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A three-echelon supply chain for economic growing quantity model with price- and freshness-dependent demand: Pricing, ordering and shipment decisions

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ABSTRACT

The demand for perishable food products is often influenced by the selling price and the age of the items. This is because perishable food products have become commodities from consumers' point of view, hence, there are very little differences between competing brands. Consequently, factors like price and freshness (or age) become important determinants of consumer demand. This fact has been used to develop several models for managing perishable inventory. However, most of these models were developed from the perspective of a retailer. Today's increasingly competitive business environment has forced companies to collaborate with fellow supply chain members in an effort to improve profitability and operational efficiency. With this in mind, this article presents a model for managing inventory in a perishable food products supply chain that begins with farming operations where live inventory items are reared and ends with the consumption of processed inventory. The farming and consumption (retail) stages are connected by a processing stage during which live inventory is processed into a consumable form. Consumer demand at the retail stage is a function of the selling price and the freshness of the processed inventory. The farming, processing and retail stages are the three-echelons of the proposed supply chain aimed at maximising the joint supply chain profit. Through a numerical example, the benefits of jointly optimising the inventory replenishment policy (among all three echelons) are quantified by comparing the network performance of a joint optimisation approach (i.e. centralised) to that of an equivalent independent (i.e. decentralised) optimisation policy.

1. Introduction

1.1. Context

The management of perishable inventory items has been a subject of interest since the publication of the seminal model by Ghare and Schrader [1]. Of late, several studies have incorporated pricing decisions in perishable inventory models, for instance, Chen et al. [2], Wu et al. [3] and Feng et al. [4]. The models presented in these studies are aimed at jointly optimising the lot-size and the selling price of perishable inventory items. These three models, as well as other extensions based on them, were formulated specifically for perishable food items, and consequently, the demand rate used in these models had a few characteristics peculiar to perishable food products. The focus of this study is on two of those characteristics, which are the dependence of the demand rate on the price and the freshness of the items. These are two of the most important demand characteristics of perishable food products such as meat, seafood, fruits and vegetables. Two reasons may be adduced to the importance of these characteristics. Firstly, given the commoditised nature of groceries (and by extension food products), there is very little to differentiate between competing brands. As a result, the selling price is one of the most important factors that affect consumers' purchase decisions. Secondly, consumers prefer perishable food products when they are fresh implying that consumers are less likely to buy a particular product if it has been on shelves for longer periods because the longer it is on the shelves, the less fresh it becomes.

Although the aforementioned studies accurately depict the inventory behaviour of perishable food products, they are all focused on (and limited to) the retail end of the supply chain. Decisions affecting the price and the length of stay of perishable food items on shelves are not limited to those taken at the retail end of the chain. To account for the entire supply chain, studies by Cai et al. [5], Cai et al. [6], Wu et al. [7] and Ma et al. [8] formulated inventory control models for perishable food products with price- and freshness-dependent demand in two-

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and three-echelon supply chains. Nonetheless, hese four studies did not consider the primary source of most food products. In reality, the primary source of these products is living organisms such as crops and livestock which are reared at farms. Furthermore, products purchased at retail outlets are seldom consumed in their original form. Most of the times, they have to be transformed into a different form that is suitable for human consumption and the transformation processes often take place in food processing plants.

1.2. Purpose

This study aims to consider the implications of price and freshness (measured through the age of the item) on the inventory management policies of a multi-echelon supply chain of growing items. To this end, an integrated model for managing inventory in a three-echelon supply chain for growing items is proposed. The three echelons correspond to the farming, processing and consumption (retail) stages of a simplified value chain for perishable food products. At the farming echelon, live items are reared until their weight reaches a specific amount. During the growing period, it is assumed that a certain fraction of the growing items die as a result of illnesses and predators. Following the growth period, the items are transformed into a form that is safe for consumption. In the case of meat, the transformation process typically entails slaughtering, cutting and packaging. In the context of this study, all the tasks taking place at this echelon are collectively termed processing and they occur at a given finite rate. At the final echelon, consumer demand for the processed item is met through sales at a retail outlet, and this demand is assumed to be a function of the product's selling price and freshness, measured through the age of the product from the time of processing.

1.3. Relevance

Perishable inventory control models such as those of Wu et al. [3] and Feng et al. [4] recognised that freshness and selling price, among other factors, are important determinants of demand for perishable food products. Nonetheless, these studies, along with their various extensions, were focused entirely on optimising purchasing decisions at the retail end of the supply chain. The current increasingly competitive business climate has forced businesses to seek external sources of cost and operational efficiencies in addition to intra-organisational optimisation. For this reason, a lot of businesses have been using supply chain integration as a tool for competitiveness. Owing to the importance of supply chain management, researchers such as Wu et al. [7] and Ma et al. [8], to name a few, developed models for managing fresh produce in multi-echelon supply chains.

This study extends the concept of supply chain integration to inventory control mechanisms used in perishable food products supply chains dealing with growing items such as livestock. In essence, the study considers an end-to-end supply chain for perishable food products, with the downstream end corresponding to consumption (of processed inventory) and the upstream end corresponding to rearing (of live inventory). Seeing that considerable cost and operational efficiencies can be achieved by integrating purchasing decisions among all supply chain members [9], the model presented in this study can serve as a guideline for production and operations managers in multi-echelon supply chains for growing items when making purchasing, shipment and pricing decisions.

1.4. Organisation

Other than the introductory section, this article has five more sections. A brief survey of related inventory control models in the literature is given in Section 2. Before the model development phase which is presented in Section 4, the notations and assumptions employed are stated in Section 3. Numerical results which highlight potential practical applications of the model are given in Section 5. The article is wrapped up in Section 6 through the presentation of concluding remarks and suggestions for future research.

2. Literature Review

Three research streams within inventory modelling form the basis of this study, with each stream corresponding to an echelon of the supply chain considered. Lot sizing models for growing items, particularly during the growing period, are the basis for the farming echelon. The production portion of vendor-buyer inventory problems forms the basis of the processing echelon, whereas perishable inventory models with demand rates that depend on price and age form the basis of the retail echelon.

2.1. Lot-sizing models for growing items

Through the development of an economic order quantity (EOQ) model for items that experience a weight increase during a replenishment cycle, Rezaei [10] introduced a new class of items to inventory modelling, namely growing items. These items are not suitable for consumption at the time they are procured, so before they are used to meet demand (i.e. consumed), they are fed and consequently, enabled to grow. Examples of items that fall under this class include seafood, livestock and grains, to name a few.

Given the foundational nature of Rezaei's model [10], in terms of not accounting for shortages, quantity discounts and multiple-items, among other popular features of EOQ models, the model has been extended to account for some these shortcomings. For example, Khalilpourazari and Pasandideh [11], Nobil et al. [12] and Sebatjane and Adetunji [13] extended the model to scenarios with multiple items, shortages and quantity discounts, respectively. Khalilpourazari and Pasandideh [11] solved the multi-item variant of Rezaei's EOQ model [10] through an exact solution methodology for small problem sizes and through two semi-heuristic algorithms for medium and large problem sizes because of the proposed model's non-linearity and the presence of multiple local optimum solutions. Nobil et al. [12] extended Rezaei's work [10] by assuming that shortages are permitted and are fully backordered during the consumption (or selling) period of the replenishment cycle. Sebatjane and Adetunji [13] incorporated incremental quantity discounts to the literature on lot sizing for growing items. Since most growing items are sold as various food items downstream in retail supply chains and these supply chains are often characterised by low profit margins, quantity discounts constitute a means of improving sales revenue through increased purchase volumes. Besides the incorporation of these common EOQ extensions, other researchers have extended the model by accounting for specific characteristics of food production systems. For instance, noting that food products are often screened for quality before being put on sale, Sebatjane and Adetunji [14] considered a situation where a random percentage of the matured items is of inferior quality and as a result, it is removed from the lot and salvaged. Despite the presence of two revenue streams in this situation, one from the good quality inventory used to meet regular demand and the other from salvaging the inferior quality inventory, the overall impact of having higher percentages of inferior quality items was negative since more items have to be ordered to meet a given rate of demand. A version of this model in a multi-echelon supply chain setting was developed by Sebatjane and Adetunji [15] through the introduction of distinct farming, processing, quality inspection and consumption echelons. Another extension based on the characteristics of food production systems was presented by Malekitabar at al. [16] who considered a case study for trout fish production and developed a model for inventory management when there is a revenue sharing contract between the party responsible for growing the fish and the one responsible for selling it. Moreover, the authors compared the effectiveness of the revenue sharing contract with a revenue and cost sharing contract and found the latter to be more cost efficient. Sebatjane and Adetunji [17] developed an inventory model for a three-level supply chain for growing items with separate farming, processing and retail levels. Through the consideration of probability functions for survival and mortality throughout the growth period of the replenishment cycle, Gharaei and Almehdawe [18] incorporated item mortality to Rezaei's model [10] and consequently created a new type of EOQ model referred to as the economic growing quantity (EGQ) model. Given the presence of various illnesses and predators in food production value chains, the EGQ is more representative of an actual inventory management system for growing items (which are living organisms) because living organisms are not immune to death.

2.2. Joint economic lot sizing models

In its most basic form, the joint economic lot size (JELS) problem considers a vendor and a buyer involved in the production and selling, respectively, of a single type of item with the intention of optimising the inventory replenishment policy for both parties. The simplest form of this problem is attributed to Goyal [19] who developed the model under the assumption of an infinite production rate and a lot-for-lot production policy at the vendor. The infinite production rate assumption is not a realistic representation of typical production systems, and so, Banerjee [20] extended Goyal's [19] model to a case with a finite production rate.

The JELS has since been extended to numerous production situations. A few notable extensions include Goyal [21], Lu [22], Yang and Wee [23] and Khouja [24]. In Goyal's [21] work, the lot-for-lot production policy was relaxed and the author considered a case where the vendor produces enough items to ship to the buyer in several (i.e. an integer number) equally-sized batches at regular time intervals, but the vendor only starts shipping at the end of a production run. Lu [22] developed a coordinated inventory model under the assumption that the vendor produces items and starts shipping them as soon as enough items to make a batch have been produced (i.e. shipping and production take place simultaneously). As opposed to most models which consider only one buyer and one vendor, Lu's [22] model also incorporated multiple buyers. Given that certain items lose some of their utility over time as a result of deterioration, Yang and Wee [23] developed a model for jointly optimising the ordering policy for a vendor and a buyer manufacturing and selling, respectively, a deteriorating item. Khouja [24] compared three different inventory coordination mechanisms, namely an equal-cycle time approach, an integer-multiplier approach and a power-of-two policy, in a three-echelon supply chain with multiple vendors, multiple distributors and multiple retailers. Sarmah et al. [25], Ben-Daya and Ertogral [26] and Glock [27] carried out comprehensive literature reviews on the JELS problem.

2.3. Lot-sizing models for perishable items with price- and freshness dependent demand

Lot sizing models for items with a demand rate that is influenced by the selling price have been studied since the publication of the seminal work by Whitin [28]. Price-dependent demand is still a popular topic in supply chain modelling as evidenced by recent works by Gan et al. [29], Oliveira et al. [30] and Raza and Govindaluri [31], to name a few. In recent times, the demand rate's price dependency has been combined with various other factors. One of the more popular factors has been the freshness of the inventory items which is incorporated through the consideration of expiration dates.

The first inventory model for perishable items with a demand rate that is influenced by the item's age and selling price is credited to Wu et al. [3]. Furthermore, the demand rate was assumed to also be a function of the item's inventory level. In developing the model, the authors also assumed a non-zero ending inventory policy whereby once the inventory reaches a certain point, it is salvaged so that it is not completely wasted after its expiration date. Moreover, the capacity of the shelf space was assumed to be limited. The model was formulated as a profit maximisation problem with the cycle time, selling price and the ending inventory level as the decision variables.

Numerous researchers have built upon Wu et al.'s work [3]. For example, Chen et al. [2] formulated a model aimed at optimising not only the price, cycle time and ending inventory level, but also the available shelf space. Motivated by the fact that retailers often discount stocks of perishables when their expiration dates are approaching, Feng et al. [4] developed an inventory management model for a retailer who has a closeout sale just before the items expire. Dobson et al. [32] took a different approach to the assumption that the demand is a function of the age of the items and developed an EOQ model for a situation where customers gauge the freshness of the items before making a purchase and they can decide to either buy the item or not, regardless of its age. In addition to considering a demand rate that depends on the age, inventory level and selling price of a perishable item, Wu et al. [33] incorporated a trapezoidal type demand pattern which is representative of most products' life cycles which are characterised by an increasing rate during the introduction phase, a flat rate at the maturity phase and a decreasing rate during the decline phase. Li et al. [34] and Li and Teng [35] incorporated advance payment schemes and reference selling prices, respectively, to Wu et al.'s [3] model. In the advance payment model, the authors assumed that the supplier of the perishable items requires the retailer to pay a portion of the purchase price before receiving the order. For the model that considers reference prices, the authors assume that the selling price has a certain threshold beyond which customers are not willing to purchase the items at all. Li and Teng [36] included the length of the credit term as a third decision variable in Wu et al.'s [3] model by extending it to a case where the supplier allows the retailer to purchase the items on credit and grants the retailer a certain amount of time to settle debt.

The aforementioned studies are all limited to the retail end of the supply chain. There have been a few studies dedicated to inventory management of fresh produce. Cai et al. [5] formulated a model for optimising both the selling price and the replenishment policy in a fresh produce supply chain with a single producer, responsible for growing the produce, and a single distributor who is in charge of transporting the produce from the producer's facility to retail outlets. Cai et al. [6] developed a model for maximising profit in a fresh produce supply chain with a producer, a third party logistics (3PL) provider and a distributor under the assumption that the demand rate for the produce is stochastic and sensitive to the selling price and the freshness condition of the produce. Ma et al. [8] considered a situation where the supply chain members do not have access to the same type of information such as order lead times, demand and delivery times, to name a few. This leads to a distortion in the amount of information and this is termed asymmetric information in economic theory. Ma et al. [8] compared centralised and decentralised inventory replenishment policies in agricultural supply chains with as symmetric information provided that the demand for the agricultural products is price- and freshness-sensitive.

2.4. Gap identification and contribution

This study presents a model for managing growing inventory items in a supply chain with farming, processing and retail echelons in which the demand rate is affected by the selling price and the freshness of the processed inventory.

2.4.1. Gap identification

Table 1 provides a summary of a selection of lot-sizing models that are closely related to the model presented in this study. A vast majority of the models are for perishable food items which are commodities and are thus characterised by demand rates that are sensitive to selling price and the age of the items, among other characteristics. These models

Table 1
Characteristics of closely related inventory models in the literature

References	Types of items under study			Costs incurred for mortal items			Attributes of growth function			Demand pattern of the processed inventory				Supply chain echelon(s)		
	Growing	Mortal	Perishable	Disposal	Feeding	Holding	Linear	Non-linear	Utility of growthfunction (UGF)	Constant	Price-dependent	Freshness-dependent	Stock-dependent	Farming	Processing	Retail
Wu et al. [3]			✓								✓	✓	✓			✓
Dobson et al. [32]			✓								✓	✓	✓			✓
Feng et al. [4]			✓								✓	✓	✓			✓
Li et al. [34]			✓								✓	✓	✓			✓
Wu et al. [33]			✓								✓	✓	✓			✓
Li and Teng [35]			✓								✓	✓	✓			✓
Li and Teng [36]			✓								✓	✓	✓			✓
Rezaei [10]	✓							✓		✓						✓
Khalilpourazari and Pasandideh [11]	✓							✓		✓						✓
Nobil et al. [12]	✓						✓				✓					✓
Sebatjane and Adetunji [13]	✓						✓			✓						✓
Sebatjane and Adetunji [14]	✓							✓		✓						✓
Malekitabar at al. [16]	✓	✓		✓				✓		✓				✓		✓
Sebatjane and Adetunji [17]	✓							✓		✓				✓	✓	✓
Gharaei and Almehdawe [18]	✓	✓		✓	✓	✓	✓		✓	✓						✓
Sebatjane and Adetunji [15]	✓							✓		✓				✓	✓	✓
This study	✓	✓	✓	✓				✓			✓	✓		✓	✓	✓

were developed from the perspective of a retailer and therefore, did not account for the preceding stages in the supply chain. A small fraction of the literature is dedicated to models for fresh produce in multi-echelon supply chains. However, these models do not explicitly consider growing items as the primary source of fresh food products under study. The production of perishable food items often involves several stages. In the most simple supply chains, these stages are often the rearing of live inventory, the processing of the live inventory into a consumable form and the selling of the consumable (or processed) inventory to end consumers. From the table, it is evident that there is currently no lot-sizing model for growing and perishable items that considers the dependence of the demand rate on both item price and freshness in an integrated manner with growing items as the primary source of the chain. The echelons of the supply chain are the rearing (farming), processing and consumption (retail) stages of the proposed supply chain. Considering that supply chains are intricate networks with multiple echelons, it is important to study lot-sizing models in multi-echelon supply chains because they are more representative of real life inventory systems.

2.4.2. Contribution

Effective inventory management in food supply chains is very crucial, not only because it ensures that consumable products are available to meet consumer demand at the right time and right price, but also because a well-managed inventory system has the potential to significantly reduce operational costs. Any form of cost saving, regardless of its magnitude, is important in food production systems because they are often characterised by relatively low profit margins.

The proposed model represents a simplified version of an end-to-end food production chain with separate farming, processing and retail stages. Based on the literature review, this is the first attempt at developing a multi-echelon growing items inventory model of this nature which also takes into account freshness and price dependent demand. The novelty of the model lies in the fact that it incorporates the following features simultaneously to the literature on lot sizing models:

- Separate farming, processing and retail operations with the common goal of jointly maximising profit.
- The demand rate at the retail level is a function of freshness and selling price.
- At the retail level, the inventory has a maximum life time (or expiration date) which is the main determinant of the inventory's freshness index.

3. Notations and assumptions

3.1. Notations

The following notations are adopted throughout this study:

- D Demand rate for processed inventory in weight units per unit time (a function of the selling price and the freshness index of the processed inventory)
- R Processing rate in weight units per unit time
- $w(t)$ Weight of an item at time t
- w_0 The newborn weight of each item
- w_1 The maturity weight of each item
- p_v Procurement (purchasing) cost per weight unit of live newborn inventory
- K_f Farmer's setup cost per cycle
- c_f Farmer's feeding cost per weight unit per unit time
- m_f Farmer's mortality cost per weight unit per unit time
- T_f The duration of the farmer's growth period
- p_f Farmer's selling price per weight unit of live mature inventory
- K_p Processor's setup cost per cycle
- h_p Processor's holding cost per weight unit per unit time
- n The number of shipments from the processor to the retailer per unit cycle of the processor
- $I(t)$ The weight of the processed inventory at time t
- L The expiration date (or shelf life) of the processed inventory
- $F(t)$ Freshness index of the processed inventory at time t (a function of the expiration date)
- T_p Processor's cycle time
- p_p Processor's selling price per weight unit of processed inventory
- K_r Retailer's ordering cost per cycle
- h_r Retailer's holding cost per weight unit per unit time
- T Retailer's cycle time
- y The number of items in the retailer's lot
- x Fraction of the live items which survive throughout the farmer's growth period
- $f(x)$ Probability density function of x
- Q_1 The weight of the items in the retailer's lot (i.e. $Q_1 = xyw_1$)
- p Retailer's selling price per weight unit of processed inventory
- a Maximum size of the market for processed inventory (or asymptotic level of demand attainable when the selling price is considered most favourable to customers)
- b Price elasticity of the demand rate
- α The items' asymptotic weight
- β constant of integration
- λ Exponential rate of growth for the items
- ϑ_f Profit-sharing ratio at the farming echelon
- ϑ_p Profit-sharing ratio at the processing echelon
- ϑ_r Profit-sharing ratio at the retail echelon

3.2. Assumptions

The supply chain under consideration has three echelons and there is a single member at each echelon. Figure 1 is a depiction of the proposed inventory system. The inventory profile at the uppermost portion of the figure shows the changes to the weight of the ordered live items at the farming echelon. The middle portion of the figure depicts the weight of the processed inventory as the live items are slaughtered, prepared and packaged (i.e. processed). The lowermost portion of the figure also shows the processed inventory at the retail echelon.

At the farming echelon, a farmer procures ny live newborn items and rears them. Given that the initial weight of each live item at the time the farmer receives the order is w_0, the weight of all the newborn items ordered, nQ_0, is therefore equal to nyw_0. The items' growth function is approximated by

$$w(t) = \frac{\alpha}{1 + \beta e^{-\lambda t}}, \tag{1}$$

which is the logistic function where α is the items' asymptotic weight, β is the integration constant and λ is the exponential rate of growth for the items. This function is chosen because of its distinctive "S"-shape which is reminiscent of the growth pattern of livestock [37]. The farmer rears the live items for a period of T_f time units. When this period ends, the weight of each item would have reached the maturity weight w_1. The live items have a survival rate of x [i.e during the growth period, x percentage of the initially ordered newborn items survive throughout the growth period while $(1 - x)$ percentage of the initially ordered newborn items die during the growth period]. This implies that the weight of all the surviving ordered mature items (nQ_1) is therefore

$$nQ_1 = xnyw_1. \tag{2}$$

This entire lot is then transferred to the processing plant. The live mature items are scheduled to arrive at the processing plant just as the processor starts a new processing cycle of duration T_p. Based on the inventory system profile for the entire supply chain, as given in Figure 1, for every single processing cycle, the farmer sends one shipment of live items to the processor. For convenience in planning, the farmer and the processor's cycle times are synced to be of equal

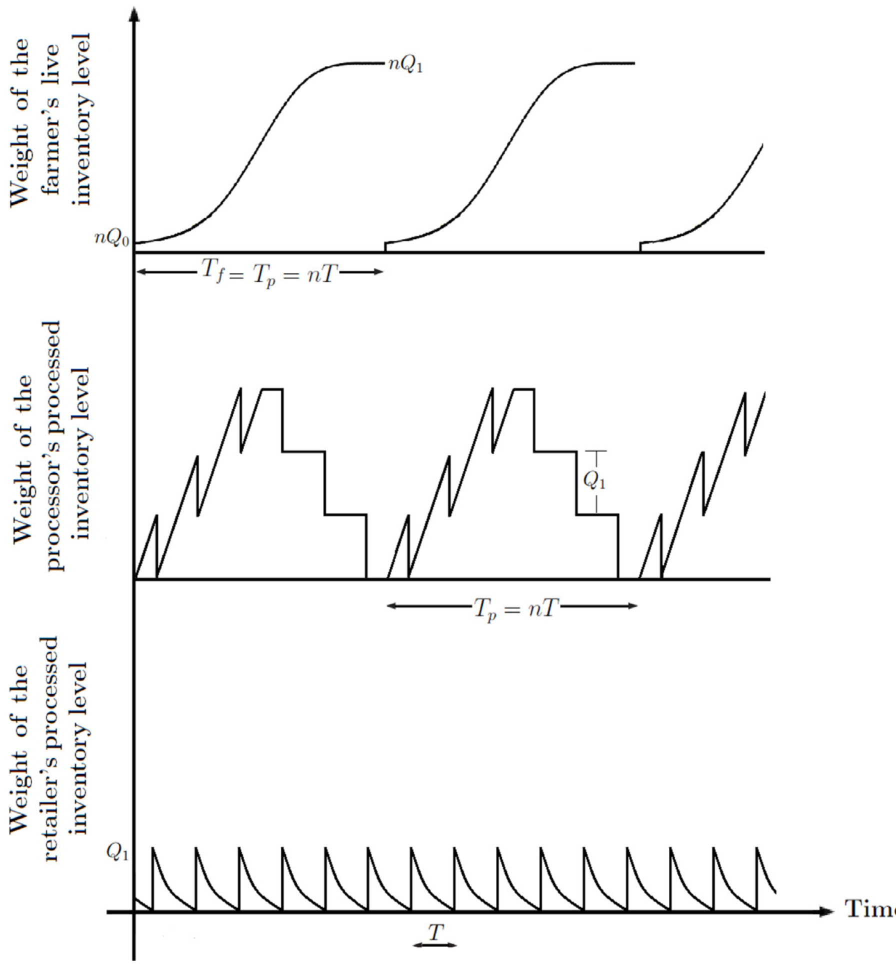


Fig. 1. Behaviour of the weight of the live inventory at the growing facility, the weight of the processed inventory at the processing plant and the weight of the processed inventory at the retail outlet.

duration, thus

$$T_f = T_p. \tag{3}$$

This means that the maturity weight of the live items, w_1, depends on the duration of the processing cycle.

At the processing echelon, the live inventory items are processed at a rate of R and they are transformed into processed inventory which is used to meet consumer demand at the supply chain's next echelon. The processing rate, R, is assumed to be a deterministic constant that is greater than the demand rate D. Consequently, processing does not take place for the entirety of the processor's cycle. This is because the weight of the processed inventory accumulates at a rate of $R - D$ and therefore, the demand can be met without having to continuously process the live items throughout the whole cycle. In essence, the processor's cycle can be divided into two portions: when there is processing and when there is no processing of items. During the processing time, the live inventory is processed and shipped to the retailer in equally-sized batches weighing $Q_1 = xyw_1$. Both processing (of the live inventory) and shipping (of the processed inventory to the retailer) take place simultaneously during this time. This implies that processor starts shipping to the retailer once they have processed enough inventory to make up a batch (of weight Q_1). During the non-processing time, the processor continues to ship batches of processed inventory to the retailer without having to process because processed inventory would have accumulated during the processing time of the cycle since $R > D$. Granted that the processor receives a lot weighing nQ_1 from the farmer and ships it to the retailer (after processing it), in equally-weighted batches (each with a weight of Q_1) and at equally-spaced time intervals, of duration T, the processor therefore makes n deliveries of processed inventory throughout a single processing cycle with a duration T_p. This implies that the retailer's cycle time, T, is an integer multiple (in this instance the integer is n) of the processor's cycle time. Hence,

$$T_p = nT. \tag{4}$$

This implies, based on Equations (3) and (4), that $T_f = T_p = nT$. Likewise, the maturity weight of the live items is determined by replacing t with nT in Equation (1).

At the final echelon, the retailer receives orders of processed inventory from the processor at regular time intervals of duration T in order to meet the consumer demand rate (for processed inventory) of D. Each order of processed inventory that the processor ships to the retailer weighs Q_1. The demand rate is assumed to be affected by the items' selling price and freshness index. Classic economic and marketing theories affirm that the sales of an item are influenced by its selling price, among other factors. In essence, lower prices tend to spike the sales of an item and for this reason, the demand rate is assumed to be an exponentially decreasing function of the price. This is in accordance with studies by Feng et al. [4], Wu et al. [33] and Feng and Chan [38], to name a few. Hence,

$$D \propto ae^{-bp}, \tag{5}$$

where a represents the maximum size of the market for the processed inventory (asymptotic level of demand attainable when the selling price is considered most favourable to customers), b is the price elasticity of the demand rate and p is the retailer's selling price per weight unit of the processed inventory. All three variables are positive numbers and thus, $ae^{-bp} > 0$.

Another aspect that affects the demand for perishable food products is the freshness of the items. A vast majority of consumable food products have shelf lives that are often expressed as expiration or sell-by dates which essentially represent the maximum life times of those products. The printed expiration dates affect consumers' likelihood to make purchases. In essence, a consumer's likelihood of purchasing an item diminishes as the item ages (i.e. as it gets closer to its expiration date). Wu et al. [3] (as well as subsequent models spun off from that particular model) used the Aarhenius equation to represent the freshness index of items. Therefore,

$$F(t) = \frac{L - t}{L}, \tag{6}$$

where L is the maximum shelf life or expiration date of the item. From Equation (6), the item is at its freshest (i.e. 100% freshness index) at $t = 0$ and it reaches its minimum freshness level of 0% at its expiration date L. The processed inventory is no longer suitable for consumption at its maximum shelf life meaning that the duration of the retailer's replenishment cycle cannot be greater than the shelf life (i.e. $L > T$).

In accordance with Chen et al. [2], Wu et al. [3] and Feng et al. [4], Equations (5) and (6) are combined to formulate the demand as a multiplicative function of the selling price (in this case, per weight unit) and the freshness index of the inventory. Hence, the demand rate is

$$D = \left(ae^{-bp}\right)\left(\frac{L - t}{L}\right), \quad 0 \le t \le T. \tag{7}$$

The proposed inventory control system is feasible when $R > D$. Since the demand rate varies with time, the only way to guarantee that this condition is met is by ensuring that the maximum possible demand rate does not exceed the processing rate. From Equation (7), the demand rate reaches its maximum value when the inventory is at its freshest (i.e. $t = 0$) and the retailer's selling price is zero (i.e. $p = 0$). This means that the maximum possible demand rate is a and therefore, $R > D$ can be expressed as $R > a$.

4. Model formulation

The proposed inventory control model in the three-echelon supply chain system is formulated as a profit maximisation problem. All three members of the supply chain have a common goal of improving the supply chain's profit by reducing the costs associated with managing inventory across the chain. Each member's profit is calculated by subtracting the costs associated with managing inventory from the revenue generated from the sales of the inventory.

Consumer demand is for the processed inventory and this particular inventory, tracked at the processor's and the retailer's facilities, incurs purchasing, setup (or ordering, in the case of the retailer) and holding costs. On the other hand, the live inventory which is tracked at the farmer's facility incurs purchasing, setup and feeding costs, with the last cost being dependent on the weight of the item.

The model's objective function is the total supply chain profit and its decision variables are retailer's cycle time, the retailer' selling price and the number of batches of processed inventory shipped to the retailer per processing cycle.

4.1. The retail echelon

The start of the retailer's replenishment cycle is marked by the receipt of an order for processed inventory weighing Q_1. This inventory is displayed on shelves at the retail outlet and it can only be kept for a specified amount of time, known as the expiration date. Once this date has elapsed, the inventory can no longer be used to meet consumer demand. Figure 2 is a representation of the changes that occur to the weight of the retailer's inventory throughout the cycle.

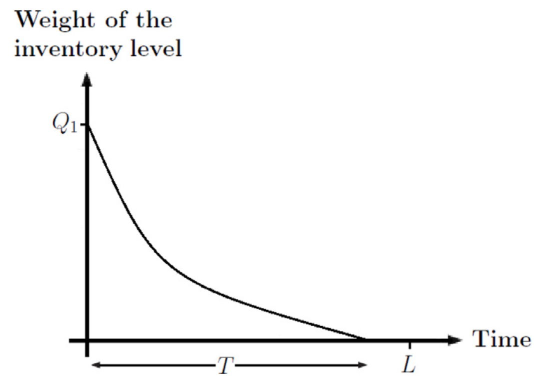


Fig. 2. The retailer's processed inventory system behaviour

During the course of a replenishment cycle, the weight of the retailer's processed inventory is depleted due to consumer demand. As a result, the weight of the retailer's processed inventory is governed by the differential equation

$$\frac{dI(t)}{dt} = -D = -\left(ae^{-bp}\right)\left(\frac{L - t}{L}\right), \quad 0 \le t \le T. \tag{8}$$

Equation (8) can be re-arranged into

$$dI(t) = \left(ae^{-bp}\right)\left(-1 + \frac{t}{L}\right)dt, \quad 0 \le t \le T. \tag{9}$$

Integrating the left and the right hand sides of Equation (9) leads to

$$I(t) = \left(ae^{-bp}\right)\left(-t + \frac{t^2}{2L}\right) + C. \tag{10}$$

Since the weight of the processed inventory at the retailer reaches zero at time T, the boundary condition $I(T) = 0$ is binding. Through substitution, it follows that

$$C = -\left(ae^{-bp}\right)\left(-T + \frac{T^2}{2L}\right) = \left(ae^{-bp}\right)\left(T - \frac{T^2}{2L}\right). \tag{11}$$

By substituting Equation (11) into Equation (10) and re-arranging the terms, the weight of the retailer's processed inventory level is determined as

$$I(t) = \frac{(ae^{-bp})}{2L}\left[t^2 + 2L(T - t) - T^2\right] \tag{12}$$

Given that the retailer receives an order weighing Q_1 at the start of each cycle (i.e. $t = 0$), the boundary condition $I(0) = Q_1$ is binding. Through substitution, it follows that

$$Q_1 = I(0) = \frac{(ae^{-bp})(2LT - T^2)}{2L}. \tag{13}$$

Granted that $Q_1 = xyw_1$, the equivalent number of items in the retailer's lot is thus

$$y = \frac{(ae^{-bp})(2LT - T^2)}{2Lxw_1}. \tag{14}$$

The retailer's cyclic holding cost (i.e. during the time period $[0, T]$) is determined using Equation (12) as

$$HC_r = h_r \int_0^T I(t)\,dt = h_r\left[\frac{(ae^{-bp})(3LT^2 - 2T^3)}{6L}\right]. \tag{15}$$

The retailer's cyclic profit function is defined as the cyclic total revenue less the sum of the cyclic ordering, purchasing and holding costs. It follows that

$$TP_r = \frac{p(ae^{-bp})(2LT - T^2)}{2L} - \frac{p_p(ae^{-bp})(2LT - T^2)}{2L} - K_r - \frac{h_r(ae^{-bp})(3LT^2 - 2T^3)}{6L}. \quad (16)$$

The first term in Equation (16) represents the cyclic revenue and it is the product of the selling price per weight unit charged to consumers (p) and the weight of processed items sold per cycle (Q_1). The second term is the cyclic purchasing cost and it is defined as the product of the weight of processed items purchased from the processor (Q_1) and the price that the processor charges for the inventory (p_p). The third term denotes the fixed cost associated with placing an order during each cycle while the last term is the cyclic holding cost from Equation (15).

The retailer's total profit per unit time is determined by dividing their cyclic profit by their cycle duration T and thus,

$$TPU_r = \frac{(ae^{-bp})(2LT - T^2)(p - p_p)}{2LT} - \frac{K_r}{T} - \frac{h_r(ae^{-bp})(3LT^2 - 2T^3)}{6LT}. \quad (17)$$

4.2. The processing echelon

The processor is responsible for transforming the live inventory into consumable processed inventory. When a new processing cycle starts, the processor receives an order of live items weighing nQ_1 from the farmer and processes the entire order at a rate of R. Throughout the cycle, the processor delivers n shipments of processed inventory to the retailer. The shipments are all of equal weight, meaning that they each weigh Q_1. The behaviour of the processor's processed inventory level is depicted in Figure 3a which is redrawn into Figure 3b for ease of computing the area under the graph. This method of redrawing the inventory system profile is adapted from a version of the JELS problem formulated by Yang et al. [39].

The processor's cyclic holding cost is computed by multiplying the holding cost per weight unit by the area under the processor's inventory system which essentially shows the processor's time-weighted inventory level. The area under the graph in Figure 3b is thus

$$\begin{aligned} \text{Area}_p &= \text{Processor's time-weighted inventory} \\ &= \frac{nQ_1^2}{2R} + Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right) + 2Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right) + \cdots \\ &\quad + (n-1)Q_1^2\left(\frac{1}{D} - \frac{1}{R}\right) \\ &= \frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{1}{D} - \frac{1}{R}\right). \end{aligned} \quad (18)$$

The demand rate in Equation (18) is a function of the retailer's selling price p. If p is held constant, then the demand rate in each cycle interval T is equal, and since T is used as the time basis for the analysis, all demands for all time intervals can be aggregated for ease of derivation. Hence, the processor's holding cost per cycle becomes

$$HC_p = h_p\left[\frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{T}{Q_1} - \frac{1}{R}\right)\right], \quad (19)$$

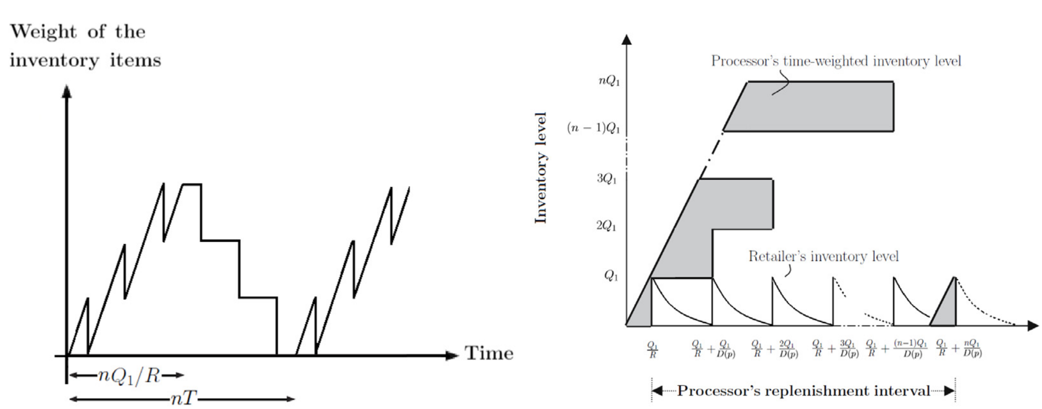
after replacing D in Equation (18) with Q_1/T so that all the terms are expressed in terms of T which is one of the model's decision variables. The expression for D as given in Equation (7) is not used because it varies with time and this becomes problematic when solving the model. Instead, a static approximation of D is used. Since the retailer receives orders of processed inventory weighing Q_1 at equally-spaced time intervals of duration T in order to meet a demand rate of D, the retailer places $\approx D/Q_1$ orders per unit time. This means that the retailer's cycle time $T \approx Q_1/D$. Likewise, $D \approx Q_1/T$.

The processor's profit per cycle is defined as the cyclic revenue minus the sum of the cyclic setup and holding costs. Thus,

$$TP_p = p_p nQ_1 - p_f nQ_1 - K_p - h_p\left[\frac{nQ_1^2}{2R} + \frac{n(n-1)Q_1^2}{2}\left(\frac{T}{Q_1} - \frac{1}{R}\right)\right] - h_p\left(\frac{n^2Q_1^2}{2R}\right). \quad (20)$$

The first term in Equation (20) represents the processor's cyclic profit and it is determined as the product of the weight of the processed inventory sold to the retailer in a single processing run (nQ_1) and the price (per weight unit) that the processor charges the retailer for the processed inventory (p_p). The second term denotes the processor's procurement cost per cycle and it is computed by multiplying the weight of the mature live inventory that the processor procures from the farmer (nQ_1) and the price that the farmer charges for the inventory (p_f). The third term denotes the fixed cost of setting up the processing facility at the beginning of each processing cycle. The fourth term represents the the cyclic holding cost as determined in Equation (19). The last term in Equation (20) represents the additional holding costs incurred by the processor as a result of warehousing the incoming live items (from the farming echelon) prior to processing. The weight of the items is nQ_1 and these items are warehoused for processing portion of the processor's cycle. The processing portion has a duration of nQ_1/R as shown in Figure 3a and consequently, the holding cost per cycle is computed as the products of cost of warehousing a single weight unit of inventory per unit time (h_p), the weight of the items to be warehoused (nQ_1) and the duration of time spent by the items in warehousing (nQ_1/R).

Dividing Equation (20) by the processor's cycle time, $T_p = nT$, yields



(a) Original. (b) Redrawn (modified from Yang et al. [39]).

Fig. 3. The processor's processed inventory system behaviour.

an expression for the processor's total profit per unit time. After substituting Q_1 with Equation (13), the expression becomes

$$\begin{aligned} TPU_p = & \frac{(ae^{-bp})(2LT - T^2)(p_p - p_f)}{2LT} - \frac{K_p}{nT} \\ & - \frac{h_p(n+1)}{2TR}\left[\frac{(ae^{-bp})(2LT - T^2)}{2L}\right]^2 \\ & - \frac{h_p(n-1)}{2T}\left[\frac{(ae^{-bp})(2LT - T^2)}{2L}\right]^2 \\ & \times \left[\frac{2LT}{(ae^{-bp})(2LT - T^2)} - \frac{1}{R}\right]. \end{aligned} \quad (21)$$

4.3. The farming echelon

Whenever the farmer's replenishment cycle begins, ny live day-old newly born items are procured and grown to maturity. The items are deemed mature when the weight of each item reaches w_1 after T_f time units. This means that the weight of all the items in the farmer's lot would be $nQ_1 = xnyw_1$ by the time they are transferred to the processing plant. This is up from an initial purchase weight of $nQ_0 = nyw_0$ for all the ordered items. Figure 4 depicts the growth trajectory of the items at the farmer's growing facility.

The farmer's cyclic feeding cost (during the time period $[0; T_f = nT]$) is defined as the product of the feeding cost per weight unit, c_f, and the area under the graph of the growth period as given in Figure 4. It is assumed that the farmer incurs feeding costs for successfully growing the items to maturity. This implies that the feeding cost is incurred only for the fraction of items that survive throughout the growth period. Therefore,

$$\begin{aligned} FC_f &= c_f \int_0^{nT} xnyw(t)\,dt \\ &= c_f xny\left\{\alpha nT + \frac{\alpha}{\lambda}\left[\ln(1 + \beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}. \end{aligned} \quad (22)$$

In addition to the feeding costs (incurred for the live items), the farmer incurs mortality costs associated with disposing the dead inventory items. This particular cost is incurred for the fraction of items that do not survive throughout the growth period, i.e. $(1 - x)$. Hence,

$$MC_f = m_f(1 - x)ny\left\{\alpha nT + \frac{\alpha}{\lambda}\left[\ln(1 + \beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}. \quad (23)$$

The farmer's profit per cycle is therefore

$$\begin{aligned} TP_f = & \; p_f nQ_1 - p_v nQ_0 - K_f \\ & - ny[c_f x + m_f(1 - x)] \\ & \left\{\alpha nT + \frac{\alpha}{\lambda}\left[\ln(1 + \beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}. \end{aligned} \quad (24)$$

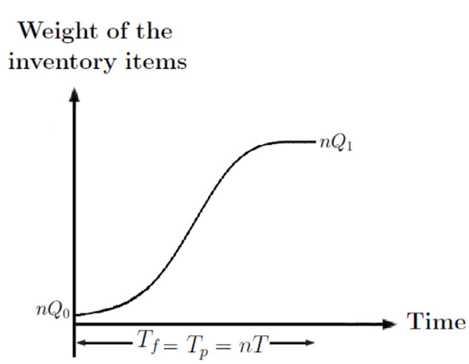


Fig. 4. The farmer's live inventory system behaviour

The first term in Equation (24) denotes the farmer's revenue per cycle and it is computed by multiplying the price (per weight unit) that the farmer charges to the processor for the live mature inventory (p_f) by the weight of the lot that the farmer sells to the processor (nQ_1). The second term is the cyclic procurement cost and it is computed as the product of the price (per weight unit) that the farmer is charged for the live newborn inventory (p_v) by their initial supplier and the weight of the lot that the farmer receives from their initial supplier (nQ_0). The third term is the fixed cost of setting up a new growing cycle while the last term is the sum of the cyclic feeding and mortality costs as determined from Equations (23) and (24).

In order to determine the farmer's profit per unit time, the profit per cycle, as given in Equation (24), is divided by the duration of the replenishment interval, $T_f = nT$, and the result becomes

$$\begin{aligned} TPU_f = & \frac{p_f(ae^{-bp})(2LT - T^2)}{2LT} - \frac{p_v w_0(ae^{-bp})(2LT - T^2)}{2LTw_1} - \frac{K_f}{nT} \\ & - \frac{[c_f x + m_f(1 - x)](ae^{-bp})(2LT - T^2)}{2LTxw_1} \\ & \left\{\alpha nT + \frac{\alpha}{\lambda}\left[\ln(1 + \beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}, \end{aligned} \quad (25)$$

after replacing Q_0 with yw_0 and substituting Q_1 and y with Equations (13) and (14), respectively.

At the end of the growing period (i.e. at time T_f), the weight of each initially order item would have reached the maturity weight of w_1. Therefore, the maturity weight can be determined from Equation (1) by substituting $w(t)$ and t with w_1 and $T_f = nT$, respectively. The maturity weight is thus,

$$w_1 = \frac{\alpha}{1 + \beta e^{-\lambda nT}}. \quad (26)$$

By substituting Equation (25) into Equation (25), the farmer's profit per unit time can be rewritten as

$$\begin{aligned} TPU_f = & \frac{p_f(ae^{-bp})(2LT - T^2)}{2LT} \\ & - \frac{p_v w_0(ae^{-bp})(2LT - T^2)(1 + \beta e^{-\lambda nT})}{2LT\alpha} - \frac{K_f}{nT} \\ & - \frac{[c_f x + m_f(1 - x)](ae^{-bp})(2LT - T^2)}{2LTx} \\ & \times \left\{nT + \frac{1}{\lambda}\left[\ln(1 + \beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}. \end{aligned} \quad (27)$$

4.4. Centralised supply chain

In formulating the model for the proposed multi-echelon inventory system, it is assumed that all three echelons work together to maximise the total profit generated across the supply chain. In doing so, pricing, order replenishment and shipment decisions are centralised and are thus taken for the benefit of the entire supply chain.

4.4.1. Problem formulation

The total profit generated across the entire supply chain is the sum of the profits generated at each of the three echelons. Therefore, the total supply chain profit per unit time, TPU_{sc}, is the sum of Equations (17), (21) and (27). The mathematical formulation of the proposed inventory system is thus

$$\begin{aligned}\text{Max:}\quad TPU_{sc} = &\frac{(ae^{-bp})(2LT-T^2)p}{2LT} - \frac{K_r}{T}\\ &- \frac{h_r(ae^{-bp})(3LT^2-2T^3)}{6LT}\\ &\times \frac{K_p}{nT} - \frac{h_p(n+1)}{2TR}\left[\frac{(ae^{-bp})(2LT-T^2)}{2L}\right]^2\\ &- \frac{h_p(n-1)}{2T}\left[\frac{(ae^{-bp})(2LT-T^2)}{2L}\right]^2\\ &\times\left[\frac{2LT}{(ae^{-bp})(2LT-T^2)} - \frac{1}{R}\right]\\ &\times \frac{p_v w_0(ae^{-bp})(2LT-T^2)(1+\beta e^{-\lambda nT})}{2LT\alpha} - \frac{K_f}{nT}\\ &- \frac{[c_f x + m_f(1-x)](ae^{-bp})(2LT-T^2)}{2LTx}\\ &\left\{nT + \frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}\\ &\text{s.t:}\quad n\in\mathbb{Z}.\end{aligned} \tag{28}$$

The constraint in Equation (28) is that the number of shipments of processed inventory delivered by the processor to the retailer is a positive whole number. This constraint makes the problem readily solvable because it is not possible for the processor to make non-integer deliveries to the retailer.

The survival rate, x, of the live items during the farmer's growth cycle is assumed to be a random variable with a known probability density function, given by $f(x)$. Therefore, the expected value of Equation (28) is

$$\begin{aligned}\text{Max:}\quad E[TPU_{sc}] = &\frac{(ae^{-bp})(2LT-T^2)p}{2LT} - \frac{K_r}{T}\\ &- \frac{h_r(ae^{-bp})(3LT^2-2T^3)}{6LT}\\ &\frac{K_p}{nT} - \frac{h_p(n+1)}{2TR}\left[\frac{(ae^{-bp})(2LT-T^2)}{2L}\right]^2\\ &- \frac{h_p(n-1)}{2T}\left[\frac{(ae^{-bp})(2LT-T^2)}{2L}\right]^2\\ &\left[\frac{2LT}{(ae^{-bp})(2LT-T^2)} - \frac{1}{R}\right]\\ &- \frac{p_v w_0(ae^{-bp})(2LT-T^2)(1+\beta e^{-\lambda nT})}{2LT\alpha} - \frac{K_f}{nT}\\ &- \frac{\{c_f E[x] + m_f E[1-x]\}(ae^{-bp})(2LT-T^2)}{2LTE[x]}\\ &\left\{nT + \frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}\\ \text{s.t:}\quad & n\in\mathbb{Z}.\end{aligned} \tag{29}$$

4.4.2. Solution procedure

The values of T, n and p that maximise $E[TPU_{sc}]$ are determined through the following iterative procedure:

Step 1 Set n to 1.
Step 2 Find the values of T and p that maximise Equation (29).
Step 3 Increase n by 1 and find the values of T and p that maximise Equation (29). Carry on to Step 4.
Step 4 If the latest value of $E[TPU_{sc}]$ increases, go back to Step 3. If the value of $E[TPU_{sc}]$ decreases, the previously calculated value of $E[TPU_{sc}]$ (along with the corresponding T, n and p values) is the best solution and if this case, carry on to Step 5.
Step 5 End.

4.4.3. Theoretical results

The concavity of the expected total supply chain profit ($E[TPU_{sc}]$) with respect to the model's three decision variables, namely, the retailer's cycle time (T) and selling price (p) and the number of shipments of processed inventory delivered to the retailer per processing cycle (n), is investigated in two ways. Firstly, the concavity of $E[TPU_{sc}]$ in T for fixed values of p and n is proven. Secondly, the concavity of $E[TPU_{sc}]$ in p and n for a fixed T value is also proven. Together, these two results show that the model's objective function (i.e. $E[TPU_{sc}]$) is concave and that there are unique T, p and n values that maximise this objective function.

Theorem 1. *For all $p > 0$ and $n > 0$, $E[TPU_{sc}]$ is a concave function of T. Therefore, a unique value of T that maximises $E[TPU_{sc}]$ exists.*

The proof of the theorem is given in Appendix A.

Theorem 2. *For all $T > 0$, $E[TPU_{sc}]$ is a concave function of both p and n. Therefore, unique values of p and n that maximise $E[TPU_{sc}]$ exist.*

The proof of the theorem is given in Appendix B.

4.5. Decentralised supply chain and centralised supply chain with a profit-sharing agreement

The proposed three-echelon supply chain system is optimised centrally, as shown by the objective function given in Equation (29), meaning that the optimal solution to the problem is aimed at maximising the profit for all three members in the supply chain. However, a centralised optimisation approach might benefit certain members more than others and this has the potential to discourage members from fully collaborating with each and using a centralised replenishment policy that benefits the whole supply chain. To counter this, a profit-sharing agreement might be put in place as a way of ensuring that the benefits derived from supply chain collaboration are equitably shared among all supply chain members. If the supply chain members do not collaborate, each might want to locally optimise decisions at their facility, and consequently, inventory replenishment decisions will be decentralised.

To quantify the benefits of (or lack thereof) supply chain collaboration, it suffices to compare the decentralised approach with the centralised approach.

4.5.1. Decentralised supply chain

For the decentralised case, each of the three supply chain members is working towards maximising their (individual) profits. Based on the structure of the proposed supply chain, the retail echelon faces consumer demand for processed inventory, which is a function of the selling price and freshness condition of the inventory. To meet the end-user demand (for processed inventory), the retailer orders processed inventory from the processor. Likewise, the processor orders live inventory from the farmer to meet the retailer's order. Based on this supply chain structure, the retail echelon optimises its (individual) decision variables, namely, p and T, based on Equation (17). Then, these decisions are passed down to the processing echelon, where the number of shipments of processed inventory delivered to the retail echelon per processing run (n) is optimised based on Equation (21). These decisions are then passed down to the farming echelon, whose objective function is given in Equation (27).

4.5.2. Centralised supply chain with a profit-sharing agreement

For the centralised supply chain structure, inventory replenishment, shipment and pricing decisions are centralised and are thus taken for the benefit of all supply chain members. Centralised supply chain structures often result in improved profits for the entire supply chain, but the individual profits of some of the supply chain members might reduce when compared to the decentralised structure. This might discourage some of the members from integrating their inventory replenishment and shipments decisions with the rest of the supply chain members. To ensure that all supply chain members are on board with the centralised decision-making process, an incentive scheme is introduced, specifically, a profit-sharing agreement is put in place to ensure that each of the members is incentivised to participate in the centralised decision-making case. Under a profit-sharing agreement, profits are shared among the supply chain members based on a ratio (called the profit-sharing ratio) for each of the members. The profit-sharing ratio for each member is described according to the profit contribution made by each echelon in the decentralised case. The profit-sharing ratios for each of the three supply chain echelons are therefore described as

$$\vartheta_f = \frac{E\,[TPU_f]}{E\,[TPU_{sc}]}, \tag{30}$$

$$\vartheta_p = \frac{E\,[TPU_p]}{E\,[TPU_{sc}]}, \tag{31}$$

$$\vartheta_r = \frac{E\,[TPU_r]}{E\,[TPU_{sc}]}, \tag{32}$$

where each of the profits represents the profits generated under the decentralised supply chain structure.

After obtaining the profit-sharing ratios for each of the supply chain echelons, the supply chain is optimised centrally and then the profit is divided based on the profit-sharing ratio (which represent the contributions, in percentage terms, made by each of the three echelons to the total supply chain profit).

5. Numerical results

A numerical example that considers a poultry growing, processing and retail system in a three-echelon supply chain is used to solve and analyse the proposed inventory control model. The example makes use of the following parameters: L= 4 days; R = 320 kg/day; K_f = 7 500 ZAR; c_f = 0.5 ZAR/kg/day; m_f = 0.6 ZAR/kg/day; K_p = 5 000 ZAR; h_p = 0.5 ZAR/kg/day; K_r = 1 000 ZAR; h_r = 1 ZAR/kg/day; p_v = 12.5 ZAR/kg; p_f = 17.5 ZAR/kg; p_p = 35 ZAR/kg; w_0 = 0.06kg; a = 275 kg/day; b = 0.03 kg/ZAR; α = 6.87 kg; β = 120; λ = 0.14 /day. The fraction of items which survive throughout the farmer's growth cycle, x, is assumed to be a random variable that is uniformly distributed over [0.8, 1] with a probability density function given by

$$f(x) = \begin{cases} 5, & 0.8 \le x \le 1 \\ 1, & \text{otherwise.} \end{cases}$$

This implies that

$$E\,[x] = \int_{0.8}^{1} 5x \quad dx = 5\left[\frac{(1^2 - 0.\,8^2)}{2}\right] = 0.9$$

The example is solved using the Solver function in Microsoft Excel and the results are presented in Table 2 for both the centralised case and the decentralised case. The expected profit for the centralised case is 814.09 ZAR/day, while for the decentralised case, the expected profit is 467.49 ZAR/day.

Table 2
Results from the example

Objective functionand decision variables	Decentralisedsupply chain	Centralisedsupply chain
$E[TPU_{sc}]^*$ (ZAR/day)	467.49	814.09
p^* (ZAR/kg)	69.42	45.47
n^* (shipments)	12	15
T^* (days)	2.584	1.831
w_1^* (kg)	2.68	1.91

5.1. Centralised supply chain structure

Since the centralised supply chain structure results in higher profits, it is recommended that the three supply chain members adopt the centalised approach.. The optimal values of the decision variables (i.e. p^*, n^* and T^*) in the centralised supply chain structure are used to determine the ordering and shipment policies to be followed by all three supply chain members. When a new cycle starts, the farmer should order ($ny \approx$) 740 newborn items with a total weight of (nQ_0=) 48.24 kg. After ($T_f = nT$=) 27.385 days, the items would have reached the maturity weight and the total weight of the live inventory would be (nQ_1=) 1 413.16 kg. Based on the optimal growth period, the optimal maturity weight of each item should be (w_1^*=) 1.91 kg at the end of the growth period. The farmer should then send the live inventory to the next echelon where it is transformed into processed inventory. During the processing cycle, the processor should deliver (n=) 15 shipments of processed inventory to the retailer, with each shipment weighing (Q_1=) 41.215 kg, at regularly spaced time intervals of (T=) 1.831 days. The retailer should sell the processed inventory at a price of (p=) 45.47 ZAR/kg. The farmer and the processor should start new cycles every ($Tf = T_p = nT$=) 27.385 days. If this policy is followed, the supply chain should expect to make a profit of about 814.09 ZAR/day.

5.1.1. Sensitivity analysis

The relative importance, in terms of impact on the objective function and the three decision variables, of some of the model's input parameters is investigated through a sensitivity analysis. Since the centralised supply chain structure performs better than its decentralised counterpart, the sensitivity analysis is conducted only for the centralised case. The results from the analysis are summarised in Table 3 from which the following note-worthy observations are drawn:

- The parameters that affect the demand rate, namely a, b and L, have the greatest impact on the objective function and the three decision variables.
- As a increases, $E[TPU_{sc}]$ increases. To understand this response, it is important to recall that a is the asymptotic level of demand attainable when the cost is considered most favourable to customers. In essence, a is the maximum size of the market for the processed inventory. As the size of the market increases, the retailer has to replenish the processed inventory more frequently (i.e. reduce the cycle time) because of the increased potential customer base. By so doing, the processed inventory is kept much fresher than it would have been if it was replenished less frequently which spikes consumer demand further. When consumer demand is increased and the market is large, the retailer can charge higher prices which increases revenue. While increasing the selling price negatively affects consumer demand, the negative effect is cushioned by the positive effects brought by the larger market size and the more frequent replenishment cycles which ensures that the inventory does not get close to its expiration date. To take advantage of this observation, management should increase their marketing (or advertising) spend which will increase their potential customer base. In the short term, this will increase costs but the long term benefits of having a larger potential customer base will outweigh the initial marketing spend.
- As b increases, $E[TPU_{sc}]$ decreases. b is the price elasticity of the demand rate which represents consumer's sensitivity to the selling price. Higher values of b imply that consumers are more price-conscious. Therefore, when b increases, the model responds by lowering the retailer's selling price (in an effort to increase demand) and ordering frequency (in an effort to reduce fixed costs). Lower selling prices lead to reduced revenue and less frequent ordering means that the processed inventory is kept in stock for much longer which reduces its freshness and by extension it's demand. Management can take advantage of this observation by targeting consumers who are less price-conscious in their marketing activities.

Table 3
Sensitivity analysis of various input parameters

	%change	Retailer'scycle time (T^*)		Number ofshipments (n^*)		Retailer'sselling price (p^*)		Total supply chain profit ($E[TPU_{sc}^*]$)	
		days	% change	shipments	% change	ZAR/kg	% change	ZAR/day	% change
Base		**1.831**		**15**		**45.47**		**814.09**	
h_r	-40	1.836	+0.3	15	0	45.16	-0.7	832.08	+2.2
	-20	1.833	+0.1	15	0	45.31	-0.3	823.06	+2.1
	+20	1.829	-0.1	15	0	45.62	+0.3	805.17	-1.1
	+40	1.826	-0.3	15	0	45.77	+0.7	796.30	-2.2
K_r	-40	1.411	-22.9	19	+26.7	45.16	-0.7	1 064.03	+30.7
	-20	1.612	-12.0	17	+13.3	45.39	-0.2	930.98	+14.4
	+20	1.996	+9.0	14	-6.7	45.65	+0.4	709.09	-12.9
	+40	2.168	+18.4	13	-13.3	45.76	+0.6	612.83	-24.7
h_p	-40	1.760	-3.9	17	+13.3	43.30	-4.8	970.49	+19.2
	-20	1.792	-2.1	16	+6.7	44.43	-2.3	889.02	+9.2
	+20	1.877	+2.5	14	-6.7	46.41	+2.1	744.77	-8.5
	+40	1.854	+1.3	14	-6.7	47.45	+4.6	680.76	-16.4
K_p	-40	1.767	-3.5	15	0	45.18	-0.6	888.20	+9.1
	-20	1.800	-1.7	15	0	45.32	-0.3	850.82	+4.5
	+20	1.794	-2.0	16	+6.7	45.83	+0.8	778.37	-4.4
	+40	1.822	-0.5	16	+6.7	45.98	+1.1	743.80	-8.6
c_f	-40	1.772	-3.2	16	+6.7	44.23	-2.7	896.80	+10.2
	-20	1.768	-3.4	16	+6.7	44.96	-1.1	854.67	+5.0
	+20	1.830	-0.1	15	0	46.19	+1.6	774.66	-4.8
	+40	1.829	-0.1	15	0	46.92	+3.2	736.08	-9.6
K_f	-40	1.798	-1.8	14	-6.7	44.82	-1.4	927.48	+13.9
	-20	1.784	-2.6	15	0	45.25	-0.5	1 869.42	+6.8
	+20	1.808	-1.3	16	+6.7	45.91	+1.0	761.01	-6.5
	+40	1.850	+1.0	16	+6.7	46.13	+1.5	709.75	-12.8
a	-40	2.439	+33.2	14	-6.7	47.73	+5.0	139.97	-82.8
	-20	2.035	+11.1	15	0	46.42	+2.1	462.91	-43.1
	+20	1.623	-11.4	16	+6.7	45.04	-0.9	1 184.52	+45.5
	+40	1.513	-17.4	16	+6.7	44.60	-1.9	1 568.98	+92.7
b	-40	1.294	-29.3	20	+33.3	67.12	+47.6	2 579.87	+216.9
	-20	1.563	-14.6	17	+13.3	53.49	+17.6	1 452.75	+78.5
	+20	2.065	+12.3	14	-6.7	40.38	-11.2	415.02	-11.2
	+40	2.331	+27.3	13	-13.3	36.89	-18.9	150.15	-81.6
L	-40	1.424	-22.2	20	+33.3	45.58	+0.2	507.09	-37.7
	-20	1.640	-10.4	17	+13.3	45.50	+0.1	690.09	-15.2
	+20	1.966	+7.4	14	-6.7	45.56	+0.2	905.13	+11.2
	+40	2.107	+15.0	13	-13.3	45.59	+0.3	975.27	+19.8

- As L increases, $E[TPU_{sc}]$ increases. In addition to maximising the expected profit, the model aims to ensure that the processed inventory does not expire and so when the inventory can last for longer periods of time (because of increased L values), the model prompts the retailer to order less frequently (i.e. increase the cycle time) because the risk of expiration is reduced. By so doing, the retailer would receive fewer shipments (of relatively larger sizes). While this reduces the fixed costs, it reduces demand because of reduced freshness since the inventory will be kept in stock for a relatively longer period (of time) because of less frequent ordering. However, this negative effect is outweighed by the positive effect of the reduced fixed costs. To take advantage of this observation, management should invest in preservation technologies such as (more advanced) refrigeration which has the potential to prolong the shelf life of the processed inventory. Once again, the initial investment will be large in the short term, but the long term benefits will outweigh this initial investment.
- When any of the fixed costs (i.e. K_r, K_p and K_f) increase, $E[TPU_{sc}]$ decreases. To reduce the fixed costs, the model's response is to reduce the replenishment frequency (i.e. increase the cycle time) by placing larger orders. This leads to increased holding costs because the processed inventory will spend more time in stock. This inadvertently reduces consumer demand because if the inventory is kept in stock for longer periods, its freshness levels decreases which negatively affects the demand.

5.2. In-depth comparison between the centralised and decentralised supply chain structures

The proposed inventory model advocates for the integration of ordering, shipment and pricing decisions among all supply chain members (i.e centralised supply chain structure). This is because organisations have realised that significant cost savings can be achieved through collaboration and integration of certain decisions, such as inventory replenishment policies, with all the supply chain members [9]. In order to investigate the benefits of (or lack thereof) integrating inventory decisions with all parties in the supply chain, the proposed supply chain system (which calls for the integration of inventory replenishment policies) is compared with an alternative policy which does not encourage supply chain integration (i.e. decentralised supply chain structure). The results from this analysis are presented in Table 4.

For the decentralised supply chain structure, the retail echelon which faces consumer demand for processed inventory, optimises its cycle time (T) and selling price (p). Then these are passed along to the

Table 4
Comparison between centralised and decentralised supply chain structures

Variables	Decentralisedsupply chain	Centralisedsupply chain
$E[TPU_r^*]$ (ZAR/day)	386.29	672.68
$E[TPU_p^*]$ (ZAR/day)	56.18	97.85
$E[TPU_f^*]$ (ZAR/day)	25.02	43.55
$E[TPU_{sc}^*]$ (ZAR/day)	467.49	814.09

next echelon (i.e. processing), where the number of shipments of processed inventory delivered to the retailer during a single processing run (n). These decisions are then passed along to the farmer. Under this supply chain structure, the retailer's expected profit amounts to 386.29 ZAR/day while the processor and the farmer should expect profits of 56.18 ZAR/day and 25.02 ZAR/day, respectively.

Under the centralised supply chain structure, inventory replenishment and pricing decisions are centralised and are thus taken for the benefit of all supply chain members. Under this structure, the supply chain should expect to make a profit of 814.09 ZAR/day, which is higher than the profit obtained for the decentralised supply chain. To ensure that all supply chain members are on board with the centralised decision making process, a profit sharing agreement is put in place to ensure that each of the members is incentivised to participate. The profit sharing ratios utilised for the farming, processing and retail echelons are described in Equations (30), (31) and (32), respectively. The profit sharing ratios in the example are ϑ_f=0.0535, ϑ_p=0.1202 and ϑ_r=0.8263 for the farming, processing and retail echelons, respectively. When a profit sharing agreement is put in place (in the centralised case), the individual profits generated by each of the supply chain members is higher than those obtained when replenishment decision are decentralised.

The results show that the centralised supply chain structure, which encourages collaboration among all supply chain members, is better at maximising supply chain profit than its decentralised counterpart. Therefore, it is highly recommended that all supply chain members should integrate their inventory replenishment and shipment decisions because of the increased profit through centralised decision making (with a profit sharing agreement).

6. Concluding remarks

6.1. Conclusions

Perishable food products constitute a significant portion of grocery retail sales. Considering the commoditised nature of grocery items and the fact that retailers often carry various brands of the same type of perishable food product, the selling price and the freshness condition of the product become important catalysts for consumer demand. Several studies in the literature have proposed inventory models for perishable perishable products whose demand rate depends on the selling price and the freshness or expiration date of the products. The common denominator among the vast majority these previous studies that considered the demand rate's price and freshness dependency has been a focus on the retail end of the supply chain. In reality, retailers do not exist in isolation, they have suppliers and their suppliers might also have suppliers.

This study presents a model for managing inventory in a three-echelon supply chain for growing items. The echelons include a farming operation where items are reared under the assumption that some of the items might die (as a result of, for example, illnesses or predators); a processing plant where the live items are processed to get them into a form that is suitable for human consumption and; a retail outlet where consumer demand is met. The demand rate is affected by the selling price and the expiration date of the processed inventory. The significance of the proposed model lies in the fact that it in more representative, when compared to previous studies in the literature, of an actual perishable food supply chain. This is because it not only accounts for pricing policies and expiration dates at the retail stage, but also the preceding farming and processing stages (i.e. it considers an end-to-end supply chain for perishable products). The most important characteristics of the proposed model are the integration of replenishment and shipment policies among all supply chain members and the demand rate's dependency on the selling price and the expiration date. The importance of these characteristics are quantified through numerical experimentation.

6.2. Suggestions for future research

Despite being more representative of an actual end-to-end supply chain for growing items, the model presented in this study still makes use of several assumptions that can restrict its potential practical applications. The model can be extended in several ways that can enrich its potential applications. Four broad groups of possible areas for further exploration are identified, namely, EGQ supply chains, incentive policies, game theory and soft computing applications. Firstly, the three-echelon supply chain model presented in this study can be extended by incorporating some of the attributes of the EGQ model such as the use of utility of growth functions (UGF) for both the live and the mortal grown inventory items. Additionally, specific characteristics of growing items such as waste by-products, reproduction and illnesses, to name a few, can be incorporated as a way of developing more realistic EGQ models in multi-echelon supply chains. Incentive policies represent another possible area for future extensions. Due to the relatively low profit margins in food retail, members of food supply chains often utilise incentive strategies such as pre-payment agreements, buy-back contracts and trade credit financing to improve profits. Consequently, the current literature can be enriched by incorporating some of these incentive policies to the model presented in this study. A third possible area of future research is the application of various game theoretical methods, such as Stackelberg and Shapley value approaches, when analysing the centralised supply chain. The performance of these game theoretic approaches can be compared to the proposed inventory system to determine if profits can be optimised further. Lastly, soft computing methods, specifically metaheuristic algorithms, can be used to solve complex versions of the proposed supply chain setup. These complex version can involve multiple farmers, processors and retailers in the supply chain as well as multiple growing items and multiple consumable processed inventory products.

CRediT authorship contribution statement

Makoena Sebatjane: Conceptualization, Methodology, Writing - original draft. **Olufemi Adetunji:** Supervision, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proof of Theorem 1

Proof. For ease of computation, the model's objective function, as given in Equation (29), can be rewritten in terms of w_1 as

$$E[TPU_{sc}] = \frac{(ae^{-bp})(2LT-T^2)p}{2LT} - \frac{K_r}{T} - \frac{h_r(ae^{-bp})(3LT^2-2T^3)}{6LT}$$
$$\frac{K_p}{nT} - \frac{h_p(n+1)}{2TR}\left[\frac{(ae^{-bp})(2LT-T^2)}{2L}\right]^2$$
$$-\frac{h_p(n-1)}{2T}\left[\frac{(ae^{-bp})(2LT-T^2)}{2L}\right]^2$$
$$\times\left[\frac{2LT}{(ae^{-bp})(2LT-T^2)} - \frac{1}{R}\right]$$
$$-\frac{p_v w_0 (ae^{-bp})(2LT-T^2)}{2LTw_1} - \frac{K_f}{nT}$$
$$-\frac{\{c_f E[x] + m_f E[1-x]\}(ae^{-bp})(2LT-T^2)}{2LTE[x]w_1}\left\{\alpha nT\right.$$
$$\left.+\frac{\alpha}{\lambda}\left[\ln(1+\beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\}. \tag{A.1}$$

The fact that w_1 is a function of T does not have an impact on the concavity of $E[TPU_{sc}]$ with respect to T because w_1 is always positive. It is not possible for the maturity weight of the items to be be a negative value.

For settled values of p and n, the first and second derivatives of $E[TPU_{sc}]$, as given in Equation (A.1), with respect to T are

$$\frac{\partial E[TPU_{sc}]}{\partial T} = -\frac{p(ae^{-bp})}{2L} + \frac{K_r}{T^2} - \frac{h_r(ae^{-bp})(3L-4T)}{6L} + \frac{K_p}{nT^2}$$
$$-\frac{h_p(n+1)(ae^{-bp})^2(T-2L)(3T-2L)}{16L^2R}$$
$$+\frac{p_v w_0(ae^{-bp})}{2Lw_1} + \frac{K_f}{nT^2}$$
$$+\frac{\{c_f E[x] + m_f E[1-x]\}(ae^{-bp})}{2LE[x]w_1}$$
$$\left\{\alpha nT + \frac{\alpha}{\lambda}\right.$$
$$\left.[\ln(1+\beta e^{-\lambda nT}) - \ln(1+\beta)]\right\} \tag{A.2}$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial T^2} = -\frac{K_r}{T^3} - \frac{2h_r(ae^{-bp})}{3L}$$
$$-\frac{K_p}{nT^3} - \frac{h_p(n+1)(ae^{-bp})^2(3T-4L)}{8L^2R}$$
$$-\frac{K_f}{nT^3} < 0 \tag{A.3}$$

Given that the second derivative of $E[TPU_{sc}]$ with respect to T is negative, as shown in Equation (A.3), it is apparent that $E[TPU_{sc}]$ is a concave function of T for any settled values of $p > 0$ and $n > 0$. This means that there is a unique T value that maximises $E[TPU_{sc}]$. □

Appendix B. Proof of Theorem 2

Proof. For compactness, Equation (A.1) can be written in terms of Q_1 (i.e. Equation (13)) as

$$E[TPU_{sc}] = \frac{pQ_1}{T} - \frac{K_r}{T} - \frac{h_r(ae^{-bp})(3LT^2-2T^3)}{6LT} - \frac{K_p}{nT}$$
$$-\frac{h_p(n+1)Q_1^2}{2TR} - \frac{h_p(n-1)Q_1^2}{2T}\left(\frac{T}{Q_1} - \frac{1}{R}\right)$$
$$-\frac{p_v w_0 Q_1}{TE[x]w_1} - \frac{K_f}{nT}$$
$$-\frac{\{c_f E[x] + m_f E[1-x]\}Q_1}{TE[x]w_1}$$
$$\times\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln(1+\beta)\right]\right\}. \tag{B.1}$$

The fact that Q_1 is a function of p does not have an impact on the concavity of $E[TPU_{sc}]$ with respect to p and n because Q_1 is always positive since it is not possible for the retailer to receive an order of processed inventory with a negative weight. Recall, from Equation (13), that $Q_1 = (ae^{-bp})(2LT - T^2)/2L$. Given that a, b and p are all > 0, ae^{-bp} will always be > 0. Furthermore, it is not possible to have negative time duration and thus, L and T are > 0. Since the retailer can not sell the processed past its expiration date, $L > T$, and thus, $2LT - T^2$ will always be > 0. Therefore, Q_1 will always be positive.

For a settled value of T, the first and second derivatives of $E[TPU_{sc}]$, as given in Equation (B.1), with respect to n and p are

$$\frac{\partial E[TPU_{sc}]}{\partial n} = \frac{K_p}{n^2T} - \frac{h_pQ_1^2}{2TR} - \frac{h_pQ_1^2}{2T}\left(\frac{T}{Q_1} - \frac{1}{R}\right) + \frac{K_f}{n^2T} - \frac{\{c_fE[x] + m_fE[1-x]\}Q_1}{TE[x]} \times \left\{nT + \frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda nT}) - \ln(1+\beta)\right]\right\} \tag{B.2}$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial n^2} = -\frac{K_p}{n^3T} - \frac{K_f}{n^3T} \tag{B.3}$$

$$\frac{\partial E[TPU_{sc}]}{\partial p} = \frac{Q_1}{T} + \frac{h_rabe^{-bp}(3LT^2 - 2T^3)}{6LT} \tag{B.4}$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial p^2} = -\frac{h_rab^2e^{-bp}(3LT^2 - 2T^3)}{6LT} \tag{B.5}$$

$$\frac{\partial^2 E[TPU_{sc}]}{\partial n\partial p} = 0 \tag{B.6}$$

The quadratic form of the Hessian matrix of $E[TPU_{sc}]$ as given in Equation (A.1) is therefore

$$[n \quad p]\begin{bmatrix} -\frac{K_p}{n^3T} - \frac{K_f}{n^3T} & 0 \\ 0 & -\frac{h_rab^2e^{-bp}(3LT^2 - 2T^3)}{6LT} \end{bmatrix}\begin{bmatrix} n \\ p \end{bmatrix} = -\frac{K_p}{nT} - \frac{K_f}{nT} - \frac{h_rab^2p^2e^{-bp}(3LT^2 - 2T^3)}{6LT} < 0. \tag{B.7}$$

Since the quadratic form of the Hessian matrix is negative, $E[TPU_{sc}]$ is a concave function of $n > 0$ and $p > 0$ for any given value of T. This means that $E[TPU_{sc}]$ is a concave function of n and p for a settled value of T and therefore, unique values of n and p that maximise $E[TPU_{sc}]$ exist. □

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