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Grade 12 Students' Proficiency in Solving Probability Problems Involving Contingency Tables and Tree Diagrams

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Knowledge of probability is perceived as valuable in the 21st century. This knowledge enables people to understand and make informed decisions relating to uncertain events that occur in real life. Hence, many educational authorities have considered the inclusion of the teaching of probability important in their school curricula. This study investigated Grade 12 students' proficiency in solving probability problems using contingency tables and tree diagrams as aids. This study employed cross-sectional survey research design, and a mixed method research approach. Data was collected from 342 secondary school students who were conveniently selected from six schools in the KwaZulu-Natal province in South Africa using an achievement test. The study revealed that most of the students (97% and 91%) scored below 50% in solving probability problems involving the use of tree diagrams and contingency tables respectively. The findings show that the students were not proficient in the use of tree diagrams and contingency tables to solve probability problems. The implications of the findings of this study for teacher training and professional development, and textbook publication are discussed and recommendations made.

Keywords: contingency table, mathematics, probability, students' proficiency, tree diagram

INTRODUCTION

Probability is the study of random or uncertain events. The knowledge of probability enables people to understand and make informed decisions relating to uncertain events that occur in real life (Nanatha, 2017). This knowledge helps in understanding disciplines like financial mathematics, insurance, industrial quality control, genetics, quantum mechanics and the kinetic theory of gases (Brown & Wong, 2015). In view of

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the importance of knowledge of probability, educational authorities in many countries have recognised the need for probability literacy (Batanero et al., 2016) and have included probability in the school curriculum.

In South Africa, probability was one of several new topics added to the school mathematics curriculum, the Curriculum and Assessment Policy Statement (CAPS), in 2011. In the CAPS document, probability and statistics form part of the ten main topics for Grades 10-12 mathematics (Department of Basic Education (DBE), 2011). However, the addition of this topic to the school mathematics curriculum was not without challenges because most of the teachers teaching mathematics had never studied probability during their school days or even at tertiary education level (Makwakwa, 2012).

According to the National Senior Certificate Examination Diagnostic Reports (DBE, 2015; 2016; 2017), students' performance in probability in the Grade 12 school certificate examinations from 2015 to 2017 show that students find the topic challenging. Similarly, Mutara and Makonye (2016) found that many students struggle to grasp the concept of probability.

Various studies have given different reasons to account for students' challenges in probability. For example, Batanero and Diaz (2012) argue that students struggle to solve probability problems because probability requires interpretation and understanding of the context, which many students find abstract. Some decades ago, students' struggles in solving probability problems was attributed to wrong intuitions, biases and primitive conception of the topic (Freudenthal, 1973; Shaughnessy, 1992). More recently, Anastasiadou and Gagatsis (2007) assert that poor knowledge foundation created by primary school and mathematics teachers who, in most cases, lack specific preparation in statistics education are the cause of students' struggle in statistics and probability. According to Makwakwa (2012), the reason might be that the teachers teaching the topic do not, themselves, fully understand the concept. They therefore share with their students their misconceptions, as opined by Paul and Hlanganipai (2014). Hirsch and O'Donnell (2001) argue that students' poor performance in probability could be linked to their misunderstanding of the laws of probability or errors that they construct from violations in the application of these laws.

Many techniques can be used to solve probability problems. These techniques include fundamental counting principles, Venn diagrams, contingency tables, and tree diagrams. This study investigated Grade 12 students' proficiency in solving probability problems using contingency tables and tree diagrams as aids. The research questions posed in this study were: How proficient are Grade 12 students at solving probability problems using (i) Contingency tables and ii) Tree diagrams?

BACKGROUND

In the South African Grades 10 – 12 mathematics curriculum, students are expected to be able to use contingency tables and tree diagrams as aids when solving probability problems where events are not necessarily independent (DBE, 2011).

Contingency Table

A contingency table is a frequency distribution table cross-classified by two or more variables (Grant, 2017). A contingency table presents a summarised frequency distribution of a population or sample that is classified according to statistical variables (Roca & Batanero 2006). The table may comprise different rows and columns with the simplest consisting of two rows and two columns which are presented in a matrix or grid form. The numbers displayed in contingency tables give the frequency of each data point.

Solving probability problems from the data presented in contingency tables has been found to be a challenge for many students. For example, Santos and Dias (2015), in a study with future educators in Portugal, found that the participants had difficulties computing probabilities from the data presented in contingency tables. Estrada and Díaz (2006) explored the proficiency of pre-service mathematics teachers in computing probabilities from contingency tables and also found that the pre-service teachers had difficulties in computing probabilities based on the data in contingency tables. Similarly, Makwakwa (2012) revealed that Grade 11 students in South Africa have difficulties in using contingency tables to solve problems related to mutually exclusive events, as well as dependent and independent events. Some studies have documented the cause of learner difficulty in the use of contingency tables, for instance, Falk (1986) opined that student difficulty in the use of contingency tables lay in their difficulty identifying the differences between the conditional probabilities P(A/B) and P(B/A). This view is supported by Einhorn and Hogarth (1986) as they stated that students misinterpret the conjunction and confuse joint and conditional probability. Similarly, Roca and Batanero (2006) found that students struggle with the reading and computing of probabilities from the two-way contingency table.

Tree Diagram

A tree diagram is used to display all the possible outcomes of an event and is used to summarise the probabilities associated with a sequence of random events. The branches emanating from any given start point represent all the possible outcomes in a sample space (Nguyen, 2015). Each branch is labelled according to the probability of the event occurring given the events that have previously happened. Hence, the sum of probabilities from each set of branches must be equal to one. Each path from the start of the tree to the end defines an outcome in the sample space. The outcomes defined by the paths are mutually exclusive (Scarrott, 2011).

The use of a tree diagram assists students in conceptualising and understanding probabilities; it is a useful tool for calculating the probabilities of events as well as determining sample space through organised counting (Nguyen, 2015). It is also useful for both conditional probability problems as well as those related to sequential events (e.g. roll a die, flip a coin) (Zahner & Corter, 2010). In the CAPS mathematics curriculum, students are expected to use a tree diagram in solving questions involving both dependent and independent events, and to list sample spaces.

METHOD

This research explored Grade 12 students' proficiency in solving probability problems related to using or analysing contingency tables and tree diagrams. This study employed a cross-sectional survey research design (Creswell, 2015) and mixed methods approach to address the identified problem. A cross-sectional survey involves different participating groups of people who differ in a variable of interest but share other characteristics such as socio-economic status, educational background and ethnicity. It allows the researcher to measure outcomes and exposures in the study participants at the same time (Setia, 2016). A mixed methods research approach enables the researcher to use both quantitative and qualitative methods to provide a better understanding of the research problem (Creswell, 2015).

Participants

The research was conducted in the Nongoma district in the KwaZulu-Natal province, South Africa. The district and province were conveniently sampled because of their proximity to the researcher collecting the data for this research. Invitations to participate in the research were extended to ten schools in the district, but only seven schools agreed to participate in the research. However, one of the schools was found to be socio-economically different to the other six schools, hence the school was excluded from the analysis reported here. It was adjudged that the six schools would give a fair picture of the overall students' proficiency in solving probability problems in the district. Hence, the choice of the six schools was purposive. There were 342 participants (180 girls and 162 boys) from the six secondary schools who took part in this study.

Instrument and Procedures

The instrument for data collection was an achievement test developed by the researchers. A pen-and-paper test was used to ascertain the students' proficiency in the use of contingency tables and tree diagrams as aids in solving probability problems. The questions demanded constructed responses, with some questions requiring short answers and others long answers. The construction of the test was guided by the Grades 10–12 mathematics curriculum assessment guidelines (DBE, 2011). The test covered aspects of probability, namely, the use of contingency tables and tree diagrams to solve probability problems. There was no time constraint in the test in order to allow students to perform to the best of their abilities by removing all forms of constraints and any possibility of errors made due to time pressure. To ensure the validity of the test and the marking guide, three experts in the field of mathematics education moderated the tests. One of the experts was a mathematics subject advisor working with the Department of Basic Education and the other two were senior mathematics educators and markers. These experts evaluated the mark allocation of each question, the language used, the content covered and also the classification of the questions according to the aspects of probability taught. They made recommendations regarding the wording of the questions and the mark allocation. They also judged the level of alignment of each question with the curriculum by using a 3-point rating scale (1 = not aligned; 2 = fairly aligned; 3 = extremely aligned). All of the questions were retained because they were judged by the experts to extremely align with the curriculum. The instrument was pilot-tested in another school that did not participate in the main study. The results from the pilot test were used to compute the test's reliability. The test-retest method was used to ascertain the reliability of the instrument and a reliability coefficient of .869 was obtained. According to Madan and Kensinger (2017) a reliability coefficient of 0.8 and above is considered to be very good.

Data Analysis

The data were analysed in a two-fold manner. Firstly, descriptive statistics involving the minimum, the maximum, the mean and standard deviations of the students' results in using contingency tables and tree diagrams as aids to solve probability problems, and also the frequencies of students' who scored below 50%, from 50% to 79% and above 80% are presented. Secondly, content analyses of the students' solutions to the questions were carried out to determine if the solutions were completely correct, partially correct, or completely incorrect. Then the frequencies of the completely correct, partially correct, and completely incorrect solutions are presented for each question.

FINDINGS

The descriptive statistics of the students' percentage achievement in the test are presented in Table 1.

Table 1
Descriptive Statistics of the Students' Scores

	N	Min	Max	Mean	SD	Frequency distribution of scores		
						Below 50%	50 -79%	80 -100
Use tree diagram	342	9	100	39	1.757	332 (97%)	7 (2%)	3 (1%)
Use of contingency table	342	9	70	43	0.656	311 (91%)	31 (9%)	0 (0%)

The descriptive statistics of the students' scores in the test (see Table 1) showed that no student scored between 80% and 100% in the use of contingency tables to solve probability problems. The results also show that most of the students in the study (97% and 91%) scored below 50% in the use of tree diagrams and contingency tables to solve probability problems respectively.

Proficiency in Solving Probability Problems Using a Tree Diagram

The question that required the use or analysis of tree diagrams was Question 3: Thandeka has a bag containing 5 green balls and 7 red balls. Two balls are picked at random from the bag one after the other.

Illustrate the information on a tree diagram if:

3.1.1 The first ball was replaced before the second ball was picked.
3.1.2 The first ball was not replaced and the second ball was picked.
3.2 Find the probability that the balls selected were of different colours in Question 3.1.1.

3.3 Find the probability that the two balls picked were of the same colour in Question 3.1.2.

3.4 Find the probability that at least one of the balls picked was green in Question 3.1.1.

3.5 For a number of experiments, provide any two ways by which one can determine whether a tree diagram drawn is correct or wrong.

For Question 3.1.1, the students were supposed to draw a tree diagram showing the outcomes for each trial when the first ball is replaced and before the second ball is picked. The students were expected to draw two branches for each trial because there were two different coloured balls in the bag. They were also expected to indicate the probabilities of outcomes in the branches (see Figure 1).

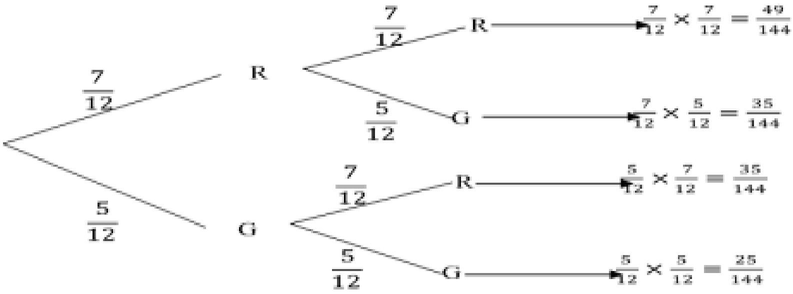
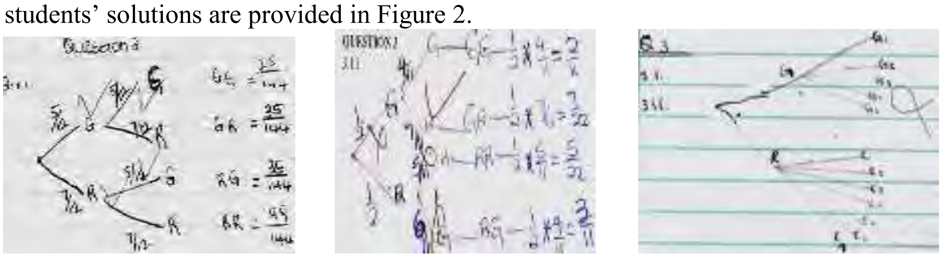


Figure 1
The Solution to Question 3.1.1

The results for Question 3.1.1 revealed that 210 (61%) of the students provided completely correct answers, 59 (17%) provided partially correct, 70 (20%) provided completely wrong and three (1%) did not attempt the question. Some examples of the students' solutions are provided in Figure 2.



a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

Figure 2
Examples of Students' Solutions to Question 3.1.1

Figure 2b shows the partially correct answer of a student. The student was able to draw the branches as well as determine the number of experiments performed. However, the student got the probabilities on the first trial wrong, as well as the probabilities on the second trial. In Figure 2c, the student got the question completely wrong because he/she was unable to draw a correct tree diagram.

For Question 3.1.2, the students were expected to draw a tree diagram when selections of balls are not replaced after the first trial. The total number of balls in the bag was expected to decrease by one after the first selection (see figure 2).

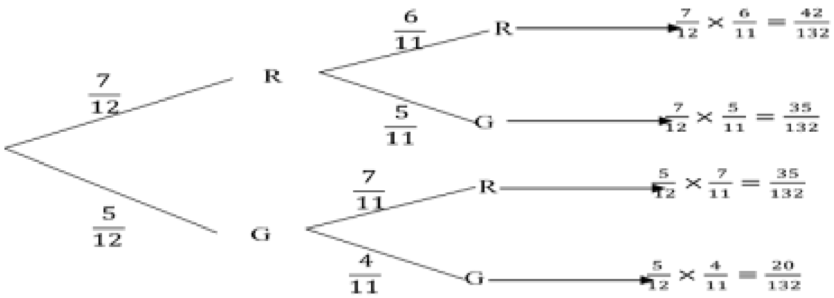
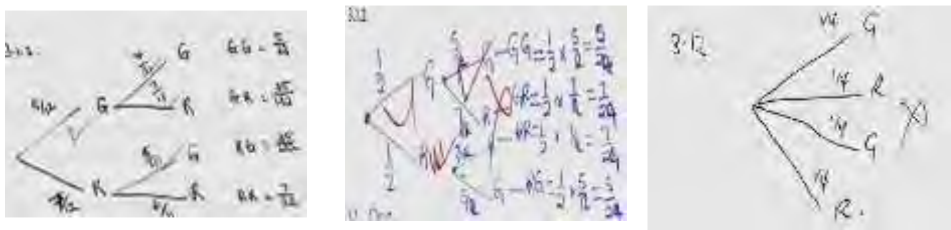


Figure 3
Solution to Question 3.1.2

The results revealed that 218 (64%) of the students provided completely correct answers, 56 (16%) provided partially correct answers and 68 (20%) provided completely wrong answers. Some samples of the students' solutions to Question 3.2 are provided in Figure 4.



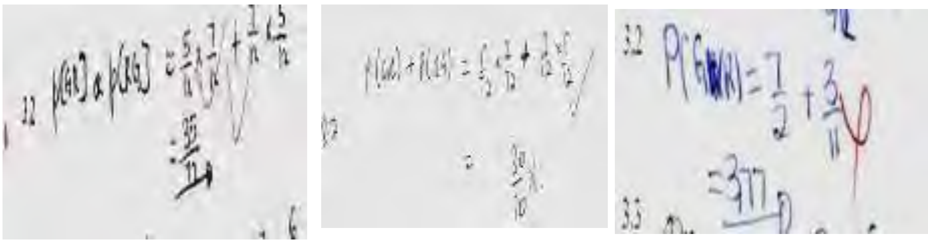
a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

Figure 4
Examples of Students' Solutions to Question 3.1.2

Figure 4b shows that the student was able to draw the tree diagram but had difficulty identifying the different probabilities in both experiments. In Figure 4c, the student lacked understanding in drawing a tree diagram i.e. knowing the different branches as well as the experiments performed, thus he/she got the question completely wrong.

In Question 3.2, the students were expected to compute P (R∩G/R) + P (G∩R/G) from the tree diagram to arrive at the correct answer by making reference to Question 3.1.1. The results revealed that 54 (16%) students provided completely correct answers, eight

(2%) provided partially correct answers and 280 (82%) provided completely wrong answers.

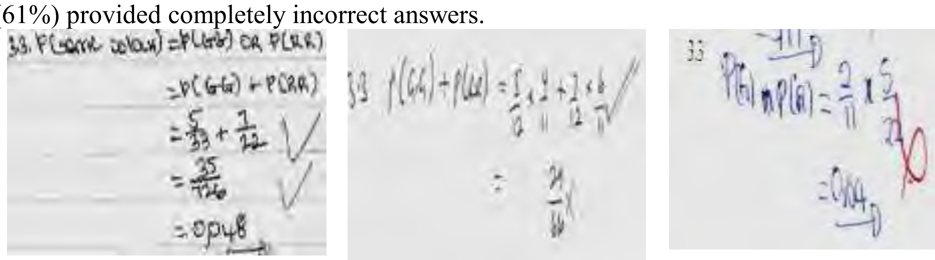


a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

Figure 5
Examples of Students' Solutions to Question 3.2

The students who failed to answer the question correctly might have gotten the tree diagram in Question 3.1.1 wrong, or they did not understand the question. Figures 5c shows that the student did not understand the basic law of probability as indicated by having a probability greater than 1 i.e. 7/2 (3.77). The student in Figure 5b used the correct formula but the computation was incorrect, leading to an incorrect final answer.

In Question 3.3, the students were expected to analyse the question as follows: P (of selecting the same colour) = P (G∩G/G) or P (R∩R/R), making reference to Question 3.1.2, (selection without replacement). The results showed that 11 (3%) students provided completely correct answers, 122 (36%) provided partially correct and 209 (61%) provided completely incorrect answers.

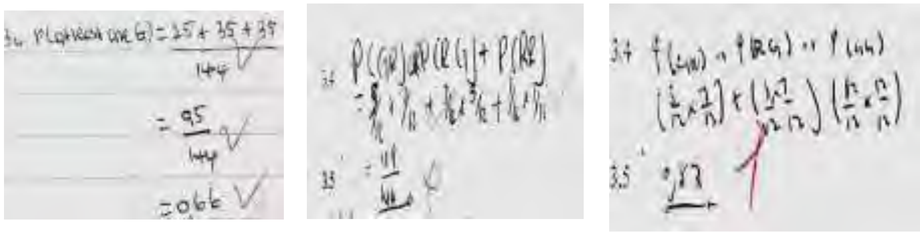


a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

Figure 6
Examples of Students' Solutions to Question 3.3

The student that got Question 3.3 partially correct in Figure 6 had computation difficulties.

In Question 3.4, the students were expected to add P (R∩G/R), P (G∩R/G) and P (G∩G/G) to obtain the answer (making reference to Question 3.1.1). The students could have also used the formula P (at least 1 green) = 1 - P (G′∩G′) to reach the desired answer. The results revealed that five (1.4%) students provided completely correct answers, 13 (3.8%) students provided partially correct answers and 324 (approximately 95%) students provided completely incorrect answers.



a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

Figure 7
Examples of Students' Solutions to Question 3.4

In Question 3.5, the students were expected to provide the following answers: the sum of all probabilities after the last selection on the branches of a tree must be one e.g. in Question 3.1.1 49/144 +35/144 +35/144 +25/144 =1. The sum of all probabilities of the branches from one node should be equal to one. For example, in Question 3.1.1 the sum of probabilities on the first node is 7/12 +5/12 =1

The results showed that only two students, representing one percent of the students who took part in the study, got completely correct answers, while 78 students (23%) got partially correct answers and 262 (76%) got completely incorrect answers for this question. Students who were able to provide both answers had it completely correct, those that provided only one had it partially correct and those that had both incorrect had it completely incorrect.

A summary of how the students performed in the questions relating to tree diagrams is presented in Table 2.

Table 2
Summary of the Students' Performance in the Tree Diagram Questions

	Completely correct solutions	Partially correct solutions	Completely incorrect solutions	Did not attempt the question
3.1.1	210	59	71	2
3.1.2	218	56	68	-
3.2	54	8	280	-
3.3	11	122	209	-
3.4	5	13	324	-
3.5	2	78	262	-

Proficiency in solving probability problems relating to the use of a contingency table

The question that required the use or analysis of a contingency table was Question 4. The question was: Each of the 200 employees of a company wrote a competency test. The results are represented in a contingency table (Table 3).

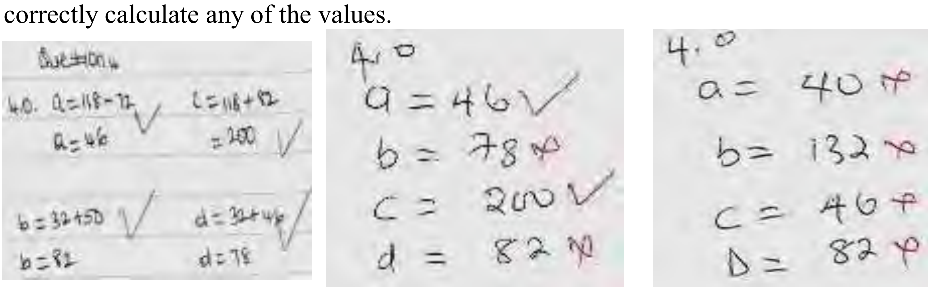
Table 3
Contingency Table

Gender	Pass	Fail	Total
Male	A	32	D
Female	72	50	122
Total	118	B	C

4.0 Find the values of A, B, C and D.
4.1.1 Are the events Pass and Fail mutually exclusive?
4.1.2 Explain your answer to Question 4.1
4.2 Show that passing/failing the competency test is independent of gender
4.3 Give any alternative solution to Question 4.2
4.4 Calculate the probability that a student selected at random was a male who passed or a female.

In Question 4.0, the students were expected to add the values in each row and equate these to the totals in the respective rows to find the missing values. A similar exercise was expected to be done for each column. For example, the equation A+ 72=118 could be used to find the value of A.

The results revealed that 309 (90%) students were able to correctly calculate the values of A, B, C, and D, while 21 (6%) were able to correctly calculate some of the values (not all of them) and 12 students, accounting for 4% of the participants, could not correctly calculate any of the values.



a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

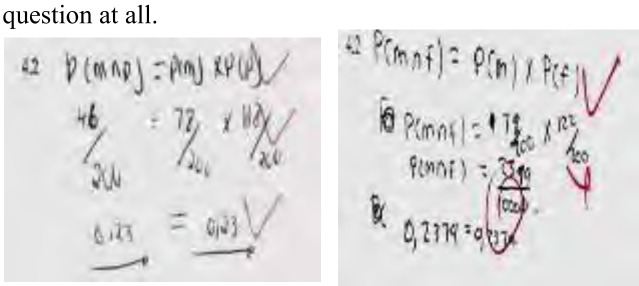
Figure 8
Examples of Students' Solutions to Question 4.0.

The correct answer to Question 4.1.1 was "yes". The results revealed that 243 (71%) of the students correctly answered the question, while 99 (29%) failed the question.

In Question 4.1.2, the students were expected to provide the following reasons: (i) Because P (F∩P) = 0, the event Fail and the event Pass cannot occur at the same time, and (ii) The events are disjointed. The results revealed that 204 (60%) students provided completely correct answers to the question and 138 (40%) provided incorrect answers.

In Question 4.2, the students were expected to test for any of the conditions: i) P(Man ∩ Fail) = P (Man) × P(Fail), ii) P(female ∩ fail) = P(female) × P(Fail), iii) P(Man ∩ Pass)

= P(Man) ×P(Pass), and iv) P(female ∩ Pass) = P(female) × P(Pass). The results revealed that two (1%) students provided completely correct answers, 35 (10%) partially correct answers, 300 (87%) completely wrong answers and five (3%) did not attempt the question at all.

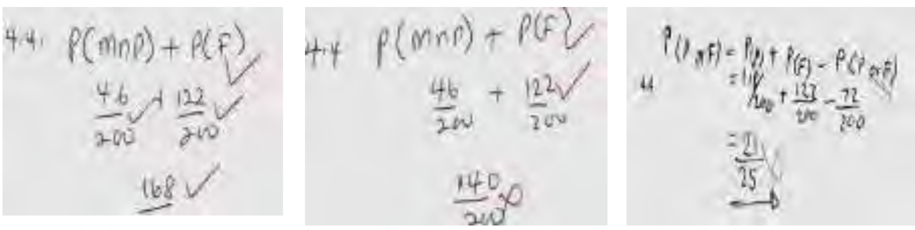


a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

Figure 9
Examples of Students' Solutions to Question 4.2

In Question 4.3, the students were requested to provide an alternative solution to Question 4.2. The students could have provided any of the alternative solutions to Question 4.2 except the one already provided to answer the previous question. For example, those students who provided the solution P (Man ∩ Fail) = P (Man) ×P(Fail), in Question 4.2 could have provided any of: i) P(female ∩ fail) = P(female) × P(Fail), ii) P(Man ∩ Pass) = P(Man) ×P(Pass), and iii) P(female ∩ Pass) = P(female) ×P(Pass). The results revealed that only four students (approximately one percent) provided completely correct answers, 21 (10%) provided partially correct answers to the question, while 314 (92%) provided completely incorrect answers and 3 (1%) did not attempt the question at all.

In Question 4.4, the results revealed that only two students (1%) got completely correct answers, six (8%) got partially correct answers and 330 (96%) got completely incorrect answers.



a) Completely correct answer b) Partially correct answer c) Completely incorrect answer

Figure 10
Examples of Students' Solutions to Question 4.4

A summary of the students' performance in questions relating to contingency tables is presented in Table 4.

Table 4
Summary of the Students' Performance in the Contingency Table Questions

	Completely correct solutions	Partially correct solutions	Completely incorrect solutions	Did not attempt the question
4.0	309	21	12	-
4.1.1	243	-	99	-
4.1.2	204	-	138	-
4.2	7	35	300	5
4.3	4	27	314	3
4.4	3	12	330	-

DISCUSSION

The study investigated the proficiency of Grade 12 students in the use of contingency tables and tree diagrams to solve probability problems. The study showed that the average scores of the students in the use of tree diagrams and contingency tables to solve probability problems were 39% (SD=1.8) and 43% (SD=0.7) respectively. Most of the students (97% and 91% respectively) scored below 50% in solving probability problems involving the use of tree diagrams and contingency tables. These findings indicate that the students were not proficient in the use of tree diagrams and contingency tables to solve probability problems.

While 61% of the students were able to illustrate the outcome of an event using a tree diagram only 2% and 3%, respectively, were able to use the tree diagram to correctly compute the probabilities of an event when the two balls selected were of different colours and of the same colours. This means that although many of them could draw a tree diagram to represent the outcome of an event, they could not interpret the diagram and use the interpretation to solve conditional probability problems. This shows a lack of conceptual understanding of tree diagrams. The findings of this study were consistent with those of Mutara and Makonya (2016) and Makwakwa (2012) that students struggle with the use of a tree diagram in solving probability problems.

That some of the students provided probabilities greater than one shows that they did not have a full understanding of the laws and principles of probability. This is in agreement with Hirsch and O'Donnell (2001), who found that when students do not understand the laws of probability, they form misconceptions through informal experiences outside the classroom. The identification of the number of branches, the computation of probability when the items are not replaced, and basic algebraic computations might be the cause of the high percentage in the category of partially correct solutions, as argued by Priyani and Ekawati (2018). These authors explain that misunderstanding in basic algebraic concepts could lead to errors, thereby leading to low proficiencies.

The students' overall average score of 43% in the use of contingency tables to solve probability problems is an indication that the students struggled with the use of contingency tables to solve probability problems. On the questions regarding the use of the data in a contingency table to determine whether the events are dependent or independent, 99% of the students could not provide complete correct solutions to the questions. This means that, as was the case with the questions relating to the use of tree diagrams to solve probability problems, the students lacked fundamental understanding of contingency tables in probability. This finding corroborates the findings of other past studies (e.g. Estrada & Díaz, 2006; Makwakwa, 2012) that many students have difficulties solving probability problems using the data from contingency tables.

The students' difficulty in showing if two events presented in a contingency table were independent or not could be due to confusion between conditional probability and joint probability, as observed in other studies (e.g. Estrada & Díaz, 2006; Santos & Dias, 2015). Some of the students in this study computed the product of the probabilities of male and of passing or of female and of passing, suggesting that they confused conditional probability with joint probability. Alternatively, others computed the probabilities of mutually exclusive events, perhaps assuming that passing (or failing) and gender were mutually exclusive events. It is possible that the students failed to understand and correctly interpret the meaning of the term 'independent of gender' as it has been shown by some studies (e.g. Groth, Butler & Nelson, 2016; Watson, 2011) that one major challenge faced by students in learning probability is the understanding of terminology.

CONCLUSION

This study investigated Grade 12 students' proficiency in solving probability problems involving the use of contingency tables and tree diagrams. The study showed that the students were not proficient in the use of tree diagrams and contingency tables to solve probability problems. Even though many of the students could draw tree diagrams, they did not understand the concepts involved. Similarly, the students had difficulties in computing probabilities from the data in contingency tables. The findings of this study provide evidence with practical implications for the teaching of probability.

In teaching probability, teachers should employ teaching methods like questioning, classroom discussions, and problem solving. These methods allow students to construct knowledge and gain a deeper insight into the concepts being taught. The teacher-centred approach to teaching that is mostly used by teachers in mathematics often causes students to learn algorithms superficially without understanding the concepts behind the algorithms. In addition, the researchers recommend that teachers always explain the precise meaning of probability terms to students in their teaching. This will possibly enhance students' understanding of the terms, and consequently their proficiencies in solving probability problems.

Since the teaching of the topic is new to many teachers, teachers may benefit from workshops on the content and method of teaching probability. Such workshops might enhance teachers' understanding of contingency tables and tree diagrams and their use in solving probability problems. Textbook authors and publishers are also advised to pay close attention to activities that involve tree diagrams and contingency tables in their textbooks to give students and teachers more exposure to the concepts. Activities and questions in these textbooks should help the users thereof to understand the concept of tree diagrams and contingency tables and how they could be used to solve probability problems.

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