered independently, but together constitute a meaningful understanding of mathematics. Therefore, learning activities must be designed to develop both optimally.

For the purpose of this study, procedural fluency is used in combination with conceptual understanding to represent the relational end of the continuum of understanding of Van de Walle et al. (2013), and in which procedural fluency without conceptual understanding represents the opposing or instrumental end of the continuum.

Benadé (2013) remarks that a student's experience of mathematical concepts is essential to their future learning, and will influence the transition from secondary to tertiary mathematics education (Hourigan and O'Donoghue, 2007). If a student is taught at secondary level to focus on the procedural aspect of mathematics, they will find it challenging to deal with problems that necessitate conceptual understanding (Benadé, 2013). She finds that students struggle to answer questions that require creative reasoning or reasoning that is founded on a relational and not just an instrumental understanding of mathematics. Benadé (2013:119) recommends "active engagement" as crucial to facilitate students' proper understanding of mathematics.

Long (2005), while teaching a general course on mathematics at tertiary level to students of education, attempts to utilise the constructs of conceptual understanding and procedural fluency to analyse the mathematical skills of future teachers. She mentions that it is not always obvious whether students are truly grasping a concept instead of merely applying procedural knowledge. What she does notice is that some students are able to apply different strategies to solve a problem, which can be interpreted as conceptual understanding, based on the definition of Kilpatrick et al. (2001). On the other hand, if students can apply a procedure but struggle when the context of the problem is changed, a lack of conceptual understanding becomes evident. Her observation about conceptual understanding is reflected in Engelbrecht, Harding and

Potgieter's (2005) definition of conceptual understanding of mathematical concepts as the ability to apply concepts to "a variety of situations".

Long (2005:62) finds that in order to stimulate a "flexible understanding" when teaching mathematics, the complex relationship between procedural fluency and conceptual understanding requires that the conceptual must support the procedural, while the procedural should provide a platform for the conceptual.

Long's research further provides an example of how the five strands identified by Kilpatrick et al. (2001) together describe mathematical ability. The meaningful understanding of mathematical concepts integrated with procedural fluency, in combination with strategic competence (the ability of using different strategies), flexible (or adaptive) reasoning, and positive beliefs (or a productive disposition) constitute mathematical ability (Kilpatrick et al., 2001).

The importance of the other three components of mathematical ability - strategic competence, adaptive reasoning and a productive disposition - is not underestimated, and is discussed later.

2.3.3 From surface learning to deeper learning. According to Benadé (2013) the transition from secondary to tertiary mathematics education comprises a move from basic mathematical reasoning or surface learning to a higher level of reasoning or deeper learning. Selden (2005) notes that at secondary level mathematics learning is characterised by surface learning, but at tertiary level students are expected to demonstrate deep learning. According to Clark and Lovric (2009), the transition from surface to deep learning is most challenging for students.

Biggs (1999) adopts a student-centred approach to teaching and notes that the lecturer needs to contemplate the depth of understanding required from the student when designing teaching/learning activities. According to Bransford et al. (2000), teaching approaches should employ learning activities that must not only address prior understandings and metacognitive approaches to learning, but must also encourage a deep understanding of content (Bransford et al., 2000).

According to Biggs (1999), deeper learning or higher-order cognitive processes are stimulated through higher levels of student activity. By designing learning activities aligned with clear learning outcomes aimed at intense student involvement, optimal learning is encouraged. Good teaching is described as guiding the student towards deep learning, or in the words of Biggs (1999:58): "Good teaching is getting most students to use higher cognitive level processes that the more academic students use spontaneously. Good teaching narrows the gap."

In Figure 2.2 Biggs (1999:58) classically illustrates that higher levels of student activity, created through interactive in-class activities, will motivate the "non-academic Robert" to adopt higher-order cognitive processes like "relating, applying and even theorising" that the "academic Susan" would have used in passive learning conditions. The implication for my study is that through the incorporation of PRS sessions higher order cognitive processes can be encouraged in students that are not necessarily academically inclined.

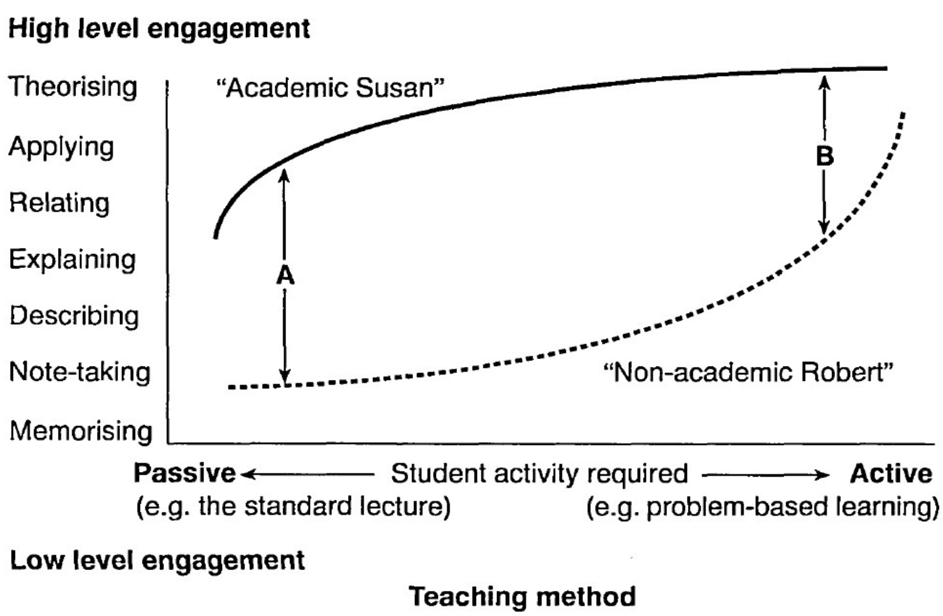


Figure 2.2: Student engagement and student activity (Biggs, 1999:58*)*

In the mathematics classroom strategic competence is the ability to *apply* and *relate* mathematical knowledge and understanding to solve mathematical problems, while adaptive reasoning is the ability to think logically, *explain*, justify and *theorise* (Kilpatrick et al., 2001). Schoenfeld (2007) analyses the five components of mathematical ability developed by Kilpatrick et al. (2001), in order to advise assessment practices. According to Schoenfeld (2007), "good problem solvers are flexible and resourceful" and these attributes are of a much higher order than merely "reproducing standard content on demand". In the context of the higher-order cognitive processes posited by Biggs (1999), higher-order cognitive abilities – such as strategic competence, i.e. the ability to apply, and adaptive reasoning, i.e. the ability to explain or even theorise – are encouraged through student engagement.

2.3.4 Students accept the responsibility for their learning. The fifth component of mathematical ability as identified by Kilpatrick et al. (2001), a productive disposition, is the ability of the student to view mathematics as meaningful, strengthened by their belief in their ability to do mathematics.

In order to get students to take responsibility for their own learning, strategies to improve their sense of commitment must be identified and incorporated (Elton, 1996). Elton (1996:63,64) further mentions that students are motivated by both extrinsic and intrinsic factors. The most important extrinsic factor for the student is "examination preparation" and the most important intrinsic factor from the perspective of the lecturer is "interest in the subject of study". "Examination preparation" refers to whether students have a sense that they are well-prepared for the examination and "interest in the subject of study" refers to the level of interest that students have in the subject matter. Elton (1996) mentions that while lecturers tend to attempt to motivate students by trying to make the subject more interesting, they should rather pay attention to students' sense of preparedness for the examination. He stresses that this does not imply that the lecturer teaches for the examination, but rather that attempts must be made to realistically improve students' confidence in their ability to pass the examination.

According to Clark and Lovric (2009), although the lecturer has a responsibility towards students to ensure that all resources are available and used optimally, students must be frequently reminded that the responsibility to transition successfully from secondary to tertiary mathematics education lies solely with them, the students.

Cazes, Guedet, Hersant and Vandebrouck (2006) study the implementation of online resources into the teaching and learning of mathematics at university level, through the lens of the didactical contract. They hypothesise that the online component, called EEB (E-Exercise Bases) will influence students' behaviour from a didactical point of view. They find that EEB does not create an *adidactic* learning environment, as proposed by Brousseau, but rather an environment of support. The contract defined by the online resources requires the student to participate and to accept responsibility, but they can choose their own path midst the exercises.

In the current study, technology (in the form of personal response systems) is implemented in a teaching/learning environment and its influence on the didactical contract and students' sense of responsibility is analysed. Hence personal response systems and related pedagogies are discussed.

2.4 Personal Response Systems and Personal Response System-based Pedagogy

According to Bransford et al. (2000), an effective learning environment should be student-centred, characterised by activity on the part of the student, and in which the lecturer facilitates learning. Hattie (2015) reports about the applicability of his Visible Learning research on tertiary education. He comments that the main objective of the lecturer, in a student-centred class, should be to obtain information about the impact of their teaching on the students. According to Hattie (2015), this objective implies that the lecturer should provide information about what successful learning looks like, utilise formative assessment and student errors for providing information about students' learning. He mentions that personal response systems can be used to engage students in large lectures. Kay and LeSage (2009) review literature about the benefits and challenges of using personal response systems in teaching/learning. They mention that personal response systems can be used to actively engage students, and to encourage students to interact with peers and actively debate their misconceptions.

Laws, Sokoloff and Thornton (1999) prefer active engagement strategies over traditional instruction for improving conceptual understanding in physics. Beatty (2004), an experienced teacher of physics, advocates the use of personal response systems to create a firm understanding of fundamental concepts. The potential value for improved understanding lies in the associated active learning pedagogy (Beatty et al., 2006; Kennedy, Cuts and Draper, 2006; Lantz, 2010; Martyn, 2007; Retkute, 2009) and by assisting the development of conceptual understanding, the development of meaningful cognitive constructs of material or learning is enhanced (Lantz, 2010).

Although research reports on the use of personal response systems in physics teaching are rife (Retkute, 2009), research reports that focus on the use of personal response systems in mathematics education are limited (Retkute, 2009). As mentioned in Chapter 1, Dangel and Wang (2008) note that personal response systems can be used to encourage deep learning or get students to use higher level cognitive processes.

Rubin and Rajakaruna (2015) explore the potential of using personal response systems to scaffold higher-order thinking word problems in a mathematics class and emulate the thought processes behind solving such problems. They found that although students showed an increased motivation and success in attempting higher-order thinking word problems, the process proved to be time-consuming. They theorise that, although time-consuming, the methodology might inspire students to take ownership of their own learning.

2.4.1 Design of personal response questions. According to Beatty, Gerace, Leonard and Dufresne (2006), the effectiveness of personal response system (PRS) technology depends strongly on the quality of the questions used. The properties of good PRS questions (Beatty, 2004) differ from the properties of good examination or test questions and the design of questions should be the focus of lecture planning. Beatty et al. (2006) advocates that every question should be asked with three objectives or goals in mind: “a content goal, process goal and metacognitive goal” (Beatty et al., 2006:31). The content goal concerns the concept or concepts that the question will address. The process goal refers to the cognitive skill that the question will assess, and the metacognitive goal encapsulates the belief(s) about the discipline and the learning of the discipline that the question aims to emphasise (Beatty et al., 2006:32).

Sullivan (2009) provides guidelines for the design of meaningful PRS questions and acknowledges that the principles for designing meaningful multiple-choice questions should be considered when designing PRS questions. An analysis of these principles reveals that questions must be based on a content goal and “a single specific mental behaviour” (Sullivan, 2009:340). She emphasises the importance of targeting a particular learning objective as content goal. Principles for setting multiple-choice questions worth mentioning are: limit reading; make sure all distractors are likely answers, and “use typical errors of students” (Sullivan, 2009:341) to write distractors.

In order to ensure that a three-sided goal is embedded in each question, Beatty et al. (2006) provide tactics for designing effective PRS questions. Some general tactics or strategies mentioned by researchers (Beatty, 2004, Beatty et al., 2006 and Dufresne, Gerace, Mestre and Leonard, 2005) are briefly discussed. Distractors used in multiple-choice questions should include possible mistakes made by students and the option “none of the above”, to provide for responses not considered (Beatty et al, 2006). Since the most prevalent response(s) can be immediately detected through the bar chart of students’ responses (Beatty et al, 2006) the students’ mistakes can be identified and discussed. Questions designed to create conflict (Beatty et al, 2006) can have the following properties: multiple possible correct answers included or “deliberate ambiguity” contained in the question. To test understanding of a concept, Dufresne et al. (2005) recommend the use of a different context from the one that was used to learn the concept. Beatty (2004:10) recommends “comparison questions” to focus student attention by setting similar simple questions with a slight difference in order to make students aware of their understanding. Another tactic is to design a question (called misconception magnets by Cline, Zullo and VonEpps, 2012) in order to elicit and discuss anticipated misconceptions or assumptions (Beatty 2004) and improve conceptual understanding (Bruff, 2010).

Class discussions inspired by the use of these questions are essential to all the above-mentioned strategies for the design of PRS questions (Sullivan, 2009), especially in a mathematics classroom (Kenwright, 2009). The reasoning behind correct and incorrect answers needs to be discussed in order to ensure that the pedagogical goals behind the questions are realised (Beatty et al., 2006). A good question initiates discussion and is cause for substantial learning, and a wide distribution in the displayed answer bar chart could be an indication of a good question (Beatty 2006).

Kalajdzievska (2014), in an attempt to frame teaching and learning strategies for large mathematics classrooms, recommends the use of “erroneous examples”. According to Kalajdzievska (2014), the practice of identifying and correcting mistakes made by other students can also be used to stimulate metacognition in students. In Kalajdzievska’s

2014 study, the student is provided with a partially completed response to an examination question, which deliberately contains at least one mistake. The student must provide the correct solution to the question, identify the mistakes and provide explanations to elaborate the identification. Bruff (2010) refers to peer-assessment questions as questions that represent the work of students to be critiqued or assessed by other students. He notes that the use of personal response systems allows for constructive criticism through anonymity.

Kislenko (2005) uses incorrect solutions to create conflict among students and elicit discussion. Though the value of this strategy for designing PRS questions has not explicitly been observed, it definitely has potential to prompt discussion through the use of personal response systems.

Dangel and Wang (2008) provide a theoretical framework for designing PRS questions to encourage deeper learning. The framework integrates, as mentioned earlier, the level of student activity to include deep learning with principles for effective pedagogical practice, as identified by Chickering and Gamson (1987). Dangel and Wang (2008:100) advocate the use of PRS questions in “communicating high expectations”, one of the seven essential elements of good pedagogy as defined by Chickering and Gamson (1987), by designing questions to facilitate students’ capacity for higher-order activities, such as applying, analysing, evaluating and creating. Rubin and Rajakaruna (2015) explore the potential of using clickers to teach the reasoning processes behind higher-order word problems. They scaffold higher-order word problems that require multiple steps of innovative reasoning into multiple clicker questions, in order to direct the students’ reasoning. According to Rubin and Rajakaruna (2015), students’ level of engagement in higher-order word problems in examinations increased as a result.

2.4.2 Pedagogy for using personal response systems. Beatty and Gerace (2009) describes Technology-enhanced formative assessment (TEFA) as a comprehensive pedagogy for using personal response systems to teach science and mathematics that is deeply rooted in research on education, based on four core principles and described in detail. According to TEFA, learning is viewed from a constructivist point of view, and includes the need to address fluency in the language of the discipline, as well as student beliefs and attitudes. Beatty and Gerace (2009) mention that beliefs determine attitudes and attitudes govern behaviour. To quote Beatty and Gerace (2009:151): "In order to influence students' classroom behaviour, we should seek to elicit, interact with, and influence their underlying beliefs". The TEFA pedagogy attempts to influence foundational beliefs, by creating opportunities for students to reconstruct knowledge, exercise dialogical discourse and develop habits towards self-directed learning (Beatty and Gerace, 2009).

Deep and challenging questions are used to introduce concepts and initiate discussion, since concepts are best introduced when students see a purpose for their use (Beatty and Gerace, 2009). By eliciting and allowing students to discuss their individual ideas, by "scaffolding" their efforts, without helping them avoid the conflict and struggle underlying sense-making, instruction becomes student-centred and learning is enhanced (Beatty and Gerace, 2009:156). According to Cline et al. (2012), the initial discussion provoked by PRS questions, can be facilitated by the lecturer in a Socratic way, by calling on students to provide explanations and not agreeing or disagreeing with any of the explanations. Through formative assessment, classroom discussion and/or the provision of complete solutions, the process of learning is strengthened.

Researchers (Beatty and Gerace, 2009; Bruff, 2010; Sullivan, 2009) refer to Eric Mazur's peer instruction method as an effective technique for using personal response systems in teaching science. PRS questions are spread throughout lectures, students respond to a question by voting, time is allowed for peer discussion and then the students vote once more on the same question, followed by a class discussion. Bruff (2010) notes the value of peers explaining difficult concepts, but emphasises that the lecturer's closing statement(s) must focus on providing reasons behind correct or incorrect answers.

2.4.3 Implications of ongoing research. Researchers comment that findings about the potential value of personal response systems are mostly based on student perceptions of learning (Barragues et. al, 2011; Kay and LeSage, 2009; King and Robinson, 2009; Retkute, 2009). According to Kay and LeSage (2009), several fundamental shortcomings exist in research on the use of personal response systems e.g. a bias towards using qualitative data.

To counteract a bias towards qualitative and circumstantial research, triangulation of methods is suggested (Kay and LeSage, 2009). Instead of only focusing on students' opinions of learning, reliable and validated instruments should be used to triangulate data (Kay and LeSage, 2009; Kennedy, Cutts and Draper, 2006).

Dangel and Wang (2008:99) analyse research on the use of personal response systems in higher education and find that the principle of using personal response systems to recognise diverse abilities and/or knowledge needs to be studied further (Dangel and Wang, 2008).

Beatty and Gerace (2009) acknowledge the lack of empirical evidence to illustrate the efficacy of TEFA in directly influencing student beliefs and attitudes.

2.5 Flipped Classroom Instructional Model

Sun, Zie and Anderman (2018) note that the flipped classroom model of teaching is not a new instructional model but has recently moved to the forefront of innovation in education due to the increased accessibility of technological developments. The flipped classroom consists of two chief components: individual learning outside the classroom and interactive learning inside the classroom (Cronhjort et al., 2018; Love, Hodge, Grandgenett and Swift, 2014; Sun et al., 2018). The modern flipped classroom model is interpreted by some researchers (Bishop and Verleger, 2013; Sun et al., 2018) to imply the use of online tutorials or lessons to constitute the first component of the model, but Abeysekera and Dawson (2015) condense the definition of a flipped classroom to imply all teaching approaches where the transfer of knowledge is moved outside the classroom, class time is utilised towards active learning, and students have to complete learning activities before and after the class.

Love, Hodge, Grandgenett and Swift (2013) describe a flipped mathematics classroom where central concepts are learned outside the classroom and in-class activities are aimed at deeper learning through interactivity. Sun et al. (2018) design a flipped mathematics classroom based on Krathwohl's (2002) interpretation of Bloom's Taxonomy, where the learning activities to be performed outside the classroom target cognitive skills like remembering and understanding, and the higher level cognitive skills of applying, analysing, evaluating and creating are the focus of activities inside the classroom. Although there is evidence of the effectiveness of the flipped classroom model in engaging students, there is also evidence that the model is detrimental to the learning of some students. Sun et al. (2018) agree with Cronhjort et al. (2018) that the flipped classroom encourages students to take responsibility for their own learning and that they become more self-directed. They advise strategies on the design of a flipped classroom aimed at supporting and motivating students.

Abeysekera and Dawson (2015) argue that the flipped classroom model has the potential to improve student motivation. In an attempt to establish a rationale for the model, they propose that student motivation is improved through creating “a sense of competence, autonomy and relatedness” (Abeysekera and Dawson, 2015:4). Competence refers to a student’s belief in their ability to learn knowledge and skills, autonomy is their desire to be in control, and self-sufficient and relatedness describes the need to belong in the social setting of a learning environment. They reason that the flipped classroom can improve a student’s intrinsic and even extrinsic motivation by creating conducive learning environments that enable them to feel competent, autonomous and relate to their peers. Sun et al. (2018) design of a flipped mathematics classroom, the lecturer should create opportunities for small successes, to help students build their competence or belief in their own abilities. Another strategy suggested by Sun et al. (2018) is to design learning activities so that students can see how their peers solve mathematics problems, in order to increase the relatedness of the mathematics classroom.

This brings us back to the work of Yoon et al. (2011) who conducted interviews with students at a New Zealand university to determine their perceptions and expectations about teaching and learning in a first year mathematics course. They find that students value the opportunity to work with peers (in small groups) because it allows the opportunity to observe other students’ approaches to solving problems, increases their confidence in their own abilities, and develops their own understanding. Their research supports the findings of Abeysekera and Dawson (2015) that the in-class component of the flipped classroom can positively impact students’ competence, autonomy and relatedness.

Vicens (2017) reports on using “clicker-based peer instruction” to create a flipped classroom. By “clicker-based peer instruction” he infers teaching through combining personal response systems with peer instruction. The students use personal response systems to respond to PRS questions. Should student responses be unsatisfactory, the bar chart showing student responses is not revealed. Students are allowed to discuss the question with a peer and vote for a second time. Vicens repeats the strategy throughout his lectures with key concepts as learning objectives.

2.6 Precis

In Chapter 2 literature related to the didactical contract and the transition from secondary to tertiary education is reviewed. The theoretical and conceptual frameworks of the study are outlined in Chapter 3.

Chapter 3: Theoretical and Conceptual Frameworks

The theoretical and conceptual frameworks of a study guide how research is conducted, and ensure the credibility of the research (Adom, Agyem and Hussein, 2018). The theoretical framework provides a foundation rooted in accepted theories or a theoretical outline for the research. In the words of Adom et al. (2018:438) the researcher "is expected to make a unique application of the selected theory so as to apply the theoretical constructs to his/her study." On the other hand, the conceptual framework is a summary of the concepts relevant to the research problem (Maree and Van der Westhuizen, 2012) and depicts how the concepts are linked (Imenda, 2014). The theoretical framework is deductive whereas the conceptual framework is inductively constructed (Imenda, 2014).

The theoretical framework is firstly presented in this chapter by discussing the philosophical and pedagogical underpinnings of the study. The chapter concludes with the conceptual framework of my study and the definition of the concept of a Time-out session.

3.1 Philosophical Underpinning

Creswell (2014) suggests that the researcher elaborates on the philosophical ideas that influence the research approach. To elaborate philosophical ideas is to explore the fundamental "beliefs that guide action" (Creswell, 2014:6). In the same way that student beliefs govern student behaviour, the beliefs of the researcher govern investigation and interpretation.

3.1.1 Ontological assumptions. My ontological assumptions guide my research. My viewpoint is that reality is constructed by individuals, as mentioned by Maree and Van der Westhuizen (2012). This viewpoint is best described by the statement: Beliefs about mathematics and mathematics teaching/learning are constructed by individual students.

3.1.2 Epistemological assumptions. According to Creswell (2014), four epistemologies or worldviews that underlie research, namely postpositivism, constructivism, transformativism and pragmatism, are generally discussed in literature. The four epistemologies are briefly discussed below, in order to highlight the epistemological assumptions underlying my study.

Constructivism. According to the constructivist worldview, people construct their own realities and develop values based on their individual experiences (Creswell, 2014). The focus of the researcher is to understand the meanings others attach to the world and to base their findings as much as possible on the participants' views of the situation being studied. Creswell (2014) also mentions that a human's views are influenced by their social perspective and that views develop through social interaction.

My focus is to understand and attempt to influence student beliefs. The underlying assumption is that an individual student attaches meanings or develops beliefs based on their experiences in the social context of mathematics teaching/learning.

Postpositivism. According to Jansen (2012), reality is objective from the positivist point of view, while Creswell (2014) states that the notion of absolute truth applies. The postpositivist worldview dictates that there is no absolute truth when studying humans, but the principles of cause and effect apply. Postpositivist assumptions are generally associated with quantitative studies where the researcher collects data to substantiate or refute a theory (Creswell, 2014).

Central to my study is the didactical contract, an agreement between the lecturer and students in a teaching/learning environment. The didactical contract is characterised by the three categories of beliefs that the participants (lecturer and students) hold about each other. The aim of the study is to gauge the expectations of students entering university and determine whether a teaching intervention – using personal response systems – can influence student beliefs. For the purpose of gauging student beliefs before and after the intervention, a postpositivist worldview is adopted.

Transformativism. The transformative worldview is directed towards change for marginalised individuals (Creswell, 2014) and the view is mostly motivated by political issues linked with social injustices.

In my study I focus on the beliefs that first year students may harbour about teaching/learning. Although it would have been useful to distinguish between the beliefs of students from different socio-economic backgrounds , it is not within the scope of the current study because the study is focused on the predominant and not various beliefs of students transitioning from secondary to tertiary mathematics education.

Pragmatism. According to Creswell (2014), pragmatism as a worldview cannot be described in terms of one philosophy, because it is oriented towards real-world practices. The researcher accentuates the research problem and uses all possible approaches to comprehend the problem. Pragmatists acknowledge the existence of research in a social context.

My perspective is pragmatic, because my focus is to explore the practice of incorporating personal response systems in a purposeful way into the mathematics teaching/learning environment, which qualifies as a social context. This implies that I am not committed to one philosophical assumption, but multiple assumptions for the purpose of solving the research problem.

3.1.3 Methodological assumptions. According to Creswell (2014), pragmatism is a valid epistemological assumption for mixed methods research. Creswell posits (2014:11) that a pragmatist researcher determines the "*what* and *how*" of the research and has the freedom to refine the research approach that best serves the purpose of the research. Mixed methods approaches incorporate both qualitative and quantitative data into the research design, whereas quantitative methods aim to observe and gauge information numerically and qualitative methods aim to interpret the meanings of participants' responses in context (Creswell, 2014).

I explore the use of personal response systems to influence the didactical contract in a mathematics classroom. The intervention consists of regular PRS sessions incorporated into the teaching approach. The didactical contract is described in terms of three categories of beliefs, based on literature, to inform my attempt to compare students' initial beliefs about the didactical contract with their post intervention beliefs through the use of quantitative methods. The goal here is not necessarily to identify general findings about the population, but to measure whether student beliefs shifted due to the intervention. Because the didactical contract is characterised by participants' subjective beliefs, qualitative methods need to be incorporated to meaningfully interpret participant beliefs. The quantitative and qualitative data are integrated using convergent parallel mixed methods, because the goal is to determine whether and how the didactical contract has shifted post intervention (Ivankova et al., 2012).

Once the researcher has decided upon the methodology or mode of inquiry, it is necessary to describe the research strategy or approach (Maree and Van der Westhuizen (2012). The research approach influenced by my philosophical assumptions is discussed in Chapter 4, but first I include a brief discussion of the pedagogical underpinning of my study.

3.2 Pedagogical Underpinning

A study aimed at addressing student beliefs about mathematics and mathematics teaching and learning, requires the researcher to explore several pedagogical beliefs. Three important views need to be addressed i.e. the nature of mathematics, the learning of mathematics by students, and the teaching of mathematics by the lecturer.

3.2.1 The nature of mathematics. Yadav (2017) and Ziegler and Loos (2014) agree that Courant and Robbins (1941) failed to adequately answer the question "What is Mathematics?" in their famous book. In an attempt to answer this important question, Yadav (2017) concludes that mathematics is the "study of assumptions, its properties and applications", while Ziegler and Loos (2014:4) posit that the question "does not need to be answered to motivate why mathematics should be taught". As mentioned before Kilpatrick et al. (2001:116) describe mathematical ability in terms of "five strands of mathematical proficiency":

1. Conceptual understanding: Comprehension of mathematical concepts, operations, and relations
2. Procedural fluency: Skill in carrying out procedures flexibly, accurately, efficiently and appropriately
3. Strategic competence: Ability to formulate, represent, and solve mathematical problems
4. Adaptive reasoning: Capacity for logical thought, reflection, explanation, and justification

5. Productive disposition: Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with diligence and belief in one's own ability

The five strands are described as "interwoven" (Kilpatrick et al., 2001:116) to illustrate their interdependence, and that together they constitute strength in mathematical ability.

My pedagogical assumptions about the teaching and learning of mathematics are based on this description of Kilpatrick et al. (2001): Mathematics constitutes a combination of skills, each of which has to be acknowledged when teaching mathematics. Since beliefs about mathematics are regarded as one of the constituents of mathematical ability, I value the importance of influencing student beliefs when teaching.

3.2.2 Learning mathematics. Van de Walle et al. (2013) examines the construction of mathematical knowledge from a constructivist point of view. According to Van de Walle et al. (2013), students attain knowledge in relation to existing structures of knowledge. Figure 3.1 illustrates this process. The grey dots represent concepts that are interconnected to constitute an existing structure of knowledge. A new concept (white dot) is incorporated in relation to the existing structure.

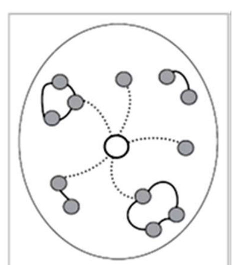


Figure 3.1: The construction of knowledge (Van de Walle et al., 2007:5)

As mentioned in Chapter 2, Skemp (1976) defines instrumental understanding of mathematics as the ability to remember and use rules, formulas and methods, whereas relational understanding is the ability to understand how rules, formulas and methods must be applied, because the concepts behind them are meaningfully connected. It can be said that procedural fluency (or skill in using mathematical methods, rules and formulas) without conceptual understanding is instrumental understanding. According to Van de Walle et al. (2013), if students are not taught to make connections, the only other option is to memorise rules, formulas and procedures separately. He therefore places instrumental and relational understanding on the opposite ends of the continuum of mathematics learning (Figure 2.1).

Besides Constructivism, Van de Walle et al. (2013) also base their philosophy of mathematics learning on Sociocultural theory, where the students develop meaning by working with "more knowledgeable others" (Van de Walle et al., 2013:6). According to Van de Walle et al. (2013), this theory proposes that learners have their own zone of proximal development. The concept of the Zone of Proximal Development (ZPD) was defined by Vygotsky in 1978 (Eun, Knotek and Heinig-Boynton, 2008:134; Wass, Harland and Mercer, 2011:318) as "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under guidance or in collaboration with more capable peers." This definition makes it evident that collaboration with peers and/or "more knowledgeable others" is essential for the construction of knowledge. The scaffolding or support provided by peers and the teacher are important to attain a perceived unattainable goal (Wass et al., 2011).

Besides Constructivism and Socioculturism, a further two theories of learning are regarded by Cottrill (2003) as central to the learning of mathematics, namely Behaviourism and the Theory of embodied mathematics. Behaviourism is based on the premise that student learning manifests as student behaviour conditioned by the learning environment (Cottrill, 2003), while at the centre of the Theory of embodied mathematics is the belief that the learning of mathematics concepts are embodied and understood by means of metaphors (Núñez, Edwards and Matos, 1999).

My beliefs about the learning of mathematics are briefly discussed. According to constructivism, students' pre-existing knowledge, skills and beliefs determine what they recognise and learn (Bransford et al., 2000). Using this as a departure point, I subscribe to the belief that students' pre-existing knowledge, skills and beliefs must be elicited and challenged to optimise learning in the mathematics classroom. Also, the distinction between instrumental and relational understanding identified by Skemp (1976) and illuminated by Van de Walle et al. (2013) is used in my study to describe student beliefs about mathematics learning. Although student beliefs about mathematics learning in secondary mathematics education are mainly directed towards instrumental understanding, relational understanding must be encouraged in tertiary mathematics education.

In my study, the design and incorporation of personal response questions aimed at influencing student beliefs are based on the didactical design principles that developed from Brousseau's theory of didactical situations (TDS). In didactical design the aim is to create an *adidactical* teaching/learning situation, where student learning develops in spite of the lecturer. From the viewpoint of Sociocultural theory, the teaching/learning situation represents "more knowledgeable others", since didactical design principles guides the design and implementation of the envisioned teaching/learning activity. Although the lecturer is another version of "more knowledgeable others", their role is restricted to that of facilitator. From the viewpoint of the Zone of Proximal Development (ZPD), peer involvement supports learning and is utilised when incorporating personal response systems for the purpose of my study.

As researcher I realise that student learning is an individual endeavour and that mathematical understanding is subject to the student's experience or even embodiment of the world, but I also acknowledge that student learning becomes visible through observing student behaviour. The implication here is that patterns of behaviour provide information about the nature of student learning.

According to Dweck (2008), a student's mindset influences their success in learning. She distinguishes between a fixed and growth mindset, and finds that a growth mindset encourages learning and development. Dweck (2008) goes on to state that a fixed mindset is characterised by the belief that intellectual ability is fixed and cannot develop, whereas a growth mindset is characterised by the belief that intellectual ability can develop through effort. Dweck (2008) further finds that by attempting to influence students' mindsets, their mathematical learning can be enhanced. In my study I acknowledge that student beliefs about their own mathematics ability and the process of learning influence their ability to adapt to tertiary mathematics education, and should be redirected if necessary.

3.2.3 Teaching mathematics. Freudenthal views mathematics as a "human activity" (Gravemeijer and Terwel, 2000:777) in which *mathematising* takes centre stage. Students must be provided with opportunities to *mathematise*, to make problems from reality mathematical or to make mathematics more meaningful. He advocates that mathematics is not a "ready-made-product", but should be the product of teaching, and students can only benefit from collaboration (Gravemeijer and Terwel, 2000:780). According to Freudenthal, the learning process is discontinuous (Gravemeijer and Terwel, 2000) and student learning can be observed by creating discontinuities. Students must be guided in "phenomenologically rich situations" to the point that they realise they are responsible for the mathematical content (Gravemeijer and Terwel, 2000:787).

Cobb (1988) examines the conflict between constructivist beliefs about learning and the transmission mode of teaching, characterised by the belief that knowledge is transmitted when teaching mathematics. According to Cobb (1988), two goals motivate teaching when employing the constructivist view of learning: the construction of complex conceptual structures and intellectual independence. He acknowledges that intellectual independence implies skills like metacognition and productive beliefs about mathematics and mathematical ability. The practice of teaching for transmission is characterised by "imposition", whereas the two goals of a constructivist view mentioned above go counter to this practice by supporting teaching through "negotiation" (Cobb, 1988:99-101).

Cobb (1988:95) mentions that besides encouraging students to be creative and informed problem solvers, one of the teacher's principal responsibilities should be to "engage students in activities that give rise to genuine mathematical problems for them". Van de Walle et al. (2013) mentions that to teach for relational understanding is to pay attention to student ideas, create opportunities for mathematical discourse, encourage students to utilise multiple approaches, view mistakes as learning opportunities and demonstrate that mathematics makes sense.

My beliefs about mathematics teaching are briefly discussed. In my study I realise the importance of creating meaningful learning opportunities based on the nature of the mathematical content. My view is that through guided engagement with the mathematical content and the incorporation of discontinuities in the learning process, student learning is optimised, and students are motivated to take responsibility for their own learning. I believe that the design and implementation of teaching/learning events directed by epistemological, pedagogical and metacognitive goals, combined with peer discussion and personal response systems, can provide opportunities for meaningful student engagement with mathematical problems, mathematical discourse and ultimately negotiation for learning. An objective of my study is to encourage relational understanding. The design of PRS questions and the pedagogical design of the PRS sessions aim to use student ideas and possible mistakes, introducing multiple approaches, and incorporating peer discussion to encourage mathematical discourse directed towards sense making.

Dweck (2008) finds that educators can assist students to develop a growth mindset by informing them about the growth potential of the brain, by emphasising the value of mistakes and the importance of effort, and by providing them with feedback about their own learning. In my study I attempt to utilise PRS sessions to influence student beliefs about their own ability by designing PRS questions to include possible student mistakes as distractors. By creating opportunities for students to discuss their answers to PRS questions with their peers, effort towards understanding is encouraged. Immediate feedback is provided in the form of a bar chart of student responses to PRS questions and a reveal of correct and incorrect answers.

This concludes the theoretical framework of my study. In order to explain the research approach in Chapter 4, the conceptual framework is discussed next.

3.3 Conceptual Framework

As mentioned earlier the conceptual framework depicts the concepts relevant to the research problem (Maree and Van der Westhuizen, 2012) and the connections between these concepts (Imenda, 2014). The concepts or constructs relevant to my study were identified in Chapter 2, and will now be highlighted and refined to constitute the conceptual framework of my study.

Researchers (Benadé, 2013, Hourigan and O' Donoghue, 2007; Pepin, 2014; Yoon et al., 2011) observe that the didactical contract in secondary mathematics education is characteristically teacher-centred, with a focus on procedural fluency, learners demonstrating surface learning and basic reasoning, and learning being the responsibility of the teacher. On the other hand, students at tertiary level are expected to focus on conceptual understanding, to demonstrate deep learning and higher-order reasoning, and to take responsibility for their own learning in a student-centred learning environment. The didactical contract in secondary mathematics education, as compared to the didactical contract envisioned in tertiary mathematics education, is summarised in Table 3.1.

Table 3.1: The didactical contract in secondary versus tertiary mathematics education

	Secondary mathematics education	**Tertiary mathematics education**
1.	Teacher-centred (Hourigan and O'Donoghue, 2007)	Student-centred (Biggs, 1999; Bransford et al, 2000; Yoon et al., 2011)
2.	Focus on procedural fluency (Benadé, 2013)	Teaching for conceptual understanding (Selden, 2005, Yoon et al., 2011)
3.	Surface learning (Selden, 2005) and basic reasoning (Benadé, 2013)	Deep learning (Selden, 2005) and higher-order reasoning (Benadé, 2013)
4.	Learning (or the truth) is the responsibility of the teacher (Benadé, 2013)	Learning (or the truth) is the responsibility of the student (Pepin, 2014)

Mathematics learning in secondary mathematics education can collectively be described as instrumental understanding, where the focus is the skill of using mathematical methods, rules and formulas (Skemp, 1976; Van de Walle et al., 2013), resulting in surface learning and basic reasoning.

At tertiary level students are expected to demonstrate conceptual understanding combined with procedural fluency, hence relational understanding (Skemp, 1976; Van de Walle et al., 2013) aimed at deeper learning and higher-order reasoning. Henceforth, mathematics learning is described as mainly constituting instrumental understanding in secondary mathematics education, as opposed to relational understanding in tertiary mathematics education. It must be said that the word mainly is used, because the above is a generalised description of secondary mathematics education that is not without exception.

Table 3.2 provides a refined version of Table 3.1, with the three main characteristics of the didactical contract redefined in term of Centredness (1), Mathematics learning (2), and Responsibility for learning (3).

Centredness refers to beliefs about the nature of the mathematics classroom; Mathematics learning refers to beliefs about the nature of mathematics learning; and Responsibility refers to beliefs about the responsibility for learning. It should be said that by describing tertiary mathematics education as student-centred, the implication is not that all mathematics classrooms are student-centred at tertiary level, but that the mathematics classroom conducive to student learning is envisioned by me, the researcher, as student-centred. As for the other characteristics, the implication is that the contract is mainly characterised by the said characteristics, not defined by the said characteristics. The summary of Table 3.2 is for the purpose of identifying and negotiating student beliefs in the mathematics classroom that can be influenced towards optimising learning.

Table 3.2: Three categories of beliefs that characterise the didactical contract in mathematics education

	Characteristic	**Secondary mathematics education**	**Tertiary mathematics education**
1.	Centredness	Teacher-centred	Student-centred
2.	Mathematics learning	Learning is instrumental understanding	Learning is relational understanding
3.	Responsibility for learning	Learning is the responsibility of the teacher	Learning is the responsibility of the student

Figure 3.2 is included to explain how I envision the use of personal response systems in the mathematics education of students entering university for the first time. The use of PRS sessions will be directed towards influencing student beliefs and encouraging a shift in the didactical contract from secondary to tertiary mathematics education, used to influence student beliefs about the centredness of the mathematics classroom, mathematics learning and the responsibility for student learning. Ideally a shift in the didactical contract can be described from being characteristically teacher-centred to

being student-centred, from being focused on instrumental understanding to being focused on relational understanding, and from the teacher being held responsible for learning to the student being held responsible for learning.

I aim to influence student beliefs by creating a discontinuity in the traditional mathematics lecture – or a change-up as described by Middendorf and Kalish (1994) – through the use of personal response systems. The discontinuity is motivated by the goal of engaging students to demonstrate effort towards learning in a student-centred learning environment. Typical student mistakes are used as distractors when designing the relevant PRS questions, and provide a foundation to encourage deep conceptual understanding and higher-order reasoning or relational understanding. The student-centred learning environment, where the role of the lecturer is restricted to that of facilitator, aims to encourage students to take responsibility for their own learning.

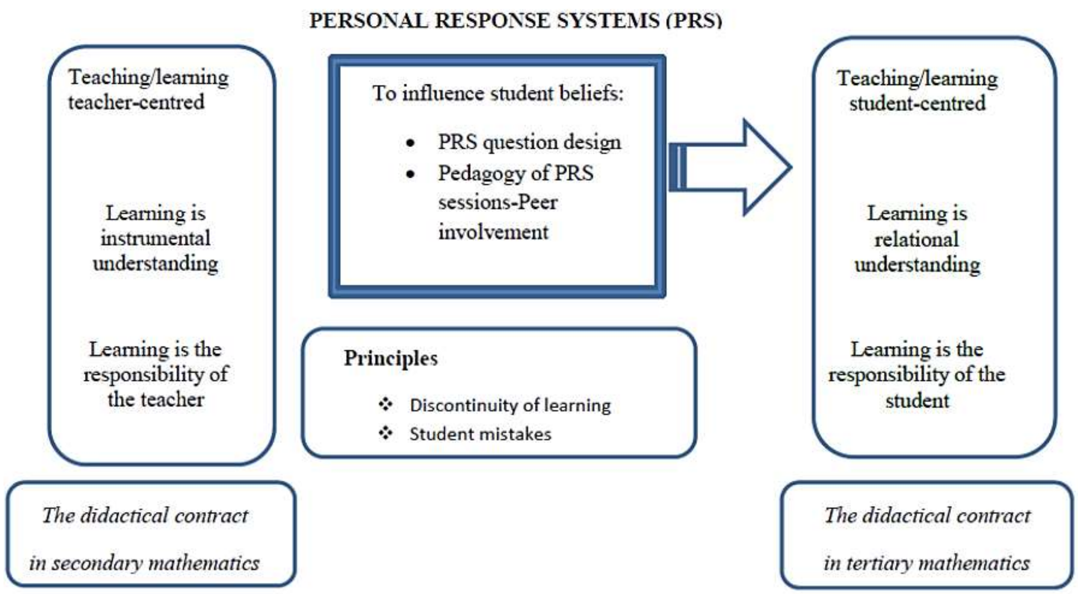


Figure 3.2: Conceptual framework of my study

In order to use personal response systems to influence student beliefs about mathematics teaching/learning, the design of PRS questions and the pedagogy of the PRS sessions have to be considered and carefully planned. The principle of discontinuity led me to the idea of incorporating PRS sessions in order to vary or take time out from the traditional transmission mode of teaching, and the concept of Time-out sessions was born.

3.3.1 The Time-out session. During a Time-out session, students respond to PRS questions designed for relational understanding in a student-centred learning environment and they discuss their answers with their peers while the role of the lecturer is minimised. Figure 3.3 depicts the traditional mathematics classroom as compared to a classroom incorporating Time-out sessions. In the figure, the lecturer is represented by a coloured circle, while students are depicted by open circles. In a traditional mathematics classroom the lecturer transmits knowledge while students mostly adopt the role of passive listeners. In a mathematics classroom with Time-out sessions, the traditional transmission mode lectures are varied with personal response systems providing feedback about learning (depicted by means of a bar chart in Figure 3.3) and students are given the opportunity to learn from their peers.

The idea of Time-out sessions was mainly inspired by Brogt (2007) who outlines the idea of Lecture-Tutorials (LTs) in an introductory astronomy classroom, aimed at engaging students and fostering positive attitudes towards learning. The LTs are short classroom activities of 10 to 20 minutes, incorporated daily into traditional lectures. The LT does not contain enough information to substitute the lecture, but questions are designed to identify student difficulties with astronomy and provide opportunities to resolve these difficulties. During these LT sessions the students work in pairs, so peer instruction is involved; the teacher uses Socratic dialogue; time is strictly managed; and no answers are provided at the end of the session. The lecture resumes after the session and students are encouraged to consult with the lecturer outside the classroom to discuss solutions and correct answers. Consultation is further motivated by the inclusion of a similar question or questions in a future test or examination. Brogt (2007) reasons that the opportunity created for students to reason a problem by themselves, encourages deeper learning and enhances students' confidence in their own ability (self-efficacy).

The pedagogy of Beatty and Gerace i.e. TEFA, (2009); Mazur's peer instruction and the successful use of LTs in the astronomy classroom (Brogt, 2007) inspired the pedagogical design of PRS sessions, to be called Time-out sessions for the remainder of the study. The name is based on the idea that students take time out from the traditional mathematics lecture, where the lecturer transmits knowledge and the students mostly listen passively.

To elaborate on the structure of the Time-out sessions: the PRS question (or sequence of questions), exploring relevant mathematical knowledge, is presented in class. Students are allowed to individually solve the problem(s) and answer each question by voting. The bar chart(s) of student answers are displayed without revealing the correct answer(s). Time is then allowed for students to discuss their answer(s) with peers, working in pairs. Students can also consult with the lecturer, but Socratic dialogue is maintained. Time is restricted and after a while the students are allowed to revote on the exact same questions as presented in the first round. The second vote is then followed by a brief discussion, where the lecturer provides the answer(s) and/or highlights important principles, but no complete discussion of the reasoning behind answers is provided. Students are then motivated to work together and consult outside the classroom in order to prepare for the following week's class test, which will include a question or questions based on the Time-out session.

What follows is a further underpinning of the concept of the Time-out session, based on existing research. The Time-out session allows for discontinuity or a change-up of the lecture, as recommended by Middendorf and Kalish (1994). They recommend changing up lectures (or varying teaching style) to keep students' attention and incorporate active engagement or effort on the part of the student into the lecture. Since the teaching approach is shifted from being lecturer-centred to being student-centred ((1) in Table 3.2), a discontinuity in the didactical contract is created. This discontinuity or break holds opportunities for students and the development of their learning strategies (Pepin, 2014). The design of the PRS questions, together with the notion that no complete solutions are provided during the Time-out session, aims to create conflict within the student. Students construct knowledge through their attempts to resolve conflict experienced during the learning process (Kislenko, 2005). By incorporating peer discussion towards a revote, dialogical discourse is encouraged. Since future questions are based on results from Time-out sessions, students are motivated to consult and search for proper understanding of underlying concepts. Students' relational understanding is encouraged through the creation of conflict, student attempts to resolve the conflict through peer involvement, and the use of similar future questions ((2) in Table 3.2).

To analyse how the Time-out sessions inspire students to take responsibility for their own learning ((3) in Table 3.2) let's consider Knowles' (1975) views on self-directed learning. Knowles (1975) sees self-directed learning as the basic competence of learning on one's own. Chee, Divaharan, Tan and Mun (2011) identify three aspects of self-directed learning. The first is taking ownership of learning, the second is management and monitoring of learning and the third is the extension of one's own learning. The Time-out session creates opportunities for active learning and students taking ownership of their own learning. Collaboration between students and the opportunity for consultation outside the classroom allow for the management and self-monitoring of learning and the class test question(s) of the following week allows for the extension of own learning. By providing students with solutions, without discussing in detail the reasoning behind solutions, the extension of own learning is further encouraged. The class test as motivational factor emphasises the institutionalisation of knowledge, as mentioned by Artigue (2009).

Initially I considered a written test to partially constitute a Time-out session. To explain why a written test would not contribute to student learning and development, the Zone of Proximal Development (ZPD) mentioned in Section 3.2.2.2, needs to be considered.

A possible written test as part of a Time-out session necessitated me to consider the influence of the third voice of the ZPD on student development (Eun, Knotek and Heining-Boynton, 2008). Instead of a first vote, the students would complete the test before being allowed time to discuss the solutions and then vote on some, or all the test questions. The rationale for this consideration was to motivate students to commit to a Time-out session. Since the goal of a Time-out session is mainly to stimulate self-directed learning and not impede student development, I decided to forfeit the idea of including a written test. By not basing the Time-out session on a written test, the risk of the influence of a third voice impeding the process of development is minimised.

To summarise, the design of the Time-out session aims to create a student-centred learning opportunity, encouraging students to develop relational understanding and take responsibility for their own learning. Also, by creating opportunity for students to reconstruct their knowledge, exercise dialogical discourse and develop habits towards self-directed learning, the goal of influencing students' underlying beliefs can be realised, as mentioned by Beatty and Gerace (2009).

A Time-out session is now fundamentally conceptualised. The design of PRS questions and the pedagogy of the Time-out sessions are further discussed in Chapter 4.

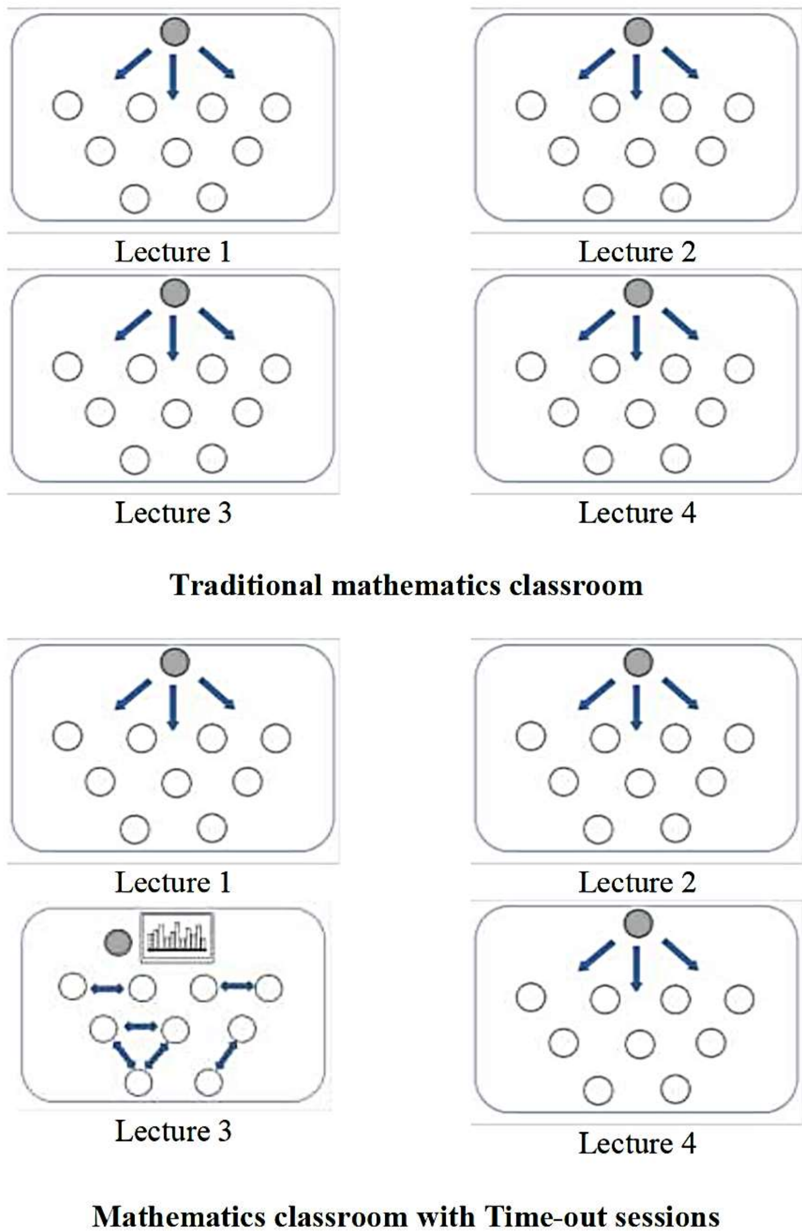


Figure 3.3: Teaching with Time-out sessions

3.4 Precis

In Chapter 3 the philosophies underlying my approach to research are discussed in order to elaborate the theoretical framework of my study. Also, findings from literature are synthesised to constitute the conceptual framework of my study. The frameworks form the basis of the research approach of my study, discussed in Chapter 4.

Chapter 4: Research Approach

According to Creswell (2014), the research approach or plan of a study, is determined by the researcher's philosophical assumptions and implicates decisions about a research design and research methods. The research design implies the "procedures of inquiry" and the research methods include the "methods of data collection, analysis and interpretation" (Creswell, 2014:3). The research question provides focus and is a determining factor in deciding the research approach (Jansen, 2012). The purpose of this chapter is to illuminate the research approach.

4.1 Primary Focus of the Study

The characteristics of the didactical contract in secondary mathematic education are compared to that of the contract in tertiary mathematics education as described by literature and summarised in Tables 3.1 and 3.2. The primary focus of the study is the didactical contract in tertiary mathematics education and the renegotiation of the didactical contract at tertiary level in order to assist students to transition successfully from secondary to tertiary mathematics education. For the remainder of the chapter Table 3.2 will be used to reference the didactical contract at tertiary level.

4.2 Research Problem

First year students enter university with definite beliefs about mathematics and mathematics teaching and learning, which are generally based on their experiences from secondary mathematics education. Since these beliefs govern their behaviour in the mathematics classroom, their fundamental beliefs must be challenged. By challenging student beliefs and renegotiating the didactical contract at tertiary level, students are supported to successfully transition from secondary to tertiary mathematics education. It is the responsibility of the lecturer to implement student-centred or interactive learning activities aimed at renegotiating the didactical contract.

In large classrooms the challenge to create a student-centred learning environment takes on new meaning. Technology in the form of personal response systems provides opportunities to do so, but to meaningfully incorporate personal response systems for the purpose of renegotiating the didactical contract necessitates exploration.

4.3 Objective of the Study

The objective of the study is to use personal response systems to negotiate the didactical contract at tertiary level by challenging and reconstructing first year students' beliefs about mathematics and mathematics teaching/learning. This objective can be further elaborated as using personal response systems to influence student beliefs about the centredness of the mathematics classroom, mathematics learning and their responsibility for their own learning.

4.4 Research Design

From a pragmatic point of view, a research design of mixed methods, incorporating both quantitative and qualitative methods, will achieve a comprehensive analysis of the research problem. According to Creswell (2014), the convergence of data strengthens the findings of the research and compensates for the limitations of other forms of data. In opposition to the closed-ended questions of a questionnaire, (focus group) interviews allow for data to emerge by means of open-ended questioning (Creswell, 2014). In the approach of the convergent parallel mixed methods design, qualitative and quantitative data are collected and analysed separately and then compared to confirm or disprove findings (Creswell, 2014).

To explore how personal response systems can be used to cause a shift in the didactical contract in tertiary mathematics education, quantitative and qualitative data are used. To gauge the nature of the didactical contract based on predetermined categories of student beliefs, quantitative data generated is collected through a questionnaire. To determine whether an intervention utilising personal response systems can influence the didactical contract, the questionnaire is re-administered after the intervention. To strengthen the findings and allow for a more comprehensive understanding of the nature of the didactical contract, qualitative data generated by focus group interviews conducted after the intervention, are incorporated.

Figure 4.1 is included to represent the timeline of my study. A questionnaire is used to gauge students' beliefs at the beginning of the first semester of their first year (referred to as Survey 1). The questionnaire is redistributed at the end of the semester (Survey 2), after the intervention consisting of so-called Time-out sessions, to ascertain whether a change in student beliefs can be discerned. At around the same time, focus group interviews are conducted to allow for more elaborate probing of student beliefs.

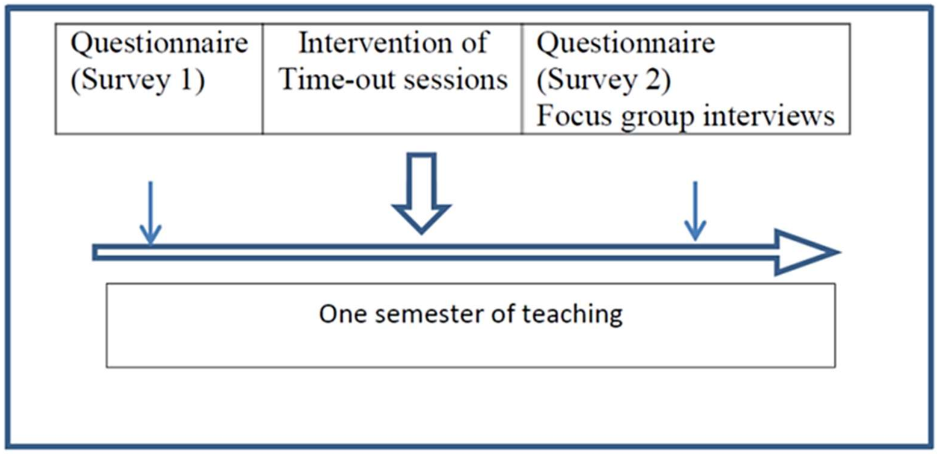


Figure 4.1: Research design

To use personal response systems to influence the didactical contract implies didactical design and pedagogical design considerations.

4.5 The Intervention

According to Artigue (2014), the aim of didactical design experiments is to provide useful and evidence-based recommendations for the practice of mathematics education. Designing didactical situations in which "knowledge is at stake" (Herbst and Kilpatrick, 1999) necessitates a consideration of the nature of the intervention. Didactical design researchers (Balachef, 1990; Kislenko, 2005) observe that mistakes made by students are the most visible symptoms of their conceptions and hold potential for future learning, showing rather what they do know, instead of what they don't know. Kislenko (2005) acknowledges the relevance of the influence of peers on student beliefs and refers to the fact that students' knowledge is constructed through their attempts to resolve conflict experienced during a didactical situation. The aim of the study – considered against the backdrop of the significance of mistakes, the resolution of conflict and peer influence – inspired the design of the intervention. The design is founded on two principles.

The first of the two principles is the design and use of personal response system (PRS) questions to highlight student mistakes and convince students of the correct answer through peer involvement, and to create and resolve conflict within students. The second principle is the use of personal response systems to create a discontinuity in the teaching/learning environment, as depicted in Figure 3.3. The nature of the PRS questions and the so-called Time-out sessions based on these two principles, are further discussed.

4.5.1 Design of PRS questions. By incorporating strategies recognised by researchers (see Section 2.4.1) to design goal-oriented PRS questions directed at encouraging relational understanding (2), I can attempt to influence a shift in the didactical contract as summarised in Table 3.2. Distractors include possible mistakes and the option "none of the above" to provide for responses not anticipated by the lecturer. As a result, student mistakes can be highlighted and discussed. In order to create conflict, PRS questions are designed to be deliberately ambiguous and/or elicit misconceptions. Another strategy is to base questions on incorrect examples and encourage metacognitive activity.

The discussion initiated by PRS questions is conducted in a Socratic way (Cline et al, 2012), where the lecturer's role is minimised but students are encouraged to explain their views. Once the students reach consensus, the teacher clarifies and simplifies the results, which brings us to the pedagogical design.

4.5.2 The Time-out sessions. In contemplating the use of personal response systems to influence student beliefs about mathematics teaching/learning and as a result renegotiate the didactical contract, we have to consider pedagogy. The following has to be kept in mind: How to incorporate PRS questions in order to transform the mathematics classroom from being lecturer-centred to being student-centred (1); to encourage a shift from instrumental to relational understanding (2) and influence students to take responsibility for their own learning (3).

The concept of a Time-out session designed to influence student beliefs through creating discontinuity and exploring student mistakes, was discussed in Chapter 3 and is mentioned here. For a Time-out session PRS questions are designed based on the relevant content, with distractors including typical student mistakes. Frequent Time-out sessions are incorporated to vary the traditional mathematics lecture, to create discontinuity and a student-centred learning environment that aims to achieve relational understanding and encourage students to take ownership of their learning.

The incorporation of the conceptualised Time-out sessions necessitated some experimentation. For this purpose, while teaching the applied calculus module in the first semester of 2017, PRS questions were set and used in accordance with the pedagogical design of the Time-out session described in Chapter 3. For the pilot study Time-out sessions were incorporated into lectures on three separate occasions and these three sessions are respectively referred to as Pilot 1, Pilot 2 and Pilot 3. The pilot study provides evidence for refining the concept of a Time-out session, mentioned in the next section and discussed in Chapter 5.

4.6 Phases of the Research Design

When designing teaching/learning interventions on the basis of didactical design principles supported by TDS, a structure is prescribed (Artigue, 2014). The successive phases of preliminary analysis, design and *a priori* analysis, implementation, *a-posteriori* analysis and validation are discussed.

The preliminary phase (Phase 1) or "preliminary analysis of an epistemological, cognitive and didactical nature" (Artigue, 2009) implies an observation of all the aspects of the mathematics classroom in its initial condition. The next phase involves the design or conception of the didactical situation, combined with the *a priori* analysis (Phase 2), which assumes an essential role in the whole process (Artigue, 2014), where the researcher postulates their assumptions and designs the process of data collection (Kislenko, 2005). Balachef (1990) mentions that it is important to be able to predict (*a-priori*) students' possible mistakes. The implementation phase (Phase 3) is where the intervention comes into action (Artigue, 2014). In the *a-posteriori* phase (Phase 4) data are compared with the posed assumptions. According to Artigue (2014), comparison between the *a priori* and *a-posteriori* analysis of classroom practices provide the basis to validate (Phase 5) the implementation of the intervention. The validation process can include a range of validation instruments i.e. pre-test and post-test, questionnaires and interviews (Artigue, 2014).

The phases of the research design were distinguished after the pilot study mentioned, and are discussed on the basis of a diagram of procedures (see Table 4.1).

Table 4.1: Diagram of procedures inherent to study

Phase no	**Procedure**	**Explanation**
Phase 1	Preliminary analysis	Questionnaire on beliefs about mathematics, mathematics teaching/learning distributed amongst students (Survey 1) participating in the intervention.
Phase 2	Design and *a-priori* analysis	Design of the intervention, consisting of weekly Time-out sessions. Every Time-out session is based on the pedagogical principle of voting for PRS questions, followed by peer discussion and a second vote. Worksheets 1 and 2, an LMS test and PRS question(s) are designed on the basis of content, cognitive and metacognitive goals; the behaviour of students is modelled against the nature of the questions and distractors. Data analysis compares the first and second vote data.
Phase 3	Implementation	Time-out sessions are incorporated into lectures.
Phase 4	*A-posteriori* analysis	Analysis of the PRS data generated by each Time-out session: comparing PRS data from the first vote with the second.
Phase 5	Validation	<u>Validation of Time-out sessions:</u> A comparison between *a-priori* and *a-posteriori* analysis to determine the validity of each Time-out session.
		<u>Validation of the intervention:</u> The questionnaire redistributed (Survey 2). Questionnaire indexes compared for Survey 1 and 2 Focus group interviews of stratified purposefully sampled students.
Foot notes Phases 2 to 4 have to be to be completed for every Time-out session. To determine the validity of the intervention: Triangulation of qualitative and quantitative data -Qualitative data: Focus group interview to gauge students' experience of the Time-out sessions -Quantitative data: Statistical analysis of the questionnaire data (Survey 1 and 2)		

For Phase 1, a Likert-scale questionnaire on beliefs about mathematics and mathematics teaching/learning is employed at the beginning of the first semester of the students' first year in 2018 (Survey 1). An analysis of student responses to the questionnaire (Survey 1) provides empirical evidence of the students' beliefs, which is the point of departure for addressing the research question.

As part of Phase 2, two worksheets based on the relevant learning objective(s) are designed. The first worksheet is posted on the university's Learning Management System (LMS) a few days in advance, and the students write an LMS test on the content of the first worksheet on the day before the Time-out session.

The second worksheet is handed out after the Time-out session, to be completed and checked against a memorandum in the students' own time. PRS questions are designed so that students' conceptions and/or possible misconceptions can be addressed in the context of the Time-out session. Phase 2 includes the design of the worksheets, LMS test and PRS questions of each Time-out session. This phase includes the *a-priori* analysis of each question, where the distractors to every question are analysed and students' anticipated responses modelled. The rationale for using the worksheets is to motivate students to prepare for the Time-out sessions in order to further motivate them to take responsibility for their own learning. The decision was made after the pilot study and is based on the idea of incorporating the Flipped classroom teaching model as explained in Chapter 5.

The next phase, Phase 3, is the implementation phase, where the Time-out session is incorporated into one of the four weekly lectures. The Time-out session is conducted based on the pedagogical principles outlined earlier in Chapter 3 and aims to create a student-centred learning environment. The role of the lecturer is minimised, and an opportunity is created for students to utilise peer input to evaluate their answers to the

PRS questions, before they vote again. The second worksheet is then handed out, to be completed in the students' own time. The worksheet includes the PRS questions of the Time-out session and students are informed that the second worksheet will not be discussed in class, but that the memorandum will be posted on the LMS. The intervention collectively consists of the Time-out sessions incorporated into one of the four weekly lectures throughout the semester.

In Phase 4, the *a-posteriori* phase, an analysis of PRS data is used to assess the success of each Time-out session in stimulating relational understanding, and to answer Research question 2: PRS data from the first voting opportunity is compared to PRS data from the second voting opportunity, in order to judge the effectiveness of the Time-out session in encouraging relational understanding. A significant improvement in the percentage of correct responses from the first to second vote is interpreted as improved learning or learning towards relational understanding. It should be mentioned that Phases 2 to 4 are repeated for every Time-out session.

Phase 5 is the validation phase and is based on a comparison between the *a-priori* (Phase 2) and *a-posteriori* (Phase 4) analysis. The purpose of this phase is twofold, namely to validate every Time-out session, and to validate the success of the Time-out sessions (or intervention) in influencing students' beliefs about mathematics and mathematics teaching/learning or renegotiating the didactical contract. The validation of the intervention is based on focus group interviews and a redistribution of the questionnaire used in Phase 1 (Survey 2). The focus group interviews are conducted, and the questionnaire redistributed near the end of the semester after the Time-out sessions have been concluded. A comparison of questionnaire results (Survey 1 and 2) by means of the three indexes (see Chapter 5) and the information gauged from focus group interviews are triangulated to conclude the study and essentially answer Research questions 1, 2 and 3. Figure 4.2 provides a summary of the research design.

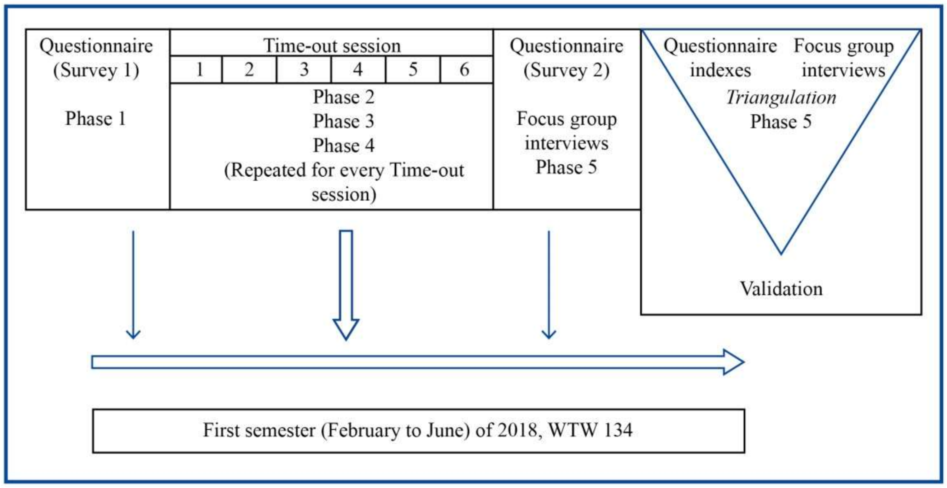


Figure 4.2: Phases of the research design

4.7 Questionnaire

According to Maree and Pietersen (2012), the questionnaire design is essential to research, since it is the instrument utilised to generate data. The following has to be considered when designing a questionnaire: appearance of the questionnaire, sequence and wording of questions and the response categories (Maree and Pietersen, 2012c). The questionnaire should appear "user-friendly" and be concise (Maree and Pietersen, 2012c:160), similar questions must follow a "logical order" and the wording must be unambiguous. The use of language must be comprehensible, formulated as statements and "leading questions" must be avoided (Maree and Pietersen, 2012a:160). According to Maree and Pietersen (2012c), closed questions are easier to answer and more practical to analyse. The disadvantages are that respondents can answer despite misunderstanding the question or not having an opinion. The Likert-scale provides an ordinal measure of a respondent's opinion; for example, the respondent can disagree or agree with a statement. According to Maree and Pietersen (2012c), the Likert-scale is quite useful when the researcher wants to measure a construct. For this purpose, values are assigned to two (or four or more) response options and a total score is calculated by adding each respondent's values corresponding to the questions related to the construct keeping in mind that all statements are "stated in the same direction" (Maree and Pietersen, 2012c:168).

A Likert-scale questionnaire on beliefs about mathematics and mathematics teaching/learning (Annexure B) was designed during the second semester of 2017, with the possibility of four answers to choose from: strongly disagree, disagree, agree and strongly agree. The questionnaire was based on a questionnaire developed by Benadé (2013), which had been adapted from a questionnaire used by Nieuwoudt (1998), that aimed to evaluate students' and lecturers' beliefs about the nature of mathematics.

The aim of the questionnaire used in this study was to obtain information about students' beliefs about mathematics and mathematics teaching/learning, in order to provide the researcher with a point of departure for addressing Research questions 1, 2 and 3. Existing questionnaire statements (Benadé, 2013) were evaluated against the conceptual framework of the current study (as expounded in Table 3.2) and new questions were formulated where necessary. For this purpose three constructs or categories of statements were identified i.e. Centredness (1), Mathematics learning (2) and Responsibility for learning (3).

The conceptual framework supporting Benadé's questionnaire (2013) needs to be highlighted and reconciled with the conceptual framework of the current study. The statements from Benadé's questionnaire represent one of the three views of mathematics as mentioned in Chapter 2: the Platonist, Instrumentalist and Dynamic view. Upon analysis of the questionnaire results, Benadé (2013) finds that students hold an instrumentalist view. According to the instrumentalist view, mathematics teaching/learning is lecturer-centred and directed towards procedural fluency (Benadé, 2013). She mentions that students "consider their learning of mathematics as the responsibility of the lecturers" (Benadé, 2013:114). Her findings can be summarised based on the abovementioned three categories of my study: mathematical teaching/learning is lecturer-centred (1), mathematics learning is procedural fluency or instrumental understanding (2) and learning is the responsibility of the lecturer (3). She mentions that the goal in teaching is to encourage a dynamic view of mathematics. Based on the dynamic view, teaching/learning is student-centred (1), mathematics is about relational understanding (2) and learning is the responsibility of the student (3). In Table 4.2 the categories of my questionnaire are reflected against the conceptual framework developed by Benadé (2013).

Tables 4.3.1, 4.3.2 and 4.3.3 provide a categorical summary of the 24 statements, referred to as Questions, included in my questionnaire and are included after Section 4.8 for purposes of layout. The asterisks indicate the statements that were adapted from the questionnaire of Benadé (2013:69-70).

The questionnaire was piloted in 2017 and the findings are discussed in Chapter 5. The reliability and validity of the instrument are discussed in Section 4.11.

Table 4.2: Categories of questionnaire related against conceptual framework of Benadé (2013)

Categories of questionnaire	**Conceptual framework of current study**		**Conceptual framework of Benadé (2013)**	
	Secondary mathematics education	**Tertiary mathematics education**	**Instrumentalist view of mathematics**	**Dynamic view of mathematics**
Centredness (1)	Teaching and learning is teacher-centred	Teaching and learning is student-centred	Teaching and learning is lecturer-centred	Teaching and learning is student-centred
Mathematics learning (2)	Focus on procedural fluency	Teaching for conceptual understanding	Emphasis is on procedural fluency	Emphasis on conceptual understanding
	Surface learning and basic reasoning	Deep learning and higher-order reasoning		
Responsibility for learning (3)	Responsibility of the teacher	Responsibility of the student	Responsibility of the teacher	Responsibility of the student

4.8 Ethical Considerations

Creswell (2014) mentions that ethical considerations are essential to a research study, since research includes people and research must be trustworthy. Since the study was conducted at the University of Pretoria, ethical clearance was obtained from the relevant faculty, the Faculty of Natural and Agricultural Sciences (reference number EC180212-174) as mentioned in Chapter 1. During the process of data collection the students were informed about the purpose of the study and that completion of the questionnaire and participation in the focus group interviews were voluntary and anonymous. The interviews were conducted by an experienced moderator and tape recorded to ensure that the rich data generated by the focus group interviews were reported truthfully and not influenced by the presence of the researcher. For the purpose of analysing focus group interviews, multiple perspectives and contrary findings were reported, while the anonymity of students was protected by numbering instead of naming the participants. To further maintain ethics the study is reported honestly and transparently and the relevant data will be stored for at least five years.

Table 4.3.1: Questions identified by keywords and categorised under Centredness (1)

Views about mathematics		**Statement keywords**	**Question number**
Teaching and learning as lecturer-centred	The lecturer as expert teaches, demonstrates, explains, transfers knowledge and refrains from challenging students. The students remain passive, listen and respond to lecturer's instructions; comfort levels of students protected by the lecturer.	Lecturer demonstrates the correct method to solve problems.	2.2*
		Student needs to listen carefully.	2.3*
		Lecturer does not leave students confused.	2.4*
		Lecturer conveys knowledge, tests whether knowledge was transferred.	2.5*
		Problems given to students should be solved easily with content provided in class time.	2.11*
Teaching and learning as student-centred	Lecturer is facilitator and stimulator of learning.	The learning of mathematics is best achieved if students battle with mathematics.	1.11
		Students can find methods to solve problems without the help of the lecturer.	2.1*
		Lecturer supports students to discover concepts.	2.8
		Students are encouraged to actively solve problems in class.	2.9
		Lecturers should encourage students to find various ways to solve problems.	2.10*

Table 4.3.2: Questions identified by keywords and categorised under Mathematics learning (2)

Views about mathematics		Statement keywords	Question
Emphasis on instrumental understand-ding	Mathematics is a bag of tools; made up of unrelated facts, rules and formulas.	Set of facts, rules and formulas.	1.1*
		Rules to be accepted and remembered, no explanation.	1.6*
		Students struggle because they do not know the correct rule or formula	1.7*
Emphasis on relational understand-ding	Mathematics is relational understanding, the creation of meaningful conceptual structures.	To be able to do mathematics is to have understanding of mathematical concepts behind rules and formulas.	1.3
		Mistakes repeated indicate a lack of understanding.	1.4
		To be able to explain answers is more important than the answer.	1.9*

Table 4.3.3: Questions identified by keywords and categorised under Responsibility (3)

Views about mathematics		Statement keywords	Question
Lecturer responsible for learning	Responsibility of the lecturer to see that students master the subject.	Student passes mathematics by attending all classes.	1.12
		Lecturer is responsible for the student's learning.	2.6
		Lecturer conveys the importance of knowledge.	2.7
		Lecturer lightens the burden of learning.	2.12
Student responsible for learning	Learners build their own knowledge through active involvement.	Responsibility to attend all mathematics classes.	1.2
		Responsibility of the student to prepare before attending classes.	1.5
		Responsibility to clarify confusion.	1.8
		Responsibility to engage continuously.	1.10

4.9 Alternatives Considered

My study necessitated a few practical considerations. For the purpose of personal response systems, clickers were used since the devices were prescribed by the majority of modules for which the sample students were enrolled. Since most of the venues at the university were not equipped with Wi-Fi at the time of the study, the use of Smartphones and other devices was not an option.

It became evident early on in the study that a control group and randomised sampling was also out of the question due to logistical limitations. Four different lecturers were responsible for lecturing the four different lecture groups, and the students' timetable was the determining factor with lecture group allocation. To use another lecture group as a control group would introduce too many extraneous factors, for example a difference in lecturing style or the size or composition of the lecture group, which could influence the outcome of the study. The main purpose of a control group is equality, to minimise influential factors and isolate the factor(s) to be measured (Maree and Pietersen, 2012b). Conclusions about the intervention would then be based on inequality in the experimental and control group.

Initial planning was to incorporate the Time-out sessions on a weekly basis, but due to time constraints and pending tests, they were only incorporated during six of the thirteen lecture weeks.

According to Ivankova, Creswell and Plano Clark (2012:277), two characteristics determine a specific mixed methods approach, namely "timing" and "mixing". Timing refers to the order of data collection and analysis. In the case of my study the parallel collection and analysis of quantitative and qualitative data allow for an in-depth analysis of student beliefs after the Time-out sessions. The mixing of the data describes the procedure of integrating the two types of data. For my study the purpose is to relate the two types of data to better comprehend student beliefs or the didactical contract in tertiary mathematics education. Therefore, the data are integrated or converged at the data analysis stage. The convergent parallel or triangulation mixed methods design where data are collected, analysed and integrated concurrently or triangulated supports the aims of my study (Ivankova et al., 2012).

4.10 Case Study

The term "case study" can be used to denote a research method or a unit of analysis (Nieuwenhuis, 2012) and implies a constrained system. A case study is an organised probe into a real-life event (or set of events) aimed at describing an identified phenomenon, where the margins between context and phenomenon are unclear (Nieuwenhuis, 2012). According to Nieuwenhuis (2012), the data gathered can include qualitative and quantitative data.

My study is a case study of the didactical contract in tertiary mathematics education, where the contract is explored in the context of a first year mathematics classroom at the University of Pretoria.

4.11 Reliability and Validity

According to Nieuwenhuis (2012), the reliability and validity of a research instrument, in this case the questionnaire, is vital for research. The reliability and validity of my questionnaire are discussed, after which further considerations related to the qualitative research and the trustworthiness of my study are explained.

4.11.1 Standardisation of the questionnaire. According to Pietersen and Maree (2012a:215), "reliability is the extent to which a measuring instrument is repeatable and consistent" or the degree to which an instrument will yield similar results if re-administered. The Cronbach's alpha coefficient can be used to measure the internal reliability of an instrument and is calculated for items that represent the same construct (Pietersen and Maree, 2012a). The more the similar items correlate, the closer the alpha coefficient will be to one. It is generally agreed that a value of 0.7, or 0.6 in the instance of exploratory research (Hair, Black, Babin and Anderson, 2014), is the minimum acceptable value for internal reliability (Pietersen and Maree, 2012a).

In Tables 4.2.1 to 4.2.3 the items or questions of the questionnaire are related to the conceptual framework of the study and categorised into three categories or constructs accordingly: Centredness, Mathematics learning and Responsibility (for learning). To determine the reliability of the questionnaire the Cronbach's alpha coefficient for the three categories of the questionnaire were calculated as three separate values. The data from Survey 1, with sample size 59 (n=59), was used for this purpose and Table 4.4

provides a summary of the three alpha coefficient values. Note that the sampling for my study is discussed in Section 4.12.

Table 4.4: Cronbach's alpha coefficient per category (n=59)

Item Category	Number of items	Alpha coefficient
Centredness	10	0.678
Mathematics learning	6	0.232
Responsibility for learning	8	0.777

The alpha coefficients for the Centredness and Responsibility for learning constructs imply low to moderate reliability in the case of the Responsibility for learning construct. A concern is the very low value of 0.232 for the Mathematics learning construct. On closer inspection of the items categorised under Mathematics learning and after considering the findings of the pilot questionnaire (Tables 5.5.1 to 5.5.3), it became evident that the wording of item 1.6 (see Annexure B) can potentially be the cause for misinterpretation: "Many rules in mathematics simply have to be accepted and remembered, there is not an explanation for it". The ambiguity of the item is most probably caused by the negative phrase "there is not really an explanation for it". It is possible that students realise that explanations behind mathematics rules do exist, but from an instrumentalist point of view mathematics learning does not necessitate the explanation of rules. The wording of the item can be improved as follows: "Many rules in mathematics simply have to be accepted and remembered, without explanation". When reworded, the ambiguity of the item is minimised and the item describes mathematics learning as instrumental understanding. If the student agrees, the implication is that mathematics is mostly the application of rules, without explanation or mathematics is instrumental understanding. If the student disagrees, the student realises that mathematics is not only the application of rules without explanation, or that mathematics is relational understanding.

Item 1.7 (see Annexure B) is another ambiguous item: "If students struggle to solve a mathematics problem, it is usually because they do not know the correct rule or formula". In an attempt to improve on the negative wording of the phrase "it is usually because they do not know the correct rule or formula" the item can be reworded as follows: “Students struggle with a mathematics problem, because of a lack of knowledge of rules and formulas”.

With both items 1.6 and 1.7 the inclusion of a negative phrase causes the direction of the items to be opposite to the direction of the other items in the questionnaire, which could be the cause of the low alpha coefficient. Reverse scoring is the practice of reversing the data values for an item (or items), so that the statement becomes positive in relation to other items. The alpha coefficient improves significantly (from 0.232 to 0.466) if both items 1.6 and 1.7 are reverse scored. If item 1.6 is deleted, the alpha coefficient improves to a highest value of 0.513.

The alpha coefficient results led to the decision to omit item 1.6 and reverse score item 1.7, when analysing the questionnaire data of my study.

The validity of an instrument is the level to which the instrument measures what it was intended to measure (Pietersen and Maree, 2012a). The questionnaire was designed to determine students’ beliefs about mathematics and the teaching and learning of mathematics. Relevant questions (11 out of the 24 items) for the study were adapted from the questionnaire developed by Benadé (2013). Benadé’s questionnaire aimed to explore students’ (and lecturers’) beliefs about the nature of mathematics, and was peer revised by specialists in the field (Benadé, 2013) to establish the face validity of the instrument. An exploratory factor analysis was subsequently conducted to establish the construct validity of the questionnaire.

For the current study the conceptual framework – established through a thorough analysis of the literature – served as the compass for including 13 ‘new’ items in the questionnaire. The content validity of the questionnaire or the level to which the instrument measures all aspects of the targeted constructs (Pietersen and Maree, 2012a) was retained by having the instrument analysed by two experts in the field, and piloting the questionnaire. The pilot led to some questions being reworded in order to clearly distinguish between opposing viewpoints and is discussed in Chapter 5.

4.11.2 Credibility and trustworthiness. According to Nieuwenhuis (2012), the practices of multiple methods and triangulation increase the trustworthiness of qualitative research. In my study qualitative data are incorporated with quantitative data to address the research problem. The trustworthiness of the study is maintained through the process of multiple methods and triangulation. In my study I also attempt to clarify my bias by reflecting openly on how my interpretation can possibly be influenced by my dual roles as lecturer and researcher. The methods used to select possible participants or sampling methods also contribute towards the trustworthiness of the study.

4.12 Sampling

The research population consisted of 598 out of approximately 1300 students enrolled for WTW 134, a first year mathematics service module, presented to mostly biological science students in the first semester of 2018 at the University of Pretoria, with a prerequisite of 50% for mathematics in grade 12 (a student's final year in secondary education). The students enrolled for WTW 134 constituted four lecture groups lectured by four different lecturers, including my lecture group (598 students). I incorporated the Time-out sessions during the lectures of my lecture group.

The questionnaire was voluntarily completed at the beginning of the semester (Survey 1) by 271 WTW 134 students and at end of the semester (Survey 2) by 59 students from my lecture group. A total of 59 students had completed both Survey 1 and 2 and the data generated was used to analyse the reliability of the questionnaire. The sampling method is the non-probability method of convenience sampling (Maree and Pietersen, 2012a), where the participants of the study were conveniently accessible to the researcher.

Students from the university's Department of Mathematics and Applied Mathematics write two so-called semester tests (term tests) during the semester. The semester tests are formally scheduled within the faculty, written under examination conditions and contribute substantially (70% combined, in the case of WTW 134) to the semester mark (term mark) of a student. For the focus group interviews I used the probability sampling method of stratified random sampling (Maree and Pietersen, 2012a), where the strata were formed based on students' performance on the first of two semester tests (or term tests) written during the first semester of 2018. The students were allocated to the different strata proportionally; the proportions were calculated based on the performance of the population in the first semester test, and students from the various strata were invited to voluntarily participate. The reason for using Semester test 1 was mainly due to the fact that Semester test 2 was scheduled very late in the semester.

Table 4.5 is included to represent the different strata and the number of students invited from each stratum. To make provision for some students choosing not to respond, the number of students per stratum was increased by 40 percent. The students were invited by email to participate in focus group interview sessions; in total thirteen students agreed and participated in two separate focus group interview sessions.

Table 4.5: Stratifying the population and sample

Percentage for Semester test 1	Students in my lecture group (N=598)	Proportion of population	Number of students invited	Number of participants	Proportion of sample
(0,40)	139	0.2	15	4	0.3
[40,65)	361	0.6	38	6	0.5
[65,100]	98	0.2	10	3	0.2

The relevance, and possible limitations, of the size of the sample is discussed in Chapter 6, where the study is concluded.

4.13 Data Collection

When collecting data for the convergent parallel mixed methods design a possible issue is the size of the sample for both the qualitative and quantitative data (Creswell, 2014). The data for the qualitative data collection are most likely smaller than that for the quantitative data collection and the challenge, according to Creswell (2014), is to resolve this inequality. A strategy used by researchers is to not consider the difference in sample sizes, because the perspective of qualitative and quantitative methods is different and from each perspective a satisfactory interpretation is given (Creswell, 2014).

4.13.1 Phase 1 (Survey 1) and Phase 5 (Survey 2). For Phase 1 the questionnaire (discussed in Section 4.7) was posted on the Learning Management System (LMS) of the university four weeks after semester 1 commenced (Survey 1) and again during the penultimate week of the semester (Survey 2) to partially constitute Phase 5 of the study. Initially with Survey 1, all students of WTW 134 were encouraged to complete the questionnaire voluntarily, but for Survey 2 the students from my lecture group that also completed the questionnaire at the beginning of the semester, were earmarked.

4.13.2 Phase 4 (PRS data for first and second vote). For every Time-out session the bar chart of student responses for the first and second vote were recorded and compared, in order to constitute Phase 4 of the study and to be used in Phase 5 to validate every Time-out session and illustrate the effectiveness of the intervention.

4.13.3 Phase 5 (Focus group interviews). Thirteen students from my lecture group participated in one of two separate focus group interviews. The interviews were conducted one week before the end of the semester. In Table 4.5 the number of students that participated in the focus group interviews is identified per stratum, and the proportion of each stratum is represented. During the interviews my co-supervisor and Education Consultant in our faculty, Dr. CJ Louw, acted as moderator. The first focus group consisted of five students and the second of eight students. The interviews were audio recorded, for the purpose of being transcribed at a later stage. The motivation for the audio recording is to ensure that a realistic representation of the group interaction is captured for qualitative data analysis (Nieuwenhuis, 2012). From Table 4.5 it is evident that the number of students that voluntarily participated in the focus group interviews constituted the sample in proportions almost equivalent to the proportions of the identified strata of the population.

The purpose of focus group interviews is to utilise group dynamics to attain an in-depth understanding of participants' opinion of the relevant topic (Nieuwenhuis, 2012) or in the case of the current study, the Time-out sessions. The strategy behind the focus group interviews was to start with broader questions and systematically direct the participants' attention to the research question(s).

Qualitative data generated by the focus group interviews were triangulated with the quantitative data generated by the questionnaire to conclude the study.

4.14 Analysis of Data

In the convergent parallel mixed methods design, the data are merged by means of a "side-by-side" comparison, where the researcher first reports the quantitative statistical results and then deliberate the qualitative findings that either confirm or refute the statistical results (Creswell, 2014). To maintain the reliability and validity of the study discrepancies are reported.

4.14.1 Analysis of questionnaire data. For every student in the intervention a value was associated with each of the four options of the Likert-scale questionnaire. A value of 1 was associated with "strongly disagree", a 2 with "disagree", a 3 with "agree" and a 4 with "strongly agree". The three indexes, the Centredness Index (*C*), Mathematics learning Index (*M*) and Responsibility Index (*R*) were calculated from the data of Survey 1 and Survey 2 and are discussed in detail in Chapter 5.

To determine the influence of the Time-out sessions on student beliefs, descriptive statistics of the three beliefs indexes for Survey 1 were compared with the descriptive statistics of the three beliefs indexes calculated for Survey 2. Box and whisker plots of the three beliefs indexes for Survey 1 and 2, showed that the data was non-normally distributed, informing the decision to proceed with nonparametric statistical methods to confirm a possible shift in student beliefs. The nonparametric test for comparison of two variables in a single sample, the Wilcoxon signed rank test, was used to compare the Centredness Index (C), the Mathematics learning Index (M) and the Responsibility Index (R) for Survey 1 with that of Survey 2. With the Wilcoxon signed rank test "the null hypothesis is that the median of the difference scores is zero" (Pietersen and Maree, (2012b:231). In the case of all three indexes the hypotheses are one-sided, meaning we expect the particular index value to be higher after the intervention than before. Since the significance (two tailed) values for the three indexes, calculated by means of the Wilcoxon signed rank test, are for a two-sided hypothesis, where we expect the index values of before and after the intervention to differ, the significance (two-tailed) values are divided by two to represent the p-value for a one-sided hypothesis (Pietersen and Maree, 2012b). If the p-value for each of the three one-sided hypotheses is less than 0.05, then the hypotheses have a statistical significance of 0.05. The findings and statistical analysis of the questionnaire data of the current study are further discussed in Chapter 5.

4.14.2 Analysis of focus group interviews. The group interaction of the focus group interviews allowed for an in-depth probe into what students believe about mathematics teaching and learning. The recordings were transcribed by me in question-by-question format. Upon transcription topics central to the research questions were coded by means of open coding and themes were identified. The analysis of the qualitative data aimed to support the analysis of the quantitative data of the study and to answer the research questions about student perceptions and beliefs. It should be mentioned that the purpose of the focus group interviews as qualitative data was not to generalise findings, but to attain insight into students' attitudes and beliefs.

Both deductive and inductive methods were used to analyse the focus group interviews. For the deductive analysis the conceptual framework of the didactical contract in secondary and tertiary mathematics education was used to evaluate student beliefs categorically i.e. beliefs about the centredness of teaching/learning events, the nature of mathematics learning and the responsibility for student learning. After analysing the transcribed data by means of the abovementioned framework, the data were coded, in search of possible emerging themes not included in the conceptual framework.

4.15 Precis

Chapter 4 provides a detailed account of the research approach of my study. The research findings are elaborated in Chapter 5 against the five phases of the research design.

Chapter 5: Results and Discussions

In the previous chapters, the research problem is highlighted against the backdrop of teaching and learning in a first year mathematics classroom where students experience conflict because of a discontinuity in the didactical contract, when transitioning from secondary to tertiary mathematics education. It is explained that students' beliefs about mathematics teaching and learning are based on their beliefs about mathematics and founded on their experiences from secondary mathematics education. To assist students to successfully transition, teaching/learning opportunities must be created that challenge student beliefs, and allow the didactical contract to be renegotiated. The research approach of my study is to explore first year students' beliefs about mathematics teaching/learning, or the didactical contract in a first year mathematics classroom. The exploration aims to arrive at a better understanding of the didactical contract and strategies for negotiating the contract, in order to assist students to successfully transition from secondary to tertiary mathematics education. Creswell (2014) suggests using mixed methods, or triangulating the comprehensive findings of quantitative research with the detail of qualitative research to better understand the research problem.

The purpose of this chapter is to provide a comprehensive account of the results of my study. The following are discussed: Pilot study (5.1), Study outline and timeline (5.2), Preliminary analysis (5.3), Validating the Time-out sessions (5.4), Questionnaire indexes (5.5) and Focus group interviews (5.6)

5.1 Pilot Study

The Time-out session as initially conceptualised is described in Chapter 3, and mentioned in Chapter 4. In the first semester of 2017 (the year before conducting my study) I was teaching the same calculus module (WTW 134) to a group of 674 students, one of four different lecture groups taught by four different lecturers. I piloted my current study by incorporating Time-out sessions into one of my weekly lectures on three separate occasions during the first semester of 2017. These three sessions provided the basis for refining the concept of the Time-out session and are respectively referred to as Pilot 1, Pilot 2 and Pilot 3. Figure 5.1 is included to provide a timeline for the pilot study.

The incorporation of PRS sessions to create a student-centred teaching/learning environment focused on deep, conceptual (or instrumental) understanding and directed towards the students taking ownership of their learning, necessitated experimentation. The focus of Pilot 1 was to explore the effect of the first and second vote of the conceptualised Time-out session on student learning. For Pilot 2 the focus was the design of PRS questions for deep learning and for Pilot 3 the design of PRS questions for conceptual understanding.

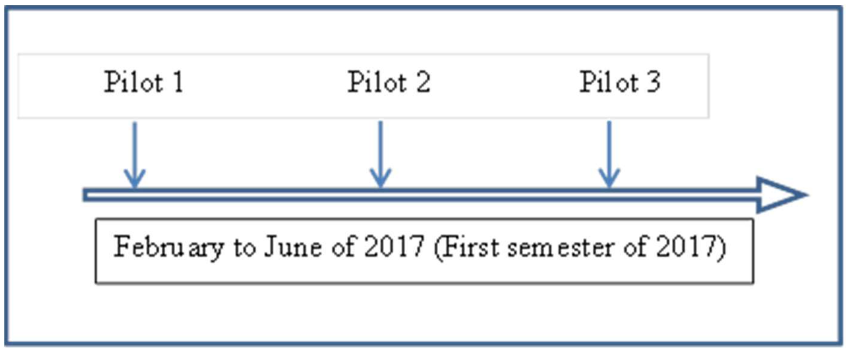


Figure 5.1: Timeline for the pilot studies

5.1.1 Pilot 1 (first versus second vote). Beatty and Gerace (2009:153) advocate that PRS questions are used to set the stage for a productive learning situation and not "just to assess previous instruction or gather data for future instruction". Four PRS questions were used at the beginning of the first semester of 2017 to gauge students' understanding after doing self-study of a study unit. The four questions were incorporated in a Time-out session, where time was allowed for peer discussion after the first and before the second vote. A summary of results is provided in the form of a table of correct response percentages for the second vote as compared to the first vote (see Table 5.1). The following was observed: The percentages of correct responses improved markedly from the first vote to the second vote. The percentages increased by between 20% and 32%.

For the second vote a fifth question, similar to the preceding questions, was included. The aim behind this 'new' question was to observe whether knowledge attained through the Time-out session was transferrable, and whether students would be able to apply the knowledge in a new context. Hence for Question 5 the students voted only once, and after one attempt 51% of the students had chosen the correct response.

Table 5.1: Percentage of correct responses of Time-out session called Pilot 1

	Percentages of correct responses	
Question	**First vote**	**Second vote**
Question 1	66%	92%
Question 2	69%	89%
Question 3	50%	72%
Question 4	40%	72%

The results of Pilot 1 allowed me to draw some insights about the intervention. In the context of the preceding study units the self-study unit did not pose real challenges, but served as an opportunity to recall and apply previously attained knowledge. The first two questions were based on rules of differentiation to be identified and utilised and the questions did not necessitate comprehensive calculations. Therefore, students supposedly used the peer discussion (or information about the majority vote provided by the bar chart of student responses) to determine the correct answers, but did not attempt to gain deeper understanding. A comparison of the results of Question 5 with the results of the other questions supports this claim. Knowledge supposedly attained did not prove to be immediately transferrable and surface learning prevailed, since the goal behind the first four questions was to assess whether students had done self-study and not to challenge and encourage student learning.

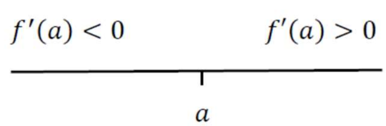
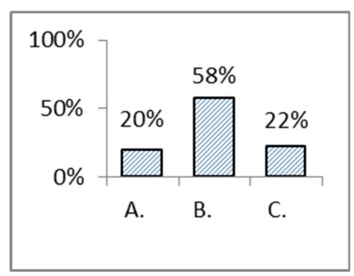
From my perspective as researcher, the students' behaviour changed the moment the Time-out session was initiated, which might be interpreted as the students experiencing a break in the didactical contract, but judging by the nature of their behaviour the majority of the students were not truly invested in the learning opportunity. A lesson learned was not to reveal the bar chart of student responses after the first vote, so that students do not have the option of following the majority vote. Also, the second vote could be followed by an inquiry as to what might have influenced a change in response the second time around.

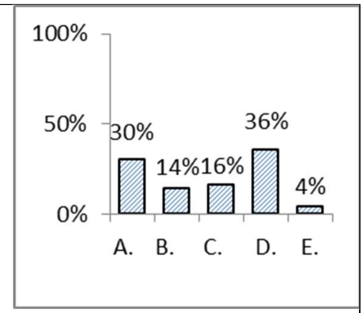
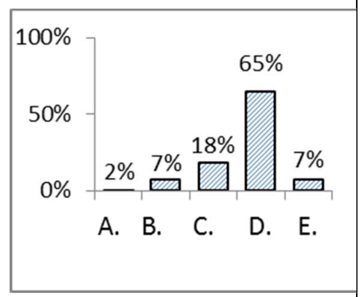
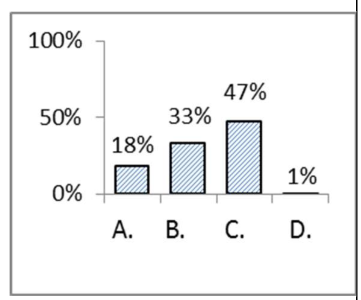
Most importantly, for learning to be meaningful, it is essential to design questions that challenge students and create conflict directed towards deeper learning. To quote Beatty and Gerace (2009:153) effective PRS questions can be used to "call students' attention to gaps in their understanding, raise dilemmas for them to wrestle with, and challenge the limits of their context-dependent knowledge".

5.1.2 Pilot 2 (designing questions for deeper learning). After conducting the Time-out session called Pilot 1, I shifted my focus to the design of challenging questions, aimed at encouraging deeper learning. The questions used in Pilot 2 were not incorporated into a Time-out session due to time constraints, but were used to constitute formative assessment.

The PRS session called Pilot 2 consisted of four questions that were integrated into the lecture without the pedagogical practise of a Time-out session. The questions were revealed, the students voted, and the bar charts of student responses were immediately revealed. The questions and bar charts of student responses are summarised in Table 5.2, with the correct answer to each question circled.

Table 5.2: Questions and results of Pilot 2

Question no	**Question** (The correct answer is circled)	**Bar chart of student responses**
1	Given the function $y = f(x)$. The domain of f is $(-\infty, \infty)$ and the domain of f' is $(-\infty,\ a) \cup (a, \infty)$. Use the following number line to answer the question:  Which statement(s) is (are) not true? $A.$ $x = a$ is a critical point $B.$ f has a local minimum at $x = a$ $C.$ $f''(a) > 0$	
2.1	Let $f(x) = 9\sqrt[9]{1 - x}$. Determine the domain of f. $A.$ $(-\infty, 1]$	

	$B.\ (-\infty, 1)$ $C.\ (-\infty, 1) \cup (1, \infty)$ $D.\ (-\infty, \infty)$ $E.$ None of the above	
2.2	Let $f(x) = 9\sqrt[9]{1-x}$. Find $f'(x)$. $A.\ f'(x) = -\sqrt[8]{1-x}$ $B.\ f'(x) = \sqrt[8]{1-x}C.\ f'(x) = \frac{1}{\sqrt[9]{(1-x)^8}}$ $D.\ f'(x) = -\frac{1}{\sqrt[9]{(1-x)^8}}$ $E.$ None of the above	
2.3	Let $f(x) = 9\sqrt[9]{1-x}$. Find the critical points of f. $A.$ No critical point $B.\ x = 1, f'(1) = 0$ $C.\ x = 1, f'(1)$ is not defined $D.$ None of the above	

The four questions are analysed against the design strategies recommended by researchers (Beatty, 2004, Beatty et al., 2006; Beatty and Gerace, 2009; Dangel and Wang, 2008 and Dufresne et al., 2005, Rubin and Rajakaruna, 2015). All four questions showed a wide distribution in the bar chart of student responses and seemingly provided opportunity for a meaningful class discussion (Beatty 2006). The distractors of all four questions include possible student mistakes and the option "none of the above" in Questions 2.1 to 2.3 provides for responses not considered (Beatty et al, 2006). Question 1 and Questions 2.1 to 2.3 challenge the students' "context-dependent knowledge" (Beatty and Gerace, 2009:153), since familiar questions are represented in a new context or using a 'new' function.

Question 1 aims to challenge students' understanding, or to create conflict, as recommended by Beatty et al (2006). Students' conceptual understanding is challenged when having to contemplate the existence of $f''(a)$ when $f'(a)$ is undefined, and when considering whether a function can have a critical point and a local minimum at $x = a$ without the function being concave up and/or $f''(x) > 0$. Also, students' understanding of mathematical notation is challenged by representing the domain of the derivative function as $(-\infty, a) \cup (a, \infty)$ instead of merely stating that f' is undefined at $x = a$. Question 1 complements Questions 2.1 to 2.3, since it represents the latter with only essential information included. This strategy of omitting unnecessary information is directed towards focusing student attention, as stated by Beatty (2004).

Dangel and Wang (2008) mention that for PRS questions to encourage higher-order student activity, questions must inspire students to apply, analyse, evaluate and create. The questions used in Pilot 2 collectively test the higher-order skill of evaluating, since students are expected to evaluate the critical point(s) of a 'new' function or a function not previously considered. Question 2 is broken down into multiple sub-questions or scaffolded, as recommended by Rubin and Rajakaruna (2015). The scaffolding of questions aims to support students' efforts without diminishing the conflict essential for understanding, as mentioned by Beatty and Gerace (2009). Using personal response systems to stimulate student activity at the level of creating (Dangel and Wang, 2008) would require students to use PRS technology to generate responses. As mentioned by Dangel and Wang (2008; 97) "PDAs, laptop computers, and even mobile phones offer tools for producing individually-created responses." Since most of the venues at the University of Pretoria are not currently equipped with Wi-Fi and the Mathematics Department decided to prescribe the least expensive PRS device or clicker, student activity through PRS use is currently restricted to recognising responses.

5.1.3 Pilot 3 (questions for conceptual understanding). For Pilot 3 five PRS questions designed with a specific learning objective in mind (as prescribed by Sullivan (2009)), were incorporated as part of a Time-out session, except that the bar chart of student responses was not revealed after the first vote. The first two questions aimed to test students' understanding of theoretical statements, and the last three questions tested the application of the concept theorised upon in the first two questions. Collectively the questions addressed the higher-order levels of analysing and applying; students were challenged to analyse a theoretical concept and apply the concept in terms of a scaffolded word problem. Table 5.3 provides a summary of the results in the form of a table of correct response percentages for the second vote compared to the first vote. From the results for Questions 1 and 3.1 to 3.3 it is evident that peer involvement in the Time-out session moved most students to choose the correct answers without knowing what the majority vote was (as was the case with Pilot 1). The same cannot be said about Question 2, since the wide distribution of the bar chart of student responses for the second vote revealed divided opinions about the correct response. The pedagogical goal behind Question 2 was to inspire students to analyse a general theoretical statement or rule, rather than merely accepting it at face value. Although the realisation of this goal is not apparent by means of the second vote results, the question did stimulate a meaningful class discussion about the underlying theory. The principles suggested by Crouch and Mazur (2001) for using personal response in combination with peer discussion are mentioned in Section 5.4.2 and were used to determine the effectiveness of the PRS questions of the current study. According to the principles of Crouch and Mazur (2001), the low percentage of correct responses for Question 2 could indicate an ambiguous question.

Table 5.3: Percentage of correct responses of Time-out session called Pilot 3

	Percentages of correct responses	
Question	**First vote**	**Second vote**
Question 1	48%	69%
Question 2	17%	37%
Question 3.1	78%	92%
Question 3.2	40%	76%
Question 3.3	81%	94%

To gauge whether the learning objective behind the Time-out session was realised, a multiple-choice test, consisting of two questions and based on the abovementioned learning objective, was designed to constitute the following week's class test. Students used their clickers or personal response devices to answer the test questions. Table 5.4 provides a summary of the average of correct response percentages for both questions, distinguishing between the 302 students that participated in the Time-out session or

Pilot 3 (called Participants) and the 372 students that did not attend my lectures and as a result did not participate in Pilot 3 (called Non-participants).

Table 5.4: Average percentages of correct responses of class test used to assess Pilot 3

Average percentages of correct responses	
Participants	**Non-participants**
48% (n=302)	47% (n=372)

From Table 5.4 it is evident that the Participants group did not outperform the students Non-participants group, but that the performance of the two groups correlate. The conclusion is that the Time-out session of Pilot 3 was not necessarily more (or less) effective in encouraging deeper learning. Students were informed beforehand that they would be subjected to a so-called Clicker class test, where they are required to answer questions by means of their personal response devices or clickers. Since all of the previous Clicker class tests were only used to record class attendance and not to assess student performance, the students' preparation may have been inadequate. According to Gibbs (2006), students value assessment only insofar as it has potential to generate marks. In the case of my study, the students undervalued the class test since they knew that it would not contribute substantially towards their marks.

The significance of Pilot 3 is threefold. Firstly, I recognised the importance of designing PRS questions to support learning objectives. Secondly, for students to value the Time-out sessions, their potential value for future assessments will have to be emphasised. Thirdly, the weekly Time-out sessions will have to be incorporated continuously and from the beginning of the semester. An isolated Time-out session (Pilot 3) incorporated near the end of a semester cannot provide reliable information about the value of the Time-out sessions for self-directed learning nor encourage students to take responsibility for their own learning. According to Hourigan and O'Donoghue (2007), a critical paradigm shift (focused on deeper learning) is needed for students to realise the importance of effort on their part as part of their approach to learning. Hence, in order to cause a shift in responsibility, the prominence and longevity of a new teaching/learning model is of utmost importance.

Sullivan (2009:345) references Beatty et al. (2006), who viewed the development of PRS questions as a continuous process of "writing, revision and review". All three pilot studies showed that the design of effective PRS questions is both time consuming and requiring continuous revision and refinement. It should also be mentioned that it became evident that strategies must be put in place to ensure that time allocated to Time-out sessions is used productively, and that the value of the session should be increased by, for example, providing each student with paper copies of the PRS questions.

The three pilot sessions helped me to grasp the characteristics of meaningful PRS questions that aim to encourage conceptual understanding and deeper learning. The pilot sessions also helped refine the concept of the first and second vote to constitute the Time-out sessions in a meaningful way. The main realisation was that in order to influence student responsibility and beliefs, Time-out sessions should form part of the teaching and learning approach throughout the semester.

5.1.4 Reflections on the intervention. At the end of semester 1 of 2017, the design of the intervention as a whole had to be reconsidered against the backdrop of the pilot study of the Time-out sessions.

One of the main objectives of the study, to move students to accept responsibility for their own learning juxtaposed with the students' perceived lack of commitment in Pilot 3, led me to contemplate a learning environment in which the students are motivated to take responsibility for their own learning.

Vicens (2017) advocates the use of personal response systems in a flipped classroom environment, where he adopts the model of peer instruction of Eric Mazur (Lambert, 2012) to engage students. According to researchers (Cronhjort et al., 2018; Love et al., 2014), a flipped classroom represents a classroom where the initial learning of core concepts takes place outside the classroom, and the classroom is used for interactive learning activities. Students learn outside the classroom by utilising educational resources, online or not, by watching educational videos, or reading up on the content. Love et al. (2014) mention that in the flipped classroom students are expected to be active and autonomous and take control of their own learning.

In order to guide students to take responsibility for their own learning, I decided to amend the Time-out sessions as follows. Once a week, the students have to prepare the content relevant to one or more learning objectives, using the textbook or prescribed online videos to complete a preparatory worksheet. The lecture then comprises a Time-out session of approximately 15 to 20 minutes, after which another worksheet is handed out to be completed and checked against a memorandum in the students' own time, which is not discussed in class. After the Time-out session the traditional mode of lecturing is resumed; in this way the traditional transmission-mode classroom is flipped once a week so that students are compelled to take control of their own learning.

Lucas (2009) combines peer instruction with personal response systems to create an interactive learning environment in his Calculus classroom. Similar to the model used by Vicens (2017), students respond to a multiple-choice question using their personal response systems, after which they pair up to discuss their answers and vote for a second time. He reveals the bar chart of student responses after the first vote, but only reveals the correct answer after the second vote, when an explanation is also provided.

I decided to not reveal the bar chart of student responses after the first vote, in order to prevent students from conforming to the majority vote without re-attempting the problem. The decision was also made to provide students with either paper copies of the PRS questions and/or post the questions online after the Time-out session, because students expressed the need to be able to revise the PRS questions and answers. Elton (1996) stresses the importance of the lecturer paying attention to students' sense of preparedness for the examination, as mentioned in Chapter 2.

The aim of the current study is to use the intervention of regular Time-out sessions distributed throughout the semester, to influence students' beliefs about mathematics and the teaching/learning of mathematics, and as a result renegotiate the didactical contract. The pilot of the questionnaire used to gather information about students' beliefs, is discussed.

5.1.5 Pilot of the questionnaire. The questionnaire was piloted after the Time-out sessions, while teaching the same applied calculus module to a different group of 364 students in the second semester of 2017. The responses of 151 students are summarised against the three main categories of the questionnaire i.e. Centredness in Table 5.5.1, Mathematics learning in Table 5.5.2 and Responsibility for learning in Table 5.5.3.

Table 5.5.1: Pilot of questionnaire-Questions on Centredness

Category	**Question number**	**Percentage of students that agreed**	**Percentage of students that disagreed**	**Summary of question**
Lecturer-centred	2.2	94%	5%	Lecturer demonstrates the correct method
	2.3	93%	5%	Student listens
	2.4	84%	15%	Lecturer does not leave students confused
	2.5	96%	1%	Lecturer conveys knowledge
	2.11	84%	13%	Problems should be easily solved in class time
Student-centred	1.11	71%	27%	Students battle
	2.1	76%	22%	Students solve without the lecturer
	2.8	82%	16%	Lecturer supports students to discover concepts
	2.9	92%	5%	Students are encouraged to actively solve problems
	2.10	92%	5%	Lecturer encourages students to find various ways to solve problems

Table 5.5.2: Pilot of questionnaire-Questions on Mathematics learning

Category	**Question number**	**Percentage of students that agreed**	**Percentage of students that disagreed**	**Summary of question**
Emphasis on instrumental understanding	1.1	75%	25%	Mathematics a set of facts, rules and formulas
	1.6	35%	65%	Rules accepted without explanation
	1.7	52%	47%	Students struggle because they do not know the correct rule or formula
Emphasis on relational understanding	1.3	95%	3%	A student must understand concepts behind rules and formulas
	1.4	88%	11%	Mistakes repeated indicate a lack of understanding
	1.9	80%	18%	To be able to explain is more important

Table 5.5.3: Pilot of questionnaire-Questions on Responsibility for learning

Category	**Question number**	**Percentage of students that agreed**	**Percentage of students that disagreed**	**Summary of question**
Lecturer responsible for learning	1.12	61%	37%	Student should pass if student attends classes
	2.6	85%	12%	Lecturer is responsible for learning through effective teaching
	2.7	90%	7%	Lecturer conveys the importance of knowledge for examination purposes
	2.12	82%	16%	Lecturer lightens the burden of learning
Student responsible for learning	1.2	91%	8%	Responsibility to attend all classes
	1.5	92%	7%	Responsibility to prepare before class
	1.8	88%	12%	Responsibility to clarify confusion
	1.10	97%	2%	Responsibility to engage continuously

In the case of all 24 questions, 75 percent of the students or more agreed with the relevant statement, except with Questions 1.6, 1.7 and 1.12. As mentioned earlier Question (or item) 1.6 was eventually omitted from the questionnaire.

Reflecting on the questionnaire results, I realised that some statements need to be rephrased in order to clearly distinguish between opposing viewpoints. For example, Question 2.8 aims to gauge whether students regard a mathematics classroom as student-centred. If the statement is reworded, the focus shifts from the role of the lecturer to the role of the student: Instead of "In the teaching of mathematics, lecturers should support students to discover concepts for themselves" the statement should read

"In the teaching of mathematics, the students have to discover concepts for themselves, while the lecturer provides support." Questions 2.9 and 2.10 can be similarly rephrased, as shown in Table 5.6. To oppose the statement in Question 2.6, "The lecturer is responsible for the student's learning of mathematics through effective teaching", Question 1.2 is reworded as "The student is responsible for his/her learning", while Questions 1.2 and 1.5 are consolidated to constitute a new Question 1.5. Table 5.6 provides a summary of the amended questions.

The questionnaire of the current study aims to gauge student beliefs about the Centredness (1), Mathematics learning (2) and Responsibility for learning (3) in the mathematics classroom (see Table 3.2). I realised after the pilot that the percentage responses shown in Tables 5.5.1, 5.5.2 and 5.5.3 do not provide a clear picture of student beliefs and that I have to find a way to analyse the data provided by the questionnaire in a meaningful way.

In order to evaluate how a student values peer instruction compared to being instructed by a lecturer, Lucas (2009) calculates the student's Learning Index, based on their answers to questions posed in a questionnaire. He defines the Learning Index L as the ratio $\frac{ss}{is}$, where *ss* represents the student's rating of peer instruction (student-student) and *is* represents the student's rating of lecturer instruction (instructor-student). The scores for both *ss* and *is* were calculated from the student's answers to three questions each about peer instruction and lecturer instruction, e.g. "does the instructor's lecturing help you learn Calculus?" (Lucas, 2009:223). As an answer to the question the student had to choose a number between one and five, one implying "unhelpful" and five "very helpful". If the student's Learning Index is less than one, then $ss<is$ and the student values the instruction of the lecturer more than peer instruction. If the index is bigger than one (or $ss>is$), the opposite is true.

Table 5.6: Statements amended after pilot of questionnaire

Question number	Initial question	Amended question
1.2	It is the responsibility of the student to attend all mathematics classes.	The student is responsible for his/her learning.
1.5	It is the responsibility of the student to prepare before attending mathematics classes.	By attending and preparing for classes, the student takes responsibility for his/her learning.
2.8	In the teaching of mathematics, lecturers should support students to discover concepts for themselves.	In the teaching of mathematics, the students have to discover concepts for themselves, while the lecturer provides support.
2.9	Mathematics is best taught if students are encouraged to actively solve problems in class.	Mathematics is best learned if the students actively solve problems in class.
2.10	Lecturers should encourage students to find various ways to solve problems.	In the mathematics classroom, students should be encouraged to find various ways to solve mathematics problems.

The scale used by Lucas (2009) to rate instruction inspired me to use the questionnaire data from the pilot study and calculate three indexes, the Centredness Index (C), Mathematics learning Index (M) and Responsibility Index (R) to measure students' beliefs about Centredness (1), Mathematics learning (2) and Responsibility for learning (3). I decided to associate specific values with the four options of the Likert scale, a one with "strongly disagree", a two with "disagree", a three with "agree" and a four with "strongly agree", as mentioned earlier. From Table 5.5.1 it is clear that five questions relate to the mathematics classroom being perceived (by the student) as student-centred and five questions relate to the classroom being perceived as lecturer-centred. The ratio $\frac{sc}{lc}$ then represents the Centredness Index (C) of a student, where *sc* is the score calculated from the student's responses to the student-centred questions, and *lc* is the score calculated for the lecturer-centred questions. In the same way the Mathematics learning Index *(*M) is the ratio $\frac{ru}{iu}$ with *ru* the score for relational understanding and *iu* representing the score for instrumental understanding. From Table 5.5.2 it is evident that regarding the Mathematics learning category, three questions each relate to instrumental and relational understanding. The Responsibility Index (R) employs the ratio $\frac{sr}{lr}$ where *sr* indicates the score for student responsibility and *lr* the score for lecturer responsibility. Four questions of the questionnaire relate to student responsibility and four to lecturer responsibility (Table 5.5.3). Table 5.7 provides a summary of the three indexes and their three pivotal values. In the case of the Centredness Index (C), a value less than one (C < 1) is a clear indication that students' beliefs about a mathematics classroom lean towards the classroom being lecturer-centred, since *sc* < *lc*. If the Mathematics learning Index (M) is less than one (M < 1), then *ru* < *iu,* which indicates that students believe mathematics learning to be focused on instrumental understanding. A Responsibility Index (R) of less than one (R < 1) relates to the belief that the responsibility for learning is that of the lecturer, because *sr* < *lr*.

Table 5.7: Students' beliefs indexes

Index	**Pivotal values**
Centredness Index (C)= $\frac{sc}{lc}$ with sc =student-centred and lc =lecturer-centred	1
Mathematics learning Index (M)= $\frac{ru}{iu}$ with ru =relational understanding and iu =instrumental understanding	1
Responsibility Index (R)= $\frac{sr}{lr}$ with sr =student responsibility and lr =lecturer responsibility	1

In Table 5.8 the averages of the three indexes – as calculated for the students involved in the pilot study of the questionnaire – are summarised. The average index values can be interpreted as the students believing a mathematics classroom to be lecturer-centred (C=0.96), mathematics learning to be relational understanding (M=1.32) and learning to be the responsibility of the student (R=1.12). One can only speculate that the three index values for students in the second semester of their first year might differ from that of first year students beginning the first semester, since second semester students have had the opportunity to adapt and to close the gap between secondary and tertiary education. Further statistical analysis is essential for a meaningful interpretation of the index values and conclusion of the current study.

Table 5.8: Averages of students' beliefs indexes

Index	Average	Implications of the result
Centredness (C)	0.96 (C < 1)	Lecturer-centred
Mathematics learning (M)	1.32 (M > 1)	Relational understanding
Responsibility (R)	1.12 (R > 1)	Responsibility of the student

It should be mentioned here that for the purpose of standardising the questionnaire, the Cronbach's alpha coefficients for the three categories of the questionnaire were calculated based on the data of Survey 1, collected in March 2018 (see Table 4.4). Based on a low alpha coefficient value for the Mathematics learning category, it was decided to omit Question 1.6 when analysing the questionnaire data. The interpretation of the Mathematics learning Index (M) in which Question 1.6 is omitted necessitates an exposition. Since three questions describe mathematics learning as relational understanding and a further two questions describe mathematics learning as instrumental understanding as explained earlier in Table 5.2.2, the pivotal value for the Mathematics learning Index (M) becomes 1.5, instead of 1. To explain: if students agree with all three statements pertaining to relational understanding (*ru*) and the two statements pertaining to instrumental understanding (*iu)*, hence demonstrating no bias, then $M = \frac{ru}{iu} = \frac{9}{6} \approx 1.5$. A bias towards relational understanding would be visible if at least $M = \frac{ru}{iu} = \frac{10}{6} \approx 1.\dot{6}$, whereas a bias towards instrumental understanding can be construed if $M = \frac{ru}{iu} = \frac{9}{7} \approx 1.285$. Table 5.9 is included as an updated version of Table 5.7, to summarise the three indexes used to analyse the questionnaire data of my study.

Table 5.9: Students' beliefs indexes

Index	**Pivotal values**
Centredness Index (C)= $\frac{sc}{lc}$ with sc =student-centred and lc =lecturer-centred	1
Mathematics learning Index (M)= $\frac{ru}{iu}$ with ru =relational understanding and iu =instrumental understanding	1.5
Responsibility Index (R)= $\frac{sr}{lr}$ with sr =student responsibility and lr =lecturer responsibility	1

The timeline of my study is discussed in the next section.

5.2 Study Timeline

In Chapter 4 the phases of the study are explained, and Figure 4.3 is referenced here to provide a timeline for the current study. I conducted the study from February to June 2018, while teaching Applied Calculus (or WTW 134) to first year students mostly enrolled in biological sciences at the University of Pretoria. The questionnaire was posted on the Learning Management System (LMS) at the beginning of the first semester of 2018 (Survey 1), to constitute Phase 1 of the study and gauge students' beliefs about mathematics and mathematics teaching and learning.

The intervention took the form of six consecutive Time-out sessions that were each designed (Phase 2), implemented (Phase 3) and assessed (Phase 4) throughout the semester. The objective was to incorporate regular Time-out sessions (at least one session per week) but pending tests and time restrictions allowed me to introduce only a total of six Time-out sessions throughout the semester. The first three Time-out sessions were incorporated into one of the four weekly lectures for three consecutive weeks at the beginning of the semester (28 February, 7 March, 14 March) and again for two consecutive weeks during the second term of the semester (18 April, 25 April). The last Time-out session was incorporated on 16 May, two weeks before the end of the semester. The relevant learning objectives were used to design a preparatory (pre-class) worksheet, an LMS test, PRS questions based on the pre-class worksheet and a second worksheet as part of each Time-out session. This process which formed part of every

Time-out session constituted Phase 2 or the *a-priori* phase of the study. The principles identified in Pilot 2 were applied in the design of deep and challenging PRS questions, most importantly the principle of potential student mistakes as distractors. The students had to prepare for a Time-out session by completing the pre-class worksheet and writing the test on the Learning Management System (LMS) the day before the lecture. During the lecture the Time-out session comprised voting for PRS questions, peer discussion, voting for a second time and a brief discussion by the lecturer to conclude the session. The second worksheet was then handed out to be completed and checked against a memorandum in the student's own time, but not to be discussed in class. As mentioned after Pilot 3, the aim of the Time-out sessions was to encourage students to take responsibility for their own self-directed learning. The second worksheet was included for this purpose.

The effectiveness of every Time-out session was judged based on a comparison of the data generated by the second vote as compared to data from the first vote (Phase 4). If the percentage of students choosing the correct response increased to more than 70% from the first to the second vote, then the Time-out session was deemed successful in encouraging deep conceptual learning, as noted by Crouch and Mazur (2011).

The questionnaire (Survey 2) was reposted after the intervention at the end of the semester, to be completed by students that had also completed the questionnaire at the beginning of the semester (Phase 5). Around the same time focus group interviews were conducted with thirteen students who participated in the intervention, to contribute to Phase 5 of the study. The three indexes, as explained in Table 5.9, were calculated for both sets of questionnaire data, generated by Survey 1 and Survey 2. The focus group interviews were transcribed, and themes were identified and coded. In this chapter, statistical analysis of the questionnaire data or indexes (quantitative data) and data from the focus group interviews (qualitative data) are triangulated to elaborate the results of the study and conclude the validation phase (Phase 5) of the study. In Table 5.10, the five phases of the study, as discussed in this chapter, are outlined.

Table 5.10: Five phases of the study

Section		Results	Phases
5.3		Preliminary analysis	Phase 1
5.4.1	Validating the Time-out sessions	PRS questions for learning	Phase 2
5.4.2		Implementation	Phase 3
		Comparing first and second vote	Phase 4
5.4.3		Validation	Phase 5
5.5	Validation of the study	Questionnaire indexes	Phase 5
5.6		Focus group interviews	
5.7		Triangulation	

5.3 Preliminary Analysis

Table 5.11 summarises the responses of the 59 students that completed Survey 1, responding to the 23 statements in the questionnaire (from which Question 1.6 was omitted). The percentages of students that agreed or disagreed with each statement are specified. Students mostly disagreed with the statements in Question 1.7, Question 1.12 and Question 2.8. The 54% disagreement with the statement in Question 1.7 can be attributed to the ambiguity of the question, as discussed earlier (see Section 4.11.1), and will not be further elaborated on. A majority (56%) of students disagreed with the statement in Question 1.12, "If a student attends all the classes, he/she should pass mathematics", which could be interpreted as the majority of students realising their responsibility towards learning outside the classroom. Almost half of the students (46%) disagreed with the statement in Question 2.8, "In the teaching of mathematics, the students have to discover concepts for themselves, while the lecturer provides support", which might be an indication that these students view the mathematics classroom to be lecturer-centred.

Table 5.11: Percentages for student responses to questionnaire (Survey 1)

Question	Keywords	Agree (%)	Disagree (%)
CENTREDNESS			
	Student-centred		
1.11	Students battle with mathematics	80	20
2.1	Students find methods to solve problems without the help of the lecturer	78	22
2.8	Students discover concepts, while the lecturer provides support	54	46
2.9	Mathematics is best learned if the students actively solve problems in class	88	12
2.10	Students should be encouraged to find various ways to solve problems	92	7
	Lecturer-centred		
2.2	The lecturer demonstrates the correct method	92	8
2.3	The student needs to listen carefully to the lecturer’s explanations	93	5
2.4	The lecturer does not leave the students confused	75	25
2.5	The lecturer conveys knowledge	95	5
2.11	Problems easily solved in class time	81	19

Question	Keywords	Agree (%)	Disagree (%)
MATHEMATICS LEARNING			
	Relational understanding		
1.3	A student must understand concepts behind rules and formulas	90	10
1.4	Mistakes repeated indicate a lack of understanding	80	20
1.9	To be able to explain answers is more important than correct answers	75	25
	Instrumental understanding		
1.1	Mathematics is a set of facts, rules and formulas	78	22
1.7	Students struggle because they do not know the correct rule or formula	46	54
RESPONSIBILITY			
	Student responsibility		
1.2	Student is responsible for his/her learning	93	7
1.5	By attending and preparing for classes the student takes responsibility	95	5
1.8	Student's responsibility to clarify confusion	86	14
1.10	Student's responsibility to engage continuously with mathematics	93	7
	Lecturer responsibility		
1.12	If a student attends all the classes, he/she should pass	44	56
2.6	Lecturer is responsible for learning through effective teaching	88	12
2.7	Lecturer conveys the importance of knowledge for examination purposes	92	8
2.12	Lecturer lightens the burden of learning	88	12

Table 5.12 summarises the average values of the three questionnaire indexes calculated (as detailed in Table 5.9) for each of the 59 students who participated in Survey 1. Using Table 5.9 as a guide to interpret index values, the following is observed. For the Centredness Index (C) a value of 0.94 ($C < 1$) indicates a bias towards a lecturer-centred classroom. This comment is in line with the earlier observation made about Question 2.8. An average value of 1.10 for the Responsibility Index ($R > 1$) can be interpreted as students believing that learning is their responsibility, as deduced earlier from student responses to Question 1.12. For the Mathematics learning Index (M), an average value of 1.83 ($M > 1.5$) can be interpreted as students having beliefs about mathematics learning characterised by relational understanding.

To further elaborate student beliefs about the centredness of the classroom, Questions 2.4 and 2.8 are juxtaposed. As mentioned earlier, one can infer from student responses to Question 2.8 that almost half of the students prefer a lecturer-centred classroom. Three quarters (75%) of the students agreed with the statement in Question 2.4, “a good lecturer does not leave students to experience confusion”, which can be interpreted as students believing that the lecturer is not allowed to leave students confused or conflicted. According to researchers (Bransford et al., 2000; Clark and Lovric, 2009 and Kislenko, 2005), learning or conceptual change takes place when students experience conflict in the learning process, and that it is the task of the lecturer to create opportunities for conflict. The central theme when analysing Questions 2.4 and 2.8 together, is that first year students prefer a lecturer-centred learning environment where they can remain passive without being challenged.

Table 5.12: Averages of students’ beliefs indexes (Survey 1)

Index	Average	Criteria
Centredness (C)	0.94	$C < 1$
Mathematics learning (M)	1.83	$M > 1.5$
Responsibility (R)	1.10	$R > 1$

Although the sample size prevents me from generalising my results to the broader student population, the results from Survey 1 provide me with a baseline assessment of student beliefs against which to measure the impact of the intervention. The quality of the intervention determines the success of the study hence a discussion of Time-out sessions as a meaningful intervention is motivated.

5.4 Validating the Time-out sessions

According to Margolinas and Drijvers (2015), the data informing the *a-priori* analysis (Phase 2) forms the basis for the *a-posteriori* analysis (Phase 4), and together these phases form the basis for the validation of Time-out sessions as an intervention. Before validating the Time-out sessions – by comparing the results for the first and second votes of every Time-out session (Phase 4) – the design of PRS questions that will achieve the purposes of relational understanding, deep learning and higher-order reasoning, has to be briefly discussed (Phase 2).

5.4.1 PRS questions for learning. For the Time-out sessions, PRS questions were designed with several goals in mind: a content goal (defined by a targeted learning outcome or outcomes); a process goal (defined by the process or procedure associated with the content goal), and a metacognitive goal (defined by the awareness of learning). This process constituted Phase 2 of the study and was used to ensure that the questions were directed towards conceptual and procedural understanding, or relational understanding. The principle of potential student mistakes as distractors were applied in the design of all PRS questions, to further promote deep learning and higher-order reasoning. Question 3 from Time-out session 4 is summarised in Figure 5.2 and its potential for promoting deep learning and higher-order reasoning is discussed. The PRS questions of all the Time-out sessions are then briefly analysed for their potential to promote deep learning and higher-order reasoning.

For Time-out session 4 the preparatory worksheet covered examples of left-hand sums as underestimates and right-hand sums as overestimates of total change, with the rate of change function given as a table of function values, and the function increasing on the given interval. For Question 3 of Time-out session 4, the rate of change function is again represented by means of a table of function values, but the function is both increasing and decreasing on the given interval. In Question 3 a 'new' function is used, and the context is changed so that the question challenges students to apply their conceptual and procedural knowledge. Possible student mistakes are presented by the distractors. Question 3 has potential to stimulate higher-order thinking and deeper learning, because students are encouraged to apply their knowledge. A conclusive class discussion on student mistakes can further support this purpose.

Time-out session 4: Accumulated change

Content goal/Learning objective:

Estimate total change if the rate of change is given.

Process goal:

Use Riemann sums to estimate total change.

Metacognitive goal:

Utilise strategies to comprehend and visualise Riemann sums.

Question 3

An old row boat has sprung a leak. Water is flowing into the boat at a rate, $r(t)$, given in the following table.

t (in minutes)	5	10	15	20
$r(t)$ (in liters per minute)	12	20	24	16

Calculate a lower estimate for the water that flowed into the boat during the 15 minutes.

A	5(12+20+24) liters
B	5(12+20+16) liters
C	5(20+24+16) liters
D	5(12+24+16) liters

Figure 5.2: Question 3 from Time-out session 4

According to Dangel and Wang (2008), personal response questions should be designed to promote higher-order thinking. Thompson (2008) explains that higher-order thinking is utilising knowledge in new contexts, whereas lower-order thinking implies working in familiar contexts. Dangel and Wang (2008) associate the cognitive learning outcomes – understand, apply, analyse, evaluate and create – as proposed by Anderson and Krathwohl (2001), with higher-order thinking and deeper learning. Anderson and Krathwohl revised Bloom's taxonomy, a framework for classifying learning objectives, in

2001 and redefined the objectives into active verbs, appropriately called the revised Taxonomy (Krathwohl, 2002). According to Krathwohl (2002:215), to understand is to determine "the meaning of instructional messages", to apply is to utilise "a procedure in a given situation" and to analyse is to decompose content "into its constituent parts", identifying "how the parts relate to one another".

In Table 5.13 the PRS questions used in the six Time-out sessions of the intervention are categorised according to the revised Taxonomy of Anderson and Krathwohl (2001). The PRS questions collectively stimulated higher-order thinking and deeper learning, by encouraging students to understand, apply and analyse content. Since the revised Taxonomy is a hierarchy of complexity, it can be said that all the PRS questions facilitate relational understanding.

Table 5.13: PRS questions categorised against the revised Taxonomy of Anderson and Krathwohl

	The cognitive learning outcomes (Krathwohl, 2002)					
	Remember	**Under stand**	**Apply**	**Analyse**	**Evaluate**	**Create**
Time-out 1			Questions 1 to 6			
Time-out 2		Question 1	Questions 2 to 5	Question 6		
Time-out 3			Questions 1 to 5			
Time-out 4			Questions 1 to 4			
Time-out 5			Questions 1 to 3			
Time-out 6			Questions 1 to 5			

The six Time-out sessions that together constituted the intervention are now analysed by comparing data from the first and second vote for each Time-out session.

5.4.2 Comparing first and second vote. The data from all six of the Time-out sessions are summarised in Tables 5.14 to 5.20. For each Time-out session the topic of the lecture, percentage of students that indicated that they prepared for the lecture, number of PRS questions and the cognitive level(s) of the questions are stated. The percentages of correct responses for the first and second votes are then highlighted per question.

Before each Time-out session is briefly discussed, principles mentioned by Crouch and Mazur (2001) for using personal response systems in combination with peer instruction for the teaching of calculus- and algebra-based introductory physics courses are mentioned. According to Crouch and Mazur (2001), the quality of the questions is vital for success. Questions should aim to explore essential concepts and reveal shared difficulties or typical student mistakes. Questions must also challenge the students, without being too difficult. If less than 35% of the students choose the correct answer for the first vote, then the quality of the question should be questioned as most probably being ambiguous. On the other hand, if more than 70 % of the students choose the correct answer, further discussion of the question is not necessary.

Table 5.14: Data from first and second vote of Time-out 1

Time-out 1: Periodic functions **Students prepared for the lecture**: 86% **Number of PRS questions:** 5 **Cognitive learning outcomes:** Apply conceptual knowledge		
Percentage of correct responses		
	First vote	**Second vote**
Question 1	92	96
Question 2	74	84
Question 3	53	Problem with polling
Question 4	83	94
Question 5	65	80

For the first vote of the first Time-out session (Table 5.14) the smallest percentage of correct responses was 53% for Question 3. As is often the case with technology, some problems were experienced with polling Question 3 for the second vote, but the first vote of 53% qualifies Question 3 as unambiguous, according to the principles of Crouch and Mazur (2001). For the second vote the smallest percentage of correct responses was 80% for Question 5, indicating successful questions according to the principles of Crouch and Mazur (2001).

Table 5.15: Data from first and second vote of Time-out 2

Time-out 2: Instantaneous rate of change **Students prepared for the lecture**: 88% **Number of PRS questions:** 6 (Question 6 only included as part of second voting session) **Cognitive learning outcomes:** Understand (Question 1), apply and analyse (Question 6) conceptual knowledge		
Percentage of correct responses		
	First vote	**Second vote**
Question 1	45	76
Question 2	34	49
Question 3	91	98
Question 4	58	74
Question 5	82	81
Question 6	Deliberately not polled	63

A sixth question was included in Time-out session 2 (Table 5.15), to be polled only during the second vote. The question was designed with the purpose of concluding the session and adding to the difficulty level of the PRS questions. Based on the principles of Crouch and Mazur (2001) Questions 1, 3, 4 and 5 qualified as meaningful questions, since 45% or more of the students chose the correct response with the first vote and 74% or more chose the correct response with the second vote. It can be reasoned that Question 2 was ambiguous, since only 34% of the students chose the correct answer for the first vote. Question 6 proved to be reliable, since the percentage of correct responses for the once off vote is 63%.

Time-out session 3 consisted of six PRS questions (Table 5.16), of which only five were used due to time constraints, and proved equally successful. A problem with technology prevented the lecturer from polling Question 2 as part of the first vote, but 96% of the students chose the correct answer when it was included in the second vote. For the other four questions, the lowest percentage of correct responses during the first vote was 53% and the lowest percentage for correct responses during the second vote was 82%. It became evident during Time-out session 3 that because students worked through a worksheet of some introductory examples, a meaningful discussion of a higher difficulty level was possible in the lecture.

Table 5.16: Data from first and second vote of Time-out 3

Time-out 3: Differentiation formulas for polynomials and power functions **Students prepared for the lecture**: 92% **Number of PRS questions:** 5 (6 planned, no time for Question 6) **Cognitive learning outcomes:** Apply conceptual knowledge		
Percentage of correct responses		
	First vote	**Second vote**
Question 1	67	84
Question 2	Problem with polling	96
Question 3	93	98
Question 4	53	90
Question 5	58	82

Between Time-out session 3 and 4 the lecturer took a break from the Time-out sessions, since Semester test 1 was scheduled during that time. Semester test 1 was followed by the university recess, so Time-out session 4 (Table 5.17) was incorporated approximately four weeks after Time-out session 3.

Based on the principles of Crouch and Mazur (2001) the first two questions qualify as successful questions, but Questions 3 and 4 do not necessarily qualify based on the low percentages for correct responses. With the first vote, 39% of students chose the correct answer to Question 3, but with the second vote this percentage decreased to 37%.

Table 5.17: Data from first and second vote of Time-out 4

Time-out 4: Accumulated change (an introduction to integration) **Students prepared for the lecture**: 86% **Number of PRS questions:** 4 (5 planned, no time for Question 5) **Cognitive learning outcomes:** Apply conceptual and procedural knowledge		
Percentage of correct responses		
	First vote	**Second vote**
Question 1	90	97
Question 2	64	95
Question 3	39	37
Question 4	22	53

Table 5.18 is included to provide information about the distribution of student response percentages to Question 3 for the first vote as compared to the second vote. The student responses appear to be wider distributed after the second vote.

Table 5.18: Student responses to Question 3 from Time-out session 4

	First vote	**Second vote**
A	56	43
B	39	37
C	4	17
D	1	2

As mentioned earlier in Section 5.4.1, the function used in Question 3 of Time-out session 4 (Figure 5.2) represented an unfamiliar or new context to the students. In the preparatory worksheet all examples had been an application of either a left-hand or a right-hand sum. With Question 3 the majority of students (60% in the case of both the first and second vote) chose option A or C, meaning they could not move beyond the idea of a left-hand or right-hand sum to estimate total change. The wide distribution for the second vote shows that the intended conflict created through Question 3 could not be resolved by means of peer discussion. The question served as a prompt for the lecturer to focus the concluding class discussion on the difference between left-hand and right-hand sums and under- and overestimates. From this perspective, Question 3 was successful in creating conflict and encouraging learning.

Although the percentage of correct responses for Question 4 was 22% for the first vote, this percentage increased substantially (to 53%) with the second vote and the question proved to be quite useful. It was utilised during consecutive lectures as a classic example of estimating, and later determining a definite integral in the context of an applied calculus problem. Questions 3 and 4 of Time-out session 4 provided proof that the effectiveness of PRS questions lies in their potential to create conflict and an opportunity for learning.

For Time-out session 5 (Table 5.19) only four of the five questions could be utilised due to time constraints. Question 3 proved to be challenging since the percentage of correct responses was very low for both votes (11% and then 19%). The question was once again utilised during consecutive lectures and proved to be quite useful for deeper learning, because it was used more than once in class discussions following Time-out session 5

Table 5.19: Data from first and second vote of Time-out 5

Time-out 5: The fundamental theorem of Calculus **Students prepared for the lecture**: 85% **Number of PRS questions:** 3 (4 planned, no time for Question 4) **Cognitive learning outcomes:** Apply conceptual and procedural knowledge		
Percentage of correct responses		
	First vote	**Second vote**
Question 1	76	89
Question 2	42	63
Question 3	11	19
Question 4	94	Not polled

For the last of the six Time-out sessions (Table 5.20) the lowest percentage of correct responses for the first vote were 39% and the lowest percentage for the second vote 50%. All the questions qualified if measured against the principles of Crouch and Mazur

(2001), but Questions 1, 4 and 5 were eventually discussed by the lecturer, since less than 70 % of students chose the correct answer with the second vote.

Table 5.20: Data from first and second vote of Time-out 6

Time-out 6: Linear algebra, matrix multiplication **Students prepared for the lecture**: 74% **Number of PRS questions:** 5 **Cognitive learning outcomes:** Apply conceptual and procedural knowledge		
Percentage of correct responses		
	First vote	**Second vote**
Question 1	47	56
Question 2	41	77
Question 3	45	84
Question 4	39	50
Question 5	61	68

5.4.3 Validation. The Time-out sessions appeared to be successful, because for most of the Time-out sessions the percentages of correct responses for the majority of questions increased significantly from the first to second vote, to more than 70% correct responses for the second vote. This was not the case with Question 3 and 4 from Time-out session 4, Question 4 from Time-out session 5 and Questions 1, 4 and 5 from Time-out session 6, but these questions created an opportunity for discussion and were utilised towards extended learning and metacognition during the same or even consecutive lectures.

Positive feedback from individual students and high percentages of students that prepared for the sessions (86% for Time-out 1, 88% for Time-out 2, 92% for Time-out 3, 86% for Time-out 4, 85% for Time-out 5 and 74% for Time-out 6) provided evidence of the success of the Time-out sessions in stimulating self-directed learning.

5.5 Questionnaire Indexes

The descriptive statistics comparing the beliefs indexes for Survey 1 with that of Survey 2 are summarised in Table 5.21. To determine the influence of the Time-out sessions on student beliefs, the averages of the three beliefs indexes for Survey 1 were compared with the averages of the three beliefs indexes calculated for Survey 2. The average of the Centredness Index (C) decreased from 0.94 to 0.92, the average of the Mathematics learning Index (M) increased from 1.83 to 1.91 and the average of the Responsibility Index (R) increased from 1.10 to 1.19. In light of the preliminary analysis of Section 5.3, students' beliefs at the end of the first semester of 2018 appeared to be biased towards a lecturer-centred classroom, with beliefs about mathematics learning leaning towards relational understanding and the responsibility for learning being that of the student. To determine a possible shift in student beliefs as a result of the intervention, further statistical analysis was conducted.

Table 5.21: Descriptive statistics for the three beliefs indexes of Survey 1 and 2

	Centredness Index (C)		**Mathematics learning Index (M)**		**Responsibility Index (R)**	
Descriptive	**Survey 1 (C1)**	**Survey 2 (C2)**	**Survey 1 (M1)**	**Survey 2 (M2)**	**Survey 1 (R1)**	**Survey 2 (R2)**
Average	0.94	0.92	1.83	1.91	1.10	1.19
Std deviation	0.19	0.21	0.41	0.79	0.17	0.36
Median	0.94	0.88	1.80	1.80	1.09	1.14
Minimum	0.50	0.50	1.00	0.86	0.79	0.79
Maximum	1.50	2.00	3.00	6.00	1.45	3.20

Box and whisker plots of the three beliefs indexes for Survey 1 and 2, showed that the data was non-normally distributed, hence the decision to proceed with nonparametric statistical methods to confirm a possible shift in student beliefs. In Figure 5.3, the box and whisker plot for the Responsibility Indexes of Survey 1 is given to demonstrate the non-normal distribution of data.

The nonparametric test for comparison of two variables in a single sample, the Wilcoxon signed rank test, was used to compare the Centredness Index (C), the Mathematics learning Index (M) and the Responsibility Index (R) for Survey 1 with that of Survey 2. With the Wilcoxon signed rank test "the null hypothesis is that the median of the difference scores is zero" (Pietersen and Maree, 2012b:231), as mentioned earlier.

The significance (two-tailed) values (or p-values) for the three null hypotheses related to the three different beliefs indexes, with a significance level of 0.05, are discussed. In the case of all three indexes the hypotheses are one-sided hence the significance (two-tailed) values are divided by two to represent the p-value for a one-sided hypothesis (Pietersen and Maree, 2012b). A summary of the significance (two-tailed) values (or p-values) for the three null hypotheses related to the three different beliefs indexes, with a significance level of 0.05, are displayed in Table 5.22.

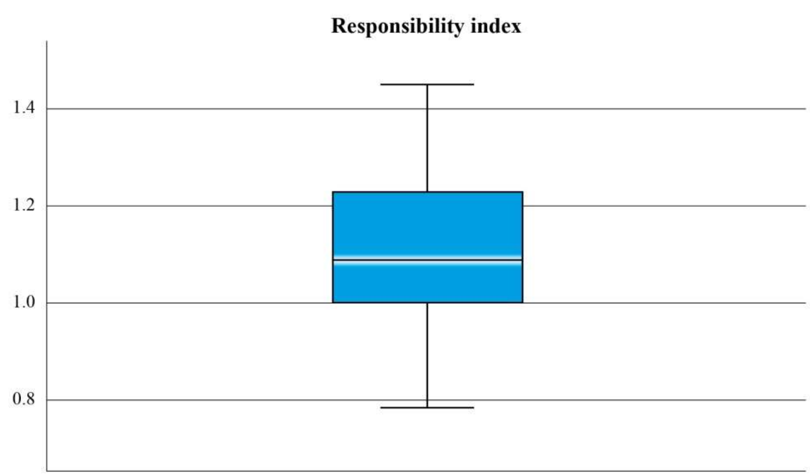


Figure 5.3: Box and whisker plot for the Responsibility indexes of Survey 1

Based on a p-value of 0.0315 for the Responsibility Index (R), the related null hypothesis is rejected, implying that the medians of the Responsibility Index for Survey 1 and Survey 2 differed significantly at the significance level of 5 percent. The other two null hypotheses are not rejected based on their respective p-values, namely 0.315 for the Centredness Index (C) and 0.256 for the Mathematics learning Index (M).

The statistical analysis of the three questionnaire indexes is summarised. Descriptive statistics provide evidence of a shift in students' beliefs about mathematics learning and taking responsibility for their own learning. The shift in students' beliefs about taking responsibility for their own learning proved to be statistically significant at 5 percent. To ascertain whether the shift can be attributed to the Time-out sessions, the qualitative data collected in the form of focus group interviews are discussed.

Table 5.22: Significance values (p-values) of null hypotheses of beliefs indexes

Indexes	**Significance (two-tailed) values** (Significance level of 0.05)	**p-value for one-sided hypothesis** (Significance level of 0.05)
Centredness (C)	0.512	0.256
Mathematics learning (M)	0.630	0.315
Responsibility (R)	0.063	0.0315

5.6 Focus Group Interviews

To analyse the transcribed data from the focus group interviews, the conceptual framework of the didactical contract in tertiary mathematics education as compared to secondary education was used, but emerging themes were also identified. The relevant themes are categorised in Table 5.23 and then discussed.

Table 5.23: Themes from focus group interviews identified and categorised

Section	**Category**	**Subcategory**
5.6.1	Shift in the didactical contract	Centredness
		Mathematics learning
		Responsibility for learning
5.6.2	Student perceptions about the Time-out sessions	Cognitive gains for students
		Social/emotional gains for students
		Graduate attributes
5.6.3	Student expectations	Mathematics teaching/learning
		The mathematics lecturer

5.6.1 Shift in the didactical contract. Evidence of a shift in student beliefs about the centredness of the mathematics classroom, the nature of mathematics learning and students taking responsibility for their own learning is discussed. Some respondents provided evidence of no shift as explained at the end of this section.

Centredness*.* When asked about whether they understood the purpose of the Time-out sessions, some respondents mentioned a perceived purpose, hence demonstrating a shift in beliefs about the effectiveness of a student-centred mathematics classroom.

R3: *I think at first no, I did not understand, but as time went by I was like oh it works, because even my semester marks improved and I could understand like better.*

R1: *I used to be very lazy solving those worksheets, but I came to realise that after I start solving those worksheets...that even my second semester* (test) *was improved.*

R9: *I feel like it was also a way for her to show us that we should prepare for lectures before we actually come to them. Not just for the one where she uploads the worksheets, but for the whole week, because she does upload the whole week's slides on a Sunday, before we come. So we actually do have time to go over the things and go through a textbook, go to external sources and see if we can figure out for ourselves. So if we go there, we don't just sit there. So, it is not the first time seeing the work for us.*

Three students from Focus group interview 1 and two from Focus group interview 2 used phrases like "force", "forced you to work" or "they still felt that they needed to finish" when referring to the Time-out sessions. These phrases demonstrate that the students are motivated by external factors, but that the Time-out sessions did not influence their fundamental beliefs about mathematics teaching/learning.

R2: *But I know of a lot of people that found it very difficult to do the self-studying. Especially then in stressful times, they did not have the time to go on YouTube and find the video or look for it in the book, but they still felt that they needed to finish the worksheet...*

R5: *I think the online worksheets actually almost to a certain extent, forced you to work or study ahead. It did not really give you much of a choice not to do it.*

R4: *Because like we, like he had mentioned in the beginning...they force you to actually be on track and to work through the things...*

R8: *...that one actually forces me to prepare for the lectures beforehand*

R6: *...I had to do it and finish it.*

With Focus group interview 2 the students were asked whether they thought that the Time-out sessions had altered their perceptions of mathematics and mathematics learning and the students responded as follows:

R7*: I think that...coming from high school, the teachers would teach us like everything, and now this, it shows me that there are a lot more work that we have to put in*.

Mathematics learning*.* The following respondents of Focus group interview 2 realised the necessity of effort towards learning and provided evidence of a shift in student beliefs about mathematics learning.

R12: *Um, my view, the thing...what I had thought before that...was wrong, that I realised was wrong...that I understood things just by listening to the lecturer in class. I was able to see how much work goes into understanding maths and actually knowing what's going on.*

R7: *Coming from high school, the teacher would teach us like everything and now this it shows me that there are a lot more work that we have to put in.*

One respondent demonstrated a shift towards the belief that mathematics learning involves conceptual understanding and other skills like utilising resources, writing mathematics and mathematical thinking, hence relational understanding.

R13: *I think it was really helpful, because like it helps you to even go for further external sources like using YouTube videos. And then it helps you also to increase your speed in terms of your skills…writing. Yeah, it helps you with many things. Even the way you think, because like you have to understand the concept, so it improves your thinking in such a way. So it is really helpful.*

Respondent 12 demonstrated evidence of a shift in beliefs about mathematics learning in a student-centred classroom towards relational understanding.

R12: *I thought that she was trying to have us see how well we would do when we are doing it ourselves. Like you were saying it is easy when you see the lecturer doing it, but then it is a wake-up call for you if you are doing it and you realise that you are not doing as well as you would have expected or it's not as easy as you would have thought.*

Responsibility for learning. A shift in student beliefs about taking responsibility for their own learning was evident from students participating in Focus group interview 2, which is discussed below. The Time-out sessions are referred to by one respondent as the "non-learning learning sessions" and by other respondents mistakenly as "clickUP tests", hereby referring to the LMS tests written in preparation for the Time-out sessions.

R9: *I enjoyed the non-learning learning sessions, because now I have to take the initiative and make sure that I do the work.*

R6: *The-the clickUP tests also shows you that you shouldn't be dependent on the teacher alone...I have to do it first, before I depend on somebody else.*

Respondents 13 and 7 quantified the responsibility of the lecturer in relation to the responsibility of the student.

R13: *I would say clickUP tests motivate me in such a way, only the lecturer does 10 percent, the 90 percent is your work.*

R7: *I think it is 40:60.*

One respondent from Focus group interview 1 referred to the Time-out sessions as being helpful, but did not experience a shift in beliefs, already realising at the beginning of the semester that learning is the responsibility of the student.

R2: *The-the...the, what are they called, these sessions were very helpful and I definitely also agree that the worksheets were helpful, but I really had that mindset at the beginning of the year, so it didn't change my view, but I just...agree that it is, it is self-study, or a lot of it is self-study, it is your own responsibility.*

5.6.2 Student perceptions about the Time-out sessions. The eight students who participated in the Focus group interview 2 scored the general usefulness of the Time-out sessions at an average of 9 out of 10. Students' positive perceptions of the Time-out sessions were identified, and categorised into three categories: cognitive gains, social/emotional gains and graduate attributes.

Cognitive gains. Participants mentioned the usefulness of the Time-out session in teaching cognitive skills i.e. preparation and self-study skills, skills in utilising resources and problem solving skills. The relevant comments are summarised in Table 5.24.

Table 5.24: Cognitive skills encouraged by the Time-out sessions

Skill	**Respondent**	**Comments**
Preparation and self-study skills	R9	*I feel like it was also a way for her to show us that we should prepare for lectures before we actually come to them. Not just for the one where she uploads the worksheets, but for the whole week.*
	R9	*Yes, it taught you how to self-study, because there are going to come a lot of times where you are going to have to self-study, where the lecturer doesn't have enough time to finish...a topic and they are going to tell you "no, that you have to self-study".*
Utilising resources	R3	*I would go on YouTube and find out what the concept is about and then I would write it down and study it and then when I went...when I wrote the clickUP test* (clicker test)*, I found that it was much (more) easier and I answered it better. And when I went to class, it was just...yeah it was easy.*
	R13	*I think it was really helpful, because like it helps you to even go for further external sources like using YouTube videos.*
Problem solving skills	R13	*They help us in such a way that we can be able, even if you have a problem, maybe if you are doing a previous question paper you have a problem, you will fight to get that problem to be solved.*

Social/emotional gains. Students commented that the Time-out sessions provided for positive social and emotional experiences in a teaching/learning environment. The relevant skills and comments are summarised:

Students commented that the involvement of their peers supported their own learning.

R2: *In class we had time to talk about it ourselves so, between the clicker tests that we did, and that gave us the opportunity to also hear an alternative way of explaining the concept for example my friend explained something to me that I did not understand or vice versa, so that was very helpful.*

R12: *She used to give us time to speak to the person next to us and that was very helpful, because you see it from the perspective of someone who is also learning it. So they, the way in which they explain if you struggle with something helps...it is an easier way of explaining, they explain the basics. So that was helpful.*

R9: *I was going to say what she said that it helps when there are other people who had to go over the work with you and they are basically on the same level as you, so they can explain in laymen's terms how to do it.*

R6: *So actually interacting with other people give you different perspectives, which will actually better your understanding other than (you) doing it your own way.*

One respondent commented on the Time-out sessions improving her confidence.

R6: *And, when you find yourself explaining to another person, it shows that you understand the work. So, it actually also builds your confidence.*

Graduate attributes. The participants identified the Time-out sessions as contributing to attributes relevant for success in their learning and studies i.e. metacognition, independent learning and a growth mindset. Student comments and the related skills are summarised in Table 5.25.

Table 5.25: Graduate attributes encouraged by the Time-out sessions

Skill	Respondent	Comments
Metacognition	R4	*They force you to actually be on track and to work through the things, so...ja it's...it helped me personally, because now I am always like on my toes, I go if I don't get this now, I'll probably never get it. Procrastination is like not an option.*
	R12	*I found that I did better in the second exercises, because I had identified my problems in the first one and I had dealt with them during the class.*
Growth mindset	R13	*You always try. You fight to understand that thing. So even if you prepare for a semester test, it will be the same thing.*
Independent learning	R13	*It helped us to become independent.*

5.6.3 Student expectations. Evidence relating to student expectations of mathematics teaching/learning and the role of the lecturer is discussed.

Mathematics teaching/learning. One participant demonstrated a resistance to the idea of demonstrating effort towards learning or struggling with mathematics.

R12: *For some...for most of it, we were able to ask people next to us and clarify our mistakes. But, then sometimes there were...you would struggle. And it's because it is self-studying and she wouldn't teach that section in class...then you would have to find time to go and consult.*

Respondent 8 appears to be intimidated by the idea of making a mistake in the context of a teaching/learning environment.

R8: *Well, that...I feel like that is, in a way, wrong, because they catch you off guard, and then you are like really scared. So, sometimes you answer the wrong answer, even though you don't intend to and you are mixed up.*

The mathematics lecturer. Students' expectations of the mathematics lecturer are categorised in Table 5.26. The only reference to student learning is made by respondents

5 and 13. The expectation of respondent 5 resonates with lecturer-centred beliefs, whereas the expectation of respondent 13 resonates with student-centred beliefs.

Table 5.26: Student expectations of the mathematics lecturer

Characteristic	**Respondent**	**Comments**
Qualification	R5	*Knowledgeable*
	R8	*Have a deep understanding*
Preparation	R7	*Well-prepared*
	R8	*The lecturer should prepare for the lecture beforehand*
	R8	*Preparing notes that supplement our textbook*
Motivation	R1	*Encourage*
	R9	*Adapt to different learners*
Communication	R2	*Be versatile, so to have alternative ways of explaining*
	R3	*The ability to convey a concept*
	R6	*Be patient*
	R7	*Enthusiastic*
	R12	*They should be approachable*
Learning	R5	*To a certain extent, but also what has to be said or what is applicable to the tests and examinations.*
	R13	*I think our lecturer should also ask questions...from learners in order to understand whether they understand a concept.*

To conclude this section, evidence of the influence of the Time-out sessions on students' beliefs about mathematics and mathematics teaching/learning, is given in the form of the (qualitative) data generated by the focus group interviews. The triangulation of the data for the purpose of answering the research questions are elaborated in the next section.

5.7 Triangulation

O'Cathain, Murphy and Nicholl (2010) describe three techniques used to integrate data at the point of triangulation, to strengthen the results from various forms of data analysis. They mention that triangulation is part of the interpretation stage of a study and mention the triangulation protocol to allow for the most comprehensive account. According to O'Cathain et al. (2010), the technique was developed for multiple qualitative methods, but can be used in mixed method studies. Farmer, Robinson, Elliot and Eyles (2006) provide an example of a triangulation protocol used for the triangulation of qualitative data in a parallel-case study design, similar to the design of my study. Farmer et al. (2006:377) mention, like O'Cathain et al. (2010), the importance of addressing the "completeness, convergence or dissonance" of key themes. O'Cathain et al. (2010) mention the use of a convergence coding matrix, where findings from various methods are listed with consideration as to whether findings agree (or demonstrate convergence) on the one hand or contradict (or demonstrate dissonance) on the other hand. The idea is to search for disagreements that can allow for a better understanding of the research question. Specifically, with a convergence coding matrix (or scheme) attention is paid to agreement, partial agreement, silence or dissonance between the data sets (Farmer et al., 2006). Agreement is derived if the sets of results demonstrate "full agreement" on both the meaning and prominence of the theme. Partial agreement occurs if agreement is evident but not on both meaning and prominence, while silence means one set of results covers the theme, but another set is silent on the theme, and dissonance implies total disagreement between the data sets (Farmer et al., 2006:383).

The findings of the focus group interviews of the current study were sorted as described by Farmer et al. (2006), to identify a combined list of the relevant key themes generated by the two focus group interviews. The purpose is to summarise the findings (see Table 5.27) so that they can be compared with the findings generated by the quantitative data of the current study, in order to establish whether the Time-out sessions contributed to a shift in student beliefs or a renegotiation of the didactical contract. The key themes relevant to the research questions of my study are represented by the statements of Table 5.27. The themes can be summarised in terms of the following question: How did the Time-out sessions influence student beliefs about the centredness, mathematics learning and the responsibility for learning?

Table 5.27: Theme frequencies and quotes from focus group interviews

Theme	Number of respondents FG 1	FG 2	Sample quotes
The Time-out sessions did influence			
student beliefs about centredness.	2	2	R3: *I think at first no, I did not understand, but as time went by I was like oh it works, because even my semester marks improved and I could understand like better.*
student beliefs about mathematics learning.		3	R7*: I think that also, like coming from high school, the teachers would teach us like everything, and now this, it shows me that there are a lot more work that we have to put in*.
student beliefs about the responsibility for learning.		4	R9: *I enjoyed the non-learning learning sessions, because now I have to take the initiative and make sure that I do the work.* R13: *I would say clickUP tests motivate me in such a way, only the lecturer does 10 percent, the 90 percent is your work.*
The Time-out sessions did not influence			
student beliefs about the centredness.	3		R5: *I think the online worksheets actually almost to a certain extent, forced you to work or study ahead. It did not really give you much of a choice not to do it.*
student beliefs about the responsibility for learning.	1		R2: *The-the...the, what are they called, these sessions were very helpful and I definitely also agree that the worksheets were helpful, but I really had that mindset at the beginning of the year, so it didn't change my view, but I just...agree that it is, it is self-study, or a lot of it is self-study, it is your own responsibility.*

Note: FG 1=focus group interview 1; FG 2=focus group interview 2

The convergence coding matrix is represented in Table 5.28 and provides a summary of the findings of Table 5.27. In Table 5.28 convergence of the qualitative results are examined for meaning and prominence of the key themes. Agreement, partial agreement, silence and dissonance are indicated and then briefly discussed. It should be mentioned here that the small sample size with focus group interviews might not be representative of the population (Nieuwenhuis, 2012). He advises more than one focus group interviews and between 5 to 12 people per group. In the case of my study Focus group 1 (FG 1) consisted of 5 students and Focus group 2 (FG 2) of 8 students.

Table 5.28: Convergence coding matrix for the nature of the didactical contract

	Convergence code			
	AG	PA	S	DA
Student beliefs shifted as a result of the Time-out sessions				
Centredness				X
Mathematics learning			X	
Responsibility for learning		X		

Note: AG=agreement; PA=partial agreement; DA=dissonance; S=silence

Two students from Focus group interview 1 and two from Focus interview group 2 indicated that their beliefs about the centredness of the mathematics classroom lean towards student-centredness, but three students from FG 1 demonstrated a lack thereof. In terms of meaning, the students of FG 1 disagree about the centredness of the mathematics classroom. Hence dissonance about centredness beliefs is concluded.

No students from FG 1 and three students from FG 2 demonstrated that the Time-out sessions contributed to their beliefs about mathematics learning. Since there is no indication that the Time-out sessions contributed to student beliefs in FG 1, silence about Time-out sessions and students' mathematics learning beliefs is construed.

Regarding beliefs about responsibility for learning, four students from FG 2 indicated that the Time-out sessions did contribute to their beliefs, but one student from FG 1 was clear about the Time-out sessions not contributing to her beliefs. In terms of meaning the students of FG 2 displayed agreement. Though disagreement was observed within one student from FG 1, the students of FG 1 and FG 2 collectively demonstrated partial agreement.

To conclude the study the qualitative data from the convergence code matrix and the quantitative data are compared. The quantitative data provided evidence of a shift in beliefs about mathematics learning being relational understanding and students accepting responsibility for their learning. From the convergence coding matrix it is evident that students from the focus group interviews believed that the Time-out sessions contributed to a shift in their beliefs about them taking responsibility for their own learning. The qualitative data of the focus group interviews confirms the observation made based on the quantitative data, that the Time-out sessions did influence a shift in student beliefs about their responsibility for learning in the desired direction.

5.8 Precis

The pilot study is discussed in Chapter 5 and the five phases of the study provide a backdrop for the exposition of the results of the study. The next chapter is devoted to discussing and concluding the results of the study.

Chapter 6: Conclusions

6.1 Introduction

In a mathematics teaching/learning environment the participants' interpretations and actions are motivated by their beliefs about mathematics and the teaching/learning of mathematics. In particular, the didactical contract or agreement about mutual responsibilities in relation to the relevant content regulates the interaction between the lecturer and students.

While teaching calculus to first year education students in 2015 and 2016, I became aware of a mismatch between the expectations of students and myself, the lecturer. The experience inspired me to explore students' beliefs about mathematics and the didactical contract in the mathematics classroom to support student learning. Several researchers (Benadé, 2013; Brandell et al., 2008; Clark and Lovric;2009, Hourigan and O'Donoghue, 2007; Pepin, 2014) observe the global phenomenon of a widening gap between secondary and tertiary mathematics education, while Yoon et al. (2009) find that by challenging student beliefs and renegotiating the didactical contract at first year level, students' transition from secondary to tertiary mathematics education can be supported. Beatty and Gerace (2009) note students' fundamental beliefs determine their classroom behaviour and define pedagogy for utilising personal response systems (PRS) to influence student beliefs in a physics classroom. The potential value of using personal response systems to influence student beliefs and renegotiate the didactical contract inspired the aim of this study. At the centre of the study is the didactical contract in secondary mathematics education as compared to the contract in tertiary mathematics education, described in terms of (1) centredness, (2) mathematics learning and (3) the responsibility for learning. The study aims to determine how PRS can be used to renegotiate the didactical contract in several ways, to redirect students' beliefs about mathematics teaching/learning from (1) lecturer-centred to student-centred, from (2) instrumental understanding to relational understanding and from (3) the lecturer being responsible for learning to the students being responsible for their own learning.

A pilot study provided the foundation for the design of the study. The intervention consisted of six Time-out sessions incorporated into the traditional mathematics lectures of biological science students over the course of the first semester of their first year at university. The design of PRS questions to be used during a Time-out session was based on the didactical design principles of Brousseau's theory of didactical situations (TDS) and pedagogy was mainly based on the principles of Beatty and Gerace (2009). For the purpose of answering the research questions, the Time-out sessions aimed to create a student-centred learning environment, stimulating relational understanding and encouraging students to take ownership for their learning. A pragmatic perspective determined the research approach, a convergent parallel mixed methods research approach. To determine whether the Time-out sessions influenced student beliefs to the point where the didactical contract could be renegotiated, a questionnaire on student beliefs was deployed at the beginning of the semester (Survey 1) and redeployed near the end of the semester (Survey 2), with focus group interviews conducted around the same time as Survey 2. Statistical analysis of the quantitative and qualitative data provided some evidence of a shift in student beliefs due to the influence of the Time-out sessions, in particular a shift in students' beliefs about (2) mathematics learning and (3) the responsibility for their learning.

Pepin (2014) examines teaching/learning (at tertiary level) through the lens of the didactical contract and notes that students in most mathematics classrooms are expected to imitate the actions of the lecturer without proper guidance. She finds that students must be supported in the development of learning strategies and that renegotiating the didactical contract affords a tool for providing support. Grønbæk et al. (2009) uphold the concept of didactically designed learning situations but mention that student-centred teaching/learning events should not dominate lectures at university. According to Grønbæk et al. (2009:91), students must also be challenged to develop the academic skill of accessing knowledge in its raw untapped form, a skill that is "indeed, something to learn, rather than a starting condition that can just be assumed." Kilpatrick et al. (2001) describe mathematical skill or proficiency as multifaceted and reason that opportunities must be created for students to develop all the strands of mathematical proficiency – that is, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. The findings of these researchers are mentioned for the purpose of addressing the research questions.

6.2 Addressing the research questions

As mentioned, the results of the study provide evidence of a shift in the didactical contract as a result of incorporating the Time-out sessions into the traditional transmission style lectures of a large mathematics classroom. At the core of using personal response systems to renegotiate the didactical contract in a first year mathematics classroom lies the creation of a student-centred learning environment. Such a student-centred learning environment is created by purposefully incorporating well-designed PRS questions with two essential principles underlying the pedagogy, namely discontinuity and cognitive conflict. The following research questions were formulated with these two principles in mind:

How can personal response systems be utilised to renegotiate the didactical contract in the mathematics classroom through influencing student beliefs about

1. the centredness of the classroom
2. mathematics learning; and
3. the responsibility for their learning?

The three research questions are addressed in the order mentioned and indicated.

How can personal response systems be utilised to renegotiate the didactical contract in the mathematics classroom through influencing student beliefs about

1. **the centredness of the classroom**
2. mathematics learning; and
3. the responsibility for their learning?

From a constructivist point of view learning is best achieved in a student-centred learning environment, but lecture halls at universities are mostly designed to support a lecturer-centred teaching approach, hosting as many as 650 students in one lecture group. As a result, large mathematics classrooms at tertiary level are predominantly lecturer-centred, characterised by traditional transmission style lectures. Personal response systems allow for the creation of a student-centred learning environment or flipped classroom so that students prepare for the lecture and the lecture is directed towards interactive learning.

To influence student beliefs about the centredness of a large mathematics classroom, student-centred learning opportunities are periodically incorporated into traditional mathematics lectures and the classroom is intermittently flipped. A PRS session is introduced to interrupt or discontinue the traditional lecture or to vary the lecturer-centred mathematics classroom so that the individual student is encouraged to depend on their own understanding of the content and the support of their peers. Through the principle of discontinuity the mathematics classroom is shifted from lecturer-centred to student-centred and opportunity is created for students to compare active learning (in a student-centred teaching/learning environment) to passive learning (in a lecturer-centred teaching/learning environment). In essence personal response systems are used to break or even "rupture" (Pepin, 2014:646) the didactical contract so that student beliefs about the centredness of the mathematics classroom are challenged and the individual student is motivated to focus on their learning and learning strategies in a teaching/learning environment. One of the students who participated in the focus group interviews demonstrated realisation of a break in contract due to the Time-out sessions and contemplation of teaching and learning strategies: *"*coming from high school, the teachers would teach us like everything, and now this … shows me that there are a lot more work that we have to put in".

In the study, Time-out sessions were only incorporated into the traditional transmission mode lectures during six of the approximately 52 mathematics lectures. The findings of the study as discussed in Chapter 5, do not reflect a significant shift in student beliefs about the centredness of the mathematics classroom. Two students spontaneously included comments about the Time-out sessions in their assessments of me, the lecturer. One of the students commented on the perceived effectiveness of the Time-out sessions while the other student commented on its ineffectiveness by stating "I do not think that the flipped classroom is as effective as traditional methods. I am lost on all content we did in the flipped classroom". It can be argued that students were polarised by the Time-out sessions and as a result some students' resistance to change was enhanced, while other students embraced the change. This observation is in line with the findings of Adams and Dove (2018). Upon comparing the learning of Calculus in a flipped classroom to learning in a traditional lecture-based classroom, they find that student beliefs about the learning of mathematics in a flipped classroom remained divided, stating that "there existed a subset that was unhappy with this instruction" (Adams and Dove, 2018:613). It is conceivable that an increase in the frequency of student-centred teaching/learning events or Time-out sessions (say at least once a week), might have an increased impact on student beliefs about centredness. One might even consider creating a predominant student-centred mathematics classroom for the purpose of influencing student beliefs about centredness, but the realisation is that a predominantly student-centred classroom at first year level might polarise students to such an extent that the transition from secondary to tertiary mathematics education is impeded and not supported.

As mentioned by Adams and Dove (2018), the successful learning of mathematics in any classroom, traditional or flipped, is dependent on the student's commitment to learning. The use of PRS to influence student beliefs about mathematics learning and responsibility for learning is discussed through respectively addressing the second and third research questions.

How can personal response systems be utilised to renegotiate the didactical contract in the mathematics classroom through influencing student beliefs about

1. the centredness of the classroom
2. **mathematics learning;** and
3. the responsibility for their learning?

When viewing teaching/learning interactions through the lens of the didactical contract, the potential of utilising student mistakes to address students' misconceptions, becomes evident. By frequently highlighting mistakes and discussing the reasoning behind correct and incorrect answers, students' misconceptions and beliefs about mathematical understanding can be adjusted. This is accomplished through the creation of cognitive conflict, defined by Assagaf (2013) as the conflict experienced when a discrepancy between new knowledge and existing constructs of understanding is observed. Cognitive conflict allows for misconceptions to be challenged and addressed so that conceptual change, a change in perception, is established (Assagaf, 2013). The study does provide evidence of conceptual change or a shift in student beliefs about mathematics learning as a result of the Time-out sessions.

To influence student beliefs about mathematics learning through the use of personal response systems requires the design of PRS questions and PRS pedagogy to be reconsidered. PRS questions are designed to encourage deep conceptual understanding and higher order thinking with possible student mistakes included as distractors. PRS pedagogy is planned to create opportunities to highlight and address students' mistakes and/or misconceptions through the creation of cognitive conflict. This realisation is essentially accomplished through pauses or discontinuity between the reveal of the PRS question, the distractors and the disclosure of the correct answer(s). For the Time-out sessions a pause between the reveal of every question and its distractors allowed individual students the opportunity to solve the problem and/or answer the question. Upon revealing the juxtaposed distractors, opportunity was created for students to vote and contemplate the correctness of their answers. Peer discussion followed by a second vote with correct responses only disclosed after the second vote, allowed the cognitive conflict to be extended and/or resolved. A conclusive class discussion allowed another opportunity for conflict resolution. By mentioning that "for most of it, we were able to ask people next to us and clarify our mistakes" a participant of the focus group interviews referred to peer-involvement and conflict resolution, while the statement *"*But, then sometimes...you would struggle" can be interpreted as the student experiencing conflict without resolve.

This brings us to the significance of addressing student beliefs about the responsibility for their learning and the challenges inherent to the learning of mathematics. Before the third research question is addressed, a last consideration is given for utilising personal response systems to influence student beliefs about mathematics learning. The flipped classroom allows an opportunity for necessary concepts to be introduced and explored outside the classroom. This allows exploration to be extended inside the classroom and the lecture to be utilised towards deeper understanding. Once again, the principle of discontinuity is essential, because of the pause incorporated between student preparation outside the classroom and the interactive in-class component. The pause allows students the opportunity to contemplate their own understanding, an opportunity that can be strengthened through the incorporation of an online assessment component or LMS test in the case of the Time-out sessions.

How can personal response systems be utilised to renegotiate the didactical contract in the mathematics classroom through influencing student beliefs about

1. the centredness of the classroom
2. mathematics learning; and
3. **the responsibility for their learning?**

The use of personal response systems to encourage students to take responsibility for their own learning implies two important considerations, the didactical contract and motivation to learn. While discontinuity in the didactical contract allows students an opportunity to contemplate their roles in the teaching/learning environment, strategies directed at student motivation allow the individual student's commitment to learning to be addressed.

When personal response systems are used to transform the mathematics classroom from lecturer-centred to student-centred, the roles of the participants (lecturer and students) are reversed so that students can evaluate their roles in the teaching/learning of mathematics. By stating "now this shows me that there are a lot more work that we have to put in" the student from the focus group interviews quoted earlier demonstrated a contemplation of their role. The same student also stated "now I have to take the initiative and make sure that I do the work" and this brings us to commitment to learn or motivation.

Motivation is an essential component of encouraging students to take responsibility for their learning. Elton (1996) suggests that strategies are directed towards enhancing students' beliefs about their abilities to pass the examination. When using PRS to influence a shift in student beliefs about their own responsibility for their learning, strategies to enhance their beliefs about being prepared for the examination must be deliberated. This was accomplished in the current study by designing PRS questions that resemble typical examination questions, encouraging students to prepare for the lecture and incorporating the PRS questions into mathematics lectures with examination conditions. Students acknowledged that the LMS test written the day before the lecture motivated them to prepare the pre-class worksheet and take ownership for their learning. As one student mentioned, referring to the LMS test as the clickUP test (hereby referring to the learning management system of the university): "I would say the clickUP tests motivate me in such a way, only the lecturer does 10 percent, the 90 percent is your work". The shift in responsibility initiated by preparing the pre-class worksheet and the LMS test was reinforced through the transferral of responsibility characterising student-centred in-class activities. Through feedback (provided through the first and second voting opportunity) the Time-out sessions allowed students to gauge their own understanding and as a result their ability to pass the examination. One student even commented on the examination strategies acquired through the experience of preparing for the Time-out sessions: "I think the clickUP-tests were good, because we had time to practice the things. And then we have 15 minutes to do a whole test, so when it comes to examinations, we'll know how much time we should be allocating to something if we don't understand it. So instead of wasting time at a question that we don't get, we can do things that we do understand, so we can get most of our marks".

A third consideration when using personal response systems to encourage a shift in student beliefs about the responsibility for their mathematics learning emerged from the study. Strategies aimed at educating students about the challenges inherent to the learning of mathematics have to be incorporated. In this study challenging PRS questions were used in the context of the lecture because the foundation for understanding was established through the pre-class worksheet and LMS test, as mentioned earlier. Through the use of challenging questions, the time allowed for students to grapple with these problems and the support of their peers students had an opportunity to experience and also reflect on the mental effort involved in solving a mathematics problem. The study does provide evidence of the perceived value of the Time-out sessions in convincing some students of the struggle associated with mathematics learning and learning. As one student from the focus group interviews put it, "I thought that she was trying to have us see how well we would do when we are doing it ourselves. Like you were saying, it is easy when you see the lecturer doing it, but then it is a wake-up call for you if you are doing it and you realise that you are not doing as well as you would have expected or it's not as easy as you would have thought." Deslauriers, McCarty, Miller, Callaghan and Kestin (2019) reason that active learning in a physics classroom is deemed ineffective by students because of the discomfort associated with increased mental effort during active learning. They suggest strategies for encouraging students to value the importance of struggling towards understanding. This study provides principles for pedagogy that aim to convince students that struggling towards mathematics learning is important.

Having addressed the research questions, a general discussion follows. The study shows that personal response systems can be utilised in support of deep conceptual learning through the use of purposefully designed questions, as noted by Dangel and Wang (2008). By incorporating well planned PRS sessions into a large mathematics classroom student involvement is intensified and higher-order learning is encouraged, as cited by Biggs (1999). The students are guided towards deep learning and the gap is narrowed, to use a phrase coined by Biggs (1999).

Lozanovski et al. (2011) also reference Dangel and Wang (2008) to argue against the use of PRS questions aimed at deep mathematics learning and/or involving serious calculations. This study disputes the findings of Lozanovski et al. (2011) and shows that the strategy of scaffolding can ease the process of extensive calculations, as demonstrated by Question 2 of Pilot 2 (see Table 5.2). Scaffolding can be used to guide students through serious calculations, by giving them direction and assisting them to navigate typical student mistakes. Through scaffolding PRS can be used to highlight and correct various mistakes made by multiple students and metacognitive skill stimulated (Kalajdzievska, 2014), a skill that is associated with intellectual independence and a productive disposition (Cobb, 1988).

This study did reveal that some students expect the lecturer to remain positive and compliant and that good teaching is sometimes equated with the skill of a clear explanation, as mentioned by Hourigan and O'Donoghue (2007), but evidence is also given of students valuing other attributes essential for successful teaching e.g. patience, versatility, adaptability etc. According to Benadé (2013), students do not value proper understanding and believe that learning is the responsibility of the lecturer, but through this study it became evident that some students do value conceptual understanding and realise their responsibilities towards learning.

Pepin (2014) remarks that all skills essential for mathematics learning should be addressed and this study shows that personal response systems can be used to support the development of essential mathematics learning skills e.g. realisation of the effort involved in the deep conceptual learning of mathematics, as mentioned by Hourigan and O'Donoghue (2007). Through this study it became evident that at tertiary level students must be challenged to develop all skills related to mathematical ability, e.g. all the strands of mathematical proficiency as mentioned by Kilpatrick et al. (2001) and the academic skill of being able to access mathematics in its unexploited form as mentioned by Grønbæk et al. (2009). The realisation is that the teaching/learning of mathematics at university, especially at first year level, should cater for all aspects of mathematical ability, an endeavour that cannot be realised in a predominant lecturer-centred or for that matter student-centred teaching/learning environment. Taylor Rice (2018) recommends strategies for frequently introducing discontinuities or pauses into lectures to allow student engagement and encourage learning. Through this study it became evident that in the mathematics classroom the motivation behind these pauses should be to encourage the learning of mathematics in all its complexity.

The value of the study lies in viewing the use of personal response systems through the lens of the didactical contract and exploring ways to employ personal response systems in a large mathematics classroom to renegotiate the didactical contract at first year level. Technology, in the form of personal response systems, is utilised to support first year students to overcome the challenges of a large mathematics classroom and to successfully transition from secondary to tertiary mathematics education. The contribution of this study to the existing body of knowledge is situated at the intersection of two important fields in tertiary mathematics education research, namely technology as "an engine driving pedagogical change" (Selden, 2005:132) and the secondary/tertiary mathematics transition. Through the pragmatic approach of the study, certain practical principles for renegotiating the didactical contract at first year level were explored. The study contributes new knowledge to the two fields of research mentioned above, but also serves to bridge the divide between theory and practice in didactical design.

The uniqueness of the study lies in evaluating the use of personal response systems in a mathematics classroom at tertiary level, through the theoretical lens of the didactical contract. In using a unique conceptual framework based on research the study is elevated from yet another study about the novel ways in which personal response systems can be used to create interactive learning opportunities to a study on how personal response systems can be used to close the gap between secondary and tertiary mathematics *education. Review of the conceptual framework of the study led the realisation that the first construct should not involve student beliefs about the nature of the mathematics classroom, but rather student beliefs about the nature of mathematics. Essentially students' beliefs about mathematics learning, the second construct, and the responsibility for their learning, the third construct, are underpinned by their beliefs about the discipline itself. The study lies the foundation for further research on utilising personal response systems to assist students to transition and universities to close the gap between secondary to tertiary mathematics education.*

To conclude, personal response systems do have the potential to play a role in renegotiating the didactical contract in the mathematics classroom, if used to support pedagogy aimed at meaningful mathematics learning in an environment conducive to individual learning. According to Hourigan and O'Donoghue (2007), students must accept that effort on their part is a crucial condition for a positive teaching and learning experience. In order to cause a shift in students' beliefs about mathematics and mathematics teaching/learning, the prominence and longevity of a new teaching/learning model is of utmost importance. The study shows that personal response systems can be utilised in the form of Time-out sessions to renegotiate the didactical contract by influencing student beliefs about mathematics and mathematics teaching/learning, if it is frequently incorporated as part of a teaching paradigm focused on exploring new means to extend the learning of mathematics beyond the classroom, and not to protect the comfort levels of the participants, i.e. both students and lecturers.

Taylor Rice (2018) references a metaphor used by a literature professor to demonstrate pauses incorporated during lectures. According to Taylor Rice (2018), the professor uses a play with various scenes to describe the pauses in the classroom. The metaphor of a play can also be used to depict the participants in a didactical situation focused on student learning, and describe their roles against the backdrop of the didactical contract. The knowledge at stake is the playwright, who determines the script, and the didactical situation is the director, who directs the actions of the actors. The didactical contract as the cast director allocates the roles, while the lecturer's performance as antagonist highlights the performance of the protagonists or students. If teaching/learning events are approached from this point of view, student learning (and not the lecturer) becomes the focus of the mathematics classroom, as is the case in a traditional mathematics classroom. The implication is that the stage must be set for student learning through the goal-directed design and practice of teaching/learning events, combined with purposeful feedback, but at tertiary level such events can only constitute one of the scenes aiming to encourage the development of essential skills.

6.3 Limitations and Recommendations for Future Research

Only 59 out of 598 students participated in completing the questionnaires for both Survey 1 and Survey 2. A possible explanation for the low responses to the survey is that the survey was redeployed too long after the intervention, at the end of the semester and before the examination, when students' investment in the lectures and the intervention was on the decline. Given such a relatively small sample, findings about student beliefs could not be generalised to the broader student population, but the findings could be useful for other researchers in similar contexts. The sample size for focus group interviews was also small, as is the case with qualitative data. The process of triangulating quantitative and qualitative data augments the findings of the study.

As mentioned earlier, only six Time-out sessions were incorporated into 13 weeks of four mathematics lectures per week. As a result the student-centred teaching/learning events did not feature enough in order to significantly influence student beliefs about the centredness of the mathematics classroom. For this purpose the frequency of the Time-out sessions can be increased to at least one session per week. It should be mentioned that for the Time-out sessions to be meaningful, planning, timing and preparation are essential.

Code, Merchant, Maciejewski, Thomas and Lo (2016) develop and utilise the Mathematics Attitudes and Perceptions Survey (MAPS) in an attempt to assist students in developing a productive disposition (or positive beliefs about mathematics and their mathematical ability). Through the survey they compare students' beliefs and attitudes to that of an expert in mathematics. If a student's response to a statement agrees with that of the expert, their response is scored 1. If the student disagrees with the expert, a score of 0 is allocated. This method of scoring holds potential value for the future analysis of the questionnaire used in this study.

Pepin (2014) uses the principle of the extended time survey to gather information about a change in the didactical contract from secondary to tertiary mathematics education. The implication here is that the students are followed from their first day well into their second year at university. This principle holds value for future research utilising my questionnaire.

My dual role as lecturer and researcher was managed throughout the study to ensure that my bias did not limit the outcomes of the study. A suggestion for future research is to separate the role of the lecturer and researcher. The challenge of such an approach is translating theory into practice, the task of bridging the gap between the researcher's conceptualisation and the lecturer's interpretation. From this point of view the duality of researcher as lecturer allows for theory and practice to serve the common purpose of aligning the design and incorporation of meaningful student-centred teaching/learning interactions in a large mathematics classroom.

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ANNEXURE A: ETHICAL CLEARANCE

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Natural and Agricultural Sciences
Ethics Committee

E-mail: ethics.nas@up.ac.za

24 April 2018

ETHICS SUBMISSION: LETTER OF APPROVAL

Ms Karin Bothma
Department of Mathematics and Applied Mathematics
Faculty of Natural and Agricultural Sciences
University of Pretoria

Reference number: EC 180212-174
Project title: Taking time out: An investigation into the use of personal response systems to renegotiate the didactical contract in tertiary mathematics education

Dear Karin Bothma,

We are pleased to inform you that your submission conforms to the requirements of the Faculty of Natural and Agricultural Sciences Ethics committee.

Please note that you are required to submit annual progress reports (no later than two months after the anniversary of this approval) until the project is completed. Completion will be when the data has been analysed and documented in a postgraduate student's thesis or dissertation, or in a paper or a report for publication. The progress report document is accessible off the NAS faculty's website: Research/Ethics Committee.

If you wish to submit an amendment to the application, you can obtain the amendment form on the NAS faculty's website: Research/Ethics Committee.

The digital archiving of data is a requirement of the University of Pretoria. The data should be accessible in the event of an enquiry or further analysis of the data.

Yours sincerely,

Chairperson
NAS Ethics Committee

ANNEXURE B: QUESTIONNAIRE

Dear student

Please complete the questionnaire by indicating with a cross whether you agree or disagree with a given statement. Participation is voluntary and anonymity will be maintained when reporting about the study. Your participation in the study is appreciated.

1. Your thoughts about mathematics and mathematics *learning*

No	Statements	Strongly disagree	Disagree	Agree	Strongly agree
1.1	Mathematics can be best described as a set of facts, rules and formulas that students have to learn.				
1.2	The student is responsible for his/her learning.				
1.3	To be able to do mathematics, a student has to understand mathematical concepts behind rules and formulas.				
1.4	Mistakes repeated by students indicate a lack of understanding.				
1.5	By attending and preparing for classes, the student takes responsibility for his/her learning.				
1.6	Many rules in mathematics simply have to be accepted and remembered, there is not really an explanation for it.				

1.7	If students struggle to solve a mathematical problem, it is usually because they do not know the correct rule or formula.				
1.8	It is the responsibility of the student to clarify confusion experienced with mathematics.				
1.9	To be able to explain answers is more important in mathematics than whether the answer is correct.				
1.10	The student has to engage continuously with mathematics throughout the semester.				
1.11	The learning of mathematics is best achieved if students battle with a mathematical problem.				
1.12	If a student attends all the mathematics classes, he/she should pass mathematics.				

2. Your thoughts about mathematics and mathematics *teaching*

No	**Statements**	**Strongly disagree**	**Disagree**	**Agree**	**Strongly agree**
2.1	Students can find methods to solve problems without the help of the lecturer.				
2.2	A good mathematics lecturer always demonstrates the correct method to solve problems.				

2.3	To be successful in mathematics, a student needs to listen carefully to the lecturer's explanations.				
2.4	A good lecturer does not leave students to experience confusion.				
2.5	The role of the mathematics lecturer is to convey knowledge to the student and to test whether knowledge was transferred.				
2.6	The lecturer is responsible for the student's learning of mathematics through effective teaching.				
2.7	The lecturer should convey the importance of knowledge for examination purposes.				
2.8	In the teaching of mathematics, the students have to discover concepts for themselves, while the lecturer provides support.				
2.9	Mathematics is best learned if the students actively solve problems in class.				
2.10	In the mathematics classroom, students should be encouraged to find various ways to solve mathematics problems				
2.11	Mathematics problems given to students should be solved easily with content provided in class time.				
2.12	A good mathematics lecturer lightens the burden of learning for the student.				