A novel single composite Shewhart-EWMA control chart for monitoring the process mean

Short running head: Single CSEWMA monitoring scheme

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Abstract

Statistical process monitoring (SPM) literature recommends the combination of Shewhart and exponentially weighted moving average (EWMA) schemes to improve the ability of the standalone Shewhart and EWMA monitoring schemes in detecting small to large shifts. The resulting scheme is named combined (or composite) Shewhart-EWMA (CSEWMA) scheme. In this paper, a new single composite Shewhart-EWMA (denoted as SCSEWMA) scheme for monitoring the mean when the process parameters are known or unknown is proposed using an additive weighted model. The flexibility of the proposed scheme is made possible by an additional weighing parameter that regulates its sensitivity towards shifts of different sizes. The new scheme is compared to the existing Shewhart, EWMA and CSEWMA $\bar{X}$ schemes and the results reveal the superiority of the proposed scheme over the latter schemes. Simulated and real-life data are used to demonstrate the application and implementation of the proposed scheme.

Keywords: Additive weighted model, EWMA, estimated process parameters, Monitoring scheme, Overall performance, Shewhart, Composite Shewhart-EWMA.

1. Introduction

The overall concept of modern monitoring schemes was established in the 1920s by Walter A. Shewhart working at Bell Telephone; see for example, Shewhart[1]. Shewhart-type monitoring schemes are memoryless schemes that only use recent information to decide whether the process is in-control (IC) or out-of-control (OOC). This makes them faster in detecting large changes (or shifts) in the process parameter. However, Shewhart-type schemes are known to be relatively slow in detecting small and moderate shifts. To compensate the weakness of Shewhart-type schemes, Page[2] and Roberts[3] introduced the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) monitoring schemes, respectively. These two monitoring schemes are memory-type schemes that use past and recent information to decide whether the process is IC or OOC. The CUSUM and EWMA schemes are considered as popular alternative of a Shewhart monitoring scheme that are utilised when small to moderate shifts are of interest; see Montgomery[4]. Their setback is that, due to their inertia, they are relatively slow in detecting large shifts in the process. After the introduction of the Shewhart, EWMA and CUSUM schemes, many authors have developed more advanced and enhanced monitoring schemes; see for instance, Daudin[5], Mosquera and Aparisi[6], Abbas et al[7], Abbasi et al[8], Shamma and Shamma[9], Lucas and Saccucci[10], Abujiya et al[11,12], Zaman et al[13], Ali and Haq[14,15], Mabude et al[16] and Huang et al[17].

An efficient monitoring scheme is expected to detect small to large shifts as quickly as possible. One of the possible techniques to enhance the sensitivity of a monitoring scheme towards small to large shifts is the combination of memoryless and memory-type schemes such as the composite Shewhart-EWMA and Shewhart-CUSUM monitoring schemes (see, for example, Lucas[18], Klein[19], Capizzi and Masarotto[20], Shamsuzzaman et al[21] and Freitas et al[22], just to cite a few). Lucas[18] proposed a combined Shewhart-CUSUM monitoring scheme for detecting small to large shifts in the process mean - see also, Klein[19]. Lucas[18] showed that the control limits of the Shewhart component help to detect large shifts as quickly as possible and those of the CUSUM component help to detect small and moderate shifts quicker. Therefore, the resulting Shewhart-CUSUM scheme is efficient for monitoring small to large shifts in the process parameters. Abujiya et al[11] proposed an enhanced Shewhart-CUSUM scheme for monitoring shifts in the process mean, and Capizzi and Masarotto[20] developed a Shewhart-EWMA scheme with estimated process parameters. Shamsuzzaman et al[21] proposed an algorithm to optimise the design of the composite Shewhart-EWMA $\bar{X}$ chart for monitoring the entire range of the shifts in the process mean. More recently, a case study on water consumption in toilet flush devices in a public university building using Shewhart, EWMA and composite Shewhart-EWMA control charts was presented by Freitas et al[22].

The above-mentioned monitoring schemes use *two separate* charting statistics to decide whether the process is IC or OOC. This makes them more difficult to implement as operators prefer simpler models of just using one charting statistic. In this study, a new flexible *single* composite Shewhart-EWMA scheme for monitoring the process mean is developed using an additive weighted model (i.e. based on a single charting statistic instead of two separate charting statistics). The flexibility, attractiveness and strength of the new scheme is based on an extra weighing parameter that regulates its ability to detect shifts of different sizes.

Since in real-life applications, the process parameters are usually unknown and need to be estimated, hence, the effect of parameter estimation is also considered in this study. That is, many authors have advocated that the estimation of the process parameters deteriorates significantly the performance of monitoring schemes (see the review papers by Jensen et al[23], Psarakis et al[24] and Does et al[25]). For instance, Aly et al[26] analysed the performance of simple linear profile monitoring schemes when the process parameters are unknown and they concluded that the estimation error decreases and the IC *ARL* approaches the desired value only for large Phase I sample sizes when using sample estimates instead of known parameters. Therefore, the proposed monitoring scheme will be designed and evaluated under both the assumptions of known and unknown process parameters, and the effect of the sample sizes will also be investigated.

The remainder of this paper is organised as follows: Section 2 starts with a brief introduction of the existing classical Shewhart, EWMA and composite Shewhart-EWMA $\bar{X}$ monitoring schemes. In Section 3, the theoretical and mathematical backgrounds of the proposed monitoring scheme are provided under the assumptions of known and estimated process parameters. The IC and OOC run-length performances are investigated in Section 4 for both parameters known and unknown. In addition, the OOC performance of the new scheme is compared to some existing counterparts. Section 5 uses simulated and real-life data to demonstrate the implementation and application of the proposed monitoring scheme. Concluding remarks and future research ideas are provided in Section 6.

2. Brief description of the existing monitoring schemes

2.1 Shewhart $\bar{X}$ monitoring scheme

Assume that, when the process is IC, the quality characteristic $\{X_{tj}; t \geq 1; j = 1, 2, \ldots, n\}$ is a sequence of samples of independent and identically distributed (i.i.d.) observations from a $N(\mu_0, \sigma_0^2)$ distribution where μ_0 and σ_0 are the IC process mean and standard deviation parameters, respectively, which are assumed to be known. In case of a shift in the process, the process mean shifts from μ_0 to μ_1 ($\mu_1 = \mu_0 + \delta\, \sigma_0$) where δ ($\delta \neq 0$) represents the change in the process mean expressed in standard deviation. At each sampling time, the mean or charting statistic of the Shewhart $\bar{X}$ scheme is given by

$$\bar{X}_t = \frac{1}{n}\sum_{j=1}^{n} X_{tj}. \qquad [1]$$

The upper and lower control limits (UCL and LCL) of the Shewhart $\bar{X}$ scheme are mathematically defined by

$$UCL/LCL = \mu_0 \pm k\frac{\sigma_0}{\sqrt{n}}, \qquad [2]$$

respectively, where k ($k > 0$) is the Shewhart control limits constant which is chosen such that the IC average run-length (ARL_0) is equal to some large desired value such as 370.4. Thus, the Shewhart $\bar{X}$ scheme gives a signal at time *t* if the charting statistic defined in Eq [1] plots beyond the control limits defined in Eq [2].

2.2 EWMA $\bar{X}$ monitoring scheme

The charting statistic of the EWMA $\bar{X}$ scheme at time *t*, denoted as Z_t, is given by

$$Z_t = \lambda\bar{X}_t + (1-\lambda)Z_{t-1}, \qquad [3a]$$

where λ ($0 < \lambda \leq 1$) is the EWMA smoothing parameter, the starting point $Z_0 = \mu_0$ and $\bar{X}$ is defined as in Eq [1], hence Eq [3a] can also be written as:

$$Z_t = \lambda\textstyle\sum_{j=0}^{t-1}(1-\lambda)^j\bar{X}_{t-j} + (1-\lambda)^t Z_0. \qquad [3b]$$

The time-varying (or exact) control limits of the EWMA $\bar{X}$ scheme at time *t* are defined by

$$UCL_{E_t}/LCL_{E_t} = \mu_0 \pm L_E\sigma_0\sqrt{\frac{\lambda}{(2-\lambda)n}(1-(1-\lambda)^{2t})}, \qquad [4]$$

respectively, where L_E ($L_E > 0$) is the EWMA control limit constant which is chosen such that the attained ARL_0 is equal to some prespecified value such as 370. The EWMA $\bar{X}$ scheme gives a signal if Z_t plots beyond the control limits defined in Eq [4], i.e. $Z_t \geq UCL_{E_t}$ or $Z_t \leq LCL_{E_t}$.

Note that when the process has been running for a long time (i.e. $t \rightarrow \infty$), the expression $(1-(1-\lambda)^{2t})$ from Eq [4] converges to 1; therefore, the asymptotic control limits are simply defined by

$$UCL_A/LCL_A = \mu_0 \pm L_E\sigma_0\sqrt{\frac{\lambda}{(2-\lambda)n}}, \quad [5]$$

respectively. To conserve space, in this paper, we will only focus on the time-varying case.

2.3 Composite Shewhart-EWMA $\bar{X}$ monitoring scheme

The composite Shewhart-EWMA (denoted as CSEWMA) $\bar{X}$ scheme is the combination of the standalone Shewhart and EWMA $\bar{X}$ schemes discussed in subsections 2.1 and 2.2, respectively. The CSEWMA $\bar{X}$ scheme signals an OOC situation at time t if one of the following conditions is satisfied.

(i) $\bar{X}_t \geq UCL$ or $\bar{X}_t \leq LCL$, or

(ii) $Z_t \geq UCL_{E_t}$ or $Z_t \leq LCL_{E_t}$.

Thus, the parameters λ, k and L_E must be chosen such that the attained ARL_0 is equal to the prespecified ARL_0 value.

3. The proposed single composite Shewhart-EWMA $\bar{X}$ monitoring scheme

In the previous section, it is shown that the existing CSEWMA $\bar{X}$ scheme uses two charting statistics to decide whether the process is IC or not. In this section, we develop a new CSEWMA scheme based on a single charting statistic when the process parameters are assumed known (i.e. Case K) and unknown (i.e. Case U), and we investigate its IC and OOC performances. To make the new scheme more flexible, an extra weighing parameter denoted by ω $(0 \leq \omega \leq 1)$ is also introduced.

3.1 Design of the proposed scheme

The charting statistic of the proposed single CSEWMA $\bar{X}$ scheme (henceforth denoted by SCSEWMA) is developed using an additive weighted model which is mathematically defined by

$$W_t = (1-\omega)\bar{X}_t + \omega Z_t, t = 1,2,3, \ldots, \quad [6]$$

where $\bar{X}_t$ and Z_t are defined in Eqs [1] and [3a], respectively.

Note that when $\omega = 0$ in Eq [6], then the proposed SCSEWMA $\bar{X}$ scheme reduces to the classical Shewhart $\bar{X}$ scheme; however, when $\omega = 1$, it reduces to the classical EWMA $\bar{X}$ scheme. Thus, the proposed scheme borrows the strengths of the standalone Shewhart and EWMA $\bar{X}$ control charts separately and it is flexible through the weighing parameter ω.

The expected value and variance of the SCSEWMA $\bar{X}$ statistic W_t in Eq [6] for Case K are given by

$$E(W_t) = \mu_0 \quad [7a]$$

and

$$Var(W_t) = \left((1-\omega)(1-\omega+2\lambda\omega) + \frac{\lambda\omega^2}{2-\lambda}(1-(1-\lambda)^{2t})\right)\frac{\sigma_0^2}{n}, \quad [7b]$$

respectively. The derivations of the mean and variance (i.e. Eqs [7a] and [7b]) of the SCSEWMA $\bar{X}$ charting statistic are provided in Appendix A.

Therefore, the time-varying control limits are given by

$$UCL_{W_t}/LCL_{W_t} = \mu_0 \pm L_W\sqrt{Var(W_t)}, \quad [8]$$

where $L_W (L_W > 0)$ is the SCSEWMA control limit constant which is chosen such that the attained ARL_0 is equal to some prespecified ARL_0. The SCSEWMA $\bar{X}$ scheme gives a signal when W_t plots beyond the control limits defined in Eq [8]. Otherwise, the process is said to be IC.

Note that when the process has been running for a very long time, i.e. $t \to \infty$, the term $(1-(1-\lambda)^{2t})$ converges towards one so that the asymptotic variance of W_t is reduced to:

$$Var(W_t) = \left((1-\omega)(1-\omega+2\lambda\omega) + \frac{\lambda\omega^2}{2-\lambda}\right)\frac{\sigma_0^2}{n}. \quad [9]$$

Next, for Case U, the proposed SCSEWMA $\bar{X}$ scheme is implemented in two regimes known as Phase I and Phase II. The process parameters μ_0 and σ_0 are estimated in Phase I, and these parameters are used to calculated the control limits of the SCSEWMA $\bar{X}$ scheme. Thereafter, in Phase II, the control limits found in Phase I are used to continuously monitor the process. That is, In Case U, the IC process parameters μ_0 and σ_0 are estimated in Phase I using m reference samples each of size n when the process is deemed to be IC. The unbiased estimators for μ_0 and σ_0 are defined by

$$\hat{\mu}_0 = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n} X_{ji}}{mn} \quad [10a]$$

and

$$\hat{\sigma}_0 = \frac{\sqrt{\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}(X_{ji}-\bar{X}_j)^2}{m(n-1)}}}{c_{4,m}}, \quad [10b]$$

respectively; where $\{X_{ji}: j=1,\ldots,m \text{ and } i=1,\ldots,n\}$ is a sequence of IC Phase I observations which follow a $N(\mu_0, \sigma_0^2)$ distribution, with $\bar{X}_j = \sum_{i=1}^{n} X_{ji}/n$ and the un-biasing constant is given by $c_{4,m} = \frac{\sqrt{2}\,\Gamma\left(\frac{m(n-1)+1}{2}\right)}{\sqrt{m(n-1)}\,\Gamma\left(\frac{m(n-1)}{2}\right)}$; see for example Abbas[27]. Thus, in Case U, the mean and variance of the SCSEWMA $\bar{X}$ charting statistic are given by

$$E(W_t) = \hat{\mu}_0 \quad [11a]$$

and

$$Var(W_t) = \left((1-\omega)(1-\omega+2\lambda\omega) + \frac{\lambda\omega^2}{2-\lambda}(1-(1-\lambda)^{2t})\right)\frac{\hat{\sigma}_0^2}{n}, \quad [11b]$$

where $\hat{\mu}_0$ and $\hat{\sigma}_0$ are defined in Eqs [10a] and [10b], respectively. Consequently, the control limits of the SCSEWMA $\bar{X}$ scheme, $\widehat{LCL}_t$ and $\widehat{UCL}_t$, are computed by substituting Eqs [11a] and [11b] in Eq [8].

3.2 Run-length performance metrics

To evaluate the performance or sensitivity of a monitoring scheme, the SPM literature widely recommends the use of the characteristics based on the run-length such as the average run-length (*ARL*), the standard deviation of the run-length (*SDRL*) as well as the percentiles of the run-length (*PRL*) where the 50^{th} *PRL* represents the median run-length (*MRL*). The run-length variable represents the number of rational samples plotted on the scheme before it gives an OOC signal for the first time. Table 1 shows how to compute the Cases K and U characteristics of the run-length of the proposed monitoring scheme using Monte Carlo simulations.

Note that the run-length distribution can be computed using two different approaches generally known as the zero-state and steady-state modes. In zero-state, it is assumed that a significant change in the process occurs when the process starts. In other words, the process starts in an OOC state. However, in steady-state, the process starts IC and a significant change occurs at a random time after it has been running for a while. The zero-state and steady-state modes are used to characterise the short-term and long-term run-length properties of a monitoring scheme. In this paper, the main focus is on the zero-state properties of the proposed scheme. Nevertheless, in Section 4.3.2, the steady-state performance of the proposed scheme is compared to the zero-state one.

Table 1. Computation of the Cases K and U run-length characteristics of the proposed SCSEWMA $\bar{X}$ monitoring scheme

Case K

Steps	***Search of the optimal L_W values and computation of the attained ARL_0 value for some prespecified ARL_0 value***
1	Specify the process parameters μ_0 and σ_0 (say, $\mu_0 = 0$ and $\sigma_0 = 1$), the sample size (n), the smoothing parameter λ, the weighing parameter (ω), the number of simulation (r) and the nominal (i.e. prespecified) ARL_0 value.
2	Set L_W to some value and calculate the control limits, LCL and UCL, using Eq [8].
3	At the t^{th} sampling time, generate a $N(\mu_0 + \delta\sigma_0,\sigma_0)$ distribution of size n where $\delta = 0$, say from $N(0,1)$ distribution.
4	Calculate the charting statistic W_t using Eq [6].
5	Compare the charting statistic found in Step 4 to the control limits computed in Step 2. If $W_t \in (LCL_t, UCL_t)$ then return to Step 3. Otherwise, the scheme gives a signal. Record the number of samples needed to get an OOC signal. This is one value of the run-length vector.
6	Repeat Steps 3 to 5 r times (say, 50000 times).
7	Once the run-length vector, $RL_{(rx1)}$, is obtained, calculate the ARL_0 as $ARL_0 = \frac{1}{r}\sum_{i=1}^{r} RL_i$. This value represents the attained ARL_0. PROC UNIVARIATE can be used in SAS to find other characteristics of the run-length vector (or distribution).
8	If the attained ARL_0 value is much closer or equal to the nominal ARL_0 value, records the L_W value. Otherwise return to Step 2; then increase the value of L_W if the attained ARL_0 is smaller than the nominal ARL_0 or decrease the value of L_W if the attained ARL_0 is larger than the nominal ARL_0.
	Computation of the OOC ARL value
9	Specify the shift δ (say, $\delta \in \{0.1, 0.2, 0.3, \ldots, 2\}$).
10	Use Steps 3 to 7 using the L_W found in Step 8 using $\delta \neq 0$.

Case U

Steps	***Estimation of the process parameters, search for L_W and the attained ARL_0 value for some prespecified ARL_0 value***
1	Specify the Phase I sample size (m), the Phase II sample size (n), the smoothing parameter λ, the weighing parameter (ω), the number of simulation (r) and the nominal (i.e. prespecified) ARL_0 value.
2	Generate m Phase I samples from a $N(\mu_0,\sigma_0)$ distribution, each of size n, say from $N(0,1)$.
3	Determine c_{4m} and afterwards, estimate the process parameters μ_0 and σ_0 denoted as $\hat{\mu}_0$ and $\hat{\sigma}_0$ using Eqs [10a] and [10b].
4	At the t^{th} sampling time, generate a Phase II sample from $N(\mu_0 + \delta\sigma_0,\sigma_0)$ distribution of size n where $\delta = 0$, say from $N(0,1)$.
5	Set L_W to some value and calculate the control limits, LCL_t and UCL_t, by substituting $\hat{\mu}_0$ and $\hat{\sigma}_0$ found in Step 3 in Eq [8].
6	Calculate the charting statistic W_t using Eq [6].
7	Compare the charting statistic found in Step 6 to the control limits computed in Step 5. If $W_t \in \left(\widehat{LCL}_t, \widehat{UCL}_t\right)$ then return to Step 4. Otherwise, the scheme gives a signal. Record the number of samples needed to get an OOC signal. This is one value of the run-length vector.
8	Repeat Steps 2 to 7 r times (say, 50000 times).
9	Once the run-length vector, $RL_{(rx1)}$, is obtained, calculate the ARL_0 as $ARL_0 = \frac{1}{r}\sum_{i=1}^{r} RL_i$. This value represents the attained ARL_0. PROC UNIVARIATE can be used in SAS to find other characteristics of the run-length vector (or distribution).
10	If the attained ARL_0 value is much closer or equal to the nominal ARL_0 value, records the L_W value. Otherwise return to Step 2; then increase the value of L_W if the attained ARL_0 is smaller than the nominal ARL_0 or decrease the value of L_W if the attained ARL_0 is larger than the nominal ARL_0.
	Computation of the OOC ARL value
11	Specify the shift δ (say, $\delta \in \{0.1, 0.2, 0.3, \ldots, 2\}$).
12	Use Steps 4 to 9 using the L_W found in Step 10 using $\delta \neq 0$.

Note though that the aforementioned metrics evaluate the performance of a scheme for a specific shift and not for a range of shift values or overall performance. To overcome this shortcoming, many researchers have recommended the use of the expected values of the previous characteristics such as the expected *ARL* (*EARL*), the expected *SDRL* (*ESDRL*) and the expected median run-length (*EMRL*). Mathematically, the *EARL*, *ESDRL* and *EMRL* are defined by

$$EARL = \frac{1}{\Delta}\sum_{\delta=\delta_{\min}}^{\delta_{\max}} ARL(\delta),\ ESDRL = \frac{1}{\Delta}\sum_{\delta=\delta_{\min}}^{\delta_{\max}} SDRL(\delta)$$

and [12]

$$EMRL = \frac{1}{\Delta}\sum_{\delta=\delta_{\min}}^{\delta_{\max}} MRL(\delta),$$

respectively, where the $\delta \in [\delta_{\min}, \delta_{\max}]$, Δ is the number of increments from $\delta_{\min}$ to $\delta_{\max}$ of the Riemann sum, $ARL(\delta)$, $SDRL(\delta)$ and $MRL(\delta)$ are the *ARL*, *SDRL* and *MRL* for a specific shift δ in the process parameter. In this paper, we use increments of 0.1 in the summations in Eq [12], with $\delta_{\min}$= 0.1 and $\delta_{\max}$= 2. Based on the latter, it follows that Δ=20. Note that control chart constants are determined such that the attained ARL_0 is equal or almost equal to the prespecified value of 370.4.

4. Performance analysis of the SCSEWMA $\bar{X}$ scheme

In this section, the robustness of the proposed SCSEWMA $\bar{X}$ scheme is investigated under Cases K and U. In addition, the Case K (i.e. when $m = \infty$) and Case U (i.e. when $m \neq \infty$) OOC performances of the SCSEWMA $\bar{X}$ scheme are also investigated.

4.1 IC robustness of the SCSEWMA $\bar{X}$ scheme

The IC robustness is very important in the design and implementation of a monitoring scheme in order to be certain of its shift detection capability in different situations such as the departures from the ideal (e.g. departure from the normal assumption). Balakrishnan et al[28] (see p.7299) defined the robustness as "a procedure that performs well not only under ideal conditions under which it is designed but also under the departure from the ideal". Thus, to investigate the IC robustness of the SCSEWMA $\bar{X}$ scheme, we used five different distributions, the standard normal distribution (denoted as *N*(0,1)), the Student's *t* distribution with degrees of freedom 5 and 25 (denoted as *t*(5) and *t*(25), respectively), and the gamma distribution with shape parameters $\alpha = 3$ and 20 and scale parameter β =1 (denoted as *G*(3,1) and *G*(20,1), respectively). The IC robustness is investigated when $\omega \in \{0.0, 0.1, 0.2, \ldots, 0.9, 1.0\}$ and $\lambda \in \{0.1, 0.5, 0.9\}$, $n = 5$, $m = \infty$ (i.e. Case K) and $m = 10$ and 100 (in Case U) for a prespecified ARL_0 of 370.4. From Table 2, it can be seen that overall, the SCSEWMA $\bar{X}$ scheme is not IC robust since the ARL_0 profile is not the same across all continuous distributions. However, under the *t* distribution, as the degrees of freedom increases, the SCSEWMA $\bar{X}$ scheme becomes robust. Under the gamma distribution, as the shape parameter increases, the SCSEWMA $\bar{X}$ scheme also gets more robust. In addition, for both Cases K and U, the SCSEWMA $\bar{X}$ scheme is more robust for large values of ω. For other Phase I sample sizes, the findings remain the same except for very small Phase I sample sizes (see, for example, Table 2 when m = 10) where the results fluctuate significantly because of the large variability in the run-length distribution. Note though, both the EWMA and Shewhart $\bar{X}$ schemes are not IC robust. It is worth mentioning that the EWMA $\bar{X}$ scheme (i.e. ω=1) tend towards robustness faster than the Shewhart $\bar{X}$ scheme (i.e. ω=0) under the above-mentioned situations.

Table 2. ARL_0 profile of the SCSEWMA $\bar{X}$ scheme along with the corresponding control limits constants when n = 5, $\lambda \in \{0.1, 0.5, 0.9\}$, $m \in \{10, 100\}$ (i.e. Case U) and $m = \infty$ (i.e. Case K) for different ω values under different distributions with a prespecified ARL_0 value of 370.4

		λ = 0.1						**λ = 0.5**						**λ = 0.9**					
m	***ω***	L_W	***N*(0,1)**	***t*(5)**	***t*(25)**	***G*(3,1)**	***G*(20,1)**	L_W	***N*(0,1)**	***t*(5)**	***t*(25)**	***G*(3,1)**	***G*(20,1)**	L_W	***N*(0,1)**	***t*(5)**	***t*(25)**	***G*(3,1)**	***G*(20,1)**
	0.0	3.047	371.4	118.6	324.7	292.4	399.5	3.046	370.9	112.6	321.6	292.9	363.9	3.046	369.8	114.6	314.6	300.5	375.7
	0.1	3.061	371.2	116.4	322.9	328.0	362.2	3.057	369.4	113.6	316.5	303.7	382.2	3.046	370.6	114.8	309.4	280.1	381.7
	0.2	3.065	370.7	113.3	316.4	337.8	390.5	3.056	371.2	116.3	309.8	336.5	368.1	3.047	370.8	116.1	316.8	320.3	384.0
	0.3	3.086	370.9	111.6	317.3	393.1	367.1	3.061	371.6	115.5	329.9	337.0	359.1	3.049	371.3	114.4	304.7	303.3	377.0
	0.4	3.100	371.4	114.9	305.8	423.5	372.7	3.067	371.0	114.2	320.5	347.2	379.4	3.052	369.8	116.2	314.5	317.0	368.9
10	0.5	3.123	370.0	114.4	305.3	420.6	414.3	3.077	370.7	119.3	316.7	355.6	364.9	3.056	371.4	116.4	306.0	342.4	392.6
	0.6	3.152	370.0	116.0	296.2	446.8	370.7	3.082	371.2	122.9	319.9	367.5	381.4	3.055	369.4	116.5	326.9	320.5	392.3
	0.7	3.173	371.2	117.8	305.3	381.2	363.8	3.093	370.5	125.5	316.6	395.0	389.1	3.059	370.9	118.1	321.0	320.5	385.1
	0.8	3.201	369.2	132.0	337.7	334.4	348.7	3.099	371.9	128.2	328.4	411.2	366.1	3.059	371.7	115.7	324.7	321.3	386.0
	0.9	3.188	371.1	180.5	325.1	311.3	368.3	3.100	369.9	135.8	311.5	388.7	365.9	3.061	369.9	118.8	310.0	325.9	399.2
	1.0	3.108	370.1	280.8	359.9	315.7	353.7	3.098	371.4	142.4	335.4	386.7	339.8	3.060	372.2	114.4	313.5	316.2	392.9
	0.0	3.183	369.7	138.8	321.6	176.7	318.5	3.182	370.5	136.7	316.1	1774.0	316.4	3.183	370.3	137.5	317.8	174.7	319.5
	0.1	3.186	370.9	136.1	322.8	178.2	327.2	3.181	370.9	137.2	317.6	177.6	321.3	3.182	370.8	136.5	322.6	174.7	318.0
	0.2	3.189	369.9	136.1	316.0	181.4	322.2	3.187	369.9	136.7	315.1	178.5	322.2	3.183	370.7	133.6	318.0	177.5	318.5
	0.3	3.193	370.3	137.4	320.6	187.0	325.7	3.185	369.9	137.0	318.0	182.5	321.3	3.184	369.9	134.6	323.4	178.9	320.9
	0.4	3.195	371.9	139.1	322.5	196.3	332.2	3.186	368.9	141.5	324.5	186.0	324.1	3.182	369.8	136.0	321.6	176.2	321.7
100	0.5	3.203	370.8	140.6	321.6	204.5	335.2	3.190	370.5	140.9	323.7	190.4	327.3	3.184	370.2	137.0	322.7	176.4	318.8
	0.6	3.209	371.4	145.8	327.0	221.3	336.5	3.188	370.1	144.2	324.0	196.5	328.1	3.184	369.6	136.4	316.1	177.8	317.9
	0.7	3.214	370.1	156.0	329.9	245.6	349.2	3.190	368.9	146.9	323.5	206.1	331.8	3.186	369.3	137.7	318.5	176.4	320.1
	0.8	3.203	371.9	177.6	335.6	274.2	353.9	3.189	369.7	155.4	324.5	214.7	335.6	3.184	371.1	138.2	317.3	177.5	322.2
	0.9	3.157	369.5	218.4	346.5	305.1	360.5	3.188	370.4	162.2	335.0	228.8	345.3	3.185	370.8	139.0	317.4	181.9	322.0
	1.0	3.003	369.7	290.8	361.1	334.5	361.2	3.186	370.2	175.3	337.0	240.9	345.9	3.187	371.3	140.1	318.5	181.8	323.5
	0.0	3.000	370.3	153.0	324.1	178.1	313.8	3.000	370.4	153.0	324.1	178.1	313.8	3.000	370.4	153.0	324.1	178.1	313.8
	0.1	2.999	370.3	151.7	326.7	177.9	313.0	3.001	370.4	153.1	326.2	180.2	315.2	3.000	370.8	153.0	322.6	179.8	313.5
	0.2	2.999	370.1	152.5	325.0	180.9	315.1	3.001	370.9	154.3	317.6	182.0	325.8	2.999	371.4	153.3	324.9	178.6	314.0
	0.3	2.998	370.2	152.3	322.0	183.8	315.5	2.998	371.2	154.6	324.2	183.2	313.6	2.999	370.7	152.1	324.9	179.3	314.1
	0.4	2.998	370.5	155.6	327.5	187.7	315.1	2.999	369.6	156.7	328.3	186.3	316.1	3.000	369.7	151.7	328.7	180.0	314.8
∞	0.5	2.998	370.8	157.2	328.9	193.9	321.7	2.998	370.6	158.7	330.6	190.9	320.2	2.998	371.4	151.9	322.0	179.1	312.4
	0.6	2.995	369.9	164.3	334.6	201.5	324.5	2.998	370.6	161.4	329.1	192.6	319.5	2.999	371.9	153.2	323.3	180.5	316.0
	0.7	2.985	370.7	173.1	337.0	219.0	330.6	2.996	369.9	168.5	339.6	202.8	328.5	2.998	369.4	153.4	323.1	180.6	313.0
	0.8	2.961	369.5	197.4	344.1	252.1	343.1	2.994	371.2	175.8	337.5	211.6	329.5	2.998	370.0	152.0	322.8	180.9	314.0
	0.9	2.885	369.5	247.9	354.1	303.3	357.6	2.986	370.3	182.4	338.9	220.7	333.1	2.997	370.9	152.9	322.1	179.5	315.6
	1.0	2.715	370.5	337.3	370.3	361.0	370.9	2.980	371.1	196.4	349.3	236.9	339.1	2.998	369.8	153.0	325.2	182.0	316.8

4.2 Case K performance analysis

4.2.1 Case K IC and OOC performances of the proposed scheme

In this section, specific and overall performances of the proposed SCEWMA $\bar{X}$ scheme are investigated in terms of the metrics introduced in Section 3.2. Tables 3 to 5 present the IC and OOC performances of the proposed SCSEWMA $\bar{X}$ scheme in terms of the *ARL*, *SDRL* and *MRL* when $0 \leq \omega \leq 1$, $\lambda = 0.1$, $n = 5$ and $\delta = 0$ (0.1) 2, respectively. The triplet (ω, λ, L) are selected for a prespecified ARL_0 = 370.4. The last row of these tables gives the overall performance values as described in Eq [12]. Thus, the results in Tables 3 to 5 can be summarised as follows:

- The width of the proposed scheme is between the widths of the EWMA and Shewhart $\bar{X}$ schemes, i.e. $L_E \leq L_W \leq k$.
- For small and moderate shifts in the process mean, in terms of the OOC *ARL*, *SDRL* and *MRL* profiles, the proposed SCSEWMA $\bar{X}$ scheme performs better for large values of ω, i.e. higher weight. For instance, if $\delta = 0.1$, as the weighing parameter increases (i.e. when $\omega = 0$, 0.5 and 1), we have ARL = 296, 210 and 101, respectively (see Table 3). These findings also hold in terms of the overall performance. For instance, when $\omega = 0$, 0.5 and 1, the SCSEWMA $\bar{X}$ scheme yields *EARL* = 36.7, 20.7 and 9.6, respectively (see Table 3), which shows a significant increase in the overall sensitivity of the proposed scheme as ω increases. A similar pattern is observed in terms of the *SDRL* and *ESDRL* values (see Table 4).
- For large shifts, there is 50% chance that the proposed scheme gives a signal between the first and the second subgroups; that is, the *MRL* values for large shifts varies between 1 and 2 (see Table 5).
- The higher the shift in the process parameter, the more sensitive the SCSEWMA $\bar{X}$ scheme becomes.
- The patterns in the sensitivity of the proposed scheme in terms of the *ARL*, *SDRL* and *MRL* profile is similar. This conclusion also holds in terms of the overall performance profiles.

Table 3. Case K *ARL* and *EARL* profiles of the SCSEWMA scheme along with their corresponding L_W values when ω =0 (0.1) 1, $\lambda = 0.1$ and *n*=5 for a nominal ARL_0 value of 370.4

ω → L_W →	0 3.000	0.1 2.999	0.2 2.999	0.3 2.998	0.4 2.998	0.5 2.998	0.6 2.995	0.7 2.985	0.8 2.961	0.9 2.885	1.0 2.715
Shift↓	***ARL* profile ↓**										
0.0	370.3	370.3	370.1	370.2	370.5	370.8	369.9	370.7	369.5	369.5	370.4
0.1	295.9	284.3	272.5	253.9	232.6	209.9	182.5	157.2	130.0	109.6	101.3
0.2	177.7	161.1	143.8	122.0	102.4	83.6	66.6	52.7	41.4	33.6	31.5
0.3	99.5	85.7	72.7	60.8	48.7	39.5	31.1	24.8	19.6	15.9	15.2
0.4	56.6	48.1	40.1	33.0	27.1	21.9	17.8	14.6	11.7	9.3	9.2
0.5	33.1	28.3	23.8	20.0	16.6	14.0	11.8	9.7	7.9	6.3	6.3
0.6	20.5	17.6	15.1	13.0	11.3	9.6	8.3	7.1	5.8	4.6	4.7
0.7	13.2	11.6	10.3	9.0	8.0	7.1	6.2	5.4	4.4	3.5	3.7
0.8	8.8	8.0	7.3	6.5	5.9	5.3	4.8	4.2	3.5	2.8	3.0
0.9	6.2	5.7	5.3	4.9	4.6	4.2	3.8	3.4	2.9	2.3	2.5
1.0	4.5	4.2	4.0	3.8	3.6	3.3	3.1	2.8	2.4	2.0	2.1
1.1	3.4	3.3	3.2	3.0	2.9	2.7	2.6	2.4	2.1	1.7	1.9
1.2	2.7	2.6	2.5	2.4	2.4	2.3	2.2	2.0	1.8	1.5	1.7
1.3	2.2	2.1	2.1	2.1	2.0	2.0	1.9	1.8	1.6	1.4	1.5
1.4	1.8	1.8	1.8	1.7	1.7	1.7	1.6	1.6	1.4	1.3	1.4
1.5	1.6	1.6	1.5	1.5	1.5	1.5	1.5	1.4	1.3	1.2	1.3
1.6	1.4	1.4	1.4	1.4	1.4	1.4	1.3	1.3	1.2	1.1	1.2
1.7	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.2	1.2	1.1	1.2
1.8	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1	1.1
1.9	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.1
2.0	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0
***EARL* profile →**	36.7	33.6	30.6	27.2	23.9	20.7	17.6	14.8	12.2	10.1	9.6

Note: The control limits constants were rounded off at 3 decimal places to conserve space

Table 4. Case K *SDRL* and *ESDRL* profiles of the SCSEWMA scheme along with their corresponding L_W values when ω =0 (0.1) 1, $\lambda = 0.1$ and *n*=5 for a nominal ARL_0 value of 370.4

ω → L_W →	0 3.000	0.1 2.999	0.2 2.999	0.3 2.998	0.4 2.998	0.5 2.998	0.6 2.995	0.7 2.985	0.8 2.961	0.9 2.885	1.0 2.715
Shift↓	***SDRL* profile ↓**										
0.0	369.4	368.8	369.7	368.9	370.2	372.6	369.4	368.1	374.8	384.9	377.5
0.1	296.5	281.9	272.5	250.8	231.1	206.1	178.7	152.3	128.5	108.2	97.3
0.2	177.5	157.7	140.4	119.2	98.0	78.5	60.5	46.3	36.0	29.6	26.2
0.3	99.0	84.0	69.6	56.7	43.3	33.7	24.7	19.1	14.9	12.5	11.1
0.4	55.9	46.9	37.2	29.3	22.4	16.9	12.8	10.1	8.1	6.9	6.1
0.5	32.8	26.6	21.1	16.6	12.8	10.1	8.0	6.2	5.2	4.5	3.9
0.6	20.0	16.1	12.9	10.3	8.3	6.6	5.4	4.4	3.6	3.1	2.8
0.7	12.6	10.4	8.5	6.9	5.7	4.8	3.9	3.3	2.7	2.3	2.1
0.8	8.3	7.0	5.9	5.0	4.2	3.6	3.0	2.5	2.1	1.8	1.6
0.9	5.6	4.9	4.2	3.7	3.2	2.8	2.4	2.0	1.7	1.4	1.3
1.0	3.9	3.5	3.1	2.8	2.5	2.2	1.9	1.6	1.4	1.1	1.1
1.1	2.9	2.6	2.4	2.2	1.9	1.7	1.6	1.4	1.1	0.9	0.9
1.2	2.1	2.0	1.8	1.7	1.6	1.4	1.3	1.1	1.0	0.8	0.8
1.3	1.6	1.5	1.4	1.3	1.2	1.2	1.1	1.0	0.8	0.6	0.7
1.4	1.2	1.1	1.1	1.1	1.0	0.9	0.9	0.8	0.7	0.5	0.6
1.5	1.0	0.9	0.9	0.9	0.8	0.8	0.7	0.7	0.6	0.4	0.5
1.6	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.5	0.4	0.4
1.7	0.6	0.6	0.6	0.6	0.5	0.5	0.5	0.5	0.4	0.3	0.4
1.8	0.5	0.5	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.2	0.3
1.9	0.4	0.4	0.4	0.4	0.3	0.3	0.3	0.3	0.3	0.2	0.2
2.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.1	0.2
***ESDRL* profile →**	36.2	32.5	29.3	25.5	22.0	18.7	15.5	12.7	10.5	8.8	7.9

Note: The control limits constants were rounded off at 3 decimal places to conserve space

Table 5. Case K *MRL* and *EMRL* profiles of the SCSEWMA scheme along with their corresponding L_W values when ω =0 (0.1) 1, $\lambda = 0.1$ and *n*=5 for a nominal ARL_0 value of 370.4

ω → L_W →	**0** **3.000**	**0.1** **2.999**	**0.2** **2.999**	**0.3** **2.998**	**0.4** **2.998**	**0.5** **2.998**	**0.6** **2.995**	**0.7** **2.985**	**0.8** **2.961**	**0.9** **2.885**	**1.0** **2.715**
Shift↓	***MRL* profile ↓**										
0.0	257.0	260.0	254.0	256.0	258.0	256.0	253.0	259.0	254.0	251.0	252.0
0.1	202.0	198.0	186.0	178.0	161.0	146.0	129.0	111.0	92.0	77.0	71.0
0.2	122.0	112.0	99.0	85.0	73.0	60.0	49.0	39.0	31.0	26.0	25.0
0.3	70.0	61.0	52.0	44.0	36.0	30.0	25.0	20.0	16.0	13.0	13.0
0.4	39.0	34.0	30.0	25.0	21.0	18.0	15.0	12.0	10.0	8.0	8.0
0.5	23.0	20.0	18.0	16.0	14.0	12.0	10.0	9.0	7.0	5.0	6.0
0.6	14.0	13.0	12.0	11.0	9.0	8.0	7.0	6.0	5.0	4.0	4.0
0.7	9.0	9.0	8.0	7.0	7.0	6.0	5.0	5.0	4.0	3.0	3.0
0.8	6.0	6.0	6.0	5.0	5.0	5.0	4.0	4.0	3.0	2.0	3.0
0.9	4.0	4.0	4.0	4.0	4.0	4.0	3.0	3.0	3.0	2.0	2.0
1.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	2.0	2.0	2.0	2.0
1.1	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.0	2.0
1.2	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.0	2.0
1.3	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.0	1.0	1.0	1.0
1.4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
***EMRL* profile →**	25.3	23.7	21.6	19.6	17.3	15.3	13.2	11.2	9.3	7.6	7.5

Note: The control limits constants were rounded off at 3 decimal places to conserve space

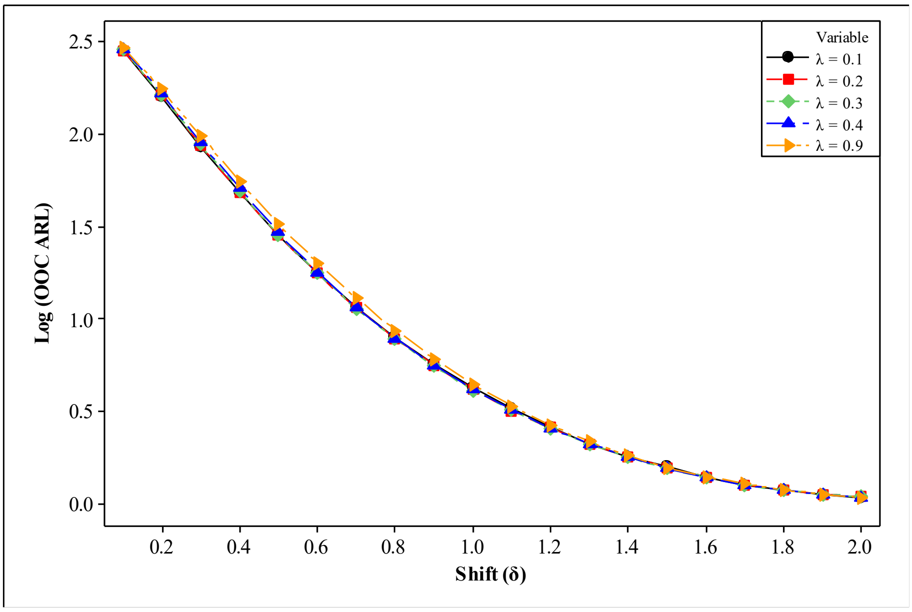
Figures 1 and 2 investigate the performance of the proposed scheme in terms of the OOC *ARL* and overall performance profiles, respectively, for different values of ω and λ. The following is observed:

(i) In terms of the OOC *ARL* profile (see Figure 1),

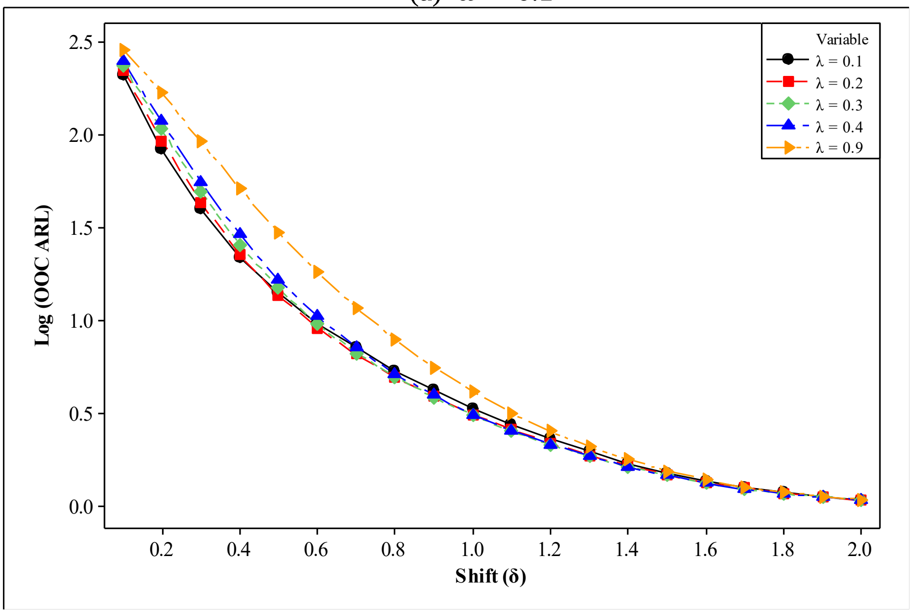
- For small values of ω, there is a slight difference in the performance of the proposed scheme for different values of λ regardless of the size of the shift in the process mean. As ω increases, the difference becomes noticeable for small and moderate shifts.
- Under small and moderate shifts, for moderate and large values of ω, the smaller the value of λ, the more sensitive the proposed scheme is. In other words, the proposed scheme performs better for small values of λ.
- For large shifts, regardless of the values of λ and ω, the performance of the proposed scheme is almost similar.

(ii) In terms of the *EARL*, *ESDRL* and *EMRL* values (see Figure 2),

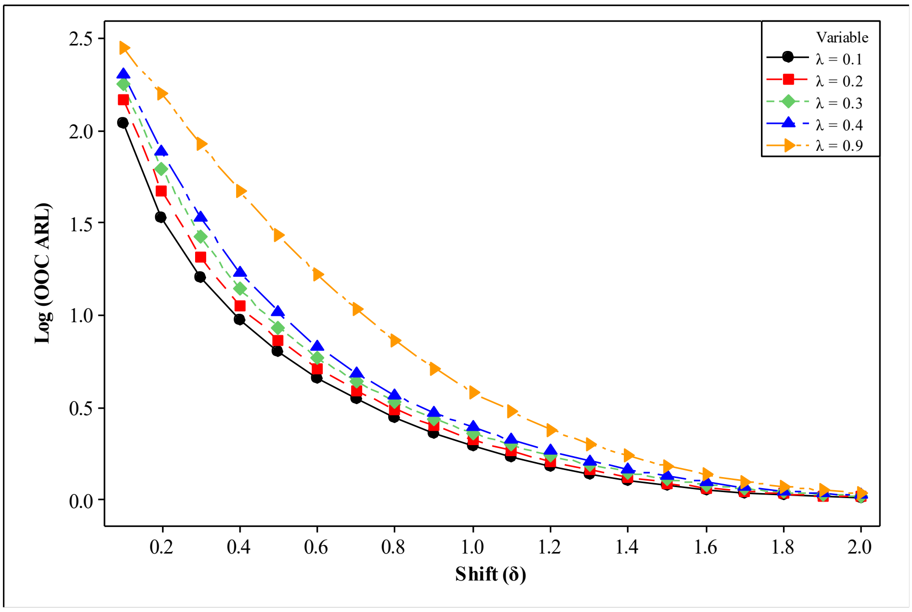
- The proposed scheme performs better for small values of λ for all values of ω except when $\omega = 0$, which is equivalent to the Shewhart $\bar{X}$ scheme, where the overall performance of the proposed scheme is the same regardless of the value of λ.
- For a fixed value of λ, the overall performance of the proposed scheme increases as ω increases.



(a) $\omega = 0.1$

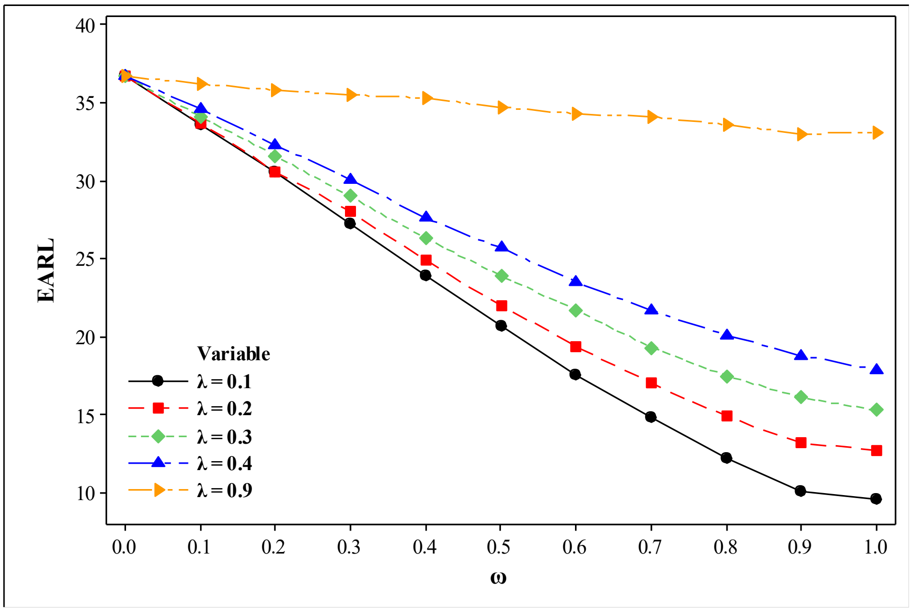


(b) $\omega = 0.5$

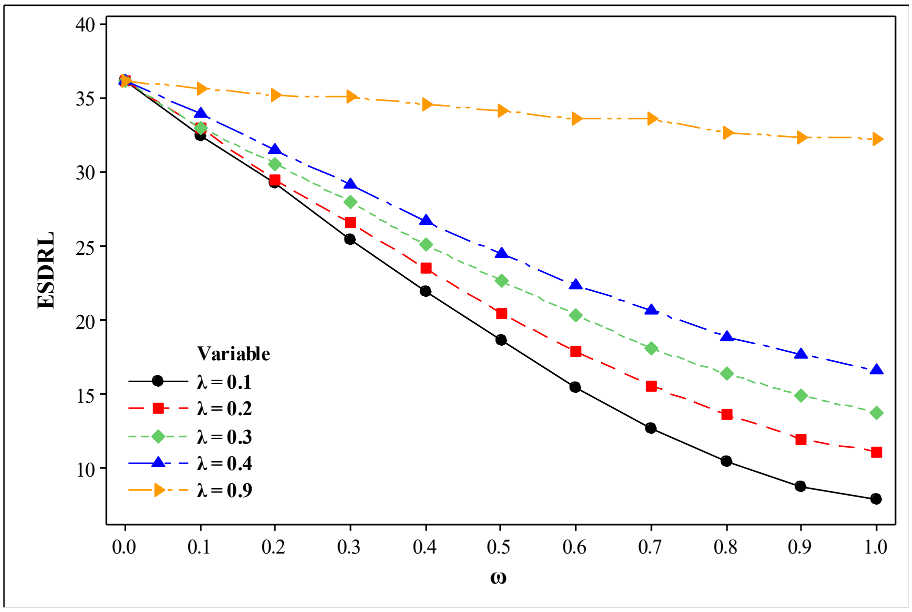


(c) $\omega = 0.9$

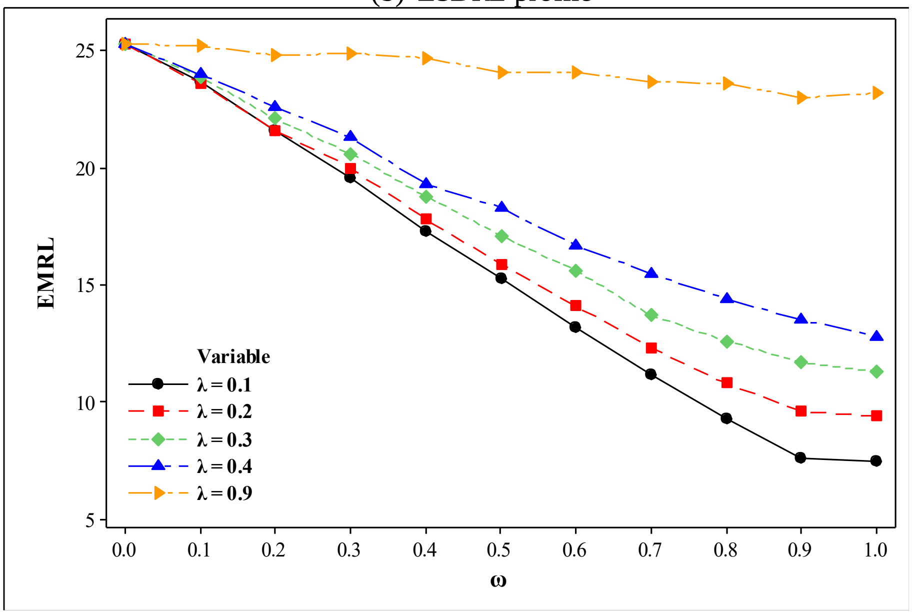
Figure 1. Sensitivity of the proposed SCSEWMA $\bar{X}$ scheme in terms of the *ARL* profile for different values of ω and λ for a prespecified ARL_0 value of 370.4



(a) *EARL* profile



(b) *ESDRL* profile



(c) *EMRL* profile

Figure 2. Overall performance of the proposed SCSEWMA $\bar{X}$ scheme when $\delta_{min} = 0.1$, $\delta_{max} = 2$, $n = 5$ for prespecified ARL_0 value of 370.4

4.2.2 Case K performance comparison of the SCSEWMA $\bar{X}$ scheme with the existing CSEWMA and CSCUSUM $\bar{X}$ schemes

In this section, the proposed scheme is compared to the existing CSEWMA and CSCUSUM $\bar{X}$ schemes. The design of the CSCUSUM $\bar{X}$ scheme entails three design parameters k, k_C and h_C as well as the control limit $H_C = h_C\sigma_{\bar{X}}$. These parameters are chosen such that the attained ARL_0 value is equal to the prespecified value of 370.4 when $n = 5$. In this comparison, we used $k_C = 0.225$, 0.5 and 0.75. The CSEWMA $\bar{X}$ scheme entails two parameters as defined in Section 2.3. The CSEWMA and SCSEWMA $\bar{X}$ schemes are investigated for $\lambda = 0.1$, 0.2 and 0.3 when $n = 5$ for a prespecified ARL_0 value of 370.4.

Tables 6 and 7 display the performance evaluation of the competing schemes in terms of the *ARL* and *MRL* profiles as well as the *EARL* and *EMRL* profiles.

From Tables 6 and 7, it can be seen that in terms of the *ARL* and *MRL* profiles, the proposed scheme with $\omega = 0.9$ is superior to the existing CSEWMA and CSCUSUM $\bar{X}$ schemes under small and moderate shifts. However, for large shifts in the process mean, the competing schemes perform almost similarly. It can also be observed that the superiority of the proposed scheme is evident in terms of the overall performance. This is also explained by small *EARL* and *EMRL* values yielded by the proposed SCSEWMA $\bar{X}$ scheme especially under small values of λ (see Tables 6 and 7).

Table 6. Case K *ARL* and *EARL* comparisons of the SCSEWMA $\bar{X}$ scheme versus the CSCUSUM and CSEWMA $\bar{X}$ schemes when n = 5 for prespecified ARL_0 value of 370.4

	CSEWMA scheme			CSCUSUM scheme			SCSEWMA scheme (ω=0.9)		
Shift	λ = 0.1	λ = 0.2	λ = 0.3	h_C=12.964	h_C=9.913	h_C=9.909	λ = 0.1	λ = 0.2	λ = 0.3
0.1	165.1	202.1	225.9	140.7	276.4	294.2	109.6	148.1	178.5
0.2	49.9	69.1	88.5	47.2	106.6	174.4	33.6	47.4	62.0
0.3	21.6	28.1	36.7	26.5	39.3	81.8	15.9	20.6	26.6
0.4	12.6	14.8	18.2	17.9	21.2	35.5	9.3	11.3	13.8
0.5	8.3	9.2	10.8	13.1	14.1	19.5	6.3	7.3	8.6
0.6	6.1	6.4	7.2	10.1	10.2	12.8	4.6	5.1	5.8
0.7	4.7	4.9	5.2	8.0	7.8	9.1	3.5	3.9	4.4
0.8	3.8	3.8	4.0	6.3	6.0	6.7	2.8	3.1	3.4
0.9	3.1	3.2	3.3	5.1	4.8	5.2	2.3	2.5	2.7
1.0	2.6	2.7	2.7	4.1	3.9	4.0	2.0	2.1	2.3
1.1	2.3	2.3	2.3	3.3	3.1	3.2	1.7	1.8	2.0
1.2	2.0	2.0	2.0	2.6	2.5	2.6	1.5	1.6	1.7
1.3	1.8	1.8	1.8	2.2	2.1	2.1	1.4	1.4	1.5
1.4	1.6	1.6	1.6	1.8	1.8	1.8	1.3	1.3	1.4
1.5	1.5	1.5	1.5	1.6	1.6	1.6	1.2	1.2	1.3
1.6	1.3	1.4	1.4	1.4	1.4	1.4	1.1	1.2	1.2
1.7	1.2	1.3	1.3	1.3	1.3	1.3	1.1	1.1	1.1
1.8	1.2	1.2	1.2	1.2	1.2	1.2	1.1	1.1	1.1
1.9	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.1
2.0	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	1.0
EARL	14.6	18.0	20.9	14.8	25.4	33.0	10.1	13.2	16.1
Control schemes constants	k = 3.068; L_E=3.203	k = 3.086; L_E= 3.239	k = 3.088; L_E = 3.249	k =3.034; k_C=0.225;	k =3.001; k_C=0.5;	k =2.999; k_C=0.75	L_W=2.885	L_W=2.934	L_W=2.959

Note: The control limit constants were round off at 3 decimal places to conserve space

Table 7. Case K *MRL* and *EMRL* comparisons of the SCSEWMA $\bar{X}$ scheme versus the CSCUSUM and CSEWMA $\bar{X}$ schemes when n = 5 for prespecified ARL_0 value of 370.4

	CSEWMA scheme			CSCUSUM scheme			SCSEWMA scheme (ω=0.9)		
Shift	λ = 0.1	λ = 0.2	λ = 0.3	h_C=12.964	h_C=9.913	h_C=9.909	λ = 0.1	λ = 0.2	λ = 0.3
0.1	117.0	141.0	156.0	106.0	193.0	204.0	77.0	102.0	123.0
0.2	38.0	49.0	63.0	42.0	79.0	121.0	26.0	34.0	44.0
0.3	18.0	21.0	26.0	25.0	34.0	59.0	13.0	16.0	20.0
0.4	11.0	12.0	14.0	18.0	20.0	29.0	8.0	9.0	11.0
0.5	7.0	8.0	9.0	13.0	14.0	18.0	5.0	6.0	7.0
0.6	5.0	6.0	6.0	11.0	10.0	12.0	4.0	4.0	5.0
0.7	4.0	4.0	5.0	9.0	8.0	9.0	3.0	3.0	4.0
0.8	3.0	3.0	4.0	7.0	6.0	6.0	2.0	3.0	3.0
0.9	3.0	3.0	3.0	5.0	4.0	4.0	2.0	2.0	2.0
1.0	2.0	2.0	3.0	3.0	3.0	3.0	2.0	2.0	2.0
1.1	2.0	2.0	2.0	3.0	3.0	3.0	1.0	2.0	2.0
1.2	2.0	2.0	2.0	2.0	2.0	2.0	1.0	1.0	2.0
1.3	2.0	2.0	2.0	2.0	2.0	2.0	1.0	1.0	1.0
1.4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
EMRL	11.1	13.1	15.1	12.7	19.3	24.0	7.6	9.6	11.7
Control schemes constants	k = 3.068; L_E=3.203	k = 3.086; L_E= 3.239	k = 3.088; L_E = 3.249	k =3.034; k_C=0.225;	k =3.001; k_C=0.5;	k =2.999; k_C=0.75	L_W=2.885	L_W=2.934	L_W=2.959

Note: The control limit constants were round off at 3 decimal places to conserve space

4.3 Case U performance analysis

4.3.1 Case U IC and OOC performances of the proposed scheme

In this section, the performance of the proposed SCSEWMA $\bar{X}$ scheme is investigated when $n \in\{1,5,10\}$, $m \in\{10, 50,100\}$, $\omega \in\{0.1,0.5,0.9\}$ and $\lambda \in\{0.1,0.5,0.9\}$ for a prespecified ARL_0 = 370.4 as reported in Tables 8 to 10.

The pattern of the Cases K and U *ARL* profiles is the same; that is, as ω increases, the performance of the SCSEWMA $\bar{X}$ scheme increases as well. The performance of the SCSEWMA $\bar{X}$ scheme in terms of the *ARL*, *SDRL*, *MRL*, *EARL*, *ESDRL* and *EMRL* profiles, deteriorates when the process parameters are estimated; see Tables 8 to 10. For instance, if (ω, λ) = (0.1,0.1), n = 5 and $\delta = 0.1$, for $m = \infty$ (i.e. Case K) and m = 100 (i.e. Case U), the SCSEWMA $\bar{X}$ scheme gives a signal on the 284$^{\text{th}}$ and 299$^{\text{th}}$ samples, respectively (see Tables 3 and 8, respectively). In Case U, the proposed SCSEWMA $\bar{X}$ scheme is more sensitive for small values of λ and large values of ω.

Table 8. Case U *ARL* and *EARL* profiles of the SCSEWMA $\bar{X}$ scheme along with their corresponding L_W values when $\omega \in \{0.1,0.5,0.9\}$, $\lambda \in \{0.1,0.5,0.9\}$, *m* = 100 and *n*=5 for a prespecified ARL_0 value of 370.4

	ω = 0.1			ω = 0.5			ω = 0.9		
Shift	λ = 0.1	λ = 0.5	λ = 0.9	λ = 0.1	λ = 0.5	λ = 0.9	λ = 0.1	λ = 0.5	λ = 0.9
0.0	370.9	370.9	369.8	369.4	276.1	371.1	370.8	371.4	370.8
0.1	298.7	298.2	308.5	238.6	143.5	298.8	167.8	254.2	297.8
0.2	174.8	181.6	185.4	100.3	69.5	180.1	45.5	109.6	174.6
0.3	93.6	98.7	103.9	44.8	35.1	98.6	19.9	47.8	92.4
0.4	52.0	54.7	58.6	24.2	19.9	54.8	11.5	23.6	51.0
0.5	29.5	31.5	34.4	15.0	12.1	31.4	7.8	13.2	29.0
0.6	18.3	19.3	21.3	10.1	8.0	19.0	5.8	8.3	17.4
0.7	12.1	12.4	13.5	7.4	5.6	12.3	4.5	5.8	11.1
0.8	8.3	8.2	8.8	5.6	4.2	8.2	3.6	4.3	7.6
0.9	5.9	5.8	6.2	4.4	3.3	5.7	3.0	3.3	5.4
1.0	4.4	4.3	4.5	3.5	2.7	4.3	2.6	2.8	3.9
1.1	3.3	3.3	3.4	2.8	2.2	3.2	2.2	2.3	3.0
1.2	2.6	2.6	2.7	2.3	1.9	2.5	2.0	2.0	2.5
1.3	2.2	2.1	2.2	2.0	1.7	2.1	1.8	1.8	2.0
1.4	1.8	1.8	1.8	1.7	1.5	1.8	1.6	1.6	1.7
1.5	1.6	1.6	1.6	1.5	1.4	1.6	1.5	1.4	1.5
1.6	1.4	1.4	1.4	1.4	1.2	1.4	1.3	1.3	1.4
1.7	1.3	1.3	1.3	1.3	1.2	1.3	1.3	1.2	1.3
1.8	1.2	1.2	1.2	1.2	1.1	1.2	1.2	1.2	1.2
1.9	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
2.0	1.1	1.1	1.1	1.1	2.0	1.1	1.1	1.1	1.1
EARL	35.8	36.6	38.1	23.5	16.0	36.5	14.4	24.4	35.3
L_W	3.186	3.184	3.183	3.203	3.190	3.184	3.155	3.188	3.185

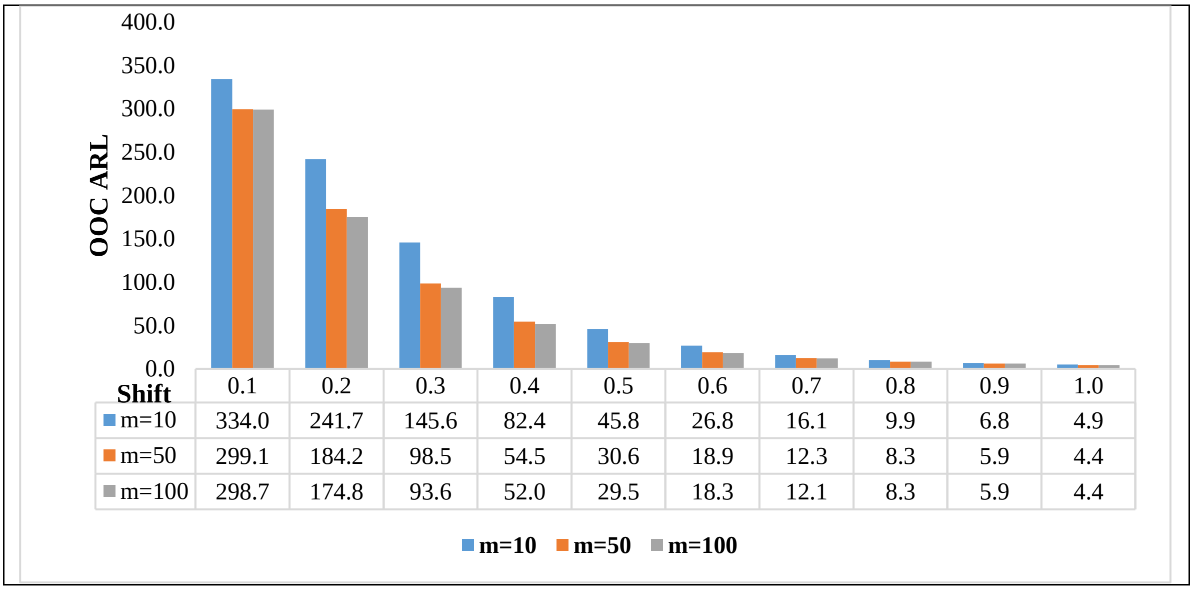
Table 9. Case U *SDRL* and *ESDRL* profiles of the SCSEWMA $\bar{X}$ scheme along with their corresponding L_W values when $\omega \in \{0.1,0.5,0.9\}$, $\lambda \in \{0.1,0.5,0.9\}$, *m* = 100 and *n*=5 for a prespecified ARL_0 value of 370.4

	ω = 0.1			ω = 0.5			ω = 0.9		
Shift	λ = 0.1	λ = 0.5	λ = 0.9	λ = 0.1	λ = 0.5	λ = 0.9	λ = 0.1	λ = 0.5	λ = 0.9
0.0	420.7	418.0	420.7	432.2	419.7	426.4	442.8	422.5	413.7
0.1	340.6	342.6	355.2	284.9	320.6	342.9	241.8	301.0	347.5
0.2	204.2	211.3	214.9	120.8	173.1	215.5	54.4	134.0	206.4
0.3	106.7	114.8	121.5	47.4	83.1	114.5	17.8	55.2	106.5
0.4	58.4	62.0	67.4	22.0	38.7	61.6	8.6	24.9	57.6
0.5	31.0	34.4	38.1	12.0	20.6	34.5	5.2	12.6	32.0
0.6	18.6	20.4	22.6	7.5	11.8	20.0	3.7	7.3	18.4
0.7	11.8	12.8	14.2	5.3	7.1	12.8	2.7	4.5	11.1
0.8	7.8	8.0	8.9	3.9	4.7	8.1	2.1	3.1	7.3
0.9	5.4	5.4	6.0	3.0	3.3	5.3	1.7	2.2	4.9
1.0	3.8	3.7	4.1	2.4	2.4	3.8	1.4	1.7	3.4
1.1	2.8	2.7	3.0	1.9	1.8	2.6	1.2	1.3	2.4
1.2	2.1	2.0	2.2	1.5	1.4	2.0	1.0	1.1	1.8
1.3	1.6	1.5	1.6	1.3	1.1	1.5	0.9	0.9	1.4
1.4	1.2	1.2	1.2	1.0	0.9	1.1	0.8	0.7	1.1
1.5	1.0	0.9	0.9	0.8	0.7	0.9	0.7	0.6	0.9
1.6	0.8	0.7	0.8	0.7	0.6	0.7	0.6	0.5	0.7
1.7	0.6	0.6	0.6	0.6	0.5	0.6	0.5	0.5	0.5
1.8	0.5	0.5	0.5	0.5	0.4	0.5	0.4	0.4	0.4
1.9	0.4	0.4	0.4	0.4	0.3	0.4	0.4	0.3	0.4
2.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
ESDRL	40.0	41.3	43.2	25.9	33.7	41.5	17.3	27.7	3.2
L_W	3.186	3.184	3.183	3.203	3.190	3.184	3.155	3.188	3.185

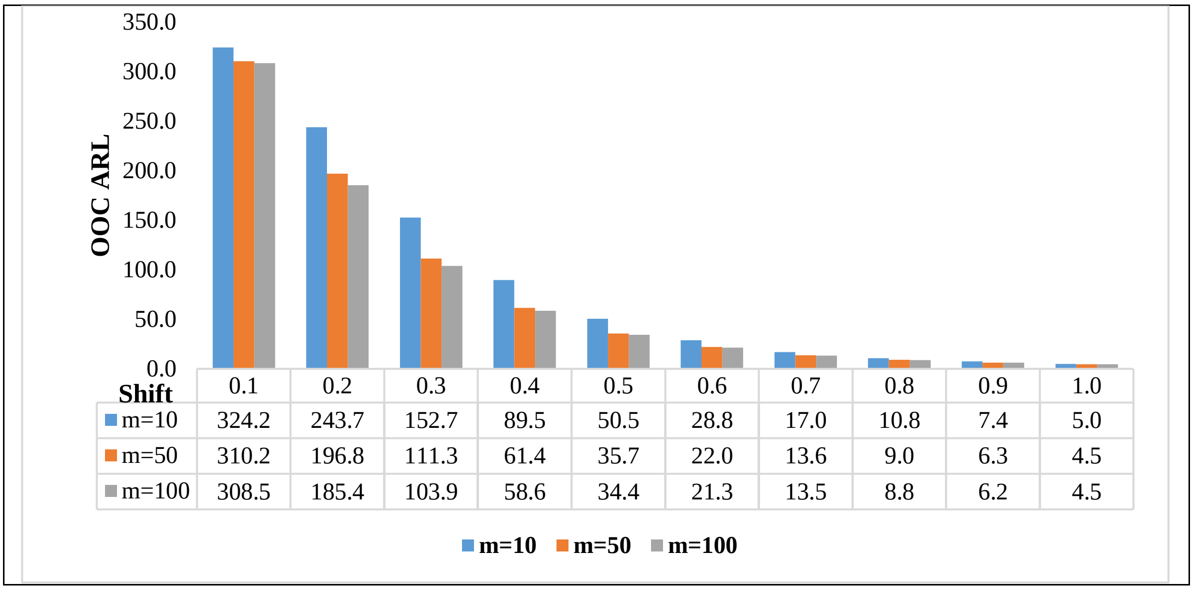
Table 10. Case U *MRL* and *EMRL* profiles of the SCSEWMA $\bar{X}$ scheme along with their corresponding L_W values when $\omega \in \{0.1,0.5,0.9\}$, $\lambda \in \{0.1,0.5,0.9\}$, m = 100 and n=5 for a prespecified ARL_0 value of 370.4

	$\omega = 0.1$			$\omega = 0.5$			$\omega = 0.9$		
Shift	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.9$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.9$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.9$
0.0	236.0	235.0	235.0	237.0	236.0	235.0	226.0	239.0	238.0
0.1	189.0	187.0	193.0	144.0	172.0	187.0	87.0	154.0	188.0
0.2	109.0	114.0	115.0	63.0	88.0	110.0	29.0	66.0	108.0
0.3	59.0	62.0	65.0	31.0	43.0	62.0	15.0	30.0	58.0
0.4	34.0	35.0	37.0	18.0	23.0	35.0	10.0	15.0	32.0
0.5	20.0	20.0	22.0	12.0	13.0	20.0	7.0	9.0	19.0
0.6	13.0	13.0	14.0	9.0	8.0	13.0	5.0	6.0	12.0
0.7	9.0	8.0	9.0	6.0	6.0	8.0	4.0	4.0	8.0
0.8	6.0	6.0	6.0	5.0	4.0	6.0	3.0	3.0	5.0
0.9	4.0	4.0	4.0	4.0	3.0	4.0	3.0	3.0	4.0
1.0	3.0	3.0	3.0	3.0	3.0	3.0	2.0	2.0	3.0
1.1	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
1.2	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
1.3	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
1.4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
EMRL	23.0	23.3	24.1	15.4	18.8	23.1	8.9	15.3	22.5
L_W	3.186	3.184	3.183	3.203	3.190	3.184	3.155	3.188	3.185

Figure 3 investigates the effect of the Phase I sample size on the Phase II performance of the proposed SCSEWMA $\bar{X}$ scheme when $\delta \in \{0.1, 0.2, \ldots, 1.0\}$, n = 5, $m \in \{10,50,100\}$ for various (ω,λ). From Figure 3, it can be observed that regardless of the values of ω and λ, the SCSEWMA $\bar{X}$ scheme performs better for large Phase I sample sizes.

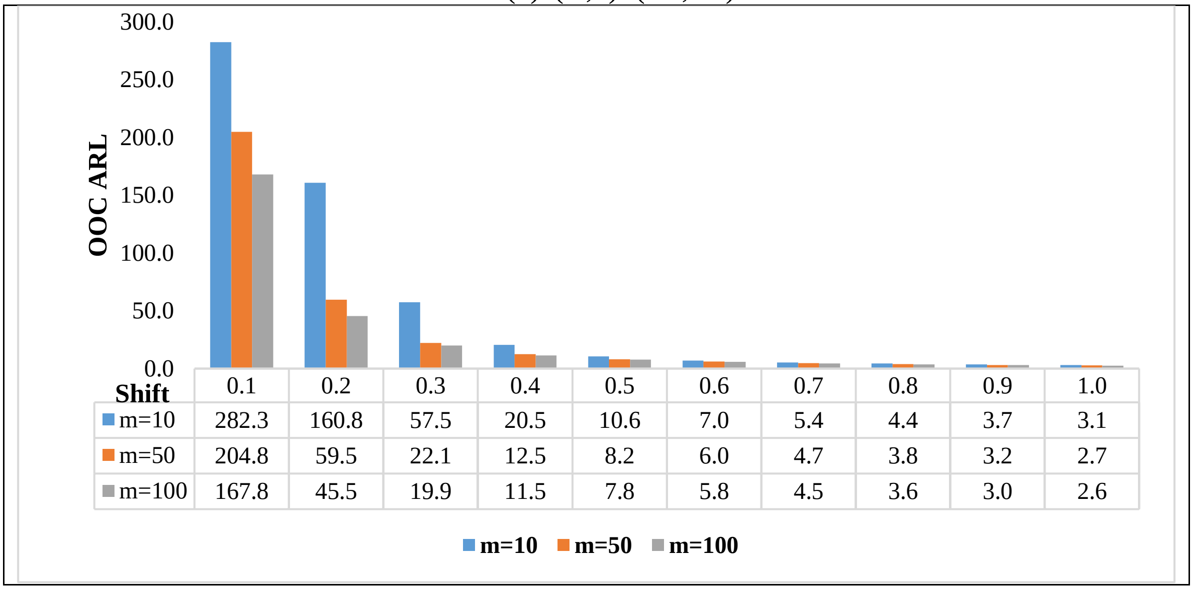


(a) (ω,λ)=(0.1,0.1)



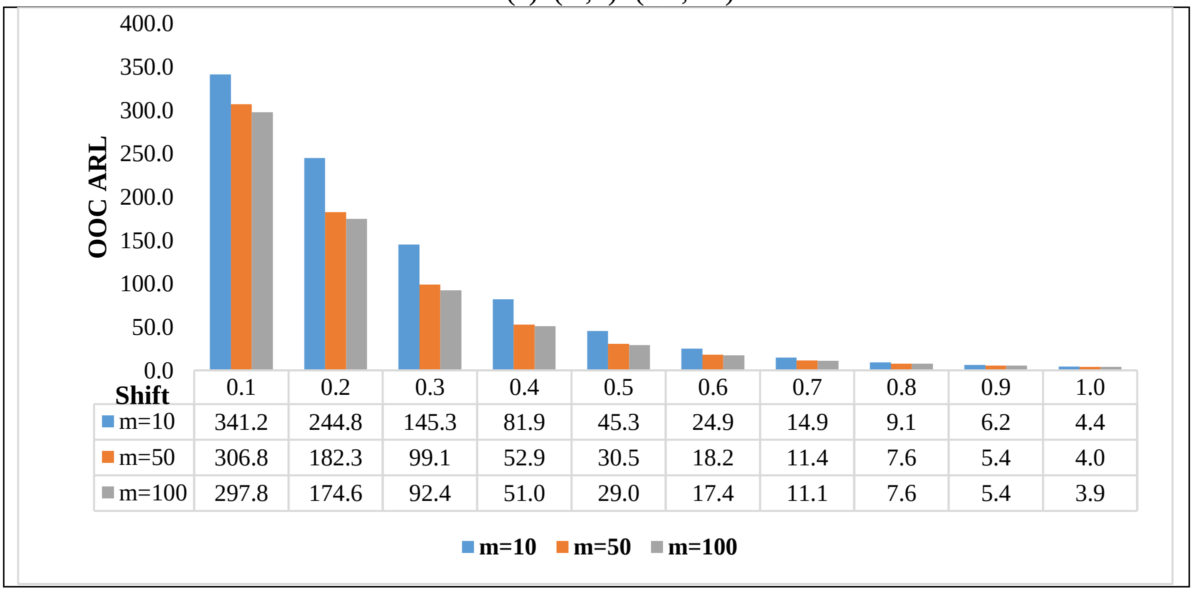
Shift	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
m=10	324.2	243.7	152.7	89.5	50.5	28.8	17.0	10.8	7.4	5.0
m=50	310.2	196.8	111.3	61.4	35.7	22.0	13.6	9.0	6.3	4.5
m=100	308.5	185.4	103.9	58.6	34.4	21.3	13.5	8.8	6.2	4.5

(b) (ω,λ)=(0.1,0.9)



Shift	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
m=10	282.3	160.8	57.5	20.5	10.6	7.0	5.4	4.4	3.7	3.1
m=50	204.8	59.5	22.1	12.5	8.2	6.0	4.7	3.8	3.2	2.7
m=100	167.8	45.5	19.9	11.5	7.8	5.8	4.5	3.6	3.0	2.6

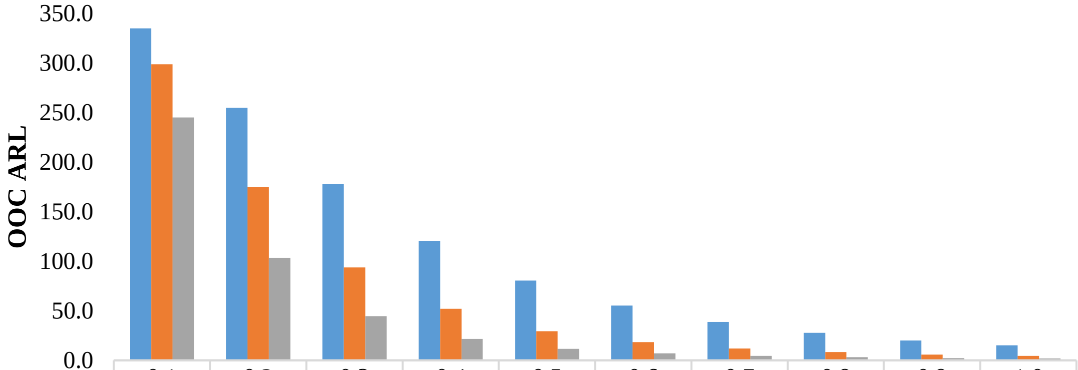
(c) (ω,λ)=(0.9,0.1)



Shift	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
m=10	341.2	244.8	145.3	81.9	45.3	24.9	14.9	9.1	6.2	4.4
m=50	306.8	182.3	99.1	52.9	30.5	18.2	11.4	7.6	5.4	4.0
m=100	297.8	174.6	92.4	51.0	29.0	17.4	11.1	7.6	5.4	3.9

(d) (ω,λ)=(0.9,0.9)

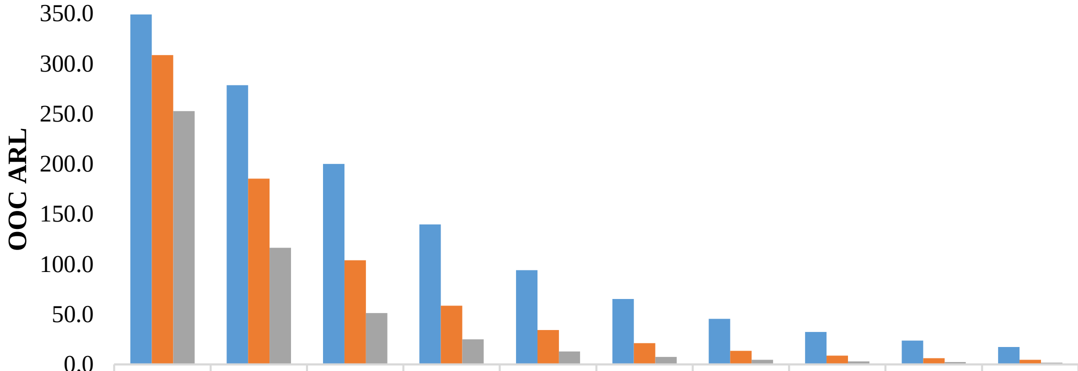
Figure 3. Effect of the Phase I sample size on the performance of the proposed SCSEWMA $\bar{X}$ scheme when $\delta \in \{0.1, 0.2, \ldots, 1.0\}$, $n = 5$, $m \in \{10, 50, 100\}$, $(\omega, \lambda) \in \{(0.1,0.1), (0.1,0.9), (0.9,0.1), (0.9,0.9)\}$ for a prespecified ARL_0 value of 370.4



Shift	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
n=2	334.6	254.7	177.6	120.6	80.6	55.1	38.9	27.8	20.0	15.1
n=5	298.7	174.8	93.6	52.0	29.5	18.3	12.1	8.3	5.9	4.4
n=10	245.1	103.3	44.5	21.7	11.8	7.0	4.5	3.1	2.3	1.8

n=2 **n=5** **n=10**

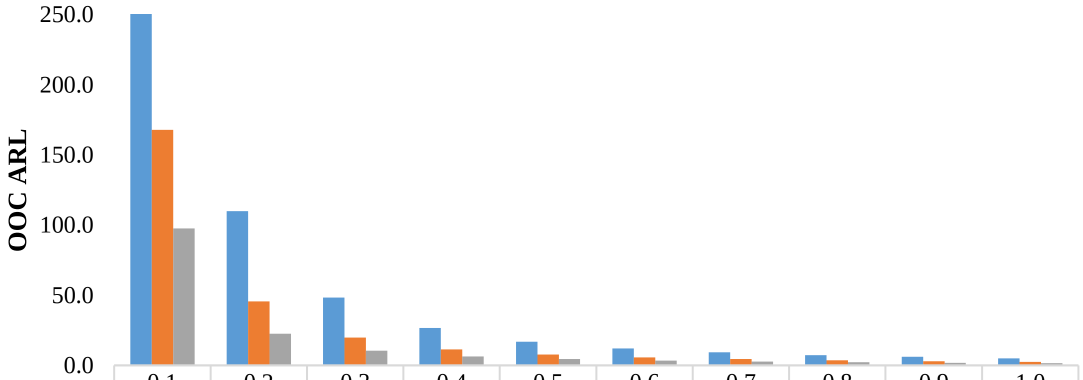
(a) (ω,λ)=(0.1,0.1)



Shift	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
n=2	349.2	278.6	200.1	139.6	93.9	65.3	45.6	32.5	23.8	17.5
n=5	308.5	185.4	103.9	58.6	34.4	21.3	13.5	8.8	6.2	4.5
n=10	252.7	116.2	51.3	25.2	13.0	7.5	4.7	3.1	2.3	1.8

n=2 **n=5** **n=10**

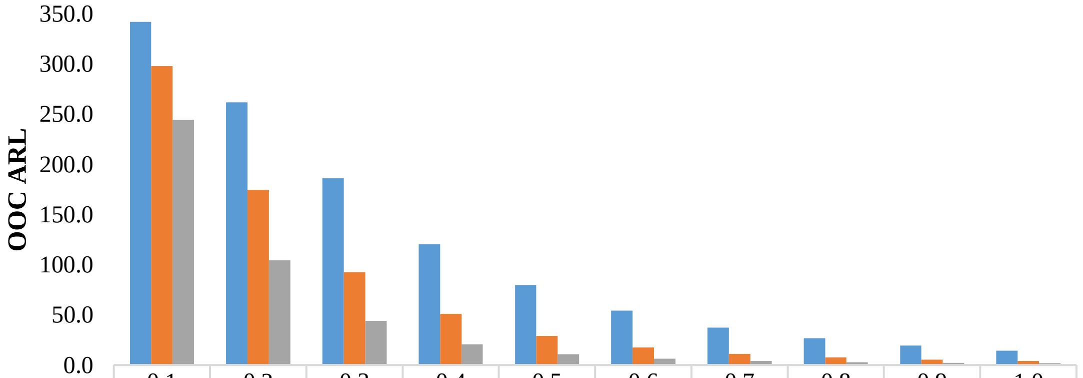
(b) (ω,λ)=(0.1,0.9)



Shift	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
n=2	250.5	110.0	48.4	26.7	17.0	12.1	9.3	7.4	6.1	5.1
n=5	167.8	45.5	19.9	11.5	7.8	5.8	4.5	3.6	3.0	2.6
n=10	97.7	22.5	10.5	6.5	4.5	3.3	2.6	2.2	1.8	1.6

n=2 **n=5** **n=10**

(c) (ω,λ)=(0.9,0.1)



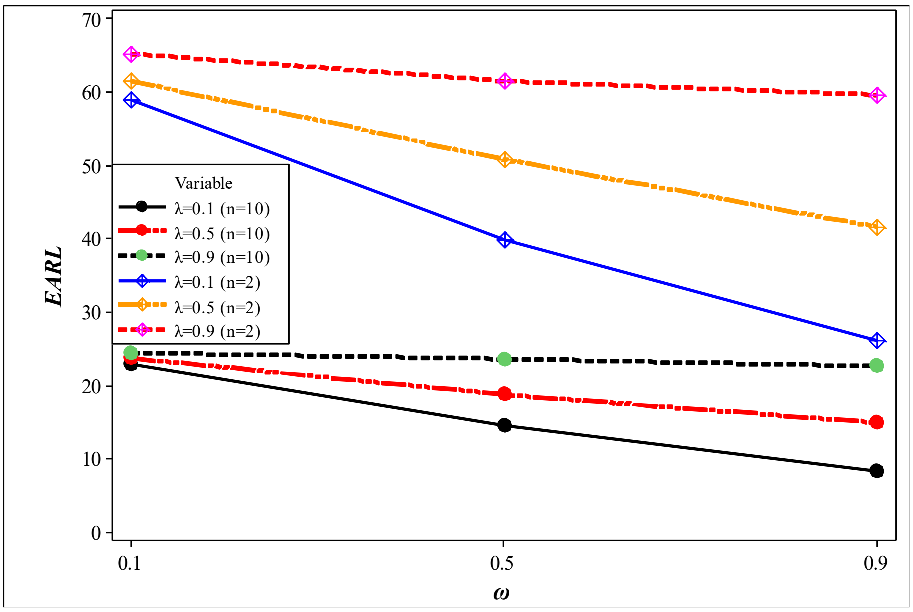
Shift	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
n=2	342.0	261.8	186.2	120.4	79.8	54.3	37.4	26.9	19.3	14.2
n=5	297.8	174.6	92.4	51.0	29.0	17.4	11.1	7.6	5.4	3.9
n=10	244.0	104.2	44.1	20.6	10.7	6.3	4.1	2.8	2.1	1.7

n=2 **n=5** **n=10**

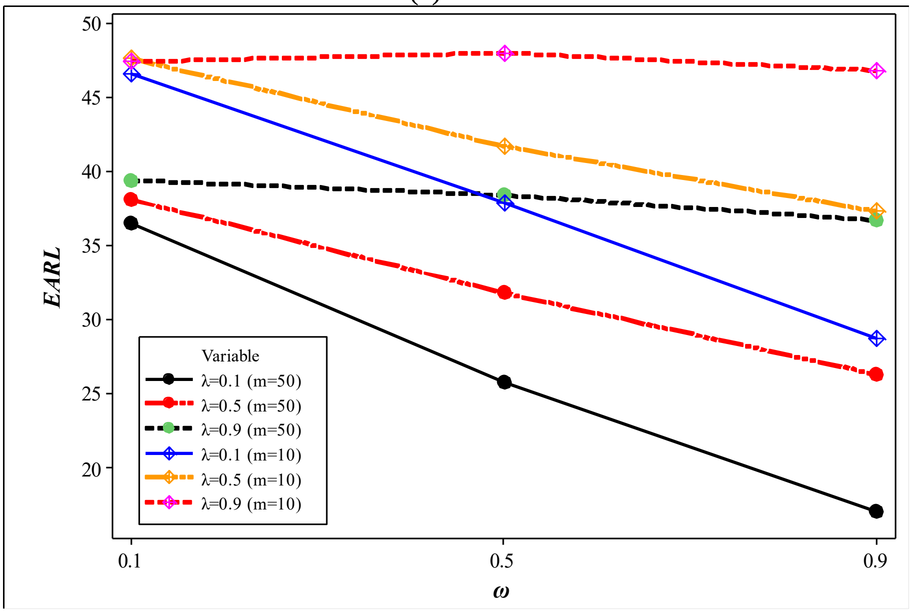
(d) (ω,λ)=(0.9,0.9)

Figure 4. Effect of the Phase II sample size on the performance of the proposed SCSEWMA $\bar{X}$ scheme when $\delta \in \{0.1, 0.2, \ldots, 1.0\}$, m = 100, $n \in \{2,5,10\}$, $(\omega,\lambda) \in \{(0.1,0.1), (0.1,0.9), (0.9,0.1), (0.9,0.9)\}$ for a prespecified ARL_0 value of 370.4

Figure 4 investigates the effect of the Phase II sample size on the performance of the proposed SCSEWMA $\bar{X}$ scheme when m = 100, $n \in \{2,5,10\}$ for various (ω,λ). For more clarity, we only display the results for $\delta \in \{0.1, 0.2, \ldots, 1.0\}$. From Figure 4, it can be seen that regardless of the values of ω and λ, the SCSEWMA $\bar{X}$ scheme performs better for large Phase II sample sizes. When both the Phase I and Phase II samples are increased, the SCSEWMA $\bar{X}$ scheme becomes more sensitive. In terms of the overall performance, Figure 5 shows that the larger the Phase I or Phase II sample, the more sensitive the proposed scheme becomes. The smaller the value of λ, the better the performance of the proposed scheme. The larger the value of ω, the more sensitive the SCSEWMA $\bar{X}$ scheme (see Figure 5).

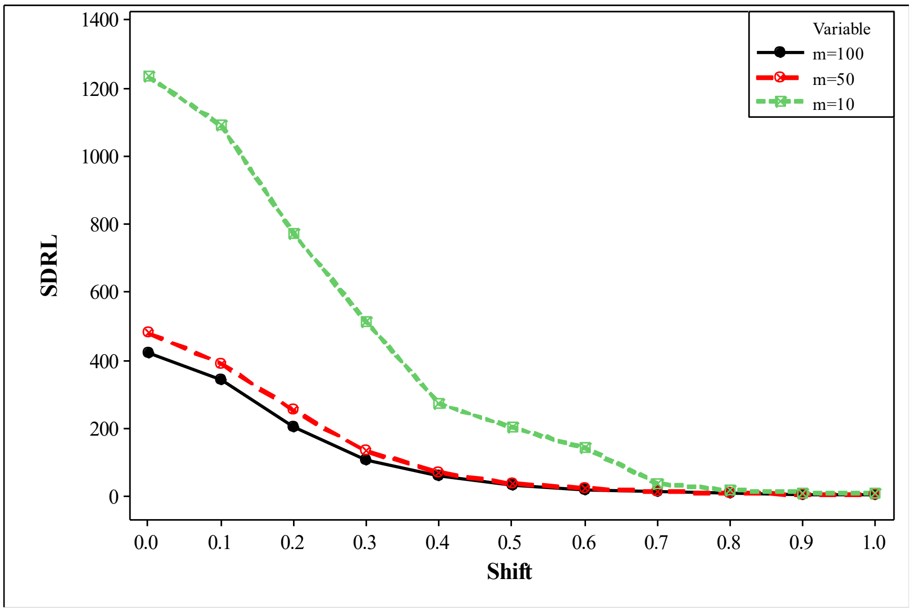


(a) m = 100

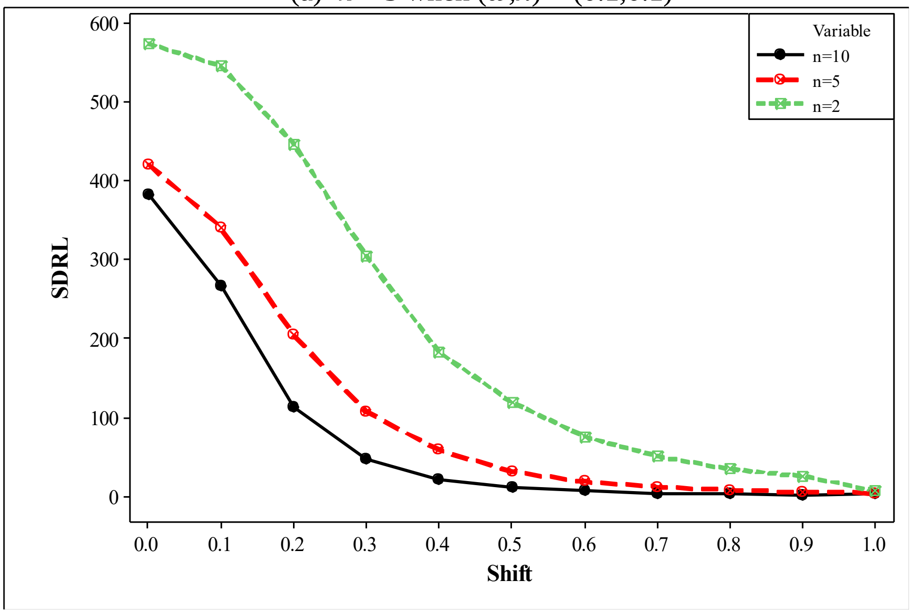


(b) n = 5

Figure 5. Effect of the Phases I and II on the overall performance of the SCSEWMA $\bar{X}$ scheme when $\omega \in \{0.1, 0.5, 0.9\}$and $\lambda \in \{0.1, 0.5, 0.9\}$ for a prespecified ARL_0 of 370.4



(a) n = 5 when (ω,λ) = (0.1,0.1)



(b) m = 100 when (ω,λ)=(0.1,0.1)

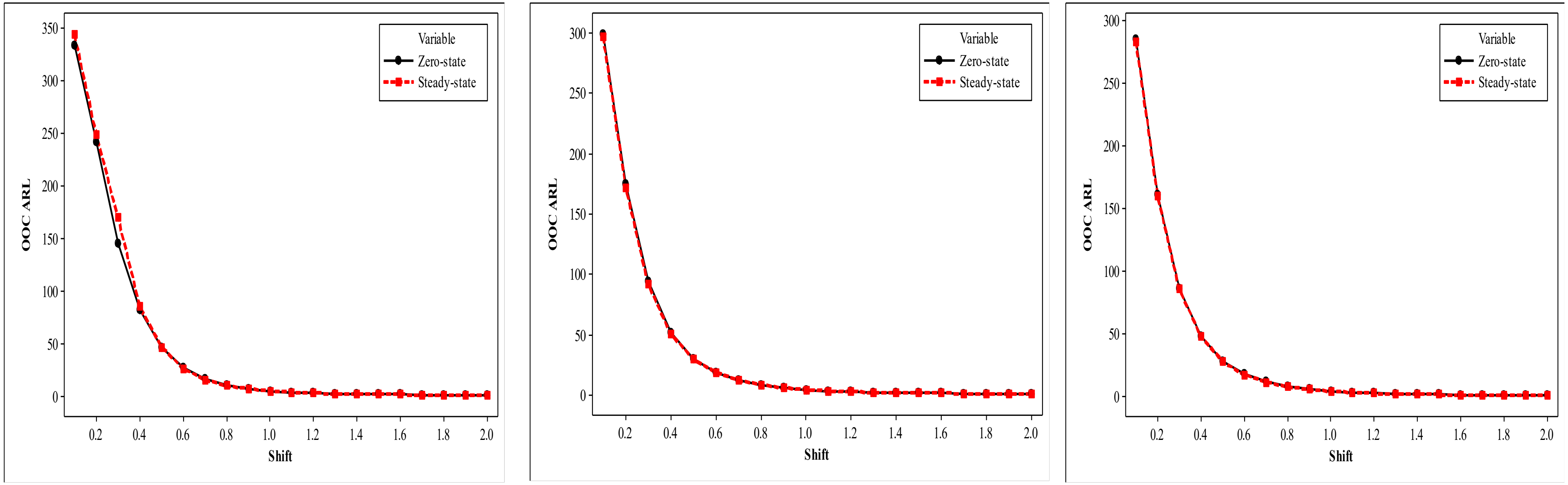
Figure 6. Effects of the Phases I and II sample sizes on the *SDRL* profile of the proposed SCSEWMA $\bar{X}$ scheme when (ω,λ)=(0.1,0.1) for a prespecified ARL_0 value of 370.4

Figure 6 investigates the effects of Phases I and II sample sizes on the *SDRL* profile and it can be seen that the proposed SCSEWMA $\bar{X}$ scheme yields larger *SDRL* values for small Phase I or/and Phase II sample size(s) regardless of the combination of (ω,λ). To conserve space, we only show the results for (ω,λ) = (0.1,0.1) (see Figure 6). Thus, the larger the Phase I and/or Phase II samples, the smaller the variation in the *ARL* values and the better the *ARL* profile.

4.3.2 Steady-state versus zero-state performance of the proposed scheme

Figure 7 compares the zero- and steady-state performances of the proposed SCSEWMA schemes in terms of the *ARL* profile for different shifts when $(\omega,\lambda) = (0.1, 0.1)$, $m = 10$, 100 and ∞ for a nominal $ARL_0 = 370$. From Figure 7, it is observed that zero- and steady-state performances of the proposed scheme are almost equal except for small Phase I sample size with small shift values (i.e. $\delta < 1$); see Figures 7 (a)-(c).

Figure 8 presents the steady-state overall performances of the proposed SCSEWMA scheme in terms of the *EARL* profile when $(\omega, \lambda) =$ (0.1, 0.1), (0.5,0.5) and (0.9,0.9), and $(\delta_{min}, \delta_{max}) =$ (0.1, 2) for a nominal $ARL_0 = 370$. From this figure, it can be seen that the proposed scheme is more sensitive in Case K compared to Case U. In Case U, the larger the Phase I sample size, the more sensitive the proposed scheme is.



(a) Case U when $m = 10$ (b) Case U when $m = 100$ (c) Case K (i.e. $m = \infty$)

Figure 7. Zero-state and steady state performances of the proposed CSEWMA scheme in terms of the ARL profile when $(\omega, \lambda) = (0.1, 0.1)$, $m = 10$, 100 and ∞ for a nominal $ARL_0 = 370$

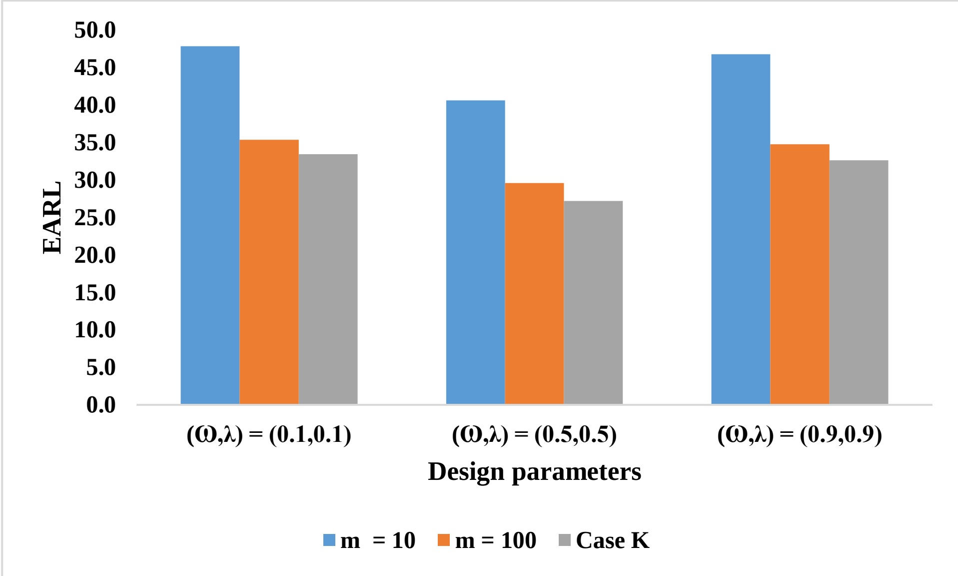


Figure 8. Steady-state overall performances of the proposed CSEWMA scheme in terms of the *EARL* profile when $(\omega, \lambda) = (0.1,\ 0.1)$, (0.5,0.5) and (0.9,0.9), and $(\delta_{min}, \delta_{max}) = (0.1,\ 2)$ for a nominal $ARL_0 = 370$

4.3.3 Case U performance comparison of the SCSEWMA $\bar{X}$ scheme with the existing CSEWMA and CSCUSUM $\bar{X}$ schemes

In Table 11, the Case U performance of the proposed SCSEWMA $\bar{X}$ scheme with $\omega = 0.9$ and 1 is compared to the ones of the existing CSEWMA and CSCUSUM $\bar{X}$ schemes when $m = 100$, $n = 5$ and $\lambda \in \{0.1, 0.5, 0.9\}$. From Table 11, it can be seen that for small to moderate shifts, in terms of the *ARL* profile, the proposed SCSEWMA $\bar{X}$ scheme with $\omega = 1$ performs better followed by the CSEWMA $\bar{X}$ scheme for small values of λ. However, for large shifts, the competing schemes considered in this paper perform almost the same. In terms of the overall performance (i.e. *EARL* values), the proposed SCSEWMA $\bar{X}$ scheme with $\omega = 0.1$ outperforms the competing schemes. The performance of the competing schemes in terms of the *SDRL* and *MRL* profiles as well as the *ESDRL* and *EMRL* profiles yield similar findings (this is not shown here to conserve space).

Table 11. Case U *ARL* and *EARL* comparisons of the SCSEWMA $\bar{X}$ scheme versus the CSCUSUM and CSEWMA $\bar{X}$ schemes when m =100 and n = 5 for prespecified ARL_0 value of 370.4

	CSEWMA scheme			**CSCUSUM scheme**			**SCSEWMA scheme (ω=0.9)**			**SCSEWMA scheme (ω=1)**		
Shift	λ **= 0.1**	λ **= 0.5**	λ **= 0.9**	h_C**=9.762**	h_C**=8.973**	h_C**=9.164**	λ **= 0.1**	λ **= 0.5**	λ **= 0.9**	λ **= 0.1**	λ **= 0.5**	λ **= 0.9**
0.1	177.9	253.7	297.5	153.69	273.4	302.6	167.8	254.2	297.8	157.6	246.3	291.6
0.2	48.0	105.8	169.3	43.9	109.0	181.5	45.5	109.6	174.6	40.3	102.2	169.9
0.3	20.0	45.5	91.2	22.73	39.6	83.3	19.9	47.8	92.4	17.6	43.3	91.1
0.4	11.5	22.2	48.5	15.33	20.5	35.8	11.5	23.6	51.0	10.2	21.5	49.1
0.5	7.6	12.4	28.4	11.5	13.3	19.2	7.8	13.2	29.0	6.9	12.1	28.3
0.6	5.5	7.9	16.8	9.16	9.7	12.3	5.8	8.3	17.4	5.0	7.7	17.1
0.7	4.3	5.5	10.8	7.59	7.3	8.7	4.5	5.8	11.1	4.0	5.4	10.9
0.8	3.4	4.1	7.3	6.42	5.8	6.6	3.6	4.3	7.6	3.2	4.1	7.2
0.9	2.8	3.3	5.2	5.45	4.6	5.1	3.0	3.3	5.4	2.7	3.2	5.1
1.0	2.4	2.7	3.8	4.68	3.8	3.9	2.6	2.8	3.9	2.3	2.6	3.8
1.1	2.1	2.3	3.0	4.01	3.1	3.2	2.2	2.3	3.0	2.0	2.3	3.0
1.2	1.9	2.0	2.4	3.42	2.5	2.6	2.0	2.0	2.5	1.8	2.0	2.4
1.3	1.7	1.7	2.0	2.93	2.1	2.1	1.8	1.8	2.0	1.7	1.7	2.0
1.4	1.5	1.6	1.7	2.51	1.8	1.8	1.6	1.6	1.7	1.5	1.6	1.7
1.5	1.4	1.4	1.5	2.16	1.6	1.6	1.5	1.4	1.5	1.4	1.4	1.5
1.6	1.3	1.3	1.4	1.86	1.4	1.4	1.3	1.3	1.4	1.3	1.3	1.4
1.7	1.2	1.2	1.3	1.64	1.3	1.3	1.3	1.2	1.3	1.3	1.2	1.3
1.8	1.2	1.2	1.2	1.45	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
1.9	1.1	1.1	1.1	1.32	1.1	1.1	1.1	1.1	1.1	1.2	1.1	1.1
2.0	1.1	1.1	1.1	1.22	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
EMRL	14.9	23.9	34.8	15.2	25.2	33.8	14.4	24.4	35.3	13.2	23.2	34.5
Control schemes constants	k = 3.467; L_E=3.194	k = 3.707; L_E= 3.204	k = 3.184; L_E = 3.624	k =3.769; k_C=0.225;	k =3.197; k_C=0.5;	k =3.181; k_C=0.75	L_W=3.155	L_W=3.188	L_W=3.185	L_W=3.004	L_W=3.177	L_W=3.182

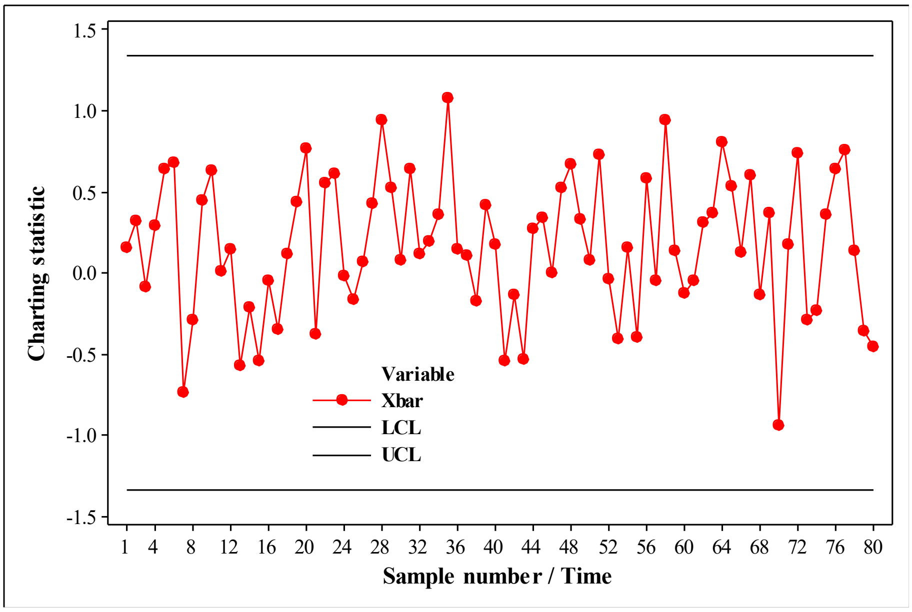
Note: The control limit constants were rounded off at 3 decimal places to conserve space

5. Illustrative examples with known and unknown process parameters

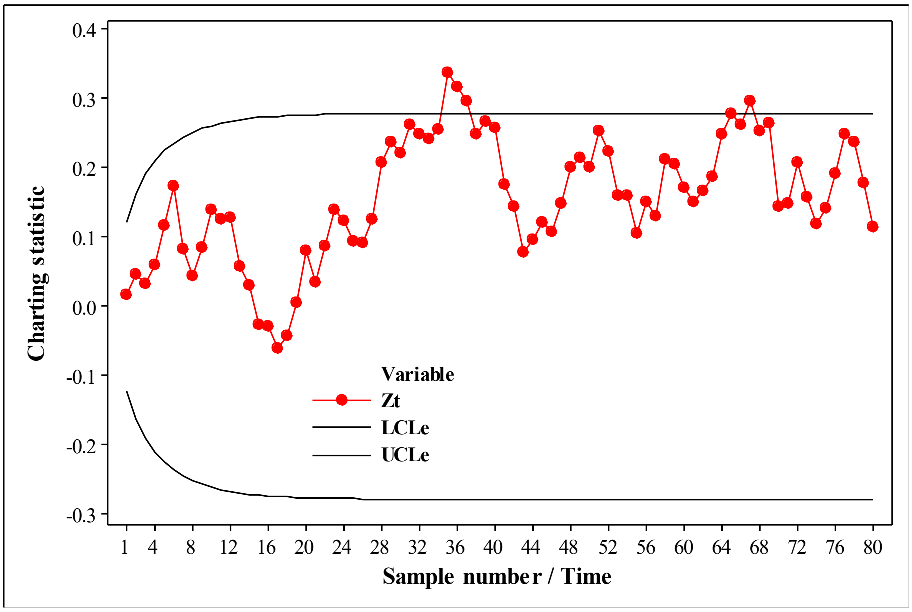
In this section, we use simulated and real-life data to demonstrate the application and implementation of the proposed SCSEWMA $\bar{X}$ scheme along with the associated Shewhart and EWMA $\bar{X}$ schemes for both Cases K and U.

5.1 Case K example: Simulated data

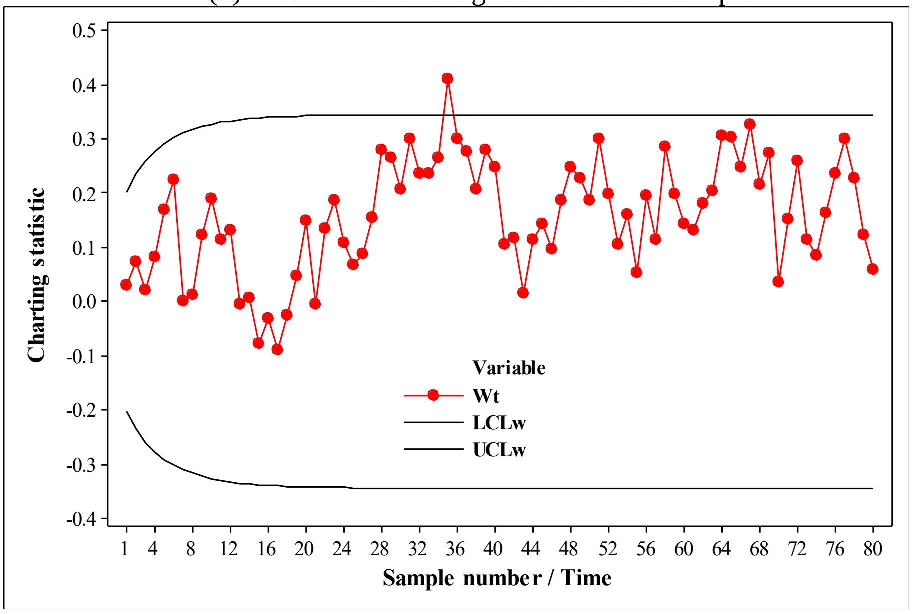
In this example, we assume that the observations are from the standard normal distribution. However, for monitoring purpose, eighty samples of size 5 are simulated from a normal distribution with a shift in the process mean of 0.25 standard deviation corresponding to a small shift; that is, $X_{tj} \sim N(0.25,1)$. The Shewhart, EWMA and SCSEWMA $\bar{X}$ schemes are designed for a prespecified ARL_0 = 370.4. The control limit constants are listed in Table 2 (keep in mind that the Shewhart and EWMA $\bar{X}$ schemes are equivalent to the SCSEWMA $\bar{X}$ scheme with $\omega = 0$ and 1, respectively). Therefore, for $\omega = 0$, 0.9 and 1, with $\lambda = 0.1$, it is found that $k = 3$, $L_W = 2.885$ and $L_E = 2.715$ which are the control limit constants for the Shewhart, SCSEWMA (with $\omega = 0.9$) and EWMA $\bar{X}$ schemes, respectively. The plots of the charting statistics of the Shewhart, EWMA and SCSEWMA $\bar{X}$ schemes are shown in Figure 9. From Figures 9 (a)-(c), it can be seen that the Shewhart $\bar{X}$ scheme does not give a signal. However, both the EWMA and SCSEWMA $\bar{X}$ schemes give a signal on the 35th sample. This shows that the proposed SCSEWMA $\bar{X}$ scheme borrows the strengths from both the Shewhart and EWMA $\bar{X}$ schemes. In this particular case, the EWMA and SCSEWMA $\bar{X}$ schemes outperform the Shewhart $\bar{X}$ schemes in monitoring small shifts in the process mean.



(a) Shewhart scheme does not signal



(b) EWMA scheme signals on the 35th sample



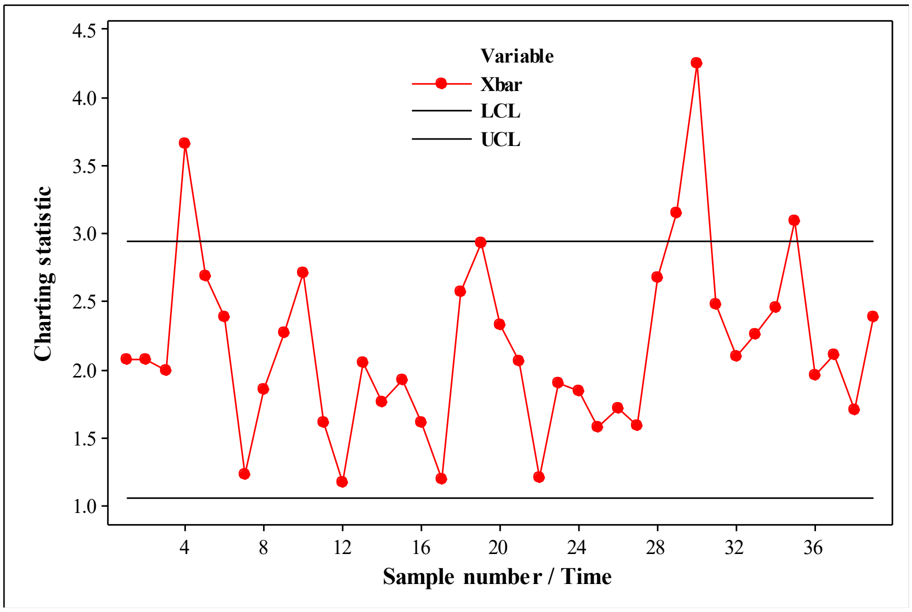
(c) SCSEWMA scheme signals on the 35th sample

Figure 9. Monitoring scheme of simulated data when $\delta = 0.25$, $n = 5$ and $\lambda = 0.1$ for a prespecified ARL_0 of 370.4

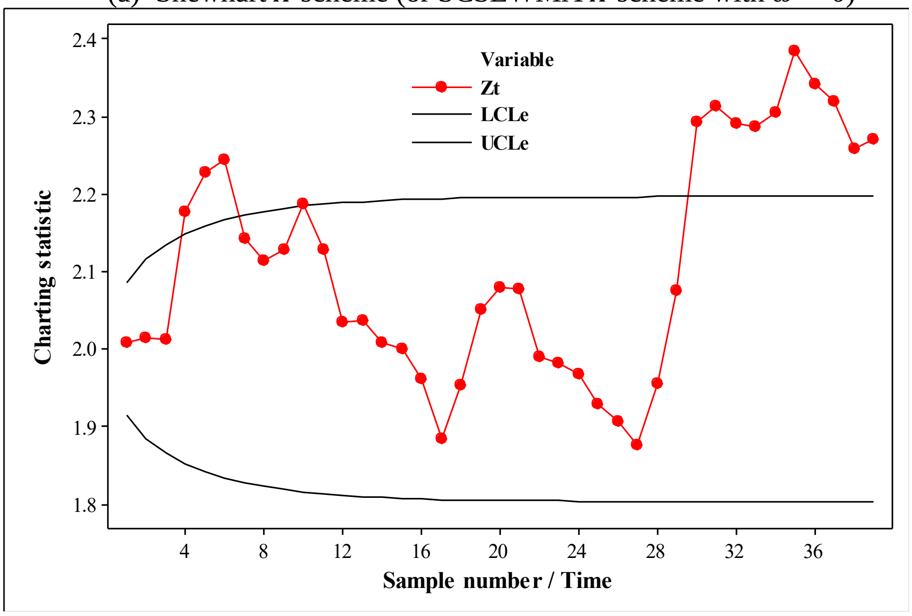
5.2 Case K example: Monitoring the level of silica concentrate in iron ore

In this example, real-life data are used to demonstrate the application and implementation of the proposed scheme. Froth flotation approach is used to enhance the iron concentration of low-grade iron ores. Low-grade iron ores contain high concentration of impurities, such as silicon dioxide or in short, silica (these are quartz or sand), phosphorus and alumina containing minerals – which are undesired. Froth flotation process is an effective approach to remove impurities. Mukherjee et al[29] showed that the quality characteristic of interest in these data is the percentage of silica concentrate that remains as an impurity at the end of the froth flotation process. A high level of silica concentrate in the iron ore is undesirable as it is not suitable to be further processed into steel. It is therefore imperative to monitor the percentage of silica that is present on each iron ore sample at the end of the flotation process.

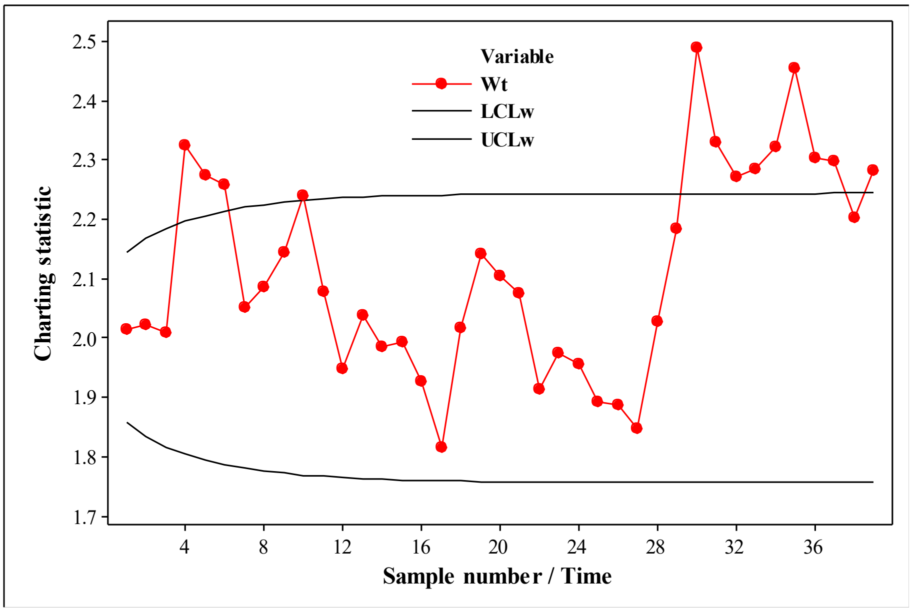
In this example, it is assumed that the IC process mean and variance are known and equal to 2 and 1, respectively, with no loss of generality and the goodness of fit test for normality was not rejected at 5% level of significance. The data provide a set of 39 samples each of size 10 to be monitored. The Shewhart, EWMA and SCSEWMA $\bar{X}$ schemes are implemented for a prespecified ARL_0 of 370.4 when n = 10 for which k = 3, L_E = 2.715 and L_W = 2.885, respectively. The plots of the Shewhart, EWMA and SCSEWMA $\bar{X}$ schemes are shown in Figure 10. It can be seen that each of the three monitoring schemes gives a signal on the fourth subgroup.



(a) Shewhart $\bar{X}$ scheme (or SCSEWMA $\bar{X}$ scheme with $\omega = 0$)



(b) EWMA $\bar{X}$ scheme (or SCSEWMA $\bar{X}$ scheme with $\omega = 1$)

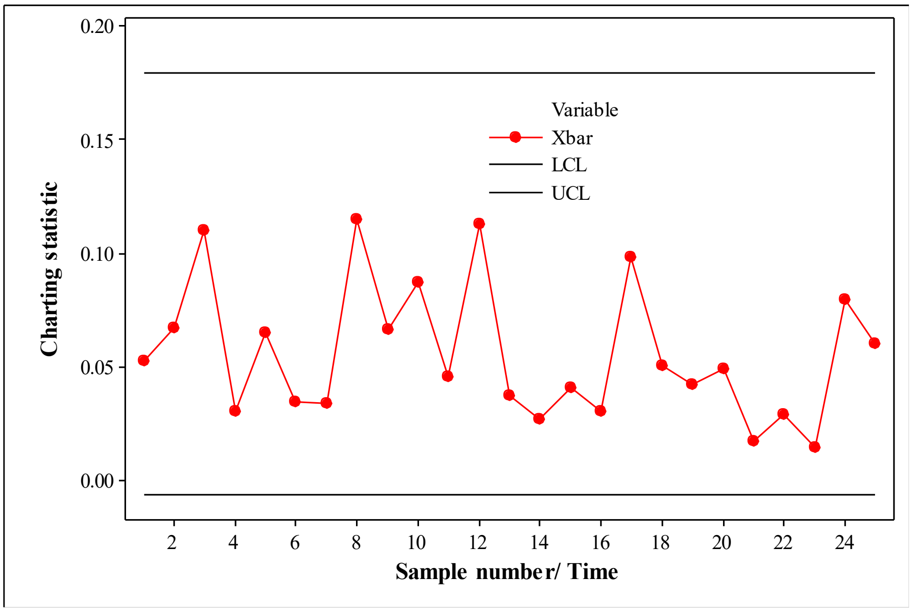


(c) SCSEWMA $\bar{X}$ scheme with $\omega = 0.9$

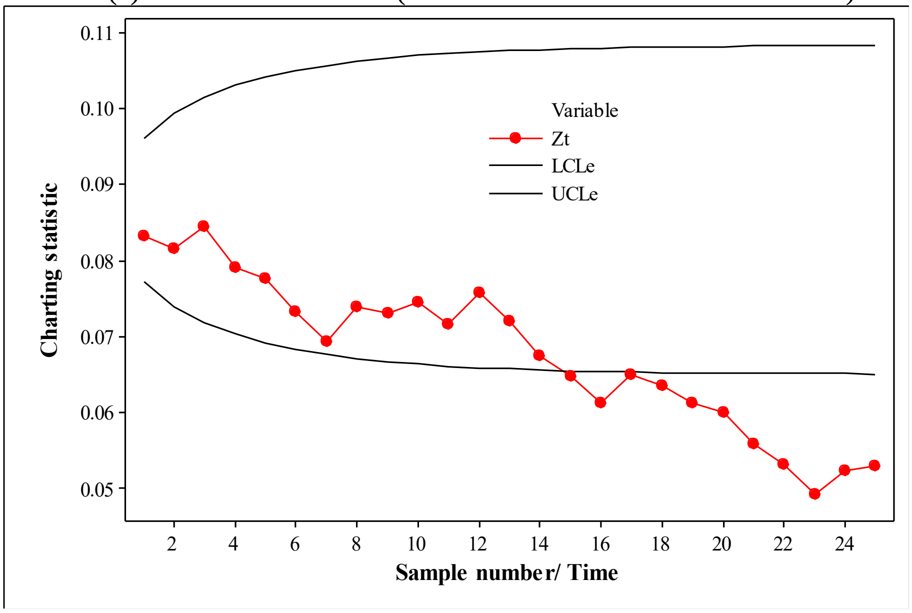
Figure 10. Case U CSEWMA $\bar{X}$ monitoring scheme of the silica data when $n = 10$ and $\lambda = 0.1$ for a specified ARL_0 value of 370.4

5.3 Case U real-life example: Real-time online purchasing intention

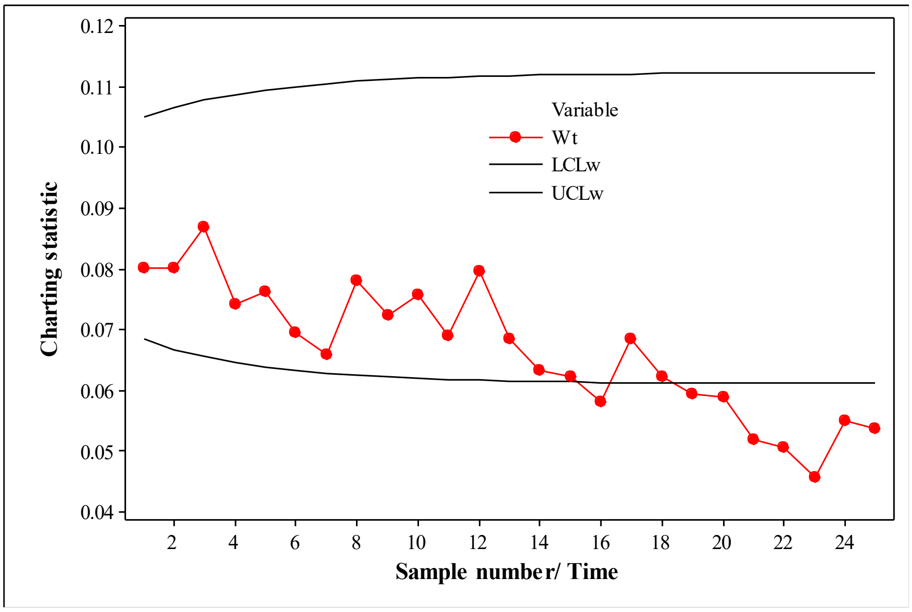
In this section, the dataset from Sakar et al[30] is used to demonstrate the application and implementation of the proposed SCSEWMA scheme with estimated process parameters. The data contain information about real-time online shoppers purchasing intention. The dataset consists of several features (or categories); namely, administrative, administrative duration, informational, informational duration, product related and product-related duration representing the number of different pages visited by the user in a session and time spent in each of these page categories. The other variables like exit rates, bounce rates and page value are metrics measured by Google Analytics for each page in the e-commerce site are also provided. In this example, we only focus on the exit rates, which represent the metric measured when the user leaves the page. The dataset contained 12330 sessions; however, through filtering the data, we were left with 530 data points. The schemes under consideration are implemented in two phases. In Phase I, 60 samples of size 5 are selected when the process is considered to be IC; the mean and standard deviation are estimated to be equal to 0.08668 and 0.6932, respectively, and $c_{4m} =$ 0.998959. In Phase II, 25 subgroups each of size 5 are monitored. The plotting statistics of the schemes under consideration, based on exit rates data, are shown in Figure 11. It can be observed that the Shewhart scheme which corresponds with the SCSEWMA scheme with $\omega = 0$, does not signal in the prospective phase. However, when $\omega = 0.9$ the proposed scheme gives a signal on the 16th sample in the prospective phase and the EWMA scheme which corresponds with the CSEWMA scheme with $\omega =$ 1, gives a signal on the 15th sample. These results reveal the flexibility of the proposed scheme and its superiority for large values of ω.



(a) Shewhart $\bar{X}$ scheme (or SCSEWMA $\bar{X}$ scheme with $\omega = 0$)



(b) EWMA $\bar{X}$ scheme (or SCSEWMA $\bar{X}$ scheme with $\omega = 1$)



(c) SCSEWMA $\bar{X}$ scheme with $\omega = 0.9$

Figure 11. Case U CSEWMA $\bar{X}$ monitoring scheme of the exit data when $n = 5$, $m = 60$ and $\lambda = 0.1$ for a specified ARL_0 value of 370.4

6. Concluding remarks

This paper proposes a new CSEWMA $\bar{X}$ scheme using a single plotting (or charting) statistic with known or unknown process parameters for monitoring the process mean. It is denoted as the SCSEWMA $\bar{X}$ scheme. As for other parametric schemes, it was observed that the SCSEWMA $\bar{X}$ scheme is not IC robust and its performance deteriorates when the process parameters are estimated. However, the larger the Phase I or Phase II sample, the more efficient the proposed scheme is. Compared to the existing Shewhart, EWMA and CSEWMA $\bar{X}$ schemes, the new scheme is more flexible through an extra weighing design parameter. Moreover, the Shewhart and EWMA schemes are particular cases of the proposed SCSEWMA scheme when the weighing parameter is equal to 0 and 1, respectively. A comparative study of the SCSEWMA $\bar{X}$ scheme and the existing CSCUSUM and CSEWMA $\bar{X}$ schemes shows that the proposed SCSEWMA $\bar{X}$ scheme is superior in monitoring small to moderate shifts and equivalent to the existing CSCUSUM and CSEWMA $\bar{X}$ schemes in monitoring large shifts in the process mean. Operators in industrial and non-industrial organizations are advised to use the newly proposed SCSEWMA $\bar{X}$ scheme instead of the existing CSEWMA $\bar{X}$ scheme.

Researchers who are interested in developing similar schemes can also look at the design of the SCSEWMA scheme using other statistics such as the variance, coefficient of variation, etc., under symmetric and non-symmetric distributions. Researchers can also look at the design of the composite Shewhart double or triple EWMA scheme using a single charting statistic.

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Appendix A

In this appendix, we show how the properties of the SCSEWMA scheme are derived.

A.1 The mean of the SCSEWMA statistic

$$W_t = (1-\omega)\bar{X}_t + \omega Z_t, t = 1,2,3,...,$$

where

$$\bar{X}_t = \frac{1}{n}\sum_{j=1}^{n} X_{tj} \qquad \text{[A.1]}$$

and

$$Z_t = \lambda \bar{X}_t + (1-\lambda)Z_{t-1}.$$

Then,

$$\begin{aligned} W_t &= (1-\omega)\bar{X}_t + \omega[\lambda\bar{X}_t + (1-\lambda)Z_{t-1}] \\ &= (1-\omega)\bar{X}_t + \omega\lambda\bar{X}_t + \omega(1-\lambda)Z_{t-1} \\ &= (1-\omega+\omega\lambda)\bar{X}_t + \omega(1-\lambda)Z_{t-1}. \end{aligned}$$

Since, the EWMA $\bar{X}$ statistic is defined as

$$Z_t = \lambda\sum_{j=0}^{t-1}(1-\lambda)^j\bar{X}_{t-j} + (1-\lambda)^t Z_0, \qquad \text{[A.2]}$$

W_t can then be written as

$$W_t = (1-\omega+\lambda\omega)\bar{X}_t + \lambda\omega(1-\lambda)\sum_{j=0}^{t-2}(1-\lambda)^j\bar{X}_{t-j-1} + \omega(1-\lambda)(1-\lambda)^{t-1}Z_0, \qquad \text{[A.3]}$$

$$\begin{aligned} E(W_t) &= \left[1-\omega+\lambda\omega+\lambda\omega(1-\lambda)\frac{1-(1-\lambda)^{t-1}}{1-(1-\lambda)} + \omega(1-\lambda)(1-\lambda)^{t-1}\right]\mu_0 \\ &= [1-\omega+\lambda\omega+\omega(1-\lambda)(1-(1-\lambda)^{t-1}) + \omega(1-\lambda)(1-\lambda)^{t-1}]\mu_0 \\ &= [1-\omega+\lambda\omega+\omega(1-\lambda) - \omega(1-\lambda)(1-\lambda)^{t-1} + \omega(1-\lambda)(1-\lambda)^{t-1}]\mu_0 \\ &= [1-\omega+\lambda\omega+\omega(1-\lambda)]\mu_0 \\ &= \mu_0. \end{aligned}$$

Therefore,

$$E(W_t) = \mu_0. \qquad \text{[A.4]}$$

A.2 The variance of the SCSEWMA statistic

From Eq [A.3], the variance of the SCSEWMA statistic is derived as follows:

$$Var(W_t) = Var\left[(1-\omega+\lambda\omega)\bar{X}_t + \lambda\omega(1-\lambda)\sum_{j=0}^{t-2}(1-\lambda)^j\bar{X}_{t-j-1} + \omega(1-\lambda)(1-\lambda)^{t-1}Z_0\right].$$

We know that the $Cov(X_j, X_i) = 0\ \forall j \neq i$ since the observations are i.i.d. and that

$$Var(Z_0) = Var(\mu_0) = 0.$$

Thus,

$$Var(W_t) = \left[(1-\omega+\lambda\omega)^2 + \lambda^2\omega^2(1-\lambda)^2\left[\frac{1-(1-\lambda)^{2(t-1)}}{1-(1-\lambda)^2}\right]\right]\frac{\sigma_0^2}{n}$$

$$= \left[(1-\omega+\lambda\omega)^2 + \frac{\lambda}{2-\lambda}\omega^2(1-\lambda)^2(1-(1-\lambda)^{2t-2})\right]\frac{\sigma_0^2}{n}.$$

Therefore,

$$Var(W_t) = \left[(1-\omega+\lambda\omega)^2 + \frac{\lambda}{2-\lambda}\omega^2(1-\lambda)^2(1-(1-\lambda)^{2t-2})\right]\frac{\sigma_0^2}{n}. \qquad [A.5]$$

Note that Eq [A.5] can also be written as:

$$Var(W_t) = \left[(1-\omega+\lambda\omega)^2 - \lambda\omega^2\left(\lambda - \frac{(1-(1-\lambda)^{2t})}{2-\lambda}\right)\right]\frac{\sigma_0^2}{n}. \qquad [A.6]$$

Thus, from Eq [A.5], when

(i) If $\omega = 0$,

$$Var(W_t) = \frac{\sigma_0^2}{n}.$$

(ii) If $\omega = 1$,

$$Var(W_t) = \left[\lambda^2 - \lambda\left(\lambda - \frac{(1-(1-\lambda)^{2t})}{2-\lambda}\right)\right]\frac{\sigma_0^2}{n}$$

$$= \left[\lambda\frac{(1-(1-\lambda)^{2t})}{2-\lambda}\right]\frac{\sigma_0^2}{n}.$$

Therefore,

$$Var(W_t) = \frac{\lambda\sigma_0^2}{(2-\lambda)n}(1-(1-\lambda)^{2t}).$$

Note that Eqs [A.5] and [A.6] can also be simplified to:

$$Var(W_t) = \left[(1-\omega)(1-\omega+2\lambda\omega) + \frac{\lambda\omega^2}{2-\lambda}(1-(1-\lambda)^{2t})\right]\frac{\sigma_0^2}{n}. \qquad [A.7]$$

Note that in Case U, the IC process mean (μ_0) and variance (σ_0^2) are substituted with their unbiased estimators i.e. $\hat{\mu}_0$ and $\hat{\sigma}_0^2$.