# Inventory management for the in-flight catering industry: A case of uncertain demand and product substitutability

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# Executive summary

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A case of uncertain demand and product substitutability

by

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The in-flight catering industry is a major contributor to food wastage. This wastage is a direct result of the deliberate overproduction of in-flight meals to protect against meal shortages and dissatisfied passengers. With the global strive towards sustainability and the resulting impact of wastage on a company's corporate image, in-flight catering companies need a solution that strives to achieve zero waste and a 100% passenger satisfaction level.

This dissertation evaluates the value of combining product substitution and demand uncertainty within an inventory decision-making model as a potential solution opportunity for the wastage dilemma faced by the in-flight catering industry. The decision-making model's purpose is to assist in-flight caterers to make improved decisions regarding the quantity of each meal type to produce for the specific flight under consideration. The model developed is defined as a stochastic multi-objective Mixed-Integer Programming (MIP) model with fixed recourse and two-way, stock-out based, partial consumer-driven (static) product substitution. The model relies on the output of a forecasting model, that consists of a time-inhomogeneous Markov Chain and a multiple regression model, to forecast the probability distribution of a flight's aggregate meal demand. Due to the lack of available data from public sources, synthetic data is generated to evaluate the model developed.

The model is compared against three alternative models that lack either demand uncertainty, product substitution or both to validate the value of including these elements in the decision-making model. The comparison results indicate the inclusion of the passenger load uncertainty improves the model's average reliability to achieve a 92% *minimum* Passenger Satisfaction Level (PSL) with at least 9.2%. Furthermore, it is shown that the stochastic passenger load model produces an average of 2.2 fewer surplus meals per flight instance at the expense of a 3.3% lower reliability when including the substitution behaviour of passengers. This substitution model's superior waste minimisation is attributed to the model's inherent risk-pooling capabilities, and further analysis shows that the value of product substitution increases when the model becomes more constrained. It is, therefore, concluded that the value of product substitution depends on the in-flight caterer's bias towards maximising either reliability or performance.

**Keywords:** in-flight catering, food wastage, product substitution, stochastic programming, pre-emptive goal programming, time-inhomogeneous Markov Chain, forecasting, inventory decision-making model, synthetic data.

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*Have I not commanded you? Be strong and courageous. Do not be afraid; do not be discouraged, for the Lord your God will be with you wherever you go.*
*– Joshua 1:9 –*

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# Acronyms

**DOW** Day of Week

**MDP** Markov Decision Process

**MAE** Mean Absolute Error

**MAPE** Mean Absolute Percentage Error

**MIP** Mixed-Integer Programming

**PSL** Passenger Satisfaction Level

**OR** Operations Research

**RP** Recourse Programming

**RMSE** Root Mean Square Error

**SAA** South African Airways

**SPGA** Sample Path Gradient Algorithm

**TBL** Triple-Bottom Line

**TPM** Transition Probability Matrix

**TPM-D** Transition Probability Matrix using Differences

**TPM-APL** Transition Probability Matrix using the Absolute Passenger Loads

# Chapter 1

# Introduction

In-flight caterers are responsible for producing food and beverages for each passenger on-board a flight. The meal provided has a dual purpose; not only does it satisfy the hunger of on-board passengers, but it is also used to distract stressed passengers and to manage their behaviour (McCool, 1995). Additionally, some airlines use in-flight catering as part of their competitive strategy due to its positive impact on passenger satisfaction (Teoh and Singh, 2018). This includes offering various meal types such as vegetarian and Halal meals. The importance of in-flight catering cannot be denied but, unfortunately, it is creating a significant sustainability issue due to excessive waste generation. In 2017, airlines generated 5.7 million tons of cabin waste of which at least 20% consisted of *untouched* food and beverages (IATA, 2019).

Food waste is a financial burden for any company and an emerging environmental and social concern. This statement is especially true in the in-flight catering industry, where surplus meals must be discarded due to stringent health policies and legislations. This wastage tends to end up in landfills or at incinerations sites, resulting in the release of unwanted greenhouse gasses. Furthermore, as mentioned by Sambo and Hlengwa (2018a), the world is becoming a concrete jungle and food is increasing in scarcity due to the growing population and the farming industry becoming less attractive. Consequently, airlines and in-flight catering companies have a responsibility to minimise food wastage, especially when considering the rapid growth of the air transport industry.

The extent of in-flight waste is far-reaching as in-flight catering companies and airlines worldwide are struggling to maintain acceptable levels of waste. The wastage dilemma results from the over-catering strategy being followed − caterers tend to inflate meal orders to mitigate the risk of meal shortages, passenger dissatisfaction and costly flight delays resulting from inaccurate meal demand predictions. This meal order inflation leads to high numbers of excess meals that are discarded as waste. These high quantities of waste negatively impact the corporate image of the catering company and their ability to attract customers due to the global strive for sustainability (Lasaridi et al., n d; Teoh and Singh, 2018). Evidently, in-flight catering companies are faced with two conflicting objectives. The primary objective is to maximise passenger satisfaction to attract and maintain passenger loyalty. The secondary objective is to minimise waste resulting from excess in-flight meals to improve the company's profitability, competitiveness and corporate image.

This dissertation recognises the need for an inventory decision-making model that can determine the most efficient in-flight meal order quantities for a specific flight under consideration. As a potential solution, this dissertation develops and investigates an inventory decision making model with uncertain demand and product substitution.

## 1.1 The extent of in-flight waste

The limited research available regarding in-flight food waste at *international* and *South African* airlines and airline catering companies are summarised in this section to emphasise the magnitude of the in-flight food waste issue worldwide. The research hints at the positive relationship between in-flight waste and demand uncertainty.

### 1.1.1 International

Thamagasorn and Pharino (2019) analysed a halal food production process at an airline catering company in Thailand. Based on the results of the study, only 63.0% of the procured food is used to produce meals served to passengers, 24.3% is kept as buffer stock, and the remaining 12.7% is wasted during production. The authors warn that food waste can increase if buffer stock is not managed appropriately. If the stock is not served to passengers before the *use-by* date, it must be discarded due to strict food safety standards and policies. The same holds for surplus meals. As such, the extent of pre-consumer waste is highly dependent on inventory management and demand fluctuations. Megodawickrama (2018) agrees with this statement. The author conducted a similar study at an in-flight catering company in Sri Lanka and observed a negative linear relationship between pre-consumer food waste and the daily meal demand. During peak periods, demand is somewhat stable as flights are frequently fully-booked and the number of meals required is, therefore, close to the flight's capacity. Accordingly, over-catering is limited, which leads to less waste generation. During the observed period, July to October 2017, the daily kitchen waste fluctuated between 1 200 kg to 2 200 kg.

Li et al. (2003) conducted a post-consumer waste composition analysis of in-flight services at Cathay Pacific Airways Limited during 1996 to 1997. The study focused on fully loaded flights with 313 economy class passengers and found that an average of 160.2 kg of food is wasted on a long-haul flight. Appallingly, this total includes 112.2 kg of *untouched* food. This wastage could potentially indicate that the meal supply was greater than the total meal demand. This statement is motivated by the work of Goto et al. (2004). The authors investigated the meal ordering performance at Canadian Airlines by studying a single flight over a six month period. In this study, roughly 75% of the flights were over-catered except when the passenger load was very low or at maximum capacity. The authors estimated the cost resulting from these excess meals to be $1.8 million annually. Similar results were also obtained by Blanca-Alcubilla et al. (2019), who performed a related study on 147 flights that travelled between November and December in 2016. It was estimated that the total in-flight waste distribution for a plane landing at the Barajas airport would consist of 23% *opened* meals and 10% *unopened* meals. El-Mobaidh et al. (2006) investigated in-flight waste for Egypt Airline's economy class passengers. The authors concluded that 284 tons of post-consumer food, with an energy potential of 2.56 tera joule, is thrown away annual. Tofalli et al. (2018) also observed alarmingly high waste generation.

### 1.1.2 South Africa

The only literature available regarding food waste at a South African airline catering company was conducted by Sambo and Hlengwa (2018b). The author used *Air Chefs* as a case study to investigate the relationship between post-consumer food waste and waste management policies within the airline catering industry.

Air Chefs is the wholly-owned catering entity of South African Airways (SAA), the country's national carrier airline. The catering company was established in 1986 and grew to be the leading airline catering company in South Africa, supplying 11 million meals annually (South African Airways, 2018). Unfortunately, Air Chefs has been struggling with food waste issues in the past decade. In 2013, Skiti (2013) reported on three separate occasions where Air Chefs had thrown away various food items. On one occasion, 435 cinnamon crumpets, 400 cream coffee cakes, 1 200 mince cannelloni meals, 32 kg halloumi cheese, 1 620 carrot cakes and 14 kg of mustard beef were dumped in a three-day period. Authorities claimed that the waste resulted from too much food being ordered, theft, fraud and insufficient inventory management. The author states that an intervention team was hired to improve the situation but was dismissed two months thereafter, thereby providing no confidence that the problems were addressed appropriately. Based on the work of Sambo and Hlengwa (2018b), the food waste problem is still present. The authors investigated the quantity and monetary value of post-consumer food waste for all SAA economy class flights returning to Johannesburg and concluded that, during a four-day period, food worth R 285 355.10 is wasted. The top five wasted commodities were 4 283 desserts, 4 186 starters, 3 821 frilled rolls, 2 622 yoghurts and 2 396 croissants.

This waste issue is especially alarming considering SAA's dire financial situation. SAA has reported financial losses since 2011 and officially entered business rescue in December 2019 (Daniel, 2021). However, the impact of in-flight food waste stretches beyond economic consequences. This is explained in the following section.

## 1.2 The impact of in-flight waste on corporate sustainability

Society is becoming more fixated on environmental and social concerns. This creates various obligations and expectations for companies worldwide as their environmental profile is becoming an essential part of their overall reputation (Lasaridi et al., n d). As such, to ensure corporate sustainability, companies must consider the *full cost* of doing business. Based on the Triple-Bottom Line (TBL) theory, this can be achieved by focussing on three key performance areas: the company's actual *profit*, and its impact on *people* and the *planet*.

Food waste is common for most businesses operating in the hospitality industry and, unfortunately, has an adverse effect on the corporate sustainability of these businesses. Below is a brief description of the impact of food waste on each TBL performance area:

**Profit:** Food wastage negatively impacts a company's profitability. According to Lasaridi et al. (n d), the profit loss resulting from wastage exceeds the purchase cost of the food commodities, since caterers will be unable to recover the add-on costs associated with labour, water, energy and waste disposal throughout the *entire* supply chain. Sambo and Hlengwa (2018a) reported a few additional *hidden* costs, including opportunity costs, lost material, time and risk and liability costs. Considering South Africa's weak economy and the competitive markets, companies such as SAA cannot afford to throw away resources.

**People:** In 2015, around 25.2% of South Africa's citizens lived below the food poverty line (Maluleke, 2019). The food poverty line refers to the amount of money a person needs to afford the minimum required daily energy intake. In 2015, this value was as low as R441 per month, yet unaffordable to a quarter of the country's citizens. The causes thereof include poverty, food insecurity and inadequate access to food sources. These causes highlight the immorality of food wastage. Alarmingly, it is estimated that approximately one-third of edible food is wasted (Thamagasorn and Pharino, 2019), whilst half would still be edible (Sambo and Hlengwa, 2018a).

**Planet:** Most of society is unaware of the adverse consequences of dumping food waste at landfill sites. Due to its moisture content, various harmful gasses, such as carbon dioxide ($CO_2$) and methane ($CH_4$), are released into the atmosphere. These gasses are known causes of the greenhouse effect. According to Thamagasorn and Pharino (2019), the environmental consequences of food waste also includes depletion of soil fertility and the loss of resources such as land and water. Lasaridi et al. (n d) agree, stating that food waste generates 8% of the global greenhouse gas emissions and consumes 30% of all the water used by agriculture.

The impact of surplus in-flight meals extends further than food waste. Although the weight of one meal might be considered negligible, the collective impact thereof can be significant. It is intuitive that an aircraft's weight has a drastic influence on its fuel burn. Some airlines have applied interesting strategies to minimise fuel cost, including reducing seat thickness, minimising in-flight magazines, removing meal trays and asking passengers to visit the lavatory before departure to ensure lighter bladders (Anon., 2018). According to Lynes and Becken (2002), a medium-sized international airline can annually save roughly USD 500,000 ($\pm R$ 8.5 million) in fuel cost alone by removing 100 kg of surplus meals per flight.

Surplus meals also result in the waste of packaging materials. Naturally, the wastage thereof is an additional expense for any company and will negatively affect the company's *profitability*. The *planet* is also impacted due to excessive pollution caused by the production of the packaging unit, as well as the biodegradability issue of the materials used. Ultimately, the pollution caused has adverse health effects on the country's *people*. To combat this issue of packaging waste, some companies are exploring more sustainable options. For instance, a design studio in the United Kingdom known as PriestmanGoode, designed a partially edible and plastic-free dinnerware alternative (Leedham, 2020). Their 100% biodegradable concept consists of coffee ground, algae, banana leaves and crispy wafers. Additionally, Air New Zealand is testing the use of *edible* cups known as 'twicce' (Bryant, 2019). These cups are vanilla flavoured and can be used to serve coffee and ice cream. Unfortunately, these cups can further increase the food wastage issue faced by airlines as excess, expired and uneaten cups will have to be discarded. It is, therefore, no surprise that some airlines encourage and award discounts to passengers who bring their own reusable travel mugs (Kollau, 2019).

In agreement with Sambo and Hlengwa (2018a), waste is undoubtedly unsustainable and irresponsible from an economic, social and environmental point of view. Although the in-flight catering industry is starting to partake in the war on waste, their focus is mostly set on reducing single-use plastics while food is seen as a disposable commodity. Furthermore, the disposal of food at landfill site is the worst course of action based on the *food waste hierarchy*. Given the extent and negative impact of food waste, it is surprising that this issue has received minimal attention.

## 1.3 The food waste hierarchy

The food waste hierarchy, depicted in Figure 1.1, is a framework used to identify and prioritise alternative courses of action to minimise and control food waste. According to Papargyropoulou et al. (2014), this framework considers the three dimensions of sustainability, which are identical to the above-mentioned TBL performance areas.


Figure 1.1: The food waste hierarchy and strategies (Adapted from Papargyropoulou et al. (2014)).

Currently, it is believed that most in-flight food waste end up in landfill sites. This *disposal* method is the least preferred method for waste management. The *recovery* and *recycle* methods are concerned with re-purposing food waste into animal food, composting or to act as a source for energy recovery. Although these methods are more environmentally friendly and can potentially generate a small income, they are not considered socially responsible. Consequently, food *re-use* for human consumption, such as donations, is ranked higher on the food waste hierarchy. Unfortunately, food donations can be problematic for companies, especially in the airline catering industry. For instance, Air Chefs used to donate surplus food to various charities but stopped doing so in 2013 due to transparency issues in the charity selection process (Skiti, 2013). Furthermore, Sambo and Hlengwa (2018a) explain that food donations can also present a high risk to the company's reputation; if the charities do not follow correct food safety procedures and food poisoning occurred, the catering company might be held liable. The authors also state that governments tend to avoid food banks as it can reveal poverty and the government's inability to create jobs for the country's people.

Waste *prevention* has the largest area in the food waste hierarchy as it is undoubtedly the most sustainable approach. Therefore, the first course of action should aim to prevent food waste at the source, such as preventing overproduction. This approach will be the most beneficial for the in-flight catering industry as the overproduction of in-flight meals is believed to be the primary cause of this industry's wastage dilemma. The overproduction of in-flight meals is herewith referred to as the '*over-catering strategy*'.

## 1.4 The over-catering strategy

The in-flight catering industry has unique characteristics differentiating it from the typical catering industry, such as high production rates, off-site meal production and the time-sensitivity of order deliveries (McCool, 1995). These characteristics create additional challenges for in-flight caterers. The most prominent is the *on-time* delivery of meal orders in the *exact* quantity required for each *specific* meal type. It is, therefore, of utmost importance that caterers plan an effective production schedule.

In-flight caterers have to start the planning, procurement and production ahead of flight departure due to the lengthy nature of the food production process. This is a challenging task as the number of passengers that will board the flight is unknown until a few hours, or even minutes, before flight departure (Hasachoo and Masuchun, 2016; Megodawickrama, 2018). This challenge is intensified when the airline offers a variety of in-flight meals because doing so increases the level of uncertainty. Accordingly, the planning must be based on *estimated* demand. The estimated demand is usually derived from the number of tickets already booked, a forecast based on historical data and the experience of the catering company (Goto et al., 2004).

As more information becomes available closer to departure, the caterer must frequently adjust the estimated demand and, subsequently, the production schedule. In the study conducted by Hasachoo and Masuchun (2016), forecasting errors accounted for 53.17% of the total adjustments made, followed by 28.07% resulting from changes in customer requirements. These changes require great production schedule flexibility, which most caterers cannot guarantee due to production lead times and various resource constraints, such as time and capital. Consequently, adjustments could lead to production disruptions, meal shortages and ultimately, costly flight delays. These consequences will negatively impact passenger satisfaction and the reputation of the airline and catering company. To compensate for the inaccurate demand estimations, in-flight caterers turn to over-catering as a means of protection against potential shortages and flight delays (Thamagasorn and Pharino, 2019).

The undesired trade-off for this over-catering strategy is high levels of post-consumer waste in the form of surplus meals. It also contributes to increased pre-consumer waste due to the supply chain bullwhip effect. Wang and Disney (2016) define '*the bullwhip effect*' as '*the phenomenon where order variability increases as the orders move upstream in the supply chain*'. This increase in order variability is analogous to the movement of a whip when flicked - the whip's wave pattern *amplifies* in a chain reaction (Daniel, 2019). For this reason, the phenomenon is also known as '*demand amplification*'. The following example illustrates how the bullwhip effect increases pre-consumer waste; Assume the true demand for apples served on-board a flight is 20. Due to the over catering strategy, the catering company ordered 30 apples to protect against demand fluctuations. Similarly, the farmer supplying the apples will increase the order with 20% (36 apples in total) to ensure that at least 30 good quality apples arrive at the catering company. Accordingly, six apples will be thrown away as pre-consumer waste and ten apples will be discarded as post-consumer waste. While pre-consumer waste cannot always be avoided completely, it can be reduced by lowering the amount of over-catering. It is clear that the over-catering strategy is biased towards the primary objective of maximising passenger satisfaction and overlooks the secondary objective of waste minimisation.

In conclusion, the over-catering strategy is a coping mechanism for the high levels of meal demand uncertainty present during the meal planning and production phases and the unreliability of current forecasting methods. These extreme levels of uncertainty create various forecasting difficulties, which explains why forecasting errors were the leading cause of production schedule changes in Hasachoo and Masuchun (2016)'s study. It is clear that the in-flight catering industry is in need of an inventory decision-making model that incorporates meal demand uncertainty to aid catering companies in making more informed decisions, and to prevent the over-inflation of meal order quantities. This *stochastic* model must be able to generate the desired solutions in real-time as it is expected that a catering company has to plan and cater for multiple flights a day. To reduce the impact of demand uncertainty, the concept of *product substitution* will also be investigated.

## 1.5 Project description

Most major airlines offer a variety of complimentary meals on their in-flight menu as part of their competitive strategy. For passengers without specific dietary requirements, the selection of meals served is substitutable; It is believed that these passengers would be willing to substitute their preferred meal (in the case of a stock-out) due to the limited availability of alternative food sources on the aircraft.

Traditional forecasting and inventory decision-making models are suboptimal when the product under consideration is substitutable because these models consider the demand for a product in isolation. However, the demand for a substitutable product is dependent on its inherent characteristics *and* on the inventory levels of possible substitutes. Also, these models are often only focused on the single-point estimate of a product's demand and, subsequently, ignore valuable information.

This dissertation identifies the *substitutability* of in-flight meals, along with the incorporation of the *meal demand uncertainty*, as the key opportunities for in-flight waste reduction caused by the over-catering strategy.

### 1.5.1 Problem statement

In-flight catering companies follow an over-catering strategy to mitigate the risks of meal shortages and flight delays. The root causes of the strategy are identified as the high level of uncertainty within the meal planning and production processes − the final passenger load of a flight and individual meal preferences are unknown up until flight departure − and the unreliability of forecasting methods. The consequence of over-catering is high amounts of leftover meals, which are frequently discarded as waste due to stringent healthy policies. This negatively impacts the catering company's TBL performance measures, their corporate image and competitiveness. This dissertation will address the following research question:

> *Will the inclusion of product substitution and demand uncertainty within an inventory decision-making model be able to help an in-flight catering company reduce waste resulting from surplus in-flight meals, while maintaining an acceptable level of passenger satisfaction?*

### 1.5.2 Research design

This dissertation will develop a multi-objective inventory decision-making model that incorporates the effect of product substitution and demand uncertainty. The model will be used to find the set of the most efficient meal order quantities for a particular flight. By following the suggested order quantities, the in-flight caterer should be confident that the desired and pre-defined *minimum* Passenger Satisfaction Level (PSL) will be achieved. This will reduce the need for the over-catering strategy. In addition, the most efficient meal order quantities will further maximise the PSL, if possible, while minimising the number of surplus meals.

The model will exploit the risk-pooling effect resulting from substitutable products. Simply put, instead of having *dedicated* safety stock for each meal option, safety stock will be *shared* among meal options. This will reduce the meal order quantities required and, ultimately, the number of surplus meals produced. The most efficient meal order quantities chosen by the model will depend on the level of meal demand uncertainty and the substitutability between in-flight meals.

The decision-making model will consist of a stochastic and multi-objective Mixed-Integer Programming (MIP) model with fixed Recourse Programming (RP) and two-way, stock-out based, partial-consumer driven product substitution. It will also incorporate forecasting using a time-inhomogeneous Markov Chain. The latter is included to predict the probability distribution of the aggregate meal demand ahead of the particular flight.

The model will be evaluated based on its reliability, performance and timeliness. The model must be reliable to ensure a reasonable level of confidence to prevent the over-catering strategy. Its performance must be satisfactorily to indicate that the model is worthwhile and able to meet the primary and secondary objective. Lastly, it must be able to generate solutions within a reasonable time to ensure that it is suitable for the in-flight catering industry as a caterer must plan for multiple flights per day.

### 1.5.3 Research methodology

The theoretical area of this dissertation is related to *Operations Research (OR)*. Rajgopal (2004) defines OR as a '*scientific approach to solving problems*', whereby the key characteristics of a problem is translated into a model that can be analysed to obtain an optimal solution. The author proposes a seven-step approach to address OR problems which is mainly followed in this dissertation. However, an additional step titled '*Data generation*' is included due to the unavailability of the required data from public sources. The relevant steps are described below in sequential order.

**Problem definition:** This project addresses the waste issue experienced by in-flight caterers. More specifically, this dissertation is aimed at the development of an inventory support model that can identify the set of the most efficient meal order quantities for a specific flight under consideration. Doing so will ensure that the respective in-flight catering company can make informed decisions regarding the order quantity of in-flight meals and the expected outcome. The most challenging factors in the problem considered are the stochastic nature of a flight's passenger load and the individual meal choices of the passengers.

**Data collection:** Various literature is investigated to identify a suitable model solution. This will include the investigation of existing in-flight waste reduction strategies, forecasting models, product substitution models and OR techniques. The main findings are discussed in the literature review.

**Model formulation:** The problem is translated into a stochastic mathematical decision-making model with two conflicting objectives: (1) maximise passengers satisfaction and (2) minimise surplus inventory. While considering the substitution behaviour of passengers, the model must find the set of meal order quantities that would best satisfy the objectives after meeting the *minimum* PSL requirement, given that the meal demand is uncertain. The demand uncertainty is captured using a capable forecasting model.

**Data generation:** The model developed is trained and tested using synthetic data due to the unavailability of public data relating to a flight's booking process. Generating the synthetic dataset for the numerical example requires a sound understanding of the causes of uncertainty within the in-flight meal order processes and the booking behaviour of passengers.

**Model solution:** The model formulated is transformed into its deterministic equivalent so that it can be solved using standard optimisation software. This is achieved using recourse programming. In addition, the chosen forecasting model is trained using the data generated. This includes a small-scale sensitivity analysis to identify the most efficient parameters that will maximise the accuracy of the forecasting model.

**Validation and output analysis:** The model is evaluated by comparing its reliability, performance and timeliness with similar models that (1) do not allow substitution, (2) neglects passenger load uncertainty and (3) do not allow substitution and neglects the passenger load uncertainty. The purpose of the four models is to evaluate the benefit of including product substitution in the model, as well as the benefit of explicitly modelling the meal demand uncertainty.

The evaluation process consists of solving each model using LINGO 18.0 for a specific flight instance, *minimum* PSL requirement and target weight combination. Thereafter, the output of each model is applied to various possible realisations of the particular flight instance to quantify the model's average reliability and performance. This process is repeated for 16 flight instances and 2 *minimum* PSLs and five target weight combinations. Based on the results, the best all-round decision-making model is identified. The recommended model is further validated against a simple approach that does not require the development of an inventory decision-making model.

### 1.5.4 Expected contribution

Teoh and Singh (2018) claim that no previous study is explicitly concerned with optimising in-flight meal order quantities by considering both aspects of supply and demand. The most relevant study found is that of Goto et al. (2004). The authors developed an optimal meal ordering policy for in-flight caterers but assumed only one available meal type per flight. As such, the expected contribution of this dissertation is to bridge the gap between the supply and demand of in-flight meals sustainably. This is achieved by developing a multi-objective decision-making model that can identify the optimal meal order quantities for the set of meals offered on the in-flight menu of a particular flight by utilising the concept of product substitution and stochastic programming. To the authors best knowledge, this is the first study that attempts to exploit the risk-pooling capabilities of a substitution model to reduce in-flight waste resulting from surplus meals

The successful development of the decision-making model could help to improve the sustainability, competitiveness and corporate image of in-flight catering companies by reducing surplus meals (waste). The model will have global applicability and could easily be adapted for other industries within the hospitality sector.

### 1.5.5 Outline of dissertation

This dissertation is structured as follows: A comprehensive literature review is presented in Chapter 2 in which the suitable models and solution techniques are identified and explored. The suggested model is discussed Chapter 3 and formulated as two interconnected parts: (A) the inventory decision-making model and (B) the forecasting model. The model formulation pinpoints the input data requirements, which is generated and presented in Chapter 4. The synthetic data is then used in Chapter 5 and Chapter 6 to obtain, respectively, the model solution and results. Recommendations and future research opportunities are summarised in Chapter 7, followed by the concluding remarks.

# Chapter 2

# Literature review

This chapter presents the literature review conducted to identify a suitable solution model. The existing in-flight waste reduction strategies are investigated first and include the forecasting models used in the airline industry. Thereafter, product substitution is discussed and a suitable model is identified. Two additional Operations Research (OR) techniques are explained afterwards, followed by an in-depth description of the chosen time-inhomogeneous forecasting model.

## 2.1 Existing in-flight waste reduction strategies

A major challenge in the in-flight catering industry is to forecast the meal demand with reasonable accuracy. This is due to the excessive variation in the daily meal demand. Figure 2.1 shows an example of the variation observed by Megodawickrama (2018) at an in-flight catering company in Sri Lanka from June to October 2017.

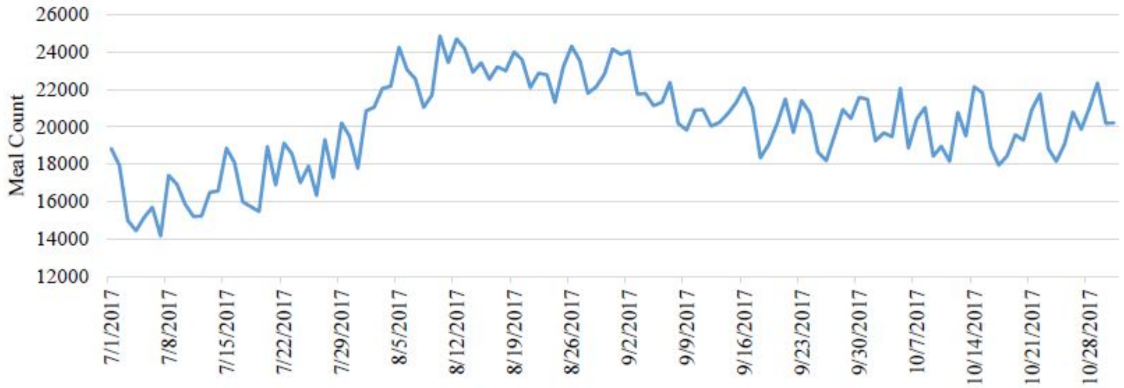



Figure 2.1: The daily meal demand variation observed at a flight catering company in Sri Lanka (Megodawickrama, 2018).

It is intuitive that a reliable meal demand forecast can greatly minimise the safety stock required. Unfortunately, due to the excessive variation inherently present in the in-flight catering industry, the accuracy of a forecasting model is limited. For this reason, the literature reviewed extended further than the frequently used forecasting models to identify a solution opportunity that utilises the unavoidable meal demand uncertainty. To identify such an opportunity, the typical forecasting models and existing in-flight waste reduction strategies must be scrutinised to find their shortcomings.

### 2.1.1 Forecasting techniques

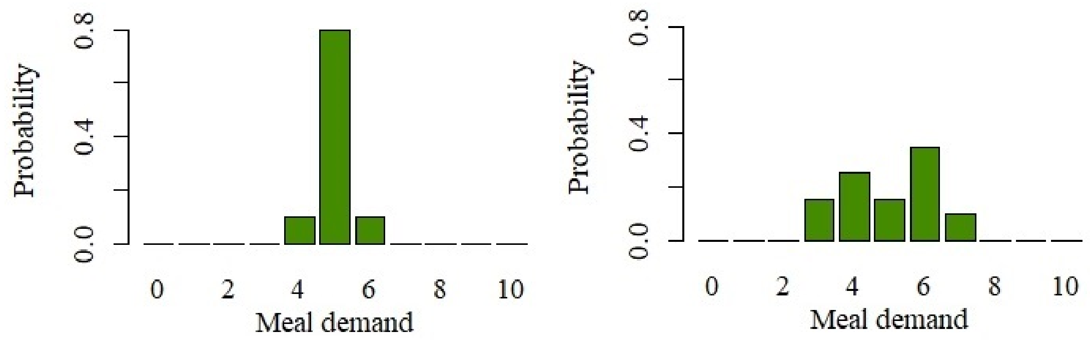
Lee (1990) state that a 10% improvement in forecasting accuracy could increase the annual passenger revenue by up to 3%. It is, therefore, no surprise that the field of forecasting in-flight passenger load is widely explored due to its undeniable importance. Various forecasting techniques exist that have proven success in their relevant study. Dantas et al. (2017) claim that casual econometric, time series and artificial intelligence are the most noteworthy forecasting approaches. Note that, as will be explained later on, this dissertation assumes that the aggregate meal demand of a flight is equal to its final passenger load.

Casual *econometric* models are concerned with the cause-and-effect relationship between the dependant variable and social, economic and service-related factors (Dantas et al., 2017). Sivrikaya and Tunç (2013) developed a semi-logarithmic regression model to predict a city's *annual* air traffic demand. The model uses geo-economics and industry-related factors as input, such as urban population, bedding capacity, travel distance by road and the number of airlines on the route.

*Time series* models depend on the correlation between past and present observation to generate a forecast. Wickham (1995) compared the relative performance of traditional time series forecasting techniques when used to predict short-term passenger demand for a *single* day. The author found the advanced pick-up method combined with exponential smoothing to be superior to the classical pick-up, regression, exponential smoothing and simple moving average methods. An important observation made is that all of these models displayed positive biases. Overestimation of passenger demand is, therefore, expected when applying these models. Zhong et al. (2016) evaluated the ability of the ARIMA, Holt-Winters and Trend analysis with decomposition models to predict *monthly* air passenger traffic and concluded that the other two models outperformed the ARIMA model.

Lastly, artificial intelligence models produce forecasts by learning and reproducing hidden patterns. A popular approach in this category is the use of artificial neural networks. Srisaeng et al. (2015) applied this technique to predict Australia's *quarterly* passenger demand on national scale. The authors demonstrated that this approach leads to more accurate results when compared with a classical regression (econometric) model. Alekseev and Seixas (2009) obtained the same conclusion when using both methods to forecast Brazil's annual air traffic.

Van Ostaijen et al. (2017) criticises some of the above-mentioned forecasting techniques by stating that valuable insight is lost when applying them. These forecasting techniques only determine a single point estimate for future stochastic events based on the expected value thereof but give no indication regarding its probability distribution. For in-flight catering companies, the meal demand distribution should be equally, if not more important, than the single point estimate. This statement is motivated with Figure 2.2.




(a) Minimal uncertainty

(b) Moderate uncertainty

Figure 2.2: Two probability distributions with an expected number of five.

Figure 2.2 depicts two different probability distribution scenarios, yet the single point estimate for each scenario is equal to five. Clearly, a caterer will follow a different strategy for each scenario. For instance, due to the minimal uncertainty in Figure 2.2a, the caterer is likely to accept the single point estimate. However, by knowing that there is a 45% chance that the demand could exceed five meals, as in Figure 2.2b, additional safety stock could be justified appropriately. The distribution of the uncertainty clearly influences the level of confidence achievable. Knowing the distribution could impact the chosen strategy to handle the uncertainty.

As a solution, Van Ostaijen et al. (2017) propose a '*dynamic booking forecasting method*' that is able to determine the expected value of a flight's passenger load and its accompanying probability distribution. This technique models the flight booking process as a time-inhomogeneous Markov chain. The authors compared the performance of their proposed model with the advanced-pickup method and the historical mean method using a real-life case study. They found their model to be superior as their model was more accurate with up to 8% lower forecasting errors, especially for short-term forecasts. This forecasting technique is, therefore, considered favourable for the in-flight meal ordering process as it is heavily reliant on accurate, short-term forecasts. Furthermore, the additional information regarding the distribution of the forecast can provide valuable insight regarding the uncertainties faced by the catering company. This insight can aid in more effective risk mitigation strategy developments and, subsequently, motivates the development of a *stochastic* inventory decision-making model.

### 2.1.2 Alternative strategies

Brochado and Freedman (2009) claim that there is a direct relationship between the meal portion size and post-consumer waste. The authors came to this conclusion by studying the effect of portion sizes on french fries consumption at a university's all-you-care-to-eat facility. Interestingly, it was also noted that portion sizes could be reduced by up to 33% before being noticed. The work of Brochado and Freedman (2009) raises questions regarding the size of in-flight meals due to the excessive in-flight post-consumer waste observed in Section 1.1. The results of Teoh and Singh (2018)'s study support the theory that in-flight meals might be too big.

Teoh and Singh (2018) developed a multi-objective model aimed at minimising food waste while maximising passenger's expectations towards in-flight catering services. In this study, the passengers were allowed to choose between a standard in-flight meal or a light meal. A light meal is similar to the standard meal but consists of a smaller portion. The authors concluded that light meals were preferred by at least 49% of the passengers on a long-haul flight, which can result in an average food weight reduction of 28 kg per flight. The benefit of reducing meal portion sizes is believed to ripple throughout the supply chain as it will reduce overall procurement and production cost (Brochado and Freedman, 2009). It will also decrease both pre- and post-consumer waste and can improve the flight's fuel efficiency.

Goto et al. (2004) addressed a similar objective as this dissertation. The authors realised that caterers are allowed to make changes to the original meal orders (as obtained from foresting) and acknowledge that the cost associated with these changes are time-dependent. Accordingly, the authors developed a decision-making model that can be used to identify an optimal meal ordering policy for a single meal type. The optimal policy suggests the most appropriate meal order quantity at a predefined decision point before departure, given that the passenger load is known at that instance. The goal thereof is to reduce the total cost of the meal ordering process; the policy weighs the trade-offs between the cost associated with meal shortages, excess meals and short-notice changes. The authors use a Markov Decision Process (MDP) to develop this optimal order policy, which includes five decision points that range between 36 hours to zero hours before flight departure.

The authors applied an aggregated model to 40 selected flights and obtained promising results. On average, the number of surplus meals reduced to 8.33 meals, whereas 10.19 excess meals were observed in actual practice. The authors argue that this result can be further improved by decreasing the bin size used to aggregate the model. The authors concluded that the appropriateness of the MDP model is dependent on the flight's duration and not beneficial for domestic flights. While the MDP model outperformed the airline's actual practice for 85.7% of the long-haul flights, the same was not true for short-haul (domestic) flights where current practice dominated 62.5% of the time. That being said, the optimal policy reduced the number of short-catered domestic flights by 37%. This indicates that an improvement was indeed observed *if* the model's objective was focused on the Passenger Satisfaction Level (PSL) instead of waste minimisation only. Thus, for domestic flights, passenger satisfaction increased at the expense of waste minimisation. These observations highlight the multi-objective nature of the in-flight catering industry and indicates the importance of considering the primary and secondary objectives simultaneously as the two objectives are conflicting.

### 2.1.3 The suggested solution opportunity

Merely reducing the overall portion size of in-flight meals is not considered a sustainable approach to reduce waste because it can negatively impact an airline's competitiveness. The risk thereof can be mitigated by offering two different meal portion sizes, as suggested by Teoh and Singh (2018). However, this approach might not be feasible due to the additional logistics involved.

Another approach to reducing waste is to improve the forecasting accuracy of the meal demand. The forecasting techniques listed and the optimal meal ordering policy approach developed by Goto et al. (2004) can be used to estimate the demand for a *single* meal type as they ignored passenger meal type selection such as beef or vegan. Rajaram and Tang (2001) reported that these subjective forecasts frequently result in average errors greater than 50%. The authors explain that this is because the demand for a product is not only dependent on its inherent qualities but is also influenced by the inventory levels of similar products. Thus, these forecasting techniques consider the demand for a product in isolation and do not consider the possibility that a customer would be willing to substitute a product. This leads to unintentional over- and under-estimation of the product's demand.

It is suspected that in-flight meal substitution is a common occurrence; Passengers have to settle for another meal choice if their preferred option is out-of-stock, otherwise, they will have to travel hungry. This substitution behaviour is believed to be the key to reducing in-flight waste by utilising the risk-pooling effect thereof.

## 2.2 Product substitution

Product substitution is essentially the act of fulfilling unsatisfied demand with alternative products. Vaagen et al. (2011) claim that it is frequently used to safeguard against future uncertainty. Accordingly, the aspiration is to incorporate product substitution into the meal order planning processes of in-flight caterers to reduce the impact of demand uncertainty. Based on intuitive reasoning, it is believed that doing so will reduce excess meals (waste) by sharing safety stock among potential substitutes instead of carrying dedicated safety stock for each meal type. This sharing of safety stock is referred to as the risk-pooling effect.

Conventional inventory decision-making models assume that a sale is lost when a product shortage occurs. This is a deviation from reality as customers are frequently willing to substitute if their preferred product is unavailable. This substitution behaviour increases the *effective* demand for substitutable products that can result in additional shortages and subsequent revenue loss. For example, Yücel et al. (2009) observed a profit decrease of 23% when ignoring the effect of product substitution, while Zeppetella et al. (2017) observed a 30% increase in lost sales. Conventional inventory decision models are, therefore, suboptimal under product substitution. It is speculated that in-flight passengers will be exceptionally open to meal substitutions as there are no other sources of food on a flight. For this reason, conventional inventory models, such as the classical Newsvendor model, could be suboptimal and inappropriate for the in-flight meal ordering processes. Furthermore, Gilland and Heese (2013) also showed that simply allocating order quantities according to a product's average fraction of the total demand will not deliver favourable results.

There is a significant amount of literature relating to inventory support models with product substitution. This is attributed to the positive impact thereof on the expected profit of the model. Rajaram and Tang (2001) state that retailers can always expect higher profits when products are substitutable. All of the literature studied agree with this statement. Unfortunately, there are contradicting observations regarding the impact of substitution on the optimal order quantities.

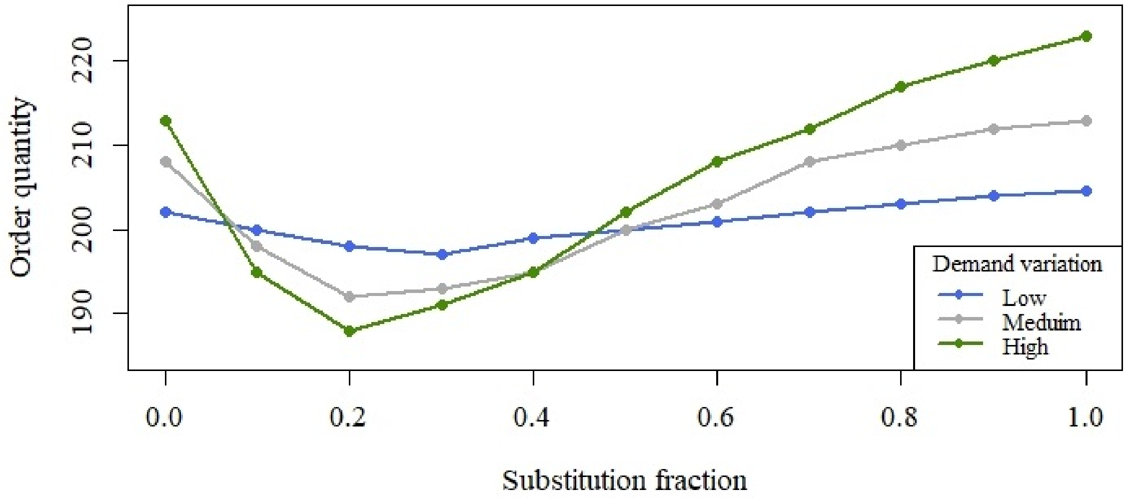



Figure 2.3: Order quantity vs subtitution fraction at different levels of demand variation for a two-product case (adapted from Vaagen et al. (2011)).

Intuitively, the optimal inventory levels should decrease due to the risk pooling effect. However, Huang et al. (2011) and Rajaram and Tang (2001) are among the few who observed inventory increases. Huang et al. (2011) claim that inventory levels and the resulting expected profit are influenced by the following model parameters:

- *Product substitutability*: Profit increases monotonically alongside product substitutability because it reduces the probability that a customer would leave empty-handed. Inventory levels can either increase or decrease depending on the substitutability of the products, as seen in Figure 2.3.
- *Demand correlation*: Rajaram and Tang (2001) and Huang et al. (2011) show that optimal inventory levels *decrease* along with demand correlation. Accordingly, substitution is most beneficial when product demand is highly *negatively* correlated. A negative correlation simply means that when one product has a high demand realisation, the other product will have a low realisation and vice versa (Netessine and Rudi, 2003). In other words, if one product's demand fluctuates positively, it follows that the other product's demand will fluctuate negatively. This negative correlation decreases the *overall* demand variation between the two substitutable products, which in turn, reduces the quantity of safety stock required (risk pooling).
- *Demand variation*: Inventory increases with demand variation due to the need for additional safety stock to protect against shortages (Yücel et al., 2009). The effect of demand variations on the order quantity is visible in Figure 2.3. The significance of product substitution increases with demand variability.
- *Newsvendor ratio*: This measure is a function of a product's selling price, purchasing cost, shortage cost and salvage value (Winston, 2004a). A high Newsvendor ratio indicates a high-profit product and, as shown by Huang et al. (2011), will result in increased order quantities. Stock levels for a low-profit item will be minimal.

Based on the above, it is encouraging to incorporate the concept of product substitution in the inventory decision-making model for the in-flight catering industry. It is reasonable to assume that passengers will be willing to substitute due to the limited availability of food on-board the aircraft. According to Kök and Fisher (2007), this willingness to substitute is an important parameter. Furthermore, in-flight meal demand is negatively correlated - the aggregate meal demand is fixed as only one complimentary meal is served to a passenger. Consequently, the demand for one meal type comes at the expense of the other meals offered on the menu. Lastly, the total and individual meal type demands are highly irregular and uncertain. This means that the effect of substitution is expected to be significant. It should also be noted that an order quantity increase does not necessarily increase the left-over inventory. It could simply indicate that the model will be able to serve more customers through improved inventory decisions.

Due to the significant amount of literature relating to product substitution, general classifications thereof will be discussed first. The problem addressed in this report is identified as a two-way, inventory-based partial consumer-driven product substitution model and relating literature is reviewed after the classification analysis.

### 2.2.1 Classification of product substitution models

Shin et al. (2015) created a thorough taxonomy of product substitution literature published between 1974 and 2013. The authors used four criteria to classify the literature: modelling objective, substitution mechanism, substitution decision-maker and the direction of substitutability. The four objectives of product substitution, also known as '*areas of decision*', are:

- *Assortment planning*: A retailer is unable to offer the entire selection of products due to resource and technical constraints. Accordingly, the retailer must choose which products to include in the assortment to maximise profit.
- *Inventory decisions*: The retailer must decide on the optimal quantity to stock for each product in the *fixed* assortment in order to minimise inventory holding costs while considering each products' substitution properties.
- *Capacity planning*: The retailer must find the optimal quantity of products to stock, given that the capacities of certain resources are restricted. This objective is an extension of inventory decision problems.
- *Pricing decision*: The retailer must determine the optimal pricing policy for each substitutable product to maximise profit. The reason being that the prices of substitutable products influence substitution behaviour.

These objectives are not mutually exclusive and joint problems are common. Based on the above, the objective of this dissertation is to find the optimal inventory levels of the in-flight meals served on a specific flight (*inventory-decision*). It is assumed that the assortment of meals offered on a specific flight's menu is predetermined and fixed as part of the airline's competitive strategy. Pricing decisions are irrelevant as the meals are assumed to be complimentary with a purchased flight ticket. The problem can be extended in future research to include assortment planning and capacity constraints, such as batch quantities or quantity discounts.

A *substitution mechanism* is the catalyst for a customer or retailers substitution behaviour. Shin et al. (2015) identified three mechanisms: assortment-based, inventory-based and price-based substitution. In *assortment*–based substitution, a customer chooses a product without knowing if the product is included in the retailer's assortment and will leave empty-handed if the product is not available (Honhon et al., 2010). In *inventory*-based substitution, the customer knows the assortment of products available, but not the individual inventory levels. If the customer's preferred product is out-of-stock, the customer might accept a substitute. For this reason, inventory-based substitution is also known as stockout-based substitution. In *price*-based substitution, the selling price of the products is the driver behind the substitution behaviour of customers.

Shin et al. (2015) define the *substitution decision-maker* as '*the active party in planning decisions for substitutable products*'. In supplier-driven substitution, the supplier decides which products should be used as substitutes for specific product demands, and it is assumed that the customers will willingly accept the chosen substitutes. Customer-driven substitution decisions are made by different parties and usually consist of a two-stage process; the supplier chooses the inventory level in the first stage, after which the customer evaluates substitution alternatives in the second stage if their first choice is unavailable. By considering consumer-driven substitution, the retailer (or manufacturer) can increase the likelihood that a customer will not leave empty-handed. Consumer-driven substitution is mostly applied to retail settings focused on maximising expected profit.

Lastly, the *direction of substitution* can be unidirectional (one-way) or bidirectional (two-way) between product pairs. In one-way substitution, product A can be substituted with product B, but not vice versa. The direction could be either upwards (substituting low-end products with high-end products) or downwards.

As will become apparent throughout this report, the in-flight meal ordering process is best classified as an inventory decision problem with two-way, stockout-based, (partial) consumer-driven substitution between meal types. This problem type and existing modelling methods are discussed in the following section.

### 2.2.2 Two-way, stockout-based, consumer-driven substitution models with multiple products

Consider a flight with 25 passengers. According to the in-flight menu, passengers can choose either a chicken, beef, fruit platter or vegan meal. The *primary* demand for each meal type is [10, 10, 3, 2] meals, while the number of meals on-board are [5, 14, 4, 2] respectively. Ideally, the flight attendant will reserve the two vegan meals for the two vegan passengers, but this requires foreknowledge regarding the actual meal demand. Realistically, the flight attendant serves passengers according to their seat number. Consequently, there is a possibility that chicken meals might reach a stock-out *before* the two vegetarian passengers were served. A passenger who initially preferred a chicken meal could then choose a vegan meal as a substitute, thereby resulting in a shortage of vegan meals that were originally even-catered. In this scenario, the *effective* demand for vegan meals was greater than two.

This example highlights the impact of the sequence of customer arrivals and their respective individual product choices. For this reason, literature is classified into two streams based on the modelling of consumer choice. A *dynamic* model considers individual consumer choices by incorporating the sequential order of customer arrivals and, therefore, acknowledges the increasing probability of stock-outs as time progresses (Smith and Agrawal, 2000). The objective function of these models are considered complex and, according to Mahajan and Van Ryzin (2001) and Netessine and Rudi (2003), might not be quasi-concave. This means that a global optimum solution cannot be guaranteed. A *static* model approximates the *dynamic* model by considering the aggregate demand behaviour of customers (Vaagen et al., 2011). It assumes that products can substitute for one another based on exogenous rules or probabilities. Yücel et al. (2009) is of the opinion that static models are easier to analyse and require fewer data.

An important component of a substitution model is the chosen consumer choice model. A consumer choice model is an analytical model that represents the decision process of a customer (Shin et al., 2015). In a *utility*-based choice model, a customer associates a unique utility (perceived value) to each product and will choose the product with the highest utility. An *exogenous* choice model explicitly specifies the product demand and the actions that will be taken when the product is unavailable through the set of first-choice probabilities ($\mathbf{q}$) and substitution rules ($\boldsymbol{\alpha}$).

According to Kök et al. (2009), exogenous demand models are most popular for stock-out based substitution models. In most of these models, the effective demand for product $i$ ($d_i^e$) is calculated through some variation of (2.1) (Vaagen et al., 2011).

$$d_i^e = d_i^p + \sum_{j \in \mathbf{I} \,|\, j \neq i} \alpha_{ij}(d_j^p - x_j)^+, \quad \forall i \in \mathbf{I} \tag{2.1}$$

The effective demand for product $i$ consists of its own primary demand ($d_i^p$) and the additional demand resulting from the anticipated stock-outs of other products in the assortment ($\mathbf{I}$). The additional demand resulting from product $j$ consists of a deterministic substitution fraction ($\alpha_{ij}$) of the product's *unmet* demand ($d_j^p - x_j$) when $x_j$ units of product $j$ was ordered. Note, $(a)^+$ denotes $max(a, 0)$.

The primary demand of product $i$ is frequently calculated as a proportion ($q_i$) of the aggregate demand ($D$) as shown with (2.2) (Yücel et al., 2009).

$$d_i^p = q_i D, \quad \forall i \in \mathbf{I} \tag{2.2}$$

These proportions are known as first-choice probabilities and can be estimated using the product's respective market share. For instance, in the example given earlier, the total meal demand is equal to the passenger load of 25 passengers. Thus, by reverse engineering (2.2), the first-choice probability of chicken meals is found to be 40% of the total demand because the primary demand thereof is ten meals.

### Dynamic substitution models

Mahajan and Van Ryzin (2001) developed a *dynamic* substitution model using a Sample Path Gradient Algorithm (SPGA) to model the sequential arrival of customers. The authors used a utility maximisation mechanism to mimic a customer's substitution behaviour. A customer associates a unique utility (perceived value) to each product and will choose the product with the highest positive utility amongst the assortment of in-stock products or will leave empty-handed. The model aims to identify the initial inventory levels that will maximise expected profit but cannot guarantee a global optimum policy. Furthermore, numerical experiments were conducted by comparing the SPGA with two naive heuristics, including the independent newsboy model, to investigate the effect of product substitution. The results are encouraging for this dissertation, as the authors observed a 20% decrease in inventory levels when substitution is incorporated. Honhon et al. (2010) claim that the SPGA model is computationally gruelling. The authors proposed a dynamic programming heuristic and demonstrated that it is superior in terms of computational speed and results in higher expected profits under certain general conditions.

Similarly, Kök and Fisher (2007) present an iterative optimisation heuristic for an assortment-planning problem in the retail industry when faced with shelf space constraints. In this study, equation (2.1) is modified to incorporate both assortment- and stockout-based substitution. The potential of the heuristic was demonstrated when it was applied to a real-world case-study, a supermarket chain in the Netherlands, and a profit increase of more than 50% was observed. Unlike other models, this heuristic explicitly penalises left-over inventory with an amount equal to the product's selling price. In this dissertation, penalising surplus meals will help to achieve the first objective - waste minimisation. In addition, the authors provide a methodology to estimate the exogenous substitution behaviour parameters and the product demand required as input.

Smith and Agrawal (2000) propose the use of a discrete Markov process to obtain an exact solution for a multi-product assortment-planning problem, such that a transition represents an arriving customer. However, they acknowledge that it is computationally infeasible for even modest-sized problems. Alternatively, the authors develop an exogenous, newsvendor-type model to approximate the solution by bounding the probability of an item being available to each customer with predefined service levels. Model development assumed that item demand has a negative binomial distribution, but the authors state that the model holds for other distribution types as well. The model is solved as a non-linear integer program. A counter-intuitive observation gained is that it is not always optimal to stock the most popular product. This observation is important as it indicates that substitution is more complex than simply increasing the inventory level of the most popular product, thereby motivating the development of an appropriate model.

Gaur and Honhon (2006) use a locational consumer choice model to determine the optimal assortment and inventory levels. In a locational model, products are viewed as a set of quantifiable attributes and customers choice is based on the product's *location* (utility) in the attribute space. Additionally, product demand is assumed to be normally distributed. Due to the inability to capture the product demand with a closed equation, the authors propose bounds for the dynamic substitution model's expected profit. The lower bound represents the scenario where customers do not substitute (traditional inventory models). The upper bound assumes that the retailer can first observe the aggregate demand and optimally allocates products to customers thereafter (static substitution). This highlights an important aspect of the static model, which will be discussed in more detail. The authors suggest a two-stage stochastic linear program to compute this upper bound.

### Static substitution models

Rajaram and Tang (2001) extended the Newsvendor model to include static substitution and developed a tractable service rate heuristic to solve the inventory decision problem. The core principle behind their heuristic is to approximate the effective demand using a service rate for each product, $\gamma_j(x_j)$, such that $(d_j^p - x_j)^+$ is replaced with $d_j^p\,[1 - \gamma_j(x_j)]$ in equation (2.1). The model is restricted in the sense that it assumes product demands are normally distributed. Using the heuristic, the authors were able to show that substitution increases the expected profit as improvements of up to 48.2% were observed when compared with the base model. Unfortunately, this improvement came at the expense of inventory increases ranging between 1% to 10%. However, the authors state that the order quantity depends on model parameters and inventory decreases are, therefore, also possible. Vaagen et al. (2011) raise a concern regarding the model, stating that the sales of a product is not bounded by its available demand. The author is of the opinion that this could explain the monotonous inventory increases observed. Vaagen et al. (2011) further warn that, while Rajaram and Tang (2001) claim to be concerned with consumer-driven substitution, the authors are solving supplier-driven substitution.

Huang et al. (2011) used the service-rate approximation approach developed by Rajaram and Tang (2001) to find the optimal inventory levels for a competitive newsboy problem with substitution. In a competitive model, each product in the assortment is managed by a different entity. The final model is one of the few models that penalises product shortages. In this dissertation, penalising meal shortages will help to achieve the second objective - high PSLs.

Yücel et al. (2009) developed a static substitution model that simultaneously considers product assortment, stocking quantities, supplier selection and shelf space restrictions to maximise expected profit in a retail setting with multiple substitution attempts. It consists of a Mixed-Integer Programming (MIP) model where demand uncertainty in equation (2.1) is modelled using a stochastic programming technique similar to Recourse Programming (RP). RP has the advantage of being able to accommodate any demand distribution and will be discussed in more detail in Section 2.3.2. Zeppetella et al. (2017) followed Yücel et al. (2009)'s consumer-driven substitution approach, but applied it to a *production* environment to develop an optimal production schedule. This implementation is unique because one-way, firm-driven substitution is usually associated with production scheduling. Consequently, the objective is to *minimise cost*, while considering both capacity and production constraints such as batch quantities and production lead times. The authors developed two models that consider (1) lost sales and (2) backlogged orders. They motivate the use of the MIP approach due to its '*rigorousness, flexibility and extensive modelling capability*'. Vaagen et al. (2011) also developed an inventory and assortment optimisation model using MIP with RP. As will be explained, the essence of their model differs from Yücel et al. (2009)'s model through the incorporation of a '*qualitative understanding*' regarding a consumer's substitution behaviour.

### Model and method selection

According to Gaur and Honhon (2006), the static model represents the upper bound of the dynamic model. The reason being that the static model optimally allocates products to customers based on the *known* aggregate demand. Gilland and Heese (2013) define this behaviour as an *ex post* allocation of product demand. But, to achieve the static model's expected profit, the products must be reserved for the assigned passenger. This is not always possible and due to the dynamic arrivals of passenger, the *true* profit obtained will likely be much lower than computed. The dynamic model incorporates this uncertainty, thereby ensuring that the *true* profit achieved will be closer to the computed expected value. Thus, the static model is less reliable and inferior to the dynamic model as it underestimates the amount of substitution that will occur (Gilland and Heese, 2013). Consequently, the *true* profit obtained will be greater when using a dynamic model.

Various literature expressed the computational complexities associated with dynamic models. As such, the question arises if it is worthwhile to model individual customer choice in an inventory model with substitution. Gilland and Heese (2013) researched this question and concluded that the benefit might be only marginal. Based on numerical experiments, the true profit obtained from the dynamic model was, on average, only 1% higher when compared with the static model. That being said, the authors warn that the performance of the static model declines with limited shelf space, high substitution fractions ($\boldsymbol{\alpha}$) and asymmetric stock-out penalty costs.

Ironically and in contrast to the given example, the static model is actually a somewhat realistic representation of the in-flight meal serving process. Thanks to today's modern flight booking technology, airlines know the preferred meal order for some of the passengers who will board the flight. This can allow them to optimally allocate a proportion of the available meals. According to Tofalli et al. (2018), doing so can save more than 30% of the food that is usually thrown away. Based on personal experience in 2018, Emirates airlines is already following a similar strategy; If a *special* meal type is booked 24 hours before departure, the meal is allocated to the passenger with a personalised sticker (Emirates.com, NA). This strategy is expected to become a norm in the near future due to the strive for customisation resulting from the fourth Industrial Revolution.

Based on the above, the benefit of using a dynamic substitution approach for the in-flight catering industry is deemed not worthwhile, especially when considering the accompanying complexities and restrictions. This dissertation will focus on developing a static model with partial consumer-driven substitution using MIP. This modelling method is chosen due to its extensive modelling capability and flexibility. This will give in-flight caterers the ability to easily expand the model to include additional constraints significant to their process. This is particularly useful for the in-flight catering industry due to different kitchen capacities and changing menus.

The decision-making model will be similar to the work of Yücel et al. (2009) and Vaagen et al. (2011) and will include a qualitative understanding of the substitution behaviour of passengers. However, the model will be expanded to addresses the multi-objective nature of the in-flight catering industry and additional uncertainty will be incorporated to improve its depiction of reality. The MIP models developed by these authors will be analysed in the following section.

## 2.3 Operations research

The chosen modelling method falls under the discipline of OR, specifically stochastic programming. A stochastic model refers to an optimisation problem with uncertain parameters. The uncertainty is represented with at least one random variable with a known probability distribution (Birge and Louveaux, 2011). The above-mentioned stochastic MIP models with product substitution and RP are analysed in this section, followed by a brief overview of RP and multi-objective programming.

### 2.3.1 Stochastic mixed integer programming models with product substitution and recourse programming

Yücel et al. (2009) and Vaagen et al. (2011) both developed a two-stage stochastic MIP model aimed at maximising the expected profit of a joint assortment planning and inventory decision problem. There are two important differences between their respective models to take note of. Firstly, Yücel et al. (2009) *explicitly* models multiple substitution attempts, whereas Vaagen et al. (2011) approximate them with a single attempt. Multiple levels of substitution represents a customer's repeated attempts to find a suitable alternative. Secondly, they differ in their definition of the substitution rules ($\boldsymbol{\alpha}$ versus $\hat{\boldsymbol{\alpha}}$) and subsequent changes to (2.1).

$$d_i^e = d_i^p + \sum_{j \in \mathbf{I} \,|\, j \neq i} \alpha_{ij}(d_j^p - x_j)^+, \quad \forall i \in \mathbf{I} \tag{2.1}$$

Recall that, as shown with (2.1), the effective demand of product $i$ ($d_i^e$) consists of its primary demand ($d_i^p$) and the *fixed* proportion of excess demand from product $j$. The exogenous substitution fraction $\alpha_{ij}$ indicates the proportion of customers who will substitute their first-choice product, product $j$, for product $i$ if their first-choice is out of stock, such that (2.3) holds.

$$\sum_{i \in \mathbf{I} \cup \{\text{No substitute}\}} \alpha_{ij} = 1, \quad \forall j \in \mathbf{I} \tag{2.3}$$

A dummy variable is included to represent a customer that is not willing to substitute. Table 2.1 provides an example of a substitution fraction matrix.

Table 2.1: An example of a substitution fraction matrix ($\boldsymbol{\alpha}$).

| | **First-choice (j)** | | | |
|---|---|---|---|---|
| **Substitute (i)** | **Chicken** | **Beef** | **Fruit Platter** | **Vegan** |
| **Chicken** | - | 0.70 | 0.15 | 0.05 |
| **Beef** | 0.50 | - | 0.05 | 0.00 |
| **Fruit platter** | 0.20 | 0.05 | - | 0.40 |
| **Vegan** | 0.20 | 0.20 | 0.30 | - |
| **No substitute** | 0.10 | 0.05 | 0.50 | 0.55 |

If chicken meals reach a stock-out, 50% of the remaining demand will be distributed to beef meals, 20% to fruit platters and vegan meals each and 10% will deny a substitute meal. A simple approach to obtain the substitution fraction matrix is through market research. Alternatively, it can be estimated using a methodology proposed by Kök and Fisher (2007).

Yücel et al. (2009) directly applied (2.1) using substitution fractions, but made modifications to include multiple levels of substitution. The authors acknowledge that substitution can lead to a loss in customer goodwill. For this reason, the authors incorporated a substitution cost in the objective function that attempts to quantify the effect thereof in monetary terms. The substitution cost is dependent on the substitution level and dramatically impacts the amount of substitution observed.

Vaagen et al. (2011) express a concern regarding the use of substitution fractions. The authors argue that real-time (dynamic) inventory data is required to calculate these values, yet it is frequently applied in a static model. In the given example, if chicken meals are not available as a substitute due to the under- or even-catering thereof, it is intuitive that the substitution fractions of the remaining meal types should increase. However, due to the static nature of the model, the substitution fractions are fixed. Yücel et al. (2009) appear to have overcome this issue through the explicit modelling of three substitution attempts where, for example, the substitution fraction for the second substitution attempt is approximated as $\alpha_{ij}\alpha_{ki}$ for each $i, j, k \in \mathbf{I}$ combination. The authors argue that considering only three levels of substitution is reasonable as the substitution fractions become negligibly small when considering more substitution attempts. Vaagen et al. (2011) propose the use of *a priori* substitutability probabilities as a solution for the concerns mentioned that do not require the explicit modelling of multiple substitution attempts that could complicate the model.

Vaagen et al. (2011) defines the *a priori* substitutability probability $\hat{\alpha_{ij}} \in [0, 1]$ as '*the proportion of customer willing to replace item j with item i*'. Stated differently, it represents the *probability* that product $j$ can be substitute by product $i$ given that product $i$ is the only substitute in-stock. The authors state that this substitution measure integrates a '*qualitative understanding of market drivers*' as it is concerned with the static similarity between the different products in the assortment and market trends. Unlike the above-mentioned substitution fractions, it is not influenced by the model's inventory decisions nor dependent on the products available when the substitution occurs. The authors state that the *a priori* probabilities can conveniently be defined by a multidisciplinary team with insightful product and market knowledge.

An example of an *a priori* substitutability probability matrix is illustrated in Table 2.2. The *a priori* probabilities do not have to be symmetric. For instance, there is a 40% probability that a passenger would accept a vegan meal as a substitute when chicken meals are out-of-stock. However, there is only a 5% chance that the reverse would occur. The reason being that the majority of the passengers that request a vegan meal will likely follow a meat-free lifestyle. Furthermore, there are two noticeable differences when comparing the *a priori* probabilities to the substitution fractions: (1) $\sum_{i \in \mathbf{I}} \hat{\alpha_{ij}} = 1 \; \forall j \in \mathbf{I}$ does not hold, and (2) a dummy variable is not required to represent unsatisfied customers.

Table 2.2: An example of an *a priori* substitution probability matrix ($\hat{\alpha}$).

| | **First-choice product (j)** | | | |
|---|---|---|---|---|
| **Substitute (i)** | **Chicken** | **Beef** | **Fruit Platter** | **Vegan** |
| **Chicken** | - | 0.50 | 0.30 | 0.05 |
| **Beef** | 0.70 | - | 0.10 | 0.00 |
| **Fruit platter** | 0.40 | 0.20 | - | 0.40 |
| **Vegan** | 0.40 | 0.00 | 0.50 | - |

By applying the *a priori* probabilities to equation (2.1) along with slight modifications, Vaagen et al. (2011) claim to be able to approximate a dynamic consumer choice model. This model will be discussed in detail in the remainder of this section as this dissertation will follow the suggested *a priori* substitutability approach. It is chosen because it corresponds with the static nature of the chosen model, requires fewer assumptions regarding actual substitution probabilities and can approximate multiple substitution attempts with a single attempt to simplify the model.

The aim of Vaagen et al. (2011)'s model is to choose the optimal production quantity, $x_i$, for each product in assortment $\mathbf{I}$ that is most likely to achieve the highest expected profit under stochastic demand. As stated, the authors model demand uncertainty using two-stage stochastic programming, similar to recourse programming. The model's objective function in deterministic form is given in (2.4).

$$\max Z = \sum_{s \in \mathbf{S}} p^s \sum_{i \in \mathbf{I}} (v_i y_i^s + f v_i t_i^s + g_i w_i^s - c_i x_i) \tag{2.4}$$

where

$$t_i^s = \sum_{j \in \mathbf{I} \mid j \neq i} z_{ij}^s, \qquad \forall \qquad i \in \mathbf{I},\ s \in \mathbf{S} \tag{2.5}$$

This function maximises the overall *expected* profit by summing together the profit of each scenario $s$ in proportions equal to the scenario's probability of occurrence, $p^s$. The expected profit of each scenario consists of the revenue received, the salvage value for excess inventory (overstock cost), minus the cost of production. The understocked cost (lost sales) is not considered by the authors.

The authors distinguish between the direct sales and substitution sales for each product $i$ in assortment $\mathbf{I}$. Direct sales, denoted by $y_i^s$, refer to the number of customers that bought product $i$ as their first-choice. The substitution sales $z_{ij}^s$ represent the number of customers who bought product $i$ as a substitute due to a stock-out of product $j$. For convenience, $t_i^s$ is total number of product $i$ sold as a substitute, as enforced by (2.5).

The selling price of product $i$ is $v_i$. However, to force the model to first meet the primary demand, a discounted-substitution factor $f \in [0, 1]$ is included for the selling of substitute products. This enforces direct sales before substitution sales, and prevents the bait-and-catch strategy where the model intentionally under-stocks a product so that the unmet demand can be satisfied with a more profitable substitute. Accordingly, it ensures partial consumer-driven substitution instead of supplier-driven substitution. With partial consumer-driven substitution, the supplier cannot force a customer to buy a substitute instead of their first-choice, but can only decide on which products to make available through safety stock as potential substitutes for the customer to choose from. Lastly, each unit of product $i$ is produced at a cost of $c_i$ and the leftover inventory $w_i^s$ is salvaged for $g_i$ per product. The objective function (2.4) is subjected to the constraints listed below

Constraints (2.6) to (2.8) are Vaagen et al. (2011)'s adaptation of (2.1). In constraint (2.6), the number of direct and substitution sales resulting specifically from the demand for product $i$, is restricted by the product's primary demand for scenario $s$, denoted by $d_i^s$. The reason being that the retailer cannot sell more than the available demand. Notice that this model does not keep track of the product shortage quantities.

$$y_i^s + \sum_{j \in \mathbf{I} \mid j \neq i} z_{ji}^s \leq d_i^s, \qquad \forall \qquad i \in \mathbf{I}, s \in \mathbf{S} \tag{2.6}$$

Constraint (2.7) limits the number of substitution sales, $z_{ij}^s$, with the deterministic proportion of excess demand of product j. The deterministic proportion is equal to the respective *a priori* substitution probability $\hat{\alpha_{ij}}$.

$$z_{ij}^s \leq \hat{\alpha_{ij}}(d_j^s - y_j^s), \qquad \forall \qquad i,\ j \in \mathbf{I},\ j \neq i,\ s \in \mathbf{S} \tag{2.7}$$

Based on the above, the effective demand for product $i$ is the sum of $y_i^s$ and $t_i^s$. Constraint (2.8) links the effective demand with the decision variable, the number of units of product $i$ produced, and the resulting excess inventory denoted by $w_i^s$.

$$w_i^s = x_i - (y_i^s + t_i^s), \qquad \forall \qquad i \in \mathbf{I}, s \in \mathbf{S} \tag{2.8}$$

Constraints (2.9) to (2.11) represent the non-negativity constraints.

$$x_i \geq 0 \text{ and integer}, \qquad \forall \qquad i \in \mathbf{I} \tag{2.9}$$

$$y_i^s,\ t_i^s,\ w_i^s \geq 0 \text{ and integer}, \qquad \forall \qquad i \in \mathbf{I},\ s \in \mathbf{S} \tag{2.10}$$

$$z_{ij}^s \geq 0 \text{ and integer}, \qquad \forall \qquad i \in \mathbf{I},\ i \neq j,\ s \in \mathbf{S} \tag{2.11}$$

While not explicitly used in Vaagen et al. (2011)'s model, (2.12) represents the authors' modification of (2.1).

$$d_i^{e,s} = d_i^s + \sum_{j \in \mathbf{I},\, j \neq i} z_{ij}^s, \quad \forall i \in \mathbf{I},\ s \in \mathbf{S} \tag{2.12}$$

The drawback of the *a priori* probabilities is that it is not able to fully describe the dependencies among the various products. For instance, consider the scenario where chicken meals are under-catered and the only available substitutes for the unmet demand is either a fruit platter or a vegan meal. Based on Table 2.2, both substitutes have a 40% probability of being chosen. However, it is not known whether or not the 40% of the passengers that would choose a fruit platter as a substitute overlap with the 40% of the passengers that would accept a vegan meal, or if these two groups of passengers are independent. Vaagen et al. (2011)'s model assumes that the groups are independent. This means that the model could assign a fruit platter as a substitute to 40% of the unmet demand, while assigning a vegan meal to another 40% of the unmet demand. Thus, the model is able to assign 80% of the unmet demand, although it is unlikely in reality that the groups are independent. The authors warn that this results in an upper bound on the expected profit of their model.

### 2.3.2 Two-stage stochastic programming with fixed recourse

Two-stage stochastic programming with fixed recourse, referred to simply as RP, is a technique used to handle uncertainty within a model. In essence, it incorporates predefined corrective actions *after* the uncertainty has been disclosed. According to Birge and Louveaux (2011), the sequence of events can be described by (2.13):

$$x \rightarrow \xi^{\gamma} \rightarrow y[x, \xi^{\gamma}] \tag{2.13}$$

In the first stage, certain decisions, denoted by $x$, must be made while the uncertainty is still present. The uncertainty is represented with a random variable $\widetilde{\xi}$ with a known probability distribution. The second stage occurs after the outcome of the uncertainty, represented with $\xi^{\gamma}$, is revealed. Consequently, second-stage decisions $y[x, \xi^{\gamma}]$ are functions of the random variable's outcome and (in some instances) the first-stage decisions. In Vaagen et al. (2011)'s model, the first-stage decisions are the order quantities for all products in the assortment as the respective primary demand thereof is unknown. The second-stage decisions and subsequent corrective actions are related to the substitution sales and the salvaging of excess inventories.

Based on the examples of Birge and Louveaux (2011) and Ahmed (2019), the generic objective function of a recourse model is given with (2.14).

$$\max Z = f(x) + E_{\widetilde{\xi}} Q(x, \widetilde{\xi}) \tag{2.14}$$

The first term represents the combined effects of first-stage decisions on the objective function, while the second term relates to the recourse actions from second-stage decisions. Note, $E_{\widetilde{\xi}}$ refers to the *expected value* of the distribution of $\widetilde{\xi}$ (Joubert and Conradie, 2005).

Regarding the in-flight catering problem, the primary meal demand is uncertain, but the probability distribution thereof is assumed to be known and fixed. Accordingly, the model should consider the trade-off between over- and under-estimation of meal quantities. For these types of problems, Birge and Louveaux (2011) suggest the introduction of two second-stage variables to represent the deviation from the actual demand. Excess inventory and product shortages are represented with $\delta^{+}[\widetilde{\xi}]$ and $\delta^{-}[\widetilde{\xi}]$ respectively. These variables can then be calculated using constraints similar to (2.15) and (2.16), given that $\widetilde{\xi}$ denotes the random demand of the product.

$$x + \delta^{-}[\widetilde{\xi}] - \delta^{+}[\widetilde{\xi}] = \widetilde{\xi} \tag{2.15}$$

$$x,\ \delta^{+}[\widetilde{\xi}],\ \delta^{-}[\widetilde{\xi}] \geq 0 \tag{2.16}$$

In Vaagen et al. (2011)'s model, the number of excess meals $w_i$ corresponds with the recourse variable $\delta^+[\widetilde{\xi}]$. However, Vaagen et al. (2011)'s model does not measure the number of shortages that occurred, meaning that $\delta^-[\widetilde{\xi}]$ is not used in the model.

Recourse programming allows the model to take corrective actions after the second-stage decisions have been made. For instance, Birge and Louveaux (2011) assume that a penalty $p^-$ is paid per unit of unmet demand and a unit of excess stock is fined with $p^+$. The second-stage value function in (2.14) that accounts for these penalties is given with (2.17).

$$Q(x, \widetilde{\xi}) = q^-[\,\delta^-(\widetilde{\xi})\,] + q^+[\,\delta^+(\widetilde{\xi})\,] \tag{2.17}$$

The constraint mentioned above captures the uncertainty within a system. However, the model must be transformed into its deterministic counterpart before it can be solved using optimisation software. This can be achieved by decomposing the probability distribution of the random variable to create a set of second-stage realisations, represented with $\mathbf{R}$. The realisation $\xi^\gamma$ represents a possible outcome of the random variable with a probability of occurrence equal to $p^\gamma$, where $\gamma \in \mathbf{R}$ and $\sum_{\gamma\in\mathbf{R}} p^\gamma = 1$. The deterministic equivalent of the overall objective function is shown in (2.18).

$$\max Z = f(x) + \sum_{\gamma\in\mathbf{R}} p^\gamma \left( q^-(\,\delta^-[\xi^\gamma]\,) + q^+(\,\delta^+[\xi^\gamma]\,)\right) \tag{2.18}$$

This requires that (2.15) and (2.16) must be replaced with (2.19) and (2.20) to account for each realisation $\xi^\gamma$ with probability $P^\gamma$, where $\gamma \in \mathbf{R}$.

$$x - \delta^+[\xi^\gamma] + \delta^-[\xi^\gamma] = \xi^\gamma, \qquad \forall \quad \gamma \in \mathbf{R} \tag{2.19}$$

$$x,\ \delta^+[\xi^\gamma],\ \delta^-[\xi^\gamma] \geq 0, \qquad \forall \quad \gamma \in \mathbf{R} \tag{2.20}$$

A major shortcoming of the stochastic model developed by Vaagen et al. (2011) is that it does not consider multiple objectives.

### 2.3.3 Multi-objective programming

The in-flight catering industry is faced with two conflicting objectives: (1) maximise the PSL of a flight while (2) simultaneously minimising waste. The two objectives are conflicting as they come at the expenses of one another. For instance, reducing meal order quantities will lessen the waste produced, however, it increases the possibility of meal shortages and, subsequently, a lower PSL. The other extreme corresponds with the over-catering strategy, where surplus meals are deliberately ordered to protect against potential shortages.

The multi-objective nature of the research question being addressed necessitates the need for multi-objective programming. The goal of a multi-objective programming model is to optimise two or more *conflicting* objectives. According to Rardin (1998), the emphasis is on finding '*efficient solutions which are optimal in a certain multi-objective sense*'. This means that the resulting *efficient* solution might be suboptimal with regards to each isolated objective, but it is the best solution overall when considering the collection of objectives and target weights given.

The proposed multi-objective programming technique is known as *pre-emptive goal programming*. This technique focuses on achieving the stated goals with predefined target weights to indicate their importance, instead of minimising or maximising specific quantities (Rardin, 1998). In pre-emptive goal programming, a target level or goal is assigned to each objective. The aim thereof is to find a solution that will achieve the predefined target levels as close as possible by minimising the sum of deviations from the target values. The advantage of this technique is that it prioritises the objectives according to how well their respective target is reached. The implementation of pre-emptive goal programming is briefly explained with the example given below.

$$\min Z = w_1\Delta_1 + w_2\Delta_2 \tag{2.21}$$

subject to

$$g_1 \leq OF_1(x) + \Delta_1 \tag{2.22}$$
$$g_2 \geq OF_2(x) - \Delta_2 \tag{2.23}$$
$$\Delta_i \geq 0, \forall i \in \{1, 2\} \tag{2.24}$$

The example contains two objectives: objective 1 and objective 2 represent, respectively, a maximisation and minimisation objective function. The variable $OF_i(x)$ denotes the value of objective function $i \in \{1, 2\}$ as a function of the decision variable $x$. Each objective is assigned a target value $g_i$ and $\Delta_i$ measures the deviation from the target value as shown in (2.22) and (2.23). Thus, the objective function of the goal program in (2.21) simply aims to minimise these deviations. An objective function value equal to zero indicates that both target values have been reached. This requires that $\Delta_i$ should be non-negative as enforced with (2.24). A positive target weight $w_i$ is assigned to each target to indicate the importance of achieving each target. Equal weights indicate that both targets are equally important.

Intuitively, the *optimal* solution desired by in-flight catering companies would result in (1) a 100% PSL and (2) zero surplus meals simultaneously. These two parameters define, respectively, the ultimate goals ($g_1$ and $g_2$) for the two conflicting objectives. The deviation from these two goals ($\Delta_1$ and $\Delta_2$) are simply (1) the passenger *dissatisfaction* level achieved and (2) the number of surplus meals produced. Accordingly, the multi-objective nature of the in-flight catering industry can be captured in the model using pre-emptive goal programming, by ensuring that the model's objective function *minimises* the weighted sum of the passenger dissatisfaction level achieved and the total number of surplus meals produced.

It should be stressed that an *efficient* solution is not always guarenteed when using pre-emptive goal programming. Rardin (1998) explains that this is because there is nothing in the goal-programming formulation to encourage further optimisation if one or more of the model's goals are achieved. To enforce an efficient solution, the author suggest adding a small multiple of each original minimisation objective and to subtract the same multiple of each original maximisation objective. An example of the resulting objective function is shown in (2.25)

$$\min Z = w_1\Delta_1 + w_2\Delta_2 + 0.001(OF_2(x) - OF_1(x)) \tag{2.25}$$

The above is, however, not required for the model being developed for the in-flight catering industry, since the 100% PSL and zero-waste targets represent the *ultimate* goals of the industry. This means that the model will always generate an efficient solution because the solution cannot be improved after achieving either one or both of the goals given.

The *efficient* solution obtainable by an in-flight catering company is strongly dependent on the primary demand for each meal type offered on the menu. The primary meal demands are obviously unknown when the caterer needs to start the meal order planning and production phases for the flight under consideration. The respective values can be estimated using equation (2.2) along with the set of first-choice probabilities and the *aggregate* meal demand forecasted.

## 2.4 Dynamic passenger load forecasting

This dissertation assumes that each passenger on-board a flight is entitled to receive one complimentary meal. This means that the flight's aggregate meal demand is equal to its final passenger load. Thus, forecasting the final passenger load is equivalent to forecasting the aggregate meal demand.

Various forecasting techniques have been discussed in Section 2.1.1. It was noted that these models ignore the distribution of the forecasted variable and, therefore, overlook valuable information. As will become clearer later on, the *distribution* of the passenger load is required in order to include uncertainty within the model being developed. Consequently, the most suitable forecasting model is that of Van Ostaijen et al. (2017) who propose the use of a time-inhomogeneous Markov chain to forecast the dynamic flight booking behaviour of passengers. This includes the probability distribution of a flight's final passenger load along with the expected value thereof.

Winston (2004b) states that a Markov chain is a special type of a discrete-time stochastic process. In essence, it represents a sequence of events where the probability distribution of the subsequent event's state, depends only on the current state. This is known as the memoryless property. The state space $\mathbb{C}$ is the set of all possible states that could occur. The transition probability $P^{i,j}$ is the probability that the process will transition from state $i$ to state $j$ in one time unit. In a time-inhomogeneous Markov chain, the transition probabilities are time-dependent; Let $P_{t_1,\,t_2}$ represent the Transition Probability Matrix (TPM) from time $t_1$ to $t_2$ such that it contains all the transition probabilities $P^{i,j}_{t_1,\,t_2}$ for each $i, j \in \mathbb{C}$ pair.

In Van Ostaijen et al. (2017)'s model, the Markov chain $\{Z_t\}$ represents the current passenger load at $t$ time units before flight departure, where $t \in \{N, N-1, ..., 0\}$ and $N$ is the forecasting horizon. Thus, $t = 0$ represents the flight's departure. The state space of $\{Z_t\}$ is restricted by $K$, the maximum number of flight bookings allowed, such that $\mathbb{C} \in \{0, 1, ...K\}$. The authors chose to use time-inhomogeneous transition probabilities. This improves the forecasting accuracy of the passenger load, since the booking behaviour of passengers is time-dependent. The memoryless property of this Markov chain is given in (2.26).

$$\begin{aligned} P^{i_t,\,i_{t-1}}_{t,\,t-1} &= \mathbb{P}[Z_{t-1} = i_{t-1} \mid Z_t = i_t,\ Z_{t+1} = i_{t+1},\ ...,\ Z_N = i_N] \\ &= \mathbb{P}[Z_{t-1} = i_{t-1} \mid Z_t = i_t] \end{aligned} \tag{2.26}$$

Lastly, as illustrated in (2.27), the authors define $\pi_t$ as the probability distribution of the passenger load at $t$ time units before the flight's departure.

$$\pi_t = (\mathbb{P}[Z_t = 0],\ \mathbb{P}[Z_t = 1],\ \dots ,\ \mathbb{P}[Z_t = K])^T \tag{2.27}$$

This probability distribution of the net passenger bookings at time $t - \beta$ can be calculated using (2.28), where $\beta \in \{1, ..., N - t\}$.

$$\pi_{t-\beta} = \prod_{l=1}^{\beta} P_{t-l+1,\,t-l} \cdot \pi_t \tag{2.28}$$

Consequently, the expected passenger load at time $t$ can be calculated using (2.29). Note, $\pi_t(i)$ refers to the $i^{th}$ element in $\pi_t$.

$$E[Z_t] = \sum_{i=0}^{K} \pi_t(i) \cdot i \tag{2.29}$$

The calculation of the probability distribution and the expected value of the passenger load at time $t$ is straight-forward. However, obtaining the required modelled TPMs are more complicated. Figure 2.4 depicts the high-level representation of the process followed by Van Ostaijen et al. (2017), Goto (1999) and Goto et al. (2004).

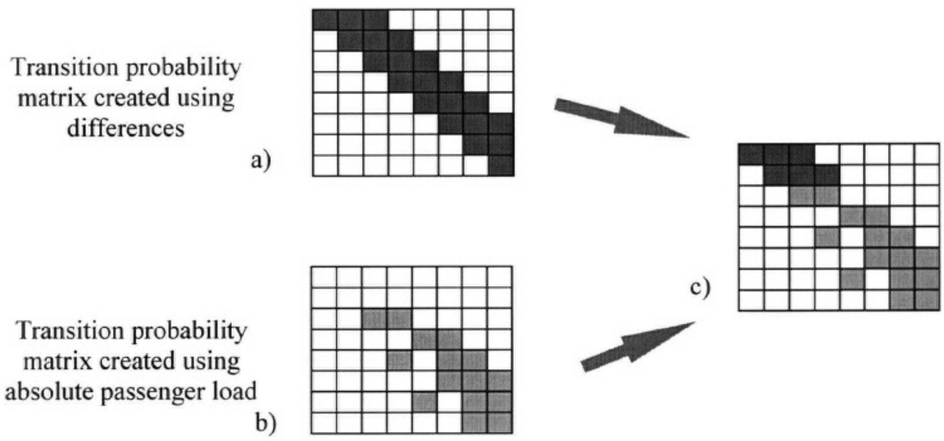



Figure 2.4: Generating a transition probability matrix (Goto et al., 2004).

In summary, a modelled TPM is estimated using (a) the differences in the passenger loads observed and (b) the frequency of the transitions observed regarding the net absolute passenger load. Goto et al. (2004) followed a similar approach to develop an optimal meal ordering policy using a MDP, which is described in more detail by Goto (1999).

To determine these matrices, historical data of the net passenger load for a specific flight at time points $t \in \{N, N-1, ...1, 0\}$ are required. The approaches suggested to generate the three matrices are discussed below.

Equation (2.28) has been emended after receiving confirmation from Van Ostaijen et al. (2017). The original equation, as provided in the source article, omitted the second subscript of $P_{t-l+1,\,t-l}$.

### (a) Transition probability matrix using differences

The Transition Probability Matrix using Differences (TPM-D), represented with $\bar{P}_{t,\,t-1}$, is based on the *differences* observed in the historical flight booking dataset. A difference is simply the change in the passenger load between two sequential pre-departure time intervals $(Z_{t-1} - Z_t)$. The matrix is then populated using the distribution of these differences without considering the current passenger load. Thus, this approach assumes that the passenger load's net change is independent of the passenger load at time $t$.

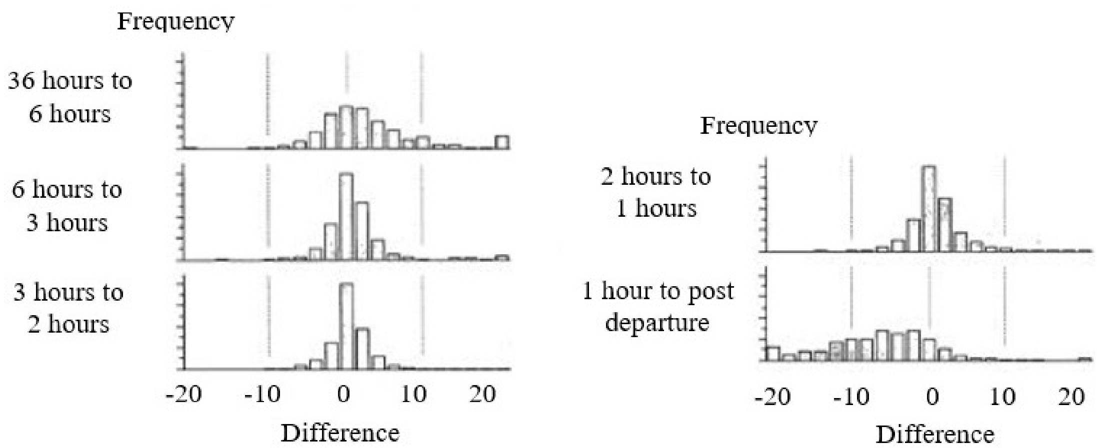



Figure 2.5: Distributions of the differences observed in a flight's passenger load between six successive pre-departure time intervals (Goto et al., 2004).

The first step in obtaining the transition probabilities is to calculate the differences in passenger load observed for each time interval. Figure 2.5 shows an example of the distributions of these differences in passenger load between six successive time intervals, as obtained by Goto et al. (2004).

Up until an hour before departure, the differences observed are normally distributed near a mean of zero with a few positive outliers. During the final hour, the normal distribution of the differences is extremely left-skewed with excessive variation. The differences in the distributions depicted demonstrate that the booking behaviour of passengers is time-dependent

As a second step, Goto et al. (2004) fitted the *trimmed* distributions of these differences with normal and shifted Poisson distributions. This allowed the authors to model the probability of observing a given change in passenger load $(Y_t)$ using (2.30).

$$\mathbb{P}[Z_{t-1}|Z_t] = \begin{cases} \mathbb{P}[Y_t \leq -Z_t] & if \quad Z_{t-1} = 0 \\ \mathbb{P}[Y_t \leq Z_{t-1} - Z_t] - \mathbb{P}[Y_t \leq Z_{t-1} - Z_t - 1] & if \quad 0 < Z_{t-1} < K \\ \mathbb{P}[Y_t > Z_{t-1} - Z_t - 1] & if \quad Z_{t-1} = K \end{cases}$$

$$\text{where } Y_t \sim Norm(\mu, \sigma) \tag{2.30}$$

The trimmed distributions represent all of the differences between the $1^{st}$ and $90^{th}$ percentile thereof. As an alternative approach, Van Ostaijen et al. (2017) maintained the initial distributions of the differences. The authors claimed that doing so reduced the forecasting errors observed.

Lastly, the probability matrix can then be populated where $\mathbb{P}[Z_{t-1}|Z_t]$ is the value of the $[Z_t, Z_{t-1}]^{th}$ entry of the matrix. The matrix might include non-zero probabilities for states that are never reached. As stated by Goto et al. (2004), this provides a contingency plan for these situations if they occur. Note that each forecasting interval will require a unique probability matrix. All of the rows in this matrix will have the same probability distribution, and as the rows increment upwards, the probability distribution will shift to the left.

The above approach assumes that the difference in passenger load is independent of the passenger load at time $t$. However, Goto et al. (2004) observed dependence during the *final* transition at one hour before departure. Consequently, the authors chose to model the *final* transition probabilities by incorporating a simple linear regression model given in (2.31), where $\beta_0$ and $\beta_1$ are the regression parameters and $\epsilon$ is the error term.

$$L_{difference} = \beta_0 + \beta_1 Z_1^* + \epsilon \tag{2.31}$$

The normal transition probabilities are then calculated using (2.30) by using the computed $L_{difference}$ value as the mean, and the standard deviation is estimated using the root mean square error ($s_e$). As seen, the *true* passenger load at one hour before departure ($Z_1^*$) is the only independent variable in the regression model. Goto (1999) considered including additional covariates, including the Day of Week (DOW), the season and the forecasted passenger load. However, these covariates were found to be strongly correlated with the actual passenger load at one hour before departure and provided negligible gain within their model. For this reason, the additional covariates were omitted from their regression model.

### (b) Transition probability matrix using the absolute passenger load

Represented by $\bar{\bar{P}}_{t,\,t-1}$, the Transition Probability Matrix using the Absolute Passenger Loads (TPM-APL) is based on the *direct transition* from one passenger load to another in successive time intervals. To populate the matrix, Goto et al. (2004) suggests the use of a frequency table for each forecasting interval, where the $(i, j)^{th}$ entry contains the total number of flights observed where the passenger load transitioned from $i$ to $j$ passengers during the particular time interval. Each element must then be divided by its row's total to obtain the desired transition probabilities.

According to Van Ostaijen et al. (2017) and Goto et al. (2004), this method requires a large data set of past passenger load observations and leads to an excessive number of zero entries. The reason being that not all transitions possible are observed in the dataset. Consequently, the resulting empirical distribution of each row in the matrix is usually non-smooth and discontinuous. However, the occurrence probability of these transitions might still be greater than zero and, therefore, motivates the combination of the TPM-D and the TPM-APL to correct the missing probabilities.

### (c) The modelled transition probability matrix

The modelled TPM is obtained by combining the TPM-D and the TPM-APL with a weighting factor $\phi \in [0, 1]$ as demonstrated in (2.32). Goto et al. (2004) states that the most appropriate $\phi$ value is flight dependent. That being said, the authors noted that higher $\phi$ values generally lead to more favourable results. The work of Van Ostaijen et al. (2017) agree with this statement.

$$P^{i,j}_{t,t-1} = \begin{cases} (1-\phi) \cdot \bar{P}^{i,j}_{t,t-1} \; + \; \phi \cdot \bar{\bar{P}}^{i,j}_{t,t-1}, & if \quad \sum_{j=0}^{K} \bar{\bar{P}}^{i,j}_{t,t-1} \neq 0 \\ \bar{P}^{i,j}_{t,t-1} & otherwise. \end{cases} \tag{2.32}$$

To help understand the generation of a modelled TPM, consider the fictitious example given earlier that is now based on the example provided by Goto et al. (2004). Assume that this flight occurs daily and has a maximum capacity of 42 passengers. Table 2.3 lists a few instances of this flight along with its known passenger load at time $t$ and time $t-1$. In this specific example, each instance of the flight had 39 passengers at time $t$. This is done deliberately in order to observe how a *single* row in the three TPMs are calculated. The row corresponds with 39 passengers at time $t$.

Table 2.3: The known passenger load at two successive time intervals for seven instances of the same fictitious flight.

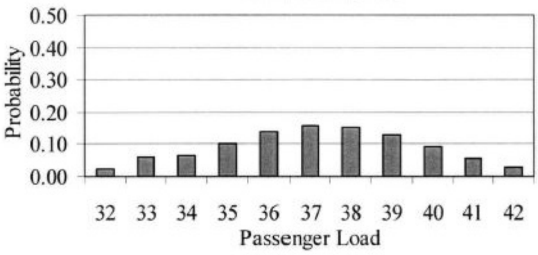
| Flight number | Passenger load at | | Difference |
|---|---|---|---|
| | time $t$ ($Z_t$) | time $t-1$ ($Z_{t-1}$) | |
| 147 | 39 | 33 | -7 |
| 202 | 39 | 35 | -4 |
| 243 | 39 | 35 | -4 |
| 294 | 39 | 38 | -1 |
| 306 | 39 | 39 | 0 |
| 354 | 39 | 39 | 0 |
| 416 | 39 | 39 | 0 |

To calculate the TPM-D for the interval $t$ to *t-1*, the differences between the passenger loads are calculated as shown in the last column of Table 2.3. Afterwards, the distribution of the trimmed differences is fitted with a normal distribution. Since the mean of these differences is roughly -2, the peak of the normal distribution should be centred around a passenger load of 37 passengers at time *t-1*. This is illustrated in Figure 2.6a.

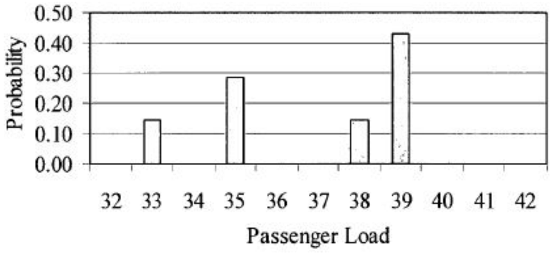
To interpret the diagram, it means that a passenger load of 39 passengers at time t has a probability of roughly 15% to change to a passenger load of 37 passengers. Thus, $\bar{P}^{39,37}_{t,t-1} = 0.15$. Similarly, the probability that the passenger load will remain the same is approximately 13%, hence $\bar{P}^{39,39}_{t,t-1} = 0.13$. These transition probabilities are calculated using equation (2.30).

Notice that the unobserved transitions, such as a transition to a passenger load of 36 passengers, also obtained a non-zero probability of occurrence. This is due to the fitting of a normal distribution to the differences, which helps to smooth out the transition probabilities.

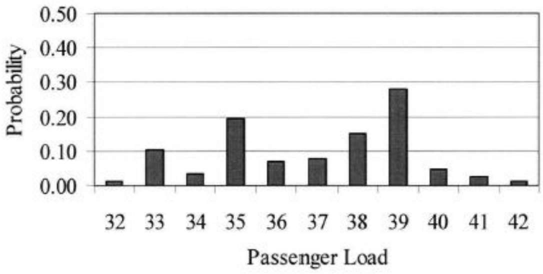
The TPM-APL load is more trivial. In the example given, the passenger load of three of the seven flight instances given remained at a passenger load of 39. Accordingly, the respective transition probability is $\frac{3}{7}$, such that $\bar{\bar{P}}_{t,t-1}^{39,39} = 0.43$. The resulting empirical distribution is depicted in Figure 2.6b. Notice that only the observed transitions have non-zero transition probabilities.




(a) Normal probability density function




(b) Empirical probability density function




(c) Modeled probability density function

Figure 2.6: An example of obtaining the transition probabilities for row 39 in each of the transition probability matrices when $\phi = 0.5$ (Goto et al., 2004).

The modelled transition probabilities are now obtained by combining the normal and empirical distributions using equation (2.32). As shown in Figure 2.6c, the distributions in this example are weighted equally because $\phi = 0.5$. Consequently, the probability that the passenger load will remain at 39 passengers at time *t-1*, denoted by $P_{t,t-1}^{39,39}$, is roughly 28%.

## 2.5 Concluding remarks

In-flight food waste reduction strategies have received minimal attention. While most include efforts aimed at improving the forecasting model used, the other two focused on reducing the impact of the uncertainty within the in-flight meal ordering process. In this chapter, *product substitution* and *stochastic programming* were identified as solution opportunities that can take advantage of the unavoidable meal demand uncertainty.

The in-flight meal ordering process is classified as a stochastic and multi-objective problem with two-way, stock-out based, partial-consumer driven product substitution. The majority of this chapter investigated suitable techniques to model the meal ordering process. The chosen approach is a stochastic MIP model with fixed RP, pre-emptive goal programming and static product substitution using *a priori* substitutability probabilities. It was noted that the model would have to be transformed into its deterministic counterpart before it can be solved using optimisation software.

The model will be dependent on the output of the chosen forecasting model, a time-inhomogeneous Markov Chain, to predict and quantify the meal demand uncertainty within the process. This forecasting model requires that the forecasting horizon must be divided into a strategic number of intervals. Each interval will require a unique modelled TPM due to the time-inhomogeneity assumption regarding the flight booking behaviour of passengers.

# Chapter 3

# Model formulation

This dissertation proposes an inventory decision-making model that incorporates the effect of product substitution and demand uncertainty. The vision for the model is to guide in-flight caterers regarding the most efficient quantity of meals to produce for a specific flight. Through its risk pooling effect, it is believed that the model will be able to reduce the number of excess meals on a flight, while simultaneously maximising the Passenger Satisfaction Level (PSL) and upholding its minimum requirement. The suggested model consists of two parts and is depicted in Figure 3.1.

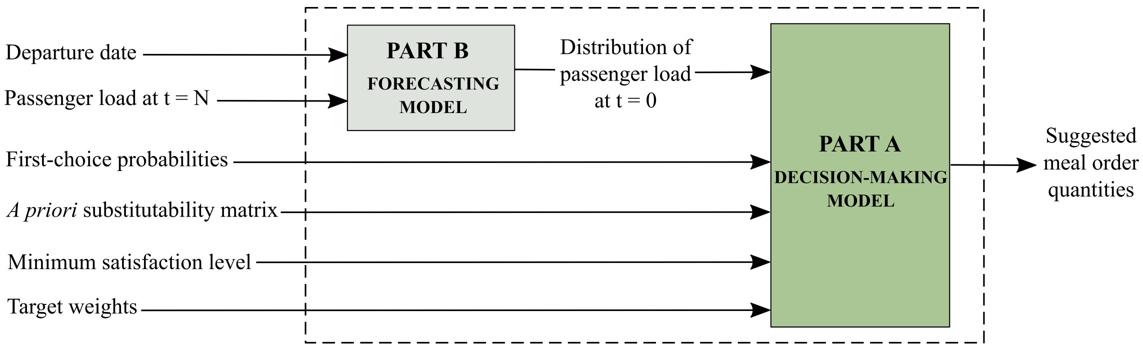



Figure 3.1: The suggested inventory decision-making model.

The output of the model is the suggested meal order quantities for each meal option available on the predefined in-flight menu for a specific flight instance. These quantities will (1) maximise the likelihood that passengers will receive their preferred meal or an acceptable substitute as an alternative to ensure that a minimum passenger satisfaction level is achieved, while (2) minimising leftover meals by considering the meal substitution behaviour of passengers and demand uncertainty. This highlights the multi-objective nature of the model. *Part A* of the stochastic model is responsible for weighting these objectives in order to generate the stated output.

*Part A* is the core of the decision-making model and will consist of a stochastic multi-objective Mixed-Integer Programming (MIP) model with fixed recourse and two-way, stock-out based, partially consumer-driven product substitution. This modelling base has extensive modelling capabilities and flexibility. This is highly desired as the product assortment (the flight's meal menu), and subsequent capacity requirements are changed frequently. Furthermore, it allows the model to be easily adapted to a specific catering company's unique production capabilities. To determine the most efficient meal order quantities, *Part A* requires five inputs:

*Probability distribution of the final passenger load* ($\pi_0$): A flight's *final* passenger load, $Z_0$, represents the aggregate meal demand as each passenger is entitled to receive a complimentary meal. This is an important input for the MIP model as it is required to derive the *primary* demand for each meal option.

However, recall that the final passenger load is unknown up until the flight's departure at time $t = 0$. Consequently, $Z_0$ must be forecasted, which creates uncertainty within the model. To handle this uncertainty and to improve the modelling accuracy, the final passenger load will be treated as a stochastic variable, denoted by $\widetilde{Z}_0$. The probability distribution thereof, represented with $\pi_0$, will be forecasted using *Part B* of the model.

*Stochastic set of first-choice probabilities* ($\widetilde{\mathbf{q}}$): These probabilities will be used to estimate the *primary* demand for each meal option with respect to the aggregate meal demand ($\widetilde{Z}_0$). The first-choice probabilities can be obtained using the ratios of meals that were booked by passengers utilising the airline's online flight booking system. Since fluctuations are possible, the set of first-choice probabilities will be treated as a stochastic variable.

*A priori substitutability matrix* ($\hat{\boldsymbol{\alpha}}$): This deterministic matrix defines the similarities between the different meal options available on the menu and explains the substitution behaviour of the passengers. Obtaining the *a priori* probabilities requires knowledge about each meal option and can be estimated using expert knowledge, experience, and market research.

*Target weights* ($w^{PSL}$ and $w^{Meals}$): The target weight $w^{PSL}$ indicates the importance of achieving a 100% PSL, while the target weight $w^{Meals}$ indicates the importance of producing zero waste. Both targets are considered equally important when the weight values are equal. The target weights will likely be flight dependent (i.e. business class vs economy class) and should be in line with the in-flight catering company and airline's competitive strategy.

*Minimum PSL* ($p^{min}$): This input represents the *minimum* PSL that must be achieved. Satisfaction levels lower than the *minimum* PSL are unacceptable, while higher satisfaction levels are desired but not required. The *minimum* PSL provides the model with an indication as to how much risk the model is allowed to take in terms of passenger dissatisfaction. For instance, a *minimum* PSL of 90% indicates that the model could risk reducing the total meal order quantity to reduce waste as a 10% dissatisfaction level is allowed. However, a *minimum* PSL of 100% forces the model to almost ignore the waste minimisation objective because there is no room for any meal shortages or substitutions.

Without the inclusion of the *minimum* PSL, the model will find the most efficient weighted balance between the primary and secondary objectives. This could result in an *expected* PSL lower than what the in-flight catering company is comfortable with as it could have a severe impact on the company's reputation. For example, an *expected* PSL of 94% might not be appropriate for a business-class flight. One approach to increase the *expected* PSL is to change the ratio between the two target weights. This will, however, require a trial-and-error approach until the correct target weights are found that will result in a more acceptable *expected* PSL. This trial-and-error approach will likely be tedious and time-consuming and is prevented through the inclusion of the *minimum* PSL requirement.

The *minimum* PSL input is flight dependent (i.e. business class vs economy class) and is influenced by the airline's strategy.

The purpose of *Part B* is to forecast $\pi_0$, the probability distribution of a specific flight's final passenger load. The chosen forecasting model consists of a time-inhomogeneous Markov Chain process, along with a regression model to improve the forecasting accuracy. The trained forecasting model will require two inputs:

*Known passenger load at* $t = N$ $(Z_N^*)$: The actual number of passenger reservations for the flight under consideration at time $t = N$.

*Departure date*: The day of the week, month and year of the flight's departure.

The forecasting model must be *trained* using historical data of the specific flight under consideration. Thus, the data required must include the passenger load at various pre-departure time points for numerous instances of the same flight. It is assumed that at least two year's of data is required to capture seasonality and growth. Unfortunately, the data required could not be obtained from industry or public data sources. For this reason, synthetic data will be generated by exploiting the known assignable causes of passenger load uncertainty and the flight booking behaviour of passengers. The synthetic data will be validated using visual inspection.

Although *Part A* is responsible for the actual decision-making aspect of the model, *Part B* is equally important due to the concept of *'garbage in, garbage out'*. Simply stated, if the output of *Part B* is highly unreliable, then the output of *Part A* will have no value. The remainder of this chapter explains the model formulation for each part in more detail.

## 3.1 Part A: Inventory decision-making model

The purpose of the inventory decision-making model is to decide on the most efficient meal order quantities for a specific flight. This model will be similar to the work of Vaagen et al. (2011). Consequently, it will consist of a MIP model with fixed recourse and product substitution where the substitution behaviour is represented with exogenous *a priori* probabilities. However, unlike Vaagen et al. (2011)'s model, the model developed will incorporate pre-emptive goal programming. This will allow the model to handle the non-monetary, multi-objective nature of the in-flight meal ordering process.

In addition, Vaagen et al. (2011)'s model assumes that the stochastic primary demand for each individual product is known. For the problem considered, each meal option's individual demands will be derived from the flight's random aggregate meal demand (final passenger load) and the set of first-choice probabilities using equation (2.2). Recall that a first-choice probability refers to the fraction of passengers who's first-choice preference is a specific meal option from the assortment of meals offered on the in-flight menu. Equation (2.2) assumes that the set of first-choice probabilities are deterministic as they represent the market share of each product. However, to further improve the accuracy of the model, this dissertation will treat the set of first-choice probabilities as random variables. The remainder of this section will discuss the model's notation and formulation.

### 3.1.1 Notation

The model consists of various decision variables, utility variables and parameters as defined in this section. The stochastic elements can be identified with a tilde ($\widetilde{A}$), while sets and vectors are indicated in bold ($\mathbf{A}$).

#### Sets

A set is a collection of elements that are grouped according to a mutual characteristic. Only one set will be used in the model:

Let $\mathbf{I}$ represent the set of meal types served on the flight under consideration, such that $\mathbf{I} \in \{1, 2, ..., n\}$ and $n$ indicates the number of meal options available.

#### Decision variable

Decision variables represent the controllable components of the model that can be manipulated to optimise the given objective function. According to Bean (2011), decision variables are used by a decision-maker to make informed decisions.

$x_i \triangleq$ the suggested order and production quantity of meal option $i \in \mathbf{I}$ for a specific flight under consideration.

#### Parameters

Parameters are known values that cannot be manipulated within the model.

$p^{min} \triangleq$ the minimum passenger satisfaction level required.

$\widetilde{Z}_0 \triangleq$ random variable describing the final passenger load (the aggregate meal demand) for the flight under consideration on the day of its departure ($t = 0$). Its probability distribution $\pi_0$ is derived in *Part B*.

$w^{PSL} \triangleq$ the target weight indicating the importance of achieving a 100% PSL.

$w^{Meals} \triangleq$ the target weight indicating the importance of producing zero waste.

$\widetilde{\mathbf{q}} \triangleq$ random vector describing the set of first-choice probabilities, such that $\widetilde{q}_i$ represents the random first-choice probability of meal option $i \in \mathbf{I}$.

$\hat{\alpha_{ij}} \triangleq$ the *a priori* probability that meal option $i \in \mathbf{I}$ can serve as a substitute for meal option $j \in \mathbf{I}$ when the latter is out of stock. Furthermore, it also represents the level of passenger satisfaction achievable when the mentioned substitution occurs.

#### Utility variables

Utility variables provide additional support to ensure that the model functions as intended, while the final value thereof is considered unimportant.

In this model, all the utility variables are second-stage decision variables. Thus, $A(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ indicates that $A$ is a second-stage variable and a function of the stochastic first-choice probability vector $\widetilde{\mathbf{q}}$ and the random aggregate demand $\widetilde{Z}_0$.

$d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \triangleq$ the primary demand for meal option $i \in \mathbf{I}$.

$\delta_i^+(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \triangleq$ the number of meals of option $i \in \mathbf{I}$ in *excess* of its effective demand.

$\delta_i^-(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \triangleq$ the number of meals of option $i \in \mathbf{I}$ in *short* of its primary demand.

$y_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \triangleq$ the number of meals of option $i \in \mathbf{I}$ assigned to satisfy its respective primary demand (direct sales). Thus, the number of passengers who will receive their first-choice meal, meal option $i$.

$z_{ij}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \triangleq$ the number of meals of option $i \in \mathbf{I}$ that is assigned to meet the unsatisfied demand for meal option $j \in \mathbf{I}$ (substitution sales). Stated differently, the number of passengers that will receive meal option $i$ as a substitute for meal option $j$. Intuitively, $z_{ii} = 0$.

$\Delta^{Meals}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \triangleq$ the deviation from the zero-waste target of the waste minimisation objective (the *total* number of surplus meals produced).

$\Delta^{PSL}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \triangleq$ the deviation from 100% PSL target of the PSL maximisation objective (the passenger dissatisfaction level).

### 3.1.2 Multi-objective stochastic MIP model with fixed recourse and product substitution

Through the implementation of pre-emptive goal programming, the objective function in (3.1) minimises the weighted sum of the deviations from the goals of the in-flight catering industry's two conflicting objectives – a 100% PSL and zero waste.

$$\min Z = w^{PSL} \cdot E_{\widetilde{\mathbf{q}}, \widetilde{Z}_0} \left[ \Delta^{PSL}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \right] + w^{Meals} \cdot E_{\widetilde{\mathbf{q}}, \widetilde{Z}_0} \left[ \Delta^{Meals}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \right] \quad (3.1)$$

Simply stated, the objective function minimises the deviation from the ideal solution. The first term of the objective function minimises the expected passenger *dissatisfaction* level, thereby indirectly maximising the *expected* PSL of the flight under consideration (primary objective). The second term minimises the expected number of surplus meals produced (secondary objective). The target weights, $w^{PSL}$ and $w^{Meals}$, indicate the importance of achieving the goals of the two objectives.

The objective function is dependent on the recourse actions of the model after the first-choice probabilities and passenger load uncertainties have been disclosed. Two recourse variables, $\delta_i^+(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ and $\delta_i^-(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$, are introduced for each meal option $i \in \mathbf{I}$ to measure the over- and under-catering thereof.

The deviation from the zero-waste target is represented with $\Delta^{Meals}(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ and is calculated using (3.2).

$$\sum_{i \in \mathbf{I}} \delta_i^+(\widetilde{\mathbf{q}}, \widetilde{Z}_0) - \Delta^{Meals}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \leq 0 \tag{3.2}$$

Since it is impossible to produce a negative amount of waste, (3.2) can be simplified so that $\Delta^{Meals}(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is equal to the total number of surplus meals produced.

The passenger *dissatisfaction* level, denoted by $\Delta^{PSL}(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$, represents the deviation from the 100% PSL target and is calculated with (3.3).

$$100 \frac{\sum_{i \in \mathbf{I}} \left( y_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) + \sum_{j \in \mathbf{I},\, j \neq i} \hat{\alpha_{ji}}\, z_{ji}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \right)}{\sum_{i \in \mathbf{I}} d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0)} + \Delta^{PSL}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \geq 100 \tag{3.3}$$

The first term in (3.3) represents the PSL. A passenger is fully satisfied when the passenger received his first-choice meal. However, when the passenger received meal option $j$ as a substitute for meal option $i$, it is assumed that the passenger's satisfaction level can be approximated with $\hat{\alpha_{ji}}$. Thus, the PSL of a flight decreases with meal shortages and substitutions. Intuitively, the maximum PSL obtainable is 100% and (3.3) can be simplified accordingly so that the $\Delta^{PSL}(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is equal to the passenger *dissatisfaction level*, calculated as 100% minus the PSL.

It is assumed that most in-flight catering companies will only consider wastage improvements *after* the *minimum* PSL is guaranteed. This PSL restriction provides confidence in the model's solutions (given that the model is deemed reliable) and, subsequently, prevents the need for the over-catering strategy. The restriction is enforced with (3.4), where $p^{min}$ represents the *minimum* PSL requirement.

$$100 - \Delta^{PSL}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \geq p^{min} \tag{3.4}$$

The model will optimise the above-mentioned objective function by selectively choosing the most efficient meal order quantity $x_i$ for each meal option $i \in \mathbf{I}$. This decision is made without knowing $d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$, the primary demand of meal option $i$. As shown in (3.5), $d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is approximated using the appropriate stochastic first-choice probability $\widetilde{q}_i$ and the random aggregate meal demand $\widetilde{Z}_0$.

$$d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \approx \lceil \widetilde{q}_i\, \widetilde{Z}_0 \rceil, \qquad \forall \qquad i \in \mathbf{I} \tag{3.5}$$

Recall that $\widetilde{Z}_0$ is equivalent to the passenger load at departure and the probability distribution thereof ($\pi_0$) is the output of the forecasting model developed in *Part B*.

Evidently, the true value of $d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is only disclosed in the second stage of the model. Once the true value is known, the model will *optimally* allocate the available meal quantities to satisfy the respective demand requirements. This allocation is carried out with (3.6).

$$d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) = y_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) + \sum_{j \in I,\, j \neq i} z_{ji}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) + \delta_i^-(\widetilde{\mathbf{q}}, \widetilde{Z}_0), \qquad \forall \qquad i \in \mathbf{I} \tag{3.6}$$

Variable $y_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ denotes the number of passengers who will receive their preferred meal choice, meal option $i$. The model will first try to satisfy the demand with $y_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ only because direct sales will minimise the passenger *dissatisfaction* level. Doing so ensures that the model exhibits *partial* consumer-driven substitution and prevents the bait-and-catch strategy. However, if meal option $i$ is under-catered, $z_{ji}(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ units of meal option $j \in \mathbf{I}$ will be used to help satisfy the excess demand. Lastly, the recourse variable $\delta_i^-(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is introduced to measure the unmet demand (shortage) of meal option $i$, if any.

Recall that the passengers' substitution behaviour is captured with *a priori* probabilities; The probability that a passenger who prefers meal option $j$ will accept meal option $i$ as a substitute is denoted by $\hat{\alpha_{ij}}$. Constraint (3.7) utilises $\hat{\alpha_{ij}}$ to estimate the maximum additional substitution demand for meal option $i$ caused by a stock-out of meal option $j$.

$$z_{ij}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \leq \hat{\alpha_{ij}}[d_j(\widetilde{\mathbf{q}}, \widetilde{Z}_0) - y_j(\widetilde{\mathbf{q}}, \widetilde{Z}_0)], \qquad \forall \qquad i, j \in \mathbf{I} \tag{3.7}$$

Variable $z_{ij}(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is bounded by the maximum substitution demand in (3.7). Additionally, $z_{ij}(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is also restricted by the available stock of meal option $i$ in (3.8).

$$x_i = y_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) + \sum_{j \in \mathbf{I},\, j \neq i} z_{ij}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) + \delta_i^+(\widetilde{\mathbf{q}}, \widetilde{Z}_0), \qquad \forall \qquad i \in \mathbf{I} \tag{3.8}$$

This constraint links the model's first-stage (left) and second-stage (right) decisions. It ensures that the model cannot allocate more meals than what was initially ordered. Additionally, the recourse variable $\delta_i^+(\widetilde{\mathbf{q}}, \widetilde{Z}_0)$ is used to measure the magnitude of over-catering when $x_i$ exceeds the uncensored *effective* demand for meal option $i$.

The model formulation assumes that all variables are positive. Furthermore, variables representing meal quantities are restricted to integer values only. This motivates the inclusion of constraints (3.9) to (3.12).

$$x_i,\ y_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0),\ d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \geq 0 \text{ and integer}, \qquad \forall \qquad i \in \mathbf{I} \tag{3.9}$$

$$\delta_i^+(\widetilde{\mathbf{q}}, \widetilde{Z}_0),\ \delta_i^-(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \geq 0 \text{ and integer}, \qquad \forall \qquad i \in \mathbf{I} \tag{3.10}$$

$$z_{ij}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \geq 0 \text{ and integer}, \qquad \forall \qquad i, j \in \mathbf{I} \tag{3.11}$$

$$\Delta^{Meals}(\widetilde{\mathbf{q}}, \widetilde{Z}_0),\ \Delta^{PSL}(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \geq 0, \qquad \forall \qquad i \in \mathbf{I} \tag{3.12}$$

## 3.2 Part B: Forecasting model

The purpose of *Part B* is to forecast the probability distribution of $\widetilde{Z}_0$, the final passenger load, given that there are $Z_N^*$ booking reservations at time $t = N$. The chosen forecasting model will consist of a time-inhomogenous Markov Chain as proposed by Van Ostaijen et al. (2017). However, the model will be adapted to allow overbooking as this is common practice in the airline industry. The required Transition Probability Matrix (TPM) for each forecasting interval will be developed by combining and adapting the approaches followed by Van Ostaijen et al. (2017) and Goto et al. (2004). The TPMs must be developed using a historical dataset for the flight under consideration. This set will be referred to as the training dataset, denoted by $\dot{\boldsymbol{\gamma}}$.

### 3.2.1 Formulating the problem as a time-inhomogeneous Markov chain

Let the time-inhomogeneous Markov Chain $\{Z_t\}$ represent the passenger load at time point $t$ before the departure of the flight, where $t \in \mathbf{T} = \{N, N-1, ..., 0\}$ and $Z_t \in \mathbb{C}$. Furthermore, let:

$N \triangleq$ the number of forecasting intervals.

$K \triangleq$ the seating capacity of the flight under consideration.

$O \triangleq$ the number of overbookings allowed before the flight's departure.

$\mathbb{C} \triangleq$ The state space of $\{Z_t\}$ that consists of all possible passenger load possibilities, such that

$$\mathbb{C} = \begin{cases} \{0, 1, ..., K + O\}, & if \quad t > 0. \\ \{0, 1, ..., K\}, & otherwise. \end{cases}$$

$P_{t,\, t-1} \triangleq$ the modelled TPM for interval $t$ to $t-1$, where element $P^{i,\, j}_{t,\, t-1}$ represents the probability that the flight's passenger load will transition from $i$ to $j$ passengers during the stated interval, where $t \in \{N, N-1, ..., 1\}$ and $i, j \in \mathbb{C}$.

$\pi_t \triangleq$ the probability distribution of $Z_t$, the passenger load at time point $t \in \mathbf{T}$.

$Z^*_t \triangleq$ the *true* and known passenger load at time point $t \in \mathbf{T}$.

The forecasting horizon is measured in hours and divided into $N$ intervals of varying lengths. Recall that $t = N$ represents the start of the forecasting horizon, while the flight's departure occurs at $t = 0$. Thus, the first forecasting interval stretches from $t = N$ to $t = N\text{-}1$. The number of intervals and the length of each interval must be decided strategically. While a large number of intervals could improve the accuracy of the model, it will also increase the model's computational requirements. The reason being that each interval requires a unique modelled TPM as it is known that the booking behaviour of passengers is time-dependent. This motivates the use of a *time-inhomogeneous* Markov Chain. Furthermore, the model being developed assumes that the state space $\mathbb{C}$ is also time-dependent due to the allowance of overbooking when $t > 0$.

Van Ostaijen et al. (2017) suggest equation (2.28) to calculate $\pi_{t-\beta}$. However, the author restricts the utility variable $\beta$ to belong to the set $\{1, ..., N-t\}$. This restriction is believed to be a potential error because it prohibits the calculation of most $\pi_{t \in \mathbf{T}}$ variables. For example, to calculate $\pi_0$ when $t = N$ requires that $\beta$ should be equal to $N$, yet it is not possible since $\beta$ is restricted to the set $\{1, 0\}$. For this reason, this dissertation will restrict $\beta$ to belong to the set $\{1, ..., t\}$.

Furthermore, there are two approaches that can be used to model a matrix, such as a TPM. Assume a scenario where the probability to transition from $Z_t = i$ to $Z_{t-1} = j$ is equal to 30%. Van Ostaijen et al. (2017) followed the approach as illustrated in Figure 3.2a, where the columns represent the current state ($Z_t$) and the rows represent the future state ($Z_{t-1}$).

| | … | i | … |
|---|---|---|---|
| … | … | … | … |
| j | … | 0.3 | … |
| … | … | … | … |

(a) Column ($Z_t$) to row ($Z_{t-1}$).

| | … | j | … |
|---|---|---|---|
| … | … | … | … |
| i | … | 0.3 | … |
| … | … | … | … |

(b) Row ($Z_t$) to column ($Z_{t-1}$).

Figure 3.2: Illustration of the two approach that can be used to model a TPM.

To ensure consistency with the matrices used in Part A of the model, this dissertation will follow the approach depicted in Figure 3.2b – the rows represent the current state ($Z_t$) and the column represent the future state ($Z_{t-1}$). This will have no impact on the outcome of the forecasting model. The reason being that the two matrices shown are simply the transpose of one another. However, due to the rules of matrix multiplication, equation (2.28) is no longer suitable and must be transformed accordingly. The resulting equation is given with (3.13), where $t \in \{N, N-1, ..., 1\}$ and $\beta \in \{1, ..., t\}$.

$$\pi_{t-\beta} = \pi_t \cdot \prod_{l=1}^{\beta} P_{t-l+1,\, t-l} \tag{3.13}$$

Equation (3.13) and the inclusion of overbooking require that $\pi_t$ must be redefined as shown in (3.14).

$$\pi_t = \begin{cases} (\mathbb{P}[Z_t = 0],\ \mathbb{P}[Z_t = 1],\ \dots,\ \mathbb{P}[Z_t = K + O]) & t > 0, \\ (\mathbb{P}[Z_t = 0],\ \mathbb{P}[Z_t = 1],\ \dots,\ \mathbb{P}[Z_t = K]) & t = 0. \end{cases} \tag{3.14}$$

Recall that the main purpose of the forecasting model is to predict $\pi_0$ using $Z_N^*$. This can be achieved using (3.13) where $t = N$ and $\beta = N$ as demonstrated below.

$$\pi_0 = \pi_N \cdot \prod_{l=1}^{N} P_{N-l+1,\, N-l} \tag{3.15}$$

Notice that $\pi_N$ is required as input for (3.15). Fortunately, it is known that the true passenger load at time $t = N$ is $Z_N^*$, meaning that $\mathbb{P}[Z_N = Z_N^*] = 1$. Consequently, $\pi_N$ can easily be derived and will consist of $|\mathbb{C}| - 1$ zero probability elements (where $Z_N \neq Z_N^*$) and one element with a probability equal to 1 (when $Z_N = Z_N^*$).

The inventory-decision making model (Part A) is only interested in the *distribution* of the final passenger load ($\pi_0$). However, the *single point estimate* thereof will be required to measure the accuracy of the forecasting model and to evaluate alternative models. The single point estimate of the final passenger load, represented with $E[\widetilde{Z}_0]$, can be calculated using equation (2.29) defined earlier, where $t = 0$.

$$E[\widetilde{Z}_0] = \sum_{i=0}^{K} \pi_0(i) \cdot i \tag{3.16}$$

Note that the single point estimate is also known as the expected value. The forecasting accuracy of the expected value will be measured using the Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE).

### 3.2.2 Deriving the transition probability matrices

Recall that the modelled TPM, denoted by $P_{t,t-1}$, can be obtained using (2.32). This equation is modified slightly to allow for overbooking as shown in (3.17).

$$P^{i,j}_{t,t-1} = \begin{cases} (1-\phi)\cdot \bar{P}^{i,j}_{t,t-1} \ + \ \phi \cdot \bar{\bar{P}}^{i,j}_{t,t-1}, & if \quad \sum_{j\in\mathbb{C}} \bar{\bar{P}}^{i,j}_{t,t-1} \neq 0, \\ \\ \bar{P}^{i,j}_{t,t-1} & otherwise. \end{cases} \tag{3.17}$$

Each element in $P_{t,t-1}$ is the weighted combination of the respective element in the Transition Probability Matrix using Differences (TPM-D), and in the Transition Probability Matrix using the Absolute Passenger Loads (TPM-APL) for the respective interval. The most efficient weighting factor $\phi$ will be chosen after conducting a sensitivity analysis for each $\phi \in \{0, 0.05, ..., 0.95, 1\}$.

#### (a) The transition probability matrix using differences

The chosen method to calculate the TPM-Ds is a combination of the two approaches followed by Van Ostaijen et al. (2017) and Goto et al. (2004). Similar to Goto et al. (2004), the TPM-D for the *final* interval will be generated using a regression analysis to capture the dependent relationship between the passenger load at the start of the final interval and the change observed. Consequently, the formulation of the set of TPM-Ds consists of three main steps:

*1) Calculate the set of differences for each forecasting interval where $t-1>0$*

The first step is to calculate the changes in the passenger load observed during each forecasting interval using (3.18).

$$Y^{\gamma}_{t,t-1} = Z^{*\gamma}_{t-1} - Z^{*\gamma}_{t} \qquad \forall \qquad \gamma \in \dot{\boldsymbol{\gamma}}, \quad t \in \{N, N-1, ..., 1\} \tag{3.18}$$

The set of differences, $\mathbf{Y}_{t,t-1}$, consists of the changes in passenger load observed during interval $t$ to *t-1* for each flight instance $\gamma$ given in the training dataset $\dot{\boldsymbol{\gamma}}$.

*2) Calculate the TPM-D for all the intervals between t = N and t = 1*

Goto et al. (2004) fitted each interval's trimmed set of differences with a normal or shifted Poisson distribution to calculate the individual transition probabilities using equation (2.30). However, Van Ostaijen et al. (2017) obtained *improved* accuracy by maintaining the initial probability distribution of the differences. A possible explanation could be that Goto et al. (2004)'s approach ignores the potentially long and thick tails of the respective distributions. An example of such a tail is visible in the first two intervals of Figure 2.5. In contrast, the drawback of Van Ostaijen et al. (2017)'s approach is that it assumes that only the differences observed are possible.

Due to the strive for exceptional accuracy, Van Ostaijen et al. (2017)'s approach will be followed. This is achieved by first finding $\hat{y}_{t,t-1}$, the probability density function of $\mathbf{Y}_{t,t-1}$. Knowing that the $[Z_t, Z_{t-1}]^{th}$ entry of the TPM is equal to $\mathbb{P}[Z_{t-1}|Z_t]$, the matrix can be populated using (3.19). This equation is derived from equation (2.30) and $\hat{y}_{t,t-1}(i)$ represents the $i^{th}$ element of $\hat{y}_{t,t-1}$.

$$\mathbb{P}[Z_{t-1}|Z_t] = \begin{cases} \sum_{i=Z_t}^{|\mathbb{C}|} \hat{y}_{t,t-1}(Z_{t-1}-i) & if \quad Z_{t-1} = 0 \\ \hat{y}_{t,t-1}(Z_{t-1}-Z_t) & if \quad 0 < Z_{t-1} < |\mathbb{C}| \\ \sum_{i=0}^{Z_t} \hat{y}_{t,t-1}(Z_{t-1}-i) & if \quad Z_{t-1} = |\mathbb{C}| \end{cases} \tag{3.19}$$

*3) Calculate the TPM-D for the final interval from t = 1 to t = 0*

To calculate the final TPM-D, Goto et al. (2004) used a simple regression analysis with a single covariate - the *known* passenger load at one hour before departure. Goto et al. (2004) claims that this variable is strongly correlated with other possible covariates and, therefore, excludes them from the regression analysis. However, in the forecasting model being developed in this dissertation, the passenger load at one hour before departure will be unknown. Consequently, four covariates will be included: the *forecasted* passenger load at the start of the last time interval ($Z_1$), as well as the flight's departure month, Day of Week (DOW) and year. It will be shown in the following chapter that the last three explanatory variables are major causes of passenger load variation. Thus, the inclusion thereof is expected to improve the forecasting accuracy of the model. The multiple regression model is given in (3.20).

$$\begin{aligned} E[\mathbf{Y_{1,0}}\,|\,Z_1 = k] = \beta_0 &+\beta_1 \cdot (k) + \beta_2 X_{Tue} + \beta_3 X_{Wed} + \beta_4 X_{Thu} + \beta_5 X_{Fri} \\ &+\beta_6 X_{Sat} + \beta_7 X_{Sun} + \beta_8 X_{Feb} + \beta_9 X_{Mar} + \beta_{10} X_{Apr} \\ &+\beta_{11} X_{May} + \beta_{12} X_{Jun} + \beta_{13} X_{Jul} + \beta_{14} X_{Aug} + \beta_{15} X_{Sep} \\ &+\beta_{16} X_{Oct} + \beta_{17} X_{Nov} + \beta_{18} X_{Dec} + \beta_{19} \cdot (year) + \epsilon \end{aligned} \tag{3.20}$$

The regression model consists of various dummy variables ($X$) to accommodate the two categorical covariates - the month and DOW. A dummy variable is equal to one if it applies to the flight instance under consideration and zero otherwise. Note that there are no dummy variables for January (month) and Monday (DOW) in order to prevent multicollinearity. Thus, a flight departing on a Monday in January is modelled as the base case. Subsequently, the regression parameters represent the change in $E[\mathbf{Y_{1,0}}\,|\,Z_1 = k]$ when the departure date of flight instance $\gamma$ deviates from the base case. If multicollinearity is not prevented, two or more variables in the regression model would be highly linearly correlated and will compromise the statistical significance of the explanatory variables (Allen, 1997).

The output of the regression model, $E[\mathbf{Y_{1,0}}\,|\,Z_1 = k]$, represents the expected change in the passenger load during the last interval if $Z_1 = k \in \mathbb{C}$. For this reason, it will be used as an approximation for the *mean* of $\mathbf{Y_{1,0}}$'s normal distribution. Its standard deviation will be estimated using the root mean square error. The normal distribution can then be used in conjunction with equation (2.30) to calculate $\mathbb{P}[Z_0|Z_1 = k]$, the value of the final TPM-D's $[k, Z_0]^{th}$ entry. This entire process must be repeated for each $k \in \mathbb{C}$ in order to populate each row in the matrix.

**(b) The transition probability matrix using net absolute passenger loads**

The TPM-APL will be obtained using the method followed by Goto et al. (2004) and Van Ostaijen et al. (2017). In summary, the matrix is calculated by constructing a transition frequency table and dividing each row with its respective total.

## 3.3 Concluding remarks

This chapter focused on the formulation of an inventory decision-making model (Part A) and a forecasting model (Part B). Together, these two models should aid an airline catering company to make informed decisions regarding the most efficient quantity of meals to order and produce for a specific flight under consideration. The model measures efficiency using the *expected* PSL obtainable and the accompanying expected number of surplus meals.

The inventory decision-making model is formulated as a stochastic multi-objective MIP model with fixed recourse and product substitution. Unlike in Vaagen et al. (2011)'s model, the model's objectives are both expressed in non-monetary terms. This is due to the difficulty in quantifying the loss in customer goodwill resulting from stock-outs of in-flight meals, and the true cost of wastage. The advantage thereof is that fewer input data and assumptions are required for the model to function. Additionally, it is believed that the use of monetary terms might cloud the intended goal of the model as it could result in the bait-and-catch strategy if the appropriate parameters are not chosen appropriately.

The inventory decision-making model is heavily dependent on the output of the forecasting model. Consequently, the forecasting model must be trained using *reliable* data from past flight observations. This means that a forecasting model cannot be shared among different flights that vary, for instance, by destination, seat capacity or ticket prices. The training of the forecasting model will be conducted in Chapter 5 and will consist of the construction of the modelled TPM's and a sensitivity analysis to identify the most favourable weighting factor.

As stated earlier, this dissertation uses a synthetic dataset to train and test both models. The assumptions and processes followed to generate the datasets will be discussed in the following chapter.

# Chapter 4

# Numerical example and synthetic data generation

The forecasting model and the inventory decision-making model formulated in Chapter 3 are applied and evaluated using a numerical example to help ease the understanding and interpretation of the model's solution and results. This requires two datasets: a training dataset to train the forecasting model and a testing dataset to evaluate the decision-making model's reliability, performance and timeliness.

The desired datasets are generated in this chapter. To ensure that these datasets are as realistic as possible, a brief analysis of the major causes of variation in a flight's passenger load is conducted. The aim of the analysis is to identify and replicate the flight booking behaviour of passengers.

## 4.1 Numerical example

The numerical example focuses on a fictitious company that is currently responsible for the provisioning of food and beverages for one domestic flight that departs daily at 9:00 AM. The flight has a seating capacity of 100 passengers ($K = 100$) and their current reservation system allows for an overbooking of five passengers ($O = 5$). Each passenger on-board the flight is entitled to receive one complimentary meal. Currently, the in-flight menu ($\mathbf{I}$) consists of a chicken meal, a beef meal, a vegan meal and a fruit platter meal option. Chicken meals are the most preferred option, followed by beef meals. Vegan meals are the least preferred option on the menu, with a maximum primary demand of 5% of the passenger load. This is visible in Table 4.1, which lists the set of first-choice probabilities ($\widetilde{\mathbf{q}}$) for five historical flights.

Through extensive market research and the use of expert knowledge, the company quantified the similarities between the meal options, as shown in Table 4.2. Recall that these similarity percentages represent the *a priori* probabilities ($\hat{\boldsymbol{\alpha}}$). For example, there is a 70% probability that a passenger who prefers a vegan meal will accept a fruit platter as a substitute. Additionally, there is also a 10% chance that the passenger would accept a chicken meal. This is possible because not all of the passengers who prefer a vegan meal as their first-choice meal are actually practising a vegan lifestyle.

Table 4.1: The set of first-choice probabilities for five historical flights of the numerical example.

| Flight nr. | Chicken | Beef | Fruit platter | Vegan |
|---|---|---|---|---|
| DF 201 | 0.54 | 0.28 | 0.16 | 0.02 |
| DF 273 | 0.51 | 0.34 | 0.10 | 0.05 |
| DF 367 | 0.46 | 0.33 | 0.18 | 0.03 |
| DF 381 | 0.58 | 0.28 | 0.14 | 0.00 |
| DF 415 | 0.45 | 0.38 | 0.12 | 0.05 |

Table 4.2: The *a priori* substitution probability matrix of the numerical example.

| | First-choice product (j) | | | |
|---|---|---|---|---|
| **Substitute (i)** | **Chicken** | **Beef** | **Fruit Platter** | **Vegan** |
| **Chicken** | - | 0.8 | 0.2 | 0.1 |
| **Beef** | 0.6 | - | 0.1 | 0.0 |
| **Fruit platter** | 0.3 | 0.3 | - | 0.7 |
| **Vegan** | 0.4 | 0.3 | 0.7 | - |

The company has to start the planning, procurement and production of in-flight meals 72 hours in advance of the flight's departure. Additional key events take place at 15 hours, six hours and one hour before take-off, and the company keeps track of a flight's passenger load at these time points. Consequently, these historical records can be used to train the forecasting model. Doing so enforces the number of intervals ($N = 4$) and the individual lengths thereof. The company's forecasting horizon is shown in Figure 4.1.

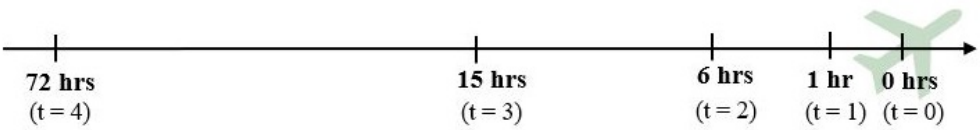



Figure 4.1: The forecasting timeline for the numberical example.

The remainder of this chapter focuses on identifying the major causes of passenger load variation and generating the desired datasets.

## 4.2 The flight booking behaviour of passengers

There is undeniably countless factors that influence the booking behaviour of passengers. This dissertation will focus on two key factors, the flight's duration and the time of travel. It also acknowledges an important pattern in a flight's passenger load during the final hour before its departure.

### Flight duration: domestic flights versus international flights

Flight duration has a major impact on the booking behaviour of passengers. Chang and Jones (2007) and Megodawickrama (2018) agree with this statement. A possible explanation is that, for international flights, most seats are booked weeks in advance for reasons such as visa requirements, early-bird discounts and bucket-list holidays. Furthermore, the cost of international flight tickets is generally significantly higher than the cost of domestic flights. This discourages impulsive and last-minute booking decisions. Because of these factors, the passenger load of international flights are not expected to change or vary drastically during the final few days before departure.

The opposite is true for domestic flights. These flights are more affordable when compared to international flights and usually do not require a passport or visa. Thus, impulsive and last-minute bookings are expected to be a frequent occurrence. This is especially applicable to passengers who travel for business. Furthermore, due to the lower airfare, passengers might be more likely to miss their flight or to cancel their bookings on short notice. It is, therefore, expected that the passenger load of domestic flights would exhibit moderate to exceptional growth and variation during the final week before departure.

As discussed earlier, Figure 2.5 shows the distribution of the differences in the passenger load observed for five consecutive time intervals as obtained by Goto et al. (2004). It is speculated that the authors studied an *international* flight because the passenger load remained somewhat stable during the 36 hours before the flight's departure. This speculation is based on the fact that the peaks of the differences in passenger load distributions depicted are located close to zero when $t > 0$.

### Time of travel: month, Day of Week (DOW) and year

Seasonality is one of the most significant causes of variation in a flight's passenger load (Megodawickrama, 2018). This is due to the variability in the tourism industry that differs for each country. The total monthly passenger traffic for domestic flights in South Africa is shown in Figure 4.2 (Airport Company South Africa, 2020). It is clear that a relationship exists between the month of travel and the total passenger traffic.

It is also evident from Figure 4.2 that the air traffic increases annually. The observed annual increases in the *total* domestic air traffic in South Africa ranged between 1.3% and 3.8%. The *monthly* growth observed between the respective years ranged between -4.8% and 7.9%. This emphasises the excessive uncertainty regarding a flight's final passenger load.

Two additional contributing factors are the DOW and the time of departure. Megodawickrama (2018) found that demand was high on Sundays, Thursdays and peaked on Saturdays. Wednesdays had the lowest demand, with at least 2 000 fewer meals required when compared to a Saturday. Danesi et al. (2017) and Koppelman et al. (2008) claim that mid-morning and late-afternoon flights are most preferred, followed by midday flights.

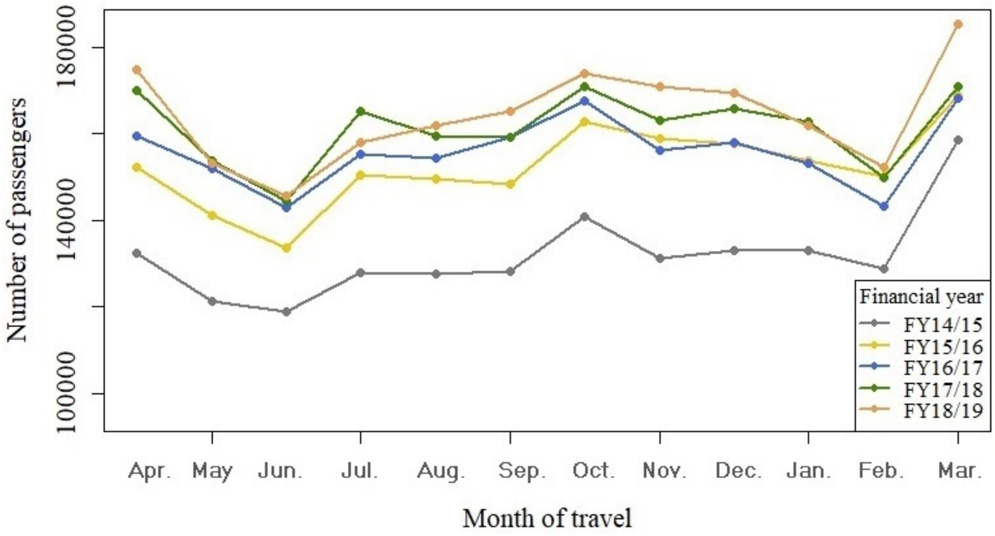



Figure 4.2: The total monthly air traffic for domestic flights in South Africa depicting a seasonality trend and annual growth (Airport Company South Africa, 2020).

The significance of the above-mentioned factors are flight dependant. For instance, Van Ostaijen et al. (2017) showed that monthly seasonality is an important factor for *tourist* flights, while the DOW is an important factor for *business* flights.

**Pattern in pre-departure passenger load during the forecasting horizon**

Goto (1999) observed the pre-departure passenger load for eight consecutive instances of the *same* flight on a single day of the week. The author's observations are depicted in Figure 4.3 where each line represents the passenger load of a flight instance with respect to the flight's capacity. Note the excessive variation even though the flight instances only differed in departure times. This highlights an additional, uncontrollable cause of variation - the human factor in the flight booking process.

At some of the pre-departure time points, the passenger load exceeds the flight's seat capacity. This is due to overbooking. Some airlines allow overbooking to maximise their revenue. These airlines rely on the observed booking behaviour trend where the passenger load sharply declines during the last hour before flight departure. This trend is clearly visible in Figure 4.3.

The trend observed indicates that a flight's passenger load will most likely decline during the last hour before departure. Goto (1999) speculate that the cause could be due to last-minute ticket cancellations, missed flights and the consequences of over-booking. The authors suspect that this last-minute variability is the key factor that makes passenger load and meal demand predictions difficult. Furthermore, Goto (1999) observed a negative relationship between the passenger load shortly before the flight's departure and the change in the passenger load observed during the final interval. In other words, the magnitude of the above-mentioned decrease is dependent on the passenger load shortly before departure.

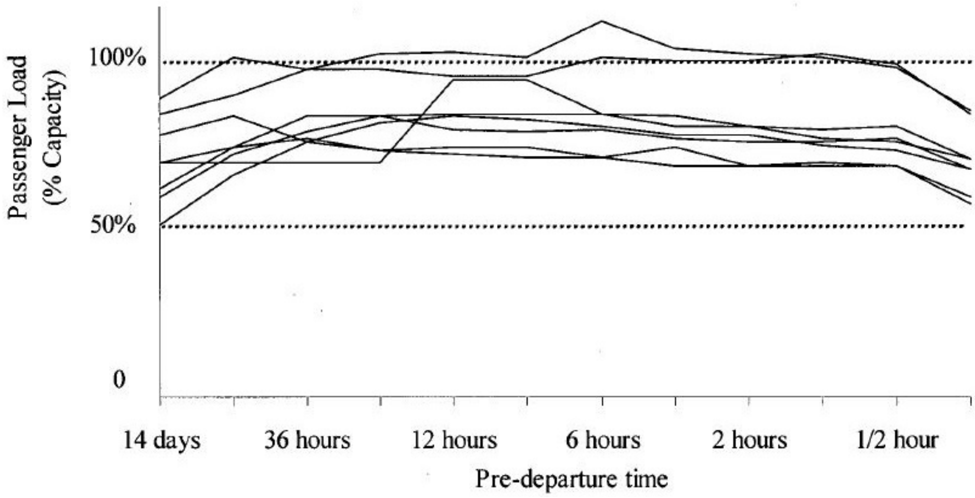



Figure 4.3: The passenger load fluctuations for eight consecutive instances of the same flight as observed by Goto (1999).

## 4.3 Synthetic data generation

This section explains the main steps followed to generate the desired datasets for the numerical example. The goal is to highlight the majority of the assumptions made regarding the flight booking behaviour of passengers, since these assumptions are expected to have a significant impact on the model's reliability and performance.

As stated earlier, two datasets are required - a training and a testing dataset. Both datasets are obtained by creating a single dataset consisting of the daily flight observations for a *five*-year period. The first three years of this dataset constitutes the training dataset, while the remaining two years will be used for testing purposes. Table 4.3 lists a few flight instances extracted from the dataset generated.

Table 4.3: A few flight instance examples from the dataset generated.

| **Number** | **Passenger load** | | | | | **Time of travel** | | | **Meal demand**[a] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | $Z_4^*$ | $Z_3^*$ | $Z_2^*$ | $Z_1^*$ | $Z_0^*$ | Day | Month | Year | C | B | FP | V |
| 52 | 40 | 56 | 65 | 66 | 65 | Wed. | Feb. | 2015 | 37 | 16 | 11 | 1 |
| 386 | 39 | 60 | 75 | 78 | 80 | Mon. | Jan. | 2016 | 38 | 28 | 12 | 2 |
| 970 | 62 | 85 | 100 | 104 | 100 | Thu. | Aug | 2017 | 58 | 23 | 14 | 5 |
| 1062 | 58 | 78 | 85 | 84 | 82 | Fri. | Nov. | 2017 | 45 | 32 | 5 | 0 |
| 1624 | 70 | 91 | 101 | 105 | 98 | Sun. | Jun. | 2019 | 54 | 37 | 4 | 3 |

[a]C = chicken, B = beef, FP = fruit platter and V = vegan.

The data generated for each flight instance includes the passenger load at each time point $t \in \{4, 3, 2, 1, 0\}$, the DOW, month and year of the flight's departure and the final meal demand. The meal demand represents the primary demand for each in-flight meal option offered. In practice, the primary demand will be challenging to obtain. A suggested approach is to estimate the demand using the pre-booked meal records of historical flights.

The first step to generate the dataset mentioned above is to create a separate record for each flight instance that departs during the five-year period. This includes assigning a DOW, month and year to each flight instance in a sequential order. The passenger load at each time point $t$ is obtained as follows:

**Passenger load at $t$=4:** To obtain $Z_4^*$, the *base* passenger load at time $t = 4$ is randomly drawn from a triangular distribution for each flight instance. The motivation behind the use of a triangular distribution will be discussed shortly. The base passenger load represents the expected passenger load for a flight departing on a Monday morning in January during the first year. However, as discussed in the previous section, the DOW, month and year of departure have a significant impact on the flight's expected passenger load. Consequently, the base passenger load must be transformed accordingly for each flight instance:

1. *Add monthly growth:* In the previous section, it was found that the average yearly growth in domestic passenger traffic ranged between 1.3% to 3.8%. This numerical example will assume a worst-case fixed annual growth of 4%, which results in a monthly growth of 0.327%, assuming that compounding is relevant. The monthly growth factor is then used to add a positive trend to the base passenger load.
2. *Add monthly and daily seasonality:* After the monthly growth has been added, the passenger load is multiplied with the flight instance's respective monthly seasonality factor. These seasonality factors are measured with respect to the base month, January, and calculated using the average passenger traffic for each month as given in Figure 4.2. The resulting pattern of the monthly seasonality factors is visible in Figure 4.4, which will be used to visually validate the data generated.

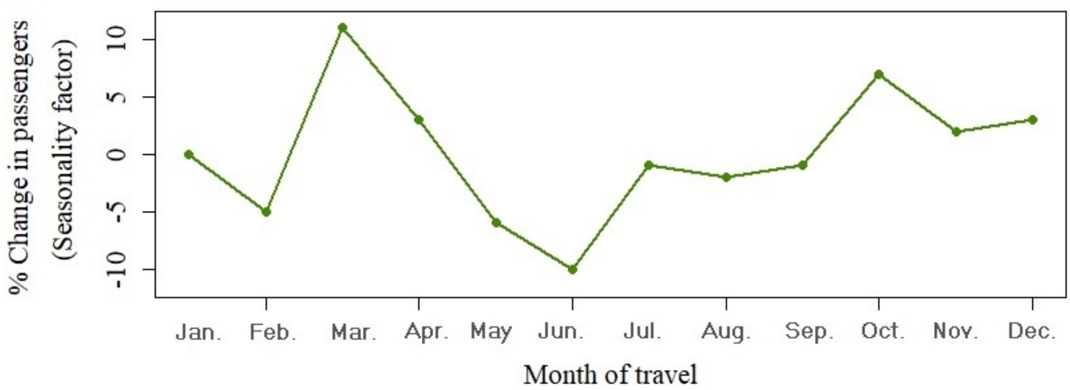



Figure 4.4: The average percentage change (seasonality factor) in the passenger traffic for domestic flights in South Africa when compared with the month of January (adapted from Airport Company South Africa (2020)).

A similar process is repeated to add DOW seasonality. The DOW seasonality factors used were assumed while knowing that demand is high on Sundays, Thursdays and usually peaks on Saturdays.

**Passenger load at $t \in \{3, 2, 1\}$:** The passenger load at time point $t \in \{3, 2, 1\}$ is calculated by simply adding a *difference* to the passenger load at time point *t-1*. The respective *differences* are also randomly drawn from a triangular distribution and multiplied with the DOW and monthly seasonality factors. Since the data is generated for a domestic flight, the triangular distributions' modes were chosen to be greater than zero to ensure that the passenger load exhibits moderate to excessive growth as expected. Figure 4.5 depicts the distribution of the differences obtained in the dataset for each forecasting interval.

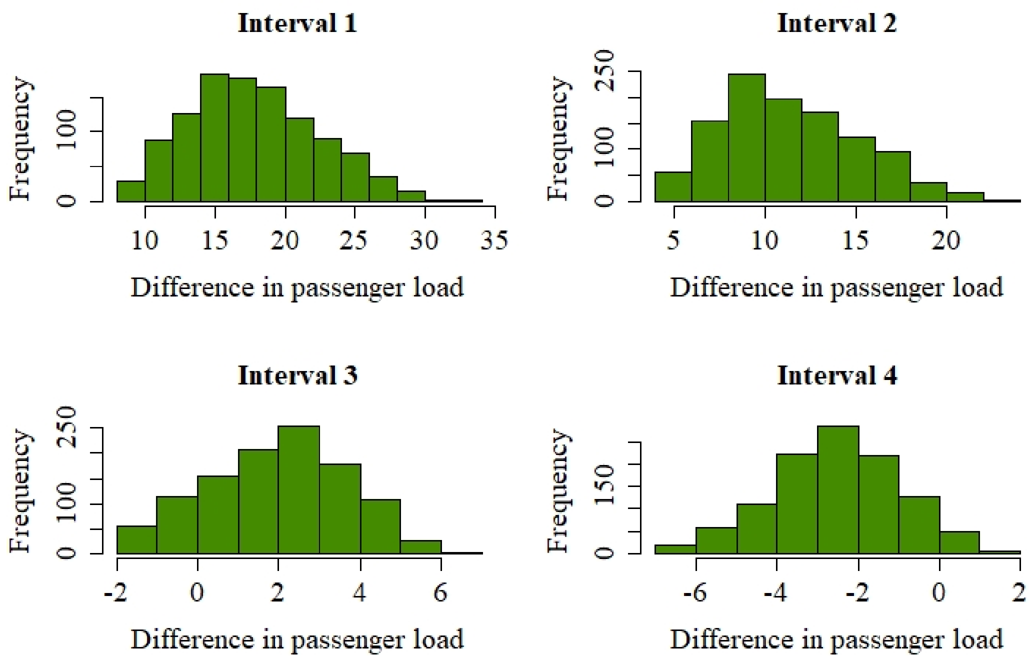



Figure 4.5: The distribution of differences for each forecasting interval in the numberical example.

**Passenger load at *t=0*:** Similar to the above, the final passenger load is obtained by adding a *difference* to the passenger load at $t = 1$, where the *difference* is sampled from a triangular distribution and multiplied with the seasonality factors. For this interval, however, the parameters of the triangular distributions will differ for each flight record to ensure a dependent relationship between $Z_1^*$ and the *differences* obtainable. Furthermore, since it is known that the passenger load is expected to decline shortly before departure, the modes of the triangular distributions should be negative. It is apparent from Figure 4.5 that the mean decline during the final interval in the dataset generated is roughly three passengers. The final passenger load for each flight instance in the dataset is shown in the top graph in Figure 4.6.

Figure 4.6 illustrates the decomposition of the dataset as a multiplicative time series. The desired monthly seasonality pattern is clearly visible and corresponds with the pattern in Figure 4.4. In addition, a strong positive trend that represents the growth in passenger traffic is also observed. These observations

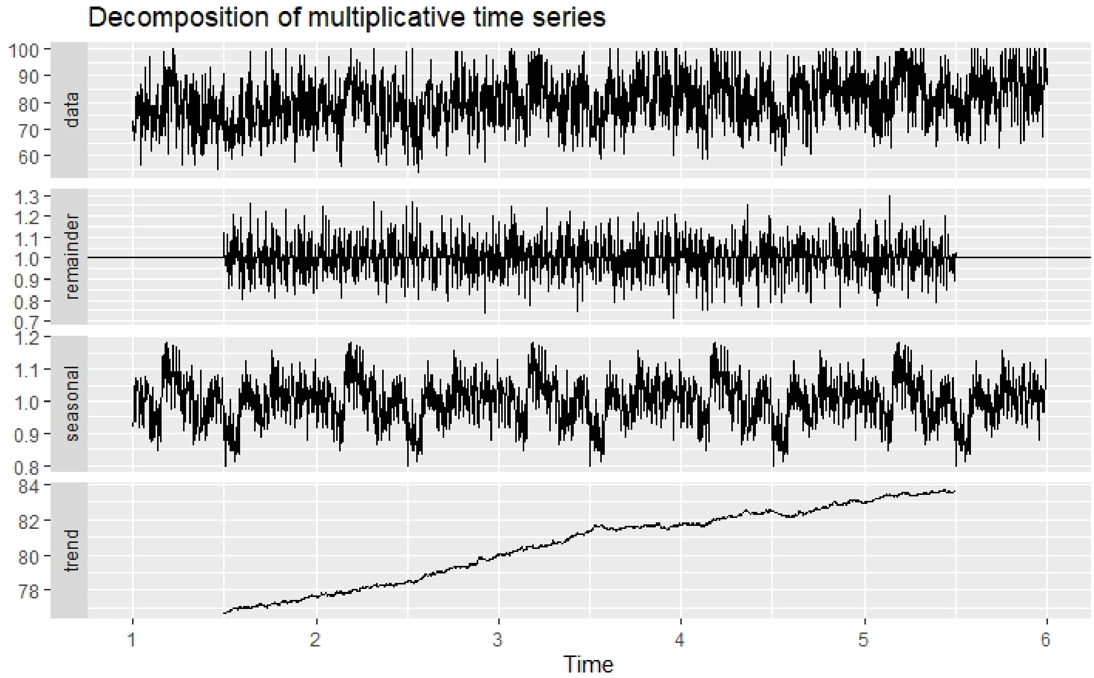



Figure 4.6: The decomposition of the final passenger loads in the dataset, which highlights the existince of the desired seasonality pattern and growth trend.

validate the authenticity of the dataset generated to some extent. Unfortunately, there is a slight pattern visible in the remainder plot that results from multiplying the *differences* with *fixed* seasonality factors. This simply means that the model's *randomness* is sharpened with the seasonality factors.

Regrettably, limited information is available regarding the expected distribution of the passenger load differences. The distributions observed by Goto et al. (2004), as shown in Figure 2.5, appear to be slightly right-skewed normal distributions when $t > 0$. Furthermore, as stated earlier, Vaagen et al. (2011) found that a higher accuracy can be obtained by maintaining the distribution of the set of differences instead of fitting a normal distribution to the set of differences when construction the Transition Probability Matrix (TPM)s. This further motivates the speculation that the expected distribution of the passenger load differences is not perfectly normally distributed, but slightly skewed. For this reason, a triangular distribution was used in the data generation process because of the ease with which the tails of the distribution can be manipulated using its *min*, *mode* and *max* parameters to model the desired skewed distribution.

**Meal demand:** The primary demand for each meal option is generated by fitting a normal distribution to the respective first-choice probabilities provided in Table 4.1. In random order, the demand for each meal type is obtained by randomly selecting a probability value from the meal option's first-choice probability distribution. The probability is then multiplied with the flight instance's final passenger load obtained previously and rounded upwards to obtain an integer value. If, at any given point, the sum of the meal order quantities exceeds the final passenger load, the current meal type's order is reduced to the maximum acceptable quantity. The process is repeated until the sum of the meal order quantities sampled is equal to the desired final passenger load. The resulting frequency distribution of each meal type's primary demand in the dataset generated is graphed in Figure. 4.7

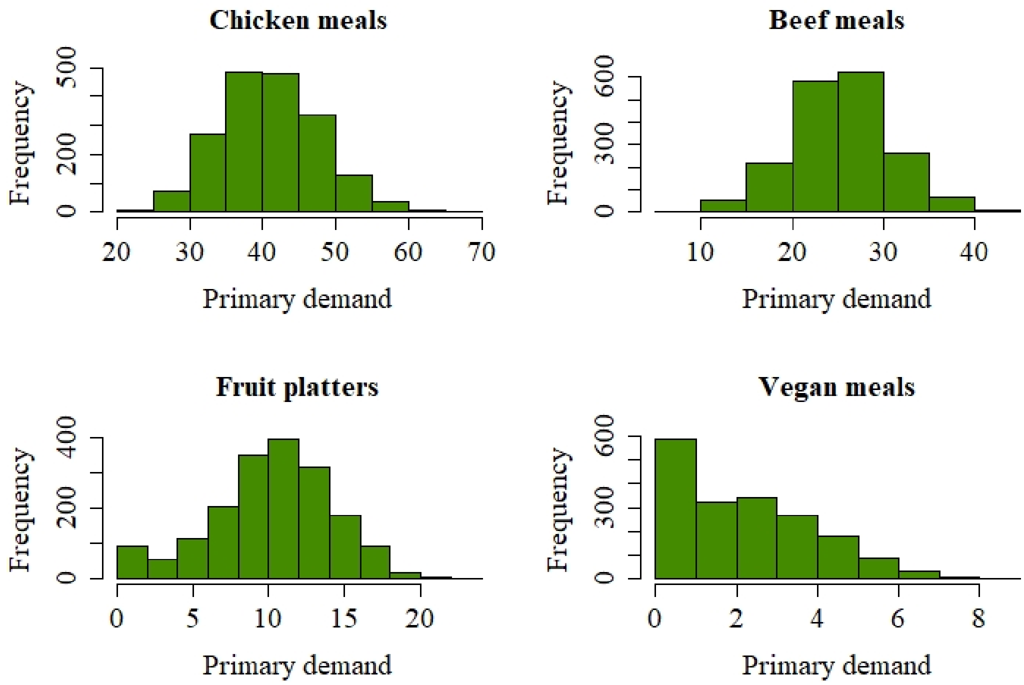



Figure 4.7: The distribution of the primary demand for each meal option offered in the numberical example.

When utilising the first-choice probabilities provided in Table 4.1, the average demand probability for chicken meals is 50.8%, followed by 32.2% for beef meals, 14.0% for fruit platters and 3.0% for vegan meals. Almost indistinguishable results are obtained when analysing the dataset generated: the average demand probability for chicken meals is 51.5%, followed by 32.1% for beef meals, 13.2% for fruit platters and 3.2% for vegan meals.

## 4.4 Concluding remarks

This chapter summarised the numerical example that will be used throughout the remainder of this dissertation to illustrate the application of the model. Furthermore, a brief overview of the processes followed to generate the required dataset is also given. The dataset generated is based on the findings obtained from analysing the flight booking behaviour of passengers in the literature. These findings included seasonality patterns, growth trends and the inherent flight booking behaviour of passengers.

# Chapter 5

# Model solution

The model formulated in Chapter 3 consists of an inventory decision-making model (Part A) and a forecasting model (Part B). The focus of this chapter is to develop the model's solution, which will be based on the numerical study discussed in Chapter 4. This solution includes the transformation of the inventory decision-making model, currently formulated as a *stochastic* Mixed-Integer Programming (MIP) model, into its *deterministic* equivalent. Furthermore, the solution will contain the derivation of the modelled Transition Probability Matrix (TPM) for each forecasting interval.

## 5.1 Part A: Inventory decision-making model

Recall that the inventory decision-making model is currently formulated as a MIP model with fixed recourse and two stochastic variables − the set of first-choice probabilities $\widetilde{\mathbf{q}}$ and the final passenger load of the flight under consideration $\widetilde{Z}_0$. As a consequence, the model must be converted into its deterministic equivalent in order to obtain an exact solution using standard optimisation software. This conversion will be achieved using recourse programming, where each of the stochastic variables is decomposed into a set of potential realisations and accompanying probabilities.

This section discusses the transformation of the stochastic model, described by equations (3.1) to (3.12), into its deterministic equivalent. To illustrate this transformation, consider the stochastic model's objective function given in (3.1).

$$\min Z = w^{PSL} E_{\widetilde{\mathbf{q}},\widetilde{Z}_0}\left[\Delta^{PSL}(\widetilde{\mathbf{q}},\widetilde{Z}_0)\right] + w^{Meals} E_{\widetilde{\mathbf{q}},\widetilde{Z}_0}\left[\Delta^{Meals}(\widetilde{\mathbf{q}},\widetilde{Z}_0)\right] \tag{3.1}$$

To convert (3.1) into its deterministic form, the stochastic passenger load $\widetilde{Z}_0$ is decomposed into a set of realisations that correspond with $\mathbb{C}$, the state space of the flight's passenger load with probability distribution $\pi_0$. Afterwards, $\widetilde{Z}_0$ is replaced with $Z_0^v$ to denote the the $v^{th}$ realisation of the random variable with discreet distribution $\{(Z_0^v = \mathbb{C}(v), p^v = \pi_0(v)),\ v \in \boldsymbol{V} = \{1, ..., |\mathbb{C}|\}\}$. Similarly, $\widetilde{\mathbf{q}}$ is replaced with $\mathbf{q}^s$, to represent the set of first-choice probabilities for realisation $s \in \mathbf{S}$ of the stochastic variable with discreet distribution $\{(\mathbf{q}^s, p^s = 0.2),\ s \in \mathbf{S} = \{1, ..., 5\}\}$. For the numerical study, the set $\mathbf{S}$ consists of the five sets of first-choice probabilities provide in Table 4.1. Each set represents a possible realisation with a probability of occurrence of 20%.

Recall that stochastic objective function minimises the weighted sum of the expected deviations from the 100% Passenger Satisfaction Level (PSL) target and the zero-waste target. The expected deviations can be approximated by summing together the deviations obtained for each individual realisation $q^s$ and $Z_0^v$ combination pair in proportion to their respective probability of occurrences, $p^s$ and $p^v$, for each $s \in \mathbf{S}$ and $v \in \boldsymbol{V}$. The resulting *deterministic* objective function is given in (5.1).

$$\min Z = \sum_{s\in\mathbf{S}} p^s \sum_{v\in\boldsymbol{V}} p^v \left(w^{PSL}\Delta^{PSL}(\mathbf{q}^s, Z_0^v) + \; w^{Meals}\Delta^{Meals}(\mathbf{q}^s, Z_0^v)\right) \tag{5.1}$$

The deviation variables, represented with $\Delta^{Meals}(\mathbf{q}^s, Z_0^v)$ and $\Delta^{PSL}(\mathbf{q}^s, Z_0^v)$, must be calculated for each unique realisation $q^s$ and $Z_0^v$ combination pair. This is achieved with (5.2) and (5.3), the deterministic equivalents of constraints (3.2) and (3.3).

$$\Delta^{Meals}(\mathbf{q}^s, Z_0^v) = \sum_{i\in\mathbf{I}} \delta_i^+(\mathbf{q}^s, Z_0^v), \qquad \forall \qquad s \in \mathbf{S},\; v \in \boldsymbol{V} \tag{5.2}$$

$$\Delta^{PSL}(\mathbf{q}^s, Z_0^v) = 100\left(1 - \frac{\sum_{i\in\mathbf{I}}\left(y_i(\mathbf{q}^s, Z_0^v) + \sum_{j\in\mathbf{I},\, j\neq i}\hat{\alpha_{ij}}\, z_{ij}(\mathbf{q}^s, Z_0^v)\right)}{\sum_{i\in\mathbf{I}} d_i(\mathbf{q}^s, Z_0^v)}\right), \qquad \forall \qquad s \in \mathbf{S},\; v \in \boldsymbol{V} \tag{5.3}$$

Note that (5.2) and (5.3) are not written in a goal-programming format as these equations are simplified. The simplification is based on the fact that the goals of the two objectives – a 100% PSL and zero waste – represent the highest performance possible and cannot be improved any further.

The remaining constraints of the deterministic model are given in (5.4) to (5.13) below. Note that $\mathbf{I} = \{1, 2, 3, 4\}$ corresponds with the {chicken, beef, fruit platter, vegan} meals options in the numerical example.

$$100 - \Delta^{PSL}(\mathbf{q}^s, Z_0^v) \geq p^{min}, \qquad \forall \qquad s \in \mathbf{S},\; v \in \boldsymbol{V} \tag{5.4}$$

$$d_i(\mathbf{q}^s, Z_0^v) \approx \lceil q_i^s\, Z_0^v \rceil, \qquad \forall \qquad i \in \mathbf{I},\; s \in \mathbf{S},\; v \in \boldsymbol{V} \tag{5.5}$$

$$d_i(\mathbf{q}^s, Z_0^v) = y_i(\mathbf{q}^s, Z_0^v) + \sum_{j\in I\, j\neq i} z_{ji}(\mathbf{q}^s, Z_0^v) + \delta_i^-(\mathbf{q}^s, Z_0^v), \qquad \forall \quad i \in \mathbf{I},\; s \in \mathbf{S},\; v \in \boldsymbol{V} \tag{5.6}$$

$$z_{ij}(\mathbf{q}^s, Z_0^v) \leq \hat{\alpha_{ij}}[d_j(\mathbf{q}^s, Z_0^v) - y_j(\mathbf{q}^s, Z_0^v)], \quad \forall \quad i, j \in \mathbf{I},\; s \in \mathbf{S},\; v \in \boldsymbol{V} \tag{5.7}$$

$$x_i = y_i(\mathbf{q}^s, Z_0^v) + \sum_{j\in\mathbf{I},\, j\neq i} z_{ij}(\mathbf{q}^s, Z_0^v) + \delta_i^+(\mathbf{q}^s, Z_0^v), \qquad \forall \quad i \in \mathbf{I},\; s \in \mathbf{S},\; v \in \boldsymbol{V} \tag{5.8}$$

$$x_i,\ y_i(\mathbf{q}^s, Z_0^v),\ d_i(\mathbf{q}^s, Z_0^v) \geq 0 \text{ and integer}, \quad \forall \quad s \in \mathbf{S},\ v \in \boldsymbol{V},\ i \in \mathbf{I} \tag{5.9}$$

$$\delta_i^+(\mathbf{q}^s, Z_0^v),\ \delta_i^-(\mathbf{q}^s, Z_0^v) \geq 0 \text{ and integer}, \quad \forall \quad s \in \mathbf{S},\ v \in \boldsymbol{V},\ i \in \mathbf{I} \tag{5.10}$$

$$z_{ij}(\mathbf{q}^s, Z_0^v) \geq 0 \text{ and integer}, \quad \forall \quad s \in \mathbf{S},\ v \in \boldsymbol{V},\ i, j \in \mathbf{I} \tag{5.11}$$

$$\Delta^{Meals}(\mathbf{q}^s, Z_0^v) \geq 0, \quad \forall \quad s \in \mathbf{S},\ v \in \boldsymbol{V} \tag{5.12}$$

$$\Delta^{PSL}(\mathbf{q}^s, Z_0^v) \geq 0, \quad \forall \quad s \in \mathbf{S},\ v \in \boldsymbol{V} \tag{5.13}$$

## 5.2 Part B: Forecasting model

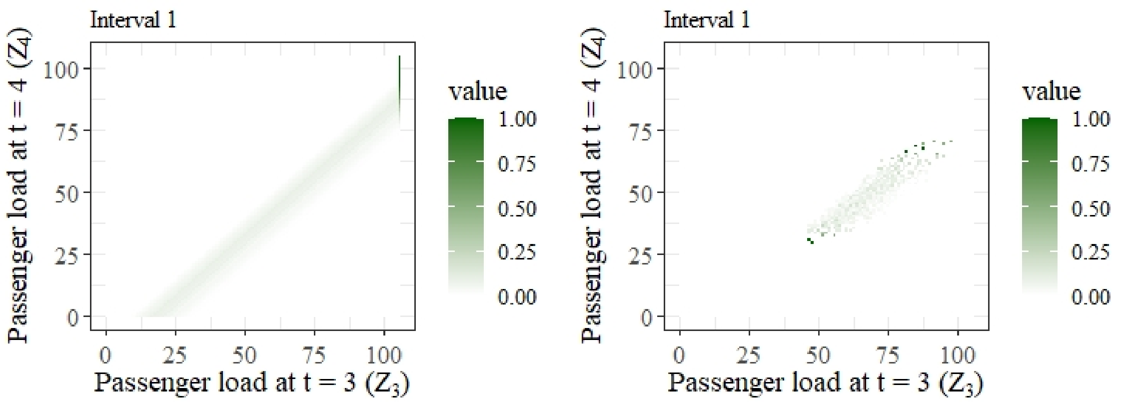
In this section, the modelled TPMs for the time-inhomogeneous Markov Chain are created using the training dataset. The TPMs are the most vital components of the forecasting model as they directly impact the accuracy of the forecast. Accordingly, a small-scale sensitivity analysis is carried out to determine the most efficient weighting factor needed to calculate the modelled TPMs.

### 5.2.1 The transition probability matrices

The dataset contains the passenger load of various flight instances at five fixed time points, thereby imposing four forecasting intervals ($N = 4$) and a need for four modelled TPMs. Key highlights of the development of these TPMs are discussed below.

#### A modelled transition probability matrix

As expressed with equation (3.17), a modelled TPM is the weighted combination of the Transition Probability Matrix using Differences (TPM-D) and the Transition Probability Matrix using the Absolute Passenger Loads (TPM-APL). Figure 5.1 depicts the latter two TPMs for the first forecasting interval in the dataset that stretches from 72 hours (*t=4*) to 15 hours (*t=3*) before the departure of the flight. There is a clear contrast between these two matrices. Note that, unless stated otherwise, the flight modelled will depart on a Wednesday morning in August 2018.




(a) Differences

(b) Absolute passenger load

Figure 5.1: The TPM using differences ($\bar{P}_{4,3}$) and the TPM using the absolute passenger load ($\bar{\bar{P}}_{4,3}$) for the first forecasting interval in the training dataset.

As explained in Section 2.4 and Section 3.2, the TPM-D utilises the probability distribution of the set of differences observed ($\hat{y}_{t,t-1}$) to estimate the change in the passenger load for a specific interval, independent of the current passenger load when $t > 0$. Consequently, each row in the matrix consists of an identical but shifted probability density function. This explains the uniform diagonal line observed in Figure 5.1a. The dark green vertical line at $Z_3 = 105$ is caused by the upper bound restriction of the passenger load and will be discussed shortly.

The TPM-APL, shown in Figure 5.1b, consists of an irregular pattern of non-zero transition probabilities. This irregular pattern is expected because these probabilities are based on the respective transition's *random* number of occurrences in the training dataset. The training dataset does not contain any $Z_4$ observations smaller than 32 or larger than 72 as the irregular pattern does not exceed these boundaries. This indicates that, based solely on the TPM-APL depicted, there is zero probability that the passenger load at 72 hours before departure ($Z_4$) will be lower than 32 passengers or exceed 72 passengers.

Figure 5.2 depicts the modelled TPM for the first forecasting interval. This modelled TPM is obtained by combining the matrices shown in Figure 5.1 using equation (3.17) with an equal weighting factor ($\phi = 0.5$).

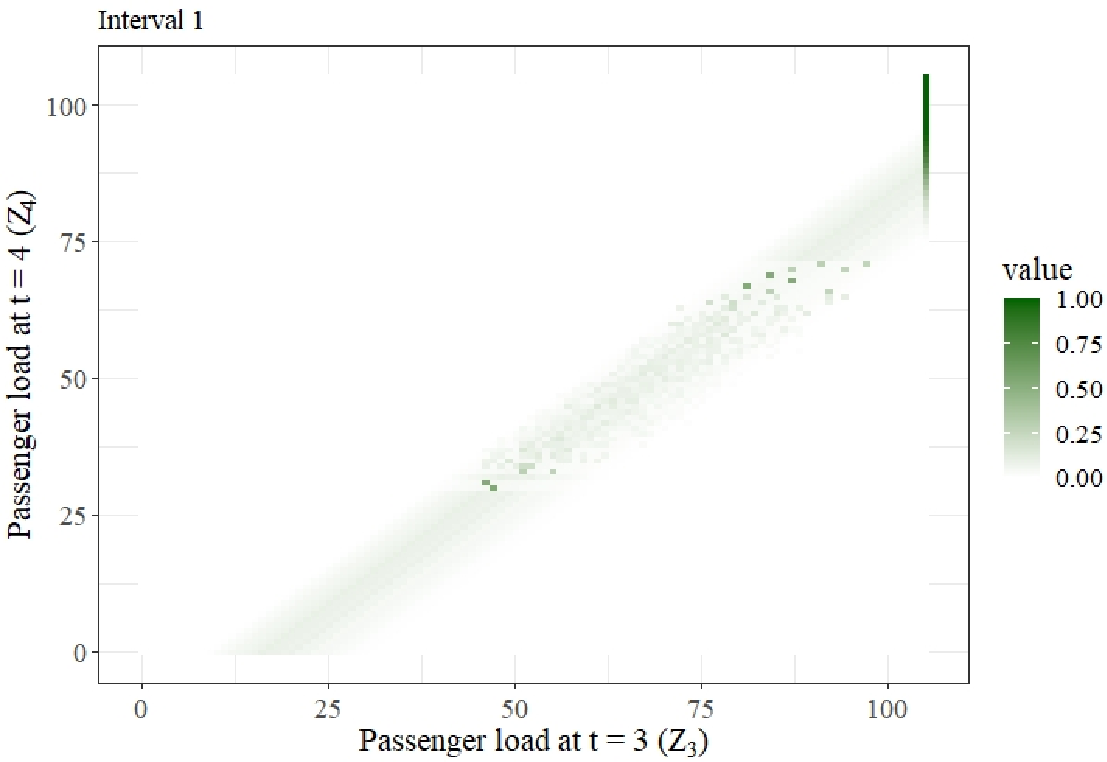



Figure 5.2: The modelled TPM for the first forecasting interval ($P_{4,3}$) when $\phi = 0.5$.

Recall that a row in a TPM represents the flight's current passenger load ($Z_t$), while the column represents the future passenger load ($Z_{t-1}$). Figure 5.3 demonstrates the transition probabilities for a (partial) *single* row in each of the TPMs discussed above. The row depicted corresponds with $Z_4 = 56$, meaning that the three graphs represent, respectively, $\bar{P}^{56,i}_{4,3}$, $\bar{\bar{P}}^{56,i}_{4,3}$ and $P^{56,i}_{4,3}$ for each $i \in \{54, ..., 105\} \subset \mathbb{C}$.

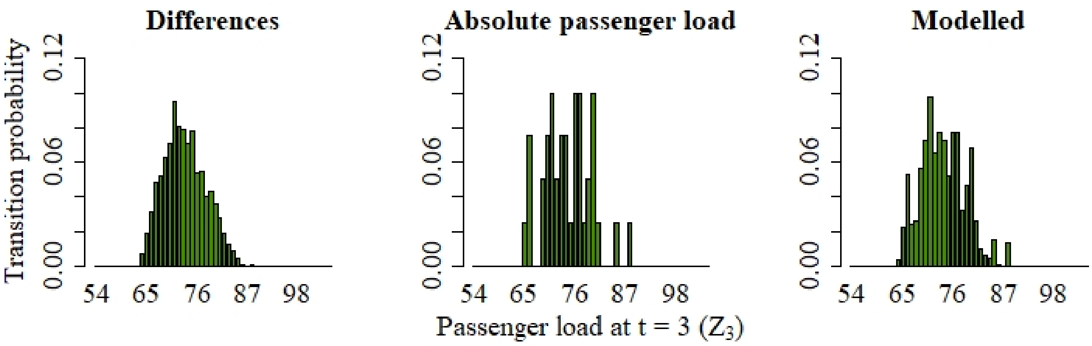



Figure 5.3: The transition probability distribution obtained from the row in each of the first interval's TPMs that corresponds with $Z_4 = 56$, given that $\phi = 0.5$.

Figure 5.3 illustrates the combination process followed to generate a row in the modelled TPM using equation (3.17). It also demonstrates its purpose – to fill the missing *absolute passenger load* transition probabilities and to smooth-out the probability distribution.

$$P^{i,j}_{t,t-1} = \begin{cases} (1-\phi)\cdot \bar{P}^{i,j}_{t,t-1} \; + \; \phi \cdot \bar{\bar{P}}^{i,j}_{t,t-1}, & if \quad \sum_{j\in\mathbb{C}} \bar{\bar{P}}^{i,j}_{t,t-1} \neq 0, \\ \\ \bar{P}^{i,j}_{t,t-1} & otherwise. \end{cases} \tag{3.17}$$

As stated, the probability distributions in each row of the TPM-D are identical but shifted, while the probability distributions in each row of the TPM-APL will vary drastically. In fact, the probability distribution in some rows of the TPM-APL might be non-existent. This occurs when no transition in an entire row of the matrices was captured in the training dataset, such that $\sum_{j\in\mathbb{C}} \bar{\bar{P}}^{i,j}_{t,t-1} = 0$ . When this happens, the respective row in the modelled TPM is set equal to the same row in the TPM-D. It is intuitive from Figure 5.1b that this will happen with certainty when $Z_4 < 32$ or $72 < Z_4$ because transitions were only observed when $Z_4 \in \{32, ..., 72\}$. Figure 5.4 provides an example by depicting the row that corresponds with $Z_4 = 80$.

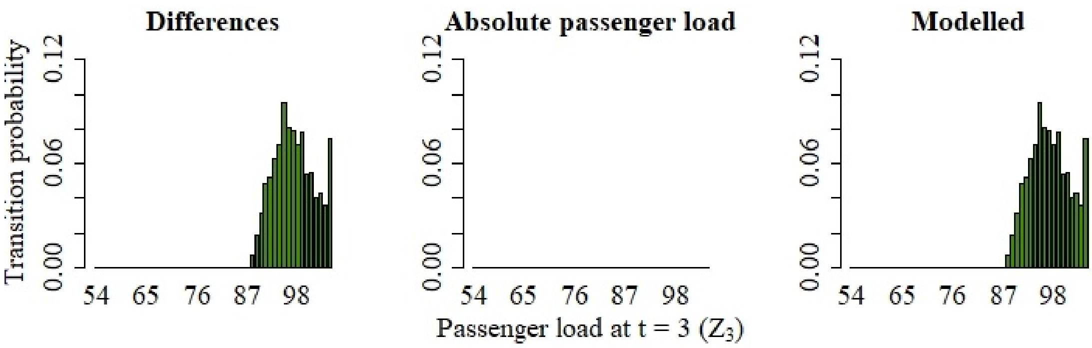



Figure 5.4: The transition probability distributions obtained from the row in each of the first interval's TPMs that corresponds with $Z_4 = 80$, given that $\phi = 0.5$.

The training data set did not contain any observations with a passenger load of 80 passengers at time $t = 4$. Consequently, the row that corresponds with $Z_4 = 80$ in the TPM-APL will not contain any non-zero transition probabilities as seen in Figure 5.4. Accordingly, the particular row in the modelled TPM is set equal to the same row in the TPM-D because $\sum_{j \in |\mathbb{C}|} \bar{\bar{P}}_{4,3}^{80,j}$ is equal to zero. This is an important detail of equation (3.17) that was not discussed previously.

Notice that the probability distribution of the TPM-D in Figure 5.4 is, as expected, identical but shifted when compared with its counterpart in Figure 5.3. However, since $Z_3$ is not allowed to exceed 105 passengers, the exceeding tail of the distribution is cumulated at $Z_3 = 105$ as enforced by equation (3.19). This results in the noticeable vertical line at $Z_3 = 105$ in the respective TPMs.

### The set of modelled transition probability matrices

The modelled TPM for each forecasting interval is shown in Figure 5.5. These four matrices constitute the time-inhomogeneous Markov Chain forecasting model. Evidently, the modelled TPMs are unique when compared with one another and thereby validates the assumption of time-inhomogeneity between the forecasting intervals.

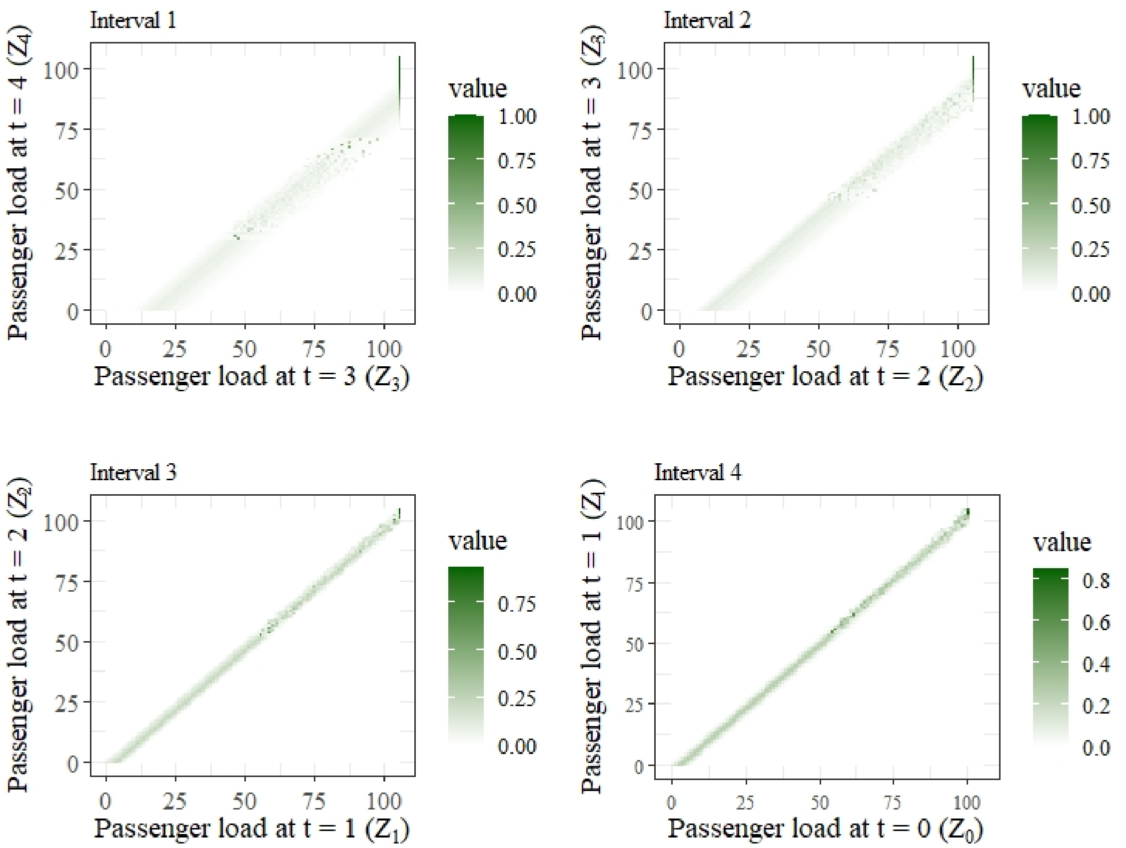



Figure 5.5: The set of modelled TPMs (with $\phi = 0.5$) for a flight instance that will depart on a Wednesday morning during the August month of year 4.

A noticeable difference between the modelled TPMs is the width of the diagonal line depicting the non-zero transition probabilities. The width of the line represents the degree of uncertainty within the respective forecasting interval. Consequently, the line's width is expected to correspond with the spread of the passenger load differences observed during the interval. From Figure 4.5, the spread of differences for the first interval is roughly 30 passengers, and eight passengers for the final interval. This explains why the width of the first interval's diagonal line is almost three times thicker than the final interval's diagonal line.

A second difference that is not immediately apparent is the x-intercept of the diagonal line. This factor relates to the expected change in the passenger load during the respective interval; During the first interval, the passenger load will increase with at least 15 passengers, while only a slight change is expected during the final forecasting interval.

Recall that the modelled TPM for the *final* forecasting interval (interval 4) has two major differences. Firstly, its TPM-D is calculated using a multiple regression model, and secondly, overbooking is not allowed. Thus, $Z_0$ cannot exceed 100 passengers. For this reason, the modelled TPM for the *final* interval is discussed below.

#### The modelled transition probability matrix for the *final* interval

As stated earlier, the probability distributions in each row of a TPM-D are identical but shifted. This statement does not hold for the final forecasting interval. This is because the probability distribution in each row of the final interval's TPM-D is calculated using a multiple regression model. This is done to account for the dependence between $Z_1$ and the change observed in the passenger load. Consequently, the probability distribution in each row of the particular matrix will vary because each row in the matrix represents a different realisation of $Z_1$. This can be observed in Figure 5.6, where the probability distribution of three different rows of the final interval's TPM-D are compared with one another.

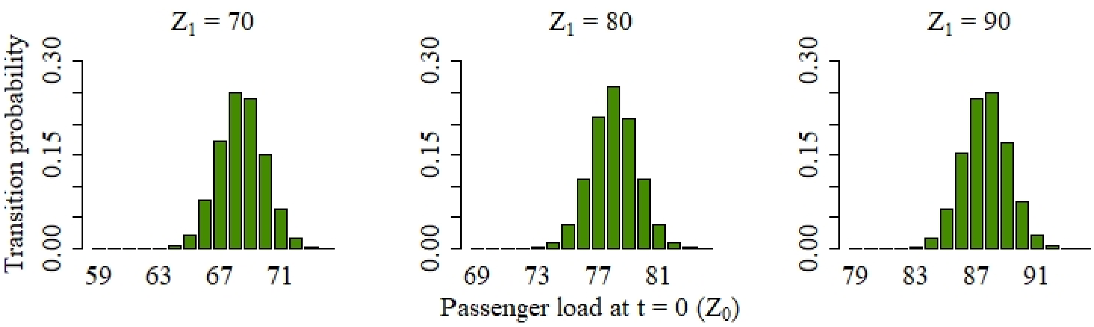



Figure 5.6: The varying probability distributions of three different rows in the final forecasting interval's TPM-D.

The regression model also includes three additional covariates - the Day of Week (DOW), month and year of the flight's departure. Accordingly, the TPM-D will be unique for each day, month and year combination. Thus, the final interval's TPM-D (and the resulting modelled TPM) will vary for flights with different departure dates.

This is validated in Figure 5.7, which compares a single row obtained from three separate TPM-Ds that vary according to the modelled flights' departure DOW, month and year as shown. The rows depicted correspond with $Z_1 = 95$.

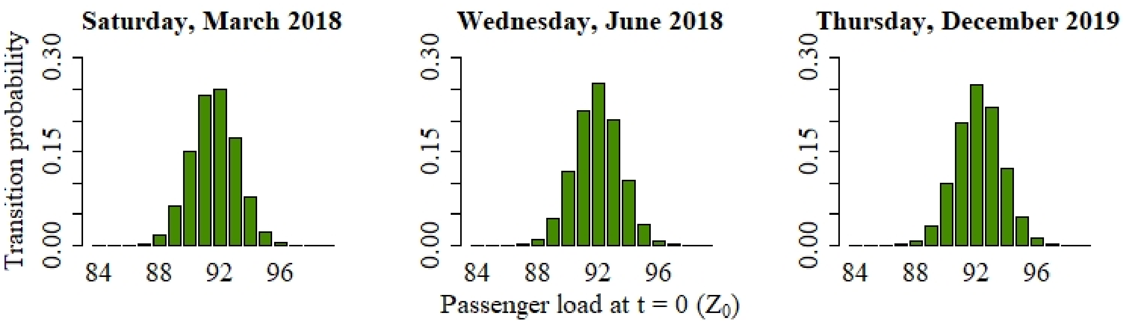
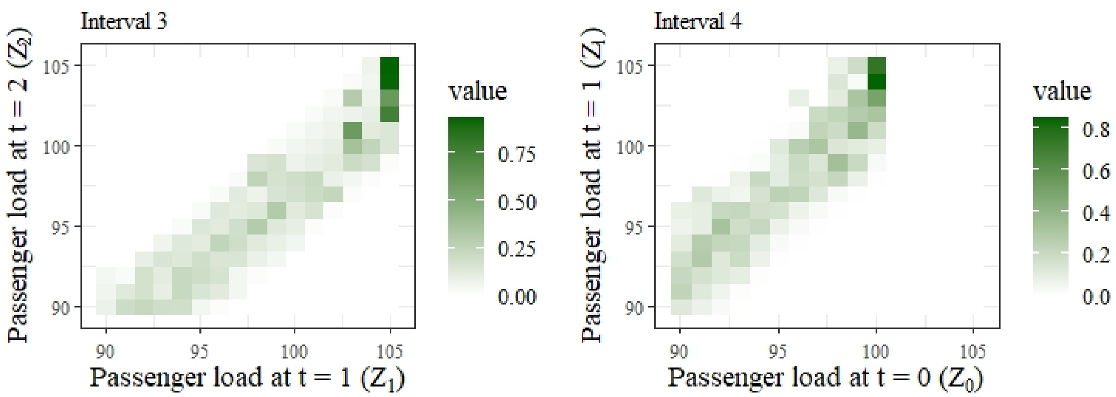



Figure 5.7: The varying probability distributions of the same row obtained from three seperate final TPM-D that differ due to the flights' departure dates.

Although the probability distributions shown in Figure 5.6 and Figure 5.7 do not vary drastically, their impact on the accuracy of the passenger load forecast is expected to be significant based on the literature reviewed.

Lastly, an overbooking of 5 passengers is allowed in the numerical example to maximise passenger revenue. However, due to the 100 passenger seat capacity of the aircraft, overbooking is not allowed during the final forecasting interval. This means that, while $Z_t \in \{4, 3, 2, 1\}$ is restricted by 105 passengers, $Z_0$ is restricted by 100 passengers. The effect of this on the modelled TPMs is shown in Figure 5.8.




(a) Allows overbooking

(b) Prohibits overbooking

Figure 5.8: An extraction from the third and final forecasting interval's modelled TPMs to demonstrate the effect of overbooking on the transition probabilities.

Figure 5.8a shows an extraction of the modelled TPM for the third interval. This TPM contains non-zero transition probabilities for some of the $Z_1$ realisations greater than the flight's seat capacity to allow for the occurrence of overbooking. The same is not true for the modelled TPM of the final interval shown in Figure 5.8b, since overbooking is not allowed.

### 5.2.2 Sensitivity analysis

The weighting factor $\phi$ is an important component in the development of the modelled TPMs, since it determines the relative weight between the TPM-D and the TPM-APL, where applicable. A high $\phi$ value tips the scale in favour of the TPM-APL.

In this section, a small-scale sensitivity analysis is conducted to determine the most favourable $\phi$ value. The sensitivity analysis will investigate the impact of $\phi$ on the accuracy of the forecasting model developed. The accuracy will be measured using the Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) of the forecast. Thus, the $\phi$ value that corresponds with the lowest MAE and MAPE values obtained will be deemed the most favourable weighting factor. The MAE and MAPE for a specific $\phi$ value will be calculated using (5.14) and (5.15).

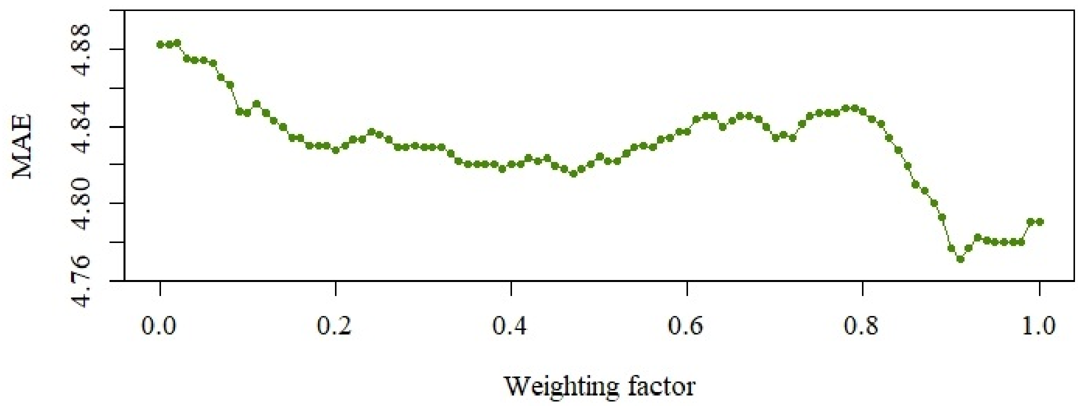
$$MAE(\phi) = \frac{1}{|\dot{\boldsymbol{\gamma}}|} \cdot \sum_{\gamma \in \dot{\boldsymbol{\gamma}}} |E[Z_0^{\gamma}(\phi)] - Z_0^{\gamma *}| \tag{5.14}$$

$$MAPE(\phi) = \frac{100}{|\dot{\boldsymbol{\gamma}}|} \cdot \sum_{\gamma \in \dot{\boldsymbol{\gamma}}} \frac{|E[Z_0^{\gamma}(\phi)] - Z_0^{\gamma *}|}{Z_0^{\gamma *}} \tag{5.15}$$

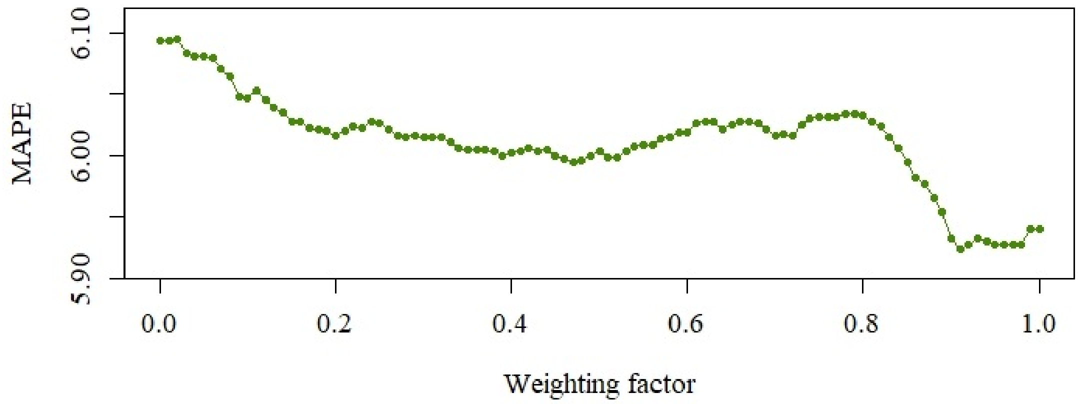
The equations given require the use of equation (3.16) to forecast $E[Z_0^{\gamma}(\phi)]$, the *expected* final passenger load for each flight instance $\gamma$ in the *testing* dataset $\dot{\boldsymbol{\gamma}}$. These equations use the set of modelled TPMs that are generated using the relevant $\phi$ value. However, recall that each flight observation will require a unique modelled TPM for the *final* forecasting interval as the flight observations differ in terms of their DOW, month and year of departure. Consequently, a unique modelled TPM for the final interval must be generated for each flight instance $\gamma$ in the *testing* dataset $\dot{\boldsymbol{\gamma}}$. Lastly, $Z_0^{\gamma *}$ represents flight instance $\gamma$'s true passenger load at $t = 0$ and is obtained directly from the dataset.

The above-mentioned process was followed to calculate the MAE and MAPE for each $\phi$ value in the set $\{0, 0.01, ..., 0.99, 1\}$. The results are plotted in Figure 5.9. Evidently, higher values of $\phi$ are generally more favourable. This conclusion is in-line with the work of Goto et al. (2004) and Van Ostaijen et al. (2017). The *most* favourable weighting factor, represented with $\phi^*$, is equal to 0.91 as it resulted in the lowest MAE and MAPE error measures. Consequently, the modelled TPMs generated earlier with $\phi = 0.5$ are now known to be suboptimal, and a new set of modelled TPMs must be generated with $\phi^* = 0.91$ to maximise the accuracy of the forecasting model. The new modelled TPMs and the output of the regression model are presented in Appendix A.

Note that the *testing* dataset had to be used to conduct the sensitivity analysis. If the *training* dataset was used, the most favourable weighting factor value would have approximated a value of one so that the modelled TPM would have been identical to the TPM-APL. This is because the TPM-APL is based on the actual transitions observed in the training dataset, meaning that it will maximise the accuracy of the forecasting model if applied in isolation to the exact same dataset. When using a different dataset, the value of the TPM-D becomes more prominent due to the occurrence of transitions that are not accounted for in the TPM-APL.




(a) Mean absolute error




(b) Mean absolute percentage error

Figure 5.9: The relationships between the weighting factor ($\phi$) and the MAE and MAPE of the forecasting model obtained from the testing dataset.

Using the *testing* dataset to identify $\phi^*$ is slightly counter-intuitive, especially considering that a *testing* dataset will not exist when the model is used in actual practise. Fortunately, it is not crucial to complete a sensitivity analysis to identify a reasonable weighting factor because, based on the work of Goto et al. (2004) and Van Ostaijen et al. (2017), it is known that high weighting factor values are generally more favourable. This is further motivated by the fact that the accuracy difference between the most favourable and a lesser favourable weighting factor value is almost insignificant. Consequently, if the in-flight catering company is unable to complete a sensitivity analysis to identify $\phi^*$, it is suggested that a weighting factor in the range of [0.90; 0.98] should be chosen.

## 5.3 Concluding remarks

This chapter presented the model solution that integrates the numerical example with the model formulated in Chapter 3. The model solution consists of two key components: (1) the deterministic inventory decision-making model that can be solved using standard optimisation software, and (2) the set of modelled TPMs that encompass the forecasting model for a specific flight under consideration.

Recourse Programming (RP) was used to derive the deterministic equivalent of the stochastic inventory decision-making model. The drawback of this approach is that it might require excessive solving times to generate a solution. This is alarming when considering the model's target audience − in-flight catering companies that require solutions in real-time to cater for multiple flights per day. It is, however, believed that the benefit of using RP outweighs this risk that could easily be addressed by reducing the number of realisations used to represent each of the stochastic variables. The benefit of RP is that the problem can be modelled with two stages, which ensures that the model is a more realistic depiction of reality. In the first stage, the caterer must decide on the number of each meal type to produce without knowing the primary demand thereof. The primary demands are revealed in the second stage and the passengers are served accordingly (recourse actions).

As stated earlier, the forecasting model is unique to a specific flight, since the booking-behaviour of passengers will vary due to factors such as the flight's duration, destination, and seasonality availability. Consequently, the forecasting model must be *trained* using historical data of the particular flight. Training the forecasting model consists of the derivation of the modelled TPMs. However, recall that the final forecasting interval's modelled TPMs depends on the particular flight observation's DOW, month and year of departure. Thus, the final forecasting interval's modelled TPM must be derived individually for each flight observation under consideration using the constant parameters of the multiple regression model.

This chapter analysed the modelled TPMs derived using an equal weighting factor ($\phi = 0.5$). However, the sensitivity analysis concluded that a weighting factor of 0.91 will likely maximise the accuracy of the forecasting model. Consequently, new modelled TPMs had to be generated using $\phi = 0.91$ to form the model solution. The new modelled TPMs and the regression model results are presented in Appendix A.

# Chapter 6

# Model results

This chapter analyses the results obtained when applying the model developed to the *testing* dataset of the numerical example. The output of the forecasting model (Part B) will be presented first because it is used as an input for the inventory decision-making model (Part A). In addition, the value of the inventory decision-making model will be validated by comparing the model developed with similar models that lack either passenger load randomness or product substitution, or both.

## 6.1 Part B: Forecasting model

The forecasting model consists of the three modelled Transition Probability Matrix (TPM)s and the regression model given in Appendix A. Recall that the purpose thereof is to forecast $\pi_0$, the probability distribution of a specific flight's final passenger load. This section discusses the results obtained after deriving $\pi_0$ for each individual flight instance in the testing dataset. Three examples are demonstrated with Figure 6.1 to Figure 6.3.

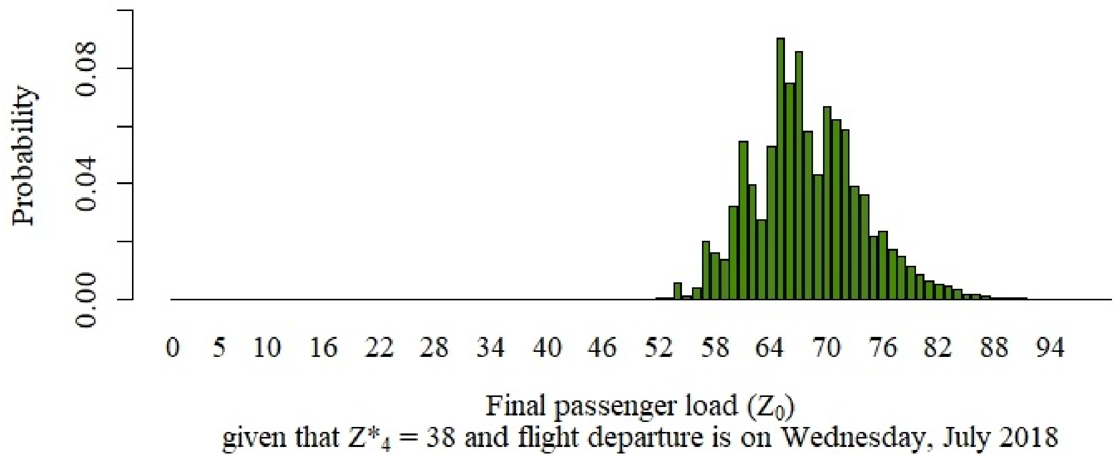



Figure 6.1: The probability distribution of the final passenger load for a flight instance instance that departs on a Wednesday in August 2018, with a passenger load of 38 passengers at 72 hours before take-off.

Figure 6.1 depicts the resulting $\pi_0$ for a flight instance that will depart on a Wednesday morning in July 2018. It is known that the flight had 38 passenger reservations at the start of the forecasting horizon ($Z_4^*$). Based on the probability distribution depicted, there is a 63.6% probability that the final passenger load will fall within the range of 65 to 75 passengers. The expected value, also known as the single point estimate, is 68 passengers. The forecasting model showed prominence for this specific flight instance, as the *true* final passenger load ($Z_0^*$) was 69 passengers.

The benefit of using $\pi_0$ instead of the expected value is demonstrated with Figure 6.2. For the flight instance shown, the *true* final passenger load is equal to 84 passengers, but the expected value thereof underestimated this outcome with seven passengers. However, when using $\pi_0$, the possibility that the passenger load could be 84 passengers is not discarded, as seen from the visual depiction of $\pi_0$.

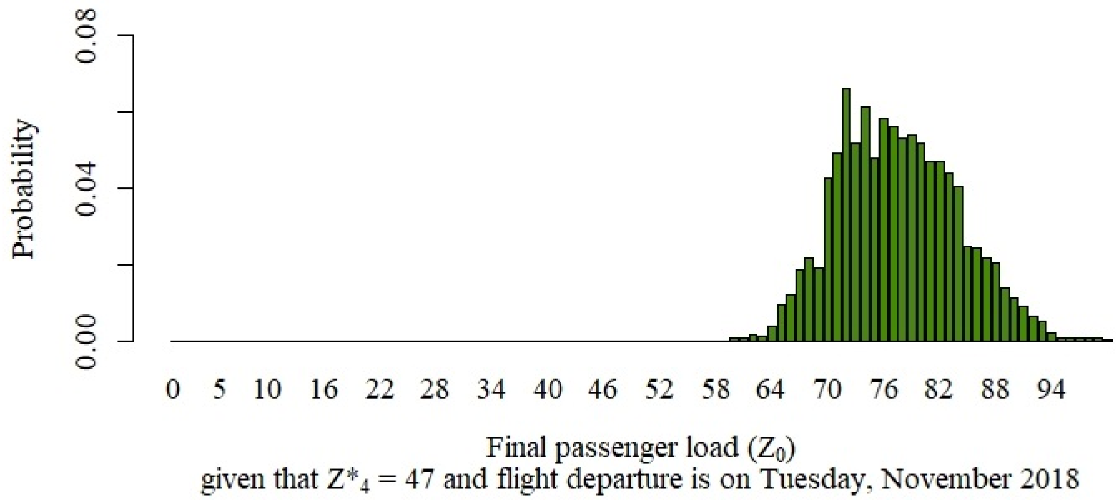



Figure 6.2: The probability distribution of the final passenger load for a flight instance that departs on a Tuesday in November 2018, with a passenger load of 47 passengers at 72 hours before take-off.

Lastly, Figure 6.3 shows the $\pi_0$ for a flight instance that departs during the high demand period – a Saturday morning in April. Based on $\pi_0$, there is a 83.1% chance that the passenger load will exceed 97 passengers. The expected value and the *true* final passenger load are both equal to 99 passengers.

The time required to train the forecasting model and to derive the respective $\pi_0$ for a flight instance is negligible. This means that the forecasting model is convenient and suitable for the in-flight catering industry, where real-time information is needed on a daily basis. Due to the rapid speed of the current model, in-flight caterers could consider expanding the forecasting model by increasing the number of forecasting intervals (N) used to further improve its accuracy. Doing so will reduce the width of the respective forecasts. Two additional considerations include increasing the size of the training dataset and adding extra significant covariates in the regression model. Possible covariates include the time of travel, the number of flights occurring on the departure day and public holidays to name a few.

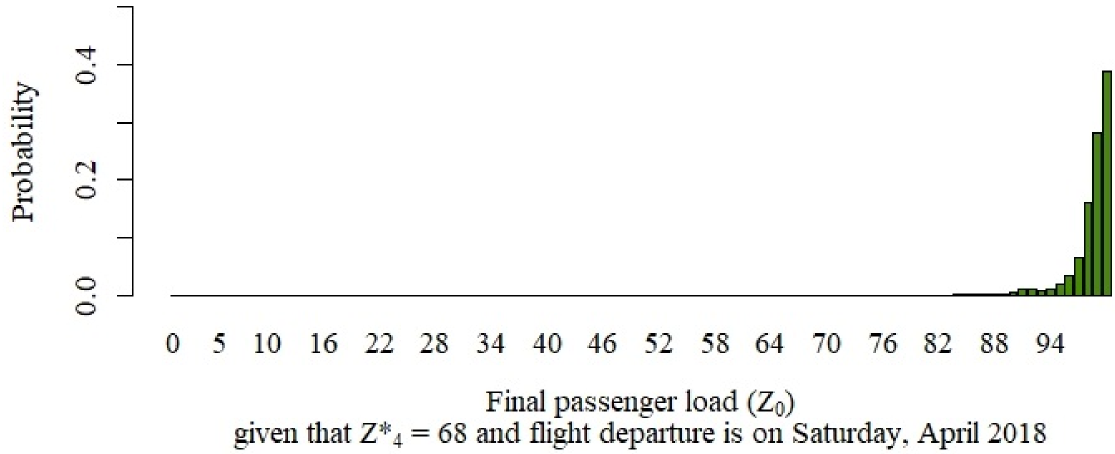



Figure 6.3: The probability distribution of the final passenger load for a flight instance that departs on a Saturday in April 2018, with a passenger load of 68 passengers at 72 hours before take-off.

Quantifying the accuracy of the forecasting model is challenging as it is difficult to capture the added benefit of using the *distribution* of the final passenger load. The most intuitive approaches to measure forecasting accuracy is to use the expected value of the final passenger load to calculate the Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE). When following these approaches, a MAE of 4.8 passengers and a MAPE of 5.9% is obtained. The resulting distribution of the forecasting errors is visible in Figure 6.4. Roughly 63% of the flight instances resulted in a forecasting error of five or fewer passengers, while 42% were underestimated with at least one passenger.

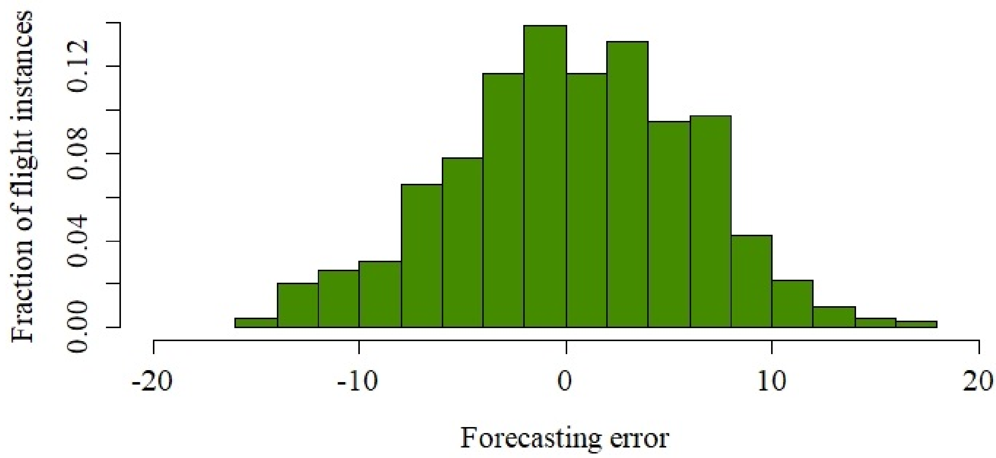



Figure 6.4: Distribution of the forecasting error when using the expected value of the final passenger load.

The benefit of incorporating the distribution of a particular flight's final passenger load instead of using its expected value is validated in the following section, which focuses on the evaluation of the inventory decision-making model developed.

## 6.2 Part A: Inventory decision-making model

This section evaluates the inventory decision-making model developed by comparing its reliability, performance and timeliness with three alternative models that lack either meal demand randomness or product substitution, or both. These alternative models complete the strategic-scenario planning matrix shown in Figure 6.5.

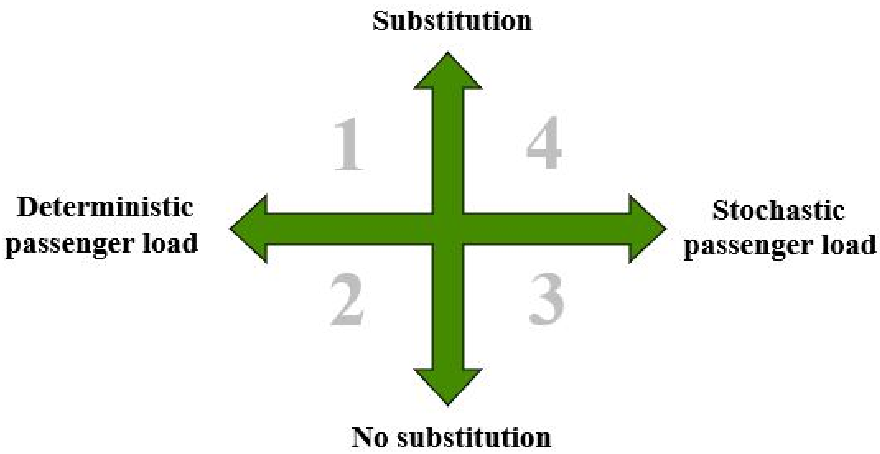



Figure 6.5: Strategic-scenario planning matrix.

The inventory decision-making model developed in this dissertation, simply referred to as the *Solution Model*, corresponds with the fourth quadrant in the strategic-scenario planning matrix. The second quadrant represents *Alternative Model 2*. In contrast with the Solution Model, this model ignores the substitution behaviour of passengers and assumes that the aggregate meal demand is deterministic. *Alternative Model 1* and *Alternative Model 3* ignore either product substitution or meal demand uncertainty as indicated with their respective quadrant. It is important to note that all four models include $\widetilde{\mathbf{q}}$, the uncertainty regarding the first-choice meal preferences of on-board passengers. Thus, all four models are stochastic models.

The alternative models are obtained by slightly modifying the Solution Model. For instance, to ignore meal substitution, $z_{ij}$ must simply be set equal to zero. Recall that this variable indicates the number of meal type $i$ used as a substitute for the out-of-stock meal type $j$, where $i, j \in \mathbf{I}$. The deterministic meal demand can be enforced by replacing $\widetilde{Z}_0$ with the *expected* meal demand instead of using recourse programming to decompose the stochastic variable. The important characteristics related to each axis of the strategic-scenario planning matrix are summarised below:

**Substitution:** $z_{ij} \geq 0, \quad \forall i, j \in \mathbf{I}$

**No substitution:** $z_{ij} = 0, \quad \forall i, j \in \mathbf{I}$

**Stochastic meal demand:** $\widetilde{Z}_0 \approx Z^b \;\forall\; b \in \{1, ..., 5\}$ [Recourse programming]

**Deterministic meal demand:** $\widetilde{Z}_0 \approx E[\widetilde{Z}_0]$ [Expected value]

The purpose of the strategic-scenario planning matrix is to determine the combined and isolated impact of *product substitution* and *meal demand uncertainty* on the decision-making model. In other words, the goal is to determine if these two elements are worthwhile and contribute towards a solution for the wastage problem faced by in-flight catering companies. This can be achieved by comparing the performance, reliability and timeliness of the Solution Model with that of the three alternative models.

### 6.2.1 Model simplification

The inventory-decision making model attempts to imitate reality by incorporating uncertainty. This uncertainty includes the aggregate meal demand of the flight under consideration and the first-choice meal preferences of passengers. To handle this uncertainty, recourse programming was used to transform the model into its deterministic equivalent by decomposing the random set of first-choice probabilities into five realisations sets, and to represent the stochastic passenger load with 101 potential realisations. Unfortunately, this model solution is accompanied by excessive computational requirements that are likely infeasible for most in-flight catering companies. The reason being that it will require expensive hardware and unconstrained solving time, which creates the need to simplify the model.

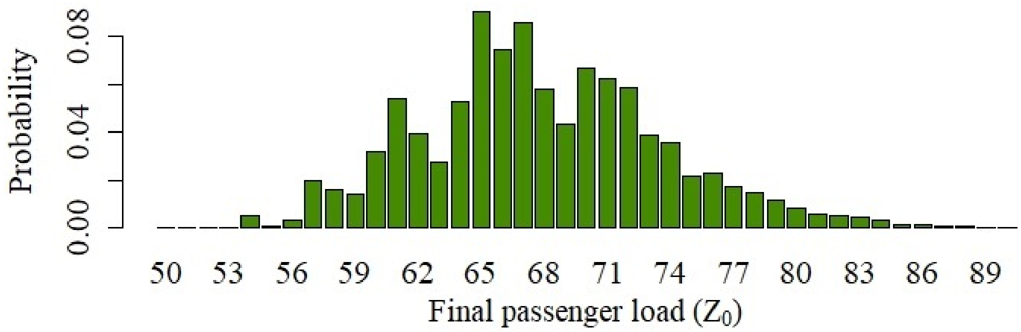
This dissertation made use of a 3.1 GHz Intel(R) Core(TM) i3-2100 CPU with 6.0 GB of RAM to solve the inventory decision-making model. After applying a trial-and-error process, the following changes were made to the model defined in Chapter 5 to try to ensure that the model is solved within a reasonable time:

**Set of first-choice probabilities:** Reduce the number of realisations to three, such that the distribution of $\mathbf{q}^s$ changes to $\{(\mathbf{q}^s, P^s = \frac{1}{3}),\ s \in \mathbf{S} = \{1, 2, 3\}\}$.

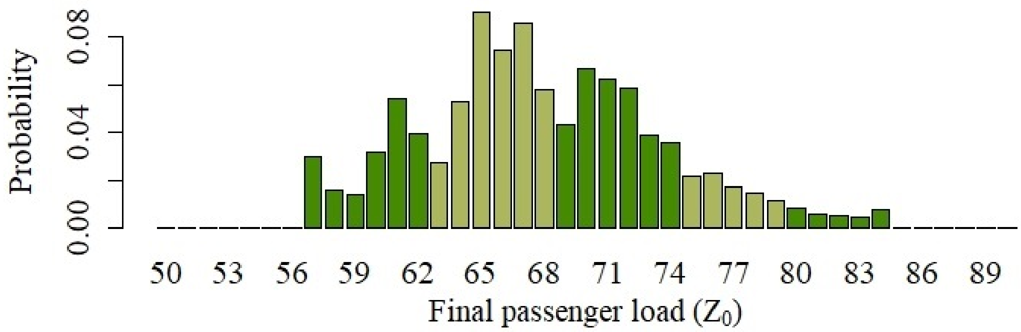
**Aggregate meal demand:** Recall that $\pi_0$ represents the probability density function for the final passenger load of a particular flight with state space $\mathbb{C}$. Since the fictitious flight in the numerical example has a maximum seat capacity of 100 passengers (K), $\mathbb{C}$ consists of 101 possible realisations when including the possibility of having zero passengers. To simplify the model, the number of possible realisations for $\widetilde{Z}_0$ is reduced to only five realisations. The process followed is described below.

As seen from Figure 6.1 through Figure 6.3, each $\pi_0$ consists of numerous transition probabilities that approximate zero. It is speculated that these transition probabilities provide negligible value. Thus, the first step is to trim both tails of $\pi_0$ that accumulates to approximately 1% on each side of the distribution. These tails are then summed together with the closest neighbouring realisation's transition probability to ensure that $\sum_{i \in |\mathbb{C}|} \pi_0(i) = 1$ still holds. This radically reduces the number of realisations. Afterwards, the remaining realisations and their respective transition probabilities are divided into five bins. The *new* realisation value associated with bin $b$, represented with $Z_0^b$, corresponds with the average of the realisations allocated to that bin. Note that the new realisation values were rounded upwards to obtain integer values.

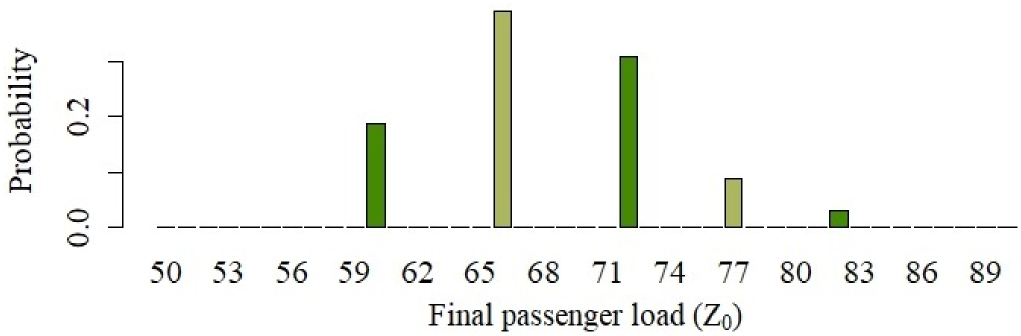
As the final step, $Z_0^v$ and $p^v$ must be replaced, respectively, with $Z_0^b$ and $p^b$ in the model solution given in Section 5.1. The discreet distribution of $Z_0^b$ is given with $\{(Z_0{}^b, p^b),\ b \in \{1, ..., 5\}\}$, where $p^b$ represents the sum of the transition probabilities allocated to bin $b$. Figure 6.6 provides a visual representation of the process described using the flight instance depicted in Figure 6.1.




(a) The probability distribution of the final passenger load ($\pi_0$) representing the probability of occurrence ($p^v$) for the 101 possible realisations ($Z_0^v$).




(b) The trimmed probability distribution of the final passenger load ($\pi_0$), colour code to indicate the five bins.




(c) The resulting five passenger load realisations ($Z_0^b$) with their respective probability of occurrences ($p^b$) that are used as input for the *simplified* inventory decision-making model.

Figure 6.6: A graphical representation of the process followed to reduce the number of the passenger load realisations required in order to simplify the model.

The MAE in the expected final passenger load obtained after applying the above changes to each flight instance increased with only 0.006. Thus, simplifying the model will slightly lower its accuracy, but the improved solving time significantly outweighs the loss in accuracy.

### 6.2.2 Model comparison process

This section briefly highlights the process followed to evaluate the models. The process starts by selecting a flight instance from the 730 flight instances in the testing dataset. The probability distribution of the flight instance's final passenger load, $\pi_0$, is then obtained using the forecasting model developed (Part B). Thereafter, $\pi_0$ is simplified to consist of only five possible realisations. For the remainder of this section, flight instance 217 will be used as an example. This flight instance's $\pi_0$ and simplified $\pi_0$ are visible in Figure 6.6a and Figure 6.6c.

The simplified $\pi_0$ is used as input into the Solution Model and each of the three alternative models. These models are then solved individually using LINGO 18.0 to obtain $\mathbf{x}$, the set of suggested meal order quantities that the respective model deems most efficient when considering the *minimum* Passenger Satisfaction Level (PSL), the two conflicting objectives and the chosen target weights. An example of the solutions obtained by each of the four models when $p^{min} = 92\%$, $w^{PSL} = 1$ and $w^{Meals} = 5$ are listed in Table 6.1.

Table 6.1: The model solutions and the expected outputs obtained when solving flight instance 217 with a *minimum* PSL of 92%, $w^{PSL} = 1$ and $w^{Meals} = 5$.

| **Model** | **Order quantity (x)** | | | | **Expected** | |
|---|---|---|---|---|---|---|
| | **C** | **B** | **FP** | **V** | **PSL (%)** | **Waste** |
| Solution Model | 41 | 24 | 11 | 2 | 97.6 | 9.3 |
| Alternative Model 1 | 33 | 23 | 10 | 2 | 92.5 | 0.0 |
| Alternative Model 2 | 36 | 21 | 10 | 3 | 93.4 | 3.7 |
| Alternative Model 3 | 42 | 25 | 12 | 3 | 99.0 | 12.9 |

Two additional outputs are also recorded in Table 6.1. These outputs represent the solution's expected outcomes, the *expected* PSL and the *expected* number of surplus meals produced when the model's solution, the suggested meal order quantities, are ordered for the particular flight instance under consideration. For simplicity, the outputs will be represented with $p^{expected}$ and $m^{expected}$ and are calculated by adding constraints (6.1) and (6.2) to the model solution formulated in Section 5.1.

$$p^{expected} = 100 - \sum_{s \in \mathbf{S}} p^s \sum_{b \in \{1,\ldots,5\}} p^b \left( \Delta^{PSL}(\mathbf{q}^s, Z_0^b) \right) \tag{6.1}$$

$$m^{expected} = \sum_{s \in \mathbf{S}} p^s \sum_{b \in \{1,\ldots,5\}} p^b \left( \Delta^{Meals}(\mathbf{q}^s, Z_0^b) \right) \tag{6.2}$$

The two expected outcomes of a model's solution will be used as benchmarks to test and compare the performance and reliability of each model.

In summary, the models' are evaluated by simulating a thousand realisations of the flight instances under consideration. Thereafter, the model's solution is applied to each realisation to analyse the objective function output, the *actual* PSL and the *actual* number of surplus meals produced. This process consists of the three main steps described below. The pseudo-code for *Step 1* is given in Algorithm 1, while the pseudo-code for the *Step 2* and *Step 3* are combined in Algorithm 2 given in Appendix B.

*Step 1 - Create a set of realisations:*

The flight instance selected is simulated a thousand times to create a set of realisations, denoted by $\mathbf{R}$. Each realisation consists of the final passenger load for the flight, as well as the primary demand for each meal type in $\mathbf{I}$. For each realisation $r \in \mathbf{R}$, the final passenger load $Z_0^{*,r}$ is obtained by sampling from $\mathbb{C} = \{0, ..., 100\}$ with probability distribution $\pi_0$. Thereafter, the primary demand for each meal type is sampled using the same process followed when creating the synthetic dataset in Chapter 4. Recall that the sum of the primary meal demands must be equal to the final passenger load.

As an example, the first realisation ($r = 1$) sampled a final passenger load of 72 passengers and the primary demands for {chicken, beef, fruit platter, vegan} meals are $\{39, 19, 10, 4\}$ meals. For the $752^{nd}$ realisation, a final passenger load of 75 passengers was sampled with primary demands equal to $\{39, 29, 3, 4\}$.

*Step 2 - Simulate the dynamic passenger demand order for each realisation:*

This step entails mimicking the random order in which passengers are served on-board the flight. In other words, it simulates the dynamic '*arrival*' of passengers. Recall from the example given in Section 2.2.2 that the dynamic arrival of passengers has a significant impact on the substitution behaviour of passengers unless meal items are pre-allocated to the passengers. For the purpose of evaluating the model, pre-allocated meals will not be considered in this dissertation to simulate the 'worst-case' scenario when airlines do not allow pre-booking of meals. Thus, while the model developed is *static*, it is evaluated on its ability to perform well in a *dynamic* environment.

For each realisation $r \in \mathbf{R}$, the dynamic order of passenger demands is obtained by sampling $Z_0^{*,r}$ meals from $\{d_1^r, d_2^r, d_3^r, d_4^r\}$ without replacement. Variable $d_i^r$ represents the primary demand for meal type $i \in \mathbf{I}$ associated with flight realisation $r$.

For the $752^{nd}$ realisation, 75 meals are drawn randomly from $\{39, 29, 3, 4\}$ units of the respective meal type. The resulting passenger demand order is {chicken, beef, chicken, ..., fruit platter, beef}. This means that the first passenger served on the flight will order a chicken meal as his first-choice, while the second passenger prefers a beef meal. The last passenger to be served will also try to order a beef meal as his first-choice, but might have to consider a substitute if beef meals are out-of-stock.

*Step 3 - Apply model solution:*

This step measures the model under consideration's ability to satisfy the dynamic demand order of each realisation $r \in \mathbf{R}$ when using its solution's suggested meal order quantities. This is done while incorporating the substitution behaviour of passengers. This means that each model is tested with the assumption that a passenger will consider a substitute meal if the passenger's first, second or third preference is out-of-stock. Ultimately, this step calculates the objective function output ($OF_r$), the *actual* PSL ($p_r^{actual}$) and the *actual* number of surplus meals ($m_r^{actual}$ ) obtained after applying the model's solution to each realisation $r$ in the set $\mathbf{R}$. The objective function output of each realisation is calculated with (6.3).

$$OF_r = w^{PSL}(100\% - p_r^{actual}) + w^{Meals}m_r^{actual} \tag{6.3}$$

As an example, assume that the Solution Model's suggested solution is applied to the set of realisations of flight instance number 217. Each realisation is evaluated with order quantities $\mathbf{x} = \{41, 24, 11, 2\}$ as the model found these quantities to be most efficient. In other words, each realisation will assume that $\mathbf{x}$ represents the meal inventory levels before passengers are served. For instance, consider the $752^{nd}$ realisation. Since the first passenger prefers a chicken meal, the stocking quantity of chicken meals will reduce to 40 meals after this passenger is served. Similarly, the stocking quantity of beef meals will reduce to 23 meals after the second passenger is served. This process will continue as long as the passenger's preferred meal is in-stock. However, notice that this realisation's primary demand for beef meals is 29 meals, while there are only 24 beef meals available on-board the flight. Similarly, vegan meals are under-catered with 2 meals. This means that *at least* seven passengers will need to consider a substitute meal. The process followed to incorporate the substitution decision process of a passenger is described below.

If the passenger's first-choice meal ($fc \in \mathbf{I}$) is out of stock, the passenger might choose his second preference. This second preference meal corresponds with the highest $\hat{\alpha}_{j,fc}$ value, where $j \in \mathbf{I}$. If the passenger's second preference (j) is in-stock, the passenger must decide if he wants to accept the substitute. This decision is modelled by randomly selecting a value in the range [0,1] with a uniform probability. If the value selected is smaller than or equal to $\hat{\alpha}_{j,fc}$, the passenger accepts the substitute and the meal is served to the passenger. If the passenger does not accept the substitute or if the substitute is also out of stock, the process is repeated to consider the passenger's subsequent meal preferences. The passenger will forfeit a meal if he does not accept any of the substitutes available, which then represents a meal shortage.

When applying the Solution Model to the $752^{nd}$ realisation, three shortages and six surplus meals occurred. Due to the under-catering of five beef meals, three passengers accepted a chicken meal as a substitute, which led to a shortage of one chicken meal. Furthermore, three passengers accepted a fruit platter as a substitute for either a beef or vegan meal, and three passenger forfeited a meal altogether. This resulted in a 93.9% *actual* PSL for the $752^{nd}$ realisation. Fortunately, the *actual* PSL exceeds the *minimum* PSL of 92%. Thus, $m_{752}^{actual} = 6$ and $p_{752}^{actual} = 93.9\%$, which resulted in a objective function output equal to 36.1 when using weights $w^{PSL} = 1$ and $w^{Meals} = 5$.

This step must be repeated for each of the three alternative models. This ensures that the models are evaluated based on the same set of realisations.

The above describes the process followed to determine the actual outputs of each model when applied to flight instance number 217 with $p^{min} = 92\%$ and target weights $w^{PSL} = 1$ and $w^{Meals} = 5$. To obtain more insight, 15 *additional* flight instances and four extra target weight combinations are selected and analysed using the exact same process. The process is also repeated without a *minimum* PSL requirement ($p^{min} = 0$) to highlight the purpose thereof. Lastly, two additional constraints are added to the decision-making models to evaluate the impact of additional constraints, such as batch order restrictions, on the models.

The 16 flight instances were strategically selected from a random sample to ensure that a variety of $\pi_0$ outcomes are analysed. The $\pi_0$ of the chosen flight instances, as obtained using the forecasting model, are depicted in Figure B.1 and Figure B.2 in Appendix B.

### 6.2.3 Model comparison results

The four models are evaluated based on each model's reliability, performance and timeliness. The reliability of a model refers to the model's ability to guarantee the *minimum* PSL required, as well as the two expected outcomes of its solutions. A model's performance relates to how closely the model is able to achieve the 100% PSL target and the zero-waste target. Lastly, a model's timeliness is concerned with the solving time and effort required by the model to generate an efficient solution. The results obtained are discussed in the remainder of this section.

#### Reliability comparison

The reliability of a model refers to the model's ability to guarantee the *minimum* PSL required, as well as the two expected outcomes of its solutions. A distinction is made between a model's *MPSL*-reliability and its *output*-reliability.

A model's output-reliability simply refers to the probability that the expected outcome(s) of a model's solution – the *expected* PSL or the expected number of surplus meals – will be achieved. It is estimated by calculating the fraction of the total flight instance realisations that achieved or further improved on the expected outcome of the relevant solution generated by the model under consideration. Three output-reliabilities are considered and the results obtained for each target weight combination analysed without a *minimum* PSL requirement ($p^{min} = 0$) are provided in Figure 6.7 at the end of the discussion of the main observations given below.

**The *expected* PSL output-reliability:** This reliability measure represents the probability that the *actual* PSL achieved will be equal to or greater than the *expected* PSL of the model's solution when applied to the specific flight instance under consideration. Thus, it represents $\mathbb{P}[p^{actual} \geq p^{expected}]$.

The subtle decreasing trend visible in Figure 6.7a indicates that this reliability measure is dependent on the target being favoured. Thus, higher reliabilities are obtained when the 100% PSL target is favoured. Alternative Model 3 obtained the highest *expected PSL* output-reliability with 68.9% when $w^{PSL} = 10$ and $w^{Meals} = 1$, as well as the lowest outcome with 40.8% when $w^{PSL} = 1$ and $w^{Meals} = 10$. Overall, this model ranked first with an average of 57.1%.

The Solution model ranked second with 55.4%, followed by the two deterministic passenger load models, Alternative Model 2 with 50.3% and Alternative Model 1 with 49.7%.

The stochastic passenger load models − the Solution Model and Alternative Model 3 − are notably superior when $w^{PSL} > w^{Meals}$ and obtained the highest reliabilities overall. This observation hints towards the benefit of incorporating demand uncertainty within the inventory decision-making model. The results obtained thus far are not promising for the substitution models − the Solution Model and Alternative Model 1 − because both of the substitution models ranked below their respective non-substitution model. Take for example the Solution Model that obtained a 1.7% lower average *expected PSL* output-reliability when compared with Alternative Model 3.

**The *expected* waste output-reliability:** This reliability measure represents the probability that the actual number of surplus meals produced when applying the model's solution to the particular flight instance, will not exceed the solution's expected value. Thus, it represents $\mathbb{P}[m^{actual} \leq m^{expected}]$.

Except for Alternative Model 1, the reliabilities obtained increase along with the relative importance of the zero-waste target. This increase is visible in Figure 6.7b. Although counter-intuitive, the opposite is true for Alternative Model 1 that ranked last in terms of the *expected waste* output-reliability, with a dreadful average of 25.3%. Alternative Model 3 obtained the highest average with 68.4%, and Alternative Model 2 and the Solution Model ranked second and third with 57.4% and 52.4%, respectively.

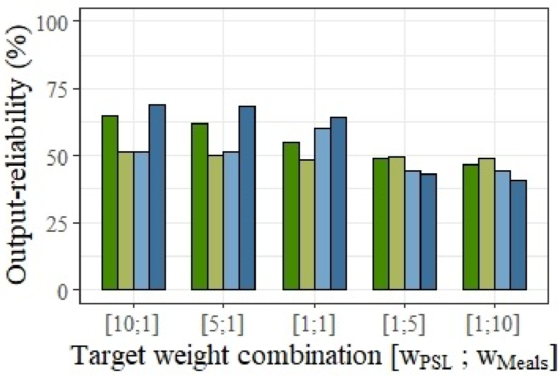
Based on the reliability ranking, the stochastic passenger load models are more reliable than their respective deterministic passenger load model. Unfortunately, the two substitution models obtained the lowest reliability outcomes.

**The overall output-reliability:** The two reliability measures discussed above consider a single expected outcome in isolation while the *overall* output-reliability measure considers both of the expected outcomes together. Accordingly, this reliability measure represents the probability that both of the solution's expected outcomes will be achieved or improved upon simultaneously when the model's solution is applied to the particular flight instance under consideration. Thus, it represents $\mathbb{P}[p^{actual} \geq p^{expected} \cap m^{actual} \leq m^{expected}]$.

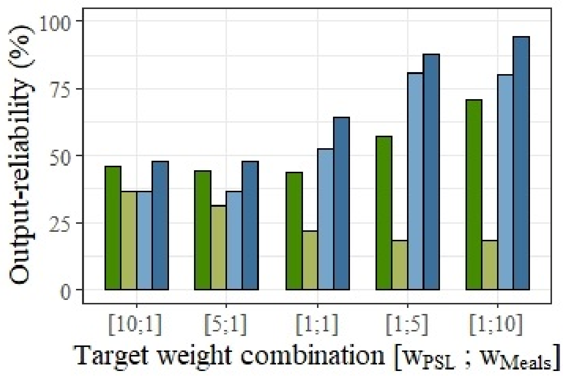
Alternative Model 3 obtained the highest *overall* output-reliability with 32.2%, followed by the Solution Model with 22.3% and Alternative Model 2 with 21.7%. Alternative Model 1 resulted in the worst reliability since only 6.4% of its realisations achieved both of the expected outcomes of the model's solution.

The *overall* output-reliabilities of the models are significantly lower than the *expected waste* and the *expected PSL* output reliabilities discussed above. Although intuitive, this indicates that it is more difficult for the model to achieve both of the expected outcomes simultaneously. If the model over-estimated the meal demand, $p^{actual}$ and $m^{actual}$ will likely be much higher than expected, which has a positive impact on the *expected PSL* output reliability. However, this over-estimation will most likely have a negative impact on the *overall* output reliability because $m^{actual}$ could exceed $m^{expected}$. Similarly, the under-estimation of meal demand will have a positive impact on a model's *expected waste* output reliability and a negative impact on the *expected* PSL and the *overall* output reliability.

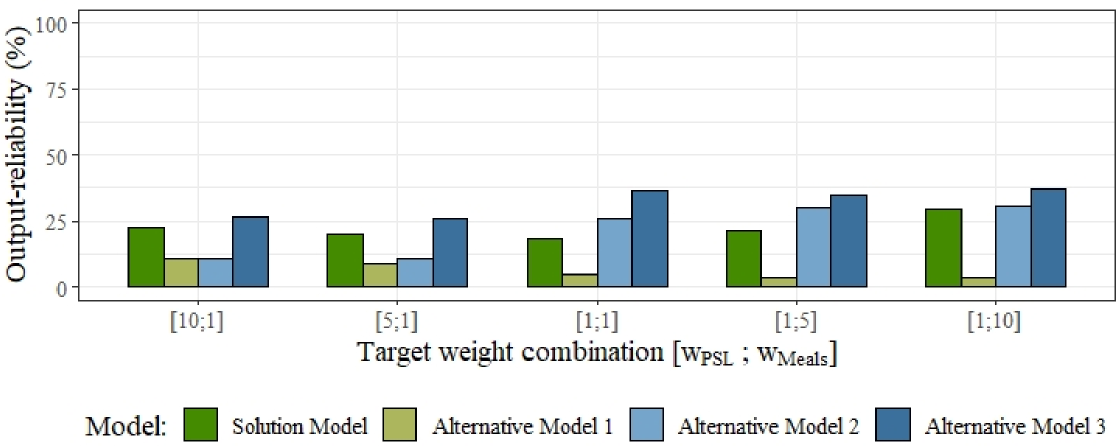
The low *overall output-reliabilities* indicate that the models frequently over- and under-estimated the meal demand. Recall that this over- and under- estimation of meal demand is a common occurrence in the in-flight catering industry due to the excessive variation and uncertainty inherently present in the flight booking processes. Thus, although care was taken to develop a suitable passenger load forecasting model, the accuracy of this model is limited by the excessive variation and the randomness within the flight booking process.




(a) The *expected PSL* output-reliability




(b) The *expected waste* output-reliability




(c) The *overall* output-reliability

Figure 6.7: The three categories of output-reliabilities per target weight combination obtained when no *minimum* PSL is required ($p^{Min} = 0$).

From the analysis of the three output-reliabilities, promising results are obtained for the inclusion of the demand uncertainty within the inventory decision-making model. The same is, unfortunately, not true for product substitution because Alternative Model 3 was found to be superior for all three output-reliabilities investigated.

Recall that Alternative Model 3 incorporates in-flight meal demand uncertainty but *ignores* the meal substitution behaviour of passengers caused by stock-outs. Figure 6.8 visually validates Alternative Model 3's superiority.

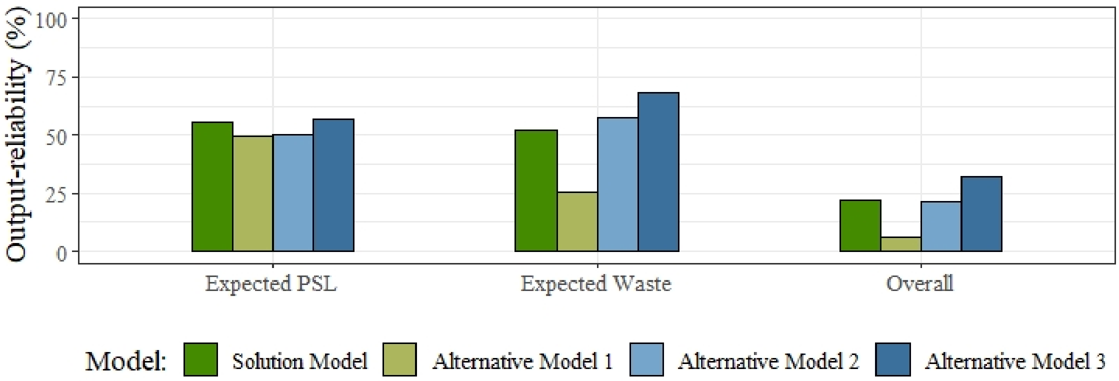



Figure 6.8: The average output-reliability when no *minimum* PSL is required.

Figure 6.8 summarises the models' *average* output-reliability for each of the three categories analysed without a *minimum* PSL requirement. The green bars represent the substitution models, while the stochastic passenger load models are distinguished with darker shades of blue or green.

A major concern regarding all four models is the generally low output-reliabilities obtained because they indicate that the expected outputs of the models' solutions are unreliable. This concern is most prominent for the *expected PSL* output-reliability. Recall that the results of this reliability measure ranged between 40.8% to 68.9%. It is argued that in-flight caterers will continue to follow the over-catering strategy out of fear of obtaining an unacceptably low PSL that could damage the company's reputation and competitiveness, *if* the *actual* PSL obtainable is not guaranteed. Higher output-reliabilities would be able to provide confidence to the in-flight catering company to trust the solutions generated by the chosen decision-making model.

The intuitive approach to obtain a higher *expected PSL* output-reliability or to achieve a desired PSL is to choose a target weight combination that drastically favours the 100% PSL target. However, identifying the appropriate target weight combination is a time-consuming process and will still not guarantee the desired outcome. This highlights the purpose of the *minimum* PSL requirement.

A positive and non-zero *minimum* PSL requirement ($p^{min} > 0$) provides a means for an in-flight caterer to ensure that an acceptable PSL will be achieved *before* the model attempts to minimise wastage. Its purpose is to compensate for the low output-reliabilities and, subsequently, to provide the necessary confidence to the in-flight catering company to reduce the need for the over-catering strategy. Its ability to fulfil its purpose is, however, dependent on the model's MPSL-reliability. The MPSL-reliability is the most important reliability measure. It represents the probability that the *actual* PSL obtained when using the model's suggested solution will be greater than or equal to the *minimum* PSL required. Stated differently, it represents the probability that constraint (3.16) will hold when the model's solution is applied to the flight instance under consideration. Recall that this constraint forces the model to generate a solution that satisfies $p^{expected} \geq p^{min}$.

The MPSL-reliability of a model is estimated by calculating the fraction of the total flight instance realisations that obtained an *actual* PSL greater than or equal to the *minimum* PSL required. This process is depicted visually in Figure 6.9 for flight instance number 217 with a *minimum* PSL requirement of 92% and target weights $w^{PSL} = 1$ and $w^{Meals} = 5$.

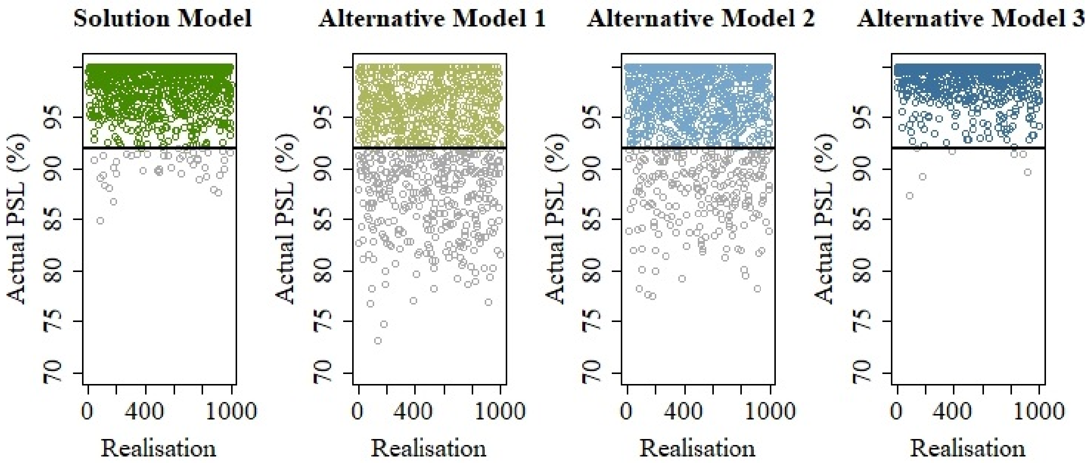



Figure 6.9: A visual demonstration of the *flight-specific* MPSL-reliability calculation for flight instance number 217 when a *minimum* PSL of 92% is required with target weights $w^{PSL} = 1$ and $w^{Meals} = 5$.

The horizontal line represents the *minimum* PSL required. The fraction of the set of realisations on or above the horizontal line represents the MPSL-reliability specific to flight instance number 217 and the chosen target weights. This process was repeated to calculate the flight-specific reliability for each of the 16 flight instances and five target weight combinations considered. The resulting (average) MPSL-reliability of each model per target weight combination is shown in Figure 6.10.

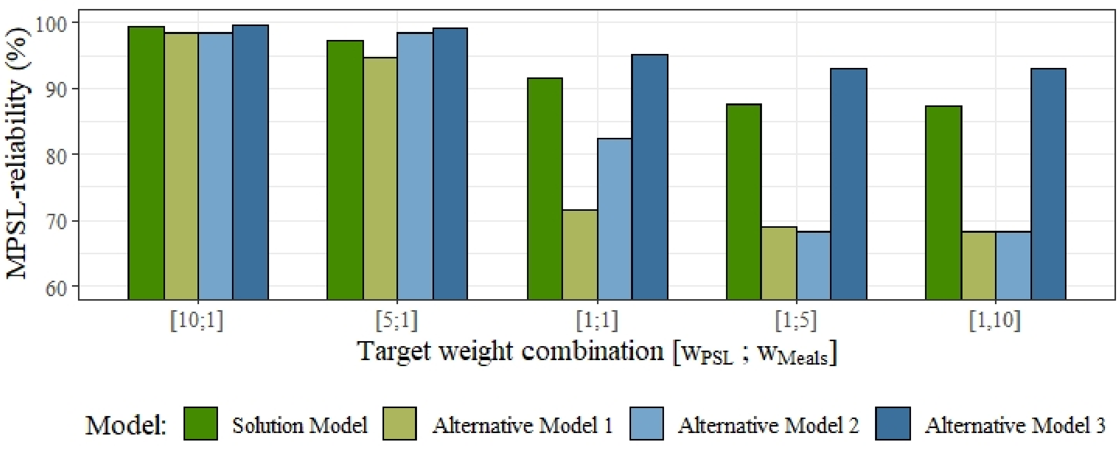



Figure 6.10: The MPSL-reliability per target weight combination analysed with a *minimum* PSL requirement equal to 92%.

Notice that the target weight combinations are ordered from favouring the 100% PSL target to favouring the zero-waste target.The decreasing trend indicates that this reliability measure is dependent on the target being favoured. A simple explanation for this observation is as follows. When $w^{PSL} > w^{Meals}$, the 100% PSL target being favoured is in-line with the purpose of the *minimum* PSL requirement and supports the ordering of additional safety stock to achieve a higher PSL that *exceeds* the minimum requirement. This provides tolerance for when the *actual* PSL obtained is lower than expected. However, when $w^{PSL} < w^{Meals}$, the zero-waste target being favoured conflicts with the *minimum* PSL requirement as it opposes the ordering of safety stock to achieve a PSL greater than the bare minimum required.

The MPSL-reliabilities of the stochastic passenger load models are exceptionally high. Both models obtained MPSL-reliabilities greater than 87% for each of the target weight combinations analysed. Overall, Alternative Model 3 obtained the highest average MPSL-reliability with an impressive 96.0%. The Solution Model followed with a sufficient average of 92.7%. The deterministic passenger load models, Alternative Model 2 and Alternative Model 1, resulted in an average of 83.2% and 80.4%.

It should be stressed that the two deterministic passenger load models obtained unsatisfactory MPSL-reliabilities below 70% when $w^{PSL} < w^{Meals}$. The cause of the deterministic passenger load models' low MPSL-reliabilities is related to the fact that these model's do not compensate for the possibility that a flight's final passenger load could exceed its expected value. On average, the final passenger load surpassed its expected value for 47.3% of the realisations simulated for each model.

The substitution models obtained either a comparable or a slightly lower MPSL-reliability when compared with each model's respective non-substitution model. The causes of the lower reliability outcomes are believed to be due to the substitution models' static nature and the models' simplifying assumption regarding the dependencies among the various substitute meals. The causes will be discussed in more detail in Section 6.2.4.

Figure 6.11 depicts the models' average output-reliability for the three categories analysed with a 92% *minimum* PSL requirement. Note that the individual outcomes per target weight combination is given in Figure B.3 in Appendix B.

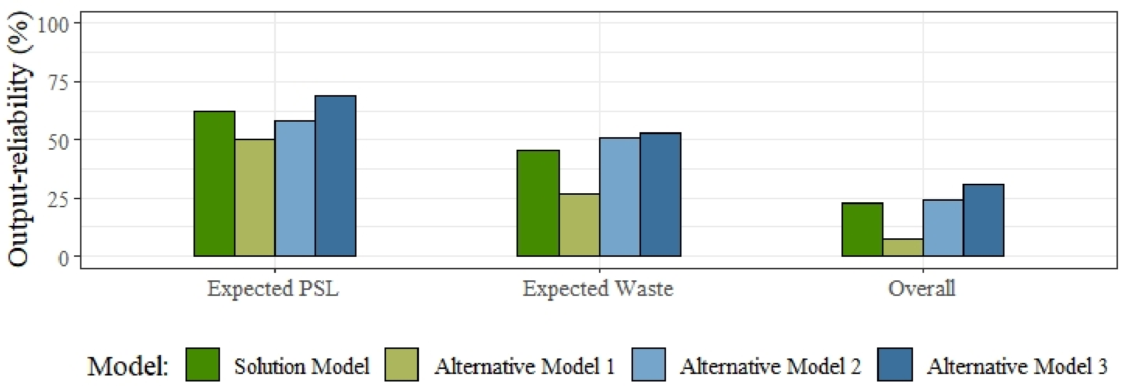



Figure 6.11: The average output-reliability when a 92% *minimum* PSL is required.

The inclusion of the *minimum* PSL requirement influences the output-reliabilities obtained. When a 92% *minimum* PSL is required, the average *expected PSL* output-reliability of the Solution Model and Alternative Model 3 are, respectively, 7.0%

and 11.6% higher when compared with the relevant reliabilities obtained without a *minimum* PSL requirement (Figure 6.11 versus Figure 6.8). Unfortunately, the average *expected waste* output-reliability of the two models are, respectively, 6.7% and 15.6% lower. The significant differences in the output-reliabilities are attributed to the fact that the models have to order additional in-flight meals to ensure that the *minimum* PSL can be achieved. This increase in the total meal order quantity ($\sum_{i \in \mathbf{I}} x_i$) reduces the number of meal shortages and substitutions but also increases the likelihood of wastage.

Despite the improvement observed in the *expected PSL* output-reliability when including a *minimum* PSL requirement, the expected outputs of the models' are still considered unreliable. This unreliability further motivates the importance of the *minimum* PSL requirement. Accordingly, the MPSL-reliability of a model is undeniably one of the most important factors to consider. This is because a model with a low MPSL-reliability will provide no value to an in-flight catering company as it will not be able to reduce the company's dependence on the over-catering strategy. For this reason, the deterministic passenger load models will not be discussed any further because the models obtained unsatisfactory MPSL-reliabilities below 70%.

### Performance comparison

The performance of a model relates to how well the model compares with the *ideal* model that would be able to ensure that the goals of the two objectives are achieved simultaneously. It is quantified by calculating the model's weighted deviation from the in-flight catering industry's ultimate targets − a 100% PSL and zero waste. Accordingly, the output of the model's objective function is indicative of the model's performance. The model that generates solutions with the lowest objective function outputs has the highest performance level.

Figure 6.12 shows the objective function output obtained for each individual flight instance analysed when using target weights $w^{PSL} = 1$ and $w^{Meals} = 5$ with a 92% minimum PSL requirement.

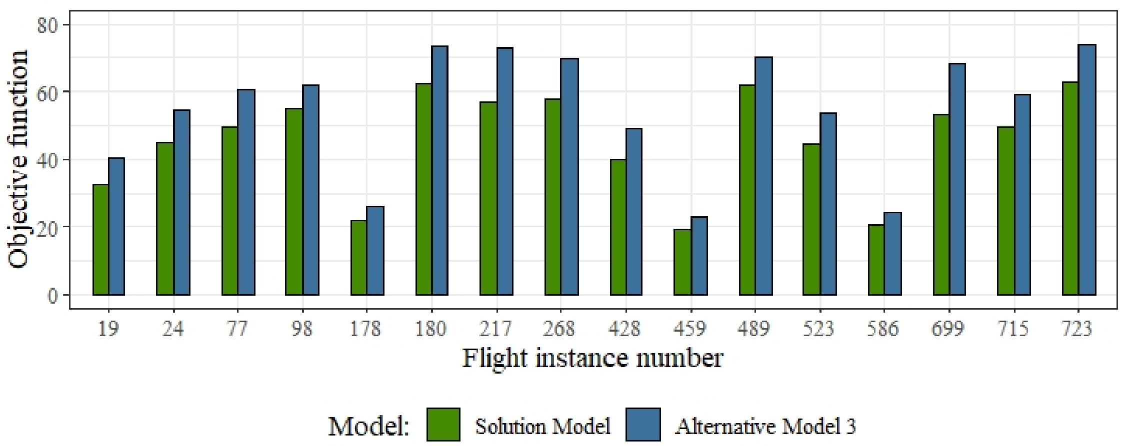



Figure 6.12: The flight-specific objective function outputs obtained when using target weights $w^{PSL} = 1$ and $w^{Meals} = 5$ with a 92% *minimum* PSL.

To clarify, the objective function value depicted for each flight instance represents the *average* objective function output for the set of realisations analysed relating to the particular flight instance, model and the stated targets weights. The objective function output for each realisation is calculated using equation (6.3). The average of the flight-specific objective function outputs is given in Figure 6.13 for each of the five target weight combinations considered.

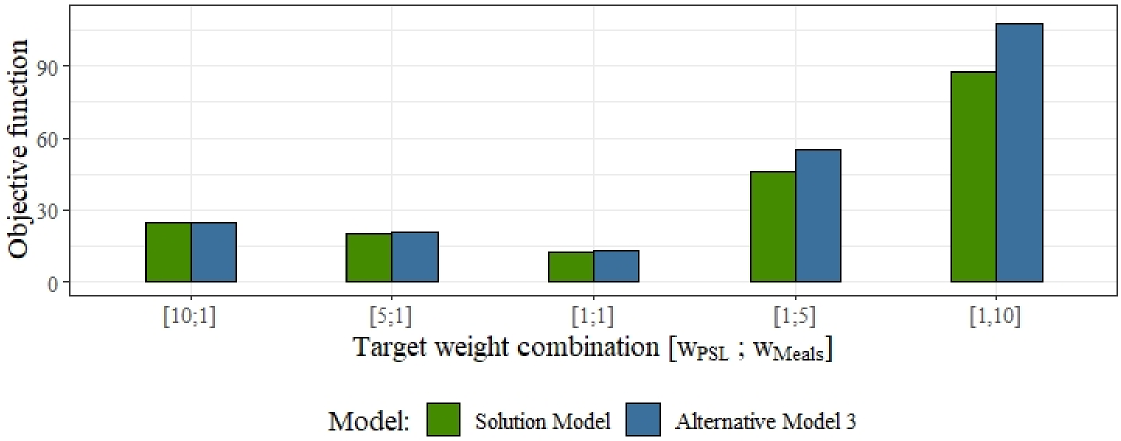
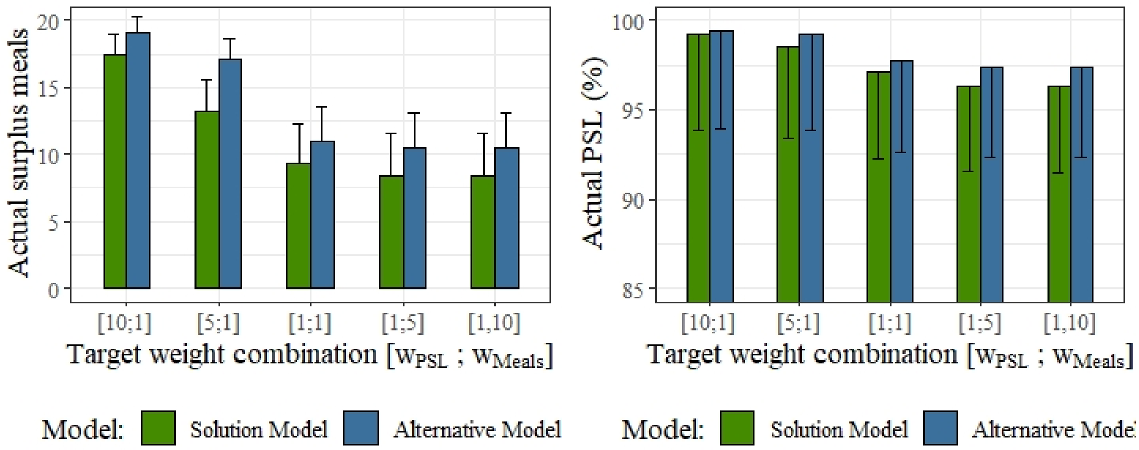



Figure 6.13: The average objective function output per target weight combination analysed with a 92% *minimum* PSL requirement.

The performance of the two models depicted are almost identical for target weights where $w^{PSL} \geq w^{Meals}$. However, the Solution Model outperforms Alternative Model 3 when the zero-waste target is favoured. The Solution Model's superior performance is attributed to the model's greater waste minimisation capabilities. As seen in Figure 6.14a, the model consistently produced the lowest number of left-over meals per flight instance.




(a) *Actual* wastage (b) Actual PSL achieved

Figure 6.14: The average of the *actual* outputs obtained per target weight combination analysed with a 92% *minimum* PSL requirement.

Figure 6.14 shows the average *actual* number of surplus meals produced and the average *actual* PSL achieved for each target weight combination analysed when $p^{min} = 92\%$. Each error line represent the average of one standard deviation from the actual outcome's average for the respective target weight combination.

Overall, the Solution Model produced 2.2 fewer surplus meals per flight instance when compared with Alternative Model 3. This highlights the substitution model's risk-pooling capabilities. The trade-off for the Solution Model's lower wastages are lower *actual* PSLs, as seen in Figure 6.14b. The *actual* PSL achieved by the Solution Model is, on average, 0.7% lower than that of Alternative Model 3. However, the benefit in terms of waste minimisation outweighs this drawback, especially when $w^{PSL} \leq w^{Meals}$, as seen with the model's superior performance in Figure 6.13.

The Solution Model's lower wastage is due to its smaller total meal order sizes, which in turn, are attributed to the model's risk-pooling capability. The model's average total meal order size is 89.1 units, while Alternative Model 3 ordered an average of 2.8 extra meals per flight instance. Recall that smaller meal orders are more desired because it reduces the amount of pre-consumer waste produced resulting from the supply chain bull-whip-effect. Pre-consumer waste production was not considered when calculating the models' performance. Subsequently, the differences between the models' performances depicted in Figure 6.13 is, in reality, more significant in favour of the Solution Model. The model's superior performance validates the value of incorporating the substitution behaviour of passengers within the decision-making model as a potential solution opportunity for the wastage dilemma faced by the in-flight catering industry.

### Timeliness comparison

While arguably not as important as a model's reliability or performance, the solving time required by the model influences the model's feasibility and usefulness in the in-flight catering industry. This is because most in-flight catering companies need to plan and cater for several flights per day. It is, therefore, important that the decision-making model should be able to generate solutions within a reasonable time with minimal effort. If the model is too time-consuming, it could become a bottleneck in the meal planning and production process and will likely be passed over by employees.

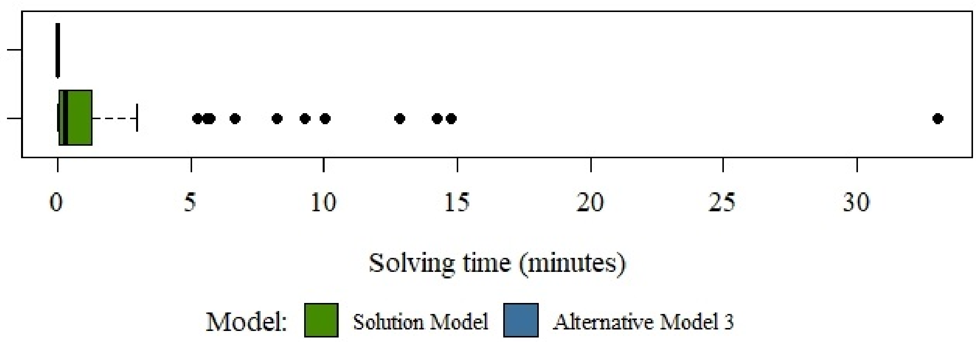



Figure 6.15: The spread in the solving time recorded.

Figure 6.15 depicts the spread in the solving time recorded for both models after solving the 16 flight instances with five different target weight combinations and a 92% *minimum* PSL requirements using LINGO 18.0. The solving time required by Alternative Model 3 is negligible (0 seconds) and ideal for the in-flight catering industry, whereas the Solution Model is slightly more time-consuming and inconsistent. This is because even though the Solution Model required less than three minutes to generate a solution for 86.3% of the flight instances analysed, various outliers occurred that required up to 34 minutes of solving time. An outlier refers to a solution that required more than 179 seconds of solving time. Unfortunately, the root cause behind the outliers observed could not be pinpointed.

The distribution of the solving time required by the Solution Model is considerably positively skewed with a median of 16.5 seconds, an average of 2 minutes and a standard deviation of roughly 5 minutes. Based on the results obtained, there is only 5% probability that the solving time required will exceed 10 minutes. However, the overall range in the solving time recorded creates additional uncertainty and *could* discourage the use of the Solution Model.

In addition to the above, the Solution Model requires more effort to generate a solution when compared with Alternative Model 3. This effort refers to the additional input required by the substitution model, namely the *a-priori* probability matrix ($\hat{\boldsymbol{\alpha}}$). This input requires various assumptions and can be considered as an additional risk due to the possibility of using inaccurate *a-priori* probabilities that could have a devastating impact on the model's reliability and performance.

### Additional batch order constraints

Economies of scale benefits, kitchen capacity limitations and batch order restrictions are common factors that must be considered by in-flight caterers during the meal order planning and production process. When present, these factors can greatly influence the most efficient solution. The models were further evaluated to confirm that the value of product substitution increases when the model is constrained by some of these factors. This evaluation required the addition of constraints (6.4) and (6.5) to the deterministic model solution discussed in Section 5.1.

$$\frac{x_i}{4} = q_i, \quad \forall i \in \{1, 3\} \tag{6.4}$$

$$q_1,\ q_3 \geq 0 \ \text{ and integer} \tag{6.5}$$

These two constraints ensure that meal option $i \in \{1, 3\}$ can only be ordered in multiple batches of four meals, where $q_i$ represents the number of batches of meal type $i \in \{1, 3\}$ ordered.

The results obtained were in favour of the Solution Model when repeating the comparison process given in Section 6.2.2 with the additional batch order constraints. The Solution Model and Alternative Mode 3 produced, receptively, 0.2 and 1.6 *extra* surplus meals per flight instance when compared with the results obtained when the batch order constraints were omitted. Subsequently, the Solution Model's performance remained relatively stable, while Alternative Mode 3's performance worsened. The change in the models' performance is visible in Figure 6.16, which depicts the models' performance obtained with and without the batch order constraints (BOC).

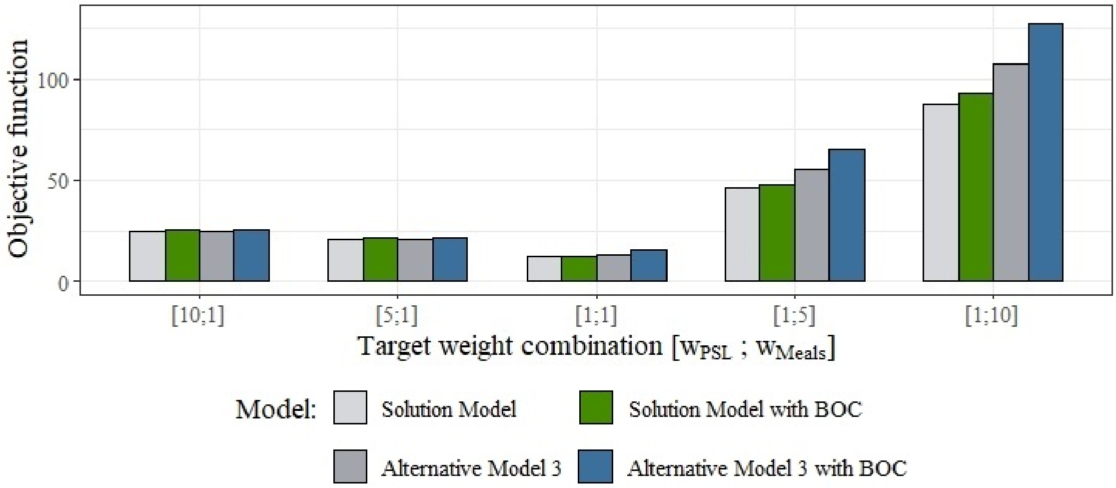



Figure 6.16: The objective function output obtained per target weight combination analysed when batch order constraints (BOC) are included ($p^{min} = 92\%$).

These results confirm that the value of product substitution increases when the model is constrained, which increase the Solution Model's appeal. Furthermore, both of the models' average MPSL-reliability increased with approximately 0.6%, and the change observed in the models' average *actual* PSL is negligible. The average solving time required by the Solution Model reduced by roughly 40 seconds, and its standard deviation improved by almost 2 minutes.

### 6.2.4 Model comparison discussion and recommendation

The purpose of the previous section was to evaluate the benefit of including meal demand uncertainty and product substitution within the inventory decision-making model developed for the in-flight catering industry. In this section, the main conclusions are summarised and discussed to reach a model recommendation.

All four models obtained low output-reliabilities with averages below 70%. The output-reliabilities obtained do not provide satisfactory confidence that an acceptable PSL will be achieved when using the solutions generated by any of the four models. Without a satisfactory confidence level, the in-flight catering company will continue to follow the over-catering strategy, which defeats the purpose of an inventory decision-making model. For this reason, a model's MPSL-reliability is considered its most important characteristic. The impact of product substitution and demand uncertainty on the MPSL-reliability of the decision-making model developed is summarised below using the axis of the strategic scenario planning matrix.

*Stochastic passenger load* [●●] *vs. deterministic passenger load* [●●]:

Recall that the stochastic passenger load models incorporate the uncertainty regarding a flight's final passenger load, whereas the deterministic passenger load models approximate the uncertainty by using its expected value. The stochastic passenger load models were found to be considerably more reliable in achieving the *minimum* PSL required. Both of the stochastic passenger load models obtained MPSL-reliabilities greater than 87% for each of the target weight combinations analysed. This outcome indicates that these models can provide a reasonable level of confidence. The same is not true for the deterministic passenger load models that obtained poor MPSL-reliabilities below 70% when the zero-waste target is favoured. These results validate the benefit of incorporating demand uncertainty within the decision-making model.

The cause of the deterministic passenger load models' poor MPSL-reliabilities is due to the fact that these models do not compensate for the possibility that the flight's final passenger load could exceed its expected value. As a consequence, the models' frequently underestimate the in-flight meal demand, which leads to unintentional meal shortages and lower than expected *actual* PSLs.

*Substitution* [●●] *vs. no substitution* [●●]:

Recall that substitution models refer to the models that consider the substitution behaviour of passengers, while the non-substitution models ignore this behaviour. Unfortunately, the substitution models were found to be less reliable in achieving the *minimum* PSL requirement when compared with each model's respective non-substitution model. Overall, the MPSL-reliability of the Solution Model is 3.3% lower than that of Alternative Model 3. Similarly, Alternative Model 1 is 2.8% less reliable than Alternative Model 2.

The Solution Model obtained an MPSL-reliability of 92.7% when $p^{min} = 92\%$. When incorporating a 5% tolerance for the *minimum* PSL requirement such that $p^{min} = 87.4\%$, the MPSL-reliability of the Solution Model increases to 98.7%, now only 0.6% lower than that of Alternative Model 3. This outcome indicates that the majority of the realisations that did not meet the 92% *minimum* PSL requirement were only slightly lower. The causes of the lower *actual* PSLs are believed to be due to the models' static nature and the simplifying assumption regarding the dependencies among substitutes.

**Static substitution models:** Recall that the solutions of a static substitution model are generated based on the assumption that the products will be optimally allocated to the customers. However, in this report, the models were tested in a dynamic and more realistic environment where the passengers are served in a random order and the aggregate demand is unknown. This means that the optimal allocation of meals could not be guaranteed. As a consequence, the *actual* PSL was frequently lower than the solution's *expected* PSL. Intuitively, this has a negative impact on the models' MPSL-reliability if the *expected* PSL was not sufficiently higher than the *minimum* PSL required.

**Dependencies among substitutes:** As stated earlier, the drawback of the *a priori* probabilities is that it is not able to fully describe the dependencies among the various substitutes. For instance, consider the scenario where chicken meals are under-catered and the only available substitutes for the unmet demand are either fruit platters or vegan meals. Based on the *a*

> *priori* probabilities provided in Table 2.2, both substitutes have a 40% probability of being chosen. The models assume that these probabilities are independent as it is not feasible to model the individual dependencies between substitutes. Accordingly, the model assumes that 40% of the unmet demand could be assigned to each substitute such that 80% of the total unmet demand is allocated. This allocation is unlikely in reality and, according to Vaagen et al. (2011), results in an *upper bound* on the expected outcome, the solution's *expected* PSL. Consequently, the *actual* PSL is frequently lower than the *expected* PSL and the *minimum* PSL.

Intuitively, the method used to approximate a flight instance's PSL also influences a model's MPSL-reliability. In the model formulation given in Chapter 3, the PSL is calculated based on the assumption that a passenger is only *fully* satisfied when the passenger received his first-choice meal. However, when the passenger received meal option $j$ as a substitute for meal option $i$, the passenger's *partial* satisfaction level is approximated with the *a priori* probability $\hat{\alpha_{ji}}$. It is argued that this assumption is realistic, since a passenger will encounter some level of dissatisfaction when the passenger has to select a substitute. However, it could be argued that this approach overestimated a passenger's dissatisfaction. For instance, based on the *a priori* probabilities given in Table 4.2, it was assumed that a passenger was only 40% satisfied when accepting a vegan meal as a substitute for a chicken meal. It is recommended that further research should be conducted to identify the true level of dissatisfaction a passenger will encounter when asked to choose a substitute meal to improve the model's accuracy. More accurate dissatisfaction levels could improve the Solution Model's MPSL-reliability, and subsequently, improve its appeal.

The evaluation of the performance and timeliness of the models were only discussed for the stochastic passenger load models due to the models' consistency in achieving remarkably high and reasonable MPSL-reliabilities for each of the target weight combinations analysed. The benefit of modelling the meal substitution behaviour of passengers within the decision-making model became evident after evaluating the models' performance with a 92% *minimum* PSL requirement. This is because the substitution model – the Solution Model – obtained the lowest objective function outputs when compared with the non-substitution model – Alternative Model 3. Thus, the solutions generated by the Solution Model result in *actual* outputs with less combined and weighted deviation from the 100% PSL target and the zero-waste target. Simply stated, the model is able to find the most favourable balance between the two weighted objectives. The Solution Model's superior performance is attributed to its impressive waste minimisation capabilities as it produced 2.2 fewer surplus meals per flight instance when compared with Alternative Model 3. While 2.2 fewer surplus meal per flight might not sound impressive, the annual cumulative impact thereof is significant from an economic, social and environmental point of view. Furthermore, due to its smaller meal order sizes, the Solution Model also has a desirable impact on pre-consumer waste production.

The Solution Model's superior performance became more significant when batch order constraints were included in the model formulation. This outcome further validates not only the benefit of product substitution, but also its applicability in the in-flight catering industry. The reason being that batch order constraints and kitchen capacity limitations, to name a few, are common factors that have to be taken into account by in-flight caterers on a daily basis.

In conclusion, the Solution Model is superior in terms of performance, whereas Alternative Model 3 is superior in terms of reliability. The recommended model is, therefore, dependent on the catering company's bias towards maximising either performance or reliability. Thus, the decision-making model should incorporate the substitution behaviour of the passenger if the in-flight caterer desires maximum performance, but should be excluded when the caterer desires maximum reliability. It should be noted that the model should include the passenger load uncertainty regardless of the in-flight caterer's bias to improve the model's reliability.

The recommendation stated above does not take into account the in-flight catering company's available resources, time constraints and risk limitations. The Solution Model is slightly more time- and effort-intensive and is accompanied by an additional risk. This could influence the in-flight caterer's bias and the model's applicability. For instance, a time-drive catering company with limited computational resources could be discouraged to choose the Solution Model because of the model's longer and uncertain solving time requirements. In contrast, an environmentally conscious in-flight catering company that aims to attract like-minded passengers might prefer the Solution Model due to its superior waste minimisation capabilities.

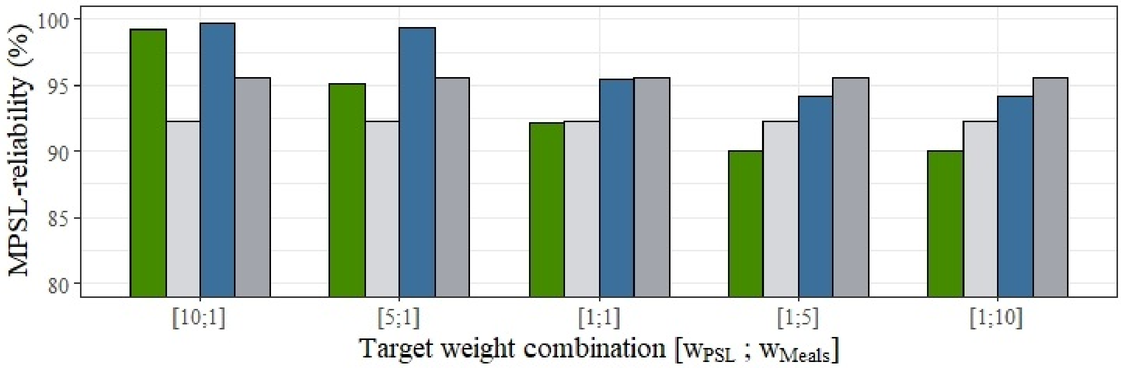
### 6.2.5 Model validation

In an attempt to validate the two stochastic passenger load models, the models were compared against a simple approach that does not require the explicit development of a *decision-making* model. In the Simple Approach Model, the meal order quantities for a specific flight is obtained by dividing the flight's expected passenger load according to the estimated market share of each meal type offered on the in-flight menu. Thereafter, the order quantities are inflated with a predefined rate, denoted by $f$, to add safety stock. The process is expressed mathematically in (6.6).

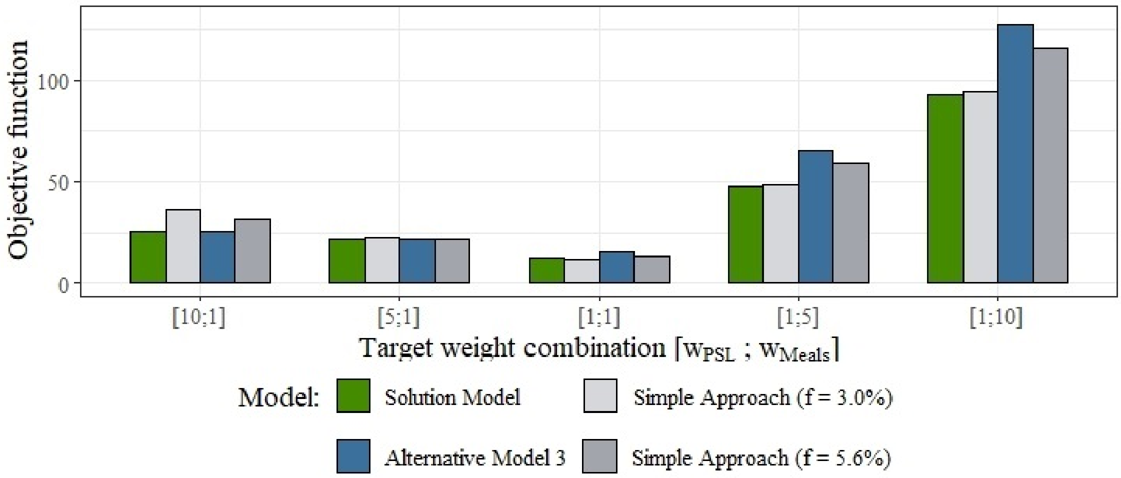
$$x_i = \lceil (1+f)\left(E[\,\widetilde{q}_i]\,E[\widetilde{Z}_0]\right)\rceil, \quad \forall i \in \mathbf{I} \tag{6.6}$$

The market share of meal type $i$ is approximated as the expected value of the meal's random first-choice probability. The meal order quantity of meal option $i \in \{1,3\}$ is further rounded upwards to the nearest multiple of four to take into account the batch order restrictions discussed in Section 6.2.3. After a trial-and-error approach, an inflation rate of 3% is chosen to ensure that the Solution Model and the Simple Approach Model's MPSL-reliability are similar when the target weights are equal. Doing so ensures that the models can be compared fairly with one another. If the MPSL-reliabilities are similar, the model with the highest performance (lowest objective function output) is superior. Figure 6.17 depicts the models' MPSL-reliabilities and objective function outputs.

Despite obtaining a 0.15% higher MPSL-reliability when $w^{PSL} = w^{Meals}$, the Simple Approach Model with $f = 3\%$ produced an average of 0.41 less surplus meals per flight instance and provided a 0.13% higher *actual* PSL when compared with the Solution Model. These results indicate that the Simple Approach Model's set of meal order quantities ($x_i \in \mathbf{I}$) is more efficient than that of the Solution




(a) The MPSL-reliability




(b) The objective function output

Figure 6.17: Outcome of the model validation process (the stochastic passenger load models vs the Simple Approach Model) with a 92% *minimum* PSL requirement.

Model. Stated differently, the Simple Approach Model with $f = 3\%$ is superior to the Solution Model. The same conclusion was reached for Alternative Model 3 after comparing the model against the Simple Approach Model with $f = 5.6\%$. The last-mentioned model is 0.13% more reliable and ensured a 0.2% higher *actual* PSL, even though it produced 1.54 less surplus meal per flight instance than Alternative Mode 3 when $w^{PSL} = w^{Meals}$.

Regrettably, since the models are not superior to the Simple Approach Model, the decision-making models developed could not be validated as a solution opportunity to lessen the wastage dilemma faced by the in-flight catering industry.

It is important to take note that the models cannot be compared when the models' MPSL-reliabilities (or performance) are not similar. To explain, consider the fact that the reliability and performance of the Solution Model are higher than that of the Simple Approach Model with $f = 3\%$ when $w^{PSL} > w^{Meals}$, as seen in Figure 6.17. The higher reliability and performance would imply that the Solution Model is superior. This is, however, not the case because the MPSL-reliability and performance of the Simple Approach Model could be improved by simply choosing an inflation rate greater than 3% when $w^{PSL} > w^{Meals}$. A higher inflation rate would increase the safety stock quantities, which in turn would increase the *actual* PSL and the model's MPSL-reliability obtained. The model's performance would then most likely improve since $w^{PSL} > w^{Meals}$. However, if the models' MPSL-reliabilities are similar, the models' performances provide an indication of the efficiency of the models' solutions relative to one another.

It should be mentioned that the solutions of the Simple Approach Model are fixed, regardless of the values of the chosen target weights. This occurs because the Simple Approach Model's solutions are not influenced by the *minimum* PSL required or the target weights assigned to the 100% PSL target and the zero-waste target. This independence explains the Simple Approach Model's uniform MPSL-reliabilities shown in Figure 6.17a. Furthermore, this independence highlights a disadvantage of the Simple Approach Model as its solutions cannot be aligned with the airline's corporate strategies.

A major advantage of the Simple Approach Model is its simplicity – it is captured with a single equation that is easy to understand. However, it is also its greatest disadvantage when various constraints have to be considered. Unlike the Simple Approach Model, the two stochastic passenger load models can be easily adapted to incorporate various constraints. Accordingly, the stochastic passenger load models should not be disregarded completely because these models could potentially be superior when more constraints have to be incorporated, or when the models are improved as discussed in the following section.

### 6.2.6 Improvement opportunity

An approach to improve the MPSL-reliability and the performance of the decision-making model is to incorporate the known pre-booked meals of the flight under consideration. Recall that most airlines allow passengers to pre-book their preferred meal ahead of the flight's departure. Including this additional information in the model can lower the uncertainty within the model. In turn, this will result in less deviation from the targets and improve the model's reliability and performance.

The first step to add the additional information is to ensure that enough of meal type $i \in \mathbf{I}$ is ordered to cover $m_i$, the known number of pre-booked meals of the respective meal type. This requires the addition of constraint (6.7).

$$x_i \geq m_i, \qquad \forall \qquad i \in \mathbf{I} \tag{6.7}$$

Since more information regarding $d_i$, the primary demand of meal type $i$, is now available, constraint (3.5) must be updated.

$$d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \approx \lceil \widetilde{q}_i \cdot \widetilde{Z}_0 \rceil, \qquad \forall \qquad i \in \mathbf{I} \tag{3.5}$$

The updated constraint is given with (6.8).

$$d_i(\widetilde{\mathbf{q}}, \widetilde{Z}_0) \approx m_i + \lceil \widetilde{q}_i \cdot (\widetilde{Z}_0 - \sum_{i \in \mathbf{I}} m_i)^+ \rceil, \qquad \forall \qquad i \in \mathbf{I} \tag{6.8}$$

The first term in the constraint represents the *known* primary demand for meal type $i$. The second term approximates the additional primary demand resulting from the remaining passengers that did not pre-book any meals. Recall that $(A)^+$ represents $max(0, A)$ and is used to correct situations where $\widetilde{Z}_0$ was underestimated.

Two additional approaches that can also be used to improve the decision-making model is to decompose the set of first-choice probabilities $\widetilde{\mathbf{q}}$ using more than three realisations, and to use more than five bins when simplifying a flight instance's $\pi_0$. These approaches can, however, increase the solving time required by the model.

## 6.3 Concluding remarks

This chapter provided an in-depth analysis of the results obtained when applying the model developed to the numerical example's testing dataset. In summary, the analysis included an inspection of the output of the forecasting model, an evaluation of four decision-making models and an attempt to validate the recommended models.

The forecasting model exhibited optimistic results. Unfortunately, the added benefit of forecasting the *distribution* of the final passenger load versus the single point estimate thereof could not be measured explicitly. It was, however, tested when evaluating the decision-making models by comparing the deterministic passenger load models with the stochastic passenger load models. Unlike the stochastic passenger load models, the deterministic passenger load models were deemed unreliable. This outcome validated the benefit of forecasting the *distribution* of a flight's final passenger load and using it to represent the uncertainty within the model.

The decision-making model developed in Chapter 5 was evaluated and validated by comparing its reliability, performance and timeliness with three alternative models to identify the impact of product substitution and passenger load uncertainty. The four models were solved using LINGO 18.0 to obtain $\mathbf{x}$, the suggested meal order quantities that the respective model deems most efficient. LINGO 18.0 is an optimisation modelling software for linear, non-linear and integer programming (LINDO Systems Inc, NA). The models developed fall under the mixed-integer linear programming category and were solved using the branch-and-bound solver to obtain the globally optimum solution.

As stated, only the two stochastic passenger load models are considered reasonably reliable, which confirmed the value of incorporating the passenger load uncertainty within the inventory decision-making model. Including product substitution has a desired impact on the model's performance and an undesired impact on its reliability and resource requirements. For this reason, the value of product substitution was deemed inclusive and dependent on the in-flight catering company's bias and requirements. The stochastic passenger load models were further evaluated against the Simple Approach Model that does not require the formulation and development of a decision-making model. Unfortunately, unfavourable results were obtained and the models could not be validated as a solution for the wastage dilemma faced by the in-flight catering industry.

# Chapter 7

# Conclusion and recommendations

In-flight catering companies worldwide are faced with the major challenge of meeting the catering requirements of a flight without knowing its final passenger load. This challenge is intensified when a variety of in-flight meal options are offered on-board the flight as it is accompanied by additional uncertainty. The high uncertainty levels within the process make it difficult to obtain reasonably accurate demand predictions using single-point forecasting models and deterministic inventory decision-making models. As a result, in-flight caterers tend to follow an over-catering strategy to mitigate the risk of meal shortages, costly flight delays and passenger dissatisfaction. The drawback of this strategy is the high number of surplus meals that must be discarded as waste. Not only is this food waste a financial burden, but it is also considered unethical from a social and environmental point of view. Catering companies are, therefore, faced with two conflicting objectives − maintain a high and acceptable level of passenger satisfaction while minimising waste resulting from excess meals.

The aim of this dissertation was to investigate if the inclusion of demand uncertainty and product substitution within an inventory decision-making model would be able to help an in-flight catering company reduce waste resulting from surplus in-flight meals, while maintaining an acceptable level of passenger satisfaction. This required the development of a suitable inventory decision-making model that can be used to determine the most efficient set of meal order quantities for a specific flight. The most efficient set of meal order quantities should, firstly, provided sufficient confidence to the catering company that the *minimum* Passenger Satisfaction Level (PSL) required by the catering company will be achieved. This will reduce the need for the catering company to implement the over-catering strategy. Secondly, the most efficient set of meal order quantities will also minimise the weighted sum of deviations from the targets of the two conflicting objectives − a 100% PSL with zero surplus meals (waste).

The model developed can be classified as a stochastic multi-objective Mixed-Integer Programming (MIP) model with fixed recourse and two-way, stock-out based partial consumer-driven product substitution. A *MIP* model was selected due to its flexibility and ease with which it can be changed to accommodate the specific needs of an in-flight catering company. *Product substitution* was incorporated through the use of *a priori* substitution probabilities to approximate the substitution behaviour of passengers. The multi-objective nature of the in-flight catering industry was addressed using *pre-emptive goal programming*, where the target weights were used to indicate the relative importance of the two targets. This ensures that the model can be aligned with the strategies of the in-flight catering company and airline. Lastly, the model incorporated *demand uncertainty* through the inclusion of the stochastic final passenger load and the random set of first-choice probabilities. For this reason, the model had to be transformed into its deterministic equivalent before it could be solved using standard optimisation software. This was achieved using *recourse programming* to decompose each stochastic variable into a set of realisations with known occurrence probabilities. Each set of realisations was simplified in size to ensure that a solution can be generated within a reasonable time when using a 3.1 GHz Intel(R) Core(TM) i3-2100 CPU with 6.0 GB of RAM. It is expected that this simplification had an undesired consequence on the model's performance and reliability. Further research is required to identify the computational resources commonly available at an in-flight catering company. Greater computational resources will require less simplification and, ultimately, could improve the decision-making model's value and appeal.

A forecasting model was also developed to estimate the probability distribution of the flight under consideration's final (stochastic) passenger load. The forecasting model consisted of a time-inhomogeneous Markov chain and a multiple regression model. Furthermore, the forecasting model was extended to allow overbooking before the departure of the flight. The drawback of the forecasting model is that it must be trained separately for flights that vary by departure date, duration, destination and passenger class, to name a few factors. Fortunately, the training of the forecasting model is almost instantaneous but requires a fair amount of historical data of past observations of the flight under consideration. A possible approach to overcome the two drawbacks could be to group flights with similar booking behaviours and seasonality trends to share the trained forecasting model. It should also be noted that, due to the modular design of the inventory support model (Part A and Part B), the chosen forecasting model could easily be replaced with an alternative model if it meets the requirements of the MIP decision-making model. It is, therefore, recommended that further research should be conducted to validate that the chosen forecasting model is the superior choice. The first phase of this validation study should focus on improving the chosen forecasting model. Potential improvement opportunities include increasing the number of intervals in the forecasting horizon and adding more covariates to the forecasting model's regression analysis. Possible covariates to consider include public holidays, the number of flights occurring on the day under consideration and the time of day when the flight is scheduled to depart.

The model developed was evaluated against three alternative models that lacked either passenger load uncertainty, product substitution or both. Together, these four models corresponded with the four quadrants of a strategic scenario planning matrix. Its purpose was to investigate the value of incorporating product substitution and demand uncertainty within the decision-making model. It was argued that the inclusion of demand uncertainty would improve the model's reliability because the additional information provides the model with the ability to compensate for demand fluctuations. Furthermore, it was argued that the inclusion of the meal substitution behaviour of passengers would allow the model to consider the sharing of safety stock among substitutable meals to reduce the number of surplus meals produced.

Favourable results were obtained for the inclusion of the uncertainty regarding a flight's final passenger load. Overall, the stochastic passenger load models were at least 9.2% more reliable in achieving the *minimum* PSL required when compared with the models that ignored the passenger load uncertainty. The deterministic passenger load models were deemed unreliable because the models obtained reliability outcomes below 70% when the zero-waste target was favoured. In contrast, the reliability of the stochastic passenger load models exceeded 87.0% irrespective of the weight assigned to the 100% PSL and zero-waste targets. These results validated the value of passenger load uncertainty and the inclusion thereof in the decision-making model is, therefore, recommend. To further improve the model's reliability, it is recommended that the model should be expanded to include the *known* number of pre-booked in-flight meals to lessen some of the uncertainty faced by the model.

The benefit of the inclusion of *product substitution* is, unfortunately, inconclusive as it depends on the in-flight catering company's bias towards maximising either reliability or performance. When the model incorporates the meal substitution behaviour of passengers, the model's performance improves while its reliability is slightly reduced. The performance improvement is accredited to the risk-pooling capabilities of the *substitution* model. On average, the substitution model produced 2.2 fewer surplus meals per flight instance when compared with the stochastic passenger load model that ignored the passenger's substitution behaviour. As a trade-off, the substitution model is 3.3% less reliable in guaranteeing a 92% *minimum* PSL requirement. Possible explanations for the model's reliability reduction relate to the model's *static* nature and the model's inability to capture the dependencies among the in-flight meals available as substitutes. This uncovers a potential future research opportunity where the comparison process should also include a *dynamic* product substitution model. Based on the results obtained, the research question stated in Chapter 1 can be answered as follows:

> *The majority of this dissertation demonstrated and validated the value of passenger load uncertainty and product substitution within an inventory decision-making model for the in-flight catering industry; Including the flight's final passenger load uncertainty will improve the decision-making model's reliability in achieving the minimum PSL required, while the inclusion of product substitution will reduce the number of surplus meals produced at the expense of a slightly lower reliability outcome. Unfortunately, the decision-making models – with and without product substitution – could not be validated as a solution opportunity for the wastage dilemma faced by the in-flight catering industry.*

Regrettably, the inventory decision-making models developed could not be validated against a simple approach that does not require the explicit development of a decision-making model. In summary, the decision-making models could not provide a higher performance at a similar level of reliability when compared with this approach. That being said, the validation process did not *sufficiently* take into account the major advantage of the decision-making model – its modelling flexibility. The decision-making model's flexibility ensures that the model can easily accommodate various constraints simultaneously that will influence the most efficient solution, and subsequently, the model's performance and reliability. This statement is validated with the change observed in the models' performances in Section 6.2.3 when two batch order constraints were added to the model formulation. Based on the results, it is expected that the decision-making model *with* product substitution will be more beneficial, and potentially superior, when the model has to incorporate additional process constraints. Subsequently, it is still inconclusive as to whether or not the inventory decision-making model developed is worthwhile, and if it can help an in-flight catering company reduce waste resulting from surplus in-flight meals while maintaining an acceptable level of passenger satisfaction. Further research is required where the validation process should consider various process scenarios with different process constraint severities and combinations.

In addition to the above, it is recommended that further research should also be conducted to identify the true level of dissatisfaction a passenger will encounter when asked to choose a substitute meal. This research should aim to address the concern, as mentioned in Section 6.2.4, that the decision-making model overestimates a passenger's dissatisfaction level when using *a-priori* probabilities. The over-estimation of a passenger's dissatisfaction hinders the decision-making model's full potential because it discourages the model to utilise the substitutability of in-flight meals.

It should be emphasised that the results and conclusions made within this report are based on synthetic data. While care was taken to generate an appropriate dataset, it is impossible to capture all of the elements that influence the flight booking behaviour of passengers. It is recommended that the work completed in this dissertation should be repeated using actual industry data with realistic process constraints to challenge and validate the conclusions made.

In conclusion, this dissertation evaluated the potential of demand uncertainty and product substitution within an inventory decision-making model as a solution opportunity for the wastage dilemma faced by the in-flight catering industry. To the author's best knowledge, no previous study has attempted to utilise the risk-pooling capabilities of a product substitution model to reduce in-flight waste resulting from surplus meals. It is, therefore, believed that this report makes a valuable contribution to the in-flight catering industry and the global strive towards sustainability.

# Bibliography


Ahmed, H. (2019). Formulation of two-stage stochastic programming with fixed recourse. *Britain International of Exact Sciences (BIoEx) Journal*, 1(1):18–21.

Airport Company South Africa (2020). Passenger and aircraft charts. Retrieved from https://www.airports.co.za/Documents/Total_National_Pax_January20.pdf. Accessed April 2, 2020.

Alekseev, K. and Seixas, J. (2009). A multivariate neural forecasting modeling for air transport–preprocessed by decomposition: A brazilian application. *Journal of Air Transport Management*, 15(5):212–216.

Allen, M. P. (1997). The problem of multicollinearity. *Understanding regression analysis*, pages 176–180.

Anon. (2018). 17 things airlines change that passengers do not notice. *Aerotime.aero*. Retrieved from https://www.aerotime.aero/aerotime.extra/23619-17-things-airlines-change. Accessed April 6, 2020.

Bean, W. L. (2011). Inventory management under uncertainty: a military application. Master's thesis, University of Pretoria.

Birge, J. R. and Louveaux, F. (2011). *Introduction to stochastic programming*, chapter Uncertainty and Modeling Issues, pages 57–70. Springer Science & Business Media, 2 edition.

Blanca-Alcubilla, G., Roca, M., Bala, A., Sanz, N., De Castro, N., and Fullana-i Palmer, P. (2019). Airplane cabin waste characterization: Knowing the waste for sustainable management and future recommendations. *Waste Management*, 96:57–64.

Brochado, C. and Freedman, M. (2009). Does reducing portion size reduce food intake and plate waste? *Journal of the Academy of Nutrition and Dietetics*, 109(9):A47.

Bryant, K. (2019). This airline is now serving edible coffee cups to reduce plastic waste. *Reader's Digest*. Retrieved from https://www.rd.com/culture/airline-edible-coffee-cups/. Accessed April 6, 2020.

Chang, Y. Y.-C. and Jones, P. (2007). Flight catering: An Investigation of the Adoption of Mass Customisation. *Journal of Hospitality and Tourism Management*, 14(1):47–56.

Danesi, A., Mantecchini, L., and Paganelli, F. (2017). Long-term and short-term forecasting techniques for regional airport planning. *ARPN Journal of Engineering and Applied Sciences*, 12(2):739–745.

Daniel, D. (2019). Bullwhip Effect. *SearchERP*. Retrieved from https://searcherp.techtarget.com/definition/bullwhip-effect. Accessed July 12, 2021.

Daniel, L. (2021). SAA could fly agian in May - if it can find new pilots. *BUSINESS INSIDER South Africa*. Retrieved from https://www.businessinsider.co.za/saa-could-fly-again-in-may-if-it-can-find-new-pilots-2021-2. Accessed February 25, 2021.

Dantas, T. M., Oliveira, F. L. C., and Repolho, H. M. V. (2017). Air transportation demand forecast through bagging holt winters methods. *Journal of Air Transport Management*, 59:116–123.

El-Mobaidh, A. M., Taha, M. R., and Lassheen, N. K. (2006). Classification of in-flight catering wastes in Egypt air flights and its potential as energy source (chemical approach). *Waste Management*, 26(6):587–591.

Emirates.com (NA). Dietary requirments. *Emirates.com*. Retrieved from https://www.emirates.com/za/english/before-you-fly/travel/dietary-requirements/. Accessed September 28, 2020.

Gaur, V. and Honhon, D. (2006). Assortment Planning and Inventory Decisions Under a Locational Choice Model. *Management Science*, 52(10):1528–1543.

Gilland, W. G. and Heese, H. S. (2013). Sequence matters: Shelf-space Allocation under Dynamic Customer-Driven Substitution. *Production and Operations Management*, 22(4):875–887.

Goto, J. H. (1999). A Makrov Decision Process Model for Airline Meal Provisioning. Master's thesis, The University of British Columbia.

Goto, J. H., Lewis, M. E., and Puterman, M. L. (2004). Coffee, tea, or?: a Markov decision process model for airline meal provisioning. *Transportation Science*, 38(1):107–118.

Hasachoo, N. and Masuchun, R. (2016). Schedule nervousness in production operations of an airline catering company: The challenge of an effective demand response program. *2016 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*.

Honhon, D., Gaur, V., and Seshadri, S. (2010). Assortment planning and inventory decisions under stockout-based substitution. *Operations research*, 58(5):1364–1379.

Huang, D., Zhou, H., and Zhao, Q.-H. (2011). A competitive multiple-product newsboy problem with partial product substitution. *Omega*, 39(3):302–312.

IATA (2019). Cabin Waste Fact Sheet. *Iata.org*. Retrieved from https://www.iata.org/pressroom/facts_figures/fact_sheets/Documents/fact-sheet-cabin-waste.pdf. Accessed September 11, 2019.

Joubert, J. and Conradie, D. (2005). A fixed recourse integer programming approach towards a scheduling problem with random data: A case study. *ORiON*, 21(1):1–11.

Kök, A. G. and Fisher, M. L. (2007). Demand estimation and assortment optimization under substitution: Methodology and application. *Operations Research*, 55(6):1001–1021.

Kök, A. G., Fisher, M. L., and Vaidyanathan, R. (2009). Assortment planning: Review of literature and industry practice. 122:99–153.

Kollau, R. (2019). Airlines are rewarding passengers who bring-their-own cup. *Airlinetrends.com*. Retrieved from http://www.airlinetrends.com/2019/07/25/airlines-are-rewarding-passengers-who-bring-their-own-cup/. Accessed September 24, 2020.

Koppelman, F. S., Coldren, G. M., and Parker, R. A. (2008). Schedule delay impacts on air-travel itinerary demand. *Transportation Research Part B: Methodological*, 42(3):263–273.

Lasaridi, K., Abeliotis, K., Manios, T., Zorpas, A., Chroni, C., and Terzis, E. (n. d.). Food waste prevention in the hospitality sector: prospects and challenges. *s. l.*

Lee, A. O. (1990). *Airline reservations forecasting: Probabilistic and statistical models of the booking process*. PhD thesis, Massachusetts Institute of Technology.

Leedham, M. (2020). The sustainable in-flight meal tray: Partially edible and 100eco-friendly. *Travel to Change the world*. Retrieved from https://traveltochangetheworld.com/edible-in-flight-meal-trays-could-the-dream-become-a-reality/. Accessed on April 5, 2020.

Li, X., Poon, C., Lee, S., Chung, S., and Luk, F. (2003). Waste reduction and recycling strategies for the in-flight services in the airline industry. *Resources, Conservation and Recycling*, 37:87–99.

LINDO Systems Inc (NA). Local optima vs. global optima. *LINGO 17 Online Users Manual*. Retrieved from https://www.lindo.com/doc/online_help/lingo17_0/local_optima_vs__global_optima.htm#:~:text=When%20LINGO%20finds%20a%20solution,feasible%20solutions%20to%20the%20model. . Accessed January 23, 2021.

Lynes, J. and Becken, S. (2002). In-flight impact: One long flight can blow your annual $C0_2$ budget. *Alternatives Journal*, 4(4):37.

Mahajan, S. and Van Ryzin, G. (2001). Stocking Retail Sssortments under Dynamic Consumer Substitution. *Operations Research*, 49(3):334–351.

Maluleke, R. (2019). Towards measuring the extent of food security in South Africa: An examination of hunger and food adequacy . Retrieved from http://www.statssa.gov.za/publications/03-00-14/03-00-142017.pdf. Accessed September 4, 2019.

McCool, A. C. (1995). *Inflight Catering Management*, chapter 3, pages 49–53. John Wiley & Son Limited.

Megodawickrama, P. (2018). Significance of Meal Forecasting in Airline Catering on Food Waste Minimization. *15th International Conference on Business Management (ICBM 2018)*, pages 1131–1144.

Netessine, S. and Rudi, N. (2003). Centralized and Competitive Inventory Models with Demand Substitution. *Operations Research*, 51(2):329–335.

Papargyropoulou, E., Lozano, R., Steinberger, J. K., Wright, N., and bin Ujang, Z. (2014). The food waste hierarchy as a framework for the management of food surplus and food waste. *Journal of Cleaner Production*, 76:106–115.

Rajaram, K. and Tang, C. (2001). The impact of product substitution on retail merchandising. *European Journal of Operational Research*, 135(3):582–601.

Rajgopal, J. (2004). Principles and applications of operations research. *Maynards Industrial Engineering Handbook.–2004.–P*, 5:11–27.

Rardin, R. L. (1998). *Optimization in operations research*, volume 166, chapter Multiobjective Optimization and Goal Programming, pages 373–401. Prentice Hall Upper Saddle River, NJ.

Sambo, N. and Hlengwa, D. (2018a). Post-flight food waste and corporate social responsibility at South Africa Airways: Perceptions of employees at Air Chefs South Africa. *African Journal of Hospitality, Tourism and Leisure*, 7(4).

Sambo, N. and Hlengwa, D. (2018b). Post-flight food waste and corporate social responsibility at South Africa Airways: Perceptions of employees at Air Chefs South Africa. Master's thesis, Durban University of Technology.

Shin, H., Park, S., Lee, E., and Benton, W. (2015). A classification of the literature on the planning of substitutable products. *European Journal of Operational Research*, 246(3):686–699.

Sivrikaya, O. and Tunç, E. (2013). Demand forecasting for domestic air transportation in turkey. *The Open Transportation Journal*, 7(1).

Skiti, S. (2013). SAA dumps mounds of food. *Sunday Times*.

Smith, S. A. and Agrawal, N. (2000). Management of Multi-item Retail Inventory Systems with Demand Substitution. *Operations Research*, 48(1):50–64.

South African Airways (2018). SAA Annual Repoprt 2017. Retrieved from https://www.flysaa.com/documents/51855150/0/SAA_IAR+2017.pdf/22db54be-b1f5-404a-99fd-d12f3fe9e56b. Accessed September 22, 2019.

Srisaeng, P., Baxter, G. S., and Wild, G. (2015). Forecasting demand for low cost carriers in australia using an artificial neural network approach. *Aviation*, 19(2):90–103.

Teoh, Lay Eng Jaspreet, K. and Singh, I. (2018). A Bi-Objective Optimization Approach for Inflight Food Waste Reduction. *E3S Web of Conferences*, 65.

Thamagasorn, M. and Pharino, C. (2019). An analysis of food waste from a flight catering business for sustainable food waste management: A case study of halal food production process. *Journal of Cleaner Production*, 228:845–855.

Tofalli, N., Loizia, P., and Zorpas, A. A. (2018). Passengers waste production during flights. *Environmental Science and Pollution Research*, 25(36):35764–35775.

Vaagen, H., Wallace, S. W., and Kaut, M. (2011). Modelling consumer-directed substitution. *International Journal of Production Economics*, 134(2):388–397.

Van Ostaijen, T., Santos, B. F., and Mitici, M. (2017). Dynamic Airline Booking Forecasting. *Proceedings of Air Transport Research*.

Wang, X. and Disney, S. M. (2016). The bullwhip effect: Progress, trends and directions. *European Journal of Operational Research*, 250:691–701.

Wickham, R. R. (1995). Evaluation of forecasting techniques for short-term demand of air transportation. Technical report, Cambridge, Mass.: Massachusetts Institute of Technology, Dept. of Aeronautics & Astronautics. Flight Transportation Laboratory.

Winston, W. L. (2004a). *Introduction to Probability models: Operations research*, volume 2, chapter Probabilistic Inventory Models, pages 138–140. Curt Hinrichs, 4th edition.

Winston, W. L. (2004b). *Introduction to Probability models: Operations research*, volume 2, chapter Makrov Chains, pages 181–190. Curt Hinrichs, 4th edition.

Yücel, E., Karaesmen, F., Salman, F. S., and Türkay, M. (2009). Optimizing product assortment under customer-driven demand substitution. *European Journal of Operational Research*, 199(3):759–768.

Zeppetella, L., Gebennini, E., Grassi, A., and Rimini, B. (2017). Optimal production scheduling with customer-driven demand substitution. *International Journal of Production Research*, 55(6):1692–1706.

Zhong, Z., Salleh, S. R., Chow, W., and Ong, Z. (2016). Studies of air traffic forecasts, airspace load and the effect of ads-b via satellites on flight times. In *Advanced Free-Space Optical Communication Techniques and Applications II*, volume 9991, page 99910B. International Society for Optics and Photonics.

# Appendix A

# The forecasting model

The modelled Transition Probability Matrix (TPM)s that encompass the forecasting model are given in this appendix, along with the results of the regression model that it used to derive the final interval's modelled TPM specific to a flight observation.

## The modelled transition probability matrices

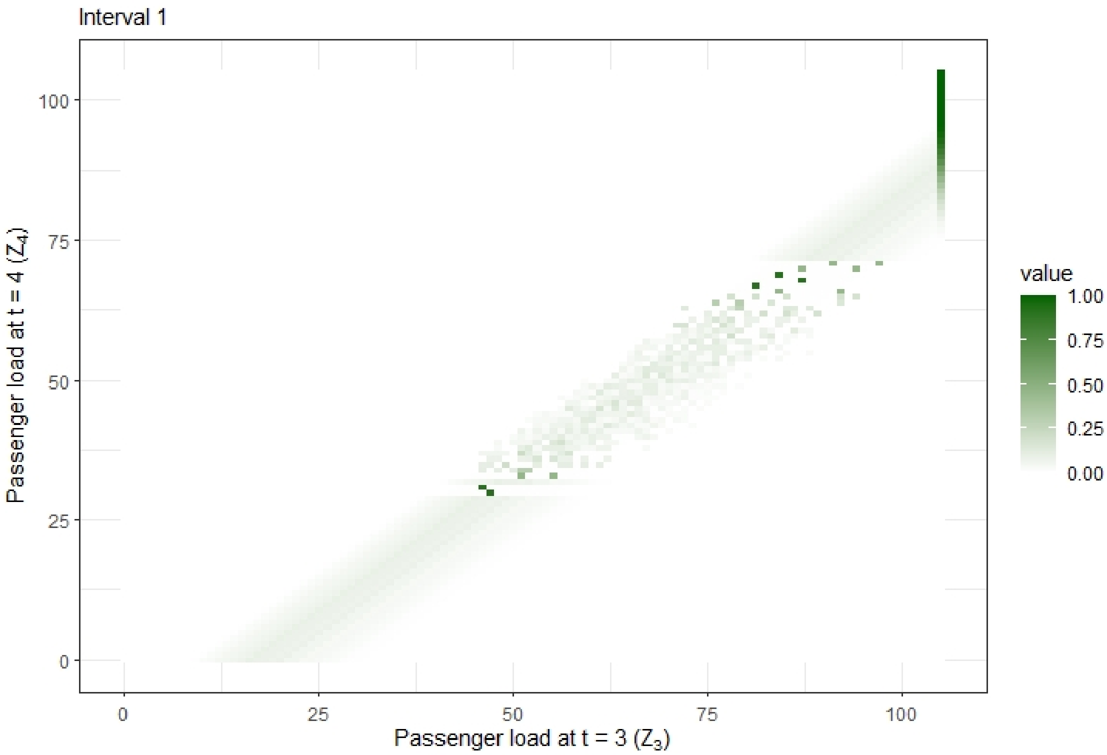



Figure A.1: The modelled TPM for the first interval ($P_{4,3}$) when $\phi^* = 0.91$.

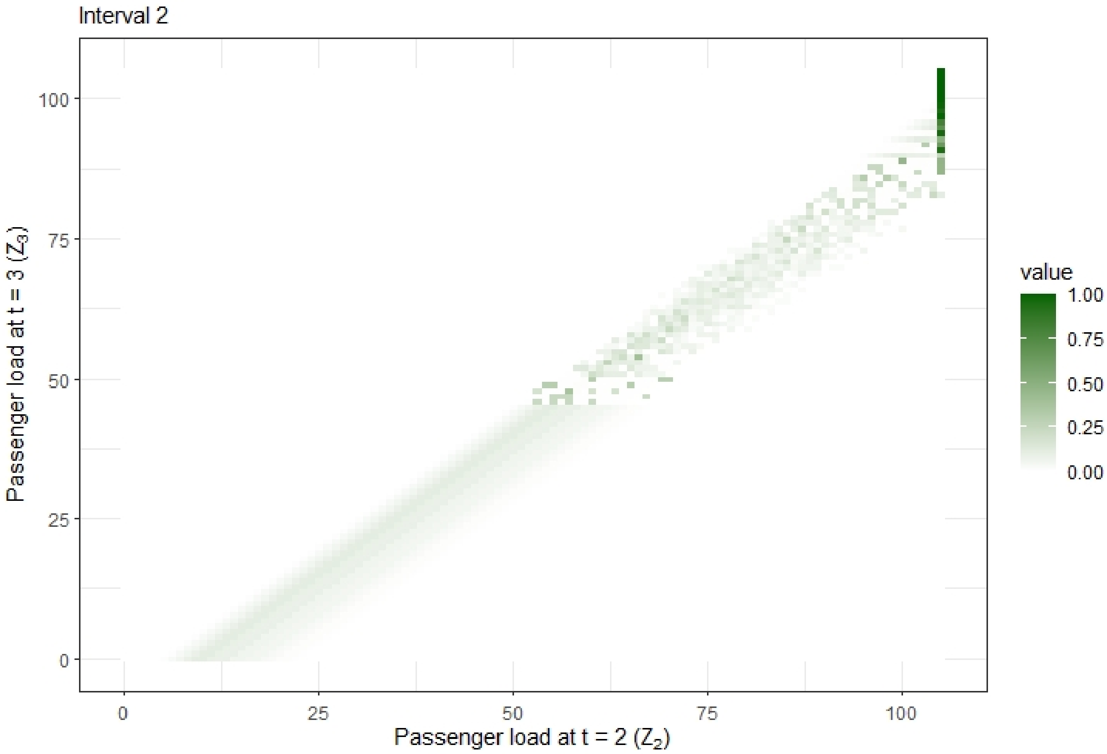



Figure A.2: The modelled TPM for the second interval ($P_{3,2}$) when $\phi^* = 0.91$.

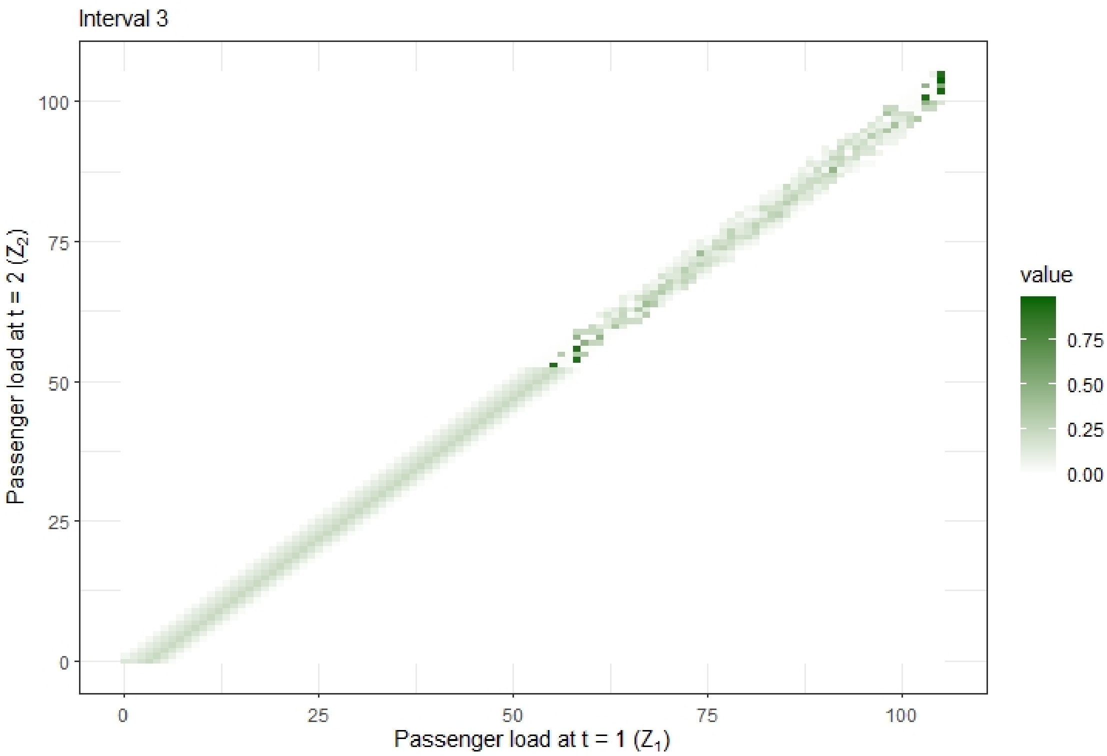



Figure A.3: The modelled TPM for the third interval ($P_{2,1}$) when $\phi^* = 0.91$.

## The regression model

Recall that the regression model is formulated as shown in (A.1).

$$
\begin{aligned}
E[Y_{1,0} \,|\, Z_1 = k] = \beta_0 & + \beta_1 \cdot (k) + \beta_2 X_{Tue} + \beta_3 X_{Wed} + \beta_4 X_{Thu} + \beta_5 X_{Fri} \\
& + \beta_6 X_{Sat} + \beta_7 X_{Sun} + \beta_8 X_{Feb} + \beta_9 X_{Mar} + \beta_{10} X_{Apr} \\
& + \beta_{11} X_{May} + \beta_{12} X_{Jun} + \beta_{13} X_{Jul} + \beta_{14} X_{Aug} + \beta_{15} X_{Sep} \\
& + \beta_{16} X_{Oct} + \beta_{17} X_{Nov} + \beta_{18} X_{Dec} + \beta_{19} \cdot (year) + \epsilon \qquad \text{(A.1)}
\end{aligned}
$$

The values of the above parameters are listed in Table A.1.

Table A.1: The parameters of the multiple regression model obtained using the training dataset.

| $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ |
|---|---|---|---|---|
| 1.20484 | -0.04065 | 0.13544 | -0.03031 | 0.11691 |

| $\beta_5$ | $\beta_6$ | $\beta_7$ | $\beta_8$ | $\beta_9$ |
|---|---|---|---|---|
| 0.09537 | -0.12318 | -0.09479 | 0.06336 | -0.37080 |

| $\beta_{10}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{14}$ |
|---|---|---|---|---|
| -0.22819 | 0.11583 | -0.14645 | 0.06945 | 0.22776 |

| $\beta_{15}$ | $\beta_{16}$ | $\beta_{17}$ | $\beta_{18}$ | $\beta_{19}$ |
|---|---|---|---|---|
| -0.06298 | -0.34846 | -0.10650 | -0.12855 | 0.05243 |

The root mean square error ($s_e$) of 1.509905 is obtained using (A.2), where $Z_0^{RM}$ denotes the estimated final passenger load obtained using the regression model.

$$
s_e = \sqrt{\frac{\sum_{\gamma \in \dot{\gamma}} (Z_0^{RM} - Z_0^*)^2}{|\dot{\gamma}|}} \qquad \text{(A.2)}
$$

# Appendix B

# Model results

## B.1 Pseudocode for model comparison process

```
Data: π_0, q
Result: Matrix containing Z_0^* and d_i ∀I for 1000 realisations
Function:
    rMatrix ← Initialize matrix for 1000 flight instance realisations
    for each realisation r in the range 1 to 1000 do
        Z_0^{*,r} = sample from {0,1,...,99,100} with probability distribution π_0
        for each meal type i ∈ I in random order do
            fc_i^{prob} = sample first-choice prob. from norm(mean(q_i), std(q_i))
            d_i^r = ceiling( fc_i^{prob} · Z_0^{*,r})
            if ∑_{j∈I} d_j^r exceeds the aggregate demand Z_0^{*,r} then
                reduce d_i^r to ensure that ∑_{j∈I} d_j^r = Z_0^{*,r}
            end
        end
        rMatrix[r, ] ← store Z_0^{*,r} and d_i^r ∀I
    end
    return rMatrix
End Function
```

**Algorithm 1:** Step 1 - Create a set of realisations for a specific flight instance with the given probability distribution $\pi_0$.

```
Data: rMatrix, x, w^{PSL}, w^{Meals}, α̂
Result: Model reliability and additional statistics
Function:
    for each realisation r in rMatrix do
        reliabilityCount = 0 for each meal type i in I do
            inStock[i] = x[i]
        end
        pOrder = sample Z_0^{*,r} meals from {d_1^r, d_2^r, d_3^r, d_4^r} without replacement
        for each passenger p from 1 to Z_0^{*,r} do
            fc = pOrder[p]
            if inStock[fc] > 0 then
                Increment y[fc] with one unit to assign meal to passenger
                Decrement inStock[fc] with one unit to remove stock
            end
            else
                while passenger did not receive a meal do
                    if no possible substitute remaining then
                        Increment underCater with one unit
                        Exit while loop
                    end
                    sub = remaining substitute with highest α̂_{sub,fc} value
                    if inStock[sub] > 0 then
                        subProbability = randomly sample from {0, ..., 1}
                        if subProbability ≤ α̂_{sub,fc} then
                            Increment z[sub][fc] to assign sub to passenger
                            Decrement inStock[fc] with one unit
                            Exit while loop
                        end
                    end
                    Remove sub as a possible substitute meal
                end
            end
        end
        Δ^r_{PSL} = calculate deviation from goal 1
        Δ^r_{Meals} = calculate deviation from goal 2
        objectiveValue^r = w_1 · dev_1^r + w_2 · dev_2^r
        if objectiveValue^r ≤ BMV then
            Increment reliabilityScore
        end
        rMatrix[r, ] ← store dev_1^r, dev_2^r and objectiveValue^r
    end
    modelReliability = reliabilityScore / 1000
    return modelReliability, rMatrix
End Function
```

**Algorithm 2:** Step 2 and 3 - Determine model reliability.

## B.2 Selected flight instances

Figure B.1 and Figure B.2 shows the final passenger load probability distributions ($\pi_0$) of the 16 flight instances (**F**) analysed. Notice that a few flight instances (such as flight instance numbers 428 and 423) look very similar. It is believed that this is the result when only a few related transitions are observed when creating the Transition Probability Matrix using the Absolute Passenger Loads (TPM-APL). Thus, these $\pi_0$ are heavily reliant on the Transition Probability Matrix using Differences (TPM-D) which results in the similar distributions observed.

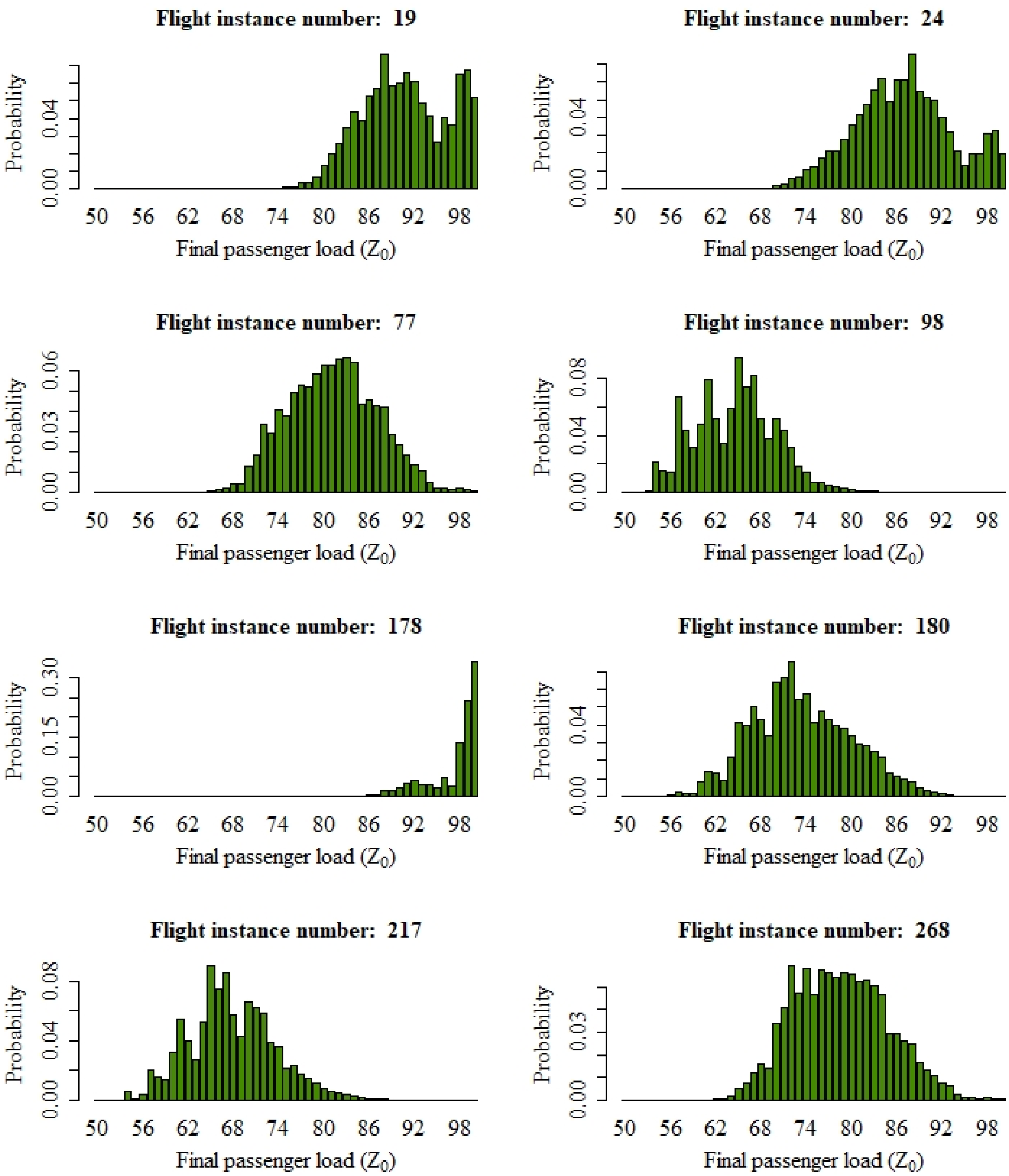



Figure B.1: The probability distribution of the final passenger load ($\pi_0$) of the first eight of the 16 flight instances analysed.

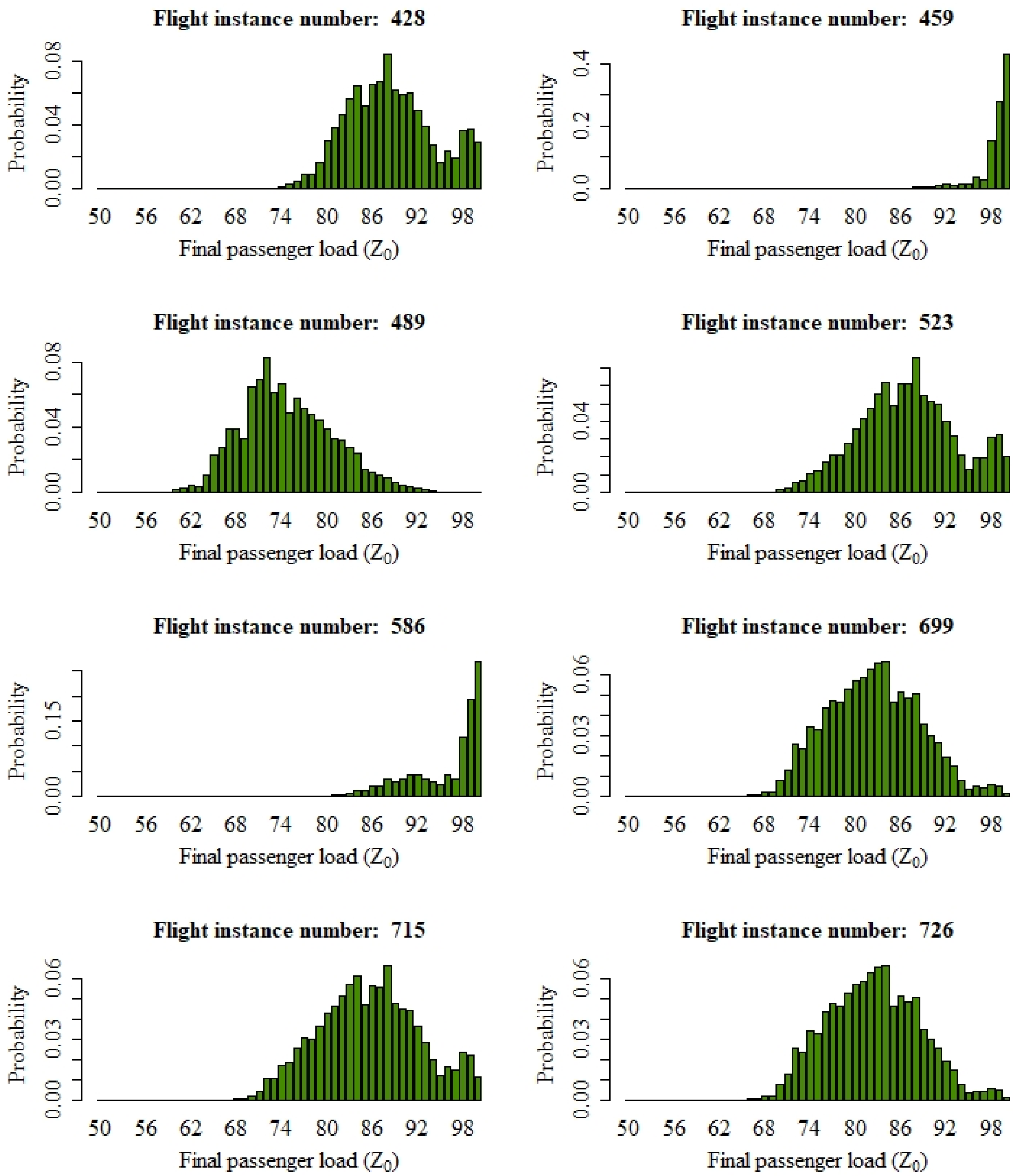
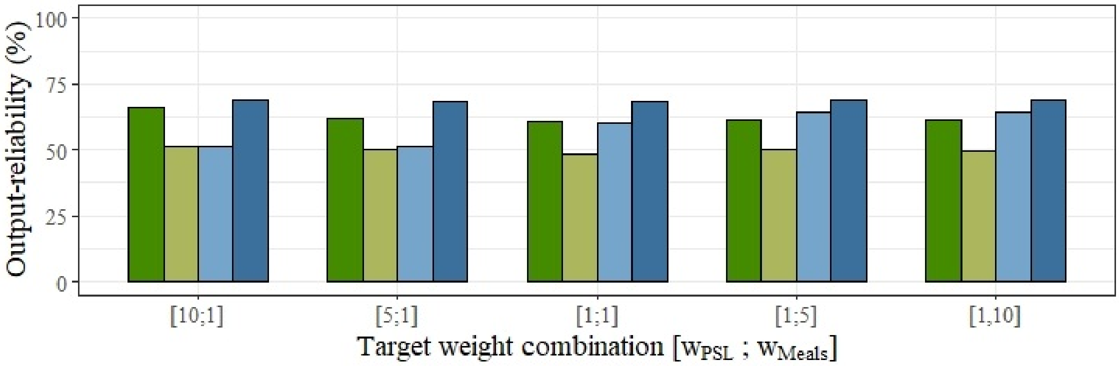


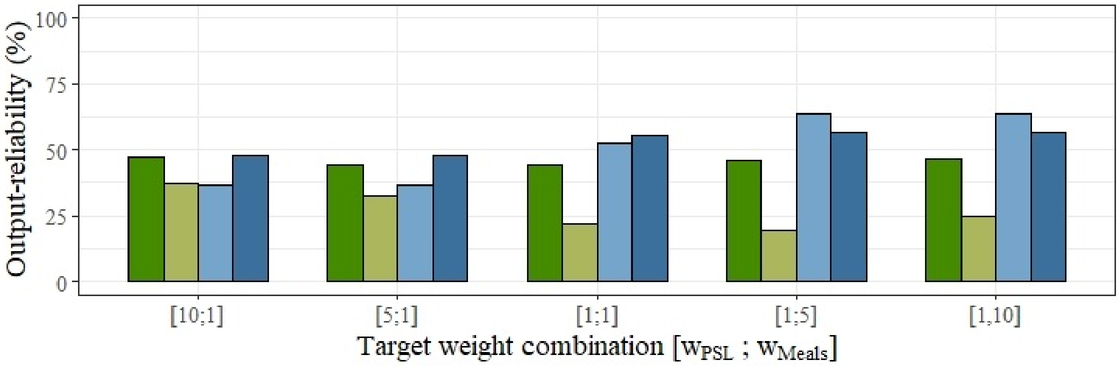

Figure B.2: The probability distribution of the final passenger load ($\pi_0$) of the last eight of the 16 flight instances analysed.

## B.3 Output-reliability results

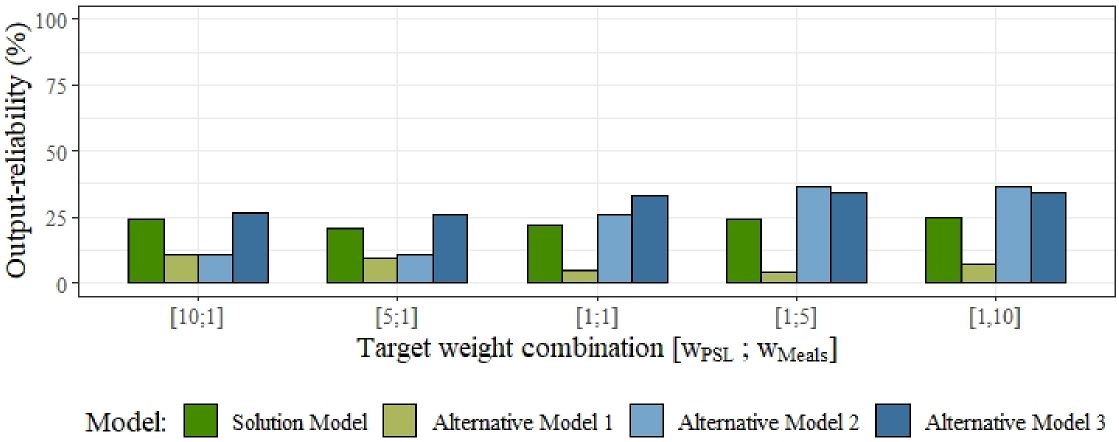
Figure B.3 presents the results obtained per target weight combination analysed for the three output-reliability measures when $p^{Min} = 92\%$.




(a) The *expected* PSL output-reliability




(b) The *expected waste* output-reliability




(c) The *overall* output-reliability

Figure B.3: The output-reliabilities per target weight combination analysed with a 92% *minimum* PSL requirement.