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An economic viability study of crop revenue insurance for the South African maize market: a statistical copula approach

by

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DECLARATION

I, Tatenda Tinashe Mutungira hereby declare that this dissertation that I submit for the Master of Commerce degree in Agricultural Economics to the University of Pretoria has not previously been submitted by me or any other person for degree purposes at any other tertiary institution.

Signature: …………………………………………………………………

Date: ……………………………………………………………………………

DEDICATION

I dedicate this dissertation to my maternal- and late paternal-grandmother for their words of wisdom, support and encouragement, and the emphasis they placed on the value of education throughout my life.

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ABSTRACT

AN ECONOMIC VIABILITY STUDY OF CROP REVENUE INSURANCE FOR THE SOUTH AFRICAN MAIZE MARKET: A STATISTICAL COPULA APPROACH

By

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The purpose of this dissertation is to conduct a viability study of a revenue-based crop insurance product, focussing on the white maize market in South Africa.

It has been established that multi-peril crop insurance (MPCI) schemes in South Africa have made losses in almost half of the 14 years between 2005 – 2018, hence the need for an alternative offering. Recent research has shown that when compared to MPCI, a revenue-based product is more viable and affordable to offer in the absence of government support, premised on an inverse relationship between the yield and prices of crops, that permits for a relatively stable expected revenue, referred to as a "natural hedge".

A statistical copula approach was applied to establish the dependence relationship between white maize yields and prices in this research. Copulas have been established as a superior dependence modelling technique because their approach moves away from the normal model assumption as depicted by the Pearson correlation relied on by insurers. Copulas are flexible modelling instruments that permit the use of different statistical distributions in the marginals of the variables which is favourable because evidence exists suggesting that yield distributions are not normally distributed contrary to the insurers modelling approach.

Alternative statistical distribution models were therefore fit to the variables to get a better representation of their marginal distributions. The marginal distributions produce cumulative distribution (CDF) values used in the copula fitting procedures. From the established copula dependence structure, a Monte Carlo simulation produced CDF values that were converted back to price and yield variates for use in comparing yield- and revenue-based crop insurance policies. The two insurance products were compared on the premium rates achieved at identical insurance coverage levels while the former is ultimately determined by the expected loss outcomes. The premium rate achieved represents the affordability of the product while the expected loss outcome resembles the level of risk from insuring a product.

Findings were that an inverse relationship exists between the variables from the three selected maize producing districts namely, Bloemfontein, Vryburg and Delmas. The best-fitting copula models were achieved from a combination of alternative marginal distributions for both variables, as well as alternatives to the Gaussian copula. Revenue-based crop insurance policies achieved lower insurance premium rates than yield-based insurance policies for Bloemfontein and Vryburg, but not Delmas. The study recommends that insures consider production region modelling and insurance products.

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ACRONYMS AND INDICES OF SPECIAL FUNCTIONS

μ : Mean
σ : Standard deviation
α : Shape 1
β : Shape 2
τ : Kendall's Tau
℃ : Degrees Celsius
ADF : augmented Dickey-Fuller
AIC : Akaike's Information Criterion
BIC : Bayesian Information Criterion
cdf : cumulative distribution function
CEC : Crop Estimates Committee
CRI : Crop Revenue Insurance
CV : Coefficient of Variation
$\boldsymbol{CvM}$: Cramer-von Mises
DAFF : Department of Agriculture, Forestry and Fisheries
FCIC : Federal Crop Insurance Corporation
FCIP : Federal Crop Insurance Program
FSCA : Financial Sector Conduct Authority
GOF : Goodness-of-fit
ICDF : Inverse cumulative distribution functions
KPSS : Kwiatkowski–Phillips–Schmidt–Shin
$\boldsymbol{KS}$: Kolmogorov-Smirnov
$\boldsymbol{ln}(\boldsymbol{P_{AEH}})$: Logged Average Expected Harvest Price
$\boldsymbol{ln}(\boldsymbol{P_{AJH}})$: Logged Average July Harvest Price
$\boldsymbol{ln}(\boldsymbol{P_{AJH}}) - \boldsymbol{ln}(\boldsymbol{P_{AEH}})$: Logged Price Change
MLE : maximum likelihood estimation
MPCI : Multi-Peril Crop Insurance
$\boldsymbol{P_{AEH}}$: Average Expected Harvest Price
$\boldsymbol{P_{AJH}}$: Average July Harvest Price
$\boldsymbol{P_{C}}$: Price Change
RMA: Risk Management Agency
rvs : random variables
LTAY : Long Term Average Yield
Tons : tonnes
t/ha : Tonnes per hectare
SAIA : South African Insurance Association
US : United States of America
USFCIP : United States Federal Crop Insurance Program

GLOSSARY

Administrative costs: Refers to all the costs associated with providing and delivering a crop insurance product.

Coverage level: The portion of production or revenue that is insured, usually expressed as a parentage.

De-risking: Taking steps to reduce the amount of risk assumed.

Expected price: This is a representative harvest price of a commodity during planting leading to harvesting time as traded by the futures contract.

Harvest price: This is taken as the average daily prices for the harvesting month of the futures price.

Indemnity payment: The amount of money the insured receives from the insurer if the realised yield/revenue is below the coverage level value

Loss ratio: These are the indemnities paid as a percentage of premiums paid by producers.

Moral hazard: A situation when a party alters their risk knowing that a third party will bear the costs associated with their behaviour

Premium: The purchase price of an insurance policy.

Premium subsidy: This is a benefit usually from the government that reduces the premium paid by farmers.

Re-insurance: A process of insurers taking an insurance policy on the risk they have underwritten.

Skewness: This speaks to the shape of a distribution of the dataset, on whether it is symmetrical, or has tails to the left or right.

Single-peril crop insurance: This is a crop insurance product that covers a single risk.

Multi-peril crop insurance: This is a crop insurance product that covers numerous risk in one policy.

CHAPTER 1

1.1 INTRODUCTION

The multi-peril crop insurance (MPCI) offering is struggling in the South African market. According to industry experts from Santam, Munich Re, Munich Re Corporate Solutions, and the Land Bank Insurance Company, MPCI is a loss-making business in South Africa (SA). Literature suggests that this is also the case for MPCI products internationally (Smith and Glauber, 2012; RMA, 2017, 2019). The SA market has experienced loss ratios above 100% in nearly half of the time, spanning over 14 years between 2005 - 2018 while the loses tend to be massive when they do occur (SAIA, 2016; Munich Re, 2018).

In the cropping seasons of 2004/05, 2005/06 and 2007/08, Munich Re stopped supporting MPCI in SA because of the continuous losses experienced, largely due to adverse weather conditions. At that time, the organisation felt that the insurers it backed in the SA MPCI market were not charging premium rates that were reflective of the risk (drought) faced, which was exacerbating the company's losses. According to role-players, amendments to the premium rates charged for MPCI saw the return of Munich Re to backing the product once again in the 2008/09 season (Munich Re, 2018).

Drought risk is the main cause of the poor performance of MPCI in SA. Due to the inherent nature of droughts, they tend to cover a large geographic area, negatively affecting a large area of crop, while often spanning over multiple seasons. Historically, SA experiences drought years that tend to be consecutive (shown in Figures 2.6 to 2.8, p.43 to 44). Because of this, historically MPCI products have been limited to 20% of the SA crop insurer's book of business, and this has been a long-standing agreement between the country's reinsurers and the insurers. This agreement was reached to satisfy what the reinsurers deemed an acceptable risk exposure for the backing of MPCI in SA while stemming out the potential systemic risk that this scheme could bring to the crop insurance industry.

Internationally, the majority of MPCI schemes receive government support whereas in SA this is not the case (Oliver Mahul and Stutley, 2010). A 2008 World Bank survey study showed that in the absence of premium rate subsidies, loss ratios exceeded 100% for 6 of the 8 high-income nations surveyed (Smith and Gaubler, 2012). For example, while relying on 2017 season's data for the United States of America (US), if the premium subsidy is left out of the loss ratio calculation, the ratio deteriorates to 144% from 53% (RMA, 2017). This means that in the absence of a premium subsidy support, for every US$1.00 collected in premium, US$1.44 would be paid out in indemnity payments by the insurer. To provide some context of an idyllic situation, according to Nieuwoudt (2000), a 95% loss-ratio is the minimum threshold to break-even in a crop insurance business. At this threshold, the MPCI scheme would at least be able to meet its insurance obligations since for every US$1.00 collected in premiums, US$0.95 is available to cover indemnity payments.

Due to the substantial drought risk, SA insurers have historically retained only a small amount of the insurance business risk they have assumed in the market and ceding the bulk of their MPCI book of business to re-insurers. The Land Bank Insurance Company typically retains 30% and cedes 70%. This means when the insurance company makes a profit on the MPCI product, which they have rarely done over the last 10 years, the bulk of the profit goes to the re-insurer, adding to the unprofitability situation of this offering.

In May 2019, the insurer Swiss Re Corporate Solutions pulled out of the SA crop insurance market. Reasons for their departure have not officially been confirmed but the dismal performance of their MPCI book is a likely contributing factor. It does not come as a surprise that some reinsurers have expressed their dissatisfaction with the SA government's stance of non-intervention in the sector, hinting at a possible withdrawal from the market if the government does not intervene with support measures.

1.2 PROBLEM STATEMENT

In comparison to single-peril crop insurance products, MPCI is significantly more expensive. MPCI is costly for two key reasons: Firstly, it is a comprehensive risk cover, that even includes the systemic type of weather risks. Secondly, MPCI schemes suffer from large administrative costs in their operations. Thus, MPCI is plagued by a combination of large losses due to systemic drought risk occurrences as well as high costs incurred to curb moral hazardous behaviour to the scheme.

From consultations with farmers, agricultural specialists, bankers and crop insurance professionals, the consensus is that SA farmers' perception of MPCI is that the product is too expensive. As a result, the farmers are reluctant and selective on taking out an MPCI policy. Consequently, uptake of MPCI fluctuates according to the season and financial position of the farmers. For instance, during the La Nina phase of high rainfall in 2009/10 and 2010/11 seasons, there was a low uptake of MPCI. However, during the El Niño drought phase of 2014/15 and 2015/16, demand for MPCI was high. This situation of variable crop insurance uptake is not ideal for insurers as they depend on the better seasons to make profits and build capital reserves that can then contribute towards indemnity payments in the bad seasons. To stem out variable insurance uptake and reduce their risk, Santam withheld MPCI policies for new clients during the El Niño phase but gave preference to their regular clients.

Furthermore, according to parties to the matter, MPCI is not structured correctly in SA. The idea of crop insurance is to leave a farmer in the same position as he/she was in before an adverse occurrence, at least in terms of production cost expenditures. However, the sentiment is that the pricing of MPCI is not aligned to the inherent risk for two main reasons. Firstly, historic yields are being overstated at the farm level to maximise potential indemnity payments. Secondly, the brokerage network benefits from having inflated production values because this leads to higher commissions received.

Despite the perceptions of SA farmers and some insurers that MPCI is expensive, the reality for the insurers is that the product is not profitable. The sentiment from the other insurers and reinsurers is that MPCI is under-priced in the market. Evidence from the underwriting experience shows loss-ratios exceeding 100% in almost half of the cases over fourteen years (2005 - 2018). Due to the poor financial performance of MPCI, there is a risk of insurers and re-insurers halting their support for these schemes in SA. As of May 2019 last year, Swiss Re Corporate Solutions is an insurer that withdrew from the SA agricultural sector largely due to losses incurred in the market. Throughout this research between 2015 – 2020, reinsurers backing MPCI in SA have been de-risking from this product i.e. reducing their MPCI reinsurance support. This has resulted in an increase in the cost of insurance for two reasons: Firstly, because insurers have fewer options to spread their MPCI book of business risk, and secondly, insurers must limit the supply of crop insurance policies. From the laws of supply and demand, a reduction in supply causes prices to increase, ceteris paribus. An increase in the MPCI costs will negatively impact crop production of farmers without adequate land collateral because this insurance is an important collateral component to accessing production finance from banks.

In 2014 the Department of Agriculture Forestry and Fisheries (DAFF) initiated research to understand global best practices for a potential Public-Private Partnership (PPP) with insurance industries. This came after the South African Insurance Association (SAIA) and other role players in the sector approached the SA government for a state-supported crop insurance scheme. The research was conducted in partnership with a specialist agricultural consultancy and the short-term insurance industry's project team, producing a proposal document for DAFF and Treasury. However, by 2020, there was still no response to the report or progress on establishing a PPP. According to the unpublished 2016 report it was established that due to a reduced amount of MPCI coverage in the market, SA banks at times are unable to disburse approved production loans for a lack of drought cover from the client (LBIC, 2016). A lack of crop insurance or finance can inhibit the risk appetite of producers to growing crops more susceptible to certain risks such a drought. An example of this situation would be an SA farmer substituting maize production with sunflowers because of the latter's higher drought tolerance but historically a lower gross margin. According to Hazell (1992) that is the exact opposite of what a crop insurance product can and should do for the producer. Assuming a farmer is profit maximising while knowing that he/she is covered from adverse occurrences beyond his control, then this producer's rational decision would lead him to produce the most profitable crop.

In ideal production financing activities, crop insurance is an important risk mitigation instrument to potential loan defaults for banks from credit provided to clients without the necessary land collateral. Therefore crop insurance makes it easier for banks to disburse production loans, as well as broadening their lending book to incorporate the smaller high-risk farmers (Hazell, 1992; Skees and Price, 2000). Without crop insurance policies, specifically MPCI, banks will end up with loan portfolios dominated by only large farmers that would typically have a stronger balance sheet. However, according to some of SA's top banks operating in agriculture namely ABSA, Standard Bank and Nedbank, as well as other role-players (personal communication), MPCI is so expensive that most commercial farmers avoid it while high drought risk areas are not covered by MPCI policies. According to bankers, in their experience, a farmer looking for an MPCI policy is often a bad farmer, looking to benefit from insurance indemnity payments. In discussions, a case was mentioned where an insurance broker stopped offering crop insurance products because it negatively affected their primary agriculture insurance business portfolio (as a result of bad blood from when a farmer's claim had been denied due to what the insurer's assessors deemed bad farming practices, resulting in the farmer cancelling his/her other policies with the broker). Given this situation, some of the banks no longer favour MPCI as collateral because if an insurer can prove bad farming practices, no claim can be made and therefore there is no guarantee of security.

The SA insurance sector needs a solution to the unsustainability problem of the prevailing MPCI product. This study investigates the viability of a crop revenue insurance (CRI) product as an alternative while utilising alternative methods to modelling crop insurance policies. In modelling crop insurance products, the global insurance industry standards assume normality in both the yield and price risk marginals, as well as the dependence relationship of these two variables, despite ample evidence suggesting better and improved ways of modelling these risks. Currently, SA insurers are pricing yield insurance policies based on historical loss experiences and not on a particular risk modelling technique.

With a better performing crop insurance product in terms of a more stable expected loss experience for the insurers, the SA government could be more inclined to support the offering. At present, the SA government does not support any crop insurance schemes whereas MPCI is rarely offered in the absence of government support mechanisms in the rest of the world (Mahul and Stutley, 2010).

1.3 PROPOSED CONTRIBUTION

In this study, CRI is proposed as an alternative to MPCI, focussing on the largest grain crop in SA which is white maize. A statistical copula approach is applied to establish the dependence relationship between white maize yields and the associated commodity price for three magisterial districts of SA. This dependence relationship is important for the viability and pricing of the different crop insurance products, while the latter is a metric used in comparing affordability. The statistical copula approach is deemed as a superior method to establishing dependence relationships that has recently gained prominence in the field of agricultural economics (Goodwin and Mahul, 2004; Goodwin, 2015) and are a shift from the common Pearson correlation measure.

This study will consider different marginal distribution models for the variables price and yield of white maize for use in the copula fitting procedures by the Elliptical and Archimedean copula families. The initial step establishes the superior fitting distribution model that best mimics the behaviour of the variables, to produce the required cumulative distribution function (CDF) values for the copula fitting procedures.

The crop insurance products are compared through the following 3 Cases that represent the progression in crop insurance risk modelling techniques:

- **Case 1** - Approach maintains normality in both marginal distribution models and dependence structure (the Gaussian copula) when modelling crop insurance risks.
- **Case 2** – Approach uses a combination of alternative marginal distributions models in modelling crop insurance risks while maintaining normality in the dependence structure (the Gaussian copula).
- **Case 3** – Approach uses a combination of alternative marginal distribution models and different copulas in modelling crop insurance risks.

The initial approach of Case 1 is the ‘benchmark model’ because that is the prevailing crop insurance industry approach. The second approach, Case 2, limits dependence modelling to the Gaussian copula but permits the use of alternative marginal distributions to assess their effect on dependence. Case 2 therefore represents the improvement in marginal distribution modelling techniques which ultimately represents the behaviour of the yield and price variables. The third approach, Case 3, allows for a combination of alternative marginal distributions and alternative copula approaches in the dependence modelling to establish the overall superior copula method. Case 3 therefore represents a combination of improvements in marginal distribution modelling techniques, as well as an improvement in the dependence relationship modelling.

Findings by Ahmed and Serra (2015) show that given identical insurance coverage levels**,** the premiums rates of CRI were lower than those of yield-based crop insurance. The reasoning behind this finding is that, as yields fall, prices rise for agricultural products and vice versa, thereby permitting a stable expected revenue for the farmer. This inverse relationship between price and yield is crucial if a CRI product is to succeed because that is how the 'natural hedge' is formed (Meuwissen, Huirne and Skees, 2003; Tejeda and Goodwin, 2008; Goodwin, 2015) and is the basis for a stable insured variable (revenue). The term 'natural hedge' refers to a situation of a stable insurable variable, 'revenue' made possible when there is an inverse relationship between crop yields and their respective prices.

The following hypotheses will be investigated:

Hypothesis 1: *There is an inverse relationship between the price and the yield of white maize in SA*.

Hypothesis 2*: Premium rates realised for a revenue-based crop insurance product are lower than those from a yield-based product.*

Hypothesis 3*: Alternative risk modelling approaches in the marginal distributions of the variables as well as in establishing dependence relationships produces a better fitting crop insurance model.*

Considering Hypothesis 1, stability in the insured variable suggests a reduced financial risk for the insurer in terms of fewer chances of making indemnity payments and the reduced size of them thereof, as evident from the lower expected losses of CRI when compared to yield-based crop insurance reported by Ahmed and Serra (2015). From

Hypothesis 2, if premiums charged for CRI insurance in SA could be lower than those of yield-based products, it would become relatively more affordable for farmers to take up crop insurance. From hypothesis 3, if alternative crop insurance risk modelling techniques give a better fitting crop insurance model, this should prompt insurance companies to improve their modelling techniques if they were not already using these techniques.

1.4 OBJECTIVES

This research has three main objectives.

- Firstly, to establish whether there is an inverse relationship between SA white maize yield and price data, and if this is indeed the case, assess whether the 'natural hedge' holds.
- Secondly, to compare the expected losses and premium rates realised from the two insurance schemes (MPCI and CRI) to make a call on the affordability of the products. Affordability is determined by a comparison of premium rates realised by the two products at identical insurance coverage levels.
- Thirdly, to assess what the effects of different marginal distribution and dependence risk modelling techniques have on the actuarially fair insurance premium rates achieved, separately and in combination (representing the progression in crop insurance risk modelling techniques). This will require comparing Case 2 results to Case 1, the 'benchmark model' as well as comparing Case 3 to Case 1.

1.5 DISSERTATION OUTLINE

In Chapter 2, the concept of crop insurance is introduced and the factors that contribute to the success or failure of the insurance products are explained. The different types of crop insurance products are introduced. An international overview of the state of crop insurance is given and then narrowed down to the SA case. Chapter 3 focusses on the research methodology that incorporates alternative marginal distribution modelling of the variables white maize yields and their prices, for use in the copula fitting procedure. Chapter 4 is the application were the variables price and white maize yields go through numerous transformations that lead to stationarity and suitability for the copula fitting procedures. Chapter 5 implements a Monte Carlo simulation to produce variates of expected prices and white maize yields that are necessary for comparing yield and revenue crop insurance products. Chapter 6 summarises and elaborates on the research findings.

CHAPTER 2

CROP INSURANCE MARKET: A GLOBAL OVERVIEW

2.1 INTRODUCTION

In this chapter, the concept and theory of crop insurance are introduced. A brief history of crop insurance is given to provide a basic understanding of where it started, what it has been used for, and how the products are evolving. Principles of insurability are discussed to understand the factors that lead to the success or failure of a crop insurance product. Factors leading to the failure of crop insurance are then discussed in-depth. A high-level summary is provided on the different but key crop insurance products on the global market, specifically their functioning and how they differ. A global overview of the recent trends that have occurred in the world's crop insurance sector is given, covering premiums collected, liability covered, and government intervention. An overview of SA's crop insurance sector is then presented with a focus on its history and current state.

2.2 CROP INSURANCE INTRODUCTION

Insurance is a way of transferring one's own risk to another for a fee called a premium. The party that assumes this risk is called the insurer. The insurer must satisfy in terms of the insurance agreement, in the event of an unexpected development that is stipulated in the insurance policy has occurred and achieved the minimum threshold that warrants an indemnity payment. It is important to understand what constitutes an insurable risk, hence the following seven key principles of insurability[1] are listed below (SchemeServe, 2014):

1. There should be numerous similar risks – insurance is primarily based on pooling risks together, therefore, having more units of an insurable risk permits insurers to have a large enough scale permitting for expected losses to approximate actual losses.
2. There must be a definable loss – referring to the ease in identifying a loss i.e. when it occurred and how it occurred.

[1] SchemeServe took these principles from the book titled "Principles of Insurance" written by Mehr and Camack in 1976. These discussed seven principles are still relevant in 2020.

3. Losses should normally occur by accident – meaning the event leading to a loss must not be intentionally influenced by the insured or benefactor of the indemnity payment.
4. Losses ought to be meaningful – the value of the insured item must be significant enough to warrant the need for insurance.
5. Premiums must be affordable – meaning premiums charged must be reasonable when considering the value of what is being insured and the value of expected indemnity payment.
6. The loss should be calculable – meaning the insurer must be able to accurately calculate losses incurred.
7. The liability should be limited – this means insurers need to limit their uptake of systemic type of risks. e.g. hurricane insurance.

Crop insurance has been used to stabilise crop production as well as the income of farmers, safeguard and stimulate investment in the agricultural sector, and aide farmers in accessing finance (Binswanger, 1986; Sherrick *et al.*, 2004; Roberts, 2005; Stutley, 2011). A crop hail insurance product was the first known and used crop insurance policy recorded from Germany in 1733 (Mahul and Stutley, 2010). The emergence of livestock insurance followed in the 1830s, initially in Germany, followed by Sweden and then Switzerland by 1900 (Smith and Glauber, 2012). Thus, crop insurance has existed for over 200 years in some parts of the world. Towards the end of the 19th century, crop insurance had been introduced to several European countries, the United States of America (US), Canada as well as Argentina (Mahul and Stutley, 2010).

Initial crop insurance policies were limited to single-peril product schemes such as hail, excessive rain, wind, or frost, offered by small mutual companies (Mahul and Stutley, 2010). These single-peril products lend themselves to easy administration from an insurer's point of view. Firstly, the damage is observed, making it easy to measure and assess losses. Over the years, insurers have developed accurate ways of calculating the damage experienced, hence Kang's (2007) comment that singe-peril crop insurance schemes lend themselves to robust premium rate calculations. Secondly, single-peril crop insurance products face little to no moral hazard risk because participants in these insurance schemes cannot create an event that warrants an indemnity payment (e.g. a hail or frost event) or influence the level of damage incurred. Therefore, it becomes less costly to administer these schemes because the insurer only incurs assessment costs in the event of the peril. Therefore, the characteristics of localised damage that is easily identifiable and measured, make the product affordable and sustainable for the insurer to offer. The adverse selection problem is catered for in this case since areas prone to hail events are identifiable. These areas will have a higher demand for that specific insurance type, which means insurers can factor into the premium rate the hail risk per region seeing that these locations have an observable loss history.

From 1899 to 1920, there are records of private companies offering MPCI in the US. MPCI, as the name suggests, is an insurance policy that is not limited to a single peril but can cover multiple risks, hence the term ‘multi-risk’. This product insures against both quantitative and qualitative losses to the crop. Quantitative losses can be measured and quantified physically whereas qualitative losses can only be assessed in the harvested crop, as the latter looks to the extent in compromised quality of the crop that reduces its market value (lower price). The first MPCI policy was written in 1899 for the Realty Revenue Guaranty Company of Minneapolis for its entire wheat crop, at a time when crop insurance coverage was limited to hail and fire. However, the company offering MPCI in 1899 subsequently cancelled this offering after a year (Gardner and Kramer, 1986). These were the early signs of complications that come with an MPCI scheme at a time when they were solely in the hands of private companies. In 1917, there were further attempts to offering MPCI policies by three joint-stock fire insurance companies in North Dakota, South Dakota and Montana (Valgren, 1922). In this setting, MPCI again failed due to a severe drought and because of the insurance companies’ inability to spread their risk, as well as covering themselves against considerable loss. In 1920, another attempt was made at offering MPCI by two fire insurance companies but with a slight variation whereby the insurer guarantees both yield and price, so essentially insuring the producer’s income (Valgren, 1922). Again, the offering failed but this time due to a larger than expected drop in crop prices resulting in massive losses for the insurers.

2.3 WHY CROP INSURANCE FAILS

To understand why crop insurance fails as a product and fails to emerge as a purely private sector offering, it is key to relook at some of the principles of insurability (covered in section 2.2). Summarised below is what Odening and Shen (2014) consider the foundations for an insurable risk at a reasonable cost:

- Many individual risks that are independent of each other.
- Loss amounts should not be enormous.
- Stationarity in the loss distributions.
- Ability to estimate loss distributions for estimated heterogeneous policyholders.
- Insurers can control for cost of moral hazard.

The above list is in line with suggestions and findings from literature as to why crop insurance fails, particularly for MPCI and can be summed into four elements:

- human factors (moral hazard and adverse selection)
- systemic weather risk
- lack of reinsurance, and
- administrative costs (Miranda and Glauber, 1997; Ozaki et al., 2008).

The listed elements above are justification for government intervention in this sector around the world.

Table 2.1 provides a summarised version of examples from the principal elements that contribute to the failure of crop insurance in the world and shall be discussed in greater detail in the coming subsections.

Table 2. 1: Key elements contributing to crop insurance failure

Elements	Examples
Systemic weather	**Drought risk** – The 2015 and 2016 El Niño drought seasons in SA. Two provinces contributing 60% of SA's maize harvest endured severe drought during the planting and crop development phase limiting the area planted to 1.9 million hectares which constitute two-thirds of the previous season's area (USDA, 2016).
Reinsurance	**Lack of Reinsurance** – Between 1899 - 1922, a lack of reinsurance support for MPCI in the US was a significant contributor to the failure of the product's policies (Valgren, 1922; Gardner and Kramer, 1986).
Human element	**Adverse selection** – Due to a superior knowledge of their production curves by US producers and ranchers, this attributed to a low insurance uptake despite premium subsidies offered to contribute to an expected indemnity payment that was on average double the value of premium rate between 1980 - 1993 (Glauber, 2004).
Human element	**Moral hazard** – US wheat farmers from Kansas who took up crop insurance policies were found to use fewer production inputs whereas those using inputs more intensively, were less inclined to purchasing crop insurance (Smith and Goodwin, 1996).
Administrative costs	Using a ratio of administrative costs to the non-subsidised premium paid *(A/P= administrative cost/premium paid),* Japan and the Philippines realised a ratio greater than 1 (Mamhot and Bangsal, 2012). This means these country's premium collected cannot even cover the administrative costs of their crop insurance programs.

2.3.1 Systemic Weather Elements

Southern Africa is getting warmer, characterised by minimum temperatures that are increasing faster than the maximum temperatures, while drought periods are becoming longer and harsher (Kusangaya *et al.*, 2014). Kruger and Sekele (2013) support this finding while stating that southern Africa is experiencing more high-temperature extremes than lower temperature extremes. Droughts, floods and extreme temperatures are all systemic weather risks and SA is mainly prone to the first (drought) (Nieuwoudt, 2000; Baudoin *et al.*, 2017). Common to these risks is the extensive geographic spread upon occurrence, affecting numerous farmers in a single period.

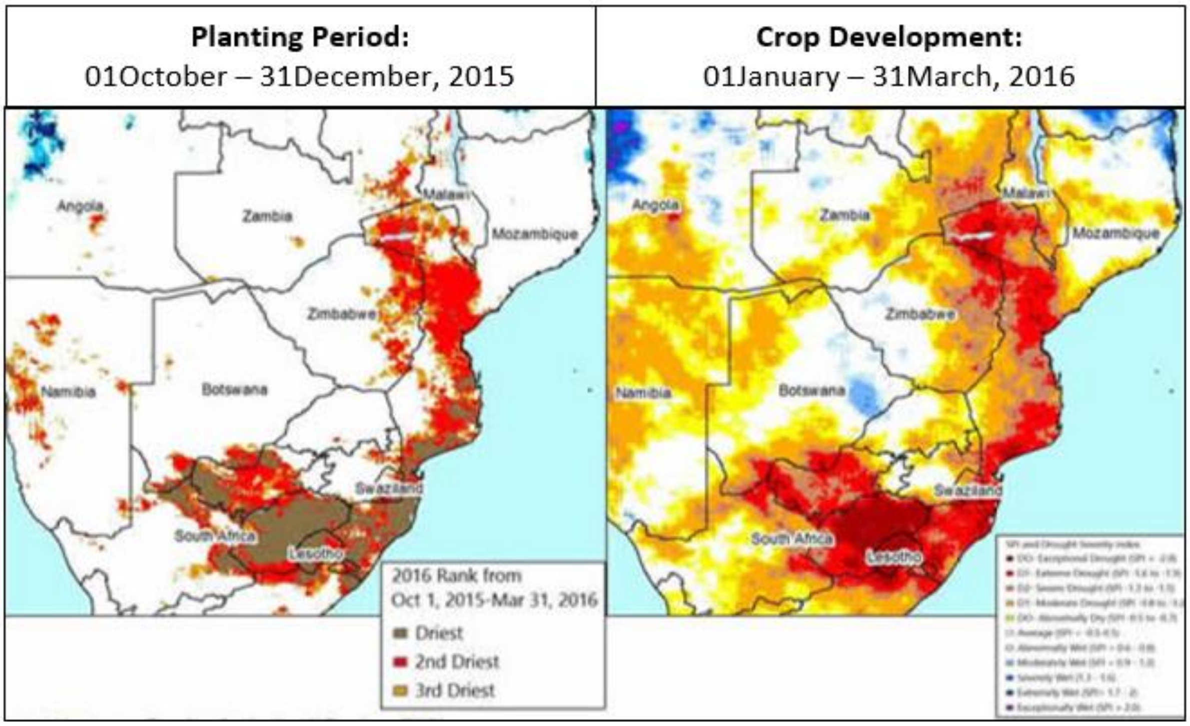


Figure 2. 1: Southern African Seasonal Rainfall Rank and Standard Precipitation Index (SPI)

Source: USDA (2016)

Notes: The seasonal rainfall rank and standard precipitation index are from the period of October 2015 to March 2016.

Figure 2.1 illustrates the extent of the 2015/16 El Niño drought that spread over three provinces in SA during the crop planting phase and eventually spread to the entire southern African region by the crop development phase. Drought is disruptive if not destructive to the development of a maize plant. In SA, the period from planting maize seed to the first emergence requires ideally warm (20℃ and 30℃ are optimal) and moist conditions (60% of soil capacity) for germination to occur after 6 to 10 days, however, dry conditions prolong this process by up to two weeks or more (ARC, 2003). Throughout the growth phase of a maize plant, every millimetre of rain consumed by the plant results in roughly 10 – 16 kg of grain while a yield of 3.15 t/ha requires 350mm to 450mm rain per annum, depending on the distribution of rainfall (ARC, 2003). Furthermore, moisture requirements are highest 2 weeks before and 2 weeks after pollination (roughly between 57 – 93 days after planting) and as the pollination process significantly influences grain production, this period is crucial (Pannar, 2020). From the drought implications shown in Figure 2.1, it is clearer to imagine what some of the yield implications will be in the respective regions under rainfed agriculture, taking into consideration that implications on yield is significant especially at the 9 to 12 leaf stage of the plant (approximately 56 days after emergence) (Pannar, 2020). In 2015/16 the Free State (FS) province was the drought epicentre, experiencing the driest spell during the planting window, followed by KwaZulu Natal (KZN) and then North West (NW) that was ranked second driest. As a result of the drought, the number of acres planted were reduced significantly (refer to Table 2.1 p.14), while typically 60% of the maize harvest comes from two of the hardest hit provinces, namely NW and FS (USDA, 2016).

Systemic and widespread weather perils make farm-level risk strongly correlated within an insured pool of the insurer's book of policies (products), thus defeating the insurer's intent of spreading loss risk across an insured portfolio. From Figure 2.1 above, the drought covered the whole of SA, thus making it difficult for the insurer offering MPCI to spread the risk of drought across multiple local policyholders. Furthermore, systemic weather risk defies most of the listed principles of insurability (from section 2.2). This is why the crop insurance industry has a significantly higher risk per unit of premium when compared to other lines of insurance, such as property liability or business insurers (Nieuwoudt, 2000). Supporting this claim, while measuring systemic risk in agriculture, Miranda and Gaubler's (1997) stochastic model produced results of a US insurer's portfolios being twenty to fifty times riskier than they otherwise would be if crop-yield losses were independent across farms.

Looking to the future and considering the climate change impact on US agricultural production, Tack et al. (2018) predict that rising temperatures will reduce average yields of maize, therefore increasing yield risk on average, resulting in higher premium rates charged. From their conservative 1 ℃ variation scenario, the annual subsidy paid would increase by 22% (a total of US$1.5 billion). From China, while investigating spatial dependence of weather events on agriculture, Okhrin et al. (2013) found that temperature has a more profound effect on crop production than precipitation, supporting findings of Lobell and Burke (2008) on the role of temperature change in crop yield variability.

The discussed adverse implications of drought and extreme temperatures on farming activities are reasons why agricultural insurance is unique to the other conventional lines of insurance offered such as property or life insurance policies. The weather's systemic nature and related catastrophic events make a strong argument for government support in the crop insurance sector. The argument is that private insurers alone cannot afford the massive losses associated with a systemic weather occurrence, which is strongly correlated in their insured pools and defies the principles of insurability.

2.3.2 The Reinsurance Element

The main condition for a successful MPCI offering is significant reinsurance or government support. Therefore the absence of reinsurance can be taken as a market failure for an insurance market, especially in the absence of government support to an MPCI offering or catastrophic risk (Skees and Price, 2000). Reinsurance is an exclusive type of insurance devoted to the primary insurer, known as the cedant, and is special for its redistributive role of the risk within an insurance sector. The concept of reinsurance is supposed to enable risk to be spread throughout the world following the principles of insurability and by doing so make undiversifiable risk diversifiable. When the risk is re-distributed on behalf of the primary insurer, a stable loss can be experienced. Therefore, reinsurance increases the primary insurers business capacity to issue more insurance policies.

However, reinsurance is limited for crop insurance schemes mainly due to systemic weather risks were catastrophic losses have always been greater than premiums collected (Pomareda, 1986). There are also suggestions that government interventions such as subsidised or free catastrophic insurance or reinsurance, have led to reduced interest and/or even undermined the emergence of private sector offerings (Miranda and Glauber, 1997). In addition to this, Skees and Price (2000) have included the availability of sophisticated methods of underwriting and understanding risk in the developed countries as a deterrence to the emergence of private sector reinsurance offerings. For example, countries with developed financial capital markets can use risk-linked securities[2] to transfer risk or rely on the use of alternative risk transfer (ART)[3] markets. While considering reinsurance, a downside is that it was meant to address diversifiable risk and not systemic risk, therefore is prone to suffer the same fate as primary insurers, hence the doubt in its ability to fully protect the primary insurer (Miranda and Gaubler, 1997).

The functioning of reinsurance is stipulated contractually between reinsurer and insurer which differs by country. Catastrophic losses are typically part of a special arrangement where the reinsurance company can limit the extent of a loss it can take and where government support exists, the latter can absorb the rest (Pomareda, 1986; Skees and Price, 2000). For example, in the US, when indemnities reach five times the size of premiums, the government absorbs all losses[4].

The first MPCI schemes offered by private companies failed due to considerable losses, suggesting a market failure due to systemic risk and a lack of reinsurance capacity (Valgren, 1922). Currently, in SA, reinsurers are de-risking from MPCI due to the systemic drought risk experienced thereby reducing the reinsurance capacity for this product. This trend in SA is likely to have an adverse effect on the future of MPCI products and as a result on crop production and lending habits of the banks (Valgren, 1922; Goodwin, 2015).

Other benefits from the reinsurers include their underwriting guidance, gained from the experience of operating with diverse clients and markets, granting them valuable knowledge to provide financial counselling (Harrison, 2010; Bednarczyk, 2014). Ultimately, reinsurance guarantees the viability of the primary insurers business.

[2] An example of risk-linked securities are catastrophe bonds (CAT) defined by the IRMI (2020) as a derivative debt investment vehicle issued by insurers and reinsurers designed to raise investor capital to cover catastrophic loss events.
[3] Alternative risk transfer markets as explained by the (IRMI, 2020a), refer to a marketplace in which non-traditional risk transfer approaches (as compared to commercial insurance) can be arranged.
[4] Taken from the 2020 Standard Reinsurance Agreement between the Federal Crop Insurance Corporation and an insurance company. Refer to the USDA website from the link: https://www.rma.usda.gov/-/media/RMAweb/Regulations/Appendix-2020/20sra.ashx?la=en

2.3.3 The Human Elements

Adverse selection and moral hazard are two significant human-based impediments to a viable private sector MPCI offering and both are a result of information asymmetry between the insured and insurers.

Adverse selection occurs when the insured has more information on their risk than the insurer and insurance will only be taken up when the value in premium paid is perceived less than the expected benefit in indemnity payments. In this case, there is information asymmetry on farm-level yield data whereby the producer has better knowledge of his/her production practices that influence the yield outcome. The insurer would have to incur a significant amount of costs to attain all the relevant and accurate farm-level yield data of a potential policyholder, which would result in higher premium rate costs. A rise in the premium rate reduces participation rates and shrinks the insured pool, which inherently increases the effect of adverse selection.

A good example of adverse selection inhibiting crop insurance uptake is from the US, between the years 1980s and 1990s were producers and ranchers instead utilised other risk-mitigating strategies (refer to Table 2.1 p.14) (Glauber, 2004). Therefore these producers and ranchers had superior knowledge of their production curves and what the insurers perceived as a low premium rate due to the subsidies provided, was not the case. Given this scenario, there is a risk of the insurer remaining with an insured pool concentrated by higher risk producers who perceive the expected indemnity as greater than the premium paid hence opting for a crop insurance policy. This scenario is exacerbated by the insurers' approach of using aggregate yield data (e.g. provincial level) in the estimation of individual yields and rates (Knight and Coble, 1997) confirming Binswanger's (1986) recommendation for the importance of localised data. However, a counter-argument by Meuwissen, Huirne and Skees (2003) find it acceptable to use aggregated data if it is correlated with individual farmer yields in the specific area where it is not practical to get all individual farmer’s yields, while that information is also highly susceptible to manipulation by producers.

Moral hazard is the second human element impediment to a purely private sector MPCI offering. Farmers can alter their farming behaviour to increase the chances of an indemnity payment received, as evident from the wheat farmers in Kansas (refer Table 2.1 p.14) (Smith and Goodwin, 1996). Ramaswami (1993) agrees with this finding, stating that insurance reduces the marginal productivity of inputs because any increase in output, lowers the expected indemnity payment, leading to a reduced input use and therefore a lower average output. These behaviours follow conventional wisdom and the fact that generally, farmers wait until the last minute to take up a crop insurance policy to gather as much information as possible on the season (Smith and Goodwin, 1996).

Around the world, insurers have planting windows for perennial crops that producers must adhere to for eligibility to participate in an insurance scheme. This limits adverse selection by producers who would only opt for an insurance policy when it is apparent that a larger than usual potential loss is looming (Skees and Price, 2000). Efforts to minimise moral hazard behaviour in crop insurance include offering a reduced coverage level but the downside is the risk reduction effect it has on taking out an insurance policy. Other types of crop insurance schemes exist such as index-based products that do not suffer from the moral hazard problems (discussed in section 2.4.2) are possible alternatives because actions needed to reduce moral hazard and adverse selection have significant cost implications on the insurer and these will be explained next.

2.3.4 The Administrative Cost Element

The last impediment to a purely private sector MPCI offering is the high administrative costs associated with the product. The costs incurred are largely in monitoring to prevent moral hazard behaviour and attaining accurate information to limit adverse selection. This information was described as an ‘investment’ by Binswanger (1986) because it has a cost to attain it and has value since the information on the policyholders’ production curves or production activities would limit information asymmetry.

An MPCI offering is costly from an administrative point of view because it is intensive in its implementation as well as monitoring. Firstly, insurers need to look at each policy for underwriting. Secondly, there is a need for skilled agronomists to assess historical farming practices before a policy is issued, as well as monitoring in-season farming activities. Therefore, before a policy can be issued, insurers need to obtain historical farm-level yield data from the farmer, soil analysis reports and the maps of the fields to be insured. Furthermore, there is in-season monitoring to ascertain the sufficiency of input application regimes (i.e. spray programmes, seed, and fertiliser applications). In SA, insurance companies send out assessors three to four times during key stages of a season to each insured field as part of their monitoring effort. For example, assessors must verify plant population at emergence, which is key in determining whether inputs were correctly and sufficiently applied during planting, and that would permit for achieving the expected yield, indicated in the insurance policy. If an occurrence has damaged the crop or could later affect the crop yield come harvest time, an assessor is required to verify this event while two assessors are required during harvesting to conclude a loss-adjustment. Furthermore, loss-adjustments are frequent under MPCI because crop yields often differ even in small local areas, thus making a further requirement for actuarial data and the respective crop insurance policy to be written at the lowest local level that is economically possible (Binswanger, 1986). Just and Weninger (1999) supported this view, finding that aggregating yield data resulted in an emphasis on region-wide variation while less attention went to farm-level variation. Beyond production level monitoring, the Unites States Federal Crop Insurance Program (USFCIP) employs data mining techniques to identify irregular claim outcomes for an investigation to curb moral hazard behaviour (Rejesus *et al.*, 2004).

One SA insurer further identified marketing costs associated with attracting and retaining clients as another significant administrative cost driver. This is because an Underwriting Management Agency (UMA) is writing crop insurance policies that are being underwritten by this insurer. Therefore, policies governing the crop insurance sector do not permit UMAs to market and sell insurance policies directly to the farmers due to a potential conflict of interest, since they are acting on behalf of an insurer. Other marketing costs are associated with the use of intermediaries which is the traditional way of distributing insurance, through for example agents and brokers (IAIS, 2018). When an insurance company does not have an agent system to sell its insurance policies, they rely on brokers who are not loyal to a specific company but are constantly looking for the best deals in the market to maximise on commission earned. This is a reason why some insurers must spend a lot on marketing their products to pull clients towards their products. Furthermore, because MPCI is inherently an expensive product means that there is a reluctance towards its uptake, and Pomareda (1986) found aggressive marketing strategies to be justified especially in the absence of government subsidy support to this scheme.

Further adding onto the administrative costs, it was identified that there is inadequate historical data on loss costs due to perils such as drought and floods, which has been established as the reason why the actuarially fair premium rate cannot be achieved (Coble and Barnett, 2013)[5]. As a result, insurers have reacted by adding to the premium rate what is called/termed a ‘load’, catering for a lack of information or uncertainty as follows: An ‘ambiguity load’, that creates a buffer for a lack of information on the expected loss calculation. A ‘reserve load’ as a contingency fund that could pay towards unforeseen losses or reinsurance. An ‘administrative cost load’ to cover administrative costs and profit margins that permits a competitive rate of return on equity. Therefore these ‘loads’ have also pushed administrative costs further up.

2.4 CROP INSURANCE PRODUCTS

Turrioz (2009) classified crop insurance products according to how the indemnity payment is determined. The three main groups are indemnity-based, index-based and a combination of yield and price measurement-based products that shall be explained below.

2.4.1 Indemnity-Based Products

These products are established on the actual loss incurred during the crop production cycle. There are two types of indemnity-based products termed the classical crop insurance products by Roberts (2005) and are popularly known as the ‘traditional crop

[5] According to Coble and Barnett (2013) when the basic conditions for insurability are adhered to, the process of establishing a premium rate for a homogeneous pool of insurable risk begins by defining: E(Loss Cost) = E (indemnity/liability). The regular approach entails use of historical experience data to accurately estimate the loss cost. Where there is enough historical experience data available to estimate the expected loss cost, the actuarially fair premium rate is equal to the expected loss cost.

insurance' products. Generally, these products fall under single/named peril or MPCI products, described in Table 2.2.

Table 2. 2: Indemnity-based products explained

Product	Description
'Damaged-based'	Based on the measure of damage to the crop. e.g. A crop hail insurance product. For an additional premium, this insurance policy can be extended to include frost, fire, and transit risk.
'Yield-based'	Centred on the crop yield-loss outcome during harvesting. e.g. A MPCI product stipulates numerous risks with the potential to reduce the expected yield and/or crop quality such as uncontrollable diseases or insect infections, floods, drought, frost etc.

To participate in these indemnity-based insurance schemes, some of the key items an SA farmer must produce are highlighted in Table 2.3 below.

Table 2. 3: Crop insurance policy contract Items

Key item	Description
Production history	Termed the Long-Term Average Yield (LTAY) is calculated over five previous seasons.
Field maps	GPS referenced maps of all fields with their names and land numbers.
Soil analysis	Copies of soil analysis reports not older than two years that are GPS referenced to the respective fields.
Area	Provide the number of hectares planted.

The LTAY represents the producer's expected yield and is important because the indemnity payment is derived from it. Only a portion of production can be insured, depending on where you are and by the insurance company, coverage ranges between 30% - 80% of the LTAY. In SA, the country's largest crop insurer offers coverages between 45% - 65%.

For a producer to take out the crop insurance policy, he/she must decide on the following[6]:

- The cover he/she would like. Therefore, must choose the Insured Yield (i.e.t/ha) and the Insured Production $(\mathrm{t/ha} \times \mathrm{ha} = \text{total production (T)})$
- The cover choices desired. This entails choosing an Insured Price (R/t) of the crop and coverage level (proportion of total production or yield e.g.45-80%).

The above two pieces of information lead to the Sum Insured value, calculated as,

$$\text{total production (T)} \times \text{Insured Price}(\mathrm{R/t}) \times \text{coverage}(\%).$$

Usually, the price chosen is one that best represents the harvest time price but during the planting period when an insurance policy is purchased by a producer.

From discussions with insurers in SA, common practice allows farmers to increase the initial insured price per ton during the cover period (provided no loss has occurred) but cannot lower it and this is permitted only within a specified time frame. Commonly found is that farmers choose to start at a low insured price then incrementally raise it accordingly as the market price direction becomes more apparent. This is a strategy used by producers to manage premium rates paid as there would be no need for a high insured value when the market indicators are not showing signs of negative price or yield risk.

In the case of a loss due to an insured peril, the following procedure leads to an indemnity payment depending on crop insurance type:

- **Crop hail** – the assessor determines the level of yield loss due to a hail occurrence and multiplies it by the Insured Prices(R/t).
- **MPCI** - the insurer takes the difference in the actual and the insured yield and multiplies it by the insured price (R/t) at harvest.

2.4.2 Index-Based Insurance Products

[6] Insights from an interview with a crop insurance specialist from Santam in February 2017.

Index-based products have an indemnity payment that is dependent on an index value derived from factors that are strongly correlated to the loss of the crop such as rainfall, temperature, wind speed or regional yield (Iturrioz, 2009).

Index-based products thrive from a cost structure perspective and ease of participation in the schemes over the traditional type of products. Firstly, the incentive to monitor participants in these schemes to limit moral hazardous behaviour is non-existent because a loss-adjustment is dependent on an index. Also, an index eliminates the need for farm-level assessments. These first two points on monitoring are significant cost drivers in the traditional crop insurance programs that an index-based product eliminates. Secondly, index-based crop insurance products do not require farm level LTAY data for participation in the insurance schemes unlike MPCI or revenue-based insurance products (Kang, 2007). Therefore this ease of participation into these index-based crop insurance schemes lends them as flexible alternatives in markets without the traditional crop insurance offerings. This makes index-based products attractive for the first time and/or emerging farmers that would not have LTAY data required in the other crop insurance schemes.

The main disadvantage associated with index-based crop insurance products is that they do not lower the risk of all participants to the program due to basis risk. Using a yield-index offering as an example, basis risk is a situation when despite experiencing yield loss at a farm-level, the area yield-loss index does not reach the minimum threshold that warrants an indemnity payment.

2.4.3 Revenue Based Insurance Products

A revenue-based crop insurance product insures the revenue stream of the farmer. Therefore, a CRI policy protects the farmer from fluctuations in revenue due to changes in price or yield, or a combination of the two (both price and yield declining).

The relationship between price and yield forms the foundation of this product, such that when production increases, prices of the product will decrease. Vice versa, when production decreases, the price of the product should increase, therefore the assumption is of an inverse relationship hence this research's ***Hypothesis 1*** (covered in section 1.3) (Meuwissen, Huirne and Skees, 2003; Ahmed and Serra, 2015). Thus, farmers face a joint risk from crop yields and crop prices. An inverse relationship between the variables price and yield makes revenue less variable when compared to the two individual insurable risks. Therefore, the product moves away from the typical approach of insuring these two joint risks (yield and price) separately and instead insure for them jointly through a revenue insurance product. This means farmers will insure their gross revenue, which is a function of the price received for a given unit of crop harvested multiplied by the total yield produced. Whereas before, farmers would insure the price of produce separately by forward selling or hedging on the futures market in conjunction with insuring the actual crop production/yield. Therefore financially, revenue insurance as a product is favourable because the producer is taking out a single crop insurance policy that caters for both price and yield risk as one, instead of insuring the two variables separately.

2.4.3.1 A Background to Revenue-Based Crop Insurance

Recent market liberalisation trends are linked to increased commodity price volatility contributing to the push factor of the emergence of revenue-based crop insurance products (Meuwissen, Huirne and Skees, 2003; Kang, 2007; Chung, 2012). Given a scenario of an increase in commodity price volatility, a farmer can experience a low gross revenue even when production is high, thereby threatening their income and the viability of their farming operation. Outside the US and Canada, foreign competition as a result of market liberalisation is reducing the domestic acreage of production in South Korea whereas European farmers are feeling the pressure of external competitive forces on production and prices, thereby threatening the viability of their domestic farming operations (Meuwissen, Huirne and Skees, 2003; Chung, 2012). The threat on the gross revenue of the farmer resulted in a need for a new type of insurance product that goes beyond ensuring production or commodity prices and that product was a revenue-based crop insurance offering.

The US and Canada have the largest and longest-running successful revenue programs in the world, making the two nations good benchmark examples. Their revenue stabilising insurance products were established in the early 1990s. Figures

2.1 and 2.2 below present a timeline of the progression of revenue-based products that have evolved towards whole-farm revenue products.

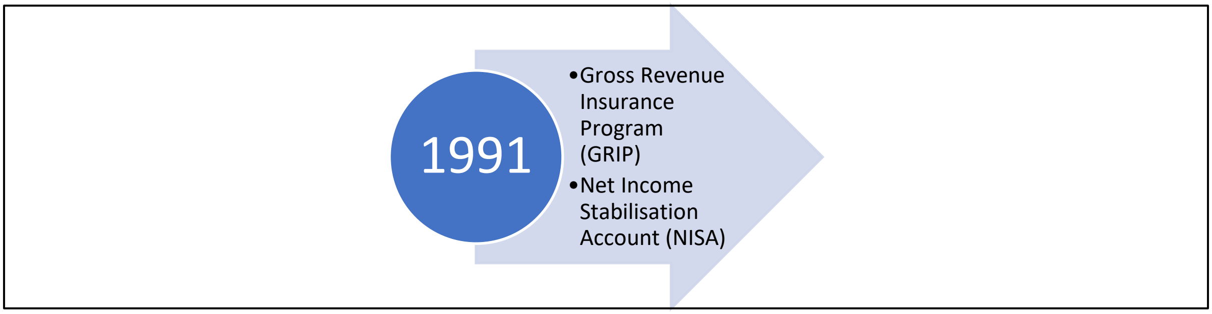


Figure 2. 2: Timeline of revenue-based crop insurance products in Canada

Source: Turvey et al. (1997)

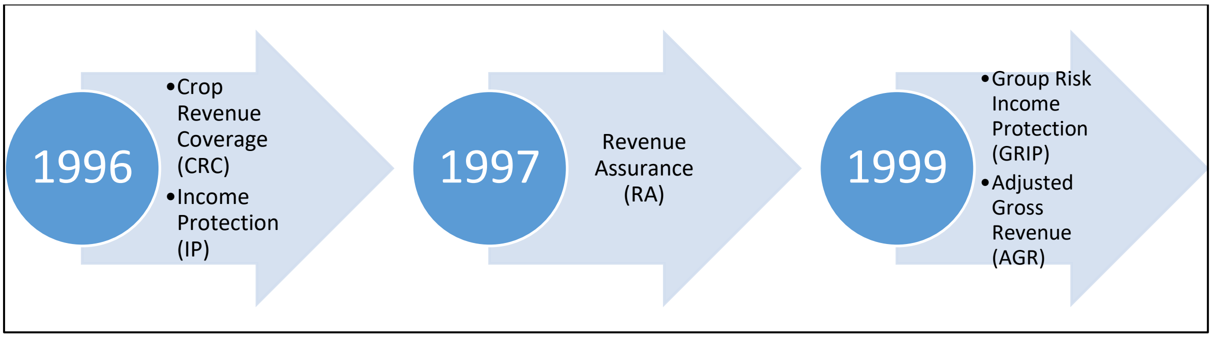


Figure 2. 3: Timeline of revenue-based crop insurance products in the US

Source: Kang (2007)

Canada has two main revenue-based products whereas the US has four, illustrated in Figures 2.1 and 2.2 respectively. Turvey *et al.* (1997) provided a detailed breakdown of Canada's revenue-based programs summarised as follows: Firstly, GRIP is a revenue insurance product that pays indemnities when market revenue falls short of the target revenue. If a shortfall occurs, the deficit is covered by the federal and provincial governments by the 65% and 35% portion, respectively. Secondly, NISA, on the other hand, is a product that provides income stabilisation utilising individual farmer accounts which encourage farmers to set aside money in high-income years for use in bad years while the government matches the funds set aside by the producers up to $250 000 per farm.

The US's initial revenue products namely, CRC, IP and RA paid indemnities when realised revenue was less than the revenue guarantee. Distinguishing features from Canada's GRIP is that the three US programs utilised the farmer's actual production history and prices as provided on the futures exchange to establish revenue. Price used is a key distinguishing feature to the US's revenue-based products and the differentiation is given in Table 2.4 below (further differentiation in Appendix A, Table A1).

Table 2. 4: Comparison of the initial revenue-based insurance products in the US

Feature	Crop revenue coverage	Income protection	Revenue assurance
Basic for insurance guarantee	Higher of: 1) APH yield * Base 2) APH yield * Harvest price **NB**. APH is Actual Production History Insurance guarantee increases when the Harvest price exceeds the Base price	APH yield * Projected price	APH yield * Projected Harvest price Harvest price option increases the guarantee when the Harvest price exceeds the Projected Harvest price

Source: Kang (2007)

From Table 2.4, IP is the only programme that does not incorporate potential benefits from seasonal changes in commodity prices. The price increase benefit from the CRC program has its limits per crop, for example, US$1.50 per bushel of maize (Kang, 2007). In the RA programme, the insured receives the full benefit in the event the fall harvest price is greater than the projected harvest price.

The US's GRIP program will pay an indemnity when the county revenue is less than the trigger revenue. Therefore, this is an index type of revenue product that utilises futures prices and county crop yields to establish expected and actual county revenue. In this program, producers are free to choose revenue protection above the expected revenue within a specified range published in the actuarial documents of the crop insurance policy (FCIC, 1998, 1999).

The AGR program allowed the producer to ensure gross revenue from all farm commodities including animal products and aquaculture, making it a whole-farm revenue product. An indemnity is paid when the adjusted gross revenue for the year is less than the insured portion of the AGR liability.

Revenue-based crop insurance products have evolved from when they were first introduced. Currently (2020), the US's Risk Management Agency (RMA) is offering four revenue-based insurance plans namely, Revenue Protection (RP), Actual Revenue History (ARH), Area Risk Protection Insurance and Whole-Farm Revenue Protection (WFRP)[7].

Table 2. 5: Comparison of four revenue-based insurance products in the US

Feature	Revenue Protection (RP)	Actual Revenue History (ARH)	Area Risk Protection	Whole-Farm Revenue Protection (WFRP)
Basic for insurance guarantee	Higher of: 1) APH *Projected Price 2) APH * harvest price	Historical producer revenues.	Average county revenues for specified crop	Revenue of the entire farming operation

Source: Shields (2015)

Table 2.5 illustrates the uniqueness of each of these revenue-based crop insurance products. ARH ensures the producer's historical revenue whereas ARP allows producers to ensure a portion of the county's revenue making it an index type of product. The latter used to be the Group Risk Income Protection (GRIP) program (Shields, 2015). WFRP as the name suggests ensures the entire farm's revenue stream from both animal and crop production. RP ensures the producer's revenue stream from a single commodity by selecting a portion of desired historical yields and a combination of a projected price and harvest price from the futures exchange, with the higher of the two prices used for the final insurance protection. There is a variation to the RP plan where one can purchase a policy known as Revenue Protection with

[7] Insurance Plans being offered in the U.S. from the USDA Risk Management Agency website, see https://www.rma.usda.gov/Policy-and-Procedure/Insurance-Plans

Harvest Price Exclusion (RP-HPE) which means the producer opts out from benefiting from potential price upswings at harvest time. Similarly, one can get an ARP with Harvest Price Exclusion.

2.4.3.2 Benefits of Revenue-Based Insurance Products

In a Spanish study by Ahmed and Serra (2014), an inverse relationship between price and yield was observed, and for the same coverage levels, revenue insurance premium rates were lower than those from traditional yield-based crop insurance products. Supporting Ahmed and Serra's findings is Meuwissen, Ruud and Skees (1999), who state that given a negative correlation between prices and yields of crops, revenue insurance premiums are supposed to be more affordable than those of stand-alone yield insurance products. From the US's experience with the Revenue Programme (RP), it was found that the offering can minimise revenue variability by up to a third in maize, soybean and wheat production, as well as increasing average per acre revenues (Motamed *et al.*, 2018).

Given the assumption of reduced premium rates from the introduction of a revenue-based insurance product over the traditional yield-based products, SA could stand to benefit from the following

- Greater participation rates - as insurance becomes relatively more affordable, demand for insurance should go up, ceteris paribus.
- Increased reinsurance capacity – with increased participation rates, it becomes relatively less risky for reinsurers to support the primary-insurers assuming the insured pool is pulling the 'good' farmers as well (following from section 1.2 arguments of adverse selection).
- Increased production – it becomes relatively more affordable for producers to assume a greater production risk.
- Lower prices of produce - as production increases, supply is growing which should result in reduced commodity prices, ceteris paribus.

The above four benefits could undo the vicious cycle that SA is currently experiencing (covered in section 2.6).

Benefits to a revenue-based crop insurance product can further be extended to welfare gains. Chung (2012) investigated welfare effects from the introduction of CRI in Korea. The results showed producer and consumer welfare increased with the introduction of CRI. While maintaining the 50% premium subsidy that exists for yield-based crop insurance, the net benefit of a CRI on society was quantified at 5.1 billion won whereas for crop yield insurance it was 1.7 billion won for the five main crops.

Chung's (2012) research supports earlier findings by Hennessy, Babcock and Hayes (1997) who investigated the efficiencies of different types of revenue insurance products over the prevailing US farmer support program of 1990. A key finding from Hennessy, Babcock and Hayes (1997) is that an equal level of financial gains could be provided to the farmer at a quarter of the cost of the deficiency payments by a farm-level revenue insurance product providing 75% coverage, for maize and soybean. In other words, consumer and producer welfare are highest under CRI as opposed to the government's 1990-deficiency payment scheme.

Additionally, Mahul and Wright's (2003) theoretical work seeking the ideal model of a revenue-based product, found that revenue insurance performed so well that the market would do away with yield-based insurance and hedging instruments. It was found optimal for a producer facing multiple sources of risk on their gross revenue to purchase a single insurance product with the ability to cover all the risks simultaneously i.e. revenue insurance. Optimality was found in that, it is more affordable to go for revenue insurance as opposed to taking out an insurance policy to cover crop or yield in conjunction with some hedging instrument to cover for price variations (Mahul and Wright, 2003).

Furthermore, Mahul and Wright (2003) elaborate on how a whole-farm revenue insurance product would be cheaper for the farmer than taking out individual revenue policies to cover for different crops, which is exactly what Stokes, Nayda and English (1997) and Goodwin and Hungerford (2014) found. Hennessy, Babcock and Hayes (1997) are in support of this too, since they found it more beneficial in terms of efficiencies when the revenue insurance is taken per portfolio as opposed to per crop, which is supported in Chung's (2012) findings.

While weighing out different insurance schemes, Meuwissen *et al.* (2007) spoke of products that move away from the traditional insurance offerings to ensuring different components of income as more desirable to the farmers. In their conclusion, revenue insurance came out as the most suitable form of aggregated income insurance to start pilot trials on, specifically if used on area yields of field crops and prices derived from the futures or spot markets which is what this study aims to do (Meuwissen, Huirne and Hardaker, 1999; Meuwissen *et al.*, 2007).

2.4.3.3 Downside to Revenue Insurance

While considering a farm level revenue-based crop insurance policy, Meuwissen, Huirne and Skees, (2003) indicated that this product suffers from information asymmetries associated with a heavy dependence on historic farm-level reports to determine guaranteed revenue. This means the LTAY or a lack of it can skew the expected revenue of the farmer that is ultimately used to calculate their coverage amount. However, this disadvantage is only relevant if the policies are based on farm-level yield data as opposed to an aggregated regional yield approach.

It is not realistic to expect the insurers to observe each policy holder's yields and prices. For this reason, Mahul and Wright (2003) delved into a scenario of designing an optimal insurance policy using estimators of individual yields and prices. This means those yield estimators used are based on aggregates of a geographic location which recalling from Meuwissen, Huirne and Skees (2003) was an appropriate approach while also relying on price indexes from the relevant agricultural commodity futures markets.

Outside of the data problems, there are modelling complexities that come with designing revenue-based crop insurance products. These types of insurance products incorporate two risks namely yield and price risk of a commodity into one policy, hence special emphasis needs to be placed on modelling this dependence relationship. One of the key complexities is in deciding on which modelling techniques to pursue in determining the dependence. Another complexity is how to incorporate the modelling of the individual risks, price and yield separately, and then combining them into the dependence modelling exercise. Furthermore, there is a complexity in deciding on the approach to determining goodness-of-fit (GOF) to risk models (covered in section 3.4.3). For instance, deciding on generic GOF statistics for statistical model versus hypothesis testing-based tests. Moreover, the actual risk modelling techniques can be computationally intensive requiring special skills and expertise (covered in section 3.4).

2.5 GLOBAL TRENDS IN AGRICULTURAL INSURANCE

In 2019, the global agricultural insurance market is estimated to have exceeded US$33 billion in premium volume and growing at a rate of 5% per annum (AXA XL, 2019). There has been tremendous growth in the global agricultural insurance premium volume in the last two decades spanning from 2007 (Iturrioz, 2009; Schwarz, 2014; AXA XL, 2018). In 2017 the global agricultural insurance market reached an estimated US$30 billion in gross premiums written (AXA XL, 2018), a 99% growth from the 2007 World Bank's survey results (Mahul and Stutley, 2010). There are a few key contributing factors to this upward trend but Barnett (2014) emphasised a larger sum insured value due to higher commodity prices and to a smaller extent increased market penetration. Findings from Italy back Barnett's argument whereby growth in insured value is not being driven by more insured hectares or farmers but by other factors, in this case, greater uptake of multi-risk contracts (Santeramo, 2018). Iturrioz (2009) agreed with Barnett and put forward his three main contributing factors to this situation: Firstly, the growth of agricultural production. Secondly, the growth in the value of assets used in agricultural production. Thirdly, a combination of increased government support and the development of new markets for agricultural insurance where the former is a significant factor in Brazil, India, and China's case.

For the recent upward trend in global agricultural insurance premiums volume, three nations have been identified as the key drivers to this growth namely, Brazil, China, and India. These nations have experienced exceptional growth in their agricultural insurance markets. According to AXA XL (2018), Brazil's agricultural insurance premium volume grew to US$1.1 billion in 2017, a 1 146% growth from 2006 levels; India's agricultural premium volume grew by 288% within a year to reach US$3.3 billion in 2017 and China's grew by 926% to reach US$7 billion in 2017 from 2006 levels. Figure 2.4[8] below illustrates the changes in premiums collected for the three nations along with the US and the aggregated world premium volume. Between the years 2007 and 2017, India had the largest growth of 2 103%, followed by China with 926% while Brazil and the US had a much smaller growth of 29% and 19% respectively. Over these ten years, China and Brazil have moved to become the second and third largest agricultural insurance markets in the world respectively (AXA XL, 2018).

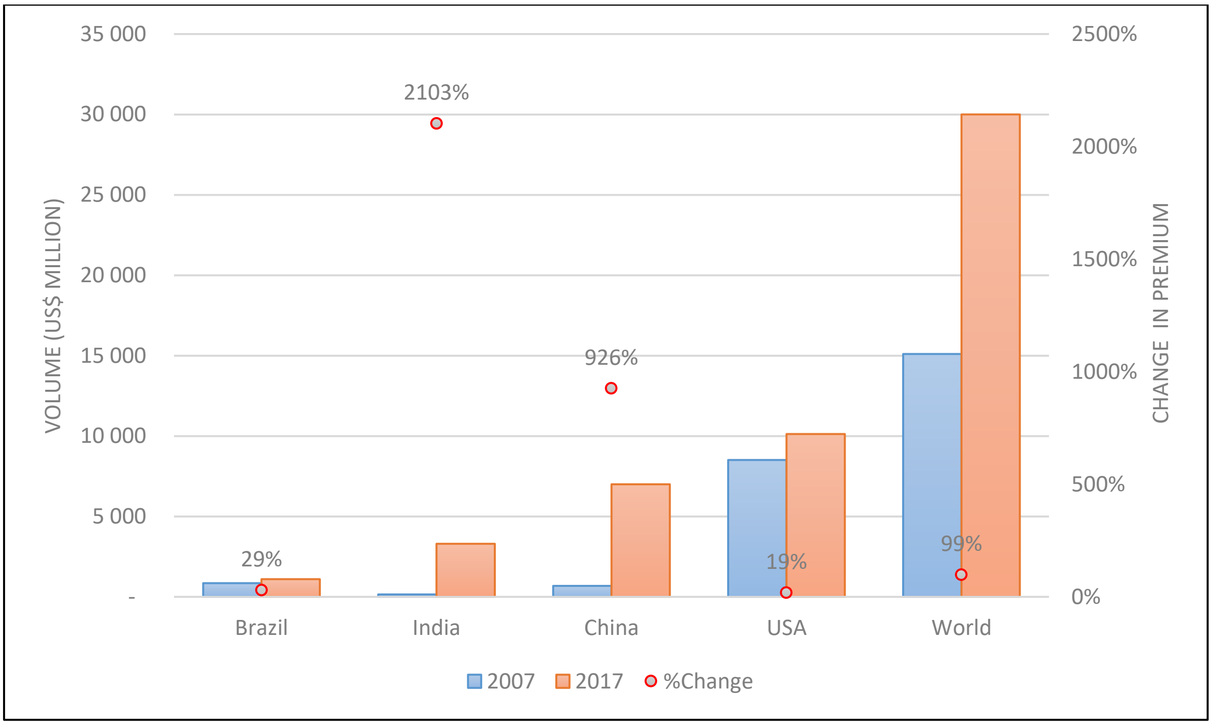


Figure 2. 4: Trends in the agricultural premium volume - 2007 vs 2017

Source: Mahul and Stutley (2010), AXA XL (2018) and RMA (2017)

Notes: Statistics compiled by combining data from the above three sources

The other notable trend is in crop insurance use and uptake. There is a gravitation towards comprehensive risk cover from the insurance risk management options available. For instance in Italy, single-peril products held 92% of crop insurance market share in 2004 whereas 10 years later, pluri-risk[9] and multi-risk contracts dominated

[8] Figure 2.4 uses three data sets: The 2007 values are sourced from the World Bank's survey work on global agricultural insurance that seemed to have inflated numbers when compared to AXA XL's (2018) numbers that were used in compiling the 2017 values for Brazil, China, India and World. The 2017 US figures come from (RMA, 2017).
[9] Pluri-risk refers to crop insurance policies in Italy ensuring farmers for three or more climatic perils, which do not have to be mutually exclusive (Santeramo, 2018)

with 73.2% and 26.8% market share respectively (Santeramo, 2018). This trend was experienced in the US far back, for instance, MPCI dominated the USFCIP's market share in 1996 when revenue-based crop insurance was introduced but in 2020 the latter accounts for above 75% of the total liability and 80% of premiums received (RMA, 2017). The move to revenue-based products is suggestive of the importance placed on income variability by farmers because of commodity price fluctuations.

Other countries on record having revenue programs at the time of the World Bank survey include Iran and Sweden (Mahul and Stutley, 2010). More recently, evidence exists showing more countries implementing revenue programs namely, Hungary, Italy, Spain, UK, and SA, were the last two nations introduced their programs in 2018 (Meuwissen, de Mey and van Asseldonk, 2018). Market statistics and uptake to these newer programs are not yet available, while this is not the UK's first attempt at such a product. The UK's initial attempt towards a revenue product was in 1998 but was cancelled after one season due to poor uptake, apparently as a result of farmers' lack of knowledge on 'derivative' type of contracts, utilising futures markets (Meuwissen, Huirne and Skees, 2003).

2.5.1 Government Intervention in Crop Insurance Markets

A key component of the crop insurance sector in both the better performing and growing markets as judged by the amount of premium collected is the level of government support offered to these schemes (OECD, 2011; Wang *et al.*, 2011, 2015; Bardaji *et al.*, 2016; Arias *et al.*, 2018; AXA XL, 2018; Santeramo, 2018). Taking a point from the reasons why crop insurance markets fail and why there are no purely private sector MPCI or any other crop insurance programs emerging, it can be said that government support is crucial in overcoming these obstacles. MPCI is rarely provided in the absence of government support (Mahul and Stutley, 2010). Thus, the support provided to insurance programs ranges from the provision of subsidies, government-backed reinsurance, underwriting support, along with lost adjustment support, all to be explained below.

The premium subsidy is the most common form of government intervention in these programs as established from the 65 nations surveyed shown in Figure 2.5 (Mahul and Stutley, 2010).

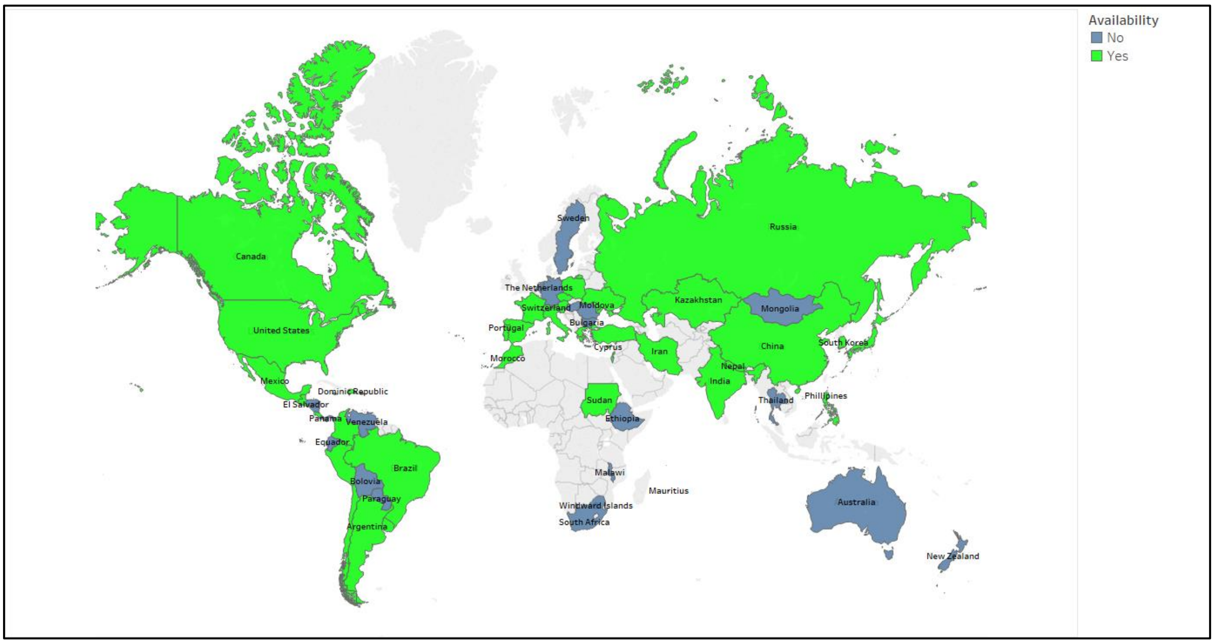


Figure 2. 5: Availability of premium subsidy support

Source: Mahul and Stutley (2010)

A subsidy is defined as a benefit given to an individual, business or institution, usually by the government. In most cases, it comes as a cash payment or a tax reduction. The purpose of a premium subsidy is to make crop insurance affordable for the farmers and assist with increasing insurance penetration. A premium subsidy intervention is considered a superior alternative to the ad hoc disaster payments that are typically used to assist farmers in the event of weather-related disasters. Firstly, Barnett (2014) found premium subsidies to be equitable because they provide the same level of support regardless of farm location and personal risk-reducing measures implemented by the producer. Secondly, premium subsidies require the farmers to have a stake in the insured risk thereby encouraging them to tackle risk more responsibly because they must absorb a portion of the cost.

The US has the largest subsidised crop insurance program in the world that is being funded by the Federal Crop Insurance Corporation (FCIC) and is popularly referred to as the USFCIP. In 2017, the USFCIP had US$106.10 billion in insurance liability, with a total premium of US$10.07 billion backed by US$6.36 billion in premium subsidies

– the burden on the taxpayer (RMA, 2017). From these 2017 figures, in the absence of premium rate subsidies, the loss ratio of the USFCIP would increase from 53% to 144% which would be an unprofitable position for the insurers[10]. Therefore, the premium subsidy is ensuring the availability and affordability of crop insurance in the USFCIP. On average, the MPCI premium subsidy in the USFCIP was 62% in 2017 (RMA, 2017). Canada's AgriInsurance Program provides an MPCI premium subsidy of approximately 60% (OECD, 2011; Canada.ca, 2020). Other examples of MPCI premium subsidy support by percentages include 46% in Austria, 60-80% in China, 65% in France, 64% in Italy, 50% in Luxembourg and 65% in Spain (Bielza, Garrido and Sumpsi, 2004; Wang *et al.*, 2011; Santeramo and Ramsey, 2017).

Governments also intervene in the crop insurance markets through the provision of administrative support to the schemes, as well as covering some of the operating expenses incurred by insurers. Barnett (2014) referred to this type of support as an indirect premium subsidy because the insurers get reimbursed by the governments when they incur these costs, and this is the current practice in the USFCIP. Examples of these costs include marketing, underwriting, sales and delivery, management, data collection and processing, legal services and claims adjustment cost, all of which tend to be high in MPCI schemes (some explained in section 2.3.4) (Coble and Barnett, 2013; Barnett, 2014). Another example is from Canada where the federal and provincial governments each pay 50% of the administrative costs (Atwood, Shaik and Watts, 2002).

When China implemented this multi-layered and multi-channelled approach towards subsidies on insurance premiums and the administrative costs support for its insurers in 2007, the result was tremendous growth in the crop insurance program. Total premiums collected in 2009 reached RMB 13.4 billion Yuan, a 1 578% increase from 2006 levels to realising a total insured amount of RMB 381 billion Yuan (Wang *et al.*, 2011). The government support to the Chinese crop insurance program reached RMB

[10] Recall, according to Nieuwoudt (2000) a 95% loss-ratio is the ideal break-even point for insurance operations.

15.8 billion Yuan[11] in 2016, a value seven times that of 2007 levels which meant more than 70% of premiums were subsidised (Wang *et al.*, 2015).

Other notable cases of the rapid growth in agricultural insurance premiums underwritten include Brazil and India (refer to Figure 2.4, p.34). The growth in Brazil's agricultural insurance market is largely attributed to the significant amount of premium subsidy support given through the Government Premium Subsidies Program (PSR) (Arias *et al.*, 2018). Between the years 2010 - 2015, premium support averaged 44% of premium volume, reaching values as high as R$1.2 billion from R$8.6million in 2005. India experienced incredible growth in just a year after receiving a 45% premium subsidy to reaching US$3.3billion in premium volume in 2017, up from the previous season's US$850 million (AXA XL, 2018).

Furthermore, some governments provide reinsurance support in partial or in full to make it a public reinsurance scheme. The US's federal government (USDA/RMA) through its Public-Private Partnership (PPP) initiatives shares losses with insurers as stipulated in the Standard Reinsurance Agreement (SRA) contract. On the other hand, Canada's federal and provincial government share the reinsurance burden of their crop insurance schemes. As reiterated in the literature and earlier discussions, private sector reinsurance cannot sustain the systemic weather risk covered by MPCI schemes, which is, therefore, the justification for public reinsurance. Without being limited to the agricultural sector, further arguments put forward is that the reinsurance sector is by far smaller than the insurance sector. In 2010, the value of assets of a top primary insurer (Axa) alone, was larger than 10 of the world's biggest reinsurers[12] combined while the market capitalization of two leading insurers (Axa and Allianz) was bigger than the entire reinsurance sector (IAIS, 2011).

This section has presented a view on the world's crop insurance products, and some of the global statistics and recent trends in the crop insurance market, the following section will focus on SA.

[11] Figure sourced from a Reuters article titled 'China issues new guidelines on agricultural insurance subsidies'. Refer to the article's link: https://www.reuters.com/article/china-agriculture-insurance/china-issues-new-guidelines-on-agricultural-insurance-subsidies-idUSL4N1FG1OP

[12] Munich Re, Swiss Re, Berkshire, Hannower Re, Lloyds, Scor, RGA, Partner Re, Transatlantic, Everest Re

2.6 AGRICULTURAL CROP INSURANCE IN SOUTH AFRICA

2.6.1 History of Insurance

In 1979, SA introduced a crop insurance product catering to drought risk. This product was supported by a 25% government premium subsidy and was offered by Sentraoes and CUAS (previously AA Mutual Agricultural Services). According to Nieuwoudt (2000), the demise of this crop insurance scheme was attributed to mainly three factors: Firstly, the premium subsidy support offered was insufficient. Secondly, the scaling down on the premium subsidy support negatively affected the viability of the program. Thirdly, there was a low uptake of crop insurance because the government provided ad hoc drought assistance to farmers which indirectly disincentivised crop insurance uptake in the market.

Twenty years later The Risk Management Pilot Program was developed in 1999 with mainly two objectives: The first of educating farmers of the five basic risks in farming. The second was to support the establishment of a robust insurance provider system that produces products tailored for SA's agricultural sector. This program was developed by the Agricultural Committee of the United States of America - Republic of South Africa (US–RSA) Bi-National Commission through USDA funding. This program was needed due to the market liberalisation trends that SA was experiencing because of the ending of apartheid-era trade sanctions. Before the market liberalisation, government support mechanisms in agriculture were administered by the commodity boards that after 1997 were no longer available as a safety net to the farmers. This marked the period as the first steps towards a purely private sector type of safety net for farmers in SA utilising crop insurance products.

2.6.2 South African Crop Insurance

SA has up to nineteen agricultural insurance companies but only four are offering crop insurance[13]. The SA agricultural crop insurance market is therefore an oligopoly. The

[13] Conclusion reached from consultations with insurance companies, crop insurance brokers and bankers. The list of nineteen agricultural insurance companies was provided by the South African Insurance Association.

four role players are, Santam, Land Bank Insurance Company, Mutual & Federal and lastly Bryte Insurance Company Limited which is a relatively new player that entered the SA market in 2017. From discussions with industry role players, Santam is the market leader that had an estimated 50% market share in 2019, followed by Land Bank Insurance company with 20-30%, then Bryte and Mutual & Federal both with an estimated 5-10%. These four are private limited companies offering three types of crop insurance products, namely single-peril, MPCI and a revenue-based product. The latter is a recent addition to the market, introduced in 2018 by Bryte while being offered through an Underwriting Management Agency (UMA) called Impact. Land Bank Insurance Company also offers its crop insurance products through a UMA call AgriSeker. These SA insurers are using a mixture of local and international reinsurance companies.

It is estimated that 30% of SA's dryland crops are insured and this is small (Weise, 2017) when compared to the USFCIP that in 2006 had up to 80% of planted hectares for the key crops insured (Dismukes and Durst, 2006). However, when considering SA's agricultural crop insurance penetration rate, 2018 achieved values of 5% which is greater than the 2.3% average of the high-income nations identified by Mahul and Stutley (2010) (Munich Re, 2018; stats sa, 2018). However, the 14-year historic average of SA's crop insurance penetration rates is 2%, which is lower than the high-income nation's levels.

Approximately one-fifth of SA's total summer grain area is covered under MPCI[14]. Over 11 years beginning with the seasons 2004/05 to 2014/15, the average area covered by MPCI and hail insurance in SA are 0.6 million ha and 1 466 million ha per year respectively, with a corresponding average risk exposure of R4 349 million and R15 450 million (SAIA, 2016; Munich Re, 2018). In 2017, Santam, the largest agricultural insurer in SA, held a total risk exposure of R2 500 million for MPCI for an estimated 0.40 – 0.50 million ha of the summer crop. At that time, Santam held more than 50% of the MPCI market share[15].

[14] Information received in a consultation with an insurance specialist from Santam.
[15] Information received in a consultation with an insurance specialist from Santam.

From the crop insurance underwritten in SA, MPCI is capped at 20% of the market share while the rest is distributed between the single peril products. This percentage represents the risk threshold that the industry can sustain in the event of a total loss. Anything greater than 20% market share for MPCI could jeopardise the viability of the overall crop insurance industry and therefore is reflective of the magnitude of risk and liability it carries in the agricultural sector. To give a perspective of the numbers, for the 2017/18 season, the total premium received for MPCI was R120 million for a total sum insured value R1 577 million (Munich Re, 2018). Pomareda (1986) states that international experience showed that indemnities should be approximately 15% of coverage and that an insurers pure risk part of the premium only should at least be as high. However, when the calculation was run for SA's MPCI market, the insurers pure risk part of premium came to 6% which is therefore not sustainable according to Pomerada's research[16].

2.6.3 The Role of MPCI

Currently (2020) crop insurance demand is largely driven by the credit providers (banks) who want their investment, the 'input cost' to be secured thereby reducing the financial risk that comes with lending to farming enterprises. For lower-middle and low-income countries, it is usually compulsory for borrowers of agricultural loans to have an agricultural insurance policy (Mahul and Stutley, 2010).

In the past, insurers in SA offered 'input cost insurance' because the farmer's land is not adequate cover (collateral) for the banks. Often the land does not belong to the farmer, or it is already heavily mortgaged, or the land is leased. For many of the new farmers, the land belongs to the government (land reform farms)[17] therefore those producing on the land cannot use it as collateral. As an alternative, banks can still provide financial support by taking a cession on the crop income. This means upon the farmer selling his/her crop, the bank is paid first and then the producer. Given the cession on the crop, the condition of the loan facility is an insurance policy, specifically

[16] This answer was an average calculation of nine years' time series data running from 2004/05 – 2014/15 seasons supplied by Munich Re.
[17] Proactive Land Acquisition Strategy (PLAS) has been active since 2006 in the Department Of Land Affairs as the state's land acquisition model for land redistribution purposes in South Africa (Department Of Land Affairs, 2006). The state acquires the land therefore becoming the owner and holds the title to the land but leases out to the previously disadvantaged people .

MPCI to cover for drought and hail implications, along with adequate hedging (price risk mitigation). In the absence of an MPCI policy and in the event the crop fails for any reason, the cession on the crop is worthless because there will be no crop to sell hence the insurance requirement. Thus, when the financial institutions are funding the farmer's production activities, an insurance policy for the crop becomes the collateral condition to the risk exposure of the financial institution where there is lack of land collateral or inadequate collateral. However, the approaches to production financing of the top four banks namely ABSA, Standard Bank, Nedbank and Landbank differ when considering the relationship between MPCI and production finance. Some prefer to loan against tangible collateral whereas others are willing to take a cession on either the crop or insurance policy or a combination of the two.

MPCI is, therefore, an important product for unlocking production finance in the SA agricultural sector particularly for producers who lack the land collateral, as well as a production risk mitigation tool. In case a peril strikes, a farmer with an MPCI policy, would at least have his input costs covered by the indemnity payment. However, the extent of that cover towards the input cost will depend on the insurance coverage level chosen. Therefore, MPCI allows the farmer to stay in business and plant the following season after the event of a peril. This product is crucial for both producers and financiers because it provides drought cover, which is a prevalent risk in the country. SA weather is characterised by wet and dry spell patterns that tend to be successive in years, particularly the dry ones.

Figures 2.6 to 2.8 provide evidence of suggestive consecutive drought periods over 20 years for the dryland (rainfed) maize production districts, Bloemfontein, Vryburg and Delmas. The rainfall data is taken for the key maize production months of October to March. From the graphs, it is evident that Delmas experiences a much more stable rainfall pattern when compared to the other regions over the 10 years starting from 2010/11 season. When the seasonal rainfall deviated below the mean for these districts, it tended to be successive, particularly for Bloemfontein and Vryburg. These deviations in rainfall are the reason why insurers want the LTAY of a client before an MPCI policy is underwritten. The LTAY is a good indication of the farm's expected yield that would otherwise be distorted had a short-term average yield been utilised due to the erratic rainfalls shown in Figures 2.7 to 2.9.

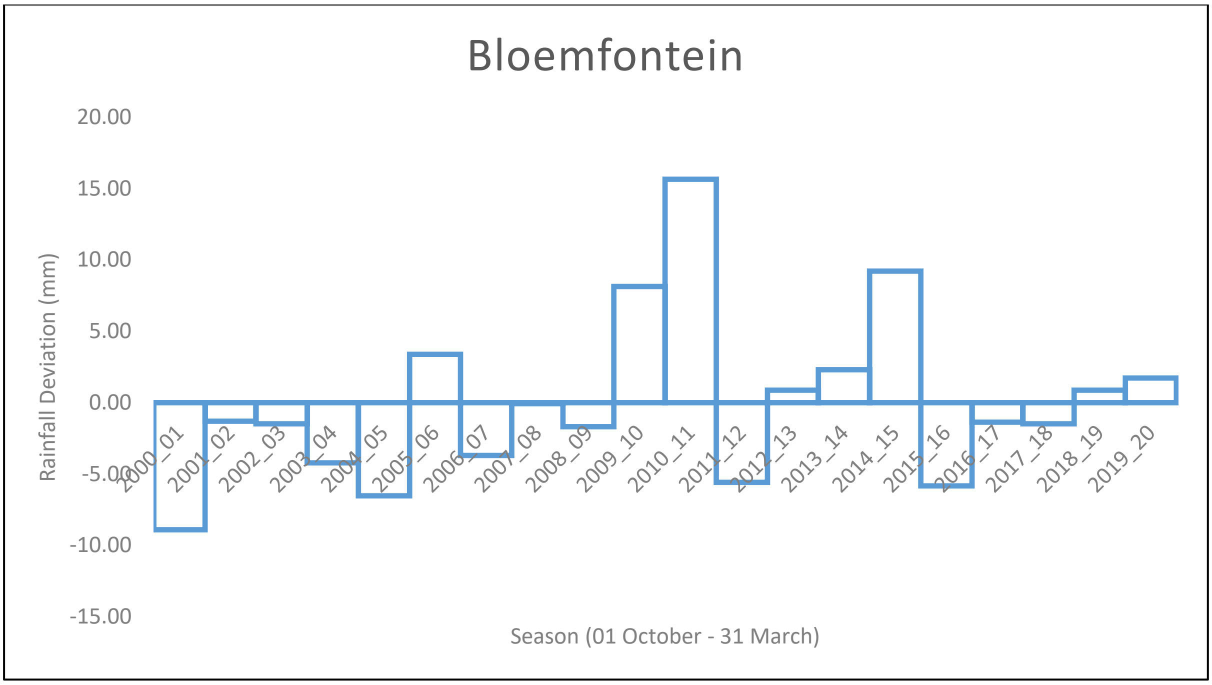


Figure 2. 6: Seasonal total rainfall expressed as the deviation (mm) from the mean for Bloemfontein

Source: Huffman et al. (2019)

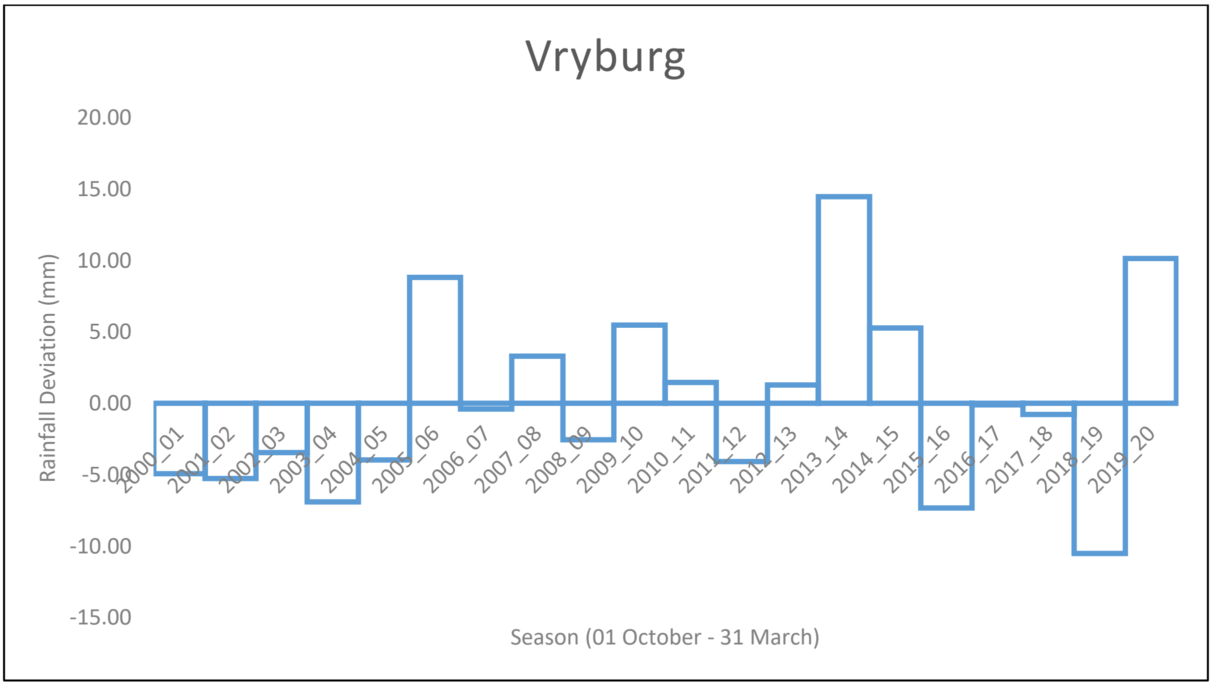


Figure 2. 7: Seasonal total rainfall expressed as the deviation (mm) from the mean for Vryburg

Source: Huffman et al. (2019)

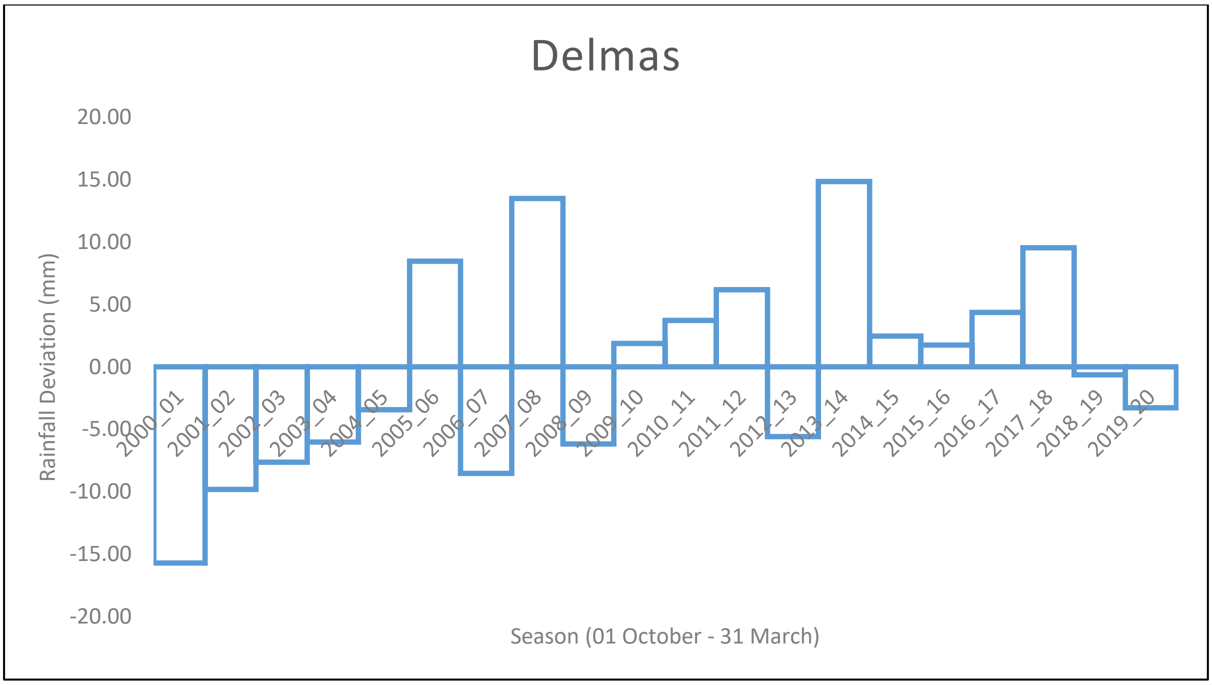


Figure 2. 8: Seasonal total rainfall expressed as the deviation (mm) from the mean for Delmas

Source: Huffman et al. (2019)

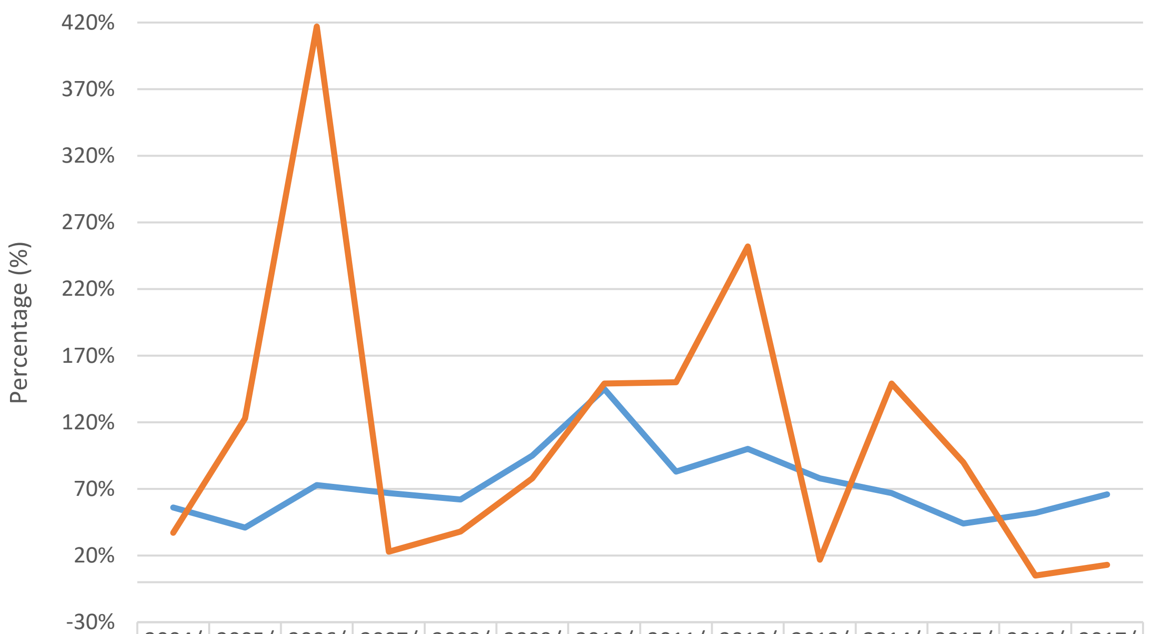
The systemic risk prevalent in agriculture, particularly drought (e.g. recent 2014/15 El Niño season) is a big problem for SA farming and the crop insurance industry as witnessed over the past 20 years (2000 - 2020). Because of drought, the insurer must have adequate money reserves readily available to make large indemnity payments in a single period due to the potential geographic spread of this risk. According to Santam, MPCI requires a minimum of six times the value of premiums received in capital reserves which is large. On the other hand, hail insurance requires just a proportion of the premium amount in capital reserves because it takes into consideration hail frequency and location of occurrence which is usually localised and occurs sporadically (Roberts, 2005). This means that a hail occurrence between farmers in a geographically diverse insurance pool will not necessarily be correlated, thereby minimising the losses incurred in that insured pool. From the 10-year average industry data available (2004/05 - 2014/15), the capital reserves needed for MPCI would be just over R24 000 million whereas for hail insurance it is less than R15 000 million (SAIA, 2016; Munich Re, 2018). The capital reserve requirement for MPCI is a significantly higher burden on the insurers, especially when considering that this product is capped at just 20% of the market share.

The risk of a drought occurrence and the implications of it thereof is why insurers will not offer MPCI policies in the drought-prone regions of SA such as the North West and parts of the Free State province such as Bloemfontein. Therefore, there is a lack of comprehensive yield cover in these regions yet there is still dryland maize cultivation occurring. Farmers in these dryer regions have managed to stay in business by emphasising cultivation methods that maximise on keeping moisture in the ground and reducing soil erosion. These practices include zero tillage and utilising no-till planters among other soil preparation methods. The result of this innovation from the farmers along with good farming practices has reduced demand for MPCI in these dryer areas. As a result, a few of the bankers and crop insurance specialists interviewed mentioned that a producer seeking an MPCI policy, from their experiences, is usually a bad client looking for an indemnity payment. Therefore, this goes back to the adverse selection and moral hazard problem discussed in section 2.3.3. Also, MPCI is struggling in SA because the better farmers are the ones moving away from the product while these are exactly the clients insurers need more of in their insured pool to reduce the adverse selection problem.

2.6.4 A Call for Intervention

Due to the systemic drought risk in SA, stakeholders in the agricultural sector, particularly the insurers, have been pushing for government intervention in the form of subsidies on premiums or as a reinsurer of last resort. In these spheres, the argument for government support is justified. To this end, the South African Insurance Association (SAIA) the representative body of the short-term insurance industry approached the government for a state-supported insurance scheme in 2014. The Department of Agriculture Forestry and Fisheries (DAFF) took the initiative to explore this possibility. DAFF tasked a specialist agricultural consultancy to discover what the global benchmark for state intervention in crop insurance is to recommend an appropriate solution. A proposal document was developed between DAFF, the hired consultants, and the short-term insurance industry team. Unfortunately, the project has been in dormancy since 2014 when DAFF and treasury were supposed to publish the final document defining a potential state-supported insurance scheme. However, as of the end of 2018, there were efforts to revive these discussions by the government with SAIA and other respective role players in the industry. In October 2020, the matter was still unresolved.

There are some suggestions for a government-backed agricultural insurer like the South African Special Risks Insurance Association (SASRIA), which is now self-sufficient, specialising in risks that private insurance companies would shun (Weise, 2017). In other words, this insurer would specialise in disaster relief arising from catastrophic risk or systemic type of risks such as droughts and floods. Other suggestions are pushing for Public-Private Partnerships (PPPs) to jointly fund for catastrophic or systemic risk cover. Munich Re is a big advocate for a PPP approach as it has successfully applied similar products in the US, Brazil, Turkey, Spain and Sudan (Drewes, 2011).



	2004/05	2005/06	2006/07	2007/08	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17	2017/18
Hail	56%	41%	73%	67%	62%	95%	145%	83%	100%	78%	67%	44%	52%	66%
MPCI	37%	123%	417%	23%	38%	78%	149%	150%	252%	17%	149%	90%	5%	13%

Figure 2. 9: Loss Ratios – MPCI vs Hail

Source: Munich Re (2018)

Simply put, no producers, insurers, reinsurers, or a group of reinsurers can afford to cover the magnitude of liability that comes with catastrophic or systemic events on their own. Figure 2.9 above illustrates the loss ratios experienced in SA over 14 years.

In extreme cases, the crop insurance sector experienced loss ratios greater than 400% (2006/07) for MPCI and 145% (2010/11) for hail. The average loss ratio for MPCI is 110% which means for every R1 collected in premiums, insurers are making an indemnity payment of R1.10 which is clearly a loss-making product. Over the past 10 years, Santam experienced a loss ratio worse than the sector's average at 126%. On the other hand, the hail insurance product in SA is profitable with an average loss ratio achieved of 80% over the 14 years. The insurers stay in business by subsiding MPCI losses from profits accrued in the better performing hail insurance policies.

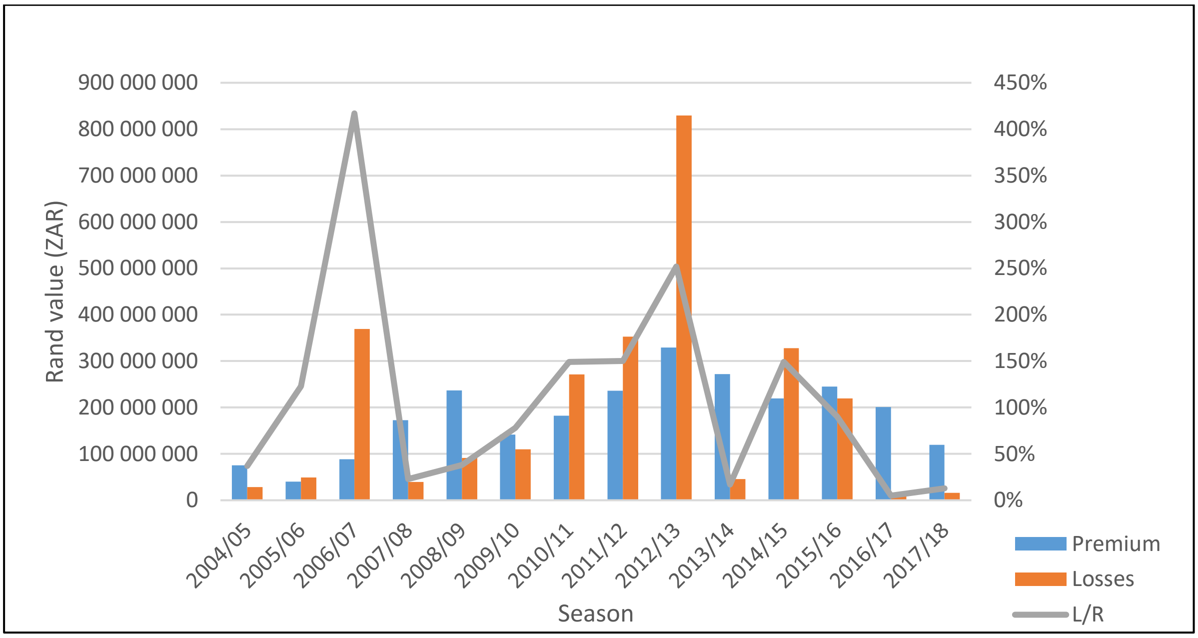


Figure 2. 10: MPCI premium volume, claims and loss ratios

Source: (Munich Re, 2018)

Figure 2.11 illustrates the premium volume against the losses incurred for MPCI over a 14-yer period. The season 2012/13 had the largest recorded loss of R830 million versus the premium of R329 million.

Given the performance of MPCI in SA over the years, the reasons why insurers have remained in that market vary. From consultations with role players, some are staying for the sake of supporting food security. Others are more optimistic that government support in the sector is near and hence want to use their presence during bad times as a bargaining tool for favourable business terms in the future. The most realistic reason given was that MPCI is a marketing tool to get farmers to take up the other lines of insurance business as well as other financial services offered by the insurers. Other lines of insurance include life insurance and other non-life insurance products such as asset and motor vehicle insurance.

It has been established that the perception towards MPCI in SA is that it is expensive. According to industry sources, the prices charged are determined by the liability taken, and as such the premium income needed to pay for potential losses and any additional 'loads' covering for information gaps (discussed in section 2.3.4). Furthermore, as fewer farmers are willing or able to take up crop insurance, mentioned previously, this pushes premiums up, making the product more expensive for farmers, resulting in a 'vicious cycle' that pushes premiums up as the insurance pool gets smaller. In fact, larger farmers who possess strong balance sheets tend to not take out MPCI policies and this does not help the efforts to grow the insured pool in SA. Insurers are struggling with varying insurance uptakes depending on the type of season (refer to section 1.2 discussing El Nina and La Niño). Therefore, there is the anti-selection problem whereby insurers experience very low uptake in good years and higher uptake in bad years also referred to as 'inter-temporal' adverse selection (Goodwin and Mahul, 2004). For these reasons, as already mentioned, Santam has become selective on who receives an MPCI policy, whiles other insurers and reinsurers have become more careful about the level of risk they assume. A suggestion to curb this varying uptake is for a multi-year insurance contract suggested by an insurance specialist, however, this is not in the market yet.

From a discussion with a crop production cost subject matter expert, in high-risk production areas of Mpumalanga and KwaZulu-Natal, a hail policy for soybean can vary between 12% - 25% of the gross production cost[18]. A hail policy in fact, and from literature, is the least expensive crop insurance when compared to MPCI thus giving some context towards the farmer's perception and sentiment towards the affordability of MPCI. Experts in the crop insurance sector estimated that MPCI policies would be double[19] that of hail. Farmers in SA are already dealing with other cost pressures such as rising fuel and fertiliser prices, along with increased costs of electricity just to

[18] Discussion with a subject matter expert working for the Bureau for Food and Agricultural Policy (BFAP).
[19] According to an industry expert who is a former banker with ABSA and Nedbank.

mention a few. There is evidence from farmers in Tzaneen in the Limpopo province ditching crop insurance to minimise their input costs[20]. Instead, these farmers are focusing on other forms of risk reduction measures at the farm level such as consolidating farm business to grow and gain economies of scale (den Hartigh, 2016). Also, the occurrence of self-insurance is growing in popularity in SA according to industry sources, whereby farmers choose to cover any potential losses using their resources such as savings (den Hartigh, 2016). Industry sources have also confirmed that farmers are willing to take chances with their crop by not insuring it and reserving insurance for their assets such as vehicles and machinery (den Hartigh, 2016).

Overall, MPCI is a crucial element in farming operations. The cost of this offering is a sign of the risk it assumes along with the cost structure associated with the product. SA needs an alternative offering that is just as good if not superior. Currently, the expensive MPCI product is geared towards commercial farmers who have the capacity and scale to justify using this product. The downside is that emerging farmers simply cannot afford it, and they are exactly the producers who need this insurance product as the majority of emerging farmers do not own their land and cannot offer land as collateral for production credit. A superior product to MPCI could motivate the intervention from the government that is currently lacking to boost both commercial and emerging farmers.

2.7 CONCLUSION

This chapter covered in detail the technical requirements necessary to achieve a successful crop insurance offering, as well as reasons why crop insurance fails in the market. While focussing on why crop insurance fails, particularly MPCI, the reasons were summed into four elements namely, systemic weather, reinsurance, human factors, and administrative costs. The key findings from this chapter were that SA's MPCI offering is struggling mainly due to the systemic drought risks that the country faces. Furthermore, the rest of the world subsidies MPCI because of the complexities arising from the elements contributing to this product's failure. However, the SA government does not subsidise MPCI in the market, leaving the private insurers to

[20] As said by subject matter expert from Naude Garrun Brokers in Tzaneen.

come up with their solutions, which in this case means taking profits from their better-performing products to subsides MPCI losses. It is important to understand these key points of this chapter because the following chapters (Chapter 4 and 5) dive into the literature review of the alternative methods and approaches pursued by this research in modelling crop insurance products. Specifically, the alternative modelling approaches could aid in reducing some of the impacts from the elements contributing to the failure of crop insurance while simultaneously promoting the elements for a successful crop insurance offering.

CHAPTER 3

MARGINAL DISTRIBUTION AND COPULA MODEL FITTING TO CROP DATA

3.1 INTRODUCTION

In this chapter, a literature review is provided on the data and methodology to be applied in this research. The literature is presented in the order that the methodology will be executed. Firstly, is an introduction of the data with an emphasis placed on data structure requirements for any modelling and analytical work to begin. Therefore, a key point being achieving stationarity in the data and how that has been done in previous studies through various transformations to the data. Secondly, the literature covers bivariate marginal distribution modelling. An elaborate discussion is given on the importance of marginal distribution model choices for the variables yield and prices of South African white maize to compare crop insurance products. The pros and cons of the different marginal distribution models are provided as relevant to the data needs of this research while examples of uses and findings from other studies are also presented. Thirdly the literature review covers the use of statistical copulas. Copulas are introduced and explanations given as to why they are the chosen method of determining dependence relationships of South African white maize yield and price data. The theory of copulas is given and is further narrowed down between the Elliptical and Archimedian copula families. An emphasis is placed on the application of the relevant copulas in this research, hence numerous examples are given of the different possible dependence structures per copula model. Lastly is a discussion on the different goodness-of-fit (GOF) criteria to be used in evaluating the marginal distribution and copula models fit on the data. The progression of these criteria over the years is provided with examples from literature.

3.2 SA MAIZE PRODUCTION

Maize is an essential staple food source in SA, as well as a key ingredient in the animal stock feed diet. Traditionally, white maize is mainly used for human consumption whereas yellow maize for stock feed. On average, SA consumes roughly 39.8% of its maize production in the manufacturing of animal feed, 37.4% on food, while 17.4% goes to exports, and the remaining 4.8% to the production of starch and glucose (BFAP, 2015). SA has three distinct rainfed maize production regions namely, the

Western regions (covering North West (NW) and Free State (FS) provinces), the Eastern regions (covering Gauteng (GP), some of FS and Mpumalanga (MP) provinces) and lastly the Kwazulu-Natal region (Haarhoff, Kotzé and Swanepoel, 2020). Figure 3.1 below illustrates the key maize producing provinces used in this study from the Western and Eastern production regions while indicating the respective magisterial districts to be analysed.

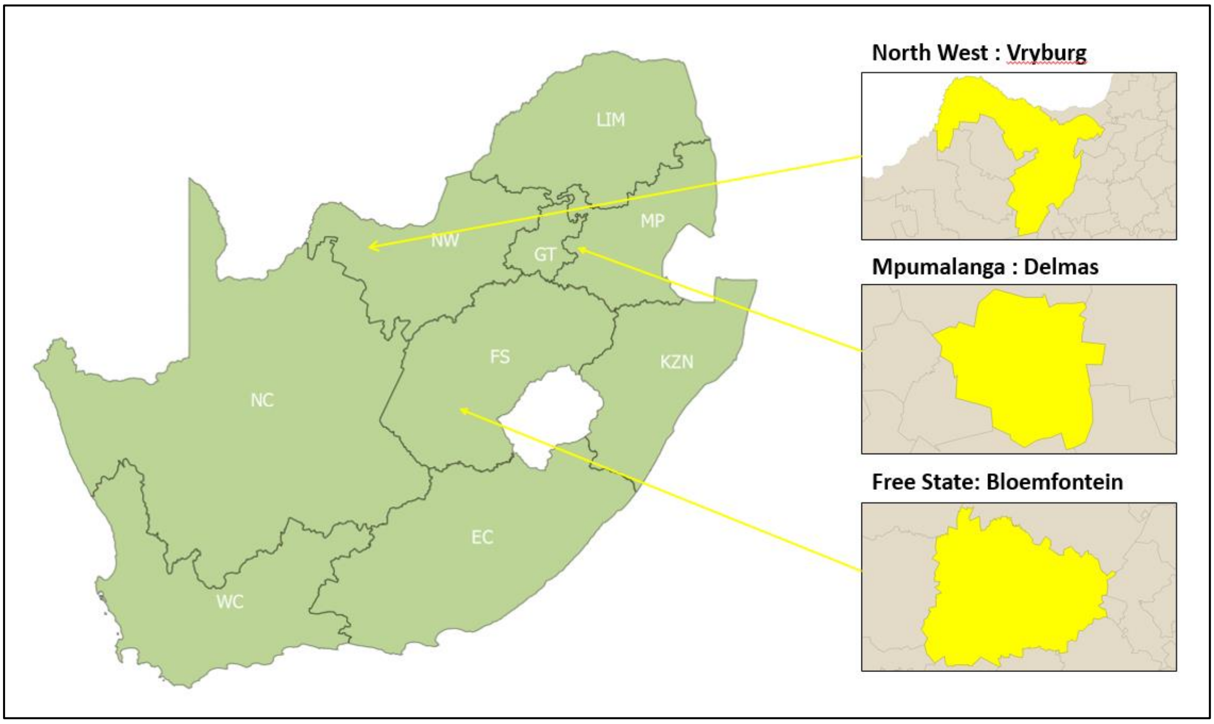


Figure 3. 1: Map of SA's key maize producing provinces

Source: Shapefiles provided PULA[21]

Figure 3.2 below illustrates a 27-year timeline of SA's maize production and key trends. There is a downward trend in the area planted from approximately 3.2 million ha in the 1990/91 season to roughly 2.7 million ha in the 2016/17 season. Over the same period, the average maize yields have increased from 2.4 t/ha to 6.37 t/ha and this is typical of crop yields due to technological advancements in varieties, as well as production methods (Duarte *et al.*, 2018). For the last 27 years, an estimated 89% of SA's maize production is situated in three provinces namely, FS, NW and MP, planting 37%, 33% and 19% respectively (GrainSA, 2020). Over the same 27 years, SA's

[21] PULA Strategic Resource Management (Pty) Ltd is a South African company that has a GIS division with functions of master data management, data collection and collation, spatial analysis and spatial mapping and reporting. Their website is: https://www.pula.co.za/index.html

average hectares planted to maize is 2.9 million hectares annually while 59% of that is white maize production and the remaining 41% is yellow maize. From the given historical hectares planted to maize production, SA has harvested on average 10 million metric tonnes (tons) of maize annually and 60% of that is white maize. Listing the provinces by the top producer and their contributions to white maize production over the stated years, it is FS (41%), NW (33%) and MP (15%), while the remaining provinces contribute an aggregate amount of 11%.

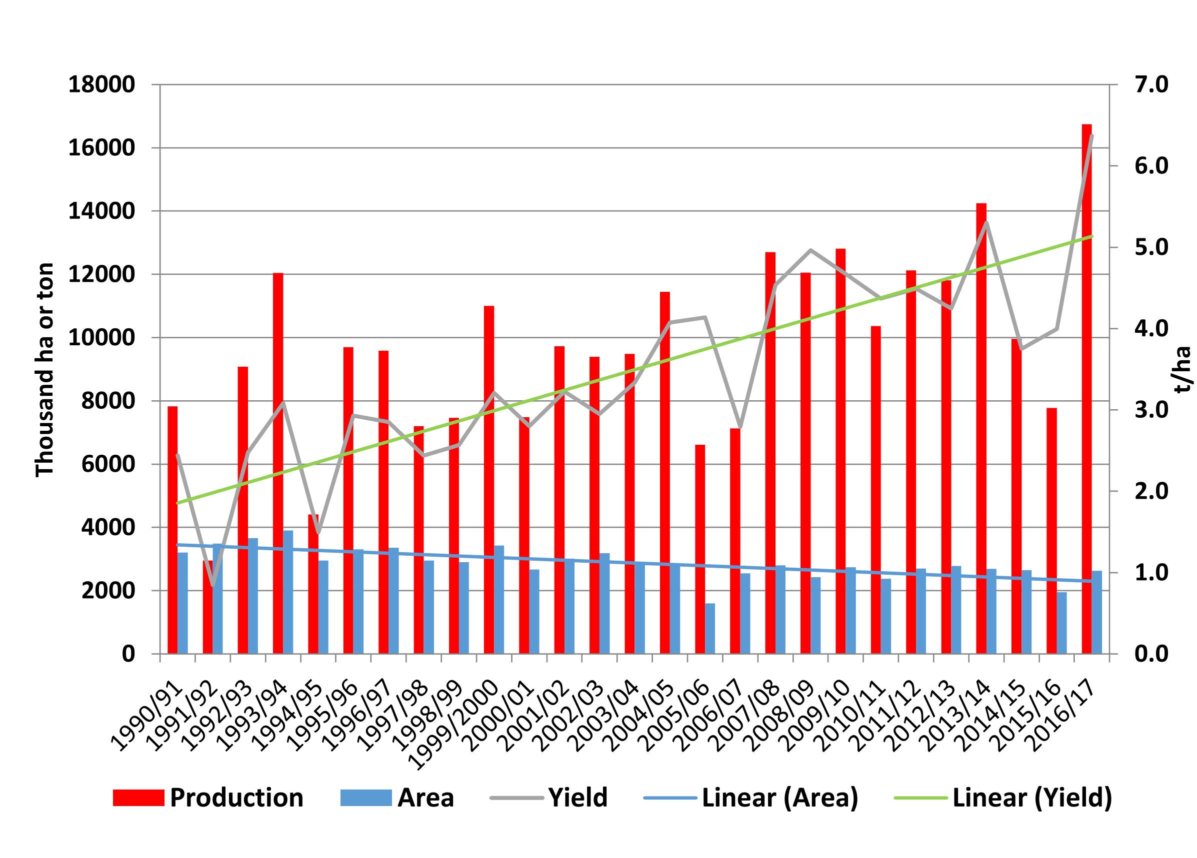


Figure 3. 2: SA's Historical Maize Area Planted, Production and Yield

Source: (GrainSA, 2020)

In this study, two variables namely white maize yield and price are investigated. The white maize yield data for the application is estimated by the Crop Estimates Committee (CEC) of the Department of Agriculture, Forestry and Fisheries (DAFF). The respective white maize prices for SA are sourced from the South African Futures Exchange (SAFEX). This study focuses on one key dryland maize producing magisterial districts from each of the key provinces namely, FS, MP, and NW.

3.2.1 White Maize Yield

For the 2017 crop season, SA planted 3.0 million ha of maize, producing 17.5 million tons of harvest (CEC, 2017). From the total area planted, 55% of it went to commercial white maize, producing 10.0 million metric tons. To estimate these cropping statistics, the CEC progresses through eight production forecasts to determine the final area planted and production (tons harvested) (CEC, 2019). The combination of total production and area planted is used to calculate the maize yields achieved. By determining the yield calculation over the area planted and not area harvested, the CEC addresses Tejeda and Goodwin (2008) concerns of maintaining a realistic view of the impact of the post-planting condition on crop yields.

The data in metric tons per hectare (t/ha) is restricted to dryland cultivation over the years 2004 - 2017 to capture the impact of natural weather elements on maize production and ultimately yields by district (CEC, 2018). Table 3.1 below illustrates the district level white maize yield data as established by the CEC while Figure 3.3 presents the graph of this data. The district-level data is available for the years 2004 - 2017 providing a total of 14 observations (per district).

Table 3. 1: District Level White Maize Yield Data

Year	Bloemfontein	Delmas	Vryburg
2004	1.21	3.4	1.89
2005	2.91	6.01	1.88
2006	4.16	7.66	1.68
2007	1.03	2.17	1.87
2008	3.29	6.93	3.34
2009	4.01	6.16	3.37
2010	4.99	5.96	3.65
2011	3.62	5.81	3.95
2012	3.97	6.58	2.29
2013	2.60	5.55	1.69
2014	3.46	5.30	4.86
2015	2.22	5.29	2.58
2016	0.65	4.52	2.10
2017	0.95	7.68	4.35

Notes: The yield data is expressed in metric tonnes per hectare (t/ha)

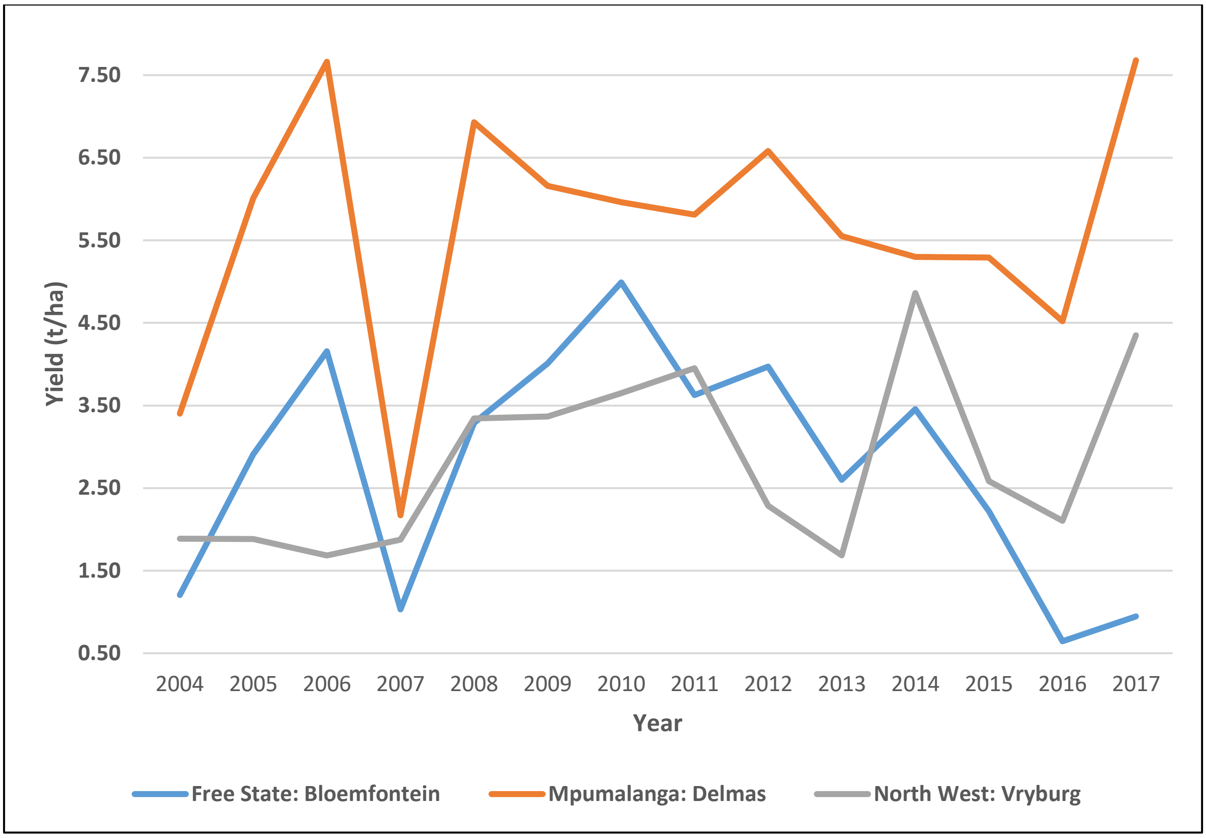


Figure 3. 3: District Level White Maize Yield Data

It is observed from Figure 3.3 that the districts' yields at times follow a similar trend but are not always in sync, with Bloemfontein and Vryburg, the dryer and more drought-prone regions seemingly more correlated. Notably, Delmas does not always follow the trend of the other two districts, especially in the years of 2008 - 2011 and 2013 - 2014 and was shown in Chapter 2, Figures 2.7 to 2.9, this district receives a relatively higher rainfall. The more recent significant drought years are seen from the dips in the graphs over the years 2013, 2015 and 2016 which is the year SA experienced the lowest amount of rainfall since 1904 (BFAP, 2016).

When analysing time-series data, stationarity in the variables is a prerequisite to avoid producing spurious results. For crop yield data, removing the trend component is a fundamental step leading to stationarity. The process of removing the trend produces yield residual values that tend to be trend stationary and are typically used as a representative sample of the yield data (Just and Weninger, 1999; Zhu, Ghosh and Goodwin, 2008). There are numerous ways of transforming the data to achieve stationarity, but it commonly involves some type of regression analysis to produce yield residual values. For instance, Goodwin and Hungerford (2014) utilised the local regression (LOESS) procedure to represent the trend of their maize and soybean yield data. On the other hand, Deng, Barnett and Vedenov (2007) utilised the log-linear trend equation for the cotton and soybean yield data.

3.2.2 White Maize Prices

To compare CRI policies, the sources of maize prices must hold characteristics of a free and unbiased price discovery mechanism that is determined by purely market forces of supply and demand (Tejeda and Goodwin, 2008; Goodwin and Hungerford, 2014; Cole and Gibson, 2010; and Marković, Veselinović and Kokot, 2016). SAFEX is a futures exchange market, a subsidiary of the JSE Limited (Johannesburg Stock Exchange) where agricultural derivatives are traded in SA, hence making it the site of price discovery and the data source. In practice, commercial farmers use futures prices for guidance on the potential maize harvest prices before planting commences to make a production decision (Zhu, Ghosh and Goodwin, 2008). Therefore, in maize production, price returns in SA are calculated over a season that overlaps between years since harvesting commences in the following year from planting. Table 3.2 introduces and explains the different maize price concepts used on SAFEX to assist in understanding the price analysis to come. The concepts to be explained include what a futures price is, the established prices during harvesting and what an in-season price change represent.

Table 3. 2: Key Concepts on Type of Prices

Type of Price	Definition
Expected Harvest Price (P_{EH})	This is a daily futures price of maize for a maize contract that expires at harvest time, which is the July contract for this study. The month of July is representative of the prime harvesting period for dryland maize produced in SA
Average Expected Harvest Price (P_{AEH})	A calculation of average daily prices of the July white maize futures contract as traded over the three months of October, November, and December of the previous year. These three months are representative of the ideal planting window for dryland maize production in SA, the maize is harvested in July the following year.
Average July Harvest Price (P_{AJH})	A calculation of the average daily prices of the July white maize futures contract as traded in July. Recalling that July is representative of the prime harvesting period for dryland maize produced in SA and thus illustrates the maize price fluctuations in a season as influenced by the bulk of the maize being supplied at the close of that season.
Price Change (P_C)	Calculated as $P_C = P_{AJH}$ - P_{AEH}. This calculation illustrates seasonal maize price fluctuations as influenced by a combination of maize stocks in the country together with weather expectations (thus P_{AEH}) and the actual amount of maize supplied during the harvesting time (thus P_{AJH}).

Table 3.3 below illustrates the different price variables used to get the price changes variables. Already stated in the previous section on yield data, stationarity is a requirement when analysing time-series data. Similarly, the price data intended for analysis needs to be stationary. This study uses price changes data which is equivalent to price returns data that tends to be stationary (Omran and McKenzie, 1999; Ho and Wan, 2002).

Table 3. 3: White Maize Futures Prices

Year	P_{AEH}	P_{AJH}
2004	987.26	881.27
2005	1002.70	600.50
2006	761.51	1397.43
2007	1182.34	1654.60
2008	1464.71	2019.69
2009	2000.94	1342.00
2010	1499.87	1103.87
2011	1365.67	1771.71
2012	1822.46	2484.35
2013	2219.68	2258.50
2014	2056.63	1705.39
2015	1934.78	3184.28
2016	2871.95	4522.00
2017	2754.00	1835.69

3.3 BIVARIATE DISTRIBUTION MODELLING OF CROP DATA

3.3.1 Distribution Options for Marginals

The choice of distribution imposed on the data is important for two reasons: Firstly, the distribution is used to generate cumulative distribution function (CDF) values for the copula fitting procedure. Secondly, the estimators of the distribution give a representation of the marginal distribution used in a Monte Carlo simulation to produce variates of yield and prices for insurance comparison purposes.

There are numerous studies with the view that crop yields are normally distributed and probably an equal amount disputing this. Atwood, Shaik and Watts (2002) consistently rejected normality in the US maize yields. When assuming normality, the crop insurance premiums were significantly under-pricing policies when compared to the empirical distribution (For example Illinois was charging only 66% of the actual empirical rate). On the other hand, Sherrick *et al.* (2004) did not explicitly test for normality but explored alternative distributions in modelling US maize and soybean yields and their resultant impact on crop insurance policy pricing. Firstly, the data was negatively skewed and had a sample kurtosis of 3.72, implying fat tails. This raised questions about the relevance of using the normal distribution that is suited for symmetric distributions and not ideal for fatter tails in the distributions. Similarly, the relevance of the lognormal distribution becomes questionable since it is more suitable for right-skewed distributions. The findings were that the Weibull and beta distributions were better at characterising the crop yield distributions when compared to the normal and lognormal. Secondly and key is that expected pay-outs for the yield insurance policies modelled by the Weibull and beta distribution were significantly higher than those of the normal and logistic models (which is also a symmetric distribution). These findings by Sherrick, *et al.* (2004) could mean that the crop insurance policies are being under-priced by the current modelling techniques. It has been found that the assumption about a distribution for a dataset is complex (especially due to data scarcity and heterogeneity of various data types) and the resultant effects of misspecification in this regard. For these reasons, Ramirez, Misra and Field (2003) employed a comprehensive approach in establishing the distribution of maize and soybean yields in the US. Their findings were that the crop yields are not normally distributed and are skewed to the left, reaffirming some of the discussed findings. Other earlier studies to have presented negative skewness in crop data include Gallagher (1987) for soybeans in the US as well as Day (1965) for oats but the latter also found positive skewness in maize and cotton in the US.

In a prominent study exploring the distribution that characterises crop yield data, Just and Weninger (1999) failed to reject normality. Instead, the authors highlighted the following three key problems found in the typical research approach to yield distributions that nullify evidence against normality:

- Misspecification of the non-random components of yield distributions.
- Misreporting of statistical significance.
- Use of aggregate time-series data to represent farm-level yield distributions.

There is no consensus on the distribution of choice that best represents crop yields. Some studies have found positive skewness in yield data while others have found it to be negative. It is known that the Risk Management Agency (RMA) assumes normality in the crop yield distributions (Goodwin, 2015). It is also known that SA crop insurers are pricing their policies by experience therefore it is unknown whether this approach takes into consideration any of the possible distribution models. However, what is clear from the literature is that the choice of distribution chosen to represent either yield or price risk has a material effect on the pricing of the risk and the presumed yield loss calculation. Accurate modelling of agricultural yield is therefore of paramount importance because the distribution that is derived from it represents the producer's loss probability and therefore is the insurable risk of the producer (Ozaki, Goodwin and Shirota, 2008). Therefore this study will unpack some of the views on the choice of distributions but with the goal of finding appropriate alternatives to the normal distribution were applicable to compare their implications in the comparison results of yield and revenue crop insurance products.

3.3.2 Application Examples

Some examples will be presented of models that have been utilised in the modelling of yield and price distributions to compare crop insurance policies, while also providing the context of these models' relevance. Sherrick, *et al.* (2004) while considering different distributions' accommodations towards skewness in crop yields, graphically illustrates that the lognormal can accommodate positive skewness, while the Weibull and beta are more flexible and can accommodate negative and positive skewness. It was also established that the beta distribution can accommodate a broader range of kurtosis than lognormal and Weibull. Goodwin and Hungerford (2014) also used the Weibull and the gamma distribution on crop yields and acknowledged their ability to cater to the negative skewness. From this same study, the gamma distribution was a better fit on the white maize yield data while the lognormal distribution was used for the price data. Another study that considered negative skewness is Tejeda and Goodwin (2008) who relied on the beta distribution.

The choice of distribution for prices is less contentious due to Black and Scholes' (1973) formula for valuing options that gained prominence as the market tool for pricing stocks. From their work, they identified the distribution of stock prices as following a lognormal distribution hence the reliance on it together with the normal distribution when modelling futures markets price data. However, Goodwin (2015) does caution against reliance on the lognormal distribution particularly in the tails of the distribution that tend to deviate from what would be implied by this preferred distribution. There have been other alternative distributions in characterising crop prices, Tejeda and Goodwin (2008) used Burr type XII distributions and compared them to those of normal and lognormal distributions. The Burr type XII distribution was chosen for its ability to better model 'higher moments' recognised in price data hence improved characterisation of skewness and kurtosis. Ahmed and Serra (2014) instead first-differenced their data and then used ARIMA and ARIMA-GARCH models for yields and price, respectively.

3.4 THE STATISTICAL COPULA APPROACH

This research is using statistical copulas in the dependence analysis of the two variables white maize yields and their prices, to compare crop insurance policies. The theory of copulas and options on their application processes will be covered in the following sections. It is important to stress that the emphasis of this section is on the application of the copulas in this research as related to their properties and qualities, and less on the mathematical/statistical development of their theory.

3.4.1 Introduction to Copulas

Copula functions have been used extensively in measuring non-linear dependence structures (Durante and Sempi, 2010; Ghosh, Woodard and Vedenov, 2011; Goodwin and Hungerford, 2014; Goodwin, 2015). Copulas functions are therefore flexible tools to model dependence relationships because they not only can model nonlinear dependence structures between variables but are also able to accommodate different families of distribution models in their marginals. This flexibility comes from their ability to separate the marginal behaviour and the dependence structure from a joint distribution function. The RMA assumes linear (Pearson) correlation between price and yield risk for crops, yet evidence exists suggesting that these risks are not always normally distributed (Black and Scholes, 1973; Atwood, Shaik and Watts, 2002; Ramirez, Misra and Field, 2003; Sherrick, Zanini, *et al.*, 2004; Ozaki, Goodwin and Shirota, 2008). Furthermore, the linear correlation is very limited because it is mostly applicable to elliptical distributions, meaning those that follow the normal distribution and other generalisations/mixtures of it thereof such as the t-distribution (Schmidt, 2006). Thus, an over-reliance on correlation comes with some shortcomings. For instance, a correlation parameter of zero does not necessarily mean the two variables are independent, only if the variables follow elliptical distributions otherwise their relationship is just not explained by this specific dependence measure. Another shortcoming from correlation is that the measure is severely undermined by outliers (Staudt, 2010). This is a significant downside when considering that the spatial correlation of crop yields is significantly stronger in times of extreme weather conditions (Ozaki, Goodwin and Shirota, 2008; Goodwin and Hungerford, 2014) while price data is characterised by skewed distributions (Tejeda and Goodwin, 2008; Goodwin, 2015). Therefore, the copula approach to establishing dependence between variables thrives from its flexibility and being a less restrictive model that accommodates many types of dependencies. In this chapter, different copula families to be used and their functions are introduced, where the focus of this study is on a bivariate case.

3.4.2 The Copula Families and Functions

A copula is a multivariate distribution function defined on the unit interval $[0,1]$ from the unit d-cube interval $[0,1]^d$ with cumulative distribution function (CDF),

$$C\big(F_1(x_1), \dots, F_d\,(x_d)\big) \tag{1}$$

and probability density function (PDF),

$$c\big(F_1(x_1), \dots, F_d\,(x_d)\big)f_1(x_1)f_2(x_2)\cdots\, f_d\,(x_d), \tag{2}$$

where

$$c\big(F_1(x_1), \dots, F_d\,(x_d)\big) = \frac{\partial^d C\big(F_1(x_1), \dots, F_d\,(x_d)\big)}{\partial F_1(x_1)\partial F_2(x_2)\cdots\partial F_d(x_d)} \tag{3}$$

for $F_i(x_i)$ and $f_i(x_i)$ as the CDF and PDF respectively, of the marginal distributions for $i\ = 1,2, \dots, d$. The CDFs of the marginal distributions are uniformly distributed, i.e. $F_i(x_i) \sim Unif(0,1)$ (See special functions in Appendix C). This study makes use of CDF, $F_i(x_i)$, from assumed continuous marginal distributions.

The following conditions must hold (Haugh, 2016) for a valid copula function $C(\cdot)$:

- $C(u_1, \dots, u_d)$ is non-decreasing in each component, $u_i,\ \ i = 1,\ \ 2,\ \ \dots, d,\ \ 0 < u_i < 1$

- $The\ i^{th}$ marginal distribution is obtained by setting $u_j = 1$ for $j \neq i$ and since it is uniformly distributed $C(1, \dots, u_i, 1, \dots, 1) = u_i$.
- For $a_i \leq b_i, P(U_1 \in [a_1, b_1], \dots, U_d \in [a_d, b_d])$ must be non-negative. This implies the rectangle inequality,

$$\sum_{i_1=1}^{2} \dots \sum_{i_d=1}^{2} (-1)^{i_1+\dots+i_d} C\left(u_{1,i_1}, \dots, u_{d,i_d},\right) \geq 0, \tag{4}$$

where $u_{j,1} = a_j$ and $u_{j,2} = b_{j.}$

Sklar's Theorem states that for every joint CDF $F(\cdot)$, there exists a copula, $C(\cdot)$: $[0,1]^d \rightarrow [0,1]$ such that, for all random variables,

$$F(x_1, \dots, x_d) = C\left(F_1(x_1), \dots, F_d(x_d)\right), \tag{5}$$

where $F_1(\cdot)$,…,$F_d(\cdot)$ are the respective marginal CDFs for random variables $X_1, \dots, X_d$.

Using the Probability Integral Transformation (Bain and Engelhardt, 1992), observations $(x_1, \dots, x_d)$ of random variables $(X_1, \dots, X_d)$ with marginals $F_1(x_1), \dots, F_d(x_d)$ respectively, can be generated via the following transformation,

$$\begin{cases} x_1 = F_{X_1}^{[-1]}(\cdot) \\ x_2 = F_{X_2}^{[-1]}(\cdot) \\ \quad\vdots \\ x_d = F_{X_d}^{[-1]}(\cdot) \end{cases} \tag{6}$$

The above is of importance to this research to simulate yield and price distributions for comparing potential crop insurance products based on dependence structures as established by the copulas.

Conversely, by defining C in terms of F and its margins (refer to equation (5)), the copula function is extracted from the multi-variate distribution function according to equation (6) and in this way, this research intends on simulating univariate variables via copulas:

$$C(u_1, \dots, u_d) = F\left(F_1^{\leftarrow}(u_1), \dots, F_d^{\leftarrow}(u_d)\right). \tag{7}$$

Because $F_1, \dots, F_d$ are assumed to be continuous distribution functions in this study given by equation (5), indicating that the joint distribution function of

$\left(F_1(X_1),\dots,F_d\,(X_d)\right)$ is a copula, denoted by $C(\cdot)$ and hence the identity in equation (5) and through the following argument $x_i = F_i^{\leftarrow}\,(u_i),\ 0 \le u_i \le 1,\ i = 1,\dots,d$, results in equation (7) (McNeil, Frey and Embrechts, 2005).

There are numerous copulas but the most common in literature belong to the Elliptical and Archimedean copula families to be considered for this study (McNeil, Frey and Embrechts, 2005; Schmidt, 2006; Tejeda and Goodwin, 2008). This study restricts itself to mainly five copulas[22]; Figure 3.4 below lists the copulas used and distinguishes them by their copula family.

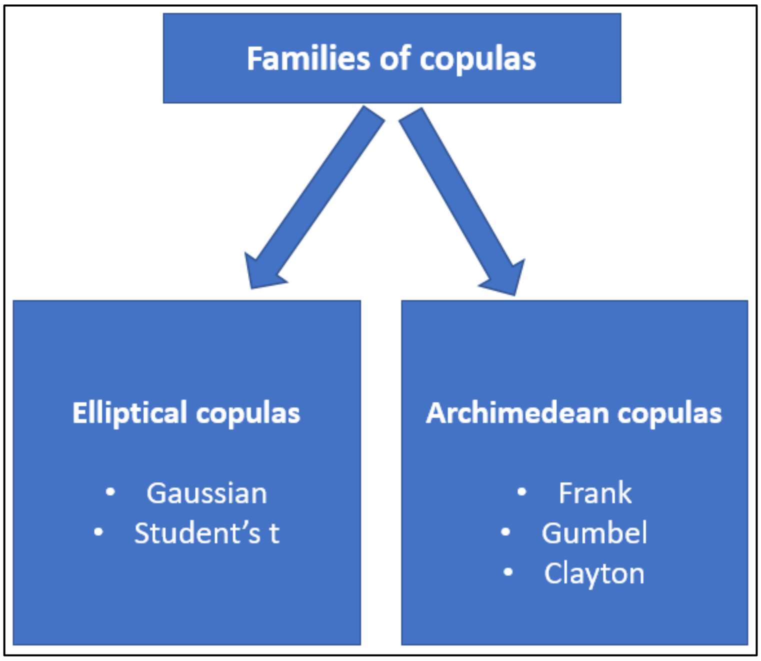


Figure 3. 4: Example Copula Families

Each copula has key features of how it models dependence differentiating it from the rest. Before getting into these features, the Frechet-Hoeffding bounds must be introduced. The Frechet-Hoeffding bounds define the boundaries that all copulas are restricted to and are given by the following inequality (Cherubini, Luciano and Vecchiato, 2004):

$$\mathbf{max}\{\boldsymbol{u}+\boldsymbol{v}-\mathbf{1},\mathbf{0}\} \le \boldsymbol{C}(\boldsymbol{u},\boldsymbol{v}) \le \mathbf{min}(\boldsymbol{u},\boldsymbol{v}) \tag{8}$$

Therefore, the lower bound is the counter-monotonicity copula given as,

$$C(u,v) = \max\{u+v-1,0\}, \tag{9}$$

[22] Other Archimedean copulas include the Joe and Ali-Mikhail-Haq copulas.

which is defined in three of the copulas namely, Gaussian, t-copula, and Frank. The upper bound is the comotonicity copula given as,

$$C(u,v) = \min(u,v). \tag{10}$$

Thus, counter-monotonicity refers to a case of perfect negative dependence whereas comonotonicity refers to perfect positive dependence, while the former is restricted to a 2-dimensional case. Given this research's Hypothesis 1, the Frechet-Hoeffding lower bounds are of interest and not the upper bounds.

Also, copulas can have the ability to measure tail dependence. Tail dependence looks at concordance in the tails of the joint distribution function for a given set of random variables (Cherubini, Luciano and Vecchiato, 2004). Simply put, tail dependence is the probability that a random variable Y takes an extreme value given that another random variable X has also taken an extreme value. Therefore, this type of dependence is found in the upper (right corner) and lower (left corner) tails of the distribution denoted by λ_u and λ_l respectively. Schmidt (2006) defined upper and lower tail dependence for random variables (rvs) as follows:
For rvs X_1 and X_2 with CDFs $F_i, i = 1,2$ the coefficient of upper tail dependence is defined by

$$\lambda_u = \lim_{q \nearrow 1} P\left(X_2 > F_2^{\leftarrow}(q) \middle| X_1 > F_2^{\leftarrow}(q)\right), \tag{11}$$

provided that the limit exists and $\lambda_u \in [0,1]$. The coefficient of lower tail dependence is defined analogously by

$$\lambda_l = \lim_{q \searrow 0} P\left(X_2 > F_2^{\leftarrow}(q) \middle| X_1 \leq F_2^{\leftarrow}(q)\right). \tag{12}$$

If $\lambda_u > 0$, it is said that X_1 and X_2 have upper tail dependence, while for $\lambda_u = 0$ it is said that they are asymptotically independent in the upper tail and analogously for λ_l.

The sections to follow will introduce the copula families, providing their denotations and dependence parameters. Copula examples will be presented with theoretical values to illustrate the effect on dependence as well as the Frechet-Hoeffding bounds. These examples will utilise scatterplots and contour plots to give a visual representation of the dependence structures. For the remainder of this study, a bivariate case is maintained (i.e., $d = 2$).

3.2.2.1 Elliptical Copulas

The Gaussian (also known as the Normal) and the t-copula (also known as the Student's *t*) belong to the Elliptical copula family that this study will focus on.

Table 3. 4: Properties of bivariate elliptical copulas

Copula name	Parameter range	Kendall's Tau (τ)	Tail dependence
Gaussian	$\rho \in (-1,1)$	$\frac{2}{\pi}\arcsin(\rho)$	0
t-copula	$\rho \in (-1,1), v > 2$	$\frac{2}{\pi}\arcsin(\rho)$	$2_{t_{v+1}}(-\sqrt{v+1}\sqrt{\frac{1-\rho}{1+\rho}})$

Source: Brechmann and Schepsmeier (2013)

From Table 3.4 above, ρ is the copula parameter representing the dependence structure and ranges between perfect negative (-1) and perfect positive (+1) dependencies. Also, the table provides the equation converting the copula parameter to Kendall's Tau (τ) as well as the tail dependence equation for the *t*-copula.

3.2.2.1.1 Gaussian copula

The Gaussian copula is restricted to radial symmetry. The following is an expression of a two-dimensional Gaussian copula:

$$C_{\rho}^{Ga}(u_1, u_2) = \Phi_{\Sigma}(\Phi^{-1}(u_1), \Phi^{-1}(u_2)), \tag{13}$$

where $\sum$ represents a 2×2 covariance matrix of $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$, Φ represents CDF of a standard normal distribution, while Φ_{Σ} represents a CDF of a bivariate normal distribution with zero mean and covariance matrix $\sum$. From Table 3.4 above, ρ is the copula parameter representing the dependence structure between the variables and is specific to that copula. The dependence structure of the Gaussian copula ranges between $\rho = -1$ and $\rho = 1$, resulting in the counter-monotonic and comonotonic copula. Figure 3.5 and 3.6 illustrate the counter-monotonic, independence and comonotonic copula when $\rho = -1$, $\rho = 0.1$ and $\rho = 0.8$, respectively. Therefore, this copula accommodates negative and positive dependence structures as shown by the contour and scatterplots. The contour plots in Figure 3.5 below illustrate radial symmetry and that the PDF peaks at its centre, the latter is also shown in Figure 3.6.

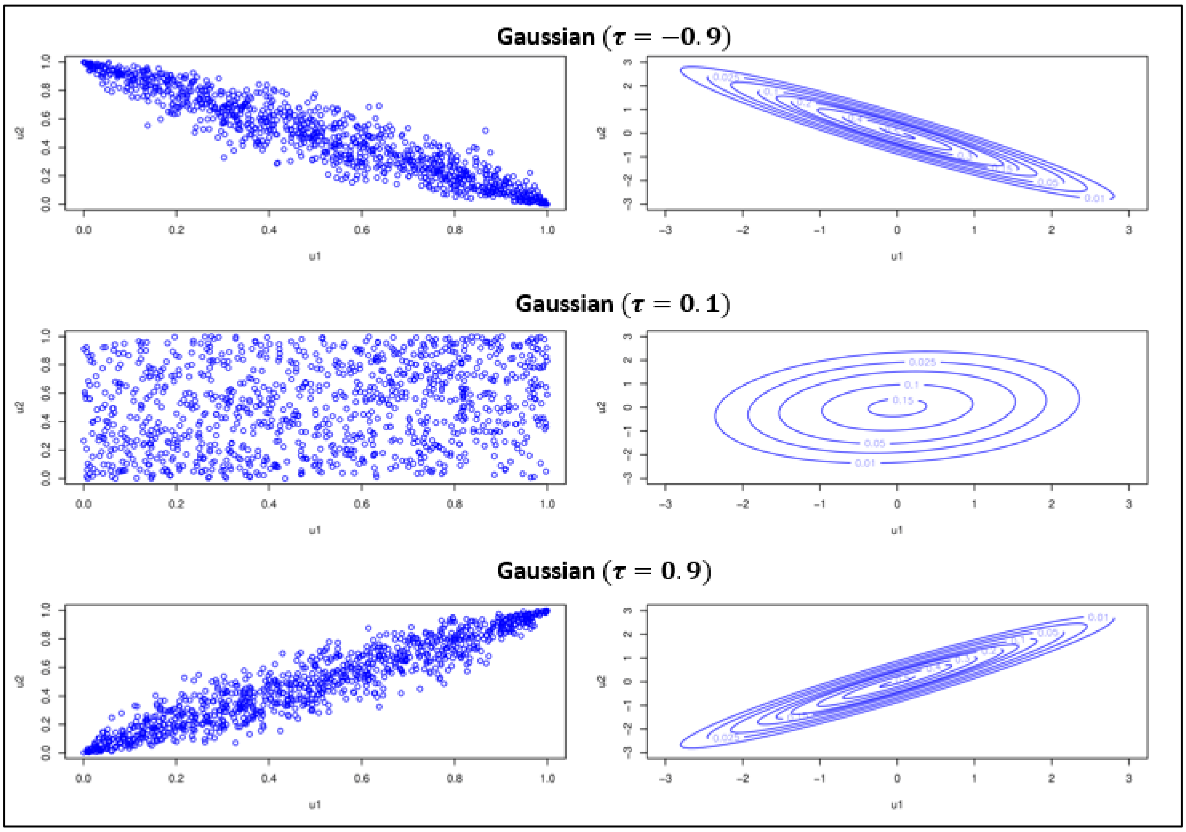


Figure 3. 5: Scatter and contour plots from three Gaussian copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Gaussian copulas with dependence parameters of $\boldsymbol{\tau = -0.9, \tau = 0.1}$ **and** $\boldsymbol{\tau = 0.9}$, respectively which is *equivalent to the copula parameters of* $\boldsymbol{\rho = -0.8}$, $\boldsymbol{\rho = 0.1}$ *and* $\boldsymbol{\rho = 0.8}$, *respectively*[23].

[23] Refer to Remark 1 (p.68) for the reason why Kendall's Tau, τ is used to express the dependence structures of the copulas in the graphs.

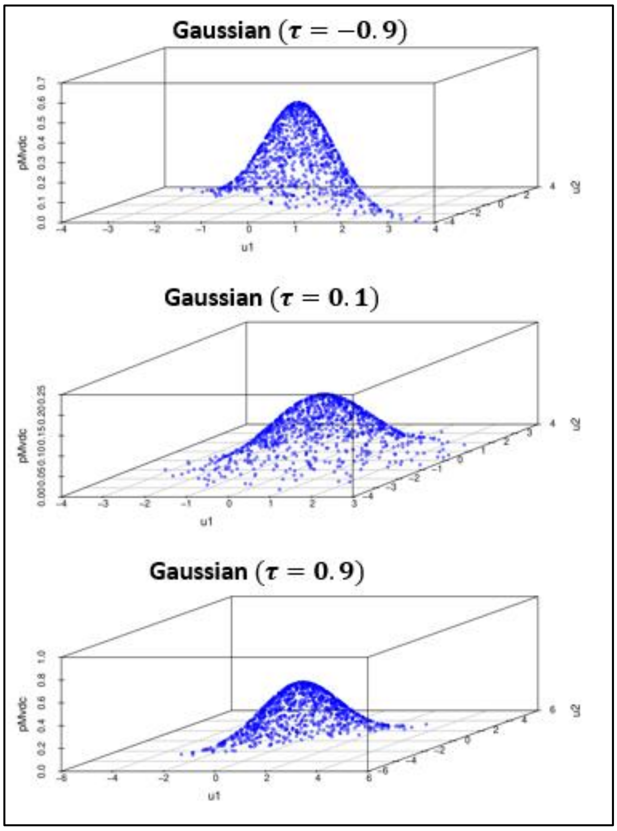


Figure 3. 6: Copula density plots from three Gaussian copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Gaussian copulas with dependence parameters of $\boldsymbol{\tau = -0.9, \tau = 0.1}$ **and** $\boldsymbol{\tau = 0.9}$, respectively which is *equivalent to the copula parameters of* $\boldsymbol{\rho = -0.8}$*,* $\boldsymbol{\rho = 0.1}$ *and* $\boldsymbol{\rho = 0.8}$*, respectively.*

In this study, the Gaussian copula function coupled with normal (yield) and normal (price) marginal distributions is referred to as the 'benchmark model', according to literature, the common actuarial practices rely on it for crop insurance risk modelling (Goodwin and Hungerford, 2014; Goodwin, 2015). Thus, because the benchmark model maintains basic assumptions of constant dependence structures between random variables yield and price therefore it is a Gaussian model. Evidence backing this Gaussian model assumption comes from Iman and Conover's (IC) procedure which is known to dominate dependence modelling in agricultural insurance (Mildenhall, 2005). Basically, to establish a Pearson correlation parameter between variables that do not follow a normal distribution model, the IC procedure, therefore, reorders the sample to have the same rank order to the distribution of interest, which is the normal distribution (Mildenhall, 2005). Therefore, the IC procedure essentially relies on algorithms that convert samples of distributions that do not follow the normal distribution model to establish a Pearson correlation structure. When the IC procedure is based on normal scores, which simply means that the reference distributions are normally distributed it is equivalent to the Gaussian copula hence why it is taken as the benchmark model (Mildenhall, 2005).

Remark 1

The figures illustrating copulas dependencies have the dependence parameter expressed as Kendall's Tau, τ because the Elliptical copula parameter ρ is only meaningful in its copula family, while τ, can be expressed in terms of copulas by the equation in Table 3.4. Therefore $\rho = -1$, $\rho = 0.1$ and $\rho = 0.8$ is equivalent to $\tau = -0.9$, $\tau = 0.1$ and $\tau = 0.9$, respectively. By expressing Kendall's τ in terms of copulas, it becomes possible to compare different copulas (and their families) dependence parameters and their respective parameter ranges.

3.2.2.1.2 t copula

The *t*-copula, like the Gaussian, is restricted to radial symmetry but can model upper and lower tail dependence. The following is an expression of the t-copula:

$$C^t_{v,\rho}(u_1, u_2) = t_{v,\Sigma}\left(t_v^{-1}(u_1), t_v^{-1}(u_2)\right), \tag{14}$$

where $\sum$ is a correlation matrix, t_v is the CDF of the one dimensional t_v distribution and $t_{v,\Sigma}$ is the CDF of the bivariate $t_{v,\Sigma}$ distribution. The dependence structure of the t-copula is the same as the Gaussian's that ranges between $\rho = -1$ and $\rho = 1$. Just like in the Gaussian case, Figure 3.7 illustrates the counter-monotonic, independence and comonotonic copulas when $\tau = -0.9$, $\tau = 0.1$ and $\tau = 0.9$, respectively. The contour plots in Figure 3.7 below illustrate radial symmetry and that the PDF peaks at its centre, the latter is also shown in Figure 3.8 below. However, the t-copula is differentiated from the Gaussian in the tails of its distributions that have a longer reach as shown in the scatter and contour plots, as well as the density plots at the same dependence structure.

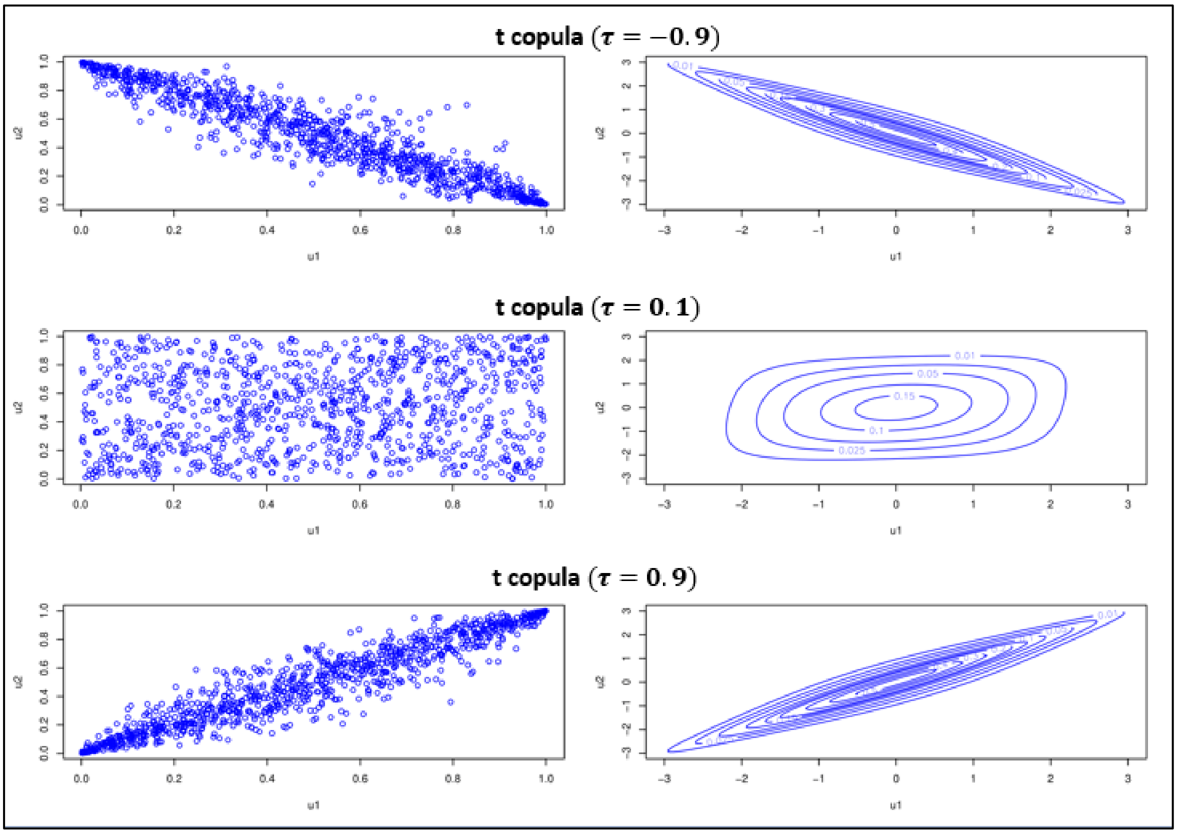


Figure 3. 7: Scatter and contour plots from three t-copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from t-copulas with dependence parameters of $\boldsymbol{\tau = -0.9, \tau = 0.1}$ **and** $\boldsymbol{\tau = 0.9}$, *respectively, which is equivalent to the copula parameters of* $\boldsymbol{\rho = -0.8}$, $\boldsymbol{\rho = 0.1}$ *and* $\boldsymbol{\rho = 0.8}$, *respectively.*

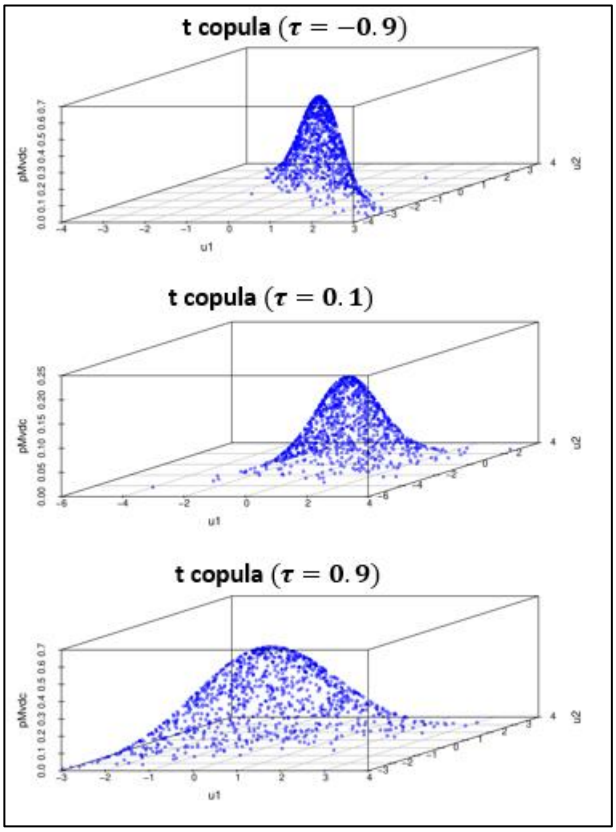


Figure 3. 8: Copula density plot from three t-copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from t-copulas with dependence parameters of $\boldsymbol{\tau = -0.9}$, $\boldsymbol{\tau = 0.1}$ *and* $\boldsymbol{\tau = 0.9}$*, respectively, which is equivalent to the copula parameters of* $\boldsymbol{\rho = -0.8}$, $\boldsymbol{\rho = 0.1}$ *and* $\boldsymbol{\rho = 0.8}$*, respectively.*

In summary of the Elliptical copula family, Figure 3.9 illustrates tail dependency as the key feature differentiating the Gaussian and t-copula from their surface plots.

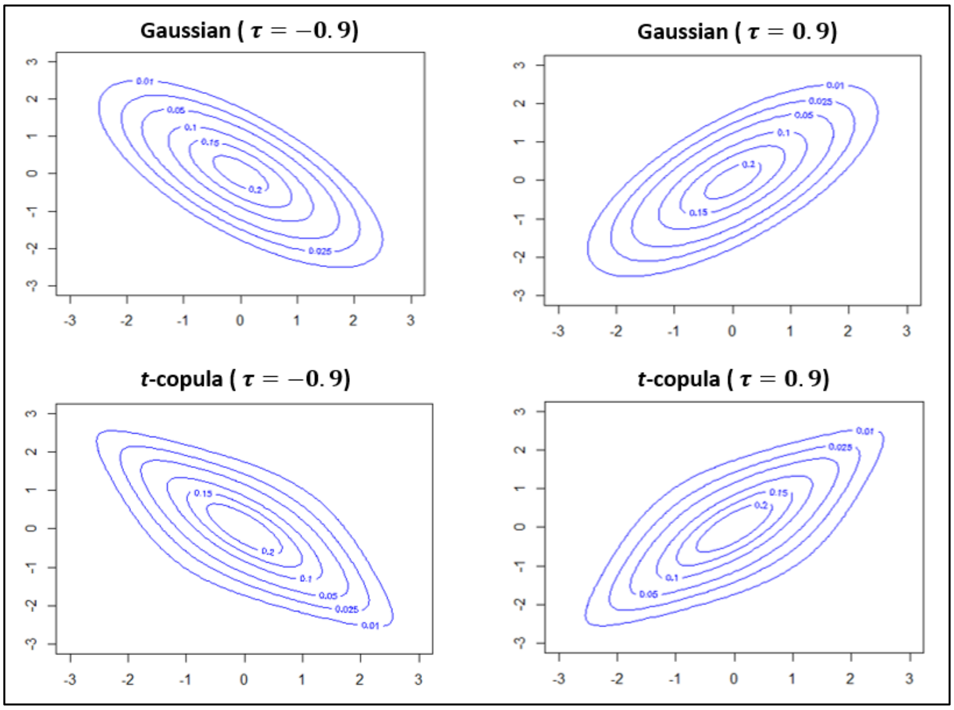


Figure 3. 9: Contour plots from the Gaussian and t-copulas for two parameters

Notes: The dependence parameters are $\boldsymbol{\tau = -0.9}$ **and** $\boldsymbol{\tau = 0.9}$, respectively which is equivalent to the copula parameters of $\boldsymbol{\rho = -0.8}$ (left column) and $\boldsymbol{\rho = 0.8}$) (right column).

From Figure 3.9 above, given a scenario of counter-monotonic and comonotonic copulas shown in the left and right columns, respectively, it is demonstrated that the t-copula has a sharper reach in the tails of the distributions given an identical dependence structure with the Gaussian copula.

3.3.2.2 Archimedean Copulas

The Archimedean copula family unlike the Elliptical family can model asymmetric dependencies. The three main copulas are Frank, Clayton and Gumbel that will be investigated further. Table 3.5 below presents a summary of the denotation and properties of the Archimedean copula family.

Table 3. 5: Denotation and properties of bivariate Archimedean copulas

Copula name	Generator function	Parameter range	Kendall's Tau (τ)	Tail dependence (lower, upper)
Frank	$-\log\left[\frac{e^{\theta t}-1}{e^{-\theta}-1}\right]$	$\theta \in \mathbb{R}\backslash\{0\}$	$1-\frac{4}{\theta}+4\frac{D_1(\theta)}{\theta}$	$(0,0)$
Clayton	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta > 0$	$\frac{\theta}{\theta+2}$	$(2^{-\frac{1}{\theta}},0)$
Gumbel	$(-\log t)^{\theta}$	$\theta \geq 0$	$1-\frac{1}{\theta}$	$(0,2-2^{\frac{1}{\theta}})$

Source: Brechmann and Schepsmeier (2013)

From Table 3.5, it is evident that the copula parameter ranges are unique to each copula. The Frank copula can accommodate negative and positive dependence structures whereas the Clayton and Gumbel are restricted to positive dependence. Also, the Clayton copula can model lower tail dependence whereas the Gumbel can model upper and lower tail dependence.

Remark 2

Further reiterating the importance of Kendall's τ while using the Archimedean copula family, Table 3.5 above shows that θ is the copula parameter representing the dependence structure between the variables while each copula has a distinct parameter range. The dependence implied by each copula is not easily comparable with another, apart from the Elliptical family with an identical copula parameter range, $\rho \in (-1,1)$, for the Gaussian and *t*-copula. Thus to make these copula parameters speak to one another and to enable researchers to compare apples with apples, literature has the implied copula dependence measures expressed in Kendall's τ (Brechmann and Schepsmeier, 2013). Kendall's rank correlation is a non-parametric measure of dependence between variables that is defined at the copula level, making it invariant under monotonic transformations (Embrechts, McNeil and Strauman, 2002). Furthermore, Kendall's rank correlation unlike Pearson's can measure nonlinear dependencies between two variables. Kendall's coefficient of dependence therefore ranges between -1 and 1, reflecting perfect negative and positive dependence, respectively. The R statistical software has a 'CDVine' Package created by Brechmann and Schepsmeier (2013) with the functions BiCopPar2Tau and BiCopTau2Par that enable programmers to link a copula parameter to Kendall's τ and vice versa. The 'VineCopula' package is an extension of the CDVine package that links the copula parameter to the Kendall's τ and vice versa as well (Stoeber *et al.*, 2019). However, for the purposes of this study, the BiCopSelect function from the VineCopula package is used to fit the copulas to the data. This function produces both copula parameters and the associated Kendall's τ as output.

3.3.2.2.1 Frank copula

The Frank copula is represented by the expression:

$$C_{\theta}^{Fr}\,(u_1, u_2) = -\frac{1}{\theta}\ln(1 + \frac{(e^{-\theta u_1}-1).(e^{-\theta u_1}-1)}{e^{-\theta}-1})\ln(1 + \frac{(e^{-\theta u_1}-1).(e^{-\theta u_1}-1)}{e^{-\theta}-1})\,, \qquad (15)$$

for $\theta \in \mathbb{R}\setminus\{0\}$. The Frechet-Hoeffding lower and upper bounds are reached when $\theta \to -\infty$ and $\theta \to \infty$ respectively. The Frank copula is unique to the rest of the Archimedean family because it has a radial symmetry shown in Figure 3.10 below and captures both positive and negative dependencies just like the Gaussian. The similarities between Gaussian and Frank is the reason why Tejeda and Goodwin (2008) compared the two modelling approaches. When compared to the *t* copula, the Frank copula cannot model tail dependence therefore it is more restricted in this regard. Figure 3.10 below illustrates the counter-monotonic, weak independence and comonotonic properties were $\tau = -0.9$, $\tau = 0.1$ and $\tau = 0.9$, respectively. The contour plots in Figure 3.10 below illustrate radial symmetry and that the PDF peaks at its centre, the latter is also shown in Figure 3.11 below.

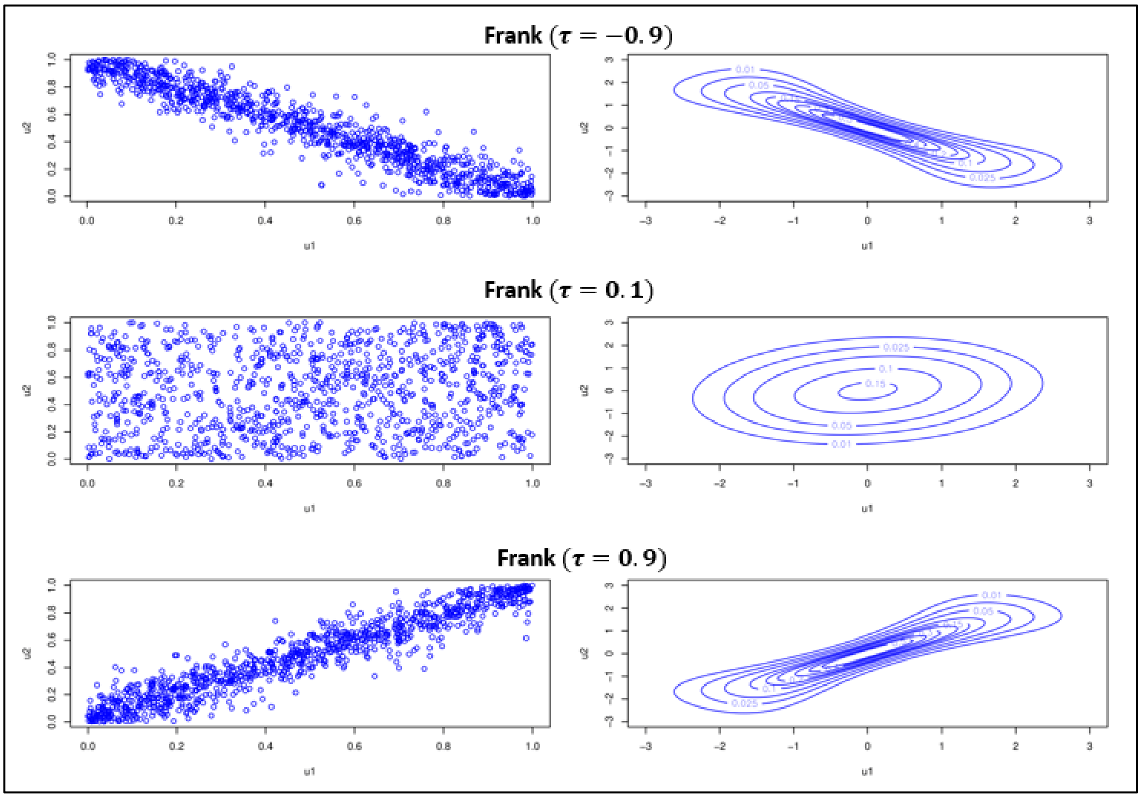


Figure 3. 10: Scatter and contour plot from three Frank copulas parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Frank copulas with dependence parameters of $\boldsymbol{\tau = -0.9, \tau = 0.1}$ ***and*** $\boldsymbol{\tau = 0.9}$*, respectively, which is equivalent to the copula parameters of* $\boldsymbol{\theta = -0.8}$*,* $\boldsymbol{\theta = 0.1}$ and $\boldsymbol{\theta = 0.8}$*, respectively.*

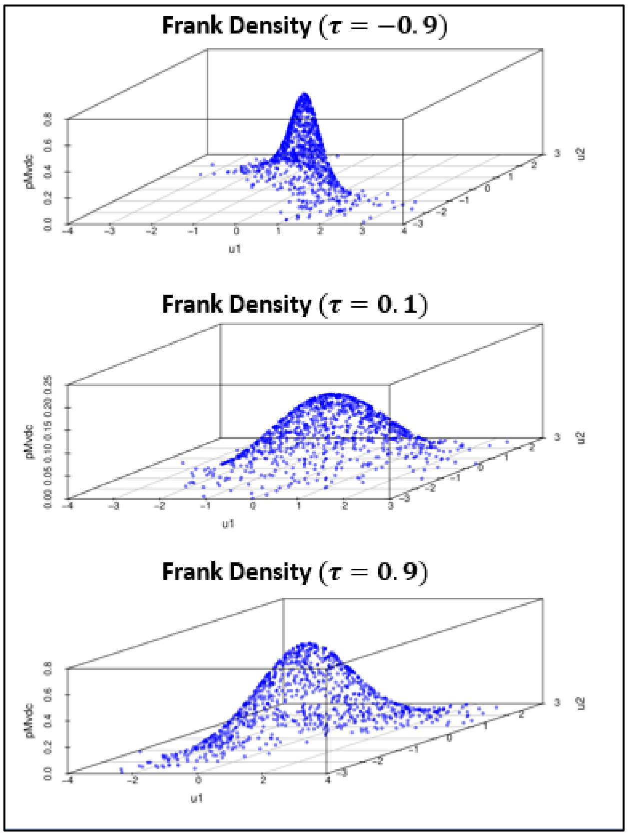


Figure 3. 11: Copula density plots from three Frank copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Frank copulas with dependence parameters of $\boldsymbol{\tau = -0.9, \tau = 0.1}$ ***and*** $\boldsymbol{\tau = 0.9}$*, respectively, which is equivalent to the copula parameters of* $\boldsymbol{\theta = -0.8}$*,* $\boldsymbol{\theta = 0.1}$ *and* $\boldsymbol{\theta = 0.8}$*, respectively.*

3.3.2.2.2 Clayton copula

The Clayton copula's closed-form expression is given as follows:

$$C_{\theta}^{Cl}\,(u_1, u_2) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1}{\theta}}\,, \tag{16}$$

where $\theta \, \epsilon \, [0, \infty)$.

As $\theta \to 0$ this results in the independence copula whereas $\theta \to \infty$ produces the comonotonicity copula. The Clayton copula exhibits greater dependence of the lower tail captured by $\theta > 0$ but due to parameter restrictions, this copula cannot model negative dependence in this current form (elaborated on under **Remark 3** p.81). This copula, therefore, ranges between independence and positive co-dependency shown in Figure 3.12 below. As the copula moves away from the independence copula when $\tau = 0.1$, the contour plot is near radial symmetry but with evidence of lower tail dependence also shown in the scatterplot by a concentration of dots. As Kendall's τ approaches the comonotonic copula, the contour plot is shown in Figure 3.12 below becomes more pear-shaped showing significant lower tail dependence that is represented by the concentrated dots in the scatterplots and represented by the PDF in Figure 3.13.

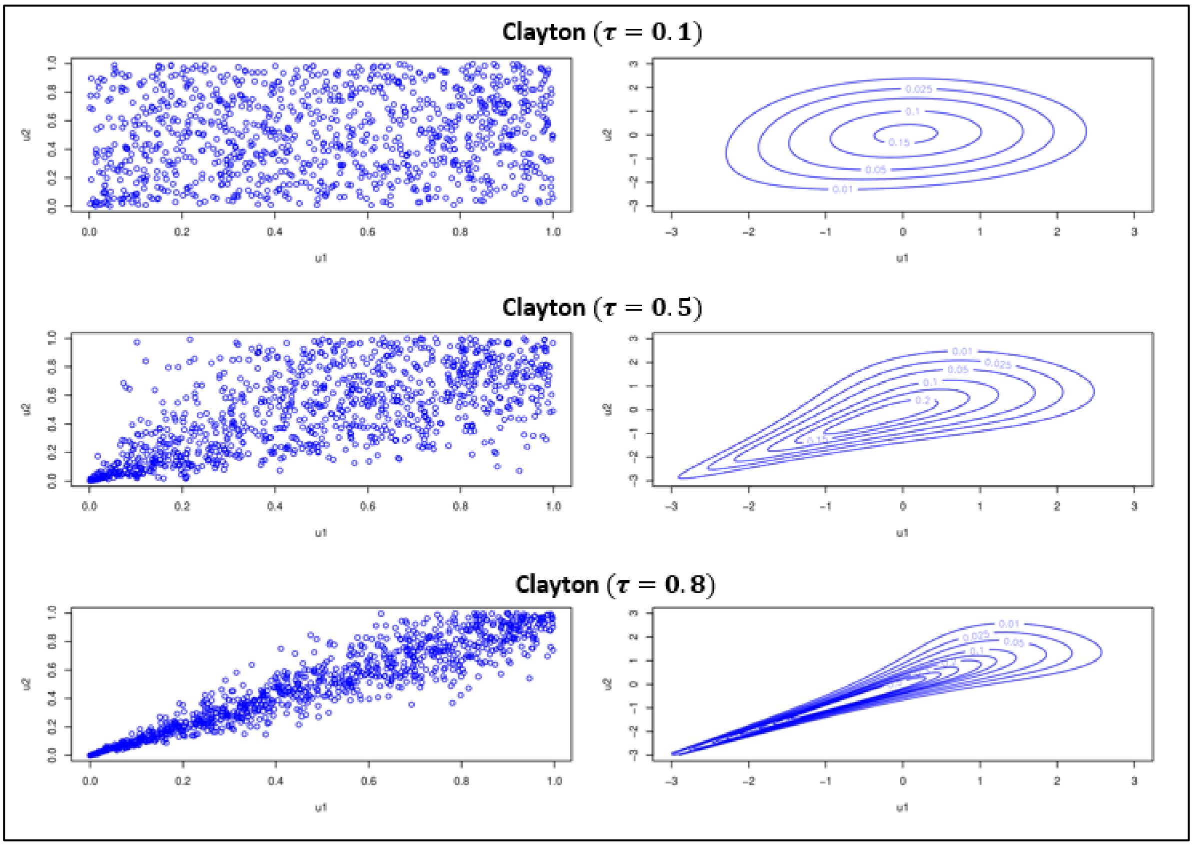


Figure 3. 12: Scatter and contour plot from three Clayton copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Clayton copulas with dependence parameters of $\tau = 0.1, \tau = 0.5$ *and* $\tau = 0.8$*, respectively, equivalent to the copula parameters of* $\theta = 0.1, \theta = 0.5$ *and* $\theta = 0.8$ *respectively.*

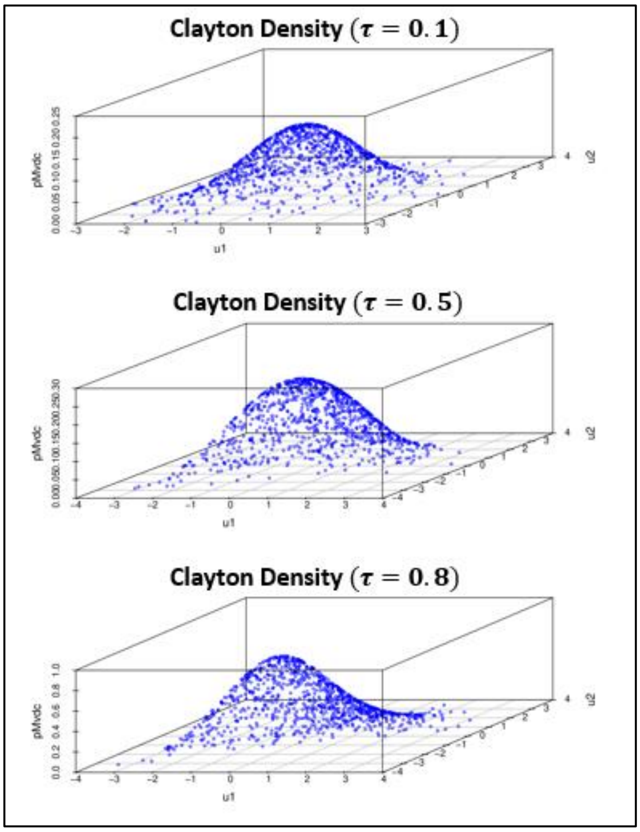


Figure 3. 13: Copula density plot from three Clayton copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Clayton copulas with dependence parameters of $\boldsymbol{\tau = 0.1, \tau = 0.5\ and\ \tau = 0.8}$*, respectively, equivalent to the copula parameters of* $\boldsymbol{\theta = 0.1, \theta = 0.5}$ *and* $\boldsymbol{\theta = 0.8}$*, respectively.*

3.3.2.2.3 Gumbel copula

The Gumbel copula is expressed as follows:

$$C_{\theta}^{Gu}\,(u_1, u_2) = \exp\left[-\left((-ln\, u_1)^{\theta} + \ (-ln\, u_2)^{\theta}\right)^{\frac{1}{\theta}},\right. \qquad (17)$$

where $\theta\ \epsilon\ [1, \infty)$.

$\theta = 1$ produces the independence copula whereas $\theta \to \infty$, results in the comonotonicity copula. Therefore, this copula ranges between independence and positive co-dependency as shown in Figure 3.14 below. As the copula moves away from the independence copula shown by $\tau = 0.1$, the contour plot is near radial symmetry but with evidence of upper tail dependence. As τ approaches the comonotonic copula ($\tau = 0.8$), the contour plot is a narrower pear-shaped showing a weaker lower tail dependency and a strong upper tail dependence, also represented by the PDF in Figure 3.15 below.

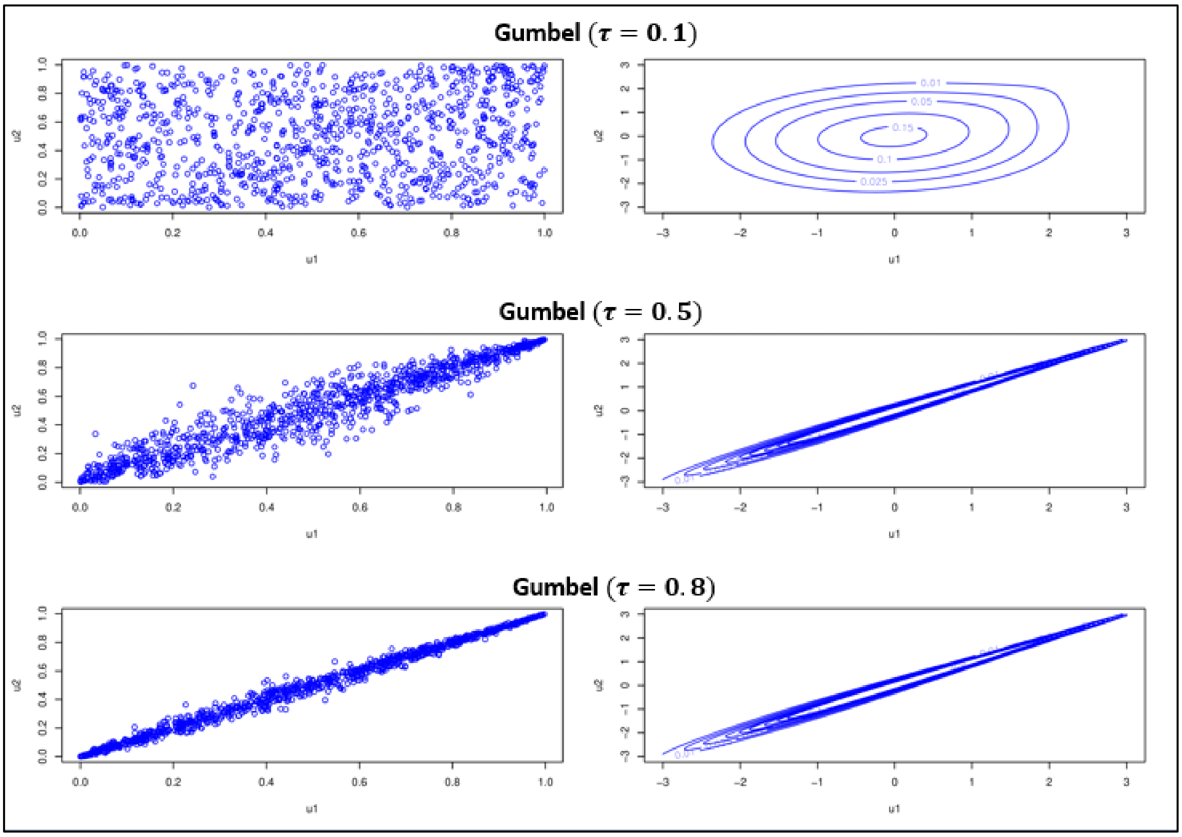


Figure 3. 14: Scatter and contour plot from three Gumbel copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Gumbel copulas with dependence parameters of $\boldsymbol{\tau = 0.1, \tau = 0.5}$ ***and*** $\boldsymbol{\tau = 0.8}$*, respectively which is equivalent to the copula parameters of* $\boldsymbol{\theta = 0.1, \theta = 0.5}$ *and* $\boldsymbol{\theta = 0.8}$*, respectively.*

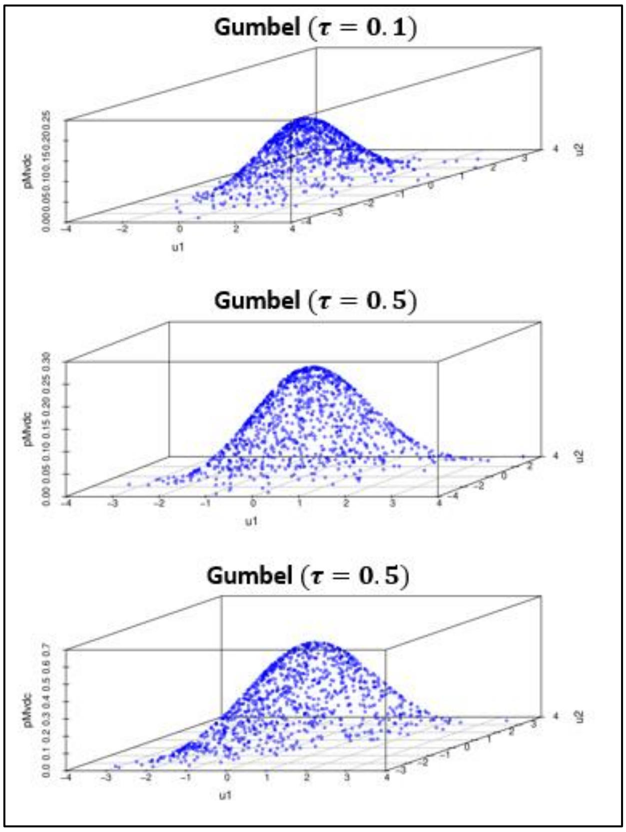


Figure 3. 15: Copula density plot from three Gumbel copula parameters

Notes: The figures are from a bivariate random sample of size 1 000 simulated from Gumbel copulas with dependence parameters of $\tau = 0.1, \tau = 0.5$ *and* $\tau = 0.8$*, respectively which is equivalent to the copula parameters of* $\theta = 0.1, \theta = 0.5$ *and* $\theta = 0.8$*, respectively.*

In summary of the Archimedean copula family, Figure 3.16 below illustrates the key distinguishing features between the copulas using a comonotonic copula example.

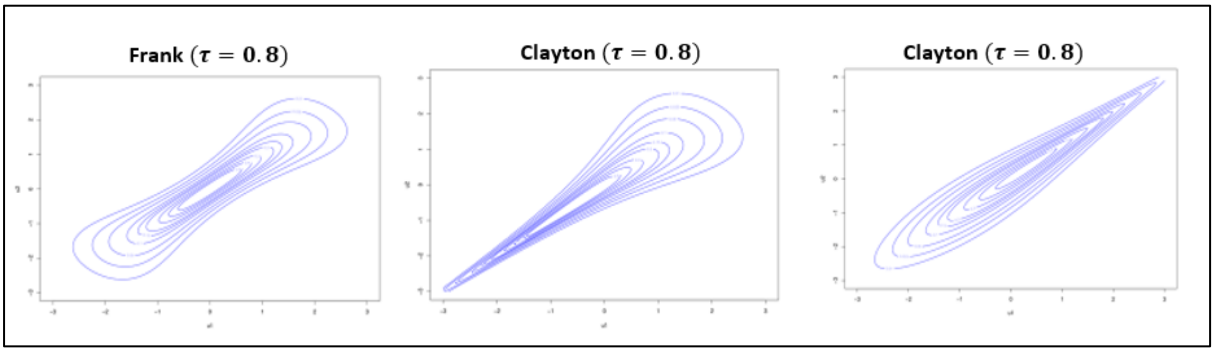


Figure 3. 16: **Contour plots for a Frank, Clayton, and Gaussian copula**

Notes: The three copulas have an identical dependence parameter of $\tau = 0.8$ *which is equivalent to the copula parameter of* $\theta = 0.8$*.*

From Figure 3.16, the Frank copula is restricted to a central tendency with no tail dependency clearly shown by its contour plot. The Clayton copula's contour plot shows a strong lower tail dependency whereas the Gumbel has a strong upper tail dependence and a weaker lower tail dependence.

Remark 3

From the Clayton and Gumbel copula descriptions provided, it has been established why these copulas cannot capture negative dependence in their natural form. However, these two copulas can be rotated 90 and 270 degrees to enable them to model negative dependence, hence the names, for example, rotated 90 degrees Clayton copula or rotated 270 degrees Gumbel copula (Brechmann and Schepsmeier, 2013). Rotating copulas is desirable to exploit some of the key features for example of the Gumbel's tail dependence modelling capabilities that are otherwise restricted from being utilised in modelling negative dependences when in their natural form. The following are distribution functions of the rotated copulas and their respective equations explaining the effect of rotating a copula on dependence measured (Brechmann and Schepsmeier, 2013):

$$C_{90}(u_i, v_i) = v_i - C(1 - u_i, v_i), \tag{18}$$

$$C_{180}(u_i, v_i) = u_i + v_i - 1 - C(1 - u_i, 1 - v_i), \tag{19}$$

$$C_{270}(u_i, v_i) = u_i - C(u_i, 1 - v_i), \tag{20}$$

where $u_i = \mathbb{P}(y_{1i} = 1)$ and $v_i = \mathbb{P}(y_{2i} = 1)$.

Figure 3.17 below therefore utilises contour plots to illustrate the effect on dependence modelled by rotating Archimedean copulas.

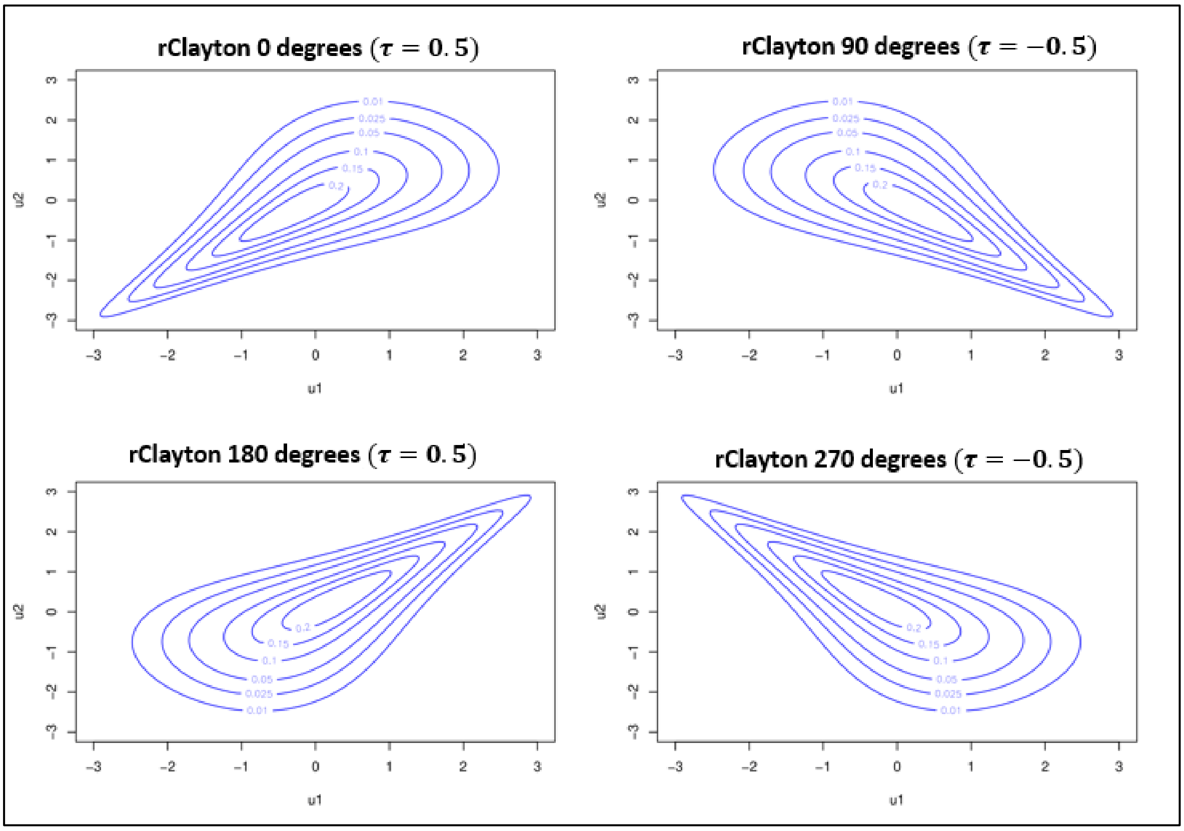


Figure 3. 17: Contour plots from a rotated Clayton copula

Notes: The Clayton copula is rotated 0, 90, 180 and 270 degrees and is modelled by standard normal margins with dependence parameters of $\tau = 0.5$ *and* $\tau = -0.5$*. A rotated Gumbel copula follows the same rationale.*

From Figure 3.17, when rotated 0 and 180 degrees, the Clayton copula can model positive dependence structures whereas, at 90 and 270 degrees, negative dependence structures are accommodated.

3.4.3 Goodness-of-fit Criteria

There are different approaches to determining the Goodness-of-fit (GOF) of copula models that are continuously evolving while literature has not presented a consensus on a standard approach. This research has looked closely at similar studies that have implemented copula approaches to dependence modelling for an application in crop insurance risk modelling. The progression in the GOF approaches has moved from simply establishing whether a statistical model fits a dataset to being accompanied by hypothesis tests assessing whether a specific copula model is appropriate for the dataset.

From the earlier simpler approaches to determining GOF for copulas, three methods have come as common namely, use of the loglikelihood value, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion of Schwarz (BIC)/Schwarz Bayesian criteria (SBC) (Tejeda and Goodwin, 2008; Zhu, Ghosh and Goodwin, 2008; Goodwin and Hungerford, 2014; Ahmed and Serra, 2015). From other finance and risk studies, Dias and Embrechts (2004), and Palaro and Hotta (2006) are also authors who have relied on the AIC as a GOF measure.

To add context on the application of copulas and common matrices of determining GOF, literature studies included the following: in a viability study of a revenue-based crop insurance product for Spain, Ahmed and Serra (2015) firstly relied on the loglikelihood values to narrow down their copula choices for further analysis. A larger loglikelihood value represents a better model fit. Zhu, Ghosh and Goodwin (2008) while designing whole farm insurance contracts relied on the typical AIC and BIC model selection methods in establishing the best fitting copula model. These two criteria were supported by the loglikelihood values of each copula. Tejeda and Goodwin (2008) analysed crop prices and yields using copulas to compare revenue-based crop insurance products by different modelling approaches followed in both marginal distributions and dependence. The two authors compared the performance of copula models based on loglikelihood and AIC values obtained. Goodwin and Hungerford (2014) applied copula methods in evaluating the degree to which weather and natural disaster risk tend to be systemic and state-dependent, and the implications they have on crop insurance and reinsurance contracts in the US. In determining the GOF of the copula models, the loglikelihood, AIC and SBC were utilised, along with the Vuong test for non-nested models.

Further context to the application of the simpler GOF tests outside of crop insurance studies comes from Dias and Embrechts (2004). The two utilised copulas to analyse the dependence structure of two-dimensional high-frequency data of foreign exchange spot rates for the US Dollar quoted against Germany and Japan's currencies. They relied on copulas to analyse conditional dependencies, as well as investigating the existence of change-points and explored structural changes in dependencies. The AIC method was used to rank copula models. On other hand, Palaro and Hotta (2006) made use of copulas in the estimation of Value at Risk (VaR) of a portfolio composed of two assets namely the Nasdaq and S&P500 stock indices. They too relied on the AIC criterion in choosing a copula model for estimating VaR.

Before this study explores other GOF approaches to copula applications, Fang, Madsen and Liu (2014) presented a good feasibility study on the application of the AIC in choosing an appropriate copula model. A key assumption is that given the true copula exists among the models being applied, the one with the smaller AIC value is supposed to be the true copula or otherwise the best one from the alternatives present. This is because the AIC measures the relative GOF of a statistical model. A downside is that the AIC method does not conduct a formal test of GOF on whether the copula is appropriate for the situation. However, the AIC method remains relevant when used under the stated assumptions in comparing different copula models fit thus allowing for distinguishing the best performing model. When the AIC method was compared to an alternative multiplier GOF test method, it emerged superior in ease of use and quickness of producing results. Further benefits highlighted accrued as a result of high computational costs associated with Cramer-von-Mises (*CvM)* test statistic particularly from a parametric bootstrapped-based GOF test, as well as the limitations associated with a larger sample size when relying on the latter approach (Kojadinovic and Yan, 2011; Fang, Madsen and Liu, 2014). Fang, Madsen and Liu (2014) explored the multiplier GOF test method as an alternative to the Kolmogorov-Smirnov (*KS*) and *CvM* tests as suggested by Kojadinovic and Yan (2011) who also explored it as an alternative.

Other approaches that moved away from the common approaches to establishing the better fitting copula model include the application of the Chen and Fen (2006) model selection by Ahmed and Serra (2015) that permitted the ranking of copulas while the goodness of fit was determined by the *KS* and *CvM* tests. From the latter tests, this is evidence of the momentum gained in the need and use for GOF tests that incorporate hypothesis tests to determine whether a copula is appropriate for the data. Studies by Christian Genest have gained prominence in alternative methods of GOF for copulas including Genest and Favre (2007) that provided two approaches, an informal method and a formal one. The informal approach is typical when establishing whether a dataset follows an assumed model by simulating values from the known theoretical models and superimposing values from a given dataset in a scatterplot. By doing so one can see whether the empirical dataset follows the same path as the theoretical model. However, this method must be used with caution as shown by Genest and Favre (2007) that when the empirical dataset is small, it can be difficult to distinguish whether the theoretical model simulated values that reproduce the same dependence structure of the empirical data. The formal approach to establishing a GOF entails using bootstrapping techniques to compute a *CvM* and *KS* test statistics and using the p-values to decide on the appropriate model. This is an improvement to earlier methods by Weng and Wells in 2000 for Archimedean copulas that had a major downside in their approach that relied on a bootstrapping techniques that were ineffective hence their failure to produce p-values. Hence as a solution the authors recommended choosing a model according to the smallest value established of the *CvM* statistic.

Formal GOF tests for copula models and the studies on them thereof is continuously evolving. For instance, Genest, Quessy and Rémillard (2006) provide an alternative method to computing p-values for different GOF test statistics based on a non-truncated version of Kendall's process whereby Wang and Well's (2000) approach had used truncated versions. Genest and Remillard (2008) provide key assumptions necessary for a parametric bootstrap approach to give appropriate estimates of p-values for GOF tests for multivariate distributions and copulas based on the *CvM* test statistic. Genest, Rémillard and Beaudoin (2009) use large Monte Carlo simulations to assess implications of sample size and strength of dependence on the level and power of blanket GOF tests for copula models. Genest *et al.* (2011) propose a GOF testing method for parametric extreme-value copulas that is based on a *CvM* test statistic. From this method, a parametric bootstrap procedure is used to estimate the test statistic while Monte Carlo simulations are used to assess the power of the test. These examples prove that the loglikelihood, AIC and BIC approach to GOF assessments can now be easily supported by an array of formal tests that incorporate the p-value in hypothesis testing.

3.5 CONCLUSION

In this chapter, there are three key points to consider going forward. The first is the choice of marginal distribution model used in modelling the data, and the second is the model used in determining the dependence relationship between white maize yield and price for use in comparing crop insurance products. The third key point is the goodness-of-fit parameters used in determining which of these models best suits the data in this research. The first two key points are important because they influence the pricing of the insurable risk, as well as the presumed yield loss calculation. It has been established in the literature provided that the choice of marginal distribution model and the choice of model used to represent dependence relationships have in some cases under-priced or over-priced insurable risks as well as resulted in over-paid or under-paid indemnities. The third key point is therefore important in minimising the chances of picking models that will misrepresent the insurable risks which is crucial if the market is to have a viable and sustainable crop insurance product. The following chapters will provide the methodology used in applying the different models, followed by the results from the application showing a comparison of MPCI and CRI.

CHAPTER 4

METHODOLOGY

4.1 INTRODUCTION

The methodology to be implemented has seven key steps that are summarised in Figure 4.1 below while the study's analytical results are presented in Chapter 5.

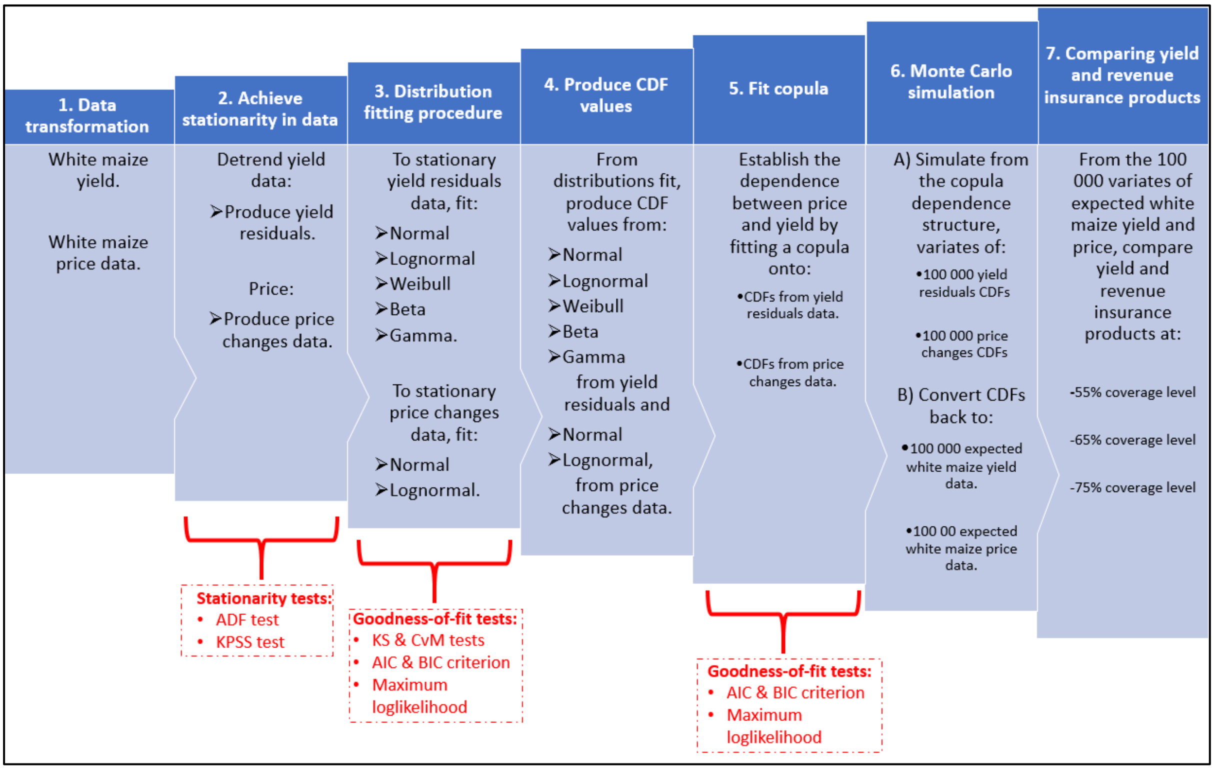


Figure 4. 1: Methodology Summary

- The first step transforms the raw SA white maize yield and price data into logged first differenced yield and logged price changes data, respectively.
- The second step highlights the stationarity requirement of the data before modelling can be done. Both datasets will go through further transformations and processes to achieve stationarity. Specifically, the logged, first differenced white maize yield data is put through a deterministic log-linear detrending technique to produce yield residual values that are typically trend stationary. The price data is turned into logged price changes data which is also typically stationary. This is achieved by taking the difference between the logged average expected harvest and logged average harvesting contract prices of white maize. Formal tests for stationarity are run utilising the Augmented-Dickey Fuller (ADF) and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests.

- In step three different families of distributions are fit onto the two stationary datasets. Before these distributions are imposed on the data, the datasets again go through different processes and transformations to accommodate all the distributions. The logged price changes and yield residuals data are translated upwards, while the latter is also parameterised to the range $0 < x < 1$ and then the distributions are fit. Different GOF tests are used to assess the fit of the different distributions on the data. GOF tests used are the AIC and BIC criterion, the *KS* and *CvM* test statistics, and the maximum loglikelihood values.
- Step four entails generating CDF values from each distribution model that was fit on the datasets.
- Step five fits different copulas to the generated CDF values to establish the dependence relationship between the two variables, yield and prices. To determine the GOF of the copulas to the data, the AIC, BIC and loglikelihood values are used, as well as incorporating hypothesis testing while relying on the p-values from the *KS* and *CvM* tests.
- Step six utilises the copula dependence structure established between the variables as well as the specified marginal distributions in a Monte Carlo simulation to produce 100 000 variates of price and yield CDF values for use in comparing different crop insurance products.
- Finally, step seven goes through numerous processes and transformation to get the 100 000 variates of price and yield CDF values back to an expected white maize yield and an average July harvest price for use in calculating expected losses and actuarially fair crop insurance premium rates for yield and revenue insurance products (MPCI and CRI respectively).

4.2 THE DATA MODELLING APPROACH

4.2.1 Modelling Maize Data

For the analysis of the white maize yield data, stationarity is a requirement covered in Step 2 of the methodology summary (refer to Figure 4.1). The following (Procedure 1) is implemented to transform the data to achieve stationarity:

Procedure 1

1. Transform the variable $Yields$ to $\log(yields)$ e.g. $ln(y_{Bloemfontein})$.
2. Transform logged yields to the first difference (resulting in one less row of observations),

$ln(yields)_t - ln(yields)_{t-1}$, where t is time in years.

3. Produce yield residual values from the following deterministic log-linear trend equations produced using excels Regression function:

$$ln(y_{District}) = \beta_0 + \beta_1 t + e_{District} \tag{21}$$

where β_i $(i = 0, 1)$ are the regression coefficients and e_j are residual errors for the specific districts.

4. Conduct an ADF and KPSS stationarity test on the yield residual values obtained.

For the white maize yield residuals, five distributions are considered, namely the normal, lognormal, Weibull, beta and gamma distribution models. The choice of these distributions follows from discussions in section 3.1 where the goal is finding an appropriate alternative to the normal distribution if it exists and to assess the performance thereof of these alternative distributions models when comparing crop insurance policies.

For the distribution model fitting, the yield residuals data is transformed, covered in Step 3 of the methodology summary (refer to Figure 4.1, p.87). To accommodate the necessary domain and support the values of all distributions, a translation by a factor of 2 units is imposed, followed by a parameterising process. Refer to Remark 4 (on p.90) which vividly explains the purpose of translating and parameterising the yield residuals. The following (Procedure 2) is implemented to transform the data to fit different distribution models on the variables:

Procedure 2

1. Translate the yield residuals by a factor of 2 (i.e. $e_{District} + 2$)
2. Transform the yield residuals such that the variables are strictly in the range [0,1] for the years 2004 to 2016 that is the time periods $t = 1, \ldots, 13$.

Therefore, the parameterizing factor is given by,

$$\text{Transformation factor } (z) = \frac{1}{\sum(e_{District}+2)}. \tag{22}$$

The equation for the transformed yield residuals in the range [0,1] is given as,

$$v_{residual} = \frac{e_{District}}{z}, \tag{23}$$

where $e_{District}$ are residual errors for the specific districts and z is the transformation factor as given in equation (22).

3. Fit the distributions to the transformed yield residual variables.
4. Generate CDF values from the specified marginal distributions.

Remark 4

From the methodology summary (refer to Figure 4.1, p.87), step 3 covers the distribution fitting process. Procedure 2 above includes the process of translating the yield residuals by a factor of 2 units followed by parameterising the values between 0 and 1 bounds. Translating the yield residuals simply adds 2 to the yield residuals values to ensure that all of the variables become positive while maintaining the shape of their graph which in this case has the effect of shifting it upwards by 2 units. The initial process of translating is done to get rid of negative values since, for example, the lognormal and gamma distributions cannot model them because they are more suited for positive skewness. Figure 4.2 below illustrates that the yield residuals values currently fall between negative and positive values. Parameterizing the values simply scales down the data so that it is restricted between 0 and 1 bounds while also maintaining the shape of the initial values' figure. Parameterizing the yield residuals between 0 and 1 is done to accommodate the beta distribution that is restricted to this domain.

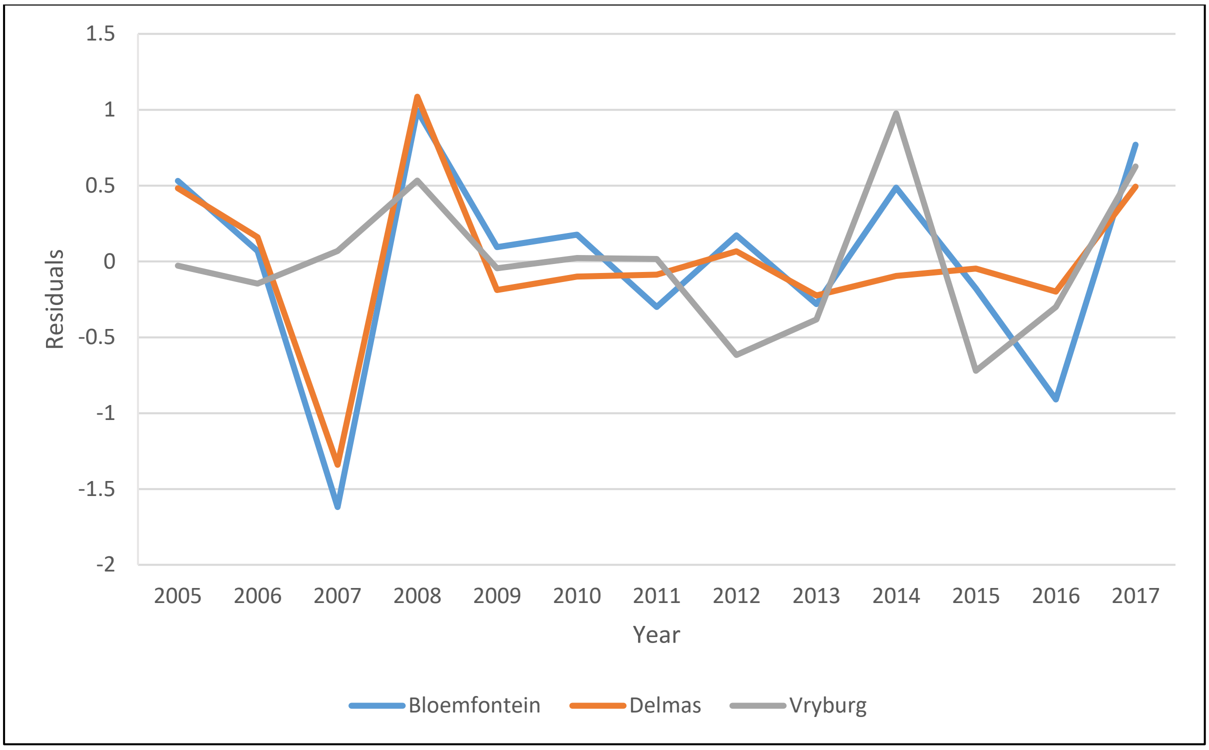


Figure 4. 2: Graph of yield residuals for the districts – Bloemfontein, Delmas and Vryburg

In Chapter 5 covering the application and results, Table 5.3 (on p.102) will illustrate the different yield residuals values, some positive and some negative which is also shown in Figure 4.2 above. Figure 5.1 (on p.101) is then given as evidence illustrating that the different transformations do not alter the shape of the graph. It should be noted that all these transformations will be undone (by Procedure 6 in section 4.4.) to get back to the original state of yield values to compare crop insurance products.

4.2.2 Modelling Price Data

For the analysis of the price data, stationarity is a requirement covered in Step 2 of the methodology summary (refer to Figure 4.1, p.87). The following (Procedure 3) is implemented to transform the data for purposes of achieving stationarity:

<u>Procedure 3</u>

1. Transform the variable P_{AEH} to $ln(P_{AEH})$ and P_{AJH} to $ln(P_{AJH})$.
2. Calculate logged price changes as $ln(P_{AJH}) - ln(P_{AEH})$.

3. Conduct an ADF and KPSS test for stationarity on $ln(P_{AJH}) - ln(P_{AEH})$.

For the logged price changes data, two distributions are considered, namely the normal and lognormal models. The choice of these distributions follows discussions in section 3.1 were the goal is finding an appropriate alternative to the normal distribution and to assess its performance in the comparison of crop insurance policies.

For the distribution model fitting, covered in Step 3 of the methodology summary (refer to Figure 4.1, p.87) the logged price changes data is transformed. To accommodate the necessary domain and support the values of the two distributions, a translation by a factor of 2 units is imposed (refer to **Remark 4** on p.90 which vividly explains the purpose of translating the yield residuals data which is also applicable to the logged price changes data). The following (Procedure 4) is implemented to transform the data to fit the normal and lognormal distributions models on the $ln(P_{AJH}) - ln(P_{AEH})$ data and generating CDF values:

Procedure 4

1. The $ln(P_{AJH}) - ln(P_{AEH})$ data is translated by 2 units to

$$(\ln(P_{AJH}) - \ln(P_{AEH})) + 2. \tag{24}$$

2. Fit the distributions to the transformed $ln(P_{AJH}) - ln(P_{AEH})$ data.
3. Generate CDF values from:
   a. normal distribution.
   b. lognormal distribution.

Remark 5

From the methodology summary (refer to Figure 4.1, p.87), step 3 covers the distribution fitting process. Procedure 4 above includes the process of translating the logged price changes data by a factor of 2 units. Translating the price changes values simply adds 2 to the logged price change values to ensure that all of the variables become positive while maintaining the shape of their graph which in this case has the effect of shifting it upwards by 2 units. This process of translating was done to get rid of negative values since the lognormal distribution cannot accommodate them whereas we want to model the data with the distribution. Figure 4.3 below illustrates that the logged price changes data has some positive and some negative values hence the need for transforming these variables as explained previously.

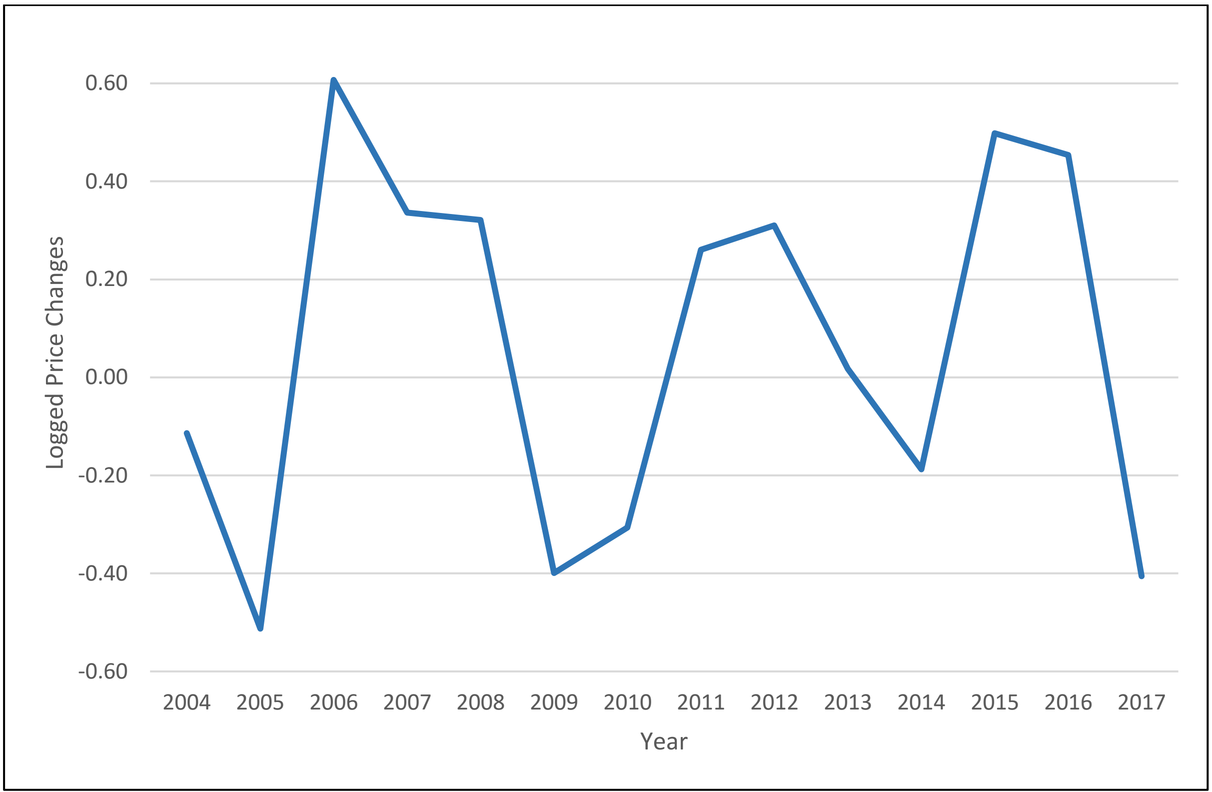


Figure 4. 3: Graph of logged price changes data

In Chapter 5 covering the application and results, Table 5.6 (p.105) illustrates the different logged price changes values, some positive and some negative which is also shown in Figure 4.3 above. Figure 5.9 will therefore be given as evidence illustrating that the different transformations do not alter the shape of the graph. It should be noted that all these transformations will be undone (by Procedure 6 in section 4.4.) to get back to the original state of price values for comparing crop insurance policies.

4.3 COPULA FITTING PROCEDURE

To establish the dependence relationship between SA white maize yield and prices, copula functions are fit to the CDF values from the specified marginal distributions. The result of this process is that each copula produces a certain dependence structure (i.e. a copula parameter and Kendall's τ), as well as different GOF parameters namely

AIC, BIC and loglikelihood values. However, the formal GOF test that incorporates hypothesis testing will still need to be done as a separate test. The following (Procedure 5) is implemented were different copulas are fit to the data and then the formal GOF test for copula models is conducted:

Procedure 5

1. Through the BiCopselect function of the Package 'VineCopula' for statistical analysis of the R statistical program, different copulas are fit to the CDF values from the transformed yield residuals and logged price changes data to establish dependence.
2. Through the gofCopula function from the Package 'Copula' and the BiCopGofTest function from the Package 'VineCopula' of statistical analysis belonging to the R statistical program, formal GOF tests of Genest, Rémillard and Beaudoin (2009), and Genest, Quessy and Rémillard (2006) are utilised.

The results of the copula procedure are presented in Chapter 5.

Remark 6

In section 3.2.2, it was established that a copula is defined on the unit interval $[0,1]$ from the unit d-cube interval $[0,1]^d$. This is the reason why most copula fitting procedures perform a function converting variables of interest into pseudo-observations i.e. standard uniform margins on the interval $[0,1]$. However, Procedure 5 above intentionally does not include that process because this research is fitting copulas with CDF values from different distributions which are naturally in the unit interval $[0,1]$.

4.4 COMPARING CROP INSURANCE PRODUCTS

To compare crop insurance products, this research needs to simulate price and yield variables to calculate expected losses and premium rates. Therefore from Procedure 5 above, a certain copula produced a certain dependence parameter and both (copula and parameter) are used in a Monte Carlo simulation to produce variates of the yield and price variables that follow the specified marginal distribution models. Essentially the Monte Carlo is producing variates that follow a certain distribution model covered and established in Procedure 2 and Procedure 3 for yields and prices respectively while relying on the copula for the dependence structure that the simulated values must follow. The actual steps leading to the comparison of crop insurance products therefore follow the process (Procedure 6) explained below:

Procedure 6

1. The copula and its established dependence structure are used in a Monte Carlo simulation to generate 100 000 CDF values from the specified marginal distributions. The probability inverse transformation (Bain and Engelhardt, 1992) is used to obtain 100 000 random variates (from generated CDF values) of the transformed yield residuals, $v_{residual_j}$ and transformed logged price changes, $v_{P_{C_j}}$ where,

$$v_{residual_j} = \frac{e_{District_j}}{z_j} \qquad (25)$$

and

$$v_{P_{C_j}} = (ln(P_{AJH}) - \ln(P_{AEH})) + 2\,, \qquad (26)$$

where z_j are the transformation factors (see equation (23)), for $j = 1, \ldots, 100\ 000$ simulated variates.

2. Obtain the original yield residuals and price change values as follows:

$$e_{District} = v_{residual_j} \times \frac{1}{\sum(e_{District}+2)}\,, \qquad (27)$$

and

$$P_C = e^{(v_{P_{C_j}}-2)}, \qquad (28)$$

where $\frac{1}{\sum(e_{District}+2)}$ is the transformation factor.

3. The final expected yield and P_{AJH_t} are respectively obtained as follows:

$$y_{District} = e^{(\beta_0+\beta_1 t+e_{District})}, \tag{29}$$

where β_i $(i = 0, 1)$ are the regression coefficients and e_j are residual errors.

$$P_{AJH_t} = P_{c_t} + P_{AEH_{t-1}} \tag{30}$$

After undoing the transformations done, the simulation procedure will have produced 100 000 variates of price changes for the year 2018. Insurers need this value to forecast an expected harvesting price ($P_{AJH_{2018}}$) for the year 2018 to price crop insurance policies for the 2017/18 maize season (planted in 2017 and harvested in 2018). Therefore, at the insurers' disposal during the planting window is the $P_{AEH_{2018}}$ while they can simulate the $P_{c_{2018}}$ from the copula dependence structure between the historic maize yield and price data. The following is the equation that is used to get the $P_{AJH_{2018}}$:

$$P_{c_t} = P_{AJH_t} - P_{AEH_{t-1}}$$

$$P_{c_{2018}} = P_{AJH_{2018}} - P_{AEH_{2017}}$$

$$P_{AJH_{2018}} = -P_{c_{2018}} + P_{AEH_{2017}} \tag{31}$$

4. The indemnity payouts for revenue and yield insurance products are calculated as follows (Ahmed and Serra, 2015):
   a. Revenue insurance indemnity payout is $\max[(\delta_j R_j^e - R_j), 0]$ $j = 1, \dots, 100000$, where, $R_j = Y_j \times P_{AJH_t}$ is total annual revenue, $R_j^e = E(R_j)$ is the expected revenue and $\delta_j \in (0,1)$ is the coverage level percentage. An indemnity payout of the value $(\delta_j R_j^e - R_j)$ is made to the farmer by the insurance company when $R_j \leq \delta_j R_j^e$ i.e. when revenue received is less than the coverage amount. Thus, the expected revenue loss computation is,

$$EL(R_{j,t}) = E\left[(\delta R_{j,t}^e - R_{j,t}) I(R_{j,t} \leq \delta R_{j,t}^e)\right] \tag{32}$$

b. A yield insurance indemnity payment is $\max\,[(\delta_j Y_j^e - Y_j)\,,0]\; j = 1,\ldots,100000$, where, Y_j is the annual white maize yield, $Y_j^e = E(Y_j)$ is the expected annual yield and $\delta_j \in (0,1)$ is the coverage level percentage. An indemnity payout of the value $\left(\delta_j Y_j^e - Y_j\right) \times P_t^e$ is made to the farmer by the insurance company when $Y_j \leq \delta_j Y_j^e$ i.e. when the yield harvested is the less than the coverage amount and where $P_t^e = E(P_{AJH_t})$ is the expected price. Thus, the expected yield loss computation is,

$$EL(Y_{j,t}) = E\left[\left(\delta Y_{j,t}^e - Y_{j,t}\right) I\left(Y_{j,t} \leq \delta Y_{j,t}^e\right)\right] \times P_t^e \tag{33}$$

5. The actuarially fair insurance premium rate for revenue and yield insurance policies is calculated as a ratio (Goodwin and Ker, 2008; Ahmed and Serra, 2014):

a.

$$\frac{EL(R_{j,t})}{E(\delta_{j,t}\, R_{j,t}^e)}\,, \tag{34}$$

for a revenue-based crop insurance policy.

b.

$$\frac{EL(Y_{j,t})}{E(\delta_{j,t}\, Y_{j,t}^e)}\,, \tag{35}$$

for a yield-based crop insurance policy.

The actuarially fair premium is an idealised calculation excluding administrative and other costs to an insurance scheme. Thus, in theory, the actuarially fair premium would equal the expected insured loss or put differently the value of expected compensation (Goodwin and Mahul, 2004; Coble and Barnett, 2013). By this reasoning, it is calculated as an insurance premium rate expressed as a ratio (or %) of expected loss to total liability which represents the unit paid in premium for each unit of liability covered (Goodwin and Mahul, 2004).

Now that a comprehensive description of Procedure 6 has been provided covering derivations of expected loss and actuarially fair premium rate calculations, the following section looks at the crop insurance comparison results.

4.5 BENCHMARK MODEL AND METHODOLOGY SUMMARY

The approaches to be implemented in modelling the marginal distributions and dependence structures of the two risks, yield and prices for SA white maize are summarised in Figure 4.4 below.

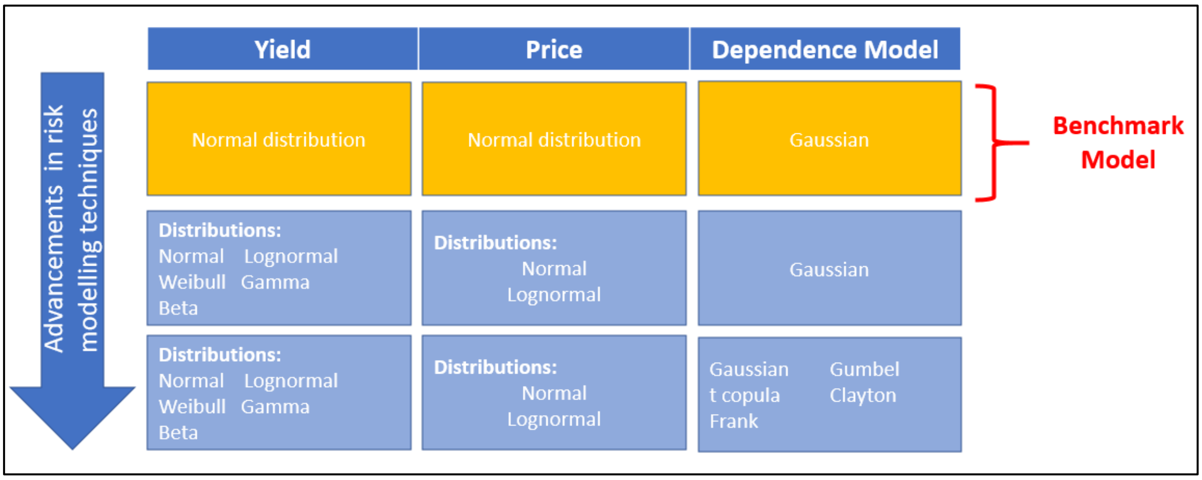


Figure 4. 4: Risk Modelling Techniques Summary

From Figure 4.4, the benchmark model distinguished in yellow consists of the normal distribution models fit onto both price and yield variables while the dependence relationship is modelled by a Gaussian copula (Goodwin, 2015). Moving further down from the benchmark model as shown by the downward-facing arrow, represents the progression in risk modelling techniques that have not necessarily been used together. While maintaining the Gaussian copula for dependence modelling, alternative models to the normal distribution for modelling yield and prices are explored (Atwood, Shaik and Watts, 2002; Ramirez, Misra and Field, 2003; Sherrick *et al.*, 2004; Tejeda and Goodwin, 2008; Goodwin and Hungerford, 2014; Goodwin, 2015). Thus, this represents ‘Case 2’ discussed in section 1.2 with the intent of assessing the implications of alternative marginal distribution models on the dependence relationship established by the Gaussian. The last approach uses a combination of alternative marginal distributions as well as alternative dependence models by including the t-copula, Frank, Gumbel, and Clayton copulas.

4.6 CONCLUSION

In this chapter, the methodology was presented while highlighting it's seven key steps. Steps one and two entailed putting the raw data namely white maize yield and prices through various processes and transformations to achieve stationarity; the two steps were covered in Procedure 1 and Procedure 3 for the yield and price data, respectively. Steps three and four added further transformations to the stationary data of yield and prices to accommodate the marginal distribution model fitting procedure which is necessary to produce CDF values as represented by a specific distribution for the copula procedure, covered by Procedure 2 and Procedure 4 for yields and prices respectively. Step 5 entailed fitting the different copula models onto the CDF data to establish the dependence relationship between the yield and price of white maize in SA and this was covered in Procedure 5 which also highlighted the necessary R functions used in the fitting process as well as in establishing the goodness-of-fit tests. Step six and seven entailed generating variates of CDF values through a Monte Carlos simulation according to the marginal distributions modelled and dependence structure established by the copula for purposes of comparing yield and revenue insurance products while this was covered in Procedure 6. Furthermore, Procedure 6 highlighted the necessary process of converting the simulated CDF values back to the initial yield and price variables, and importantly, the indemnity and premium rate calculation required to compare the different crop insurance products. Lastly in this Chapter, a detailed summary is presented highlighting the progression in crop insurance risk modelling techniques over the years that was illustrated in Figure 4.4. The following Chapter 5 will be presenting the results from executing the seven key steps of the methodology as highlighted by Procedures 1 - 6.

CHAPTER 5

APPLICATION TO THE COPULA FITTING PROCEDURE

5.1 INTRODUCTION

In this chapter, results from the application of the methodology leading to the copula fitting procedure are presented. The methodology was summarised into seven key steps of execution, covered by six procedures (refer to Chapter 4). This chapter presents results up to step 5 (up to Procedure 5) as follows:

- Step 1 - results of transforming the yield data by logging, first differencing and lastly detrending to produce yield residuals. Results of transforming the price data into price changes and logged price changes are presented.
- Step 2 - results of informal and formal stationarity verification and testing of the transformed data.
- Step 3 - implications of transforming the yield residuals and logged price changes data for the distribution model fitting procedure are shown. Results of the distribution fitting process are presented.
- Step 4 - from the different distribution models fit, CDF values are produced and presented along with their parameters and goodness-of-fit (GOF) parameters.
- Step 5 - results of the copula fitting procedure are presented showing dependence parameters obtained and GOF parameters.

5.2 DATA TRANSFORMATION

5.2.1 White Maize Yields

The following Tables 5.1 to 5.3 show the transformations that the district level white maize yield data goes through, from logging, then first differencing and finally the yield residual values produced from the detrending process (covering Procedure 1, from Chapter 4).

Table 5. 1: Logged District Level White Maize Yield Data

Year	$ln(y_{Bloemfontein})$	$ln(y_{Delmas})$	$\ln(y_{Vryburg})$
2004	0.19	1.22	0.63
2005	1.07	1.79	0.63
2006	1.42	2.04	0.52
2007	0.03	0.77	0.63
2008	1.19	1.94	1.21
2009	1.39	1.82	1.21
2010	1.61	1.79	1.29
2011	1.29	1.76	1.37
2012	1.38	1.88	0.83
2013	0.96	1.71	0.52
2014	1.24	1.67	1.58
2015	0.80	1.67	0.95
2016	-0.44	1.51	0.74
2017	-0.06	2.04	1.47

Table 5. 2: Logged First Difference District Level White Maize Yield Data

Year	$ln(y_{Bloemfontein})$_1	$ln(y_{Delmas})$_1	$\ln(y_{Vryburg})$_1
2004	N/A	N/A	N/A
2005	0.88	0.57	0.00
2006	0.36	0.24	-0.11
2007	-1.39	-1.26	0.11
2008	1.16	1.16	0.58
2009	0.20	-0.12	0.01
2010	0.22	-0.03	0.08
2011	-0.32	-0.03	0.08
2012	0.09	0.12	-0.55
2013	-0.42	-0.17	-0.30
2014	0.28	-0.05	1.06
2015	-0.44	0.00	-0.63
2016	-1.24	-0.16	-0.21
2017	0.38	0.53	0.73

Table 5. 3: District Level White Maize Yield Residual Values

Year	$e_{Bloemfontein}$	e_{Delmas}	$e_{Vryburg}$
2005	0.53	0.48	-0.03
2006	0.07	0.16	-0.15
2007	-1.62	-1.34	0.07
2008	0.99	1.09	0.53
2009	0.10	-0.19	-0.04
2010	0.18	-0.10	0.02
2011	-0.30	-0.09	0.02
2012	0.17	0.07	-0.62
2013	-0.28	-0.22	-0.38
2014	0.49	-0.10	0.98
2015	-0.18	-0.05	-0.72
2016	-0.91	-0.20	-0.30
2017	0.77	0.49	0.63

Figure 5.1 below summarises the initial transformation results of the district level white maize yield data. From left to right, the first column shows the logged data, the second column illustrates the first differencing effect, and the third column is the yield residuals. It is evident from the third column of Figure 5.1 constructed from values in Table 5.3 that yield residuals have both negative and positive values, hence the need for further transformations to accommodate the distribution models that need to be fit onto this data (explained in Chapter 4, Procedure 2 and Remark 4 on p.90).

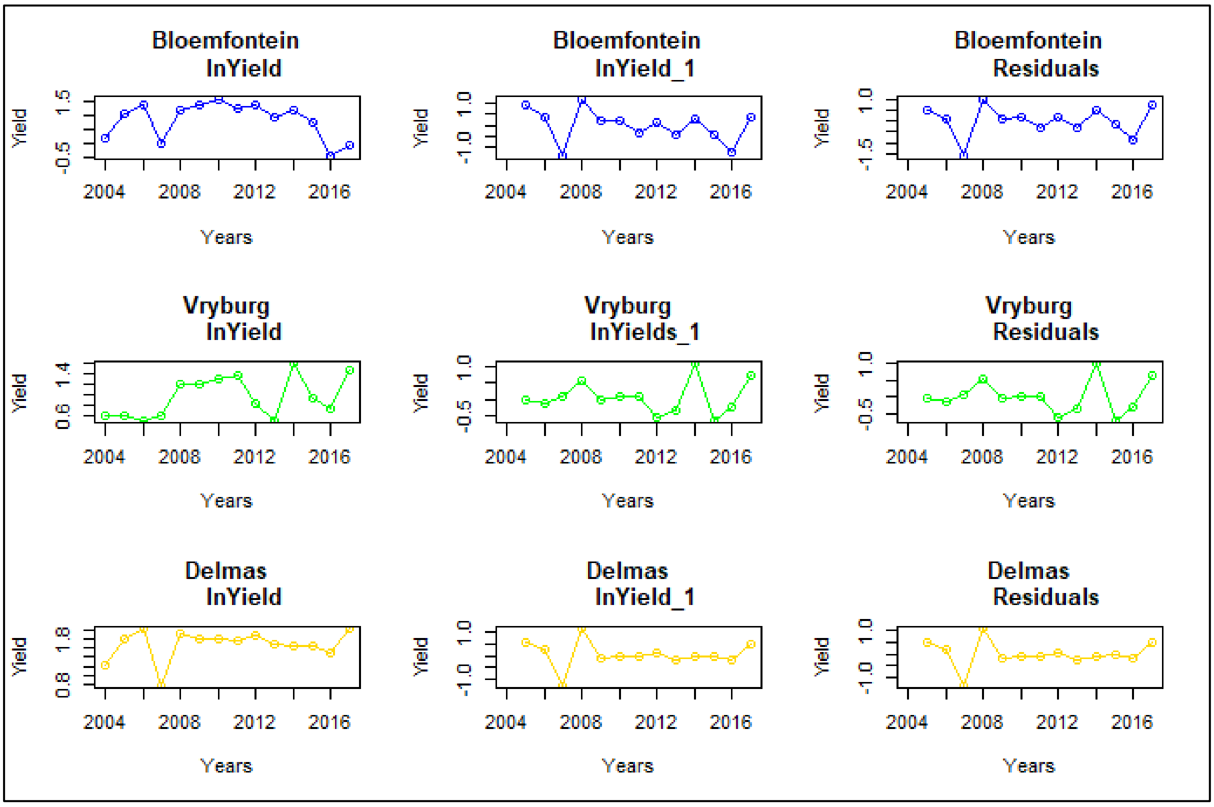


Figure 5. 1: District Level Yield Data Transformations

Procedure 1 is therefore necessary for the following two important reasons: Firstly, it removes the trend in the yield data. Secondly, it produces trend stationary yield residual values that are commonly used in analysis as a representative sample of the yield data (Just and Weninger, 1999).

The deterministic log-linear trend equations for the different district's yield data are established as:

$$y_{Bloemfontein} = 0.41t - 0.061 + e_{District}, \quad (36)$$

$$y_{Delmas} = 0.0042t - 0.092 + e_{District}, \quad (37)$$

$$y_{Vryburg} = 0.0062t - 0.021 + e_{District}, \quad (38)$$

where t is time in years.

Figure 5.2 below illustrates the residuals as produced by the log-linear detrending equations. Visually, the residuals for Bloemfontein, Delmas and Vryburg suggest a trend-stationary time series given the evidence of the series reverting about a constant mean of zero.

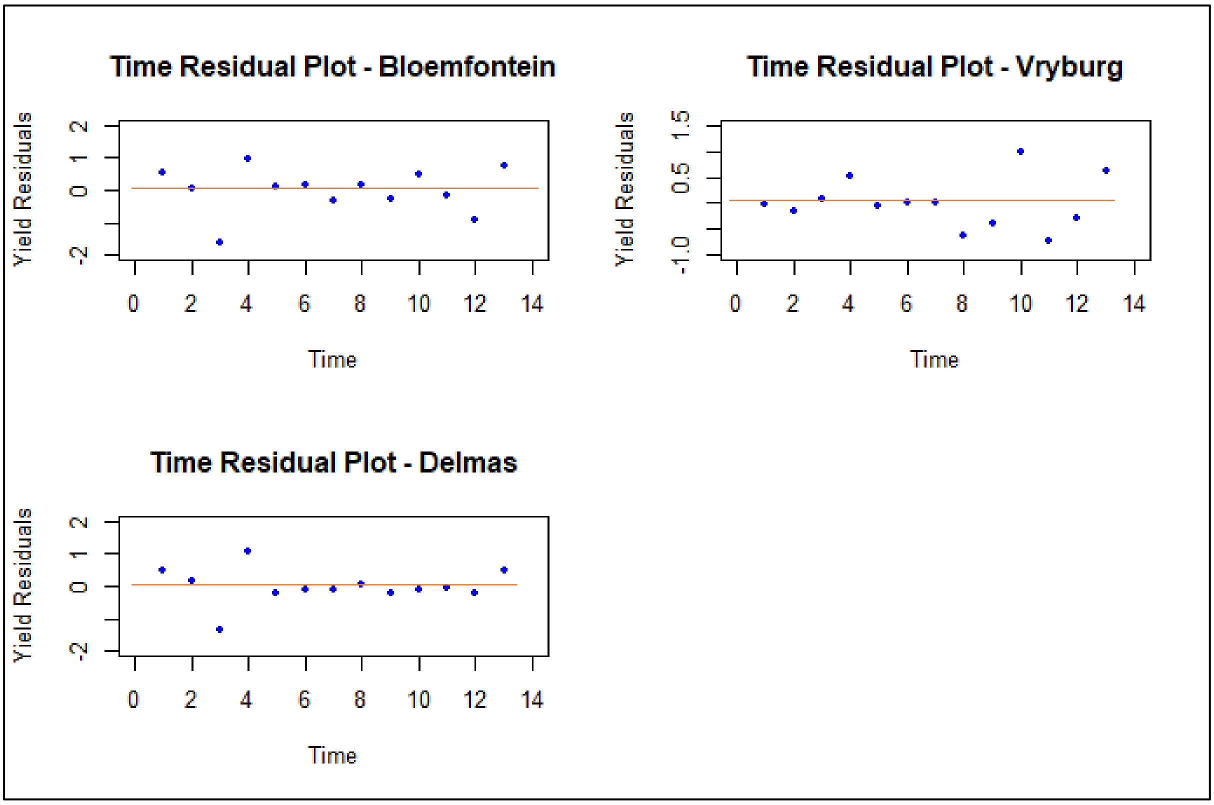


Figure 5. 2: Time Residual Plots

Table 5.4 below presents the results of the ADF and KPSS formal tests for stationarity on the yield residuals series for the three districts.

Table 5. 4: Stationarity Testing of Yield Residuals

	ADF test (P-value)	KPSS test (P-value)
$e_{Bloemfontein}$	-4.80***(0.010)	0.11*(0.100)
e_{Delmas}	-5.52**(0.013)	0.14*(0.063)
$e_{Vryburg}$	3.89**(0.049)	0.14*(0.055)

*Notes: Statistical significance levels represented by *, **, and *** for 10%, 5% and 1% respectively.*

The ADF test rejects a null hypothesis of a unit-root at a 1% level of significance for Bloemfontein and a 5% level of significance for Delmas and Vryburg in favour of the alternative trend-stationary hypothesis. However, the KPSS manages to reject the trend-stationary null hypothesis in all the districts at a 10% level of significance. The inconsistency in the results from the two tests can be attributed to the small yield data sample size. Due to the scarcity of low level aggregated yield data at a district level, this study was restricted to a limited data points of fourteen variables, while a larger sample that is aggregated at an even lower level would have been preferred for analysis. Despite these inconsistencies, the formal ADF test for stationarity allows for this research to confidently state that the yield residuals are trend stationary and this supports the initial informal stationarity suggestions from Figure 5.2.

5.2.2 White Maize Prices

The following Tables 5.5 to 5.6, and Figure 5.4 show the transformation that the different prices for white maize go through, from first getting the price changes variable, to logging the data to produce logged price changes (covering Procedure 3, from Chapter 4).

Table 5. 5: White Maize Futures Prices

Year	P_{AEH}	P_{AJH}	P_C
2004	987.26	881.27	-106.00
2005	1002.70	600.50	-402.20
2006	761.51	1397.43	635.91
2007	1182.34	1654.60	472.26
2008	1464.71	2019.69	554.98
2009	2000.94	1342.00	-658.94
2010	1499.87	1103.87	-396.01
2011	1365.67	1771.71	406.05
2012	1822.46	2484.35	661.89
2013	2219.68	2258.50	38.82
2014	2056.63	1705.39	-351.24
2015	1934.78	3184.28	1249.50
2016	2871.95	4522.00	1650.05
2017	2754.00	1835.69	-918.31

Table 5. 6: Logged Price Data

Year	$ln(P_{AEH})$	$ln(P_{AJH})$	$ln(P_{AJH}) - ln(P_{AeH})$
2004	6.89	6.78	-0.11
2005	6.91	6.40	-0.51
2006	6.64	7.24	0.61
2007	7.08	7.41	0.34
2008	7.29	7.61	0.32
2009	7.60	7.20	-0.40
2010	7.31	7.01	-0.31
2011	7.22	7.48	0.26
2012	7.51	7.82	0.31
2013	7.71	7.72	0.02
2014	7.63	7.44	-0.19
2015	7.57	8.07	0.50
2016	7.96	8.42	0.45
2017	7.92	7.52	-0.41

Figure 5.3 illustrates the transformation of the data from price changes to logged price changes form. As expected, the shape of the graph has remained relatively the same but the domain that the values fall in has been restricted by logging.

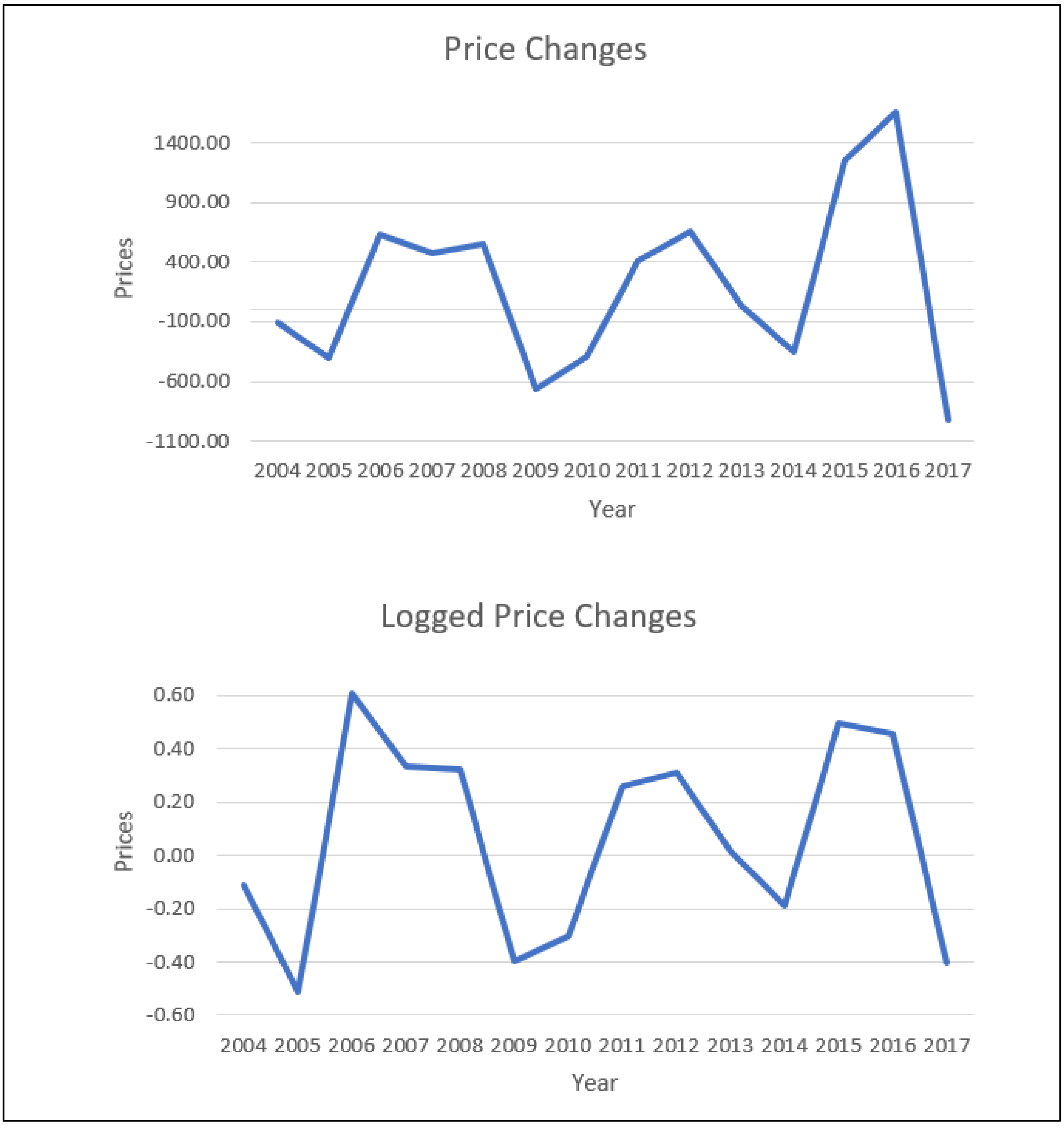


Figure 5. 3: Transformed White Maize Price Changes

The following Table 5.7 below presents the results of the ADF and KPSS formal tests for stationarity on the logged price changes data.

Table 5. 7: **Unit root testing**

	ADF (P-value)	KPSS test (P-value)
$\boldsymbol{ln}(\boldsymbol{P}_{AJH}) - \boldsymbol{ln}(\boldsymbol{P}_{AeH})$	-3.81*** (0.01)	0.08 *(0.1)

*Notes: Statistical significance levels represented by *, **, and *** for 10%, 5% and 1% respectively.*

The ADF test rejects a null hypothesis of a unit-root at a 1%, 5% and 10% level of significance in favour of the alternative stationary hypothesis since the p-value (0.01) is less than the above-mentioned level of significances. The KPSS test rejects the stationarity null hypothesis at only 10% level of significance. Again, the stationarity testing was conducted on a small sample size is possibly the reason for the inconsistencies in the stationarity testing results. Because this research is in part conducting a dependence study of yields and prices, the latter was restricted to the same sample size of the scarce yields. Despite these inconsistencies, the formal ADF test for stationarity allows for this research to confidently state that the yield residuals are stationary, and this supports the informal stationarity suggested in Figure 5.3 above.

5.3 MODELLING THE MARGINAL DISTRIBUTIONS

This section presents the results of fitting different marginal distribution models onto the transformed yield residuals and logged price changes data to produce CDF values. The variables are transformed to accommodate the domains of the different distribution models (explained in Remarks 4 and 5 in Chapter 4, p. 90 - 92). Recalling that the CDF values are important for their purpose in the copula fitting procedures as well as in simulation exercises in the latter sections and chapter.

5.3.1 Distributions for Yields

The following Figure 5.5 below illustrates the results of the transformation procedures that the yield residual values have gone through for the three districts.

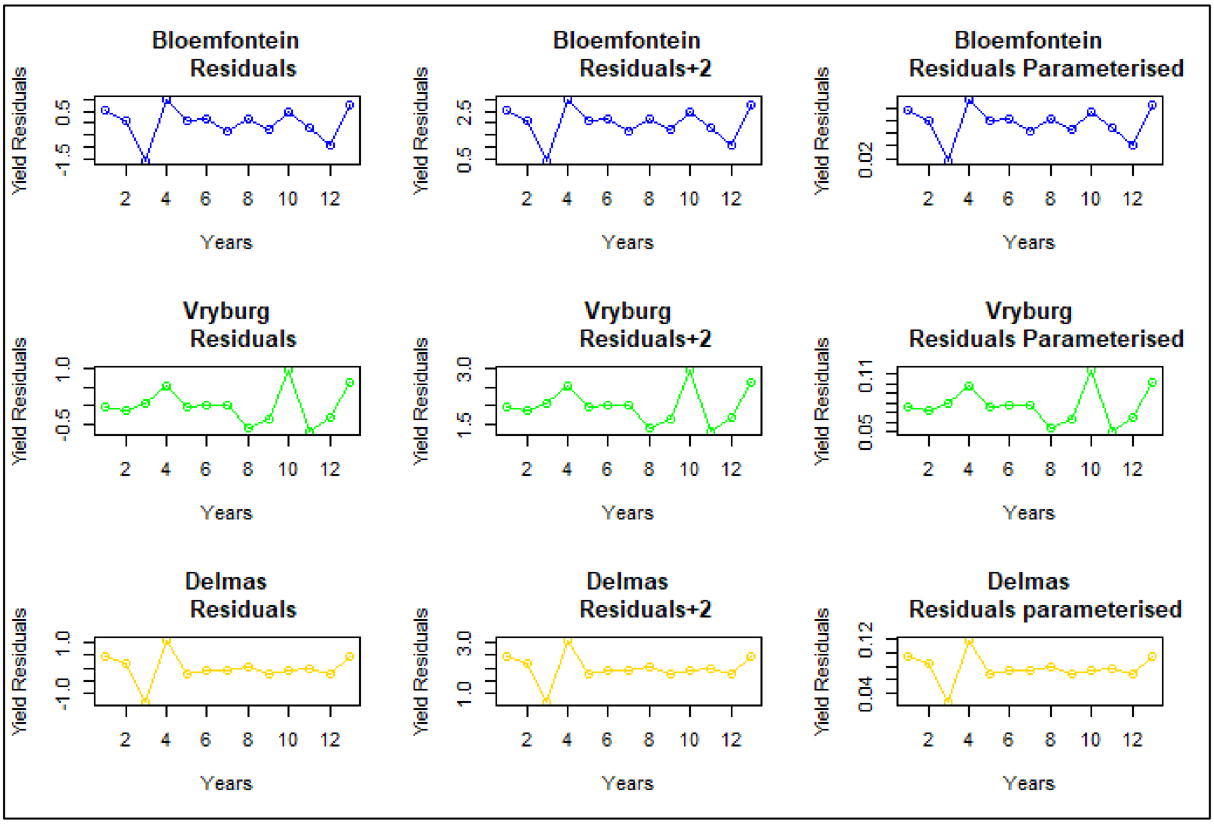


Figure 5. 4: Yield Residual Data Transformations

Notes: The first column consists of yield residuals graphs. The second column illustrates the graphs of yield residuals after translating them by a scale factor of 2. The third column illustrates the graphs of transformed yield residuals.

Figure 5.5 above illustrates the transformation procedures the yield residual values have gone through for the three districts. The graphs show that despite the transformations done to these yield residuals, the shape of the curve has not changed therefore suggesting consistency in results from any analysis to be done. From the left to the right column, the yield data has been transformed firstly into yield residuals, secondly, the yield residuals are translated by a factor of 2 and thirdly, the transformed yield residuals are then parameterised between the 0 and 1 bounds. For consistency, all the distributions were fit onto the parameterised residuals that can accommodate all five of the chosen distribution models.

Based on the AIC and BIC criterion, KS and *CvM* test statistics, and loglikelihood values, this research will distinguish the performance of fitting different distribution models to the transformed data. The following Table 5.8 presents results and

Maximum Likelihood Estimators (MLE) parameters obtained from the distribution fitting procedure on the transformed yield residual variables.

Table 5. 8: Yield Distribution Model Fitting Results

	Model	Parameter Estimate	KS test	*CvM* test	Maximum loglikelihood	AIC	BIC
Bloemfontein	Normal **(1)**	$\mu = 0.08$ $\sigma = 0.03$	0.17	0.06	29.12	-57.25	-53.12
	Lognormal **(5)**	$\mu = -2.66$ $\sigma = 0.52$	0.29	0.21	24.77	-45.44	-44.41
	Weibull **(2)**	$k = 3.44$ $\lambda = 0.09$	0.18	0.06	28.84	-53.68	-52.55
	Beta **(3)**	$\alpha_1 = 5.14$ $\beta_1 = 61.82$	0.25	0.14	26.88	-49.77	-48.64
	Gamma **(4)**	$k = 5.38$ $\theta = 69.88$	0.26	0.15	26.67	-49.35	-48.22
Delmas	Normal **(1)**	$\mu = 0.08$ $\sigma = 0.02$	0.26	0.14	32.19	-60.38	-59.25
	Lognormal **(5)**	$\mu = -2.61$ $\sigma = 0.34$	0.34	0.24	29.41	-54.82	-53.69
	Weibull **(2)**	$k = 4.23$ $\lambda = 0.08$	0.26	0.15	32.09	-60.18	-59.05
	Beta **(3)**	$\alpha_1 = 9.99$ $\beta_1 = 119.98$	0.31	0.19	30.79	-57.57	-56.44
	Gamma **(4)**	$k = 10.58$ $\theta = 137.50$	0.31	0.20	30.65	-57.30	-56.17
Vryburg	Normal **(4)**	$\mu = 0.08$ $\sigma = 0.02$	0.21	0.07	33.90	-63.80	-62.67
	Lognormal **(1)**	$\mu = -2.59$ $\sigma = 0.23$	0.17	0.05	24.29	-64.58	-63.45
	Weibull **(5)**	$k = 4.56$ $\lambda = 0.08$	0.23	0.08	33.57	-63.15	-62.02
	Beta **(3)**	$\alpha_1 = 17.50$ $\beta_1 = 209.99$	0.18	0.05	34.25	-64.49	-63.36
	Gamma **(2)**	$k = 18.97$ $\theta = 246.58$	0.18	0.05	34.26	-64.51	-63.38

Table 5.8 above provides the stated GOF test statistics for the distribution model fitting procedure followed by Figures 5.6 to 5.8 illustrating the fit of these models. According to the GOF values and test statistics from Table 5.8 above, the models per district are

ranked (number in brackets) according to the rule of thumb stating that a larger loglikelihood value is preferred whereas it is a smaller value for the AIC, BIC, KS and *CvM* value[24] representing a better fit. Therefore the overall best-fitting model (in brackets) to the transformed yield residuals values of each district are Bloemfontein (Normal), Delmas (Normal) and Vryburg (Lognormal).

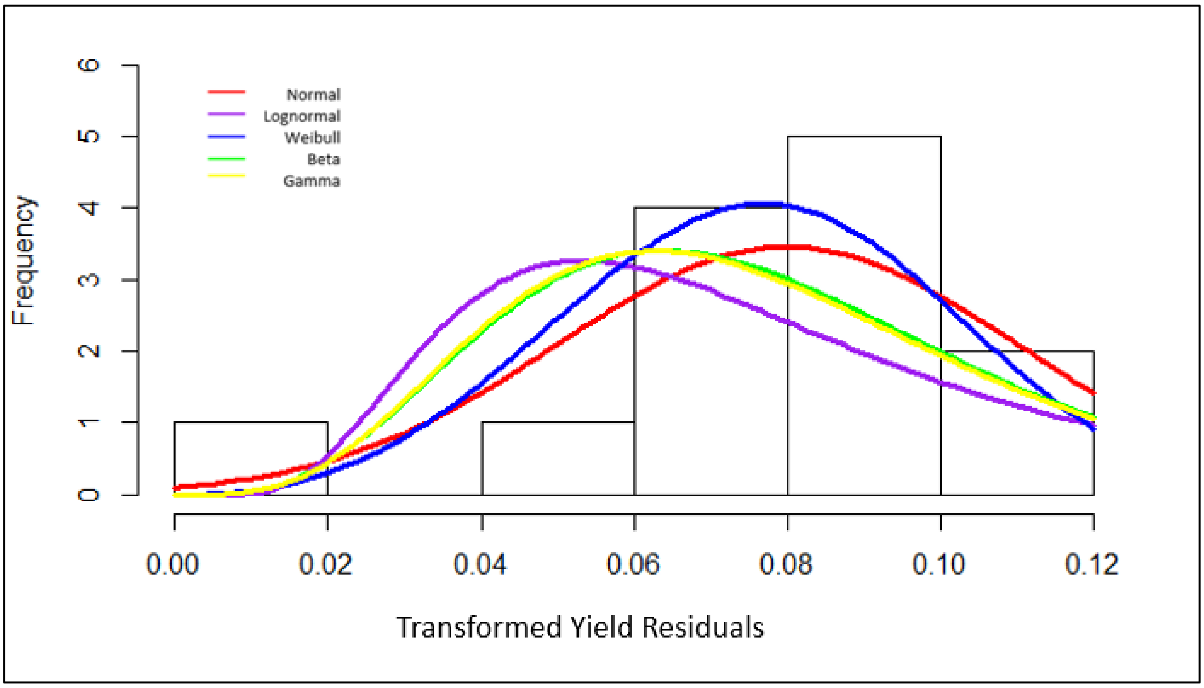


Figure 5. 5: Fitting PDFs of Marginals on Transformed Yield Residuals – Bloemfontein

[24] The KS and *CvM* tests statistic values are rounded off to 2 decimal places however when the decision on the best fitting model was made, the values were rounded off to three decimal places to avoid the ties and these correspond with the AIC and BIC values as well as loglikelihood findings.

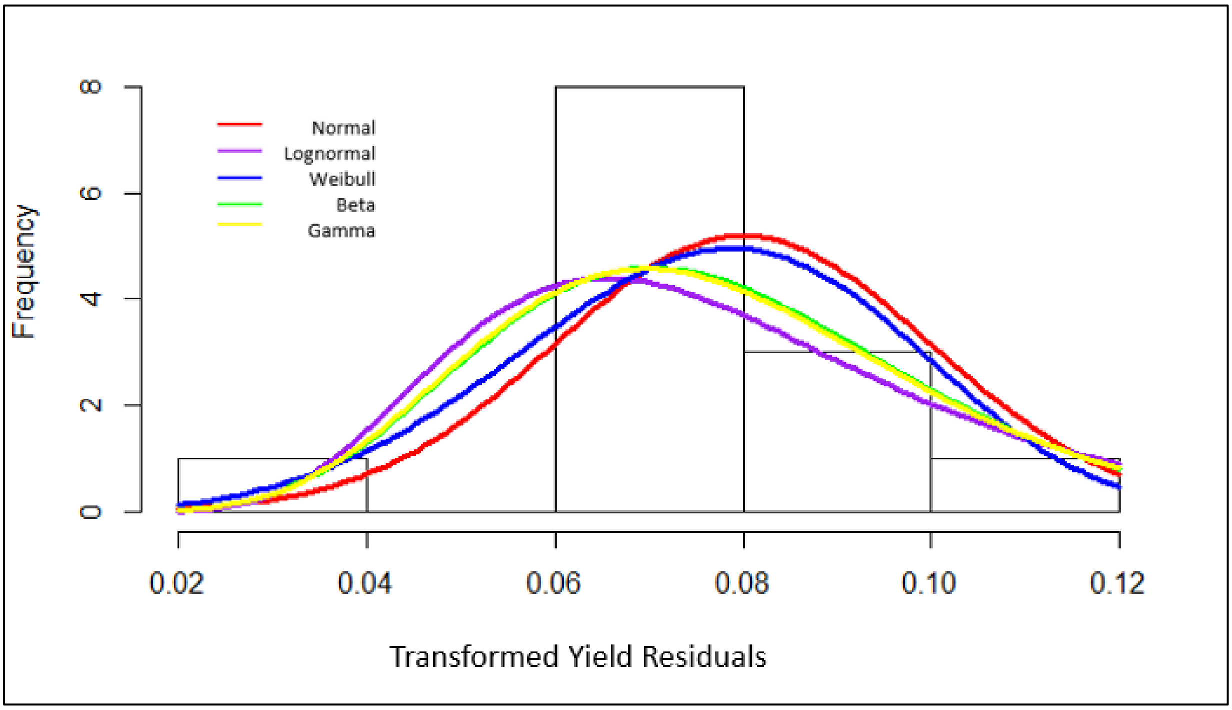


Figure 5. 6: Fitting PDFs of Marginals on Transformed Yield Residuals – Delmas

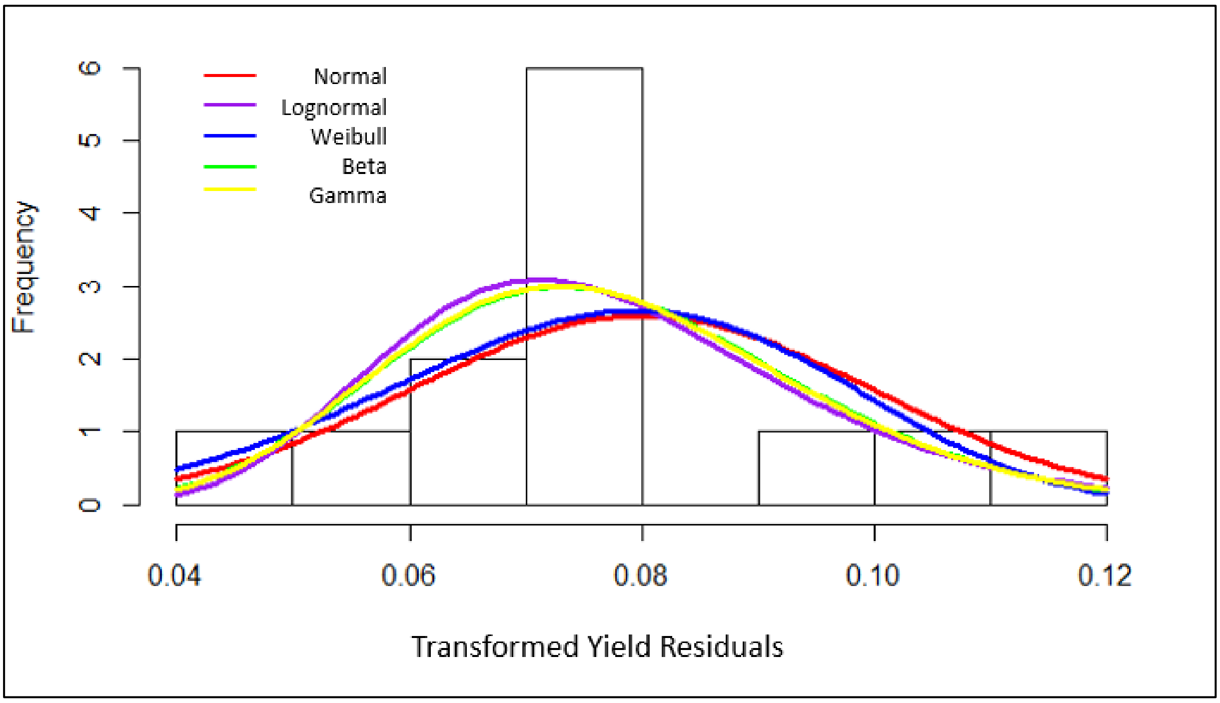


Figure 5. 7: Fitting PDFs of Marginals on Transformed Yield Residuals – Vryburg

Tables 5.9 to 5.11 below illustrate for each district, the CDF values generated by each of the marginal distribution models. The respective CDF points will be applied in the copula fitting procedure. The GOF results to these models will be kept in mind when evaluating the results of copula fitting and in comparing insurance products to assess whether distribution choice has some effect on results for SA.

Table 5. 9: Bloemfontein CDF Values

normal	**lognormal**	**Weibull**	**gamma**	**beta**
0.72	0.74	1.00	0.76	0.76
0.49	0.60	0.95	0.59	0.59
0.01	0.00	0.01	0.00	0.00
0.88	0.83	1.00	0.87	0.88
0.51	0.61	0.95	0.60	0.60
0.55	0.63	0.97	0.63	0.63
0.31	0.45	0.78	0.41	0.41
0.55	0.63	0.97	0.63	0.63
0.32	0.46	0.79	0.42	0.42
0.70	0.73	1.00	0.75	0.75
0.37	0.50	0.85	0.47	0.47
0.10	0.16	0.28	0.13	0.13
0.81	0.79	1.00	0.83	0.83

Table 5. 10: Delmas CDF Values

normal	lognormal	beta	Weibull	gamma
0.78	1.00	0.80	0.82	0.80
0.56	0.93	0.64	0.61	0.64
0.00	0.00	0.00	0.01	0.00
0.97	1.00	0.95	0.99	0.95
0.30	0.26	0.41	0.36	0.41
0.36	0.47	0.47	0.43	0.47
0.37	0.50	0.48	0.43	0.48
0.49	0.82	0.58	0.55	0.58
0.28	0.19	0.39	0.34	0.39
0.37	0.48	0.47	0.43	0.48
0.40	0.60	0.51	0.46	0.51
0.30	0.24	0.40	0.36	0.41
0.79	1.00	0.80	0.83	0.80

Table 5. 11: Vryburg CDF Values

normal	lognormal	beta	Weibull	gamma
0.42	0.52	0.50	0.47	0.51
0.33	0.41	0.40	0.38	0.40
0.49	0.60	0.59	0.54	0.59
0.81	0.87	0.87	0.86	0.87
0.41	0.50	0.49	0.45	0.49
0.46	0.56	0.55	0.51	0.55
0.45	0.56	0.54	0.50	0.54
0.09	0.07	0.08	0.12	0.08
0.19	0.21	0.21	0.22	0.21
0.96	0.97	0.97	0.98	0.97
0.06	0.03	0.04	0.08	0.04
0.23	0.27	0.27	0.27	0.27
0.85	0.90	0.91	0.90	0.91

5.3.2 Distributions for Price

Figure 5.8 below illustrates results of the transformation procedure leading to the transformed logged price changes data i.e. $(ln(P_{AJH}) - \ln(P_{AeH})) + 2$. The graph on the top is the $ln(P_{AJH}) - ln(P_{AeH})$ while the graph on the bottom is of $(ln(P_{AJH}) - ln(P_{AeH})) + 2$ after translating this data by a factor of 2. These two graphs show that despite the transformations to the $ln(P_{AJH}) - ln(P_{AeH})$ data, the shape of the curve has not changed suggesting consistency in results from any analysis to be done. A factor of 2 was used to also maintain consistency with the scale applied to the yield data.

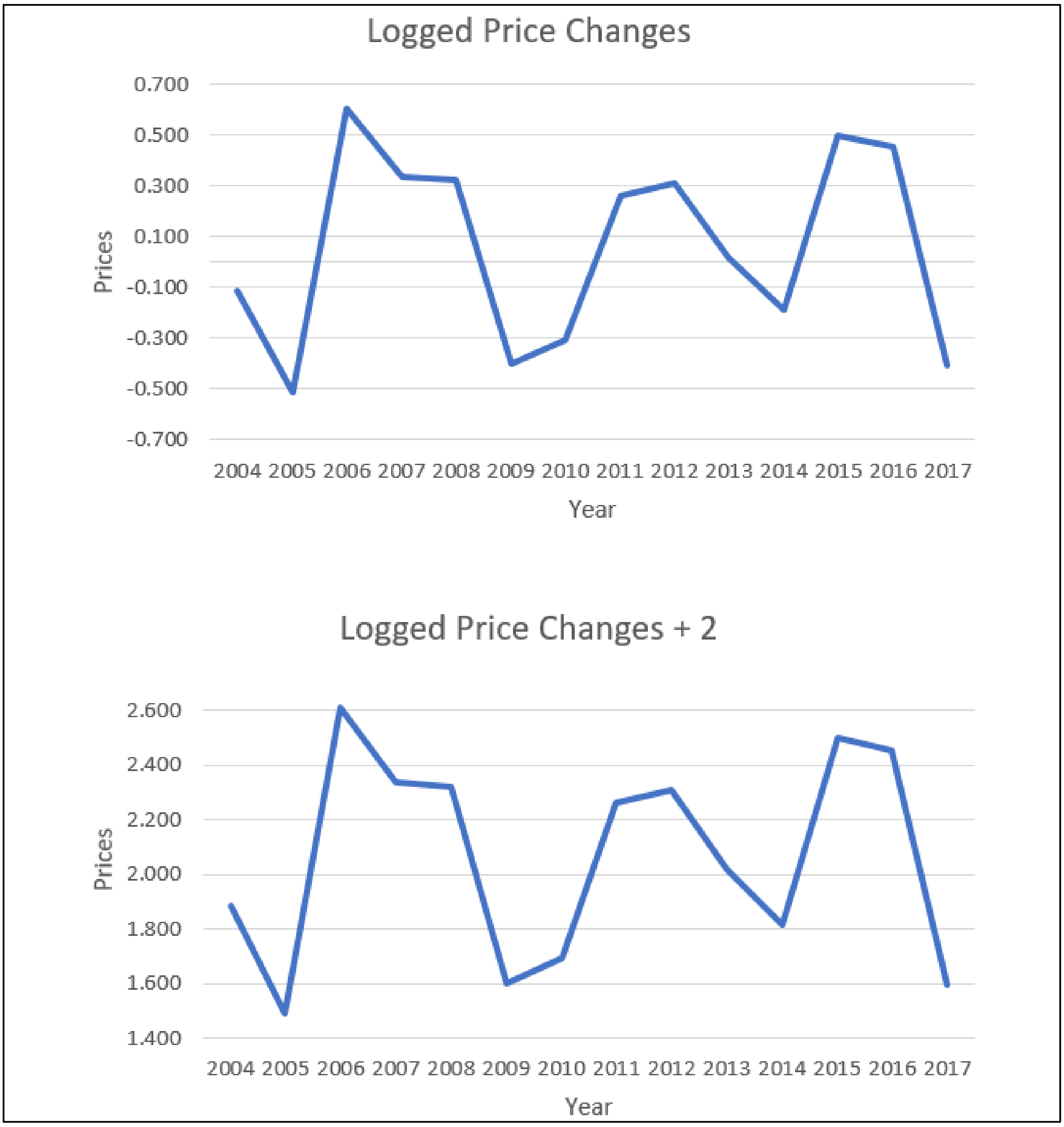


Figure 5. 8: Transformed Logged Price Changes

The following Table 5.12 presents results and MLE parameters obtained from the normal and lognormal model distribution fitting procedure on the $(\ln(P_{AJH}) - \ln(P_{AeH})) + 2$ data.

Table 5. 12: Price Distribution Model Fitting Results

	Parameter Estimate	**KS test**	***CvM* test**	**Maximum loglikelihood**	**AIC**	**BIC**
Normal **(1)**	$\mu = 2.08$ $\sigma = 0.04$	0.23	0.11	-5.78	15.56	16.69
Lognormal **(2)**	$\mu = 0.07$ $\sigma = 0.19$	0.24	0.12	-6.09	16.18	17.31

Based on the GOF test statistics namely, the AIC and BIC criterion, KS and CvM test statistics, and loglikelihood values, the normal distribution was a better fit to the $(\mathbf{ln}\,(\boldsymbol{P_{AJH}}) - \mathbf{ln}\,(\boldsymbol{P_{AeH}})) + \mathbf{2}$ data when compared to the lognormal model. Figure 5.9 below illustrates thereof the fit of these two models.

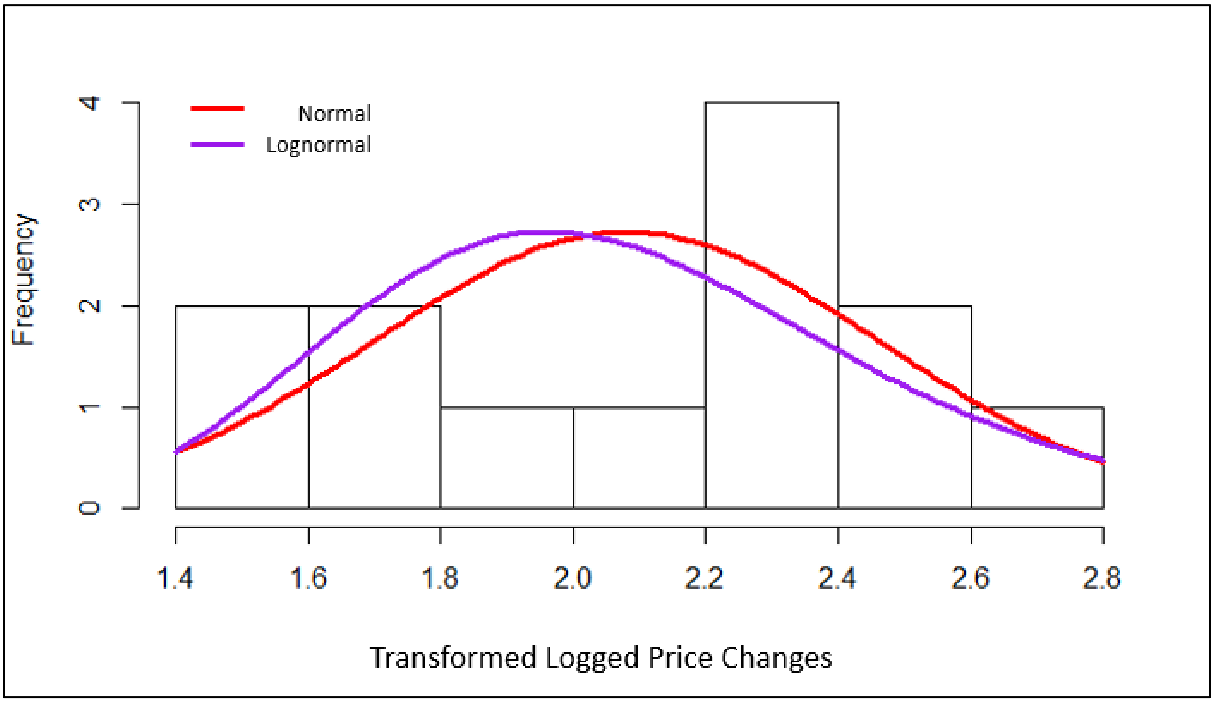


Figure 5. 9: Fitting PDFs of Marginals on Transformed Logged Price Changes

Tables 5.13 below presents the CDF values generated by the normal and lognormal marginal distribution models. The respective CDF points will be applied in the copula fitting procedure.

Table 5. 13: Price CDF Values

normal	lognormal
0.06	0.05
0.92	0.91
0.75	0.77
0.74	0.76
0.10	0.10
0.15	0.16
0.68	0.71
0.73	0.75
0.44	0.49
0.24	0.27
0.87	0.86
0.83	0.84
0.10	0.10

5.4 COPULA FITTING

This section will present the results of the copula fitting procedure on the CDF values of price and yield data of white maize. The results will be analysed as modelled by the three different cases representing the progression in marginal distribution and dependence risk modelling techniques. MLE techniques are applied in the copula fitting procedure. Just to reiterate what the three cases represent, earlier methods assumed normality in both the marginal distributions and dependence of yield and price risk thus the reliance on the Gaussian copula hence making this Case 1, the 'benchmark model' (refer to section 1.3 on Cases 1 - 3). As crop insurance risk modelling techniques improved, there was gravitation towards alternatives to the normal model in the marginals of the distribution whiles maintaining the normality assumption in the dependence structure of risks and this is covered in Case 2. Case 3 represents the progression towards a combination of an alternative to the normal model assumptions in both marginal distributions and the dependence structure of the risks hence the use of alternative copula and marginal distributions.

5.4.1 A Presentation of The Cases

Tables 5.14 to 5.16 illustrate the modelling results of the three cases for each district. From each table, the following key items are distinguished: the case, names of the superior distribution and copula model realised per case in modelling the variables, the dependence structure as given by Kendall's τ, as well as values of GOF measures namely the loglikelihood, AIC, BIC and KS test.

Table 5. 14: Results of the dependence cases – Bloemfontein

Case	Price changes	Yield residuals	Copula	τ	Loglikelihood	AIC	BIC	KS statistic (P-value)
1	Normal	Normal	Gaussian	-0.33	1.46	-0.92	-0.36	0.09(0.38)
2	Lognormal	Normal	Gaussian	-0.33	1.53	-1.06	-0.50	0.09(0.34)
3	Lognormal	Normal	Frank	-0.31	1.58	-1.15	-0.59	0.08(0.34)

*Notes: Statistical significance levels represented by *, **, and *** for 10%, 5% and 1% respectively.*

Table 5. 15: Results of the dependence cases - Vryburg

Case	Price changes	Yield residuals	Copula	τ	Loglikelihood	AIC	BIC	KS statistic (P-value)
1	Normal	Normal	Gaussian	-0.31	1.34	-0.69	-0.12	0.08(0.46)
2	Normal	Lognormal	Gaussian	-0.30	1.54	-1.08	-0.51	0.08(0.34)
3	Lognormal	Weibull	rotated Clayton 90 degrees	-0.35	1.89	-1.78	-1.22	0.56(0.93)

*Notes: Statistical significance levels represented by *, **, and *** for 10%, 5% and 1% respectively.*

Table 5. 16: Results of the dependence cases - Delmas

Case	Price changes	Yield residuals	Copula	τ	Loglikelihood	AIC	BIC	KS statistic (P-value)
1	Normal	Normal	Gaussian	-0.11	0.22	1.57	2.13	0.13**(0.029)
2	Lognormal	Gamma	Gaussian	-0.13	0.29	1.42	1.98	0.13**(0.029)
3	Lognormal	Gamma	Gaussian	-0.13	0.29	1.42	1.98	0.11**(0.029)

Notes: Statistical significance levels represented by *, **, and *** for 10%, 5% and 1% respectively.

Based on Kendall's Tau (τ) parameter established by the copula functions, there is an inverse relationship between the white maize yield and price data. The inverse relationship is stronger in the Bloemfontein and Vryburg districts but weaker in

Delmas. From the Loglikelihood, AIC, BIC and KS test statistics, this research was able to evaluate the robustness of each copula model, to be compared between the three cases. According to the KS GOF test for copula models, the null hypothesis was rejected at a 5% level of significance for Delmas but failed to reject for Bloemfontein and Vryburg. This means the data from Bloemfontein and Vryburg was appropriately modelled by the copulas chosen for Cases 1 to 3 whereas for Delmas this was not the situation. Overall, it is observed that Case 3 gives the best performing model with the smallest AIC, BIC, KS test value as well as the largest maximum loglikelihood values in all three districts. Therefore, following from Hypothesis 3, the alternative risk modelling approaches pursued indeed produce a better fitting crop insurance model. For Bloemfontein, the Frank copula function performed the best as modelled by the lognormal (price) and normal (yield) marginals. For Vryburg, it is the rotated Clayton 90 degrees copula as modelled by the lognormal (price) and Weibull (yield) marginals. For Delmas, Case 2 and Case 3 give the same best alternative model that is the Gaussian function as modelled by lognormal (price) and gamma (yield) marginals.

Please note that numerous copulas were fit to the data per case with different combinations of marginal distribution models of yield and price for SA white maize. To see all the different combinations of results, refer to Appendix C, section C.1.3. The copula models presented in cases 1 to 3 from Tables 5.14 to 5.16 above represent the best copula model for each of those cases out of the potential 180 copula model combinations tried for all three districts.

5.5 CONCLUSION

In this chapter, three of SA's district-level white maize yield and price data are analysed. The yield data was stationary after logging, followed by first differencing and then producing yield residual values from a log-linear detrending technique (see Procedure 1, Chapter 4). The price data was stationary after the different price data, P_{AJH} and P_{AEH} were logged and used to compute the logged price changes data, $ln(P_{AJH}) - ln(P_{AEH})$ (see Procedure 3, Chapter 4). Five distribution models (normal, lognormal, Weibull, beta, and gamma) were fit to the transformed yield residuals data. For the three districts, the best fitting model (in brackets) on the transformed yield residuals were as follows: Bloemfontein (normal), Delmas (normal) and Vryburg (lognormal). The normal and lognormal distribution models were fit to the transformed logged price changes data i.e. $(ln(P_{AJH}) - \ln{(P_{AEH})}) + 2$. The normal model was the most suitable to the $(\ln{(P_{AJH})} - \ln{(P_{AeH})}) + 2$ data. The copula fitting procedures were carried out to determine the dependence structure between white maize yields and prices per district while cognisant of the progression in crop insurance risk modelling techniques represented by the three cases. It was observed that Case 3 (alternative copula coupled with alternative marginal distributions) was the best fitting copula model in all three districts achieving the smallest AIC, BIC, KS test and *CvM* test values as well as the largest maximum loglikelihood values in all three districts. Given these findings, it has been established that yes, there is an inverse relationship between prices and yields of white maize in the chosen districts. Also, because Case 3 produced the better fitting copula models suggests that alternative risk modelling techniques followed produce a better crop insurance model hence the inverse relationship they established is more accurate when compared to Case 1's, the 'benchmark model'. The implications of these findings on yield and revenue crop insurance policy comparison results will be established in the next chapter.

Remark 7

With regards to tail dependence, it was established in Chapter 3, section 3.2 that some copulas can model it. This study did not establish tail dependence because the data did not lend itself to it since it did not have extreme occurrences. Also, due to data scarcity, the time series was limited to roughly 14 years. However outside of this study, in many cases, a dependence relationship may appear independent but due to a phenomenon such as extreme weather event could result in a change from independence to a tail dependence, is the reason for the importance of the concept in risk analysis.

CHAPTER 6

COMPARISON OF CROP INSURANCE PRODUCTS

6.1 INTRODUCTION

In this chapter, an economic viability study of a CRI product is conducted. The analysis of viability entails comparing CRI and MPCI products at different coverage levels. The computation of the expected losses and actuarially fair insurance premium rates is required because that is what the insurance products are compared on, as influenced by the risk modelling approach pursued. Thus, the actuarially fair insurance premium rate is indicative of the pure cost of buying a crop insurance policy. The analysis uses variates of expected July harvest prices and expected yields simulated by a Monte Carlo simulation based on the copula dependence structures established in combination with the specified marginal distribution models used. Implications of the approaches taken in crop insurance risk modelling techniques are evaluated, represented by the three cases (defined in section 1.3 and summarised in Figure 4.4, p.98). These three cases exemplify the progression in the risk modelling techniques followed.

6.2 CROP INSURANCE COMPARISON RESULTS

In this section, the results of comparing CRI and MPCI policies are presented. There are three cases per district to be analysed that represent the progression in crop insurance risk modelling techniques. The expected losses and actuarially fair premium rates (simply referred to as premium rate in the analysis) from the two crop insurance schemes shall be compared at the insurance coverage levels of 55%, 65% and 75%.

6.2.1 Bloemfontein Results

For Bloemfontein, Figures 6.1 to 6.3 illustrate the dependence relationship as depicted by the copula functions and specified marginal distributions represented by the three cases. Case 1 shows a symmetric relationship as modelled by the Gaussian copula whereas Case 2 departs from symmetry, as evident from the lower tail of the contour plot when the price and yield marginals follow a lognormal and normal distribution model respectively. Case 3, while also modelled by the lognormal (price) and normal (yield) marginal, shows a less extent of the lower tail which is justified because it is modelled by the Frank copula that focusses on central tendencies. From this analysis, alternative marginal distributions do influence the representation of dependence as shown in the comparison of Case 1 and 2 from Figures 6.1 and 6.2, respectively. Table 6.1 below will provide the extent of the implication of the different risk modelling techniques on the expected loss and premium rate results for CRI and MPCI products.

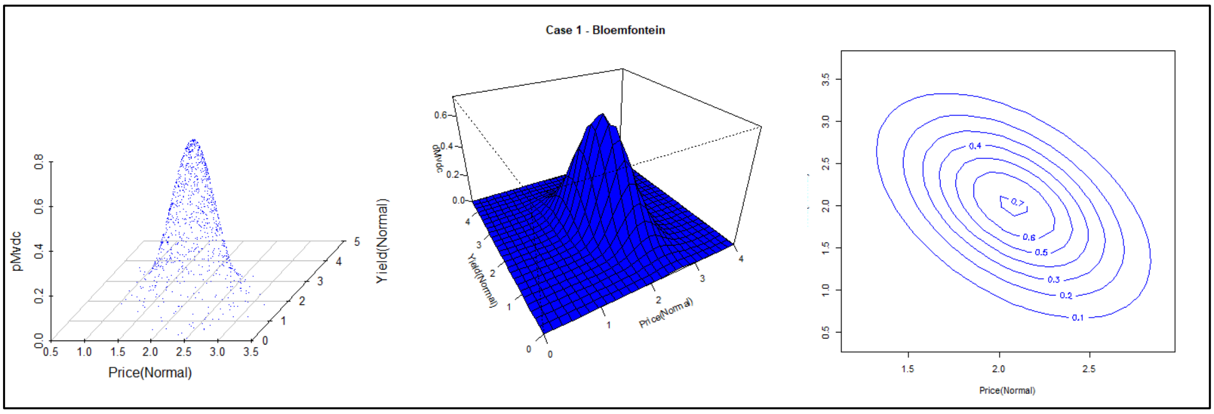


Figure 6. 1: 3D Scatter-, Surface- and Contour-plot from a Gaussian copula

Notes: The figures are from a bivariate random sample of size 1 000 modelled by normal (price) and normal (yield) marginals and simulated from a Gaussian copula with a dependence parameter of τ = -0.33.

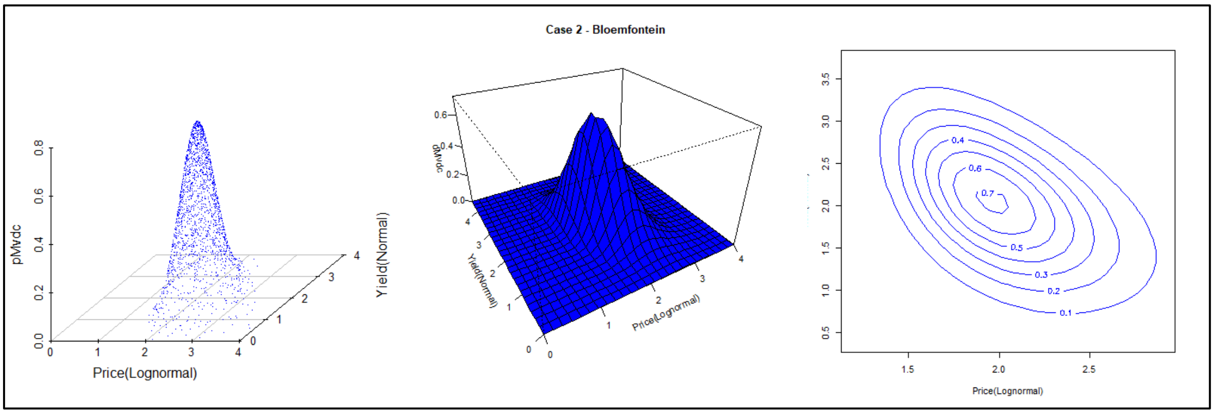


Figure 6. 2: 3D Scatter-, Surface- and Contour-plot from a Gaussian copula

Notes: *The figures are from a bivariate random sample of size 1 000 modelled by lognormal (price) and normal (yield) marginals and simulated from a Gaussian copula with a dependence parameter of τ = -0.33.*

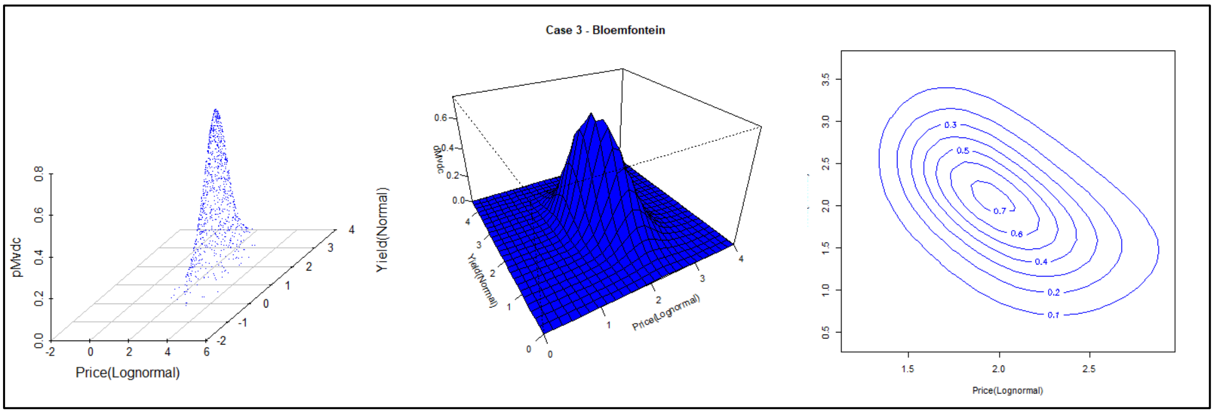


Figure 6. 3: 3D Scatter-, Surface- and Contour-plot from a Frank copula

Notes: *The figures are from a bivariate random sample of size 1 000 modelled by lognormal (price) and normal (yield) marginals and simulated from a Frank copula with a dependence parameter of τ = -0.31.*

The crop insurance comparison results between CRI and MPCI are presented in Table 6.1 below.

Table 6. 1: Bloemfontein insurance comparison results for Case 1 to Case 3

		55% Coverage		**65% Coverage**		**75% Coverage**	
		Expected loss (premium) (R/t)	**Premium rate**	**Expected loss (premium) (R/t)**	**Premium rate**	**Expected loss (premium) (R/t)**	**Premium rate**
Case 1	**MPCI**	R169.95	12.78%	R265.15	16.87%	R379.14	20.91%
	CRI	R105.85	9.19%	R176.88	12.99%	R266.46	16.96%
Case 2	**MPCI**	R170.19	12.70%	R266.04	16.80%	R380.98	20.85%
	CRI	R104.03	9.11%	R174.24	12.91%	R261.98	16.82%
Case 3	**MPCI**	R170.34	12.77%	R265.91	16.87%	R380.71	20.94%
	CRI	R117.74	10.05%	R193.08	13.95%	R287.15	17.98%

Notes: The copulas and marginals that modelled each case are as follows: Case 1 - Gaussian with normal (price) and normal (yield), Case 2 - Gaussian copula, with lognormal (price) and normal (yield) and Case 3 - Frank copula with lognormal (price) and normal (yield) marginals.

For Bloemfontein, as the coverage levels increase, so does the expected losses and premium rates realised for both MPCI and CRI, which is expected because the amount of risk being assumed by the insurer is growing. At identical insurance coverage levels, CRI achieves lower expected losses and premium rates than those of MPCI in all three cases. These findings are in line with expectations from the literature that given an inverse dependence relationship between crop yields and their prices, revenue-based crop insurance policies will be cheaper than those of yield-based crop insurance products (Meuwissen, Huirne and Skees, 2003; Ahmed and Serra, 2015).

Objective three of this study requires an assessment of the implications of the different risk modelling techniques followed, which represents the progression in crop insurance risk modelling techniques. Therefore, this requires the comparison of Cases 2 (τ = -0.33) and 3 (τ = -0.31) to Case 1 (τ = -0.33), the 'benchmark model' as follows:

- Comparing the CRI premium rates realised between Case 2 and Case 1, the assessment is as follows (refer to Table 6.1, p.123): for the insurance coverage levels of 55%, 65% and 75%, Case 2 realised the premium rates of 9.11%, 12.91% and 16.82%, respectively, which are lower than Case 1's premium rates of 9.19%, 12.99 and 16.96%, respectively. Despite that Case 1 and Case 2 have an identical dependence structure of $\tau = -0.33$ as established by the Gaussian copula, the premium rates realised were slightly different possibly due to the marginal distribution models used in representing the variables. This is a probable reason because the distribution estimates are used in simulating (predictions of) yield and price variates which are then used for calculating expected losses and premium rates. With the distribution model choice in mind, Case 1 was restricted to the normal (price) and normal (yield) distribution models whereas Case 2 had the lognormal (price) and normal (yield) distribution models.
- Comparing CRI premium rates realised between the most robust copula model, Case 3 ($\tau = -0.31$) and Case 1 ($\tau = -0.33$), the assessment is as follows (refer to Table 6.1, p.123): for the insurance coverage levels of 55%, 65% and 75%, Case 3 realised premium rates of 10.05%, 13.95% and 17.98%, respectively, which are higher than Case 1's premium rates of 9.19%, 12.99 and 16.96%, respectively. This finding is expected because recalling the basis of a natural hedge (covered in section 1.3) says that, given an inverse dependence relationship between crop yields and their prices, CRI expected losses and premium rates should be lower than those of MPCI. From the basis of a natural hedge, this means one should be able to compare different CRI

policies and the one modelled from a bigger inverse dependence structure should therefore achieve lower expected losses and expected premium rates. In this situation, Case 3 ($\tau = -0.31$), has a smaller inverse dependence structure than Case 1's ($\tau = -0.33$), hence why the formers premium rates are higher.

6.2.2 Vryburg Results

From Vryburg, Figures 6.4 to 6.6 illustrate the dependence relationship as depicted by the copula functions and specified marginal distributions represented by the three modelling cases. Case 1 shows a symmetric relationship as modelled by the Gaussian copula whereas Case 2 departs from symmetry, as evident from the lower tail of the pear-shaped contour plot when the distributions follow a lognormal (yield) and normal (price) models. On the other hand, Case 3's pear-shaped contour plot has a narrower and further-reaching lower tail as modelled by the rotated Clayton 90 degrees copula with lognormal (price) and Weibull (yield) distributions models. From this analysis, alternative marginals distributions do influence the representation of dependence as shown in the comparison of Case 1 and Case 2.

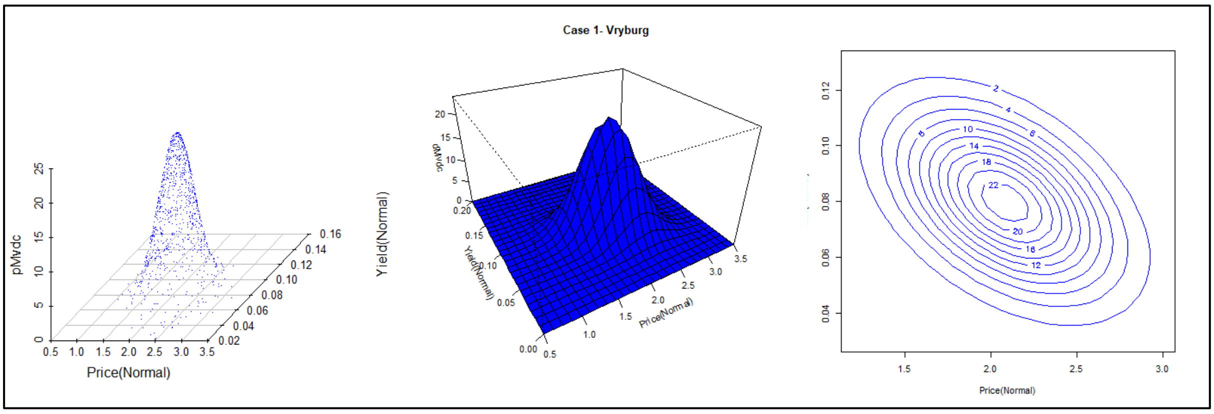


Figure 6. 4: 3D Scatter-, Surface- and Contour-plot from a Gaussian copula

Notes: The figures are from a bivariate random sample of size 1 000 modelled by normal (price) and normal (yield) marginals and simulated from a Gaussian copula with a dependence parameter of τ = -0.31.

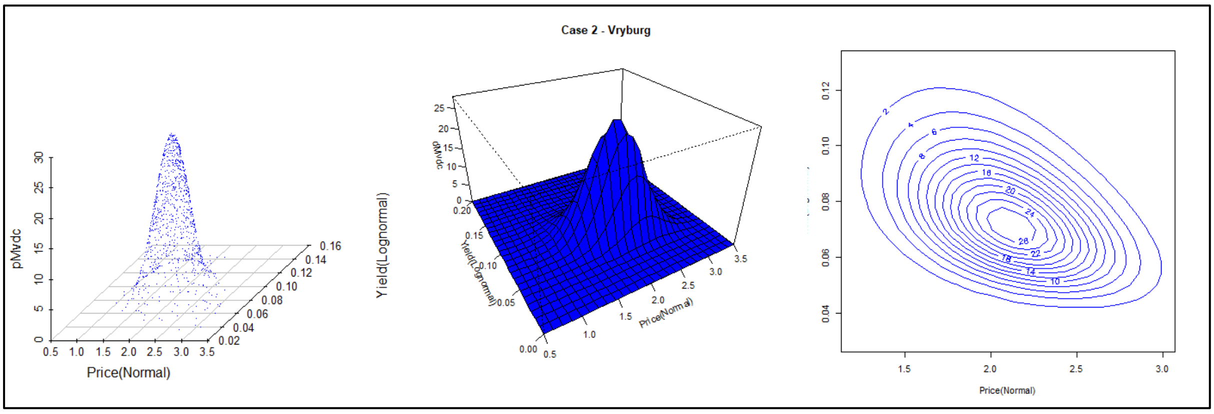


Figure 6. 5: 3D Scatter-, Surface- and Contour-plot from a Gaussian copula

Notes: The figures are from a bivariate random sample of size 1 000 modelled by normal (price) and lognormal (yield) marginals and simulated from a Gaussian copula with a dependence parameter of τ = -0.30.

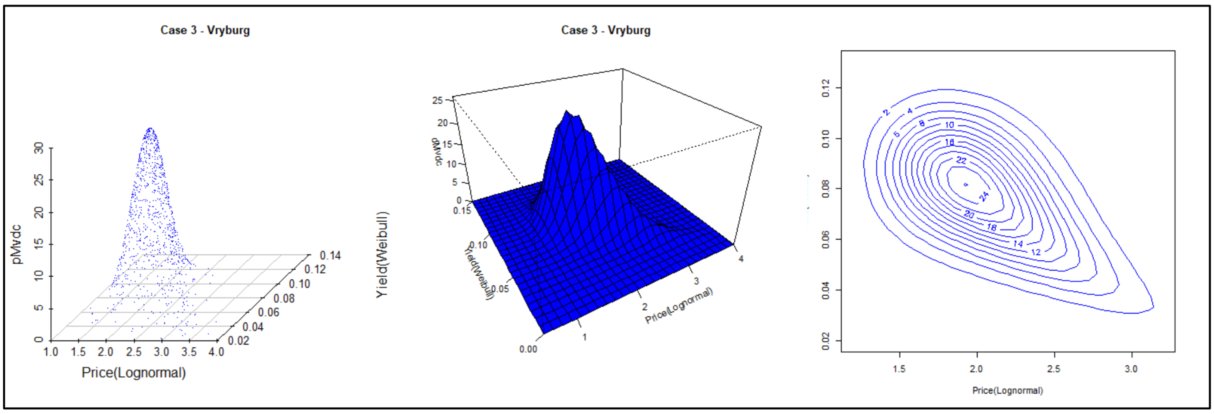


Figure 6. 6: 3D Scatter-, Surface- and Contour-plot from a rotated Clayton 90 degrees copula

Notes: The figures are from a bivariate random sample of size 1 000 modelled by lognormal (price) and Weibull (yield) marginals and simulated from a rotated Clayton 90-degrees copula with a dependence parameter of τ = -0.35.

Table 6.2 below provides the extent of the implication of the different risk modelling techniques on the expected loss and premium rates realised for CRI and MPCI products for Vryburg.

Table 6. 2: Vryburg insurance comparison results for Case 1 to Case 3

		55% Coverage		**65% Coverage**		**75% Coverage**	
		Expected loss (premium) (R/t)	**Premium rate**	**Expected loss (premium) (R/t)**	**Premium rate**	**Expected loss (premium) (R/t)**	**Premium rate**
Case 1	**MPCI**	R57.15	4.29%	R114.56	7.27%	R196.12	10.78%
	CRI	R39.68	3.26%	R85.09	5.92%	R153.04	9.23%
Case 2	**MPCI**	R27.83	2.09%	R77.87	4.95%	R159.33	8.79%
	CRI	R26.61	2.19%	R66.92	4.66%	R132.54	7.99%
Case 3	**MPCI**	R52.82	3.95%	R105.56	6.68%	R180.81	9.92%
	CRI	R24.31	2.03%	R59.81	4.22%	R119.23	7.29%

Notes: The copulas and marginals that modelled each case are as follows: Case 1- Gaussian with Normal (price) and Normal (yield), Case 2- Gaussian copula, normal (price) and lognormal (yield) and Case 3- rotated Clayton 90 degrees copula with lognormal (price) and Weibull (yield).

For Vryburg, as the coverage levels increase, so does the expected losses and premium rate realised for both MPCI and CRI, which is expected because the amount of risk being assumed by the insurer is increasing. At identical insurance coverage levels, CRI achieves lower expected losses than those of MPCI as well as premium rates in the three cases, except for Case 2's premium rate calculation at a 55% insurance coverage level. To a larger extent, these findings are in line with expectations from the literature that given an inverse relationship between crop yields and their prices, revenue-based crop insurance policies are cheaper than those of yield-based crop insurance products (Meuwissen, Huirne and Skees, 2003; Ahmed and Serra, 2015).

Objective three of this study requires an assessment of the implications of the different risk modelling techniques followed which represent the progression in crop insurance risk modelling techniques. Therefore, this requires the comparison of Cases 2 $(\tau = -0.30)$ and Case 3 $(\tau = -0.35)$ to Case 1 $(\tau = -0.31)$, the 'benchmark model' as follows:

- Comparing CRI premium rates realised between Case 2 and Case 1, the assessment is as following (refer to Table 6.2 above) for the insurance coverage levels of 55%, 65% and 75%, Case 2 realised the premium rates of 2.19%, 4.66% and 7.99%, respectively, which are lower than Case 1's premium rates of 3.26%, 5.92% and 9.23%, respectively. These findings contradict the natural hedge expectation that indirectly states that when comparing CRI policies, the larger the inverse dependence relationship between crop yields and their prices is the lower the premium rates realised. In this situation, Case 2 ($\tau = -0.30$) has a smaller inverse dependence structure than Case 1 ($\tau = -0.31$), yet Case 2 premium rates were cheaper, hence contradicting the natural hedge expectation. To explain this contradiction, it is plausible that the marginal distributions played a role. This is because the distribution parameter estimates are utilised in simulating (predictions of) yield and price variates for calculating expected losses and premium rates. Case 1's marginal distributions are restricted to the normal (price) and normal (yield) distribution models whereas Case 2 has normal (price) and lognormal (yield) distribution models. Furthermore, having explained why the Gaussian copula is most suited for normal marginal distribution models whereas Case 2 has a lognormal distribution could have also contributed to the contradictory finding (refer to section *3.2.2.1.1* explaining the 'benchmark model').

- Comparing CRI premium rates realised from the most robust copula model, Case 3 to Case 1, the assessment is as follows (refer to Table 6.2 above): for the insurance coverage levels of 55%, 65% and 75%, Case 3 realised the premium rates of 2.03%, 4.22% and 7.29%, respectively, which are lower than Case 1's premium rates of 3.26%, 5.92% and 9.23%, respectively. This finding is expected because it meets the natural hedge expectation since Case 3 ($\tau = -0.35$) has a larger inverse dependence structure between crop yields and their prices than that from Case 1 ($\tau = -0.31$).

6.2.3 Delmas Results

For Delmas, Figures 6.7 to 6.9 illustrate the dependence relationship as depicted by the copula functions and specified marginal distribution models represented by the three modelling cases for Delmas. Case 1 suggests a weak symmetric relationship as modelled by the Gaussian copula whereas Case 2 departs from symmetry, as evident from the lower tail slightly resembling a pear-shaped contour plot when the distribution models follow gamma (yield) and lognormal (price) models. Case 3's figures are identical to Case 2's because the two are from the same copula models (i.e. they have the same marginal distribution parameters and copula function). Alternative marginals distributions do influence the representation of dependence as shown in the comparison of Case 1 to Case 2 and 3.

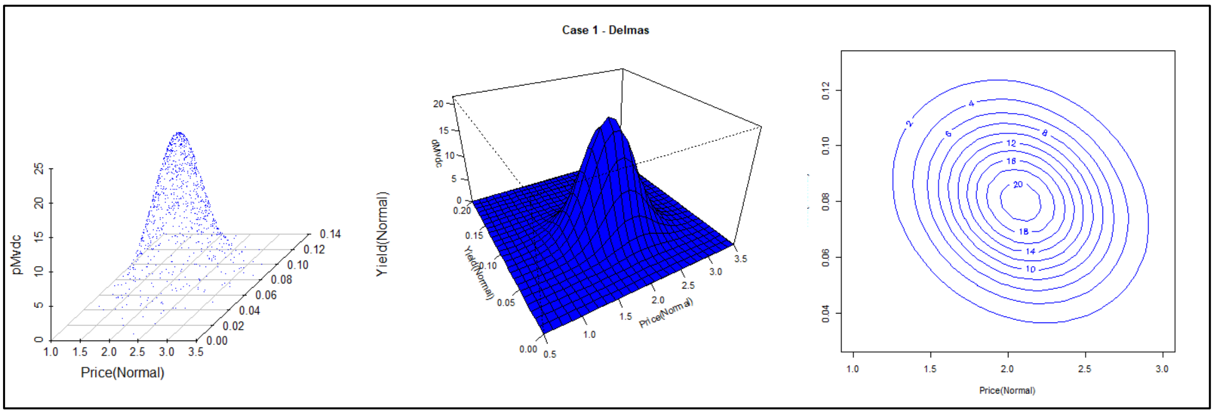


Figure 6. 7: 3D Scatter-, Surface- and Contour-plot from a Gaussian copula

Notes: The figures are from a bivariate random sample of size 1 000 modelled by normal (price) and normal (yield) marginals and simulated from a Gaussian copula with a dependence parameter of τ *= -0.11.*

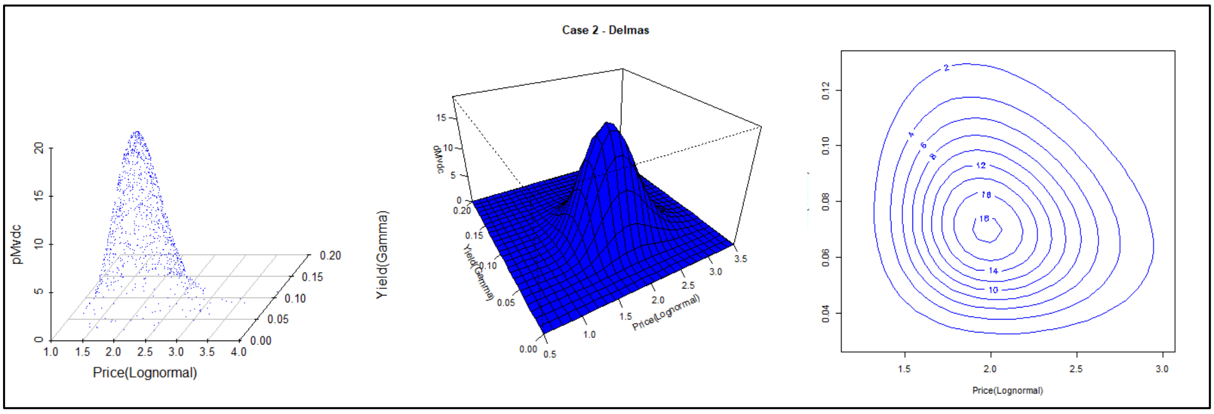


Figure 6. 8: 3D Scatter-, Surface- and Contour-plot from a Gaussian copula

Notes: The figures are from a bivariate random sample of size 1 000 modelled by lognormal (price) and gamma (yield) marginals and simulated from a Gaussian copula with a dependence parameter of τ *= -0.13.*

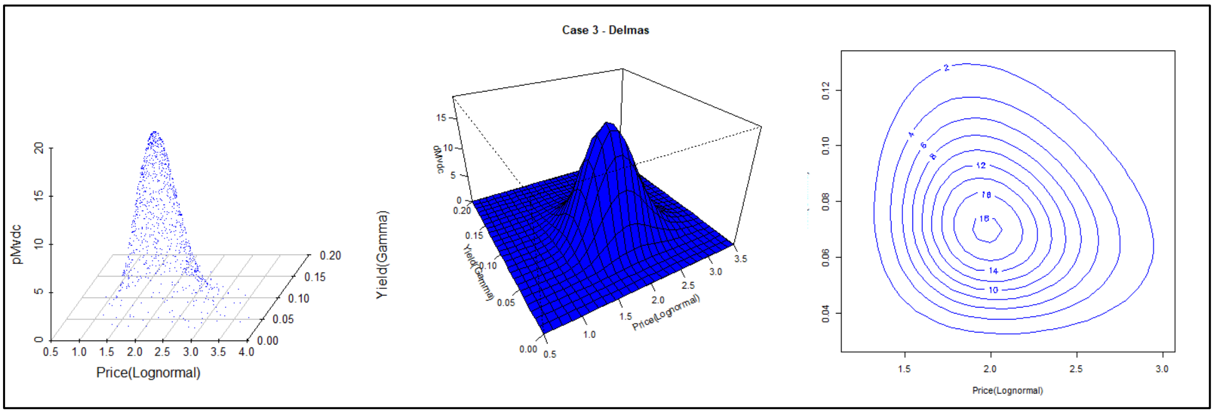


Figure 6. 9: 3D Scatter-, Surface- and Contour-plot from a Gaussian copula

Notes: The figures are from a bivariate random sample of size 1 000 modelled by lognormal (price) and gamma (yield) marginals and simulated from a Gaussian copula with a dependence parameter of τ = -0.13.

Table 6.3 will provide the extent of the implication of the different risk modelling techniques on the expected loss and premium rate results for CRI and MPCI products.

Table 6. 3: Delmas insurance comparison results for Case 1 to Case 3

		55% Coverage		**65% Coverage**		**75% Coverage**	
		Expected loss (premium) (R/t)	**Premium rate**	**Expected loss (premium) (R/t)**	**Premium rate**	**Expected loss (premium) (R/t)**	**Premium rate**
Case 1	**MPCI**	R56.54	4.26%	R113.50	7.23%	R194.59	10.78%
	CRI	R79.04	6.17%	R144.31	9.53%	R231.34	13.24%
Case 2	**MPCI**	R93.20	6.97%	R177.3	11.23%	R286.65	15.73%
	CRI	R109.98	8.75%	R193.54	13.03%	R298.5	17.41%
Case 3	**MPCI**	R86.37	6.46%	R166.46	10.54%	R271.85	14.91%
	CRI	R105.33	8.34%	R186.69	12.50%	R289.89	16.83%

Notes: The copulas and marginals that modelled each case are as follows: Case 1-Gaussian with normal (price) and normal (yield), Case 2-Gaussian copula, normal (price) and lognormal (yield) and Case 3-rotated Clayton 90 degrees copula with lognormal (price) and Weibull (yield).

For Delmas, as the coverage levels increase, so does the expected losses and premium rate realised for both MPCI and CRI, which is expected because the amount of risk being assumed by the insurer is increasing. Contrary to other studies (Meuwissen, Huirne and Skees, 2003; Ahmed and Serra, 2015), CRI insurance expected losses and premium rates are higher than those of MPCI even with the inverse dependence structure present between crop yields and their prices. These findings contradict the natural hedge expectation (covered in section 1.3) that states given an inverse relationship between crop yields and their prices, revenue-based crop insurance products are cheaper than those of yield-based insurance products (Meuwissen, Huirne and Skees, 2003; Tejeda and Goodwin, 2008; Goodwin, 2015). Therefore, this inverse relationship is supposed to be the premise on which CRI functioning permits it to be cheaper than MPCI.

Objective three of this study requires an assessment of the implications of the different risk modelling techniques followed which represent the progression in crop insurance risk modelling techniques. Therefore, this requires the comparison of Cases 2 ($\tau = -0.13$) and 3 ($\tau = -0.13$), to Case 1 ($\tau = -0.11$), the 'benchmark model' as follows:

- Comparing CRI premium rates realised between Case 2 and Case 1, the assessment is as following (refer to Table 6.3 above): for the insurance coverage levels of 55%, 65% and 75%, Case 2 realised premium rates of 8.75%, 13.03% and 17.41%, respectively, which are higher than Case 1's premium rates of 6.17%, 9.53% and 13.24%, respectively. These findings also contradict the natural hedge expectation that indirectly states when comparing CRI policies, the larger the inverse dependence relationship between crop yields and their prices is the lower the premium rates realised. However, in this situation, Case 2 ($\tau = -0.13$) has a larger inverse dependence structure than Case 1 ($\tau = -0.11$) yet the formers premium rates are higher. Because Case 3 is an identical copula model to that of Case 2, the premium rate results also contradict the natural hedge when compared to Case 1.

Delmas experienced opposite results from those of the other districts which is contrary to other studies. It is possible that given the relatively stable rainfall experience in Delmas has contributed to the disconnect in the yield and price relationship in this district which is weak thus eliminating the natural hedge concept. When comparing the dependence relationships established between crop yields and prices, Delmas' is weakly inverse whereas the other districts which are drought-prone realised stronger inverse dependence structures. The copula fitting procedure for Delmas achieved poor GOF parameters when compared to the other districts (refer to section 5.4) possibly indicating the disconnect. The hypothesis test results for the copula model GOF state that models for cases 1 to 3 for Delmas do not fit the data well hence the null hypothesis was rejected at a 1% level of significance (refer to Chapter 5, Table 5.16 on p.118). Furthermore, Delmas yields have not necessarily followed the same trends as the other districts and when they do, the magnitude of yield changes also differ. This means there could be instances where the yields in Delmas are high, but the national yield is low explaining the disconnect in this district's yield and national price relationship.

6.3 CONCLUSION

In this chapter, it was established that alternative models to the normal in the marginals of the distribution do influence the dependence relationship as modelled by the Gaussian copula functions. Because parameter estimates from the marginal distribution models are used in a Monte Carlo simulation, it is important to pick an appropriate model to avoid overestimating or underestimating the risks, which is reflected in the premium rate. When comparing insurance products, Bloemfontein and Vryburg realised lower CRI expected losses and premium rates than those of MPCI. However, comparison results for Delmas contradicted the 'natural hedge' expectation when CRI realised higher expected losses and premium rates than MPCI, even though there was an inverse dependence structure between white maize yields and prices. It has been elaborated that the potential disconnect between Delmas' yields and prices is probably largely due to the higher and more stable rainfall the district enjoys.

CHAPTER 7

CONCLUSIONS

7.1 OVERVIEW

Chapter 1 provided the context of this study's purpose to conduct a viability study of a crop-revenue insurance product for the South African white maize market. It was established that MPCI is a loss-making product in South Africa.

Chapter 2 introduced the concept of insuring crops. The transformation of crop insurance over the past 200 years was discussed, incorporating the current trends in the global industry, and then narrowed down to a South African context.

Chapter 3 introduced the literature on the crop insurance risk modelling techniques used and how these techniques have progressed over the years. The statistical copula approach was introduced as a superior dependency modelling technique. It was established that each copula function has a unique type of dependence that it can model but is flexible enough to be rotated to accommodate other types of dependences. Furthermore, copula functions were established as flexible in accommodating different marginal variable distribution model structures.

Chapter 4 provided the methodology while highlighting the seven key steps to follow in its execution. An in-depth explanation of the different procedures to be implemented was provided, as well as the context is given on why certain process and transformations to the data needed to occur to - fit marginal distributions models, for the copula fitting procedures and use of the Monte Carlo simulations.

Chapter 5 presented the results from executing the methodology up to the copula fitting procedure. A combination of alternative marginal distributions and copula functions were implemented in the crop insurance risk modelling and the results were compared to the Gaussian copula function of normal (price) and normal (yield) distributions - the industry 'benchmark model'. It was found that there is an inverse relationship between the two variables that was strong in Bloemfontein and Vryburg but weak in Delmas. In all three districts, the alternative marginal distribution combination with alternative copulas was a better fitting model than the 'benchmark model'. Given this finding, the benchmark model overestimated the inverse relationship in Bloemfontein and understated it in Vryburg and Delmas, while this relationship is important because it influences the expected loss outcome and ultimately the actuarially fair insurance premium rates achieved.

Chapter 6 presented the findings and implications of the different risk modelling techniques on the comparison results between CRI and MPCI. A Monte Carlo simulation produced variates of expected yield and price values as modelled by the different risk modelling techniques implemented. Simulated values of yield and prices were used to calculate expected losses and actuarially fair premium rates of the two products and these were the two matrices used in comparing CRI and MPCI.

7.2 FINDINGS SUMMARY

To analyse the viability of CRI for the South African white maize market, this study employed three key stages to the analysis. The initial step entailed fitting different distribution models to the yield and price marginal distributions of the districts. For Bloemfontein and Vryburg, the normal model was a better fit to the yield marginal distributions followed by Weibull and beta coming in second and third, respectively. For these two districts, it means the benchmark model will correctly model the yield variables. On the other hand, the lognormal model provided the best fit to the Delmas yield data, followed by the beta and gamma models, respectively. In the case of Delmas, the benchmark model is not the most appropriate model to represent the behaviour of the yield data of this district. The price data was better represented by the normal model over the lognormal, thus the benchmark model is appropriate for modelling South African white maize price data. An appropriate distribution model for these variables is required since the MLE parameter estimates are used in simulating (predictions of) yield and price variates for calculating expected losses and premium rates for the comparison of different crop insurance products.

The second stage of analysis entailed using copula functions to establish the dependence relationships between the yield and price data, which answers to this study's first hypothesis - ***Hypothesis 1:*** *There is an inverse relationship between the* *price and the yield of white maize in SA*. The process of the second stage of analysis was defined by the following three cases:

- **Case 1** - Approach maintains normality in both marginal distribution models and dependence structure when modelling crop insurance risks – the 'benchmark model'.
- **Case 2** – Approach uses a combination of alternative marginal distributions models in modelling crop insurance risks while maintaining normality in the dependence structure (the Gaussian copula).
- **Case 3** – Approach uses a combination of alternative marginal distribution models and different copulas in modelling crop insurance risks.

Therefore, CDF values generated from the different distribution models were utilised in the copula fitting procedure. In all three districts, ***Hypothesis 1*** holds, an inverse relationship was established between white maize yield and the price data that was stronger in Bloemfontein and Vryburg but weak in Delmas. When taking into consideration the rain patterns of the three districts over 10 years starting from the 2010 season (refer to Figures 2.6 to 2.8, p.43 - 44), Delmas exhibits the most and stable rainfall. Therefore, a weak inverse relationship in Delmas was likely attributed to the stable rainfall experienced that produces less variable yields (refer to Figure 3.3, p.55) when compared to the other districts, while the domestic maize prices are influenced by a combination of national and international harvests. Thus, for Delmas the South African maize yield and price linkage would be weakened, hence the weaker inverse relationship experienced. From the three cases, it was established that the best fitting copula model was from Case 3, followed by Case 2 and lastly Case 1 in all the districts.

The third stage of analysis entailed the use of the copula dependence structures in a Monte Carlo simulation to produce variates of white maize yield and prices as depicted by the marginal distribution models. These simulated variates were used to calculate expected losses and premium rates for each district to compare insurance products, recalling that the premium rate is the matrix determining affordability. By comparing the premium rates of MPCI and CRI, the second hypothesis is answered - ***Hypothesis 2****: Premium rates realised for a revenue-based crop insurance product are lower than* *those from a yield-based product.* For Bloemfontein and Vryburg, it was established that at the same insurance coverage levels, CRI expected losses and premium rates are lower than those of MPCI products therefore ***Hypothesis 2*** holds. These findings were expected because of the inverse dependence relationship established between yield and price which is the premise of the natural hedge concept (explained in section 1.2) and therefore also answering objectives one and two of this study that stated:

- Establish whether there is an inverse relationship between SA white maize yield and price data, and if this is indeed the case, assess whether the 'natural hedge' holds.
- Compare the expected losses and premium rates realised from the two insurance schemes (MPCI and CRI) to make a call on the affordability of the products. Affordability is determined by a comparison of premium rates realised by the two products at identical insurance coverage levels.

However, Delmas experienced results different from the other districts which is contrary to other studies because it defied the 'natural hedge' expectation and thus ***Hypothesis 2*** did not hold. Therefore, answering objectives one and two, the 'natural hedge' failed to hold despite having an inverse relationship between price and yield, resulting in CRI being more expensive than MPCI in this case. A plausible reason given as to why Delmas results were different to the other regions is due to a disconnect between the district's maize yield to the national yield, as well as the national price relationship because of the relatively more stable rainfall experienced in that district. Furthermore, the white maize price discovery mechanism for SA is also influenced by international maize supplies. Therefore, an oversupply of maize on the international market will influence SA maize prices thus also contributing to the domestic yield and price relationship disconnect that could further alienate Delmas' yield and national price relationship.

The final stage of analysis speaks to the third hypothesis - ***Hypothesis 3****: Alternative risk modelling approaches in the marginal distributions of the variables as well as in establishing dependence relationships produces a better fitting crop insurance model.* ***Hypothesis 3*** is therefore accompanied by the third objective of this study that states:

- Assess what the effects of different marginal distribution and dependence risk modelling techniques have on the actuarially fair insurance premium rates achieved, separately and in combination (representing the progression in crop insurance risk modelling techniques). This will require comparing Case 2 results to Case 1, the ‘benchmark model’ as well as comparing Case 3 to Case 1.

Therefore, to answer ***Hypothesis 3*** and the third objective of this study, the final stage of analysis compares the CRI premium rate outcomes by case as follows:

Bloemfontein

Comparing Bloemfontein results, Case 2 ($\tau = -0.33$) CRI premiums were lower than those of Case 1 ($\tau = -0.33$), the ‘benchmark model’, even though they have an identical dependence relationship. It must be noted that Case 2 price data was modelled by a lognormal distribution model whereas it was established that the normal distribution was a better fit to that data thus possibly a contributing factor to the difference in premium rates achieved. As already explained, the choice of distribution chosen influences the simulation outcomes. However, the Gaussian copula model did have Case 2 as the better fitting model when compared to Case 1 despite the distributions being a combination of normal (yield) and lognormal (price) models. The premium rates from Case 3 ($\tau = -0.31$) were higher than those from Case 1 and 2, and this was expected since Case 3 had the weaker inverse dependence structure. According to the natural hedge concept, the stronger the inverse relationship between price and yield is the cheaper the CRI insurance premium and vice versa. Taking into consideration that Case 3 is the superior copula fitting model, the rating results suggest the ‘benchmark model’ under-priced CRI policies in Bloemfontein from the data used in this study.

When restricting the premium rate analysis to the MPCI findings, the premiums for Case 3 when compared to Case 1 are lower at 55% coverage but equal at 65% coverage and greater at 75% coverage. This finding on MPCI suggests that at lower coverage levels, the benchmark model is overpricing MPCI premiums while under-pricing at higher coverage levels given the data used in this study and the results of the better fitting model which was Case 3.

Vryburg

Comparing Vryburg results, CRI premium rates for Case 2 ($\tau = -0.30$) were lower than Case 1’s ($\tau = -0.31$), the benchmark model, indirectly contradicting the natural hedge expectation since the latter has a greater inverse dependence structure. It is worth noting that for Vryburg, the normal distribution was chosen as the superior fitting model for both yield and price variables whereas the best fitting Gaussian copula was Case 2 with the distribution models of lognormal (yield) and normal (price). Again, this may be a contributing factor to the difference in premium rates achieved as explained already that choice of distribution chosen influences the simulation outcomes. Comparing Case 3 ($\tau = -0.35$) to Cases 1 and 2, the former achieves lower premium rates, and this is in line with the natural hedge expectations since Case 3 has a larger inverse dependence structure between yields and prices. Taking into consideration that the better fitting copula model realises the lowest premium rates for CRI suggests the 'benchmark model' in this case would overprice CRI policies from the data used in this study.

Restricting the premium rate analysis to MPCI findings for Vryburg, Case 3 realises lower premiums than those of Case 1 suggesting that this product is being overpriced in Vryburg when using the benchmark model based on the data of this study and the results of the better fitting model which was Case 3.

Delmas

Comparing the affordability of CRI products for Delmas, Case 2 ($\tau = -0.13$) premiums rates are relatively more expensive than Case 1's ($\tau = -0.11$), the benchmark model. This finding indirectly contradicts the natural hedge expectation that states the bigger the inverse relationship, is the lower the premium rates realised which is not the case here. Case 3 ($\tau = -0.13$) being an identical copula model to Case 2 also indirectly contradicted the natural hedge expectation when compared to Case 1. Findings from CRI comparisons for Delmas were the opposite of the other districts and contradicted expectation from other studies. A probable cause given was a disconnect in the relationship between Delmas white maize yields and the national prices because of the district's stable rainfall when compared to other top maize producing regions.

Restring the premium rate analysis to MPCI findings for Delmas, Case 3 premiums are higher than those of Case 1, suggesting that comprehensive yield insurance is under-priced by the benchmark model based on the data used in this study and the results of the better fitting model which was Case 3.

In conclusion, this study brings forth indicative results of how CRI and MPCI policies compare on expected losses and premium rates given an inverse relationship between maize yields and prices. When the inverse relationship is strong, it was established that CRI policies are indeed cheaper than those of MPCI in Bloemfontein and Vryburg. However, when the inverse relationship is weak, as was the case in Delmas, the natural hedge expectation did not hold hence MPCI policies were cheaper than CRI. Therefore, this study's outcomes are not definite conclusions on how CRI compares to MPCI but merely providing an approach on how to model revenue-based crop insurance policies.

7.3 RECOMMENDATION FROM FINDINGS

From this study, it was observed that at the same level of insurance coverage, CRI expected losses and premium rates are lower than those of MPCI policies for Bloemfontein and Vryburg. Given these findings, this study recommends that insurers should consider offering CRI in Bloemfontein and Vryburg as an alternative to MPCI. Considering that MPCI is struggling in SA for various reasons, one of them being that it is expensive, it could therefore make a business case for the insures to switch to a relatively cheaper alternative in CRI. This switch, therefore, means farmers will have an alternative to MPCI that is relatively more affordable based on the premium rates achieved. Hopefully, the difference in the cost to purchase crop insurance will incentivise farmers to buy CRI as their first choice in crop insurance risk mitigation tools. If indeed farmers give a positive response to CRI by buying the policies, the insured pool would grow in SA to possibly incorporate the lower risk clients that make an insurers book of business more profitable. The profitability of the product is desirable for the industry because insurers and reinsurers have been de-risking from MPCI due to losses incurred. Furthermore, it is a possibility that the affordability of CRI relative to MPCI could curtail the culture of variable insurance uptake in SA which has been blamed on the high costs of purchasing MPCI policies.

Another reason why this study recommends CRI to the insurers is based on the expected losses realised for Bloemfontein and Vryburg when compared to those of MPCI. It was established that given the same crop insurance coverage levels, the value of expected loses realised from CRI are less than those of MPCI policies. Given this finding, this study argues for CRI on the grounds of being a relatively more sustainable product to offer because insurers and reinsurers would be paying less in indemnity payments when compared to MPCI at identical insurance levels.

Despite the favourable results of this study on the performance of CRI in Bloemfontein and Vryburg, the systemic nature of risks found in agriculture remains. This study, therefore, recommends that the South African government through a Private-Public-Partnership (PPP) should share the responsibility of providing reinsurance for the systemic type of risks, specifically drought. Some examples were provided in Chapter 2 on how PPPs have been utilised in getting the US and Canadian governments involved in supporting reinsurance. The reason for the PPP recommendations is to ensure responsible underwriting from the reinsurers since they will have a stake in the risk assumed, while possibly bringing in private sector knowledge and efficiencies in the underwriting process. This study has recommended the government's involvement in reinsurance for specifically CRI for two reasons. The first is that based on the expected loses realised it is a relatively more sustainable offering when compared to MPCI, symbolising a responsible use of taxpayers' money. Secondly, it was established in Chapters 1 and 2 that systemic risk, especially of drought in SA, are key contributors to the failure of crop insurance products hence why insurers and reinsurers are de-risking from MPCI in SA. Therefore, the government's involvement with CRI could enhance the offering in the market. Furthermore, it is in the best interest of the government to support CRI because the traditional role MPCI has played in the market was in assisting farmers without land collateral (or inadequate collateral) to accessing production finance, while this product is fast disappearing in SA. Also, because the government holds the title deed to the land reform farms means that beneficiaries of these farms do not possess the title deed to use as collateral and without MPCI or CRI policy, it would be difficult for them to access production finance from banks or any other financial institution.

Contrary to findings in Bloemfontein, Vryburg and other studies, the natural hedge expectation was not realised in Delmas even with the inverse dependence structure between white maize yields and prices. Therefore, this study does not encourage the insurers to offer CRI as an alternative to MPCI in Delmas because if they did, they would pay more in indemnity payments while farmers would be charged a relatively higher premium rate. Given these findings, this study recommends that insurers incorporate local modelling techniques for different areas to capture much more accurate dependence relationships between crop prices and yields. This means going further than what this study's district-level data permits because this research was restricted in this regard and limited to a small dataset. Furthermore, insurers and reinsurers when modelling their risk should compare alternatives and not always rely on benchmark models because findings from this research have shown that the benchmark model was not the best.

To finalise the recommendations section, CRI performs well in Bloemfontein and Vryburg whereas MPCI is a better product in Delmas. Therefore, insurers could use a mix of these two products in SA to achieve a profitable portfolio of crop insurance products as their competitive advantage.

7.4 STUDY LIMITATIONS AND RECOMMENDATIONS

This study focussed primarily on assessing the viability of CRI based on the expected losses and premium rates achieved when compared to those of MPCI. The argument from the literature was that, given an inverse relationship between crop yields and their prices, revenue-based crop insurance products are expected to be cheaper than those of yield-based policies. However, from the crop insurance rating results of this study, Delmas defied this expectation. What stood out from Delmas compared to the other districts was its weak dependence relationship as modelled by the copulas. This research recommends that future studies investigate what the inverse dependence structure threshold is that warrants for revenue-based crop insurance expected losses and premium rates to be lower than those of yield-based policies. Also, this study was limited to a short time-series data set of fourteen points which is restrictive as a larger series is preferable to capture a longer trend. The data was limited due to this study's need for a low level of yield aggregation which was available but limited at a district level. Therefore, the findings should be interpreted while taking into consideration the short time-series utilised.

From discussions in section 2.4.3.2, there are welfare gains discussed linked to the introduction of CRI schemes. This study, therefore, recommends that future studies quantify the welfare changes from the introduction of CRI in SA. By quantifying welfare gains, the crop insurance sector could present a stronger case for government support in the industry.

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APPENDICES

Appendix A Comparison of Revenue Insurance Products

A.1 List of Tables

Table A 1: Comparison of the initial revenue-based insurance products in the US

Feature	Crop Revenue Coverage	Income protection	Revenue assurance
Unit organisation	Basic, optional, or enterprise units	Enterprise unit (all acreage of the insured crop in the county in which the insured has interest)	Basic, optional, enterprise, or Whole Farm units (all RA insurable crop acreage in the county in which the insured has an interest)
Basic for insurance guarantee	Higher of 1) APH yield * Base 2) APH yield * Harvest price Insurance guarantee increases when the Harvest price exceeds the Base price	APH yield * Projected price	APH yield * Projected Harvest price Harvest price option increases the guarantee when the Harvest price exceeds the Projected Harvest price
Maximum protection unit price increase	Maize US$1.50/bushel, Cotton US$0.70/lb. Grain sorghum US$1.50/bushel Rice US$0.05/lb. Soybeans US$3.00/bushel Wheat US$2.00/bushel	Not applicable	Not applicable
Reference commodity price	For corn, cotton, rice, soybeans, and wheat, 100% of the selected commodity contract traded on a commodity futures exchange.	For maize, cotton, soybeans, and wheat, 100% of selected commodity contract traded on a commodity futures exchange. Grain sorghum is 90% of the corn futures.	100% of selected commodity contract traded on a commodity futures exchange

	Grain sorghum is 95% of the maize futures.	Barley is 85% of the corn futures.	
Eligibility for high-risk land	High-risk land is eligible for coverage	High-risk land is not eligible for coverage	High-risk land is eligible for coverage
Coverage levels	50-75% in 5% increments, except 50-85% where 85% APH is available. CAT is not available	50-75%, except 50-85% where 85% APH is available CAT is 27.5%	65-75%, except 65-85% for Whole farm and Enterprise units. CAT is not available
Hail and fire exclusion	Not available	Not available	Not available
Insured crops	Maize, cotton, grain, sorghum, rice, soybeans, and wheat	Barley, maize, cotton, grain sorghum, soybeans, and wheat	Maize, feed barley, rapeseed, canola, soybeans, sunflowers, and spring wheat
Premium rating	APH base rate plus low price factor plus high price factor plus CRC factor	New rating model incorporating yield and price variability	New rating model incorporating yield and price variability and yield and price correlation

Appendix B Distribution Models

B.1 Other – Uniform Distribution

The probability density function of a uniform distribution is,

$$f(x) = \frac{1}{b-a}\text{, for } a \le x \le b \tag{39}$$

where a = the lowest value of x and b=the highest value of x.

The theoretical mean is given by,

$$\mu = \frac{a+b}{2} \tag{40}$$

and standard deviation by,

$$\sigma = \sqrt{\frac{(b-a)^2}{12}}\,. \tag{41}$$

The cumulative distribution function (CDF) of a uniform distribution is given by,

$$F(x) = \int_a^x \frac{1}{b-a}\,dx = \frac{x-a}{b-a},\ \text{ for } a \le x \le b. \tag{42}$$

Appendix C Referenced Results - Special Functions

C.1 Special Functions

C.1.1 Results 1 - Price Distributions

C.1.1.1 CDF of normal distribution

The cumulative distribution function (CDF) of Normal distribution is,

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\Pi}\sigma} \exp\left[-\frac{(t-\mu)^2}{-2\sigma^2}\right] dt, \sigma > 0 \tag{43}$$

Mean = μ = 1.990

Standard deviation = σ = 0.325

C.1.2 Results 2 – Yield Distributions

C.1.2.1 Normal distribution

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\Pi}\sigma} \exp\left[-\frac{(t-\mu)^2}{-2\sigma^2}\right] dt, \sigma > 0 \tag{44}$$

mean = μ = 2.00

standard deviation = σ = 0.166

C.1.2.2 Beta distribution

$$P(x \leq t) = \int_{0}^{t} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} dt \tag{45}$$

Expected value = $E[X] = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$ = 132.6677

Variance = $Var[X]$ = 2785.9795

C.1.2.3 Weibull distribution

$$F(x) = 1 - \exp\left[-\left(\frac{x-\lambda}{\alpha}\right)^{\beta}\right], \quad x \geq \lambda, \ \alpha > 0, \ \beta > 0 \tag{46}$$

$$\text{mean} = \alpha\Gamma\left(1+\frac{1}{\beta}\right) + \lambda = 15.82$$

$$\text{variance} = \alpha^2\left(\Gamma\left(1+\frac{2}{\beta}\right) + \Gamma^2\left(1+\frac{1}{\beta}\right)\right) = 2.077$$

C.1.3 Copula Fitting Results

C.1.3.1 Bloemfontein

Table C 1: Copula fitting with normal (price) and beta (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**loglikelihood**	**AIC**	**BIC**
normal	beta	Gaussian	-0.42	-0.28	1.27	-0.54	0.02
normal	beta	t-copula	-0.42	-0.27	1.13	1.73	2.86
normal	beta	Frank	-2.92	-0.30	1.48	-0.95	-0.39
normal	beta	rotated Gumbel 90 degrees	-1.43	-0.30	1.32	-0.65	-0.08
normal	beta	rotated Clayton 270 degrees	-0.9	-0.31	1.51	-1.02	-0.45
normal	beta	rotated Joe 90 degrees	-1.74	-0.29	1.31	-0.62	-0.06

Table C 2: Copula fitting with normal (price) and normal (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**loglikelihood**	**AIC**	**BIC**
normal	normal	Gaussian	-0.49	-0.33	1.46	-0.92	-0.36
normal	normal	t-copula	-0.48	-0.32	1.36	1.28	2.41
normal	normal	Frank	-2.94	-0.30	1.50	-1.00	-0.44
normal	normal	rotated Gumbel 90 degrees	-1.41	-0.29	1.14	-0.27	0.29
normal	normal	rotated Clayton 270 degrees	-0.75	-0.27	1.14	-0.27	0.29
normal	normal	rotated Joe 90 degrees	-1.59	-0.25	0.86	0.27	0.84

Table C 3: Copula fitting with normal (price) and Weibull (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	Weibull	Gaussian	-0.15	-0.10	0.73	0.54	1.10
normal	Weibull	t-copula	-0.16	-0.10	0.64	2.72	3.85
normal	Weibull	Frank	-1.31	-0.14	0.67	0.65	1.22
normal	Weibull	rotated Gumbel 90 degrees	-1.03	-0.02	0.09	1.82	2.38
normal	Weibull	rotated Clayton 270 degrees	-0.08	-0.04	0.54	0.91	1.48
normal	Weibull	rotated Joe 90 degrees	-1.01	-0.01	0.02	1.96	2.53

Table C 4: Copula fitting with normal (price) and gamma (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	gamma	Gaussian	-0.42	-0.28	1.26	-0.52	0.05
normal	gamma	t-copula	-0.41	-0.27	1.12	1.76	2.89
normal	gamma	Frank	-2.92	-0.3	1.46	-0.92	-0.36
normal	gamma	rotated Gumbel 90 degrees	-1.42	-0.3	1.31	-0.62	-0.06
normal	gamma	rotated Clayton 270 degrees	-0.9	-0.31	1.51	-1.01	-0.45
normal	gamma	rotated Joe 90 degrees	-1.74	-0.29	1.30	-0.61	-0.04

Table C 5: Copula fitting with normal (price) and lognormal (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	lognormal	Gaussian	-0.4	-0.26	1.11	-0.21	0.35
normal	lognormal	t-copula	-0.39	-0.26	0.95	2.1	3.23
normal	lognormal	Frank	-2.82	-0.29	1.30	-0.6	-0.03
normal	lognormal	rotated Gumbel9 0	-1.4	0.29	1.13	-0.26	0.31
normal	lognormal	rotated Clayton 270 degrees	-0.88	-0.31	1.37	-0.74	-0.17
normal	lognormal	rotated Joe 90 degrees	-1.71	-0.28	1.12	-0.23	0.33

Table C 6: Copula fitting with lognormal (price) and beta (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	beta	Gaussian	-0.43	-0.28	1.32	-0.64	-0.07
lognormal	beta	t-copula	-0.42	-0.28	1.18	1.63	2.76
lognormal	beta	Frank	-2.94	-0.3	1.49	-0.97	-0.41
lognormal	beta	rotated Gumbel 90 degrees	-1.42	-0.3	1.31	-0.61	-0.05
lognormal	beta	rotated Clayton 270 degrees	-0.85	-0.3	1.43	-0.87	-0.3
lognormal	beta	rotated Joe 90 degrees	-1.7	-0.28	1.22	-0.44	0.12

Table C 7: Copula fitting with lognormal (price) and normal (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	normal	Gaussian	-0.5	-0.33	1.53	-1.06	-0.5
lognormal	normal	t-copula	-0.49	-0.33	1.44	1.13	2.26
lognormal	normal	Frank	-3.04	-0.31	1.58	-1.15	-0.59
lognormal	normal	rotated Gumbel 90 degrees	-1.41	-0.29	1.16	-0.31	0.25
lognormal	normal	rotated Clayton 270 degrees	-0.73	-0.27	1.12	-0.23	0.33
lognormal	normal	rotated Joe 90 degrees	-1.57	-0.24	0.84	0.33	0.89

Table C 8: Copula fitting with lognormal (price) and Weibull (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residual s**	**Copula**	**Copula paramet er**	**Kendall 's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	Weibull	Gaussian	-0.15	-0.09	0.68	0.64	1.21
lognormal	Weibull	t-copula	-0.15	-0.1	0.62	2.75	3.88
lognormal	Weibull	Frank	-1.2	-0.13	0.57	0.87	1.43
lognormal	Weibull	rotated Gumbel 90 degrees	-1.03	-0.03	0.12	1.76	2.33
lognormal	Weibull	rotated Clayton 270 degrees	-0.08	-0.04	0.52	0.95	1.52
lognormal	Weibull	rotated Joe 90 degrees	-1.02		0.04	1.91	2.48

Table C 9: Copula fitting with lognormal (price) and gamma (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	gamma	Gaussian	-0.43	-0.28	1.31	-0.61	-0.05
lognormal	gamma	t-copula	-0.42	-0.27	1.17	1.67	2.8
lognormal	gamma	Frank	-2.93	-0.3	1.47	-0.94	-0.37
lognormal	gamma	rotated Gumbel 90 degrees	-1.42	-0.29	1.29	-0.58	-0.02
lognormal	gamma	rotated Clayton 270 degrees	-0.86	-0.30	1.43	-0.86	-0.29
lognormal	gamma	rotated Joe 90 degrees	-1.70	-0.28	1.21	-0.42	0.14

Table C 10: Copula fitting with lognormal (price) and lonormal (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	lognormal	Gaussian	-0.41	-0.27	1.15	-0.3	0.26
lognormal	lognormal	t-copula	-0.40	-0.26	0.99	2.02	3.15
lognormal	lognormal	Frank	-2.81	-0.29	1.29	-0.58	-0.02
lognormal	lognormal	rotated Gumbel 90 degrees	-1.39	-0.28	1.11	-0.21	0.35
lognormal	lognormal	rotated Clayton 270 degrees	-0.83	-0.29	1.29	-0.57	-0.01
lognormal	lognormal	rotated Joe 90 degrees	-1.67	-0.27	1.03	-0.05	0.51

C.1.3.2 Vryburg

Table C 11: Copula fitting with normal (price) and beta (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	beta	Gaussian	-0.45	-0.30	1.49	-0.99	-0.42
normal	beta	t-copula	-0.44	-0.29	1.38	1.24	2.37
normal	beta	frank	-2.42	-0.25	1.30	-0.61	-0.04
normal	beta	rotated Gumbel 270 degrees	-1.38	-0.27	1.32	-0.63	-0.07
normal	beta	rotated Clayton 90 degrees	-0.80	-0.29	1.58	-1.16	-0.60
normal	beta	rotated Joe 270 degrees	-1.62	-0.26	1.27	-0.54	0.03

Table C 12: Copula fitting with lognormal (price) and lognormal (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	lognormal	Gaussian	-0.46	-0.30	1.54	-1.08	-0.51
normal	lognormal	t-copula	-0.45	-0.29	1.42	1.16	2.29
normal	lognormal	frank	-2.43	-0.26	1.33	-0.66	-0.09
normal	lognormal	rotated Gumbel 270 degrees	-1.35	-0.26	1.23	-0.47	0.10
normal	lognormal	rotated Clayton 90 degrees	-0.72	-0.27	1.48	-0.95	-0.39
normal	lognormal	rotated Joe 270 degrees	-1.54	-0.23	1.11	-0.21	0.35

Table C 13: Copula fitting with normal (price) and gamma (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	gamma	Gaussian	-0.46	-0.30	1.50	-1.00	-0.43
normal	gamma	t-copula	-0.44	-0.29	1.39	1.23	2.36
normal	gamma	frank	-2.42	-0.25	1.31	-0.61	-0.05
normal	gamma	rotated Gumbel 270 degrees	-1.37	-0.27	1.31	-0.61	-0.05
normal	gamma	rotated Clayton 90 degrees	-0.79	-0.28	1.57	-1.14	-0.57
normal	gamma	rotated Joe 270 degrees	-1.61	-0.25	1.25	-0.50	0.07

Table C 14: Copula fitting with normal (price) and normal (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	normal	Gaussian	-0.46	-0.31	1.34	-0.69	-0.12
normal	normal	t-copula	-0.45	-0.30	1.26	1.47	2.60
normal	normal	frank	-2.36	-0.25	1.17	-0.35	0.22
normal	normal	rotated Gumbel 90 degrees	-1.43	-0.30	1.42	-0.84	-0.27
normal	normal	rotated Clayton 90 degrees	-0.95	-0.32	1.78	-1.56	-0.99
normal	normal	rotated Joe 270 degrees	-1.77	-0.30	1.59	-1.19	-0.62

Table C 15: Copula fitting with normal (price) and Weibull (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	Weibull	Gaussian	-0.45	-0.30	1.34	-0.69	-0.12
normal	Weibull	t-copula	-0.44	-0.29	1.24	1.52	2.65
normal	Weibull	frank	-2.41	-0.25	1.24	-0.42	0.14
normal	Weibull	rotated Gumbel 270 degrees	-1.44	-0.31	1.47	-0.93	-0.37
normal	Weibull	rotated Clayton 90 degrees	-1.03	-0.34	1.82	-1.65	-1.08
normal	Weibull	rotated Joe 270 degrees	-1.85	-0.32	1.69	-1.37	-0.81

Table C 16: Copula fitting with lognormal (price) and beta (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	beta	Gaussian	-0.44	-0.29	1.40	-0.81	-0.24
lognormal	beta	t-copula	-0.43	-0.28	1.30	1.41	2.54
lognormal	beta	frank	-2.38	-0.25	1.27	-0.54	0.03
lognormal	beta	rotated Gumbel 270 degrees	-1.38	-0.27	1.31	-0.62	-0.05
lognormal	beta	rotated Clayton 90 degrees	-0.82	-0.29	1.63	-1.25	-0.69
lognormal	beta	rotated Joe 270 degrees	-1.64	-0.26	1.33	-0.66	-0.09

Table C 17: Copula fitting with lognormal (price) and lognormal (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	lognormal	Gaussian	-0.45	-0.30	1.46	-0.91	-0.35
lognormal	lognormal	t-copula	-0.43	-0.29	1.34	1.32	2.45
lognormal	lognormal	frank	-2.39	-0.25	1.29	-0.59	-0.02
lognormal	lognormal	rotated Gumbel 270 degrees	-1.35	-0.26	1.23	-0.45	0.11
lognormal	lognormal	rotated Clayton 90 degrees	-0.74	-0.27	1.51	-1.03	-0.46
lognormal	lognormal	rotated Joe 90 degrees	-1.56	-0.24	1.15	-0.31	0.26

Table C 18: Copula fitting with lognormal (price) and gamma (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	gamma	Gaussian	-0.44	-0.29	1.41	-0.82	-0.26
lognormal	gamma	t-copula	-0.43	-0.28	1.30	1.40	2.53
lognormal	gamma	frank	-2.38	-0.25	1.27	-0.54	0.02
lognormal	gamma	rotated Gumbel 270 degrees	-1.37	-0.27	1.30	-0.60	-0.03
lognormal	gamma	rotated Clayton 90 degrees	-0.81	-0.29	1.61	-1.23	-0.66
lognormal	gamma	rotated Joe 270 degrees	-1.63	-0.26	1.30	-0.61	-0.04

Table C 19: Copula fitting with lognormal (price) and normal (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	normal	Gaussian	-0.45	-0.30	1.28	-0.57	0.00
lognormal	normal	t-copula	-0.44	-0.29	1.21	1.59	2.72
lognormal	normal	frank	-2.35	-0.25	1.17	-0.33	0.23
lognormal	normal	rotated Gumbel 270 degrees	-1.43	-0.30	1.44	-0.88	-0.32
lognormal	normal	rotated Clayton 90 degrees	-0.99	-0.33	1.87	-1.74	-1.18
lognormal	normal	rotated Joe 270 degrees	-1.80	-0.31	1.70	-1.40	-0.83

Table C 20: Copula fitting with lognormal (price) and Weibull (yield) marginals

Distributions		Dependence measures			GOF statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	Weibull	Gaussian	-0.43	-0.29	1.25	-0.49	0.07
lognormal	Weibull	t-copula	-0.42	-0.28	1.15	1.70	2.83
lognormal	Weibull	frank	-2.36	-0.25	1.18	-0.35	0.21
lognormal	Weibull	rotated Gumbel 270 degrees	-1.44	-0.31	1.45	-0.91	-0.34
lognormal	Weibull	rotated Clayton 90 degrees	-1.06	-0.35	1.89	-1.78	-1.22
lognormal	Weibull	rotated Joe 270 degrees	-1.88	-0.33	1.77	-1.54	-0.98

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Table C 21: Copula fitting with normal (price) and beta (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	beta	gaussian	-0.19	-0.12	0.25	1.51	2.07
normal	beta	t-copula	-0.18	-0.11	0.13	3.74	4.87
normal	beta	frank	-0.98	-0.11	0.16	1.67	2.24
normal	beta	rotated Gumbel 90 degrees	-1.14	-0.12	0.19	1.61	2.18
normal	beta	rotated Clayton 270 degrees	-0.29	-0.13	0.17	1.66	2.23
normal	beta	rotated Joe 270 degrees	-1.23	-0.12	0.17	1.67	2.23

Table C 22:Copula fitting with normal (price) and normal (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	normal	gaussian	-0.18	-0.11	0.22	1.57	2.13
normal	normal	t-copula	-0.17	-0.11	0.12	3.77	4.90
normal	normal	frank	-1.02	-0.11	0.20	1.60	2.16
normal	normal	rotated Gumbel 90 degrees	-1.10	-0.09	0.11	1.78	2.35
normal	normal	rotated Clayton 90 degrees	-0.12	-0.06	0.12	1.76	2.33
normal	normal	rotated Joe 90 degrees	-1.11	-0.06	0.05	1.91	2.47

Table C 23: Copula fitting with normal (price) and Weibull (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	Weibull	gaussian	-0.16	-0.10	0.17	1.65	2.22
normal	Weibull	t-copula	-0.16	-0.10	0.10	3.79	4.92
normal	Weibull	frank	-1.04	-0.11	0.20	1.60	2.17
normal	Weibull	rotated Gumbel 90 degrees	-1.09	-0.09	0.10	1.79	2.36
normal	Weibull	rotated Clayton 90 degrees	-0.14	-0.06	0.12	1.77	2.33
normal	Weibull	rotated Joe 90 degrees	1.10	-0.06	0.04	1.92	2.48

Table C 24: Copula fitting with normal (price) and gamma (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	gamma	gaussian	-0.20	-0.13	0.25	1.50	2.07
normal	gamma	t-copula	-0.18	-0.11	0.13	3.73	4.86
normal	gamma	frank	-0.98	-0.11	0.16	1.68	2.24
normal	gamma	rotated Gumbel 90 degrees	-1.14	-0.12	0.20	1.61	2.17
normal	gamma	rotated Clayton 270 degrees	-0.30	-0.13	0.18	1.65	2.21
normal	gamma	rotated Joe 90 degrees	-1.24	-0.12	0.17	1.65	2.22

Table C 25: Copula fitting with normal (price) and lognormal (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
normal	lognormal	gaussian	-0.06	-0.04	0.18	1.64	2.21
normal	lognormal	t-copula	-0.04	-0.04	-0.48	4.97	6.09
normal	lognormal	frank	-0.33	-0.04	0.03	1.94	2.51
normal	lognormal	rotated Gumbel 90 degrees	-1.00	0.00	0.00	2.00	2.57
normal	lognormal	rotated Clayton 90 degrees	-0.02	-0.01	0.08	1.84	2.41
normal	lognormal	rotated Joe 90 degrees	-1.00	0.00	0.00	2.00	2.57

Table C 26: Copula fitting with lognormal (price) and beta (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield resid uals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	beta	gaussian	-0.21	-0.13	0.29	1.42	1.99
lognormal	beta	t-copula	-0.19	-0.12	0.17	3.66	4.79
lognormal	beta	frank	-1.07	-0.12	0.19	1.61	2.18
lognormal	beta	rotated Gumbel 90 degrees	-1.13	-0.12	0.18	1.64	2.20
lognormal	beta	rotated Clayton 270 degrees	-0.27	-0.12	0.18	1.65	2.21
lognormal	beta	rotated Joe 90 degrees	-1.19	-0.10	0.13	1.75	2.31

Table C 27: Copula fitting with lognormal (price) and norma (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	normal	gaussian	-0.18	-0.12	0.24	1.53	2.09
lognormal	normal	t-copula	-0.18	-0.11	0.13	3.73	4.86
lognormal	normal	frank	-1.07	-0.12	0.22	1.57	2.13
lognormal	normal	rotated Gumbel 90 degrees	-1.09	-0.08	0.09	1.82	2.38
lognormal	normal	rotated Clayton 90 degrees	-0.13	-0.06	0.13	1.74	2.31
lognormal	normal	rotated Joe 90 degrees	-1.08	-0.04	0.03	1.95	2.51

Table C 28: Copula fitting with lognormal (price) and Weibull (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	Weibull	gaussian	-0.18	-0.11	0.21	1.58	2.15
lognormal	Weibull	t-copula	-0.17	-0.11	0.14	3.72	4.85
lognormal	Weibull	frank	-1.12	-0.12	0.23	1.55	2.11
lognormal	Weibull	rotated Gumbel 90 degrees	-1.09	-0.08	0.10	1.79	2.36
lognormal	Weibull	rotated Clayton 90 degrees	-0.166	-0.07	0.14	1.71	2.28
lognormal	Weibull	rotated Joe 90 degrees	-1.09	-0.05	0.04	1.93	2.49

Table C 29: Copula fitting with lognormal (price) and gamma (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	gamma	gaussian	-0.13	-0.13	0.29	1.42	1.98
lognormal	gamma	t-copula	-0.19	-0.12	0.17	3.66	4.79
lognormal	gamma	frank	-1.07	-0.12	0.19	1.62	2.18
lognormal	gamma	rotated Gumbel 90 degrees	-1.13	-0.12	0.18	1.63	2.20
lognormal	gamma	rotated Clayton 270 degrees	-0.27	-0.12	0.18	1.63	2.20
lognormal	gamma	rotated Joe 270 degrees	-1.19	-0.10	0.13	1.74	2.30

Table C 30: Copula fitting with lognormal (price) and lognormal (yield) marginals

Distributions		Dependence measure			Fit Statistics		
Price changes	**Yield residuals**	**Copula**	**Copula parameter**	**Kendall's Tau**	**Loglike lihood**	**AIC**	**BIC**
lognormal	lognormal	gaussian	-0.07	-0.04	0.24	1.52	2.09
lognormal	lognormal	t-copula	-0.07	-0.04	-0.37	4.74	5.87
lognormal	lognormal	frank	-0.44	-0.05	0.05	1.90	2.46
lognormal	lognormal	rotated Gumbel 90 degrees	-1	0.00	0	2	2.57
lognormal	lognormal	rotated Clayton 90 degrees	-0.02	-0.01	0.08	1.85	2.41
lognormal	lognormal	rotated Joe 270 degrees	-1.00	0.00	0.00	0.00	0.00

C.1.4 Theory of Goodness-of-fit

C.1.4.1 AIC and BIC Criterion

Generally, the rules of thumb are, the bigger the log-likelihood value and the smaller the AIC or BIC value, is the better the fit of a model. The AIC and BIC is defined as follows (Fang et al. 2014):

$$AIC = -2\,(maximum \log likelihood) + 2\,(number\ of\ free\ parameters)$$

$$SIC = -2(maximum\ \log likelihood$$
$$+ \ln(number\ of\ obseravation)\,(number\ of\ free\ parameters)$$

The AIC and BIC approach despite not being able to conduct a formal goodness-of-fit test is favourable because you can use the same dataset to compare different copula methods thus lending itself to ease of computation. The downside to this approach is that, if the true copula is not part of the pool, the one with least AIC/BIC value is chosen which would result in the incorrect copula method being implemented possibly.

C.1.4.2 Kolmogorov-Smirnov Goodness-of-Fit Test (*KS* test)

This is a 'distribution free' goodness of fit test (Massey, 1951) which means that one can test how well their observed data fits to a hypothetical distribution. The *KS* statistic therefore implies how well a dataset fits to a specified curve.

The *KS* test hypothesis tests are given as:

H_0: The data follows a specified distribution

H_a: The data does not follow a specified distribution

The KS test statistic is defined by Massey (1951) as follows:

$$d = maximum|F_0(x) - S_N(x)|, \quad (47)$$

where $F_0(x)$ is the assumed known specified population cumulative distribution and $S_N(x)$ is the observed cumulative step-function of a sample.

The rejection rules use the significance $\propto$ level and critical values from tables provided. The null hypothesis is rejected if *d* is greater than critical value provided in the tables.

Because the *KS* test statistic, *d* is measuring the distance between two curves, while the smaller the gap means the observed data fits well to a specified distribution is how this research manages to make a decision on which is the better fitting distribution to the yield and price data.