AN INNOVATIVE TRAVEL MODEL SYSTEM FOR WESTERN CAPE

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ABSTRACT

In 2018, the Western Cape Department of Transport and Public Works retained Bentley Systems (formerly Citilabs) and Esri SA to develop the Western Cape Land Use/Transportation Interaction model. This model provides the ability to test a wide variety of demographic and multimodal transport system changes in support of a new performance-based planning process. This model is innovative and unique in several respects:

1) It is an integrated person travel/goods movement/land use forecasting process, in which the transport system influences growth, which in turn influences the system.
2) The person travel model uses the new type of tour-based structure. Instead of zonal averages, choice probabilities are used to simulate the decisions of individual travellers.
3) Land use is modelled applying bid-rent mechanics to achieve an equilibrium between real estate supply and demand and accounts for informal development.
4) The goods movement component uses a commodity flow model, translating macroeconomic and demographic data into the flow of material by mode and estimating truck travel on roads.

Developing a new travel model usually requires significant data on demographics and existing travel patterns, which is largely unavailable in South Africa. Thus, the models are crafted from a combination of local information and behaviour transferred from other areas. The household synthesis component was calibrated from Stats SA data. Demand is estimated using several components: tour frequency, destination choice, mode choice, intermediate stops, time of day, and trip assignment. The model is developed using Cube software. As of mid-2020, the model system was calibrated, tested, and documented. The project's final year will see some enhancements, more testing, and additional staff training.

1. INTRODUCTION

1.1 Background

The Western Cape Government Department of Transport and Public Works (WCG DTPW) wanted to improve its technical analysis capabilities by developing a new travel demand model system. Although the City of Cape Town's transport directorate had been using an area-wide travel model for several years, there was no similar capability at the provincial level. Travel demand models have been used by numerous local municipalities and by provincial government on individual projects for a long time. However, the population of Cape Town and other urban areas in the province is growing at a high rate, increasing interactions between previously disconnected towns. Individual models of different areas have become obsolete. Therefore the WCG recognised the need to create a single model for the entire province which can be used for integrated planning purposes.

1.2 WCG's Approach

In many jurisdictions worldwide, some level of technical analysis capability exists, and the challenge is to determine how best to use those skills to assist decision-making for large and small public works projects. But DTPW is relatively new to the use of performance-based planning and did not have a lot of resources in this area. This agency had limited experience in the application of quantitative planning to project development.

As a result, DTPW decided to take an innovative approach: work with experts to first create a comprehensive set of transport and land use planning tools and then work within the agency (and other agencies) to encourage the acceptance of these tools and their integration into the agency's processes. Their logic was that having the planning tools in place and a new ability to answer "What if?" questions would produce a wealth of useful information that would help decision-makers find better solutions to the region's current and future transport difficulties. DTPW also believed that having robust planning tools would help foster improved cooperation with the Municipalities, especially Transport of Cape Town (TCT).

DTPW is an existing user of ArcGIS and has a relationship with the local distributor, Esri SA. Through Esri SA, DTPW discovered Citilabs, a US-based mobility analytics firm partnered with Esri that provides the **Cube** suite of integrated software specializing in travel demand analysis, including person travel, goods movement, and land use. DTPW retained Citilabs and Esri SA to develop a new travel model system, train DTPW staff, and create local expertise by working with WCG, Cape Town, local consultants, and University of Cape Town staff during a three-year development process. The work began in February 2018 and as of this writing the initial versions of all model components are complete. Various enhancements and staff training will continue throughout 2021. In October 2019, Bentley Systems acquired Citilabs, but the work has continued throughout the transition.

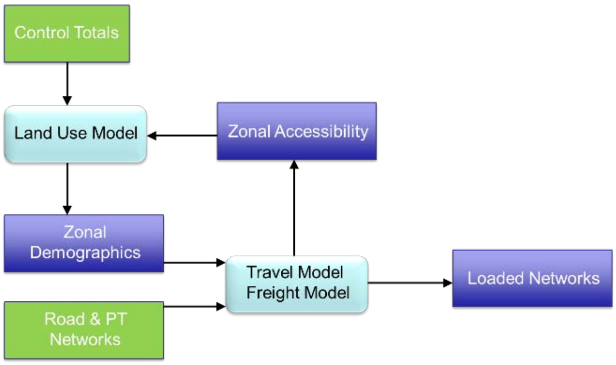
2. PROJECT SCOPE

2.1 Overall Model Structure

DTPW desired a complete model system that would cover both the City and the rest of the province. This includes the following components, organised as shown in Figure 1:

- Land use allocation model: this uses econometric analysis to allocate total provincial growth to units called small area layer (SAL) zones; it includes consideration of the extensive informal residential and commercial sector in Western Cape (through supplemental models, the SAL data is converted to data at the traffic analysis zone (TAZ) level for use in the travel demand models).
- Travel demand model: this uses the population and employment at the TAZ level along with a description of the roadway network and public transport system to estimate weekday traffic volumes and PT ridership.
- Goods movement model: this uses the forecast of employment to estimate the quantity and flow of goods by truck and rail throughout the City and province.

- Integrated transport/land use process: this is not an independent module; rather, it integrates the above three models in recognition of the fact that land use and transport affect each other and must be modelled together in a coordinated fashion.



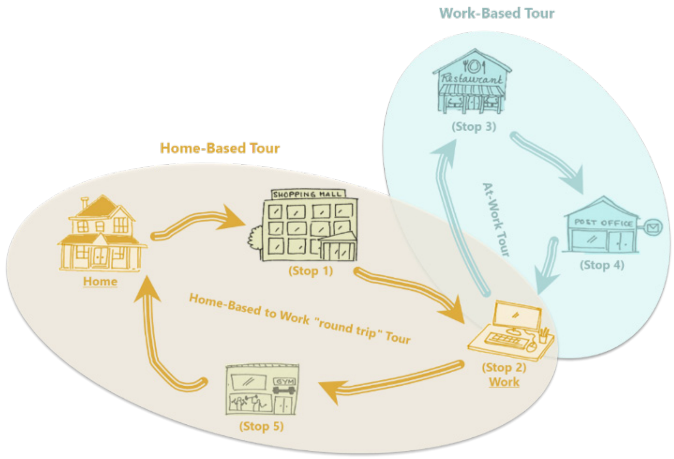
Source: Bentley Systems

Figure 1: Model Organisation

The remainder of this paper focuses on the travel demand model. Other papers presented at SATC 2021 cover other aspects of the entire model system.

2.2 Disaggregate Travel Modelling

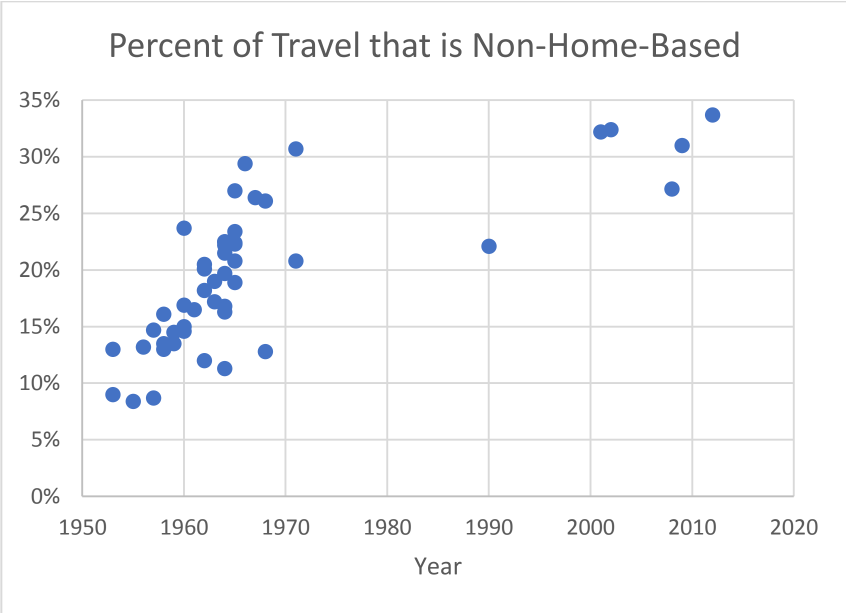
For over 60 years, the most common approach to travel modelling was the "four-step" approach, featuring trip generation, trip distribution, mode choice, and traffic assignment steps. This process treats travel as aggregate totals at the TAZ level and has the advantages of years of experience, is relatively easy to understand, and fast to run. However, with the availability of more powerful desktop computers, a new process has emerged that represents travel in a more realistic manner: as individual tours. Instead of modelling disconnected trips, the round-trip tour is the basic unit of travel, allowing for intermediate stops along the tour that represent less important activities that are conducted on the journey between home and a primary destination and back (Figure 2). Most new models are using this discrete "tour-based" approach, which estimates the behaviour of every single tour in terms of logit-based choice probabilities and uses Monte Carlo analysis to select from amongst the available options. Instead of using aggregate statistics based on average values, discrete modelling identifies the values and choices associated with each household and each tour. This improves accuracy by removing "aggregation error" (2,6).



Source: Bentley Systems

Figure 2: Tour-Based Approach

It is believed that these two key features – treating travel in terms of tours and modelling each tour in discrete fashion – make this approach more accurate and life-like. It also clearly improves the ability to estimate and understand intermediate stop behaviour, which is represented by non-home-based (NHB) travel in the aggregate trip process. Such behaviour has increased in recent years and this trend should be expected to continue, as Figure 3 indicates.



Source: Bentley Systems, based on data from source 1

Figure 3: Trend of NHB Travel

Tour modelling also responds to some of the criticisms of the aggregate process. In trip-based models, trips are largely independent of each other which is clearly unrealistic. In a tour-based model, the two halves of the tour (e.g. home-to-work and work-to-home) are logically and mathematically connected to each other. So, for example, if you rode a bus to work in the morning, you are quite likely to ride a bus home in the evening. This is not assured in an aggregate model, but it is in a tour-based model. Another issue is that discrete models are able to consider more influential factors on travel behaviour and can interconnect these factors. Many aggregate models consider the effects of HH size and income on trip generation, but there are surely other factors that are also important. Tour-based models are able to consider additional factors such as vehicle ownership, the number of workers, and accessibility. In addition, discrete models can consider key travel interactions: if a person makes a Work trip, he/she is less likely to make a Shop trip. This is because he/she has less time available for travel and is also able to make stops for shopping on the way to or from work. Another example: one of the biggest topics in transportation planning today is the potential for autonomous vehicles (AV). It is significantly easier to model AV impacts with a discrete model than with an aggregate model (3). AV ownership is just one more HH attribute that is modelled along with the other characteristics.

One of the earliest types of tour-based model is the Activity-Based Model (ABM) (2). This process attempts to model every daily activity of every person and then travel is derived from the need to connect these activities. This creates an extremely complex process that represents many relationships and constraints. Although it is arguably a realistic picture of human behaviour, the resulting model set is unwieldy and extremely resource-intensive. Development takes several years and model run time is measured in days. Many ABMs require very sophisticated and expensive computer hardware in order to keep the run times in the range of 24-72 hours.

2.3 A Simpler Method

In recent years, a new process has emerged in response to the problems of ABM complexity and long run time. This procedure, labelled Simplified Tour-Based Modelling (STM), uses the same discrete tour-based process as ABM but makes some simplifying assumptions and removes some of the constraints. This creates a process that is significantly easier to understand and develop and runs in a fraction of the time of a typical ABM, while producing essentially the same output. In this respect, it is a more reasonable and accessible improvement over the aggregate trip process and is well-suited to most urban areas (7). STM has been implemented successfully in the US and was the approach selected for the new Western Cape Travel Model (WCTM) (5).

STM has two major components: demographic (household synthesis) and travel behaviour (demand estimation). The Household Synthesis (HS) model was calibrated from Stats SA data and thus represents local conditions with respect to HH size, income, life cycle, and the number of workers and vehicles. This data was then used to create a multi-dimensional iterative proportional fitting model of HH composition in each zone. The output is one record for each HH with the zone number and the attributes listed above. *Size* is modelled as 1 to 7 persons. *Income* is modelled as a quintile: a value from 1 to 5 indicating which quintile describes the HH's total income, compared to the province as a whole. *Vehicles* includes all motor vehicles, whether owned, leased, or employer-provided, ranging from 0 to 3+. *Workers* includes all workers, whether full- or part-time, ranging from 0 to 3+. *Life cycle* describes the family composition: 1 = HH has any retired people, 2 = HH has no retired people but has any children, 3 = HH has no retired people or children. Prior research has indicated that these are all attributes that strongly influence travel behaviour (4).

Stats SA conducted a home interview travel survey in 2011 called the Nationwide Household Travel Survey (NHTS). Unfortunately, the NHTS has by now become slightly outdated and was also not conducted in a manner that would permit it to be used to directly estimate most of the demand model components. For example, it asked origin and destination details about only work and school travel. To remedy this gap, the authors took advantage of one of the key benefits of discrete models: because they are better grounded in individual travel behaviour, it is theoretically easier to transfer such models between urban areas (2). Although the US and South Africa are indeed different environments, once the model properly accounts for the differences in demographics and transport conditions, the authors believe that the way in which *similar* people *respond* to those conditions is largely the same everywhere. The authors assert that this is demonstrated by the model's validation and by its continuing ability to provide a reasonable response to hypothesised transport changes (to be documented in future papers).

The Stats SA data would be used to identify the attributes of HHs within Western Cape. The NHTS would be used to calibrate certain model components and the highway and PT assignments would be validated against local count data. In this manner, the available data could be combined with US and European experience in an efficient fashion to produce an advanced travel model that was synchronised to local conditions.

The demand component consists of six models: tour frequency, destination choice, mode choice, intermediate stop, stop location, and time of day. These sound very similar to the phases of a four-step model and that is not a coincidence. STM was developed to be familiar to planners. Each of these steps models a particular travel choice:

- Tour Frequency (TF): this is a HH's choice of how many tours to make for each purpose. It is analogous to the 4-step's trip generation step. The main variables are the HH attributes described above.
- Destination Choice (DC): this is the choice of a destination zone for each tour, out of all possible zones. It is analogous to the 4-step's gravity model. The main variables are travel time by mode, traveller income, area type, and accessibility, as well as the population and employment in each zone.
- Mode Choice (MC): this is the choice of travel mode for the entire tour. It is computed in the same manner as the 4-step but applied differently. The main variables are the time and cost of each mode, traveller income, area type, and accessibility.
- Intermediate Stop (IS): this is the choice of how many stops to make on the way to and from the main tour destination zone. There is no exact equivalent in the 4-step but it is represented by the 4-step's NHB purpose. The main variables are the tour O/D time, area type, retail density of the tour O and D zones, the total tours made by the HH, and the mode used for this tour.
- Stop Location (SL): this is the choice of the locations of any intermediate stops. The main variables are the same as those of the DC model.
- Time of Day (TD): this is the choice of the starting time period for each half-tour. Each tour has two half-tours and this process models the period for both of them simultaneously. Four time periods are used: AM peak, midday (inter-peak), PM peak, night. The main variables are the tour purpose and mode.

The trip purposes used in this model are similar to those in most travel demand models, except that there is no non-home-based purpose. This model uses Work (both full- and part-time), School (K-12), University (all tertiary education), Shop, Other, and At-Work. All tours are assumed to begin and end at the traveller's home except for At-Work. Those tours are made by workers and begin and end at the worker's usual workplace (e.g., to go to lunch, a meeting, or another worksite).

2.4 Discrete Tour Modelling

Each of these demand model components uses a *logit* structure, shown in equation 1:

$$p_i = \frac{e^{U_i}}{\sum e^{U}} \quad (1)$$

where

p_i = probability of option i being chosen

U_i = utility of option i = a0 + a1*X + a2*Y + a3*Z …

The utility of an option is a linear function of the attributes of an option. Some of those attributes are negatively associated with the option, such as time and cost, while others are positively associated with the option, such as accessibility. The objective of model calibration is to identify the attributes (X, Y, Z, …) and the coefficients (a0, a1, a2, a3, …) that produce the best fit to the available observed data. These must be variables that are logically related to the choice being modelled, that rationally lead to increases or decreases of that choice, and that can be forecasted.

There are two basic types of logit model:

- Few choices: this includes the TF, MC, IS, and TD models, in which the number of available options is relatively small. The Mode Choice component is the most well-known of these, but the other models are mathematically similar.

- Many choices: this includes the DC and SL models, in which the choice includes potentially thousands of zones. These models are slightly different in that they contain a *size* parameter, which is the natural logarithm of one or more values indicating the amount of activity in the zone (e.g., population, employment).

Note that the use of the term *choice* is figurative. It refers to one option being “selected” from the array of options available to an individual traveller. In some cases, this could indeed be an individual’s actual choice but in other cases, it could simply represent one alternative out of many. These are generally considered to be short-term choices that could in theory vary on a daily basis. (Whereas the HH Synthesis process deals more with longer-term choices such as vehicle ownership that tend not to vary day-to-day.)

The Mode Choice process also provides a good way to explain the difference between aggregate and discrete modelling. In the aggregate 4-step process, the number of person trips is estimated for an O/D pair and the mode choice logit model determines the percentage shares of person travel by auto, PT, and walking. Those percentage shares are then multiplied by the total person trips to determine the trips by mode. In a discrete model, the MC logit calculations are similar, but the resulting percentages are treated as the *probabilities* of each mode being chosen for a single tour. The mode for a particular tour is then estimated by using Monte Carlo simulation.

2.5 Monte Carlo

Monte Carlo simulation provides the mathematical mechanism by which the probabilities from a logit model are used to determine the choice for each tour. It is helpful to think of this in terms of a roulette wheel, which is why the process is named for the Casino de Monte Carlo in Monaco. A roulette wheel is divided into several wedges, all of which have equal size, so that the probability of the ball falling into each wedge is the same.

Now consider a different kind of wheel, where the size of each wedge is proportional to the probability of each choice. An example is shown in Figure 4, where a logit model of Tour Frequency has estimated the probabilities of each number of tours for a household for a specific purpose as shown. The Monte Carlo process is the mathematical equivalent of spinning the wheel and wherever it stops, that identifies the number of tours that are estimated for this household.

The wheel is “spun” for each trip purpose, for each HH. Before each spin, the probabilities (and thus the size of the wedges) are recalculated using the attributes of the HH and the model’s coefficients. In mathematical terms, a random number is drawn and compared to the cumulative share for each option, as shown in Table 1.

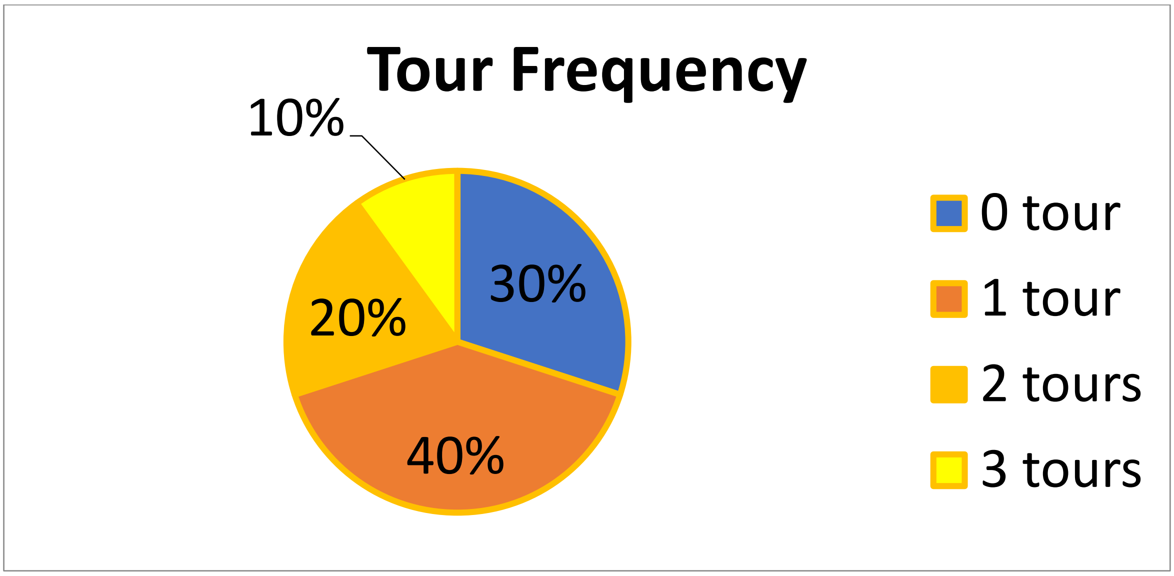


Figure 4: Monte Carlo Example

Table 1: Cumulative Probabilities for Tour Frequency Example

Number of Tours	Probability	Cumulative Probability
0	0.30	0.30
1	0.40	0.70
2	0.20	0.90
3	0.10	1.00

Assume the random number is 0.76. In Table 1, the first option whose cumulative probability exceeds 0.76 is “2 tours” and so that would be selected for this purpose for this HH. This process is repeated for all purposes and all HHs, except for the At-Work tours, which are calculated for each worker, starting from his/her usual workplace.

2.6 Model Infrastructure

Travel modelling is a very data-intensive activity that takes place within an “infrastructure” that includes a TAZ system, demographic database, roadway network, and PT network descriptions. At the start of this project, none of these items existed for Western Cape province and so a major task was to create them.

2.6.1 TAZ System

The traffic analysis zone is the basic unit of geography in all travel demand models. A TAZ (or *zone*) is roughly equivalent to a neighbourhood. These areas should be drawn to be consistent with the available data but small enough so that they can load travel onto the roadway and PT networks in a realistic fashion. Within the City of Cape Town, TCT had already developed a TAZ system and this was adopted as is. Outside the City, the zones were defined to be equal to one or more of the Enumeration Areas (EA) developed by Stats SA. Special zones called *external stations* are used to represent the roadways at the provincial border. The final system consists of 4000 zones: 3954 internal zones and 46 external stations.

2.6.2 Demographic Database

Creating this database was a significant challenge (as is documented in a separate paper) due to many factors, including data representing different years and different geographic areas, inconsistent data definitions, conflicting information, and limitations on data sharing amongst agencies. A base year of 2016 was selected as being fairly recent, but with the hope that data representing that year should now be available.

The data that are needed for each zone are defined by the model itself, by experience in other areas, by reference to the "best practice" in travel modelling, and by the availability of data items for the Western Cape. In addition, there was a difference in data availability and quality between the City and the outer areas. The following items were identified:

- Population (all ages).
- Households (occupied dwelling units, formal and informal).
- Average annual household income in rands.
- Number of jobs (formal and informal) by type: Retail, Office, Industrial, Service, Other
- K-12 enrolment.
- University/college/tertiary enrolment.

This is a reasonably small list of data items, designed to strike the best balance between model accuracy and burden on the user. Sources included various public agencies in the City and province, Stats SA, Esri, and some private data sources. This data was assembled for each zone, representing approximately 2016 conditions.

2.6.3 Roadway Network

From its prior modelling work, TCT had some roadway network databases but they were found to be insufficient for this project's comprehensive needs. A new network for the City and province was created from data obtained from HERE Technologies, a company specialising in location technology, converted to Cube format, and populated with variables representing the type of roadway (facility type), number of lanes, usage limitations (e.g., no trucks, pedestrians only, etc.), speed limit, and toll class. This was thought to represent a reasonable tradeoff between the need for greater model accuracy and minimising the burden on the user to describe the system. The network coding also allows for the modelling of high-occupancy lanes, bus-only lanes, reversible lanes, truck restrictions, and a variety of tolling and road pricing schemes. In addition, sub-models were created to determine the *area type* for each link, describing the level of adjacent development, as well as tables that define the free-flow speed and hourly capacity of each link based on its facility type and area type.

Not all streets and roads in the province were included, but enough of the system was included in the coded network to provide reasonable access to the TAZ system described above. Another factor influencing roadway development was to include enough roads to be able to adequately define the bus routes.

2.6.4 PT Network

A separate paper describes the public transport system, which consists mainly of the fixed-route, fixed-schedule bus and train services available throughout the province. This includes the MyCiti, Golden Arrow, GoGeorge, and University of Cape Town bus systems and the Metrorail system. Data was obtained to describe the routes, frequency by period, run times, and fares and this information was coded into Cube's Public Transport module to provide a reasonable representation of the PT system for a typical weekday. Frequency information is input for all four periods (AM, midday, PM, night) but the focus for modelling purposes is on the AM peak and midday operations. The fare coding generally represents the single-trip cash fare and no attempt has been made to model the extensive system of discounts and passes, although that will be the subject of future analysis. Where the bus routes use bus-only lanes on the major roads, this is included in the coding. Separate nodes and links were added to represent the rail system.

Coding the Golden Arrow Bus System (GABS) required a special effort. GABS service is operated by a private company and the service is operated in a somewhat irregular manner. Some routes have a large number of branches, some routes do not have reverse service (i.e., between AM and PM), and many routes operate only one or two bus trips daily. This pattern of service did not lend itself to straightforward representation in Cube, so a number of assumptions and simplifications were made in order to code the GABS lines.

The result of this work was a coded set of routes that permit the calculation of O/D paths and travel impedances for AM peak and midday, stratified by time component (walk, wait, ride time, number of transfers, fare).

2.6.5 Minibus-Taxi Service

Minibus-taxi (MBT) service in the province is provided by thousands of individual drivers, operating 10-14 passenger vans. Many of these drivers work for companies and other associations while others are individual drivers. These services are licenced by the province to operate on a fixed route for a specified number of hours per day, but deviating from the route to serve demand is common. MBT service is also characterised by overcrowded vehicles, unsafe driving, and haphazard operations. MBT provides service to areas that are poorly served (or unserved) by fixed-route transit and appeals mainly to a segment of the population that have limited options.

MBT is very similar to other informal transit services that exist in large cities in developing countries, such as Buenos Aires (colectivo), Mexico City (pesero), Manila (jeepney), and Bangkok (tuk-tuk). The flexible (and unpredictable) nature of the service and the large number of routes (thousands) make it extremely difficult to code this service in the same manner as the fixed-route services. An alternative method was developed to represent this service level in an approximate manner. The average walk and wait times are estimated based on population and employment density and the ride times are derived from the roadway network with an adjustment to reflect pickups and drop-offs. Fares are estimated based on trip distance. The authors assert that this simplified approach represents a reasonable balance between system detail and the burden on the model user. Its effect on the accuracy of the results requires significantly more observed data on actual MBT travel patterns for comparison.

2.7 Calibration

As noted above, no travel survey data was available that was suitable to estimate the model coefficients for Western Cape. Thus, the selected approach was to transfer an existing model (from Charlotte, North Carolina, USA) and compare the resulting travel characteristics to those that could be determined from the NHTS and other local sources, in order to modify the coefficients so as to customise the model to Western Cape as much as possible. As it turned out, surprisingly few adjustments have been needed, which supports the theory described above that discrete models can more readily be transferred between locations, compared to 4-step models. This also highlights the importance of developing an HH Synthesis model entirely from local data that describes the modelled area’s population as accurately as possible.

2.8 Validation

For most reviewers, the principal validation of the model is its ability to replicate base year link traffic volumes and PT riders by submode. Unfortunately, the PT element is still in progress, having been delayed by difficulties in obtaining internally consistent observed ridership by submode. But the highway element has been shown to have a reasonable level of accuracy.

Recent US guidance suggests the accuracy targets shown in Table 2 and these are shown to be mostly met.

Table 2: Highway Assignment Accuracy Criteria

Criterion	Target	Actual
Percent total error	±5%	+1%
Percent RMSE*	<= 40%	40.4%
Link level r^2	>= 0.90	0.909

* RMSE = root-mean-square error. This is the square root of the mean squared error (assigned volume – count) ^{2} at the link level. This is divided by the average count to obtain % RMSE.

Figure 5 shows how the percent RMSE varies by volume group. The solid lines show the average and best performance from a group of US models. The WCTM values are shown to be better than the US average at all points and better than the US best for lower volume roads. Figure 6 shows a scatterplot of links by their estimated and observed volumes. A perfect fit would have all of the points falling on the 45° line. The points fall equally on both sides of the line, indicating no bias in the estimates.

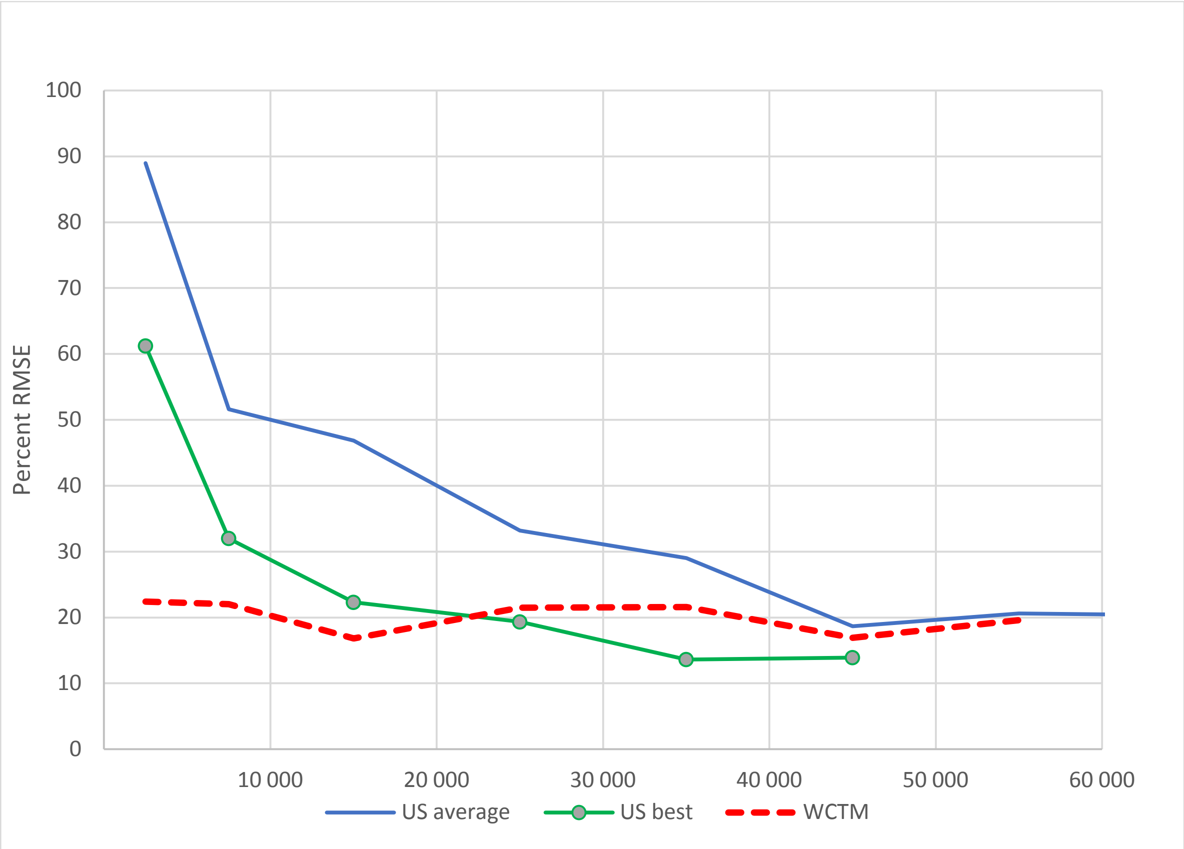


Figure 5: RMSE by Volume Group

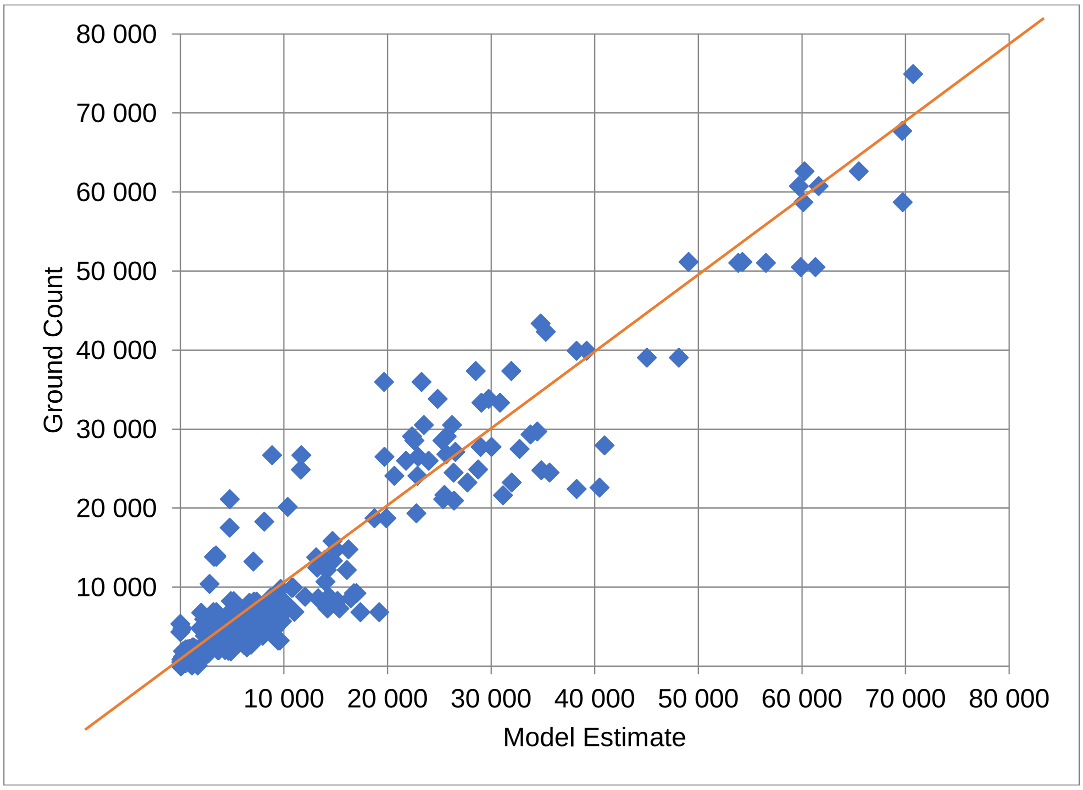


Figure 6: Link Level Assigned Volume vs. Count

While comparison to existing data is important, perhaps a more *useful* indicator of model validation is how it responds to changes in transport conditions. On-going work through 2021 is focused on performing a variety of tests to determine how the model responds to a wide variety of scenarios. This work will provide the users with added confidence in the model's value as a planning tool and the authors hope to report this information at future SATC meetings. Similarly, the land use component (which is described in a separate SATC paper) will also undergo sensitivity testing.

3. CONCLUSIONS

This work has developed an advanced travel demand model for Western Cape that represents a good tradeoff between theoretical comprehensiveness and ease of use/run time. It is based on an innovative structure that is similar to an activity-based model but is easier to understand and does not require specialised computing equipment. The travel model is integrated within a system of models that includes zonal land use and goods movement forecasting and runs in an acceptable time frame on a normal laptop computer.

This work supports a number of conclusions, including:

- It is feasible for a travel model to be assembled from a combination of calibrated and transferred components. The discrete type of model structure might indeed lend itself better to model transfer.
- The WCTM is a noticeable improvement over the conventional aggregate 4-step process but is not so complex that no one can understand how it works.
- A model that runs in 8 hours is more useful than one that takes 24 hours.
- Once you account for differences in demographics and in transport characteristics between different geographic regions, peoples' travel behavior is not that much different.

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