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A comparative analysis of generic models to an individualised approach in portfolio selection

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Executive Summary

The portfolio selection problem has been widely understood and practised for millennia, but it was first formalised by Markowitz (1952) with the proposition of a risk-reward trade-off model. Since then, portfolio selection models have continued to evolve. The general consensus is that three objectives, to maximise the uncertain Rate Of Return (ROR), to maximise liquidity and to minimise risk, should be considered.

It was found that there are opportunities for improvement within the existing portfolio selection models. This can be attributed to three gaps within the existing models. Generally, existing portfolio selection models are generic, especially in how they incorporate risk, they generally do not incorporate Socially Responsible Investing (SRI), and generally they are considered to be unvalidated. This dissertation set out to address these gaps and compare the real-world performance of generic and individualised portfolio selection models.

A new method of accounting for risk was developed that consolidates the portfolio's market risk with the investor's financial risk tolerance. Two portfolio selection models that incorporate individualised risk and SRI objectives were developed. These two models were called the risk-adjusted and social models, respectively. These individualised models were compared to an existing generic Markowitz model.

These models were formulated using stochastic goal programming. A sample of 208 companies JSE Limited companies was selected and two independent datasets were extracted for these companies, a training (2010/01/01 – 2016/12/31) and testing (2017/01/01 – 2019/12/31) dataset. The models solved were in `LINGO` using the training dataset and tested on an unknown future by using the testing dataset.

It was found that in the training period, the individualised risk-adjusted model outperformed the generic Markowitz model and the individualised social model. Furthermore, it was found that it would not be beneficial for an investor to be Socially Responsible (SR). Nevertheless, investors invest to achieve their ROR and SRI goals in the future, not in the present. Thus, it was necessary to evaluate how the portfolios selected by all three models would have performed in an unknown future.

In the testing period, both the generic Markowitz model and the risk-adjusted models had dismal performance and were significantly outperformed by the South African market and unit trusts. Thus, these models are not useful or suitable for their intended purpose. On the contrary, the social model portfolios achieved high ROR values, were SR, and outperformed the market and the unit trusts. Thus, this model was useful and suitable for its intended purpose. The individualised social model significantly outperformed the other two models. Thus, it was concluded that an individualised approach that incorporates SRI outperforms a generic portfolio selection approach.

Given its unparalleled performance and novel model formulation, the social model makes a contribution to the field of portfolio selection. This dissertation also highlighted the importance of testing portfolio selection models on an unknown future and

demonstrated the potentially horrific consequences of neglecting this analysis.

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List of acronyms

ADP Absolute Dissimilarity Percentage

CCP Chance-Constrained Programming

CVaR Conditional Value-at-Risk

ESG Environmental, Social and Governance

EV Expected Value

ILO International Labour Organisation

MOO Multi-Objective Optimisation

NSGA Nondominated Sorting Genetic Algorithm

ROR Rate Of Return

RTS Risk Tolerance Score

SARS South African Revenue Service

SR Socially Responsible

SRI Socially Responsible Investing

SRTS Self-assessed Risk Tolerance Score

TPP The Potential Portfolio

UN United Nations

VaR Value-at-Risk

Chapter 1

Introduction

Investing is the act of committing money or capital to an endeavour with the expectation of gaining an additional income or profit (Gitman et al., 2015). The practice of investing has existed for millennia with numerous methods, principles and practices for effective investing developing and evolving over time. Investors invest to achieve long-term financial goals such as building a reserve fund for retirement, repaying a mortgage early, paying university fees for their children and many others. Thus, investment is a vehicle for the achievement of one's future financial goals.

Bartering has been an agent of trade for millennia, but the earliest recorded occurrence of investing appeared in Ancient Mesopotamia around 1700 BC when the Code of Hammurabi was written and implemented. Although this is the first recorded formal use of investing, investment, as understood in today's modern society, was only established in 1602 AD when the first stock exchange, the Amsterdam Stock Exchange, opened. The establishment of the formal stock exchange allowed potential investors and businesses to connect and offered liquidity, published value, broadcast availability and lowered transaction costs. This allowed for more accessible, cheaper and standardised investing (Petram et al., 2011).

There are many different forms of investments, such as shares, bonds, investment funds, savings, options, insurance, and others. Each of these investments carry their own return, liquidity, and risk. Investments can be classified as either good or bad, where a good investment is one that will make a profit, and a bad investment is one that will result in the investor losing their money. Despite the clear difference between good and bad investments, knowing which investments are good and which investments are bad is not straightforward. An investment may appear to be a good investment when investing, but over time it is proven to be a bad investment and vice versa. This is because share prices reflect how valuable the market, and thus people, perceive a company or asset to be. These perceptions change over time, and thus cause share prices to change, which causes day-to-day volatility and uncertainty within the market. This uncertainty contributes to the risk associated with investing. Should investors make the wrong investment decisions; they will lose the money that they have invested.

To counteract investment risk and maximise the gain received, investors started investing in a collection of various investments as opposed to investing in only one. This collection of investments is known as an *investment portfolio*. Portfolio investing is based on the principle that having a combination of investments will lower the overall risk to the investor, and thus result in higher returns.

1.1 Opportunities for improvement in portfolio selection models

Deciding which assets to include in an investment portfolio is a problem as old as the practice of portfolio investing itself. The process of deciding which investments to include in an investment portfolio is known as portfolio selection. In addition to selecting the investment portfolio, portfolio selection involves deciding what amount of money should be invested in each of the selected investments. This process involves constantly balancing the potential gain that the investment portfolio can achieve and the risk that the investment portfolio will experience a loss. Over time, this problem has come to be known as the portfolio selection problem. The portfolio selection problem has been widely understood and practised for millennia, but it was first formalised by Markowitz (1952) in the article titled "Portfolio selection" which was published in the Journal of Finance. With this publication and the proposition of a risk-reward trade-off model, Markowitz (1952) revolutionised the investment landscape and ushered portfolio selection into the modern age. This theory has since become the cornerstone of modern portfolio selection and is standard practice among financial professionals and researchers. The traditional risk-reward model has continued to evolve with the development of investment knowledge and technological advancements. Today, multiple objectives such as Rate Of Return (ROR), risk, market liquidity, the number of securities in a portfolio, maximum investment proportion weighting, and Socially Responsible Investing (SRI) considerations are considered concurrently to select an investment portfolio (Steuer et al., 2007).

Despite being a well-established field, it was perceived that generally the majority of portfolio selection methods fall short by failing to account for the needs of *individual* investors. Palma-dos Reis et al. (1999) comment that portfolio selection models do not include the investor's preferences by design. This is because these models are designed with the assumption that this is the only model that can, and thus, will be used. However, as observed by Musto et al. (2015), accounting for the individual needs of investors when selecting investments is of paramount importance. These authors comment that to know the investors and deliver them *personalised* investment options is today considered to be an essential facet of a fruitful and effective advisory strategy. Furthermore, Kaiser et al. (2014) comment that it is necessary to improve the quality of financial advisory systems, which means that they need to be adapted to an investor's individual properties, needs and knowledge.

It should be noted that the works of Ballestero et al. (2012); Hallerbach et al. (2004); Kaiser et al. (2014); Musto et al. (2015) and Palma-dos Reis et al. (1999) are exceptions to the generalisation that portfolio selection models do not account for individual investors. These articles do consider the individualised nature of investors in their models. The work of Ballestero et al. (2012) incorporate the investor's targets or aspiration levels into their SRI portfolio selection model. Hallerbach et al. (2004) implement multi-attribute portfolio selection, and the attributes of the selected securities are fashioned in a way that suits the circumstances and preferences of the investor. Kaiser et al. (2014) defined a function to select a combination of investment products, such as pension funds and life insurance, for an investor based on his/her individual preferences. Musto et al. (2015) incorporates the investment goals and other individualised factors about the investor such as gender and age into their model. Palma-dos Reis et al. (1999) incorporate the investment goals, desired investment duration and purpose of investment of the investors into their model.

Despite the existence of these works, it was observed that the majority of existing portfolio selection models focus on building generalised models that encompass a broad spectrum of potential investors. One area in which this is especially evident is the consideration of risk in the portfolio selection problem.

Generally, standard practice accounts for risk by considering how reliant one stock's return is on another stock or the market as a whole, and then minimising this dependence over all the stocks being considered for inclusion in the portfolio (Abdelaziz et al., 2007; Markowitz, 1952; Steuer et al., 2007). Yet, individuals perceive and experience financial risks differently. It is this perception that may determine an appropriate composition of assets in an investment portfolio which is optimal for an individual investor in terms of risk and return (Droms, 1987). As such, it is necessary to account for this perception, or *financial risk tolerance*, when selecting investments for an investor. In addition to financial risk tolerance, individuals may also have specific goals regarding ROR, liquidity, SRI and others. Thus it is necessary to extend the standard portfolio selection model to incorporate these investor goals, allowing for individualised investment portfolios to be created.

Another area in which existing portfolio selection models are lacking is in how they are evaluated. Generally, the existing literature demonstrates how portfolio selection models can be formulated and solved using historical data. The work by Abdelaziz et al. (2007); Bhattacharyya et al. (2011); Markowitz (1952) and Steuer et al. (2007) are examples of this approach. Yet, these publications did not evaluate the future performance of the selected portfolios, and no evidence was given that the models were adequate to select investment portfolios that would have had satisfactory performance in the future. Investing is not an instantaneous process. Investors invest in the present to achieve their financial goals sometime *in the future*. Thus it is insufficient to create a portfolio selection model based purely on historical data and not validating that the portfolios produced by the model would perform as desired in the future. Furthermore, an unvalidated model can not be classified as a successful model or as useful or suitable for its intended purpose.

1.2 The rising importance of socially responsible investing

It is well-known that portfolio selection involves considering multiple objectives simultaneously. However, portfolio selection researchers disagree on the number of objectives that should be considered and which objectives should be included. Despite the contention, most researchers agree that three objectives should be considered, namely, to maximise the uncertain ROR, to maximise liquidity, and to minimise risk (Abdelaziz et al., 2007; Bhattacharyya et al., 2011; Kaiser et al., 2014). All three of these objectives are aimed at increasing the financial performance of the investment portfolio.

In recent years, there has been a shift in how investors view their investment portfolios. Traditionally, most investors are merely interested in the expected return and risk that their investments will achieve. Yet, modern investors are no longer only investing for the monetary benefit (Ballestero et al., 2012). Recent financial, social and environmental crises have led to an increased consciousness about moral and ethical factors in the world, including Environmental, Social and Governance (ESG) factors. This has led to investors becoming more aware of the moral and ethical implications of their investment decisions. Thus, more and more investors want to consider more objectives than just their expected return when investing (Hallerbach et al., 2004). As a result, investors have started selecting investments based not only on their financial aspirations but also on their moral and ethical beliefs. The practice of incorporating moral and ethical considerations when selecting investments is known as SRI (García et al., 2019).

Given its recent emergence, SRI it is not yet standard practice in the field of portfolio selection. Thus very few portfolio selection models incorporate an SRI objective or Socially

Responsible (SR) considerations. Notable exceptions to this are the articles by Ballestero et al. (2012); Hallerbach et al. (2004) and Revelli and Viviani (2015). In their article, Ballestero et al. (2012) use negative and positive screening to select SRI investment portfolios. Hallerbach et al. (2004) created a sustainability score and scored each company in their sample based on a SRI criteria to determine whether or not the companies can be considered to be SR. Their model then aimed to select the companies with the highest sustainability score. Revelli and Viviani (2015) conducted a meta-analysis to determine whether or not SRI portfolios outperform non-SRI or traditional investment portfolios.

Yet, the emerging viewpoint among investors is that investments should be SR, a model selecting investments for these investors must be adjusted to incorporate SRI. Thus, another objective, to maximise SRI, must be added to the three aforementioned objectives in portfolio selection models. From this discussion it is clear that there is opportunity for improvement in existing portfolio selection models to make these more suited within the current investment landscape.

These opportunities relate to the individualised investment goals (especially with regards to risk), model validation, and incorporation of SRI.

This raises the question as to whether a portfolio selection model can be developed that incorporates these improvements and how such a model will perform when compared to existing portfolio selection models. Consequently, the research question for this dissertation is:

> *How does the real-world performance of generic portfolio selection models compare to individualised portfolio selection models that incorporate the individual investment goals of investors, an individualised risk objective, and SRI?*

1.3 Research design

This dissertation aims to compare the performance of generic portfolio selection models, henceforth *generic models*, to individualised portfolio selection models, henceforth *individualised models*. The individualised models incorporate the individual investment goals of the investors, an individualised risk objective and an SRI objective. In order to address this aim, the following deliverables must be achieved:

1. Individualised models that incorporate the individualised investment goals of investors, an individualised risk objective, and an SRI objective must be developed for an evaluation period of three years. As portfolio selection models consider multiple conflicting objectives, some of which contain uncertainty, Multi-Objective Optimisation (MOO) and stochastic programming must be employed to achieve this deliverable.

2. The generic and individualised models must be solved using a real-world dataset. This real-world dataset is comprised of two smaller and independent three-year datasets, a historical dataset and a future testing dataset.

3. The results produced by the generic and individualised models must be validated using an appropriate validation technique and tested within a future context. The portfolios selected by the models are considered valid if they achieve the goals of the investor in the future, and not just when the portfolios are selected. The results obtained from the models must be analysed and compared. It must also

be determined whether or not the models produce market-competitive investment portfolios.

1.4 Research methodology

The portfolio selection problem is a well-known problem within the field of operations research. Thus, an operations research methodology should be used to address the research question. The design research methodology proposed by Manson (2006) for operations research is an established and highly regarded methodology within this field. It is a five-stage approach that describes how a research idea is translated into a well-developed and evaluated solution. According to this methodology, a researcher should first become *aware of the problem*. A *suggestion* should then be made for how the problem can be addressed and a tentative design formulated. The research then enters into the *development* stage where an artefact is built, after which the performance of the artefact should be *evaluated* using acceptable criteria. The results are written up and *conclusions* drawn from the research. This process is iterative as some stages may be repeated as more knowledge is gained. This five-stage approach will be followed in this dissertation.

1.4.1 Awareness of the problem

Having always had a keen interest in the world of investment and the strategies for choosing suitable investment portfolios, the researcher decided to study portfolio selection. Awareness of the problem came when the researcher identified several opportunities for improvement within the existing portfolio selection literature. Firstly, generally, existing portfolio selection models do not account for the investment goals of individual investors, but rather select investment portfolios for the goals of a "generic" investor. Furthermore, the general method of incorporating risk into the models does not take into account how an investor may feel about taking financial risks. Thus, the method does not account for the financial risk tolerance of the investors. Secondly, there has been a recent shift towards SRI. Yet, the majority of portfolio selection models do not allow an investor to select investments that are aligned with their moral and ethical beliefs and their financial aspirations. Lastly, existing portfolio selection models are generally considered to be unvalidated. This is because, with minimal exceptions, the existing portfolio selection literature presents how to develop and solve portfolio selection models, but no evidence is given that these models select portfolios that have satisfactory performance in the future. Given these observations, it became clear that there is a gap in the existing body of portfolio selection knowledge. To address this gap, a deeper understanding of portfolio selection models and how to solve them was required. Chapter 2 presents an extensive review of the existing literature on the portfolio selection model objectives, and how these models are solved.

1.4.2 Suggestion

Given the identified gaps within existing portfolio selection models, it was suggested that an individualised portfolio selection model be developed. This model should account for the individualised investment goals of the investors, have an individualised risk objective that accounts for the financial risk tolerance of the investor, and incorporate SRI. Furthermore, it was suggested that this individualised model, as well as a generic model, should be solved and tested on an unknown future so that a comparison can be made between the two approaches.

1.4.3 Development

The artefact that was created in this study is an individualised portfolio selection model. This model has several different variations and, as such, the artefact consists of two models. These models are prescriptive as their outputs recommend a possible course of action for an investor. These models have four objectives, to maximise ROR, to maximise liquidity, to minimise risk and to maximise SRI. Furthermore, the risk objective was reformulated to account for the financial risk tolerance of the investor. The models are constrained to ensure that a diversified portfolio is selected. Furthermore, a constraint was added to ensure that the full amount of the initial investment is invested in the selected portfolio. Given the multi-objective nature of portfolio selection models, an appropriate MOO method was used to solve these models. Stochastic programming was used to incorporate the uncertain ROR objective into these models. The same MOO and stochastic programming approaches used for the individualised models were used for the generic models. The data used by these models was the market data for a sample of companies which was obtained from open-source online platforms. Each model produced a set of investment portfolios and prescribed the amount of money that should be invested in each of the selected companies.

1.4.4 Evaluation

Portfolios were selected from a sample of companies listed on JSE Limited [1], henceforth the JSE, a securities exchange listed in Johannesburg, South Africa. The historical market data for these companies was collected for a specific timeframe and was divided into two smaller and independent datasets. The dataset containing the oldest data is the historical dataset, while the more recent dataset was reserved as the future testing dataset. An investor can invest in a company by buying the shares of that company. Thus, the portfolios selected by the models were portfolios consisting of shares. To determine how a portfolio of shares would have performed in an uncertain future, the ROR, liquidity, risk and social responsibility of the portfolio was determined during the future testing period and measured against the original investment goals. If the selected portfolio met or exceeded the original investment goals, the models were classified as suitable for their intended purpose.

The ROR, liquidity, risk and social responsibility values achieved by the selected portfolios during the future testing period were also used to compare the generic and individualised models. Furthermore, the performance of the selected portfolios in the future testing period was compared to the performance of the JSE all share index (market indicator) and unit trusts, in the same time period. This evaluation was performed to determine whether or not the models select market-competitive and worthwhile investments. If the performance of the selected portfolios was equal to, or greater than, the JSE all share index, they were classified to be market-competitive investments. Furthermore, if the performance of the selected portfolios was equal to, or greater than, the unit trusts, they were classified to be worthwhile investments.

1.4.5 Conclusion

Once the models and their results were evaluated, conclusions about the results and the usefulness of the models were made. Furthermore, conclusions about the comparison

[1]The Johannesburg Stock Exchange was rebranded in 2005 as JSE Limited. In this new corporate identity, the letters JSE are no longer an acronym.

between an individualised and generic portfolio selection approach were made. Awareness about new problems that arose while doing this research were discussed and introduced as avenues of future research.

The application of this design research methodology is aimed at addressing the research question posed in Section 1.2 and at adding a contribution to the portfolio selection body of knowledge. The following section discusses what this contribution is expected to be.

1.5 Expected contribution

Throughout this chapter, it has been shown that there is a clear gap in the existing body of portfolio selection knowledge. This is attributed to three factors within the existing portfolio selection models: generally they are not individualised, generally they do not incorporate SRI, and generally they are unvalidated. The contributions of this dissertation will address this gap.

It is perceived that generally, the existing, generic portfolio selection models are not individualised because they do not account for the individualised investment goals of the investors for which they are selecting portfolios. This is especially problematic within the risk objective as the models account for risk purely by considering investment market conditions and do not account for the financial risk tolerance of the investor. Investors are not all the same. They have different investment goals, financial risk tolerances and moral and ethical beliefs. By failing to account for this individuality, the generic models are rendered inadequate. This dissertation addressed this gap by formulating portfolio selection models that account for this individuality. Furthermore, a risk objective which incorporates the financial risk tolerance of the investor was proposed.

Given the recent shift towards SRI, many investors would like to be SR in their investments. Yet, generally, generic portfolio selection models are limited in this capacity as they are not designed to allow for the incorporation of SRI when selecting portfolios. Thus, investment portfolios that are aligned with the moral and ethical beliefs of the investors can not be selected. To address this gap, the individualised models that were formulated include an objective that aims to maximise the SRI of the selected investment portfolio.

Generally, generic portfolio selection models are unvalidated because they are not tested on an unknown future, and no evidence is given that they are capable of achieving the aim for which they are created. A model can not be classified as useful and reliable if it is not validated. To address this gap, the investment portfolios selected by the generic and individualised models were tested on an unknown future. These results were used to determine whether or not the models produced reliable results and if they achieve the aim for which they were developed.

1.6 Outline of the dissertation

The remainder of this dissertation continues as follows: Chapter 2 consists of a comprehensive literature study on the four portfolio selection objectives, MOO and stochastic programming methods used in portfolio selection, method for solving portfolio selection models, and the importance of model validation and model validation techniques.

Chapter 3, presents the mathematical formulation of the generic and individualised models developed in this dissertation. It is shown how these models incorporate stochasticity into the model formulation. Furthermore, the generic and individualised models are applied within a South African context by using a sample of companies that are listed on the JSE. Different software packages capable of solving these models are investigated and a suitable option selected. A discussion on how the models and their performance are evaluated is presented.

Chapter 4 presents and discusses the results produced by the generic and individualised models in the future and a comparative analysis between these results is conducted. The implications of these results within the portfolio selection field are also discussed in this chapter. Chapter 5 presents a summary of the results and conclusions of this study. The contributions of this study are presented and possible limitations and improvements for both the generic and individualised models are investigated. Finally, suggestions for further research are presented.

Chapter 2

Literature review

Since being formalised in 1952, the portfolio selection problem has continued to evolve. As it incorporates several, often conflicting, objectives and random variables, it is a complex problem requiring Multi-Objective Optimisation (MOO) and stochastic programming. From the literature, it was perceived that the majority of existing portfolio selection models *could be* unsuitable for selecting portfolios for individuals and thus requires revision to this end. Furthermore, model validation is often overlooked within the field of portfolio selection. This brings into question the reliability and usefulness of portfolio selection models and should thus be investigated. It should be noted that the only types of investment considered in this study are the shares of companies listed on JSE Limited. This securities exchange was selected as it is the largest securities exchange in Africa and is located in South Africa, the country where this study was conducted. Thus, all the literature is explained with respect to shares of companies, although many of the concepts and principles apply to other types of investments as well.

2.1 Portfolio selection objectives

Since the introduction of the Markowitz model in 1952 (Markowitz, 1952), many additional objectives have been added to the familiar risk-reward trade-off. Portfolio selection models that incorporate the traditional risk and reward objectives, and other objectives pertaining to liquidity, the number of companies in the investment portfolio, Socially Responsible Investing (SRI), and many others, have been developed. Steuer et al. (2007) identified up to 12 potential objectives that could be included in the portfolio selection model. Some of these objectives can be combined into a single objective, while others can be incorporated as constraints rather than as objectives. As such, many researchers settle on having three objectives in the portfolio selection model, namely, to maximise the uncertain Rate Of Return (ROR), to maximise liquidity, and to minimise the risk associated with investing in the selected portfolio (Abdelaziz et al., 2007; Bhattacharyya et al., 2011).

In recent years, another objective, to maximise SRI, has been added to the three aforementioned objectives. This is because investors have become aware of the moral and ethical impact that their investments have on the world, and thus wish to incorporate their moral and ethical beliefs into the portfolio selection process. Examples of such considerations may include not investing in petrochemical companies because they have a negative impact on the environment, or avoiding investment in companies with a history of human rights violations and corruption. Thus, investors want to consider not only their financial gain but Environmental, Social and Governance (ESG) factors as well when selecting investments (García et al., 2019).

The four objectives discussed, namely to maximise the random ROR, to maximise liquidity, to minimise risk and to maximise SRI, were considered in this dissertation and are explained in the following sections with the aid of an illustrative example.

2.1.1 Rate of return

The ROR is the gain or loss generated by investing in a company's shares over a specific time frame, and includes capital gains and dividends (Abdelaziz et al., 2007). An investor makes a capital gain when a share is sold for more than the purchase price. Dividends are payments made by a company to its shareholders from the profits made by the company. For any company j the random ROR $\widetilde{r_j}$ is:

$$\widetilde{r_j} = \frac{\widetilde{p_{j,t}} - p_{j,t-1} + \widetilde{d_{j,t}}}{p_{j,t-1}} \tag{2.1}$$

Where $\widetilde{p_{j,t}}$ is the random closing price of company j's shares at time t and $\widetilde{d_{j,t}}$ is the random dividends received by company j during the period $[t-1, t]$ (Abdelaziz et al., 2007; Bhattacharyya et al., 2011). The tilde indicates that the variable is random, or uncertain.

Share prices are volatile on a daily basis; they change several times a day and are rarely consistent from one day to the next. Due to this daily volatility, it is impossible to determine what the exact resale price of a share will be. Furthermore, it is unknown what dividends the investor will receive over the investment period, and thus, ROR is uncertain or random. To the contrary, the other objectives in the portfolio selection model do not have daily volatility and are thus not random. This assertion, that ROR is the only random objective in the portfolio selection model, is well documented and supported by academics and researchers (Abdelaziz et al., 2007; Bhattacharyya et al., 2011; Markowitz, 1952). To aid in explaining ROR, and the other objectives, consider the following example:

> Investor MVN is interested in investing in an investment portfolio called The Potential Portfolio (TPP) for a period of three years. She wants to achieve a 100% ROR for her investment, achieve an exchange flow ratio of 1, avoid investments that carry a risk that is beyond her risk tolerance, and be Socially Responsible (SR) in her investments.
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> A well-known investment firm, *The Firm*, has created a new investment portfolio known as *TPP*. TPP contains three JSE listed companies, namely: company ABC, MNO and XYZ. The Firm invests 20%, 30%, and 50% of its capital into each of these three companies, respectively. Upon investigation, it was found that over the last three years, these three companies had ROR, liquidity, Conditional Value-at-Risk (CVaR), and ESG rating values as given in Table 2.1.

Table 2.1: Financial information of the companies in TPP

	ABC	**MNO**	**XYZ**
ROR	100%	50%	-15%
Liquidity	0.89	1	1
CVaR	80%	15%	50%
ESG rating	1.5	2.6	3.5

In the example, if company ABC has a current price of R 130 per share, had a selling price of R 100 per share three years ago, and received a total dividend of R 70 over these three years, it would have a three-year ROR of 100% ($(130 - 100 + 70)/100 = 1$).

The ROR of an investment portfolio is the weighted sum of the ROR achieved by the companies in the portfolio. It is calculated using equation Equation (2.2).

$$\text{Portfolio rate of return} = \sum_{j}^{N} \widetilde{r}_j w_j \tag{2.2}$$

Where w_j is the proportion of the total capital (the weight) assigned to investing in company j in a portfolio containing N companies.

In the example, it was found that TPP has an ROR of 27.50%, calculated as follows: $0.2(100\%) + 0.3(50\%) + 0.5(-15\%) = 27.50\%$. Thus, if only ROR was considered, investor MVN should not invest in TPP because it has an ROR value that is far below the 100% ROR that she wants to achieve. However, ROR is only one of the multiple objectives that are considered in the portfolio selection problem. In literature, the second objective that is considered is to maximise liquidity.

2.1.2 Liquidity

Liquidity is a concept that generally refers to how easy it is to buy and sell shares without seeing a change in the price (Brunnermeier and Pedersen, 2008). When investing, an investor is interested in companies with greater liquidity (Bhattacharyya et al., 2011). This implies that an investor is interested in companies that have a similar, or higher, resale value than the purchase value. In the example, if investor MVN purchases shares in ABC for R 100 a share and upon selling those shares manages to sell them for R 100 per share, those shares would be considered perfectly liquid. If MVN manages to sell the shares for any amount above R 100, the shares are considered liquid, and MVN will make a profit on the sale. The higher the selling price of the shares are, the higher the liquidity of the shares will be. However, if MVN sells the shares for less than R 100, the shares are considered illiquid. If MVN is unable to sell the shares at all (there is no longer a demand for ABC's shares), the shares will be considered perfectly illiquid, and MVN will experience a substantial loss.

From this explanation, it is also evident that liquidity is dependent on the demand for a company's shares. The higher the demand for a company's shares, the easier it will be to sell the shares and to sell them at an increased price. Thus, the shares are liquid. However, if there is no demand for the company's shares, it will not sell, and the owner will experience a loss, making the shares illiquid.

The above describes the day-to-day effects of liquidity. However, an individual investor invests over several years rather than for several days. As such, it is essential to determine liquidity over a more extended period to select portfolios that will meet the investment goals of the investor.

Gabrielsen et al. (2011) explain that there are three characteristics which make shares liquid. These are: (i) *Depth* — shares are deep when orders are placed for the shares at a value that is both above and below the current price of the shares. (ii) *Breadth* — shares are broad when a large volume (or number) of shares are bought and sold. (iii) *Resiliency* — shares are resilient when there are many orders in response to price changes. Furthermore, in their survey, they found that there are three main categories of liquidity measures: volume-based liquidity measures, price-variability indices and measures based on transaction costs. However, these measures are designed to be used for short periods and are not scalable, rendering them unsuitable for use in this dissertation.

Abdelaziz et al. (2007) developed a liquidity measure which is a function of demand over a specified time period, called the exchange flow ratio. It is determined using Equation (2.3).

$$e_j = \frac{n_j}{n_m} \tag{2.3}$$

Where e_j is the exchange flow ratio, n_j is the number of days when the shares of company j were traded and n_m is the number of days on which any assets were traded on the market (trading days) within the analysis period (Abdelaziz et al., 2007). Thus, the exchange flow ratio reflects the proportion of days that a company's shares were traded to all the possible trading days. Thus, the exchange flow ratio is always a value between zero and one. Furthermore, as it is a proportion of the number of trading days being considered, it is scalable to any time period.

The fact that the exchange flow ratio is a scalable calculation, combined with the fact that only listed companies are considered in this investigation, makes the exchange flow ratio a suitable liquidity measure for use in this dissertation. For a company to be viable, it must have an exchange flow ratio of more than zero. The higher the exchange flow ratio is, the higher the liquidity of that company. Thus higher exchange flow ratio values are preferred.

In the example, in the last three years, the JSE was open and trading for 780 days. Yet, company ABC only sold its shares on 696 of those 780 days. Thus, ABC has an exchange flow ratio of 0.89. As with the ROR, the liquidity of a portfolio is the weighted sum of the exchange flow ratio of the individual companies. Thus, TPP has an exchange flow ratio of 0.978 $(0.2(0.89) + 0.3(1) + 0.5(1))$. This exchange flow ratio is below the exchange flow ratio of 1 that MVN would like to achieve. Thus, if only liquidity is considered, MVN should not invest in TPP.

A limitation of the exchange flow ratio is that it does not display any of the characteristics of liquidity that Gabrielsen et al. (2011) identified. This is because the exchange flow ratio only reflects the demand of the shares and does not reflect the price or volume of the shares. Thus, the depth, breadth and resiliency of the shares are not evident from the exchange flow ratio.

In the portfolio selection problem, ROR and liquidity are not the only goals that are considered. As stated in Section 2.1, most researchers agree that the portfolio selection problem has three objectives. As first presented by Markowitz (1952), risk is an integral part of the portfolio selection problem and is incorporated as an objective that should be minimised.

2.1.3 Risk

Dowd (2007) defines risk as the possibility that an investor will experience a financial loss or gain due to unforeseen changes in market prices. When investing, an individual is interested in what risk shares carry so that they can diminish or accept the risk they will be taking by investing in these shares. Yet, unlike the ROR and liquidity it is extremely difficult to measure risk. It is not something that can be observed directly and as a result it has to be inferred. For this reason, proxies are used to capture risk within portfolio optimisation problems. Palma-dos Reis et al. (1999) found that variance and standard deviation remain the most widely used measures of risk. In the traditional portfolio selection model, the proxy used to determined the risk is given by Equation (2.4).

$$\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{2.4}$$

Where w_i and w_j are the proportions of capital invested in companies i and j, and σ_{ij} is the covariance between r_i and r_j (as in (2.1)) given that there are N companies in the portfolio (Markowitz, 1952). Thus, Equation (2.4) minimises the weighted covariance of all the companies that are selected as part of a portfolio. This means that if company i's share price increases, but company j's share price decreases, the two companies will be negatively correlated and a negative covariance will be achieved. This equation aims to minimise this negative covariance. Having a negative covariance is desirable because if all the companies in a portfolio are positively correlated and these companies suddenly experience a decrease in value, then the investor will make a financial loss. On the contrary, if the the share price of some companies increases, while the share prices of some other companies decreases (thus negative correlation and negative covariance), then the investor is likely to receive a return on investment, or at the very least not experience a loss.

From this explanation it can be seen that this risk measure is also a proxy for portfolio diversification because if all the companies in a portfolio are positively correlated, then a selected portfolio is considered to be an undiversified and risky portfolio.

Given a clear understanding of what Equation (2.4) represents, it can be argued that this way of accounting for risk is not sufficient for this dissertation. This is because this method does not account for how an investor may feel about taking financial risks. Not all investors feel the same way about taking financial risks and thus their propensity towards risk, known as their financial risk tolerance, needs to be incorporated into a model that selects investment portfolios for the individual investor. Thus a new method for measuring portfolio risk is required that compares the risk associated with shares, known as market risk, and the financial risk tolerance of the investor. This dissertation proposes such a method, which results in a new risk metric called the individualised risk measure.

From the explanation of Equation (2.4) it is clear that in addition to being a proxy for risk, this equation is also a proxy for diversification. Portfolio diversification is an vital element in portfolio selection models. Thus, in addition to having the objective of minimising the risk associated with the selected portfolio, existing portfolio selection models incorporate a risk-related constraint, known as the portfolio diversification constraint. This constraint aims to reduce the overall risk of an investment portfolio by ensuring the portfolio contains multiple companies. This constraint, as well as the concepts of market risk and financial risk tolerance, and the formulation of the new individualised risk measure, are presented in the following sections.

Market risk

When determining market risk, researchers, academics, and industry practitioners use the Value-at-Risk (VaR) or CVaR methods. VaR is a statistical technique used to measure and quantify the level of financial risk within a company over a specific time frame. It can be determined at three intervals, 95%, 99% and 99.9%, which represent the certainty of the risk being calculated. For example, if VaR(95) was calculated to be 60%, it means that there exists a 5% chance that an investor will lose 60% or more of the money they invested.

VaR is a measure of losses due to “normal” market movements and is widely used in industry (Rockafellar and Uryasev, 2002). It assumes that the losses experienced by any company at any time follows a normal distribution. However, in reality, loss distributions are hardly ever normally distributed and tend to be fat-tailed distributions. Given this assumption, VaR does not take these non-normal tail ends of a distribution into account when determining risk and thus only determines how risky a company is if a tail event, which is a loss in excess of VaR, was never to occur. This makes VaR unstable and difficult to work with numerically when loss distributions are not normally distributed (Rockafellar and Uryasev, 2002). VaR produces an extensive range of potential losses, rather than one value, and it is thus challenging to account for the market risk when using this measure. This most often leads to the calculated market risk being underestimated and thus, should losses be experienced, they will be substantially greater than expected. As the majority of financial losses experienced are not normally distributed, it is necessary to use a risk metric that does account for the tail ends of a distribution (Dowd, 2007; Rockafellar and Uryasev, 2002).

CVaR is a measure that quantifies the losses that may be experienced for any distribution. Furthermore, (Rockafellar and Uryasev, 2002) shows that the CVaR has significant advantages over the VaR. Thus, CVaR is a better proxy for market risk . CVaR can be calculated at three intervals, at 95%, 99% and 99.9%, yet the interval most often used in practice is 95% because it is the most conservative (Chan, 2017). It determines the average expected loss of a company as opposed to the VaR, which produces a large range of potential losses which can be difficult to interpret. CVaR(95) indicates, with a 5% chance, what average loss an investor will experience. Moreover, it can be expressed as a minimisation formula which can easily be incorporated in optimisation problems that aim to minimise or constrain risk (Rockafellar and Uryasev, 2002). Many software packages such as `R` have built-in functions that compute the CVaR of a company, at a specified interval, for a specific time period, based on the return received by a company's shares (R Core Team, 2020). The result of the CVaR function in `R` is a value between zero and one, which is multiplied by 100 to give a percentage value.

Market risk is divided into five categories, namely, conservative risk, moderately conservative risk, moderate risk, moderately aggressive risk and aggressive risk. These risk categories indicate the level of risk involved with a company and encompasses market risk values of 0–36.73%, 36.74–44.90%, 44.91–57.14%, 57.15–65.31% and 65.32–100% respectively.

In the example, the market risk of company ABC was found to be aggressive and was calculated to be 80% using the CVaR(95) method. This indicates that there exists a 5% likelihood that an investment in ABC will result in an investor losing an average of 80% of the money that they have invested in ABC.

The market risk of an investment portfolio is the weighted sum of the individual companies' CVaR values, as expressed by Equation (2.5).

$$\text{Portfolio market risk} = \sum_{j=1}^{N} w_j \, \text{CVaR}_j \tag{2.5}$$

Where w_j is the proportion of capital invested in company j and CVaR_j is the market risk associated with company j, in a portfolio containing N companies. As with the market risk for a single company, market risk for a portfolio falls into one of the five aforementioned risk categories. In the example, TPP has a moderate market risk of 45.50% $(0.2(80) + 0.3(15) + 0.5(50) = 45.50)$.

As discussed, market risk alone is an insufficient measure for risk in the portfolio selection model as it does not incorporate the financial risk tolerance of the individual investor. As such, a new metric is required. Before a new metric can be developed, it is essential to understand precisely what financial risk tolerance is and how it is measured.

Financial risk tolerance

Hallahan et al. (2004) define financial risk tolerance as the maximum amount of uncertainty that a person is willing to accept when making a financial decision. Thus, it describes how willing an investor is to experience a financial loss should an investment be unsuccessful. Palma-dos Reis et al. (1999) comment that risk tolerance, or risk aversion, is a widely accepted component of a person's personality.

There are two methods for determining financial risk tolerance, namely self-assessed risk tolerance and a financial risk tolerance assessment. Self-assessed risk tolerance involves asking an individual to estimate their financial risk resulting in a Self-assessed Risk Tolerance Score (SRTS) for the individual. A financial risk tolerance assessment is a psychometric aptitude test in which the individual is required to give responses to different financial situations. These responses, in conjunction with other personal factors such as gender, age, marital status, level of income, level of education and current assets owned, are then used to determine a Risk Tolerance Score (RTS) for the individual (Hallahan et al., 2004).

The research of Hallahan et al. (2004) shows that the RTS is preferred over the SRTS. They indicate that the RTS is approximately 5 points higher than that estimated by the individual. This suggests that individuals usually underestimate their risk tolerance score and should they invest based on this SRTS they may overlook potentially beneficial investments because they appear to be too risky when in actuality they are not. Thus, the RTS should be used as the baseline when determining whether to invest in a certain company.

In 1999, Grable and Lytton (1999) developed and introduced a 13 question financial risk tolerance assessment. This assessment has become widely used, and its reliability and validity has remained robust since its development (Kuzniak et al., 2015). With this assessment, an RTS is measured on a scale of 20 to 69, with a higher score indicating a higher propensity towards taking risks. Furthermore, as with market risk, RTS values can be grouped into five categories, namely conservative risk tolerance, moderately conservative risk tolerance, moderate risk tolerance, moderately aggressive risk tolerance and aggressive risk tolerance. The aforementioned categories encompass RTS values of 20–38, 39–42, 43–48, 49–52 and 53–69 for the five categories respectively.

Given a clear understanding of market risk and financial risk tolerance, a new risk measure which incorporates both can be created. This individualised risk measure is presented in the next section.

Individualised risk measure

Using the financial risk tolerance scale developed by Grable and Lytton (1999) and the CVaR of the companies, the risk of an individual investor can be incorporated into the portfolio selection problem. This is achieved by comparing the market risk of the selected portfolio to the financial risk tolerance of the investor.

$$\sum_{j=1}^{N} w_j \,\mathrm{CVaR}_j \leqslant \frac{\mathrm{RTS} - 20}{49} \tag{2.6}$$

The left side of Equation (2.6) is the portfolio market risk as given in Section 2.1.3. The right side of this equation takes the RTS of an investor and normalises it to be a value between zero and one. Thus, when selecting companies for an investment portfolio, Equation (2.6) ensures that the risk associated with investing in a selected portfolio is within the investor's risk tolerance.

In this example, investor MVN had an RTS of 50 which, when normalised, equates to 0.612. As the 0.455 market risk of TPP is below the 0.612 financial risk tolerance of MVN, if considering only risk, MVN should invest in TPP.

It should be noted that although publications that incorporate the individualised risk profile of the investors are scares, some do exist. These include the work by Ballestero et al. (2012); Musto et al. (2015) and Palma-dos Reis et al. (1999). Ballestero et al. (2012) incorporate a risk aversion coefficient, determined through a test where the investor discloses his/her risk aversion, into their model that aims to select SRI portfolios.

In their article, Musto et al. (2015) propose a framework that can be used by financial advisors to propose diverse and *personalised* investment portfolios. Portfolios were selected for 1 172 real investors and the risk profile of these investors was modelled using a scale of very low, low, normal, high and very high. This risk profile was incorporated into the portfolio selection model. The results show that the return achieved by the recommended portfolios outperforms the portfolios suggested by human advisors, while meeting the risk profile of the investors.

Palma-dos Reis et al. (1999) puts forward that portfolio selection systems should have intelligent components for customising and personalising the system, based on the profiles and needs of the investors. They elicit the risk aversion of the investors using nine questions, and receive a risk aversion score between 0 and 1. This risk aversion is then incorporated into their model.

In this section an individualised risk measure incorporates both market risk and financial risk tolerance into the portfolio selection problem. Yet, this risk proxy alone is insufficient to account for risk within a portfolios selection model. As explained, portfolio diversification is an important concept within portfolio selection and as such it must be taken into consideration a portfolio selection model.

Portfolio diversification

Portfolio diversification is the practice of investing in multiple companies to reduce the overall risk of an investment portfolio over time. This practice works on the premise that by having multiple companies, should a few of these companies experience a loss, this loss will be counteracted by the gains received by the other companies in the portfolio. It is the principle of not putting all of one's eggs in the same basket when investing.

According to Statman (1987), an investment portfolio should contain at least 30 different shares to be considered a diversified portfolio. Thus, the portfolio selection model should have a constraint that ensures that no less than 30 companies are selected as part of the investment portfolio.

Risk is a well-known and researched component of the portfolio selection problem. Yet, in recent years, the portfolio selection problem had to be extended to accommodate the incorporation of factors that hold no monetary value but are based on the ethics of the investor. This trend toward investing ethically is known as SRI and is the final objective in the portfolio selection problem for this dissertation.

2.1.4 Socially responsible investing

The last objective in the portfolio selection model is to maximise SRI. It is a paradigm within investing that regards the moral, social and environmental characteristics of companies when selecting investment portfolios (Ballestero et al., 2012; Sparkes, 2008). The Global Sustainable Investment Alliance (2019) identifies seven activities and strategies with which SRI could be incorporated into portfolio selection. These are: Negative/exclusionary screening, positive/best-in-class screening, norms-based screening, ESG integration, sustainability-themed investing, impact/ community investing, and corporate engagement and shareholder action.

Negative screening Not investing in, thus excluding, companies that are believed to involve activities that may be considered harmful, controversial or "sinful". Such companies are called sin stocks and include factors such as the selling of alcohol and tobacco, gambling, and nuclear power (Ballestero et al., 2012; De Colle and York, 2009).

Positive screening Specifically investing in companies that engage in activities that are considered to be beneficial to people and the planet, such as employment diversity, renewable energy, and sustainability (Ballestero et al., 2012; De Colle and York, 2009).

Norms-based screening The practice of only investing in companies that adhere to minimum standards of business practice as prescribed by internationally accepted organisations, such as the International Labour Organisation (ILO) and the United Nations (UN) (Global Sustainable Investment Alliance, 2019).

ESG integration Identifying and including ESG factors into financial analysis and investment decision-making (Foo, 2017).

Sustainability-themed investing Investing only in companies which are specifically engaged in sustainability-themed activities, such as renewable energy, green technology or zero-waste initiatives (Global Sustainable Investment Alliance, 2019).

Impact investing Selects companies that are specifically aimed at solving social or environmental problems or encouraging community development (Louche et al., 2012).

Corporate engagement and shareholder action When shareholders use their power to direct a company's corporate behaviour (Foo, 2017).

The Global Sustainable Investment Alliance (2019) found that negative screening is the predominant SRI strategy in the world, followed by ESG integration. Furthermore, Ballestero et al. (2012) found that negative screening is the oldest and most basic SRI strategy. Despite being the oldest, most basic and most commonly used strategy in the world, many researchers argue that there are problems with negative screening, both in theory and practice (De Colle and York, 2009; Trinks and Scholtens, 2017). To explain their argument, consider the example of investing in hospitals. With negative screening, investors would never invest in hospitals because they perform abortions, provide contraceptives and use stem cells in treatments, all of which are considered to be "sinful" activities (Trinks and Scholtens, 2017). Yet, hospitals provide millions of people with life-saving and much needed medical care. Thus, it can be argued that the good outweighs the bad, and therefore hospitals are SR companies. For this reason, the second most used strategy, ESG integration, is the preferred method of incorporating SRI into the portfolio selection model.

In an international survey of asset and fund managers, Van Duuren et al. (2016) found that the predominant way in which ESG integration is achieved is through the consideration of an ESG rating when selecting companies. An ESG rating is a comprehensive measure of how well a company manages ESG issues. Agencies that determine ESG ratings look at and measure ESG factors such as biodiversity, climate change, customer responsibility, health and safety, labour standards, anti-corruption, corporate governance, risk management and many more, to determine an overall "goodness" score for a company (FTSE Russell, 2020). The JSE uses the methodology created by the company FTSE Russell (2020) which considers 14 different ESG factors, each measured by 10–35 mathematical indicators, to determine the ESG rating of each company on its exchange. Furthermore, if a company has an ESG rating of 2.5 or above, they are considered to be SR by the JSE (JSE Limited, 2019a). In the example, company ABC had an ESG rating of 1.5, which is less than 2.5, and thus ABC is not an SR company.

ESG ratings evolve over time. This is attributed to the fact that as companies become aware of how SR they are, they begin to make conscious efforts to incorporate and increase ESG factors within the company, leading to increased ESG performance. Furthermore, as companies gain access to financial resources, their financing costs tend to decrease, which in turn encourages the adoptions of SR policies (Revelli and Viviani, 2015). Thus, a company may be more SR at the end of a specific period than they were at the beginning of that period. Thus, when selecting companies based on ESG ratings, the selection should be made based on the ESG ratings of the company *at the time* of selection, rather than considering historical ESG ratings (Lundström and Svensson, 2014).

As with the other three goals, the ESG of an investment portfolio is the weighted sum of the ESG ratings of the individual companies. In the example, TPP has an ESG rating of 2.83 $(0.2(1.5) + 0.3(2.6) + 0.5(3.5))$. Thus, as TPP has an ESG rating above 2.5 it is considered to be an SR investment and MVN should invest in it.

The portfolio selection problem is a multi-objective problem and as shown throughout the preceding sections, TPP is a good investment in terms of risk and SRI, yet it is a bad investment when considering ROR and liquidity. This illustrates how selecting the ideal investment portfolio for an investor requires making trade-offs between the multiple, sometimes conflicting, objectives. In reality, it is not always possible to always meet all the objectives when selecting investment portfolios. Thus, it is necessary to evaluate trade-offs between the various objectives (Hallerbach et al., 2004). For this reason, it is necessary to use MOO, which uses mathematical techniques and methods to find the best possible solution while taking all the objectives into account. Various MOO methods can be used to solve the portfolio selection problem.

2.2 Multi-objective optimisation methods in portfolio selection

Aouni et al. (2018) reviewed the applications of methods and procedures used in the formulation and solving phases of portfolio selection problems. In the 116 publications reviewed, Aouni et al. (2018) found that 12 portfolio optimisation methods have been used to formulate these problems, with the majority of these being mathematical programming procedures. Although 12 methods were found, only four of these methods were widely applied and appeared in more than 10% of the publications reviewed. These were goal programming, compromise programming, ϵ-constraint methods and fuzzy mathematical programming. Aouni et al. (2018) found that goal programming is the most used method, appearing in 42% of the publications considered. This is followed by compromise programming with 19%. Lastly, the ϵ-constraint method and fuzzy mathematical programming are tied at 13% each. Given the widespread use of these four MOO methods in portfolio selection literature, they will be discussed and considered for use in this dissertation in the following sections.

2.2.1 Compromise programming

Compromise programming aims to find a solution that is as close as possible to the ideal solution for the problem being considered. An ideal solution is one that achieves the ideal point of the problem, which is the solution where all the objectives achieve their optimal value (Beula and Prasad, 2012). The optimal value for an objective is found by removing all the objectives from the problem except the one being considered, leaving all the constraints unchanged and then solving this new single objective problem. Thus if one of the objectives is to maximise $f(x)$, the optimal value of this objective is the maximum value of $f(x)$ that can be achieved given the problem constraints.

In many cases, because of the conflicting objectives, the ideal solution is infeasible. Thus, the solution that achieves the ideal point as closely as possible, known as a compromise solution, is sought for the problem. For this reason, in compromise programming, the objectives are formulated as constraints aimed at achieving the ideal point. To allow for a solution that does not fully achieve the ideal point, an additional variable, known as the degree of closeness, is added to the objective constraint formulation. This degree of closeness variable measures the difference between a feasible solution and the optimal value as a percentage. Finally, a single objective function is formulated that aims to minimise the sum of all the degree of closeness variables within the model, thereby minimising the total percentage by which the model solution does not meet the ideal solution (Beula and Prasad, 2012).

Within the finance industry, compromise programming has been applied to solve problems related to profit maximisation, financial strategy planning, the design and assessment of economic policies, and portfolio selection (André et al., 2008; Aouni et al., 2018; Martı et al., 2011). Some of the purported advantages of this approach are that it requires less computation than other multi-objective programming and is a more objective (as opposed to subjective) approach than, for example, goal programming. The latter characteristic is said to be advantageous because it requires less information about the model user (Gan et al., 1996).

Within the context of this dissertation, the latter point is, in fact, a significant drawback. This is because this approach is based on the assumption that the model user would always want to achieve the ideal point, which is arbitrary to the model user and is not based on any user inputs (Beula and Prasad, 2012). If the same dataset is used, the ideal point will be the same for every single investor for whom a portfolio is selected, regardless of the individual investor's investment goals. This indicates that compromise programming is a generalised approach and does not allow for the incorporation of an individual investor's investment goals. Given that this is contrary to the aim of this dissertation, compromise programming can not be used to address the research question. Although compromise programming is not a suitable MOO for this dissertation, it is only the first of four options being considered. The second option, the ϵ-constraint method, is explored in the next section.

2.2.2 ϵ-Constraint method

In the ϵ-constraint method, one objective is selected and optimised, while the remaining objectives are converted into constraints. These constraints aim at achieving some specified, or given, *epsilon* value. For example if an objective is given as minimise $f_2(x)$, it would be expressed as $f_2(x) \leqslant \epsilon_2$ when converted into a constraint. This process is repeated until all the objectives have been optimised, resulting in an evenly distributed set of Pareto-efficient solutions (Chircop and Zammit-Mangion, 2013). The ϵ-constraint method has been applied to stock market forecasting, to determine budget allocations, bank loan management and portfolio selection within the finance industry (Abdelaziz et al., 2014; Aouni et al., 2018; Mukerjee et al., 2002; Üstün and Anagün, 2015).

An advantage of this approach is that it can be applied to both convex and nonconvex problems. Yet, a challenge with this method is that it is highly reliant on the selection of the ϵ value to ensure that a feasible solution is found. If the wrong ϵ values are selected, the solution obtained will be infeasible. There are numerous candidate values for each ϵ value, and to test all of them to find the most suitable option is computationally-intensive and timely. A further disadvantage of this method is that it only allows for hard constraints. Thus, it does not allow for any violation of the constraints, which is unrealistic within real-world applications (Coleman et al., 1999).

Given that this method may result in infeasible solutions, is inflexible, is timely to implement and is computationally intensive, it is not considered to be a desirable method to use in this dissertation. Thus, it was decided that this method will not be applied in this dissertation. Given that this method will not be used in this dissertation, another candidate MOO method, fuzzy mathematical programming, must be considered.

2.2.3 Fuzzy mathematical programming

In fuzzy mathematical programming, fuzzy logic is applied within the model formulation. Fuzzy logic provides a method of incorporating human reasoning abilities into mathematical models and logic. Fuzzy numbers are used to model the random parameters, and constraints are modelled as fuzzy sets (Sahinidis, 2004). A fuzzy number indicates a connected set of possible real numbers that a value can take. Each possible value carries a weight between zero and one, which indicates how acceptable a possibility is. A weight of zero indicates that a possibility is completely unacceptable while a weight of one indicates that a possibility is completely acceptable (Khan et al., 2018). Weights are allocated to the possibilities according to a graphed function, known as a membership function. As an example, consider an investor who wants to achieve an ROR of *about* 70%. As humans, we intuitively understand what *about* 70% means, but machines do not have such intuition and thus need to be programmed to understand this concept. An example of a fuzzy number for this 70% could be $\{69.8, 69.9, 70, 70.1, 70.2\}$. If the 69.8 value has a membership function of 0.9, it means that this value is only acceptable 90% of the time.

In multi-objective fuzzy mathematical programming, all but one of the objective functions are removed from the problem, and all the problem constraints are maintained. The remaining objective function is then maximised and minimised in order to determine the upper and lower bound values of that objective function. This is repeated for all the objective functions until the lower and upper bounds of all the objectives have been found (Alimi et al., 2012). Although it is preferred to determine the upper and lower bounds of the objectives in this way, these upper and lower bounds could also be subjectively stipulated by the model user. If the objective function is to be maximised, a membership function value of zero is given to all values less than or equal to the lower bound value, while all values greater or equal to the upper bound value are given a membership function value of one. If the objective function is to be minimised, the values are allocated in the opposite order, with values less than or equal to the lower bound value having a membership function value of one. Values that are between the upper and lower bounds are given a membership function value based on the shape of the membership function. There are many different forms of membership functions, but a linear membership function is used most often (Sahinidis, 2004).

The formulas that describe the membership functions, in terms of the upper and lower bounds and the objective function values (z_i), are then set to be less than or equal to a beta variable (β_i). This beta variable is the rate, as a percentage, by which an objective function nears its optimal solution (Alimi et al., 2012). Solving for z_i in this new inequality gives a target value that the objective should aim to achieve. The objective functions are then converted into constraints which are aimed at achieving this target value. A new objective function which minimises the sum of all the β variables in the model is then formulated.

Within the finance industry, fuzzy mathematical programming has been used for corporate financial planning, personal and business credit scoring, to forecast the financial position of businesses, and portfolio selection (Aouni et al., 2018; Korol, 2012; Tarrazo and Gutierrez, 2000). There are many reported advantages to fuzzy mathematical programming. It allows vague data to be modelled by replacing the vague data with fuzzy numbers and fuzzy sets, thus enabling the modelling of real-world problems. It is used to model situations where there is limited data available by replacing vague data with fuzzy numbers and fuzzy sets. Furthermore, this method requires less data collection than other methods as variables do not need to be defined precisely (Rommelfanger, 2004). Lastly, an advantage offered by this approach that is relevant to this dissertation is that it allows for the integration of user-specified targets into the objectives.

A significant drawback of this method is that it is computationally intensive. Although the shape of the membership functions of the objective functions' constraint can be taken as linear, the shape of the membership function for the unknown variables in the portfolio selection problem are unknown. There are several different shapes of membership functions, and testing all of them to find the optimal one would require a significant amount of time and computing. Furthermore, fuzzy mathematical programming is usually applied when there is uncertainty within a problem, and there is not enough information available to determine distributions for the variables being considered. Thus, if there is not enough information available for stochastic programming techniques to be used, this method is employed. Given the fact that this method is computationally intensive and the fact that the information required for this study is known, thus stochastic programming can be used, this method will not be used in this dissertation.

After evaluating three of the four most-used MOO methods identified by Aouni et al. (2018), an appropriate method for use in this dissertation has not yet been identified. Thus, it is necessary to evaluate whether or not the fourth method, goal programming, can be used in this dissertation.

2.2.4 Goal programming

In goal programming, the objectives are formulated as constraints aimed at achieving some user-specified ideal value, or goal, rather than as objectives that must be maximised or minimised. A deficiency variable is added to each of these constraints to model the under or over achievement of the goal. Finally, a single objective function is formulated that aims to minimise the sum of all the deficiency variables within the model, thereby minimising the total goal violation within the model (Rardin, 1998). The ideal values for each constraint is a value that the model user will be satisfied to achieve, but it is not necessarily the best obtainable value (Sen and Nandi, 2012).

Zopounidis et al. (2018) found that goal programming has widespread applications within finance in areas such as capital budgeting, working capital management, financial planning, and portfolio selection. The realisation that investors evaluate their financial performance against some benchmark led to the use of goal programming within portfolio selection (Azmi and Tamiz, 2010). Abdelaziz et al. (2005) introduced the idea of using the investor's investment preferences as the ideal values. Thus, goal programming allows for the incorporation of an individual investor's goals into a portfolio selection model, which is one of the aims of this dissertation. Therefore, goal programming is an applicable technique for this dissertation. A further advantage of goal programming is that it is capable of handling a large number of variables, constraints and objectives (Sen and Nandi, 2012). A drawback of this approach is that very detailed information about the user's preferences are required (Hallerbach et al., 2004). This means that the model user has to specify *exactly* what goal they would like the model to achieve. For example, it is not sufficient for an investor to say that they would like to achieve an ROR of *about* 70%, they must be specific and state that they would like to achieve an ROR of *exactly* 70%. Despite this drawback, goal programming is still considered to be an applicable technique and will be used in this dissertation.

There are many different types of goal programming that can be used in portfolio selection. These include: lexicographic goal programming, weighted goal programming, polynomial goal programming, stochastic goal programming, fuzzy goal programming, and other variants of goal programming. The other variants of goal programming encompass a total of eleven other variations of goal programming that have been applied in the portfolio selection problem. Although there are many types of goal programming, Tamiz et al. (2013) show that all the objectives in the portfolio selection problem can be considered to be of equal importance. Thus, this is the approach that is adopted for this study.

It has been shown that goal programming is a suitable MOO technique for this dissertation. Nevertheless, it does not address the uncertainty in the portfolio selection problem. One way to address this uncertainty is to apply stochastic programming.

2.3 Stochastic programming in portfolio selection

Throughout literature, there are many stochastic programming techniques, but Masmoudi and Abdelaziz (2018) show that there are three methods which can be used to address stochasticity in portfolio selection. These are the Expected Value (EV) method, recourse programming and Chance-Constrained Programming (CCP). In each of these methods, the stochastic program is transformed into a deterministic equivalent using a stochastic programming intervention.

The EV method is often the first method applied when addressing problems that contain uncertainty (Higle, 2005). In this method, the model is converted into its deterministic equivalent, and the random variables are replaced with their EV. The EV value of a random variable is the average of values found in the distribution of the historical values of the variable being considered (Shapiro et al., 2009).

An advantage of the EV method is that EV models can be solved without additional manipulation of the variables and constraints. This approach has been applied within the field of portfolio selection by Abdelaziz et al. (2009), Zhu (2010) and others. The EV method is often applied when a value is sought for an uncertain variable based on the historical data that is available for that variable. Thus, this method is highly dependent on the historical data distribution of the uncertain variable. Within the world of investing, enormous amounts of historical data are readily available. It was found that in this dissertation, these variable follow skew-normal, uniform or triangular distributions. Example of these distributions can be seen in Appendix A. Given these clearly defined distributions, the EV method can be applied. Consequently, distributions of unknown variables, such as price and dividends, can easily be compiled and determined. For this reason, the EV method is well suited for use in the portfolio selection problem and will thus be used in this dissertation.

An alternative to the EV method is recourse programming. In mathematical models, constraints are usually written as inequalities which aim at achieving a value less than, less than or equal to, greater than or greater than or equal to some goal value. In resource programming, constraints with uncertain variables are converted into equations that are equal to the goal value. This is accomplished by adding two variables to the constraint, one that represents the over achievement of the goal and the other that represents the under achievement of the goal. Furthermore, it is then specified that both of these variables must be greater or equal to zero. Should either of these two variables be found to be greater than zero, a penalty is imposed on the model. This penality is incorporated into the model by introducing additional costs into the objective function. These costs are the cost of the under or over achievement of the constraint (Masmoudi and Abdelaziz, 2018).

A recourse model is converted into its deterministic equivalent by considering the different values that the random variable may take on, as sampled from a distribution, and the probability associated with each of these values. The equation that contains the uncertain variable is then duplicated and inserted into the model for as many times as there are values to consider. Thus, if seven values are being considered, that equation would be inserted into the model seven times. For each of these seven equations, the uncertain variable is replaced with one of the seven values that are being considered. Then a function is added to the objective function. This function consists of the probability associated with a specific value multiplied with the sum of the penalty cost that have been imposed on the model because of the over or under achievement of the goal. Again, this is done for each of the seven values being considered.

Recourse programming has been applied to the portfolio selection problem by Masmoudi and Abdelaziz (2012) and Masmoudi and Abdelaziz (2017). As with the EV method, this method is dependent on historical data distributions. Furthermore, it is highly dependent on having known penalty costs. When reviewing the literature, no portfolio selection with recourse programming studies were found that were conducted within the South African market. Thus, it could not be determined what realistic penalties costs would be for a study being conducted on a sample of South African companies. Furthermore, in certain situations, these costs could be specified by the model user. Yet, given that the model users in this study are everyday investors, most of whom have little to no understanding of the costs associated with the under or over performance of their investment goals, it was decided that this approach would not be used for this study. Given that no published penalty costs could be found for a South African study, and the impracticality of the alternative, and to avoid assuming what appropriate penalty costs may be, it was decided that recourse programming would not be used in this dissertation.

The third and final approach highlighted by Masmoudi and Abdelaziz (2018) is CCP. Usually, with mathematical models, it is assumed that all the constraints must be adhered to all of the time. This is however not the case with CCP. To aid in the explanation consider the following problem:

$$\underset{x \in X}{\text{Min}}\, f(x) \quad \text{subject to} \quad F(x, \widetilde{\xi}) \leqslant 0 \tag{2.7}$$

Where $F(x, \widetilde{\xi})$ denotes a set of constraints with known (x) and unknown $(\widetilde{\xi})$ variables. In CCP, it is not necessary to adhere to the random constraints all of the time. Random constraints are constraints that contain random, or uncertain, variables. Rather, random constraints are formulated so that the probability of meeting that constraint is above a certain percentage $\alpha \in (0,1)$ (Masmoudi and Abdelaziz, 2018; Nemirovski and Shapiro, 2007). Thus when employing CCP, the problem given by Equation (2.7) would become:

$$\underset{x \in X}{\text{Min}}\, f(x) \quad \text{subject to} \quad \text{Prob}\Big(F(x, \widetilde{\xi}) \leqslant 0\Big) \geqslant 1 - \alpha \tag{2.8}$$

Yet, CCP problems may be computationally intractable, meaning that there is no efficient algorithm available to solve these problems. One way in which CCP problems can be built to be computationally tractable is to employ the scenario approach, which is based on Monte Carlo sampling techniques. In this method, a sample of N independent observations are drawn from the known distribution of the random variable. Thus, for the problem given in (2.8), the sample would look as follows: $\xi^1, ..., \xi^N$. For each value in the sample, a constraint is added to the problem to account for the uncertainty (Nemirovski and Shapiro, 2007; Sahinidis, 2004). Thus, the problem given in (2.8) becomes:

$$\underset{x \in X}{\text{Min}}\, f(x) \quad \text{subject to} \quad F(x, \xi^\nu) \leqslant 0, \nu = 1, ..., N \tag{2.9}$$

Thus, rather than having one single constraint, the problem will have N constraints and will thereby account for uncertainty in the random constraint.

CCP has been applied within the field of portfolio selection by Abdelaziz et al. (2007), Abdelaziz et al. (2009), and Masmoudi and Abdelaziz (2017). As with the EV method, CCP is highly dependent on the historical distributions of the unknown variables. Given that it is known that the unknown variables in this dissertation follow skew-normal, uniform or triangular distributions and that the data required to determines these is readily available, CCP is an appropriate technique to address the uncertainty in the portfolio selection problem. Thus, CCP will be used in this dissertation.

After reviewing the three stochastic programming methods that can be used in portfolio selection, it was decided that the EV method and CCP would be used in this dissertation. Yet, stochastic programming methods are only employed so that a problem can be formulated in such a way that the uncertainty is accounted for and the problem can be solved. Stochastic programming is not used to solve the problem. Thus in order to obtain a solution to the formulated model, model solution approaches must be used.

2.4 Model solution

Once a MOO problem has been formulated, exact or approximation methods can be used to solve the problem. Exact approaches include branch and bound, branch and cut, and others (Applegate et al., 2007). Approximation approaches include evolutionary methods, such as a Nondominated Sorting Genetic Algorithm (NSGA), genetic algorithm with an aggregating function, customised local search, simulated annealing, tabu search and a genetic algorithm (Coello et al., 2007). Exact methods are always preferred when solving optimisation problems because they produce a globally optimal solution. However, realistically it is not always possible to find an exact optimal solution. In such cases, approximation approaches are employed.

Although both methods can be employed to solve portfolio selection problems, Aouni et al. (2018) argue that when solving portfolio selection problems, exact methods are preferred. They attributed this to the fact that in the world of financial portfolios, where billions of rands hang in the balance, most financial managers prefer not to add algorithm risk to the many other risks that need to be taken into account. Thus, exact methods are preferred in portfolio selection.

Many mathematical optimisation software programs, such as `LINGO`, `ZIMPL`, `AIMMS`, `GAMA` and `GAMS`, have built-in solvers that are capable of solving MOO problems. These programs evaluate the problem that is to be solved and decide whether an exact or approximation approach should be employed to solve the problem. For this dissertation, such a program was used to solve the generic and individualised models.

The portfolio selection models that exist in literature are solved using exact or approximation approaches. Yet, the majority of these existing models remain unvalidated and are thus impractical. In order to state that a model is useful, it must be validated. Thus, model validation is an essential component of model development.

2.5 Model validation

Model validation is the process of determining whether or not a model is performing as expected and whether the model outputs have a satisfactory accuracy given its intended use. Models are developed and built for a specific purpose, and the validity of the models should be determined with regard to the intended purpose (Pidd, 2010; Sargent, 2013). In reality, no model is entirely accurate, and thus model validation is performed to determine if a model is sufficiently accurate to provide meaningful results (Semini, 2011).

The significance of models, within many different contexts, lies in their use as decision support tools (Sargent, 2013; Thacker et al., 2004). Thus, using unvalidated models may lead to decision-makers having false confidence in the model, and thus making incorrect decisions, which may lead to disastrous consequences (Ivanescu et al., 2016). These consequences include arriving at incorrect conclusions, incurring increased costs, and reduced model use, to name a few (Pidd, 2010; Strandhagen, 1994).

The majority of existing portfolio selection models are considered unvalidated because they are developed and solved using historical data, and are then not tested on the unknown future. Noteworthy exceptions are the works published by Dastkhan et al. (2013) and Musto et al. (2015). In the article by Dastkhan et al. (2013), the portfolio selected from the New York Stock Exchange using historical share data (January 2005 – December 2006) was tested on an unknown future (January 2007 – December 2008). In the article by Musto et al. (2015), the expost performance of the suggested portfolios was evaluated. Yet, the prevailing trend is that portfolio selection models are unvalidated. Thus, it can not be stated that these models are suitable for their intended purpose, which is to select investment portfolios that will achieve the future investment goals of an investor. If the model is invalid, an unsuitable portfolio may be selected for the investor.

Sargent (2013) identifies 17 techniques that are commonly used to verify and validate models. These are animation, comparison to other models, data relationship correctness, degenerate tests, event validity, extreme condition test, face validity, historical data validation, internal validity, multistage validation, operational graphics, parameter variability-sensitivity analysis, philosophy of science methods, predictive validation, structured walkthrough, trace and the Turing test. In general, a combination of techniques is used. The techniques that should be used are selected based on the context of the problem being considered.

Given that the existing models are not tested on an unknown future, and thus it can not be stated that these models are useful for their intended purpose, they are considered to be unvalidated in this dissertation. As the aim is to compare and evaluate the *future* performance of the portfolios selected by the portfolio selection models, a validation method is required that can facilitate this future performance evaluation. Of all these techniques, historical data validation is the only technique that can accommodate testing on an unknown future and is thus the technique that will be used. Furthermore, this technique is used if historical data is available for the system being modelled (Sargent, 2013). Given that there is an abundance of historical information available for companies, historical data validation can be employed and is thus a suitable validation technique for the portfolio selection problem.

In historical data validation, historical data is collected with the intention of building and testing a model. The collected data is divided into two sets, known as the training and testing datasets. A training dataset is a set of data used to create a model and teach it how to achieve its intended purpose. A testing dataset is the set of data, independent of the training data, that is used to assess the efficacy of the model and its performance (James et al., 2013; Kuhn and Johnson, 2013).

James et al. (2013) explains that, in general, model developers and users are not interested in how well a model performs when using the training data, but are rather interested in the performance of the model when a previously unseen dataset, the testing data, is used. As portfolio selection models aim to select investment portfolios capable of achieving the future investment goals of investors, it is the future performance of the selected portfolios that is of interest and should be evaluated. Given this knowledge, historical data validation is the technique that will be used in this dissertation, and the future performance of the selected portfolios will be evaluated.

2.6 Concluding remarks

The portfolio selection problem is a widely-established problem aimed at selecting investment portfolios that will achieve some predefined investment objectives. Investors invest to achieve some *future* investment goals, and as such, it is deduced that the portfolio selection problem aims to select investment portfolios that will achieve these future goals. Four objectives should be accounted for in the portfolio selection problem, namely, to maximise the random ROR, to maximise liquidity, to minimise risk and to maximise SRI. Generally, the existing portfolio selection models do not incorporate all of these objectives. Furthermore, it could be that the current models fail to account for the individualised goals of investors, especially with regard to the risk objective. To this end, a new individualised risk objective based on the match between the market risk of selected companies and the financial risk tolerance of the investor is proposed.

Solving a problem with multiple objectives requires using MOO. Although many MOO methods can be used to solve portfolio selection models, Aouni et al. (2018) found that goal programming is the most used method within this context. Furthermore, goal programming allows for the incorporation of the individualised investment goals of investors as the ideal values can be set to these goal values, as proposed by Abdelaziz et al. (2005).

To address the uncertainty within the portfolio selection problem, especially with regard to the ROR objective, stochastic programming is used. Masmoudi and Abdelaziz

(2018) identified three stochastic programming methods which can be used in portfolio selection models, namely, the EV method, recourse programming and CCP. Given that all the data required for the use of the EV method and CCP is readily available, and the fact that recourse programming requires additional information that is not available, it was decided that only the EV method and CCP would be used in this dissertation.

Generally, existing portfolio selection models are not validated as the future performance of the selected investment portfolios are not evaluated. Given that portfolio selection aims to select investment portfolios that will achieve some predefined future investment goals, evaluating the future performance of the selected portfolios is paramount to determining whether or not the model is useful for this purpose. It was found that the validation method that Sargent (2013) names historical data validation is the most suited method for validating a portfolio selection problem. This method uses a training set of data to build the model and then tests the model performance using an independent testing dataset.

The next chapter presents the portfolio selection models that were developed in this dissertation. Furthermore, it is shown how the selected MOO, stochastic programming and model validation techniques were applied to these models.

Chapter 3

Model development

In this dissertation, three portfolio selection models will be considered: A generic risk-return Markowitz model that incorporates liquidity, a model that incorporates the Rate Of Return (ROR), individualised risk and liquidity objectives, and a model that incorporates the ROR, individualised risk, liquidity and Socially Responsible Investing (SRI) objectives. The generic Markowitz model that incorporates liquidity will act as a baseline for the comparison between a generalised and individualised portfolio selection approach. These models account for the multiple objectives by employing goal programming and address the stochastic elements using both the Expected Value (EV) and Chance-Constrained Programming (CCP) stochastic programming methods. These models will be validated using the historical data validation method and the future performance of the portfolios selected using these models will be evaluated and compared. This chapter also discusses how these models are applied within a South African context, explaining how the required data was collected, how an appropriate sample of companies was selected, and how the required distributions were determined. Finally, the software packages capable of solving the models are explored, and a method for evaluating the model results is presented. The first step in this process is determining for which time period the models should be developed, built and solved.

3.1 Evaluation period

Portfolio selection models are time-dependent as many of the variables are determined over a specified time frame. What this time frame should be is unknown.Furthermore, Revelli and Viviani (2015) show that the time frame for which an investment is made influences the financial performance of the investment. The investment world is plagued with uncertainty, and although a selected investment portfolio may be suitable for an investor at present, the unpredictable changes in the investment market may render the selected portfolio unsuitable in the future. In essence, this implies that over time, some of the companies in an investment portfolio will become more attractive, while others will become less attractive. As an investment portfolio is constructed based on the investor's goals, such financial shifts within the portfolio over time may lead to unsatisfactory investment performance for the investor. To prevent such a situation, investment professionals perform portfolio rebalancing.

Portfolio rebalancing is the process of re-evaluating an investment portfolio and selling companies that have increased in relative value and buying companies that have decreased in relative value (Willenbrock, 2011). This raises the questions of how long an investor should wait between investing in a selected portfolio and rebalancing it. This time between investing in an investment portfolio and rebalancing it is known as the holding period.

The South African Revenue Service (SARS) considers the buying and selling of shares to involve short holding periods of up to and including three years (South African Revenue Service, 2020). Investments go through cycles. At times, certain investments may appear attractive, but due to changing market conditions and world events, these investments lose their appeal and become unattractive after some time. Yet, market conditions continue to change and may improve over time, resulting in previously unattractive investments reverting to being attractive investments. Thus, rebalancing an investment portfolio too quickly, and not waiting for the markets to recover, may result in avoidable losses for the investor. Furthermore, rebalancing a portfolio can be an arduous and time-consuming task. As such, portfolio rebalancing should be performed only as often as is necessary to ensure that the investor investment goals are achieved.

For these reasons, it was decided that the maximum SARS trading holding period of three years will be used in this dissertation. Thus, the models were built, solved and evaluated using three-year time periods. Furthermore, as historical data validation is used, two independent three-year datasets are required. The three-year periods that were used are 2014/01/01 – 2016/12/31 (training set) and 2017/01/01 – 2019/12/31 (testing set). The data that was used as well as how it was obtained is discussed in Section 3.4.1

3.2 Mathematical models

This section presents the mathematical formulation for the three models that were considered in this dissertation, namely:

- The Markowitz model as proposed by Abdelaziz et al. (2005) that takes into account the standard objectives relating to ROR, liquidity and risk.
- The risk-adjusted model which has the standard ROR and liquidity objectives as well as the individual risk objective proposed in this dissertation.
- The social model that has the standard ROR and liquidity objectives, the individualised risk objective proposed in this dissertation and the SRI objective described in section 2.1.4.

The sets, variables and parameters defined below are used in all three models

Let

$\boldsymbol{J}$ be the set of the sample of N JSE Limited companies such that $\boldsymbol{J} = \{1, ..., N\}$

$\boldsymbol{T}$ be the set of years for which an investment portfolio is selected such that $\boldsymbol{T} = \{0, 1, 2, 3\}$

The decision variable is then:

$w_j \triangleq$ the proportion of the portfolio invested in company $j \in \boldsymbol{J}$

The auxiliary variable x_j is defined as follows:

$$x_j \triangleq \begin{cases} 1 \text{ if company } j \in \boldsymbol{J} \text{ is selected as part of the investment portfolio} \\ 0 \text{ otherwise} \end{cases}$$

The following variables and parameters are defined:

$\widetilde{r_j} \triangleq$ the three-year ROR (%) received by company $j \in \boldsymbol{J}$ during period $[0, 3]$

$\widetilde{d_j} \triangleq$ the three-year dividends (Rands/share) received by company $j \in \boldsymbol{J}$ during period $[0, 3]$

$e_j \triangleq$ the three-year exchange flow ratio (%) of company $j \in \boldsymbol{J}$ during period $[0, 3]$

$\sigma_{i,j} \triangleq$ the covariance between r_i and r_j during period $[0, 3]$

$c_j \triangleq$ the three-year CVaR (%) of company $j \in \boldsymbol{J}$ during period $[0, 3]$

$s_{j,t} \triangleq$ the ESG rating of company $j \in \boldsymbol{J}$ at time $t \in \boldsymbol{T}$

$\widetilde{p_{j,t}} \triangleq$ the closing price of the shares of company $j \in \boldsymbol{J}$ at time $t \in \boldsymbol{T}$

$n_j \triangleq$ the number of days that shares of company $j \in \boldsymbol{J}$ where traded during period $[0, 3]$

$n_m \triangleq$ the number of days that the JSE (market) was trading shares during period $[0, 3]$

$R \triangleq$ the ideal three-year ROR (out of one) during period $[0, 3]$

$E \triangleq$ the ideal exchange flow ratio (out of one) value during period $[0, 3]$

$C \triangleq$ the ideal portfolio covariance value during period $[0, 3]$

$T \triangleq$ the financial RTS of the investor

$S \triangleq$ the ideal ESG rating at the end of period $[0, 3]$

As explained in Chapter 2, the portfolio selection problem should consider four objectives, as given below:

$$\text{Maximise portfolio ROR} \tag{3.1}$$

$$\text{Maximise portfolio Liquidity} \tag{3.2}$$

$$\text{Minimise portfolio risk} \tag{3.3}$$

$$\text{Maximise portfolio ESG rating} \tag{3.4}$$

To solve this multi-objective problem, goal programming is used. Each of the objectives is transformed into a constraint, and a new objective function is defined. These constraints aim to achieve some ideal value, which is set by the user of the model. In the portfolio selection context, this user is the investor for whom an investment portfolio is being selected. These ideal values are the ROR, liquidity, risk and ESG rating that the investor would like their portfolio to achieve. As an example, consider an investor who would like to achieve at least a 100% ROR on their investment. The ROR objective given by Equation (3.1) is changed into the constraint given by Equation (3.5).

$$\text{Portfolio ROR} \geqslant 100\% \tag{3.5}$$

Similarly, the other three objectives are transformed into constraints. There is, however, the possibility that no solution exists for which all the ideal values can be achieved. For this purpose, deficiency variables are introduced into the formulation of the constraints. These deficiency variables measure the degree to which the constraint does not meet the goal. As an example, once again consider the ROR goal. Adding a deficiency variable to Equation (3.5), yields Equation (3.6).

$$\text{Portfolio ROR} + \text{deficiency}_1 \geqslant 100\% \tag{3.6}$$

Similarly, deficiency variables are incorporated into the other three constraints. The new objective function minimises the sum of all the deficiency variables in the model, thereby aiming to satisfy all the ideal values as closely as possible.

For the models considered in this dissertation, the following deficiency variables are defined:

$\delta_1 \triangleq$ the value by which the portfolio ROR falls short of the goal

$\delta_2 \triangleq$ the value by which the portfolio liquidity falls short of the goal

$\delta_3 \triangleq$ the value by which the portfolio risk exceeds the goal

$\delta_4 \triangleq$ the value by which the portfolio ESG rating falls short of the goal

The following sections present how the problem is formulated for the Markowitz model, the risk-adjusted model and the social model.

3.2.1 Markowitz model

The problem is formulated as follows:

$$\min z = \delta_1 + \delta_2 + \delta_3 \tag{3.7}$$

subject to

$$\sum_{j \in \boldsymbol{J}} w_j \widetilde{r_j} + \delta_1 \geqslant R \tag{3.8}$$

$$\sum_{j \in \boldsymbol{J}} w_j e_j + \delta_2 \geqslant E \tag{3.9}$$

$$\sum_{i \in \boldsymbol{J}} \sum_{j \in \boldsymbol{J}} w_i w_j \sigma_{i,j} - \delta_3 \leqslant C \tag{3.10}$$

$$\widetilde{r_j} = \frac{\widetilde{p_{j,3}} - p_{j,0} + \widetilde{d_j}}{p_{j,0}} \qquad \forall j \in \boldsymbol{J} \tag{3.11}$$

$$e_j = \frac{n_j}{n_m} \qquad \forall j \in \boldsymbol{J} \tag{3.12}$$

$$\sum_{j \in \boldsymbol{J}} x_j \geqslant 30 \tag{3.13}$$

$$\sum_{j \in \boldsymbol{J}} w_j = 1 \tag{3.14}$$

$$x_j \leqslant 1\,000\, w_j \qquad \forall j \in \boldsymbol{J} \tag{3.15}$$

$$w_j \leqslant 0.05\, x_j \qquad \forall j \in \boldsymbol{J} \tag{3.16}$$

$$w_j \geqslant 0 \qquad \forall j \in \boldsymbol{J} \tag{3.17}$$

$$x_j \in \{0, 1\} \qquad \forall j \in \boldsymbol{J} \tag{3.18}$$

$$\delta_1, \delta_2, \delta_3 \geqslant 0 \tag{3.19}$$

The objective function expressed in (3.7) aims to minimise the value out of one (as ROR, exchange flow ratio and risk are values out of one), by which the goals are not met. Equations (3.8)– (3.10) calculate the extent to which the solution falls short of each of the three goals. Equations (3.11) and (3.12) calculate the ROR and exchange flow ratio for each company respectively. To ensure that a diversified portfolio is selected the model is constrained to ensure that there are at least 30 companies in the selected portfolio, given by Equation (3.13). Equation (3.14) ensures that all the weights add up to one. Without Equations (3.15) and (3.16), the model can select a company as part of the portfolio (e.g. $x_1 = 1$) but then decide that no money should be invested in this company ($w_1 = 0$). To enforce that if a company is selected some proportion of money must be invested in that company, Equations (3.15) and (3.16) are added to the formulation. Furthermore, Equation (3.16) limits the value of any weight to five or less percent of the total capital invested. When reviewing existing investment options, such as unit trusts, it was discovered that the highest proportion invested in any single company was five percent. Furthermore, Hallerbach et al. (2004) impose the restriction that a maximum investment proportion of 5% may be invested into a single company. Thus, the upper bound of the weights was set to five percent. Equations (3.17), (3.18) and (3.19) are the non-negativity and binary constraints.

As discussed in Chapters 1 and 2, this model is not suitable to select portfolios for the individualised investors as it does not account for their financial risk tolerance. As such, this model must be adjusted to incorporate financial risk tolerance. This risk-adjusted model is given in the next section.

3.2.2 Risk-adjusted model

In the risk-adjusted model, the formulation as given in Section 3.2.1 remains the same, with the exception of Equation (3.10), which is replaced by Equation (3.20) as given below.

$$\sum_{j \in \boldsymbol{J}} w_j c_j - \delta_3 \leqslant \frac{T - 20}{49} \tag{3.20}$$

Equation (3.20) ensures that the risk associated with the portfolio, calculated as the weighted sum of the CVaR values of all the companies, is less than the normalised financial risk tolerance of the investor.

Although this model accounts for the individualised goals and financial risk tolerance of the investor, it does not allow an investor to be Socially Responsible (SR) in their investments. A model that incorporates the financial risk tolerance of the investor and selects SR companies is given in the next section.

3.2.3 Social model

In the social model, the formulation as given in Section 3.2.1 is adjusted to include an additional objective. For this purpose, an additional deficiency variable is required and the objective function as given in Equation (3.7) is replaced with Equation (3.21) as given below. As before, the objective function aims to minimise the value by which the goals are not met.

$$\min z = \delta_1 + \delta_2 + \delta_3 + \delta_4 \tag{3.21}$$

As with the risk-adjusted model, the traditional risk objective given by Equation (3.10) is replace by the individualised risk objective given in Equation (3.20).

To incorporate the SRI objective into this formulation the constraints given in Equations (3.22), (3.23) and (3.24) are added to the constraints in the Markowitz model formulation.

$$\sum_{j \in \boldsymbol{J}} w_j \left(\frac{s_j - s_{min}}{s_{max} - s_{min}} \right) + \delta_4 \geqslant \frac{S - s_{min}}{s_{max} - s_{min}} \tag{3.22}$$

$$s_{max} = \max_{j \in \boldsymbol{J}} \{s_{j,3}\} \tag{3.23}$$

$$s_{min} = \min_{j \in \boldsymbol{J}} \{s_{j,3}\} \tag{3.24}$$

$$\delta_1, \delta_2, \delta_3, \delta_4 \geqslant 0 \tag{3.25}$$

Equation (3.22) calculates the degree to which the investment portfolio does not fulfil the SRI goal of the investor. ESG ratings are not always given as a value between zero and one and as such, normalisation is required. This normalisation is done in Equation (3.22), resulting in δ_4 being a value between zero and one. Unlike with the RTS values, the maximum and minimum ESG rating values are unknown, and are thus determined using Equations (3.23) and (3.24). Equation (3.25) is the non-negativity constraint for the four deficiency variables.

The next section shows how the uncertain variables in Equations (3.8) and (3.11) are converted to their deterministic equivalents using the EV method and CCP.

3.3 Incorporating stochasticity

When using the EV method, the uncertain variables are replaced by the EV of those variables. Using this method, the following constraints are obtained:

$$\sum_{j \in \boldsymbol{J}} w_j E(r_j) + \delta_1 \geqslant R \tag{3.26}$$

$$E(r_j) = \frac{E(p_{j,3}) - p_{j,0} + E(d_j)}{p_{j,0}} \qquad \forall j \in \boldsymbol{J} \tag{3.27}$$

Where $E(r_j)$, $E(p_{j,t})$and $E(d_j)$ are the expected values of the random ROR, price and dividends variables respectively. The versions of the models that use Equations (3.26) and (3.27) are referred to as the EV models.

With CCP, multiple instances of a probabilistic variable are sampled for the uncertain variable from a known distribution. The number of instances I required to ensure that the optimal solution is feasible, at the specified reliability, is determined using Equation (3.28).

$$I = \left\lceil 2n\alpha^{-1} \ln \left(\frac{12}{\alpha} \right) + 2\alpha^{-1} \ln \left(\frac{2}{\Delta} \right) + 2n \right\rceil \tag{3.28}$$

Where n is the number of unknown variables in the model (Nemirovski and Shapiro, 2007). Furthermore, it was decided that the solution should adhere to the constraints 90% of the time ($\alpha = 0.1$), at a 95% reliability level ($\Delta = 0.05$). These parameter values were selected because Pagnoncelli et al. (2009) found that the best candidate solutions for the portfolio selection problem were found at these values.

If $\boldsymbol{B}$ is the set of I instances required, the deterministic equivalents of Equations (3.8) and (3.11) are:

$$\sum_{j \in \boldsymbol{J}} w_j {r_j}^b + {\delta_1}^b \geqslant R \qquad \forall b \in \boldsymbol{B} \tag{3.29}$$

$${r_j}^b = \frac{{p_{j,3}}^b - p_{j,0} + {d_j}^b}{p_{j,0}} \qquad \forall j \in \boldsymbol{J} \qquad \forall b \in \boldsymbol{B} \tag{3.30}$$

Both of the EV method and CCP are applied to each of the three models given in Section 3.2, resulting in a total of six models being produced.

The model formulation is now complete, and uncertainty has been addressed, yet, there is still uncertainty about what distributions the price, dividends and ROR variables follow. After investigation, conducted as explained in Section 3.4.1, it was found that these variables follow skewed-normal, uniform, or triangular distributions. Examples of these distributions can be seen in Appendix A.

It is now necessary to solve the models using suitable mathematical optimisation software. Once the models have been solved, it is necessary to evaluate the effect of the existing and proposed models on portfolio performance. For this purpose, the models were created, solved and tested within a South African context using data from companies listed on the JSE between 2010 and 2019.

3.4 Application to JSE listed companies

This section presents how the required data was collected, and a procedure for selecting the sample of companies considered by the models is given. Furthermore, it is explained how the ideal values for the goal programming formulations were determined. Various software packages capable of solving the models were explored and the most suitable option was used. Finally, a method for evaluating the performance of the models is presented.

3.4.1 Data collection and sample selection

In South Africa, publicly listed companies must, by law, make all pertinent investment information publicly available to all potential investors. Thus, share data is readily available on many public platforms such as `Google Finance` and `Yahoo Finance`. This information can be extracted into a usable format using specific functions within certain programming languages.

The `getSymbols` function, contained within the `Quantmod` package in `R` is designed for this purpose (R Core Team, 2020; Ryan and Ulrich, 2020). This function looks as follows:

```
getSymbols(symbol, from, to, src)
```

where `symbol` is the unique code, known as a ticker, used to identify a listed company on an exchange market, `from` and `to` are used to specify the time period of interest, and `src` specifies the source of the information. At present, this function can only be used to extract information from `Yahoo Finance`. As an example consider extracting the share data of the JSE listed company Anglo American Platinum Limited from `Yahoo Finance` for the training period (2014/01/01 – 2016/12/31).

```
getSymbols(symbol="AMS.JO", from="2014-01-01", to="2016-12-31", src="yahoo")
```

Running this function produces a data frame that contains the open, high, low, close, adjusted close prices, as well as the trading volume data for every day in the specified time period for the company being considered. This is the method used to extract all the share data in this dissertation. Additional investment information, such as the ESG data, can be obtained directly from the JSE.

To use this method of data collection, it is necessary to know which companies are being considered in the analysis. Furthermore, to select well-diversified portfolios that are capable of achieving the goal programming goals, the models must consider an appropriate sample of companies. The sample of companies was selected using the following procedure:

1. A list of the 344 companies listed on the JSE was obtained from the JSE website.

2. Using the `getSymbols` function in `R`, it was discovered that only 326 of the listed companies had their share data published on `Yahoo Finance` (R Core Team, 2020; Ryan and Ulrich, 2020). Thus, the sample was decreased to 326 companies.

3. As stochastic programming requires the use of probability distributions, specifically the three-year ROR distributions in this dissertation; it is necessary to have sufficient data to determine such distributions for each company. As such, various ROR values, for several three-year time periods are required. For this reason, it was decided that only companies that have been listed for at least ten years (2010/01/01 – 2019/12/31) would be considered, resulting in a sample of 208 companies.

The next set of data required to build the models was the three-year ROR distributions of the sample companies. As the training period is small, only three years, it is not possible to determine the ROR distributions using only this dataset. For this reason, a larger time period of seven years was considered (2010/01/01 – 2016/12/31). The ROR distributions were then determined as follows:

1. The ROR achieved by each of the 208 companies was determined for 500 randomly selected three-year periods between 2010/01/01 and 2016/12/31, for example:
   - ROR_1 : 2010/01/01 – 2012/12/31
   - ROR_{125}: 2011/06/14 – 2014/16/13
   - ROR_{250}: 2012/04/28 – 2014/04/27
   - ROR_{375}: 2013/11/21 – 2016/11/20
   - ROR_{500}: 2014/01/01 – 2016/12/31

2. For each company, these 500 ROR values were averaged to determine the expected ROR value to be used in the EV models.

3. In literature, many portfolio selection authors use the assumption that share returns follow a normal distribution (Abdelaziz et al., 2007; Pagnoncelli et al., 2009; Steuer et al., 2007). For this reason, it was decided to first apply the Anderson-Darling test to determine whether the three-year ROR values of the samples companies are normally distributed. It was found that none of the 208 sample companies have a three-year ROR distribution that is normally distributed.

4. Then for each company:
   - The 500 ROR values were used to determine what the parameters would be if the values followed a skewed-normal, triangular or uniform distribution.
   - Three samples were then drawn from theoretical distributions defined by the parameters.

- The theoretical samples were then compared to the original sample of 500 values using the Kolmogorov-Smirnov test.
- In additional to the Kolmogorov-Smirnov test, visual inspections of the fits were also performed. These, in combination with the p-values calculated in the previous step, were used to determine which of the three distributions should be fitted to each company's ROR distribution.

The exchange flow ratio (e_j), covariance ($\sigma_{i,j}$), CVaR (c_j) and ESG rating ($s_{j,t}$) values for each company in the sample were incorporated into the models in Section 3.2 using the data collected for the training period (2014/01/01 – 2016/12/31). The performance of the selected portfolios were evaluated using the data obtained for the testing period (2016/01/01 – 2019/12/31).

The final set of data that needed to be extracted for this dissertation was the JSE all share index data. It was found that this data was not published on `Yahoo Finance` and could thus not be extracted using the `getSymbols` function. However, it was found that this data is available on `Google Finance` and can be extracted into a useable format using the `GOOGLEFINANCE` function in `Google Sheets`. The `GOOGLEFINANCE` function looks as follows:

```
GOOGLEFINANCE(ticker, attribute, start_date, end_date, interval)
```

where `ticker` is the ticker used to identify a company, or as in this case market index, on an exchange market. The `attribute` is the specific data that the user would like to obtain, such as the opening or closing prices, or the trading volume sold. The `start_date` and `end_date` parameters are used to specify the time period of interest and `interval` is used to specify the frequency of the data; either daily or weekly. Thus to extract the daily closing price data for the JSE all share index in the testing period (2017/01/01–2019/12/31) the function looks as follows:

```
GOOGLEFINANCE("ASX", "close", "2017/01/01", "2019/12/31", "daily")
```

Running this function produces a data frame that contains that closing prices for every day in the specified time period for the market index being considered. This method was used to extract all the JSE all share index data required to evaluate the performance of the models and the model portfolios.

Given that the sample and all required data were no longer unknown elements in the models, the only remaining unknowns were the goal programming ideal values. What these values should be and how they were determined is presented in the next section.

3.4.2 Ideal values

This dissertation aims to compare a generic portfolio selection model to individualised portfolio selection models that incorporate individualised risk and SRI objectives. For this purpose, a collection of "individual" investors were simulated and their investment goals used as the ideal values in the models.

The first set of ideal values that must be specified is R. It is assumed that no investor would ever want to experience a loss when investing thus, $R \geqslant 0$. Furthermore, Bernstein (1997) found that an investor can reasonably expect an ROR of 5.4% per year, which equates to an effective ROR of 17.09% for three years, when investing in a portfolio of shares. Given this information, it was decided to run the models over differing three-year ROR values, to simulate different investors. The ROR values chosen are: $R = \{0\%, 5\%, 10\%, 15\%, 20\%\}$.

Unlike the ROR ideal value, the ideal exchange flow ratio is not set by the individual. Rather this value is calculated by solving Program 1 as proposed by Abdelaziz et al. (2007).

Program 1: Determine the ideal exchange flow ratio value

$$E = max\left\{\sum_{j\in \boldsymbol{J}} w_j e_j\right\} \tag{3.31}$$

subject to

$$e_j = \frac{n_j}{n_m} \quad \forall j \in \boldsymbol{J} \tag{3.32}$$

$$\sum_{j\in \boldsymbol{J}} x_j \geqslant 30 \tag{3.33}$$

$$\sum_{j\in \boldsymbol{J}} w_j = 1 \tag{3.34}$$

$$x_j \leqslant 1\,000\, w_j \quad \forall j \in \boldsymbol{J} \tag{3.35}$$

$$w_j \leqslant 0.05\, x_j \tag{3.36}$$

$$w_j \geqslant 0 \quad \forall j \in \boldsymbol{J} \tag{3.37}$$

$$x_j \in \{0, 1\} \quad \forall j \in \boldsymbol{J} \tag{3.38}$$

Where E is the maximum value exchange flow ratio observed for any possible portfolio chosen from the sample companies. Solving this program for the sample of 208 companies yields an exchange flow ratio ideal value of $E = 1$.

The next set of ideal values that need to be specified are the risk goals. For the Markowitz model, C is defined as the maximum portfolio covariance that can be accepted by the model without penalising the objective function. Thus, the C value is dependent on the covariance of the dataset being considered. When following a similar approach as used in Program 1, it was found that the minimum C value that could be obtained if all other objectives were ignored, was 9×10^{-5}. When solving the model at this "ideal" C value, it was found that the model was unsolvable (many of the w_j values were negative). However, the distribution of $\sigma_{i,j}$ was very wide (minimum $= -2\,107$; maximum $= 406\,842$; mean $=$ 13; median $= 9 \times 10^{-3}$). Thus, through a process of elimination it was found that the models are solvable at a C value of 1. Thus, through this procedure and to ensure that the risk goal does not unduly dominate the other objectives, the single ideal value $C = 1$ was used in the Markowitz model.

For the risk-adjusted model and social model, an investor's RTS value is used as the ideal risk value. RTS values range from 20 to 69, resulting in 50 values that must be simulated to represent all possible investors. This results in the following ideal values: $T = \{20, 21, 22, 23, ..., 69\}$.

The final ideal value that must be specified is the ideal ESG rating. The JSE specifies that if a company carries an ESG rating of 2.5 or more, it is considered to be an SR company, thus $S = \{2.5\}$ (JSE Limited, 2019a).

For the Markowitz model, with five ideal values for R, one ideal value for E and one ideal value for C, a total of five individual investors were simulated ($5R \times 1E \times 1C$). For the remaining two models, with E and S fixed, the models were executed for all enumerations of R and C resulting in 250 investors being simulated ($5R \times 1E \times 50T \times 1S$).

It is assumed that for each simulated investor, the investment goals remained unchanged from the time of portfolios selection (last day of the training period) until the end of the testing period (2016/01/01 – 2019/12/31). As investors specify the goals that they would like to achieve in the future when they select portfolios in the present, this is a valid assumption, and will not affect the usability of the models.

For each of the three models, the EV method and CCP was applied to address the stochastic elements. This results in a total of ten investment portfolios being selected by the Markowitz model (five investors × two methods). Furthermore, the risk-adjusted and social models each select 500 portfolios (250 investors × two methods). Thus, in total, 1 010 investment portfolios were selected.

Given the complexity of the problem, mathematical optimisation software is required to solve the model. The next section discusses different mathematical optimisation software packages that are available and the software that was selected and used in this dissertation.

3.4.3 Mathematical optimisation software

Upon investigation, it was found that there are various mathematical optimisation software packages available. These include `LINGO`, `ZIMPL`, `AIMMS`, `GAMA` and `GAMS`. Furthermore, several programming languages such as `R` and `Python` contain specific mathematical optimisation packages, allowing them to solve mathematical models (R Core Team, 2020). Although all of these options are adequate to solve the proposed model, many of them are complex and require advanced programming knowledge and expertise.

`LINGO` is a comprehensive tool that was designed to make building and solving mathematical optimisation models easy and efficient. It is a completely integrated package that includes a powerful language for expressing optimisation models, a graphic environment for building and editing problems, and a set of fast built-in solvers that are capable of efficiently solving the models being built (LINDO Systems Incorporated, 2018). Furthermore, it has built-in stochastic functions, allowing for the integration of stochastic variables and constraints. `LINGO` requires a license, but a full licensed version can be obtained from the developers for free, given that it will be used for academic purposes.

Given that it is easy to use, is readily available for academic research, and has all the functionality required to solve the models given in Section 3.2, it was decided that `LINGO` would be used in this dissertation. The latest version (`LINGO` 18.0) was released in November 2018, and is the version used to solve the models in this dissertation.

`LINGO` solves models using a range of solvers. Two that are applicable to this dissertation are the `Global Solver` and the `Multistart Solver`. The `Global Solver` was selected for use as it aims to find globally optimal solutions. However, in some instances, finding a globally optimal solution is not possible. In such a case, `LINGO` initiates the `Multistart Solver`. This solver generates a set of potential starting points within the solution space. A nonlinear solver is then initiated at each starting point to find the local optimum. For each multi-start, the local optimum achieved is compared to the previous local optima achieved, and it is determined which of these local optima is better. This local optimum becomes the current best solution. This process continues until the best-proven solution is found, or until the user-specified maximum number of multi-starts has been performed. Thus, the `Multistart Solver` produces an approximate solution.

The models were solved in `LINGO` on an HP Elitebook 850 computer with an intel i7 core. It was found that it takes the EV versions of the risk-adjusted and social models between 450 and 500 iterations to solve for each simulated investor. Furthermore, for these two models, LINGO required between 10 and 15 seconds to find the globally optimal solution for each simulated investor. The EV Markowitz model also took between 10 and 15 seconds, and between 1 920 and 2 500 iterations to find the globally optimal solution for each of the simulated investors. For the CCP versions of the risk-adjusted and social models LINGO required between 200 and 300 iterations to find the globally optimal solution. Furthermore, for these two models, it took between 1.8 and 4 minutes for LINGO to find the solution, for each simulated investor. For the CCP Markowitz model, LINGO used the `Global Solver` and the `Multistart Solver`. For the five investors that were simulated, $R = 0$, 5, 10, 15 and 20, LINGO required 3.5 minutes, 1 hour, 20 hours, 45 hours and 60 hours respectively to solve. These runtimes amounted to between 100 and millions of iterations.

As explained in Section 2.5, the results produced by a model can not be considered useful if the model is not validated. Thus, it is necessary to validate the models in this dissertation and determine whether or not they produce useful and market-competitive results. The next section discusses how the model results were evaluated.

3.4.4 Model evaluation

Investors invest to achieve their *future* investment goals. It was thus necessary to investigate how the selected portfolios performed in the future. Thus, for the models presented in Section 3.2 to be suitable for their intended purpose, the selected portfolios must have met or exceeded the simulated investment goals in the future.

The historical data validation technique was used in this dissertation and requires the use of two independent datasets. For this purpose, the JSE share data and ESG data for the 208 sample companies were collected for the training (2014/01/01 – 2016/12/31) and testing periods (2017/01/01 – 2019/12/31). The models were built and solved using the training dataset, and a total of 1 010 portfolios and their investment proportions were obtained. In the context of this dissertation, as per the historical data validation technique, the models were considered suitable for their intended purpose if the selected portfolios met or exceeded the future goals of the simulated investors. Although four objectives must be considered in the portfolio selection model, an investor only has two goals in the future, namely ROR and SRI. An investor will be satisfied with his/her selected portfolio if the portfolio produces a payout (ROR > 0) and is SR at the end of the investing period. This is true even if the selected portfolio happened to carry low liquidity and high risk after the investment was made. Thus it is evident that even though the liquidity and risk objectives are crucial to ensure that satisfactory portfolios are selected, they hold no bearing on investor satisfaction in the future. Thus, only the ROR and SRI performance of the selected portfolios were evaluated.

To evaluate the future performance of the selected portfolios, it was necessary to determine the ROR and ESG rating values achieved by the selected portfolios in the testing period. These values, henceforth known as the calculated portfolio values, were calculated using the testing dataset and Equations (3.39) – (3.40).

$$\text{Portfolio rate of return} = \sum_{j \in \boldsymbol{J}} w_j r_j \tag{3.39}$$

$$\text{Portfolio ESG rating} = \sum_{j \in \boldsymbol{J}} w_j s_{j,t} \tag{3.40}$$

These calculated portfolio values were then be compared to the simulated investor ROR and SRI goals to determine whether or not the selected portfolios satisfied or exceeded these goals during the testing period. If the models satisfied or exceeded the goals, it was stated that models are suitable for their intended purpose. If the goals were not satisfied, the models were not suitable for the intended purpose.

This dissertation aims to compare a generic portfolio selection model to individualised portfolio selection models that incorporate individualised risk and SRI objectives. In order to accomplish this, the calculated portfolio values of the Markowitz model, the risk-adjusted model and social model were compared. The model that had the best performance in term of ROR and ESG rating was considered the best model to use for portfolio selection.

It is important to remember that all investments are dependent on the conditions of the market during the investment period. Thus it is essential to consider the calculated portfolio values for all the selected portfolios within the context of the South African market during the testing period. The JSE all share index is a South African market index. It consists of 150 JSE-listed companies and is the largest South African index with regards to size and overall value (JSE Limited, 2019b). Thus, the JSE all share index is an excellent indicator of how the South African market performed during a specific time period. For example, suppose the JSE all share index performed poorly during the testing period. In that case, the JSE all share index indicates that the South African market experienced turmoil during that period and thus achieved unsatisfactory performance. The model portfolios were compared to this market indicator to determine how the models and the selected portfolios performed given the South African market conditions. It should be noted that the ESG rating achieved by the JSE all share index is unknown and thus any comparisons with this index were only made in terms of the ROR values achieved.

In this dissertation, a model is considered to be useful if it selects investment portfolios that are market-competitive and/or worthwhile investments. Thus, in the world of investing, even if a model may not be suitable for its intended purpose, the model may still be useful. This is the case if it is still beneficial for an investor to invest in a model portfolio, despite the investment goals not being achieved. It is beneficial for an investor to invest in a model portfolio if that portfolio is a market-competitive or worthwhile investment. A model portfolio is a market-competitive investment if it achieves an ROR that is the same or greater than the ROR achieved by the market. Furthermore, a model portfolio is a worthwhile investment if it achieves an ROR that is the same or greater than existing investment options, such as unit trusts. Thus, in addition to determining the validity of the models, it was necessary to determine whether or not they produce market-competitive and worthwhile results. For this purpose, the calculated portfolio values were compared to the performance of the JSE all share index and various unit trusts in the testing period.

Unit trusts are appropriate investment options to use to determine whether or not the models produce worthwhile portfolios because they are developed and managed with the same aims as the portfolio selection models presented in this chapter. These aims are to maximise ROR, maximise liquidity and to minimise risk. Some unit trusts do account for SRI as well by applying negative screening or ESG integration, but this is not standard practice. The results of unit trusts are reported only in terms of applicable risk category and the ROR that was achieved. Thus, any comparison between the results of model portfolios and the results of the unit trusts can only be made with regards to the ROR and risk objectives.

Twenty-five unit trusts from varying risk categories were selected for this comparison. These unit trusts were selected from five of South Africa's biggest unit trust companies, one from each company for each of the five risk categories explained in section 2.1.3. These 25 unit trusts and the testing period ROR achieved, after fees, are given in Table 3.1.

Table 3.1: The unit trusts, with their ROR values, used in the comparison

Risk category	Unit trust name	Three-year ROR achieved (%)
Conservative	Allan Gray Optimal Fund (Allan Gray, 2019c)	−5.59
	Coronation Money Market Fund (Coronation, 2019d)	25.27
	Momentum Money Market Fund (Momentum, 2019d)	24.23
	Prudential High Yield Bond Fund (Prudential, 2019c)	29.50
	Stanlib Enhanced Yield Fund (Stanlib, 2019c)	25.55
Moderately conservative	Allan Gray Stable Fund (Allan Gray, 2019e)	20.12
	Coronation Balanced Defensive Fund (Coronation, 2019a)	20.46
	Momentum Diversified Income Fund (Momentum, 2019b)	27.02
	Prudential Enhanced Income Fund (Prudential, 2019d)	23.54
	Stanlib Balanced Cautious Fund (Stanlib, 2019a)	17.76
Moderate	Allan Gray Balanced Fund (Allan Gray, 2019a)	14.77
	Coronation Balanced Plus Fund (Coronation, 2019b)	19.10
	Momentum Odyssey Moderate Aggressive (Momentum, 2019e)	16.76
	Prudential Balanced Fund (Prudential, 2019e)	20.12
	Stanlib Balanced Fund (Stanlib, 2019b)	17.49
Moderately aggressive	Allan Gray-Orbis Global Fund of Funds (Allan Gray, 2019d)	12.81
	Coronation Market Plus Fund (Coronation, 2019c)	16.43
	Momentum Aggressive Growth (Momentum, 2019a)	13.46
	Prudential Enhanced SA Property Tracker Fund (Prudential, 2019a)	−14.26
	Stanlib Global Balanced Feeder Fund (Stanlib, 2019e)	39.67
Aggressive	Allan Gray Equity Fund (Allan Gray, 2019b)	11.19
	Coronation Top 20 Fund (Coronation, 2019e)	20.12
	Momentum Equity Fund (Momentum, 2019c)	13.20
	Prudential Equity Fund (Prudential, 2019b)	15.76
	Stanlib Equity Fund (Stanlib, 2019d)	16.96

3.5 Concluding remarks

Investment portfolios are selected for a specific time frame, yet it is not clear what this time frame should be. In this chapter, it was decided that a holding period of three years, as suggested by SARS, would be used. Thus, all the models and their variables and parameters were formulated for a period of three years.

Three models are considered, the Markowitz model, the risk-adjusted model and the social model. The Markowitz model has the standard objectives relating to ROR, liquidity and risk. The risk-adjusted model has the standard ROR and liquidity objectives and the individualise risk objective proposed in this dissertation. Lastly, the social model has the standard ROR and liquidity objectives, the proposed individualised risk objective and the SRI objective. The general sets, variables and parameters used in all three models were defined, and the mathematical formulation for these three models given. Furthermore, stochasticity was incorporated into these models using the EV method and CCP.

The required share data can be extracted from `Yahoo Finance` using the `getSymbols` function in `R` (R Core Team, 2020; Ryan and Ulrich, 2020). Through a process of elimination, a sample of 208 companies was selected. For each of these companies, the three-year ROR distribution was found by comparing the distribution of 500 ROR values to a normal, skewed normal, triangular and uniform distribution and selecting the best fitting distribution. Finally, the required JSE all share index data can be extracted from `Google Finance` using the `GOOGLEFINANCE` function in `Google Sheets`.

The models are formulated using goal programming, where the objectives are converted into constraints which aim to achieve an ideal value. Thus, it is necessary to define these ideal values. As the aim is to compare generic and individualised models, it was decided that a collection of "individual" investors would be simulated and their investment objectives used as the ideal values in the models. Based on research, an appropriate set of investor investment objectives was defined for each of the four objectives.

Various software packages capable of solving the models were investigated. It was decided that `LINGO` would be used as it is easy to use and contains all the functionality required to solve the formulated models.

After solving the models, it was necessary to evaluate the performance of the selected investment portfolios and the validity of the models being considered. For this purpose, the ROR and SRI achieved by the selected portfolios in the testing period was calculated. The calculated portfolio values were compared to the simulated investor goals to determine whether or not the models are suitable for their intended purpose. These calculated portfolio values achieved by the three models were compared to each other so that a comparison could be made between the generic and individualised portfolio selection approaches. Furthermore, the ROR values achieved by the selected portfolios in the testing period were compared to the ROR achieved by the JSE all share index in the same period. This was done so that these results could be understood within the context of the South African market.

Even if the models do not meet the investor goals, they may still produce worthwhile and market-competitive investments portfolios. This is the case if the ROR achieved by selected portfolios is greater than or equal to the ROR achieved by the JSE all share index or unit trusts, in the same period. For this comparison, 25 unit trust trusts were selected.

The validation and comparisons performed in the section produced five different sets of results. These are whether or not the models are suitable for their intended purpose, which of the three models had the best performance, how the models performed within the South African market, whether or not the models select market-competitive portfolios, and whether or not the models select worthwhile portfolios. Furthermore, it was also investigated how the selected model performed in the training period. These results and their implications for the investor are presented and discussed in the next chapter.

Chapter 4

Results and discussion

In this chapter, the results obtained by solving the models in `LINGO` are presented and discussed. It is shown how the selected portfolios performed in the training period, and these results are used to determine whether or not the models are capable of selecting viable investment options. It was then evaluated how the selected portfolios performed in the unknown future. The Rate Of Return (ROR) and Environmental, Social and Governance (ESG) ratings achieved by the selected portfolios in the testing period were calculated and used to determine whether or not the models are suitable for their intended purpose. Furthermore, the future ROR performance of the selected portfolios was compared to the ROR values achieved by the JSE all share index and 25 South African unit trusts in the same period. These comparisons were made so that it could be determined whether or not the models are useful and whether they produce market-competitive and worthwhile investments. Lastly, it was determined whether or not the risk-adjusted and social models did in fact select individualised investment portfolios.

Throughout this chapter and the rest of this dissertation, the following definitions are used:

Viable investment option A model portfolio is a viable investment option if it meets or exceeds the investment goals of the investor, or has negligibly small deficiency variables, during the training period.

Suitable for its intended purpose A portfolio selection model is suitable for its intended purpose if it selects investment portfolios that meet or exceed the ROR and Socially Responsible Investing (SRI) goals of the investor in the testing period.

Market-competitive A model portfolio is market-competitive if it achieves an ROR value that is greater than or equal to the ROR achieved by the JSE all share index.

Worthwhile A model portfolio is a worthwhile portfolio if it achieves an ROR value that is greater than or equal to the ROR achieved by unit trusts.

Useful A portfolio selection model is considered to be useful if it selects investment portfolios that are market-competitive and/or worthwhile.

Take note that these definitions are specific to this dissertation. The models in this dissertation were developed and solved using a single dataset under very specific configurations and assumptions, such as the length of the holding period and the use of equal weights in the goal programming. These configurations and assumptions are considered to be the scenario for this study and are thus henceforth collectively be known as *this scenario*. Other portfolio selection studies done under different scenarios may use other definitions to evaluate their models or find that these definitions are not applicable.

4.1 Companies and portfolios selected based on the training data

Although six different models are presented in Chapter 3, it may not be necessary to evaluate the results of all six these models. This is the case if any of the models selected identical portfolios to another model, and thus had identical results. To this end, this section explores which companies the models selected, how these companies were combined in the selected portfolios and whether the models produced identical, similar or dissimilar portfolios.

4.1.1 Portfolio contributions

When analysing the selected portfolios, it was found that there were many similarities between the selected portfolios. Thus, it was decided to investigate which of the sample companies were selected, and whether or not some of the sample companies were considered to be more important, or of greater significance, than others by the models.

It is argued that if the models invest more heavily in certain companies than they do in others, then it is evident that the models favour certain companies over others, and thus the favoured companies must be better investments. Consider for example a comparison between two companies, company A and company B. If the models invested larger proportions into company A than they did in company B, then it is clear that the models favour company A and thus company A must be a better investment than company B. To determine whether or not certain of the sample companies were better investment options than others, the proportion invested into each company, w_j, was summed over all the portfolios selected by all the models (1 010 portfolios). The results of this summation can be seen in Figure 4.1.

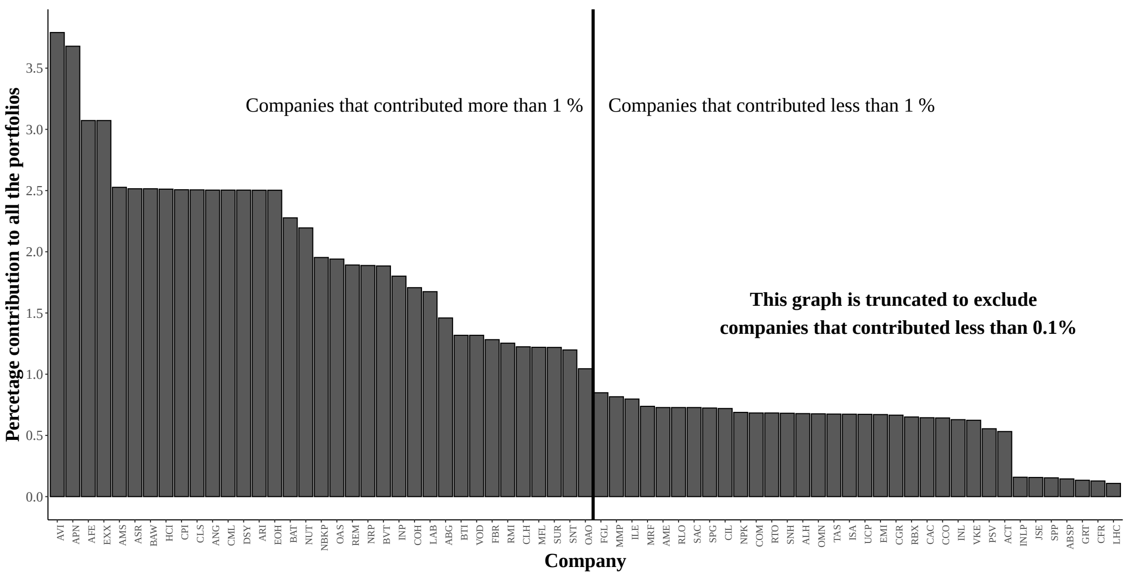

Figure 4.1: The total percentage that each of the sample companies contributed to all the selected portfolios

Almost all of the companies were selected as part of at least one model portfolio. Only two of the 208 companies were never selected. Due to the inclusion of Equation (3.16), the maximum contribution that any company could have was 5%, yet the maximum proportion achieved was 3.79%. Only 15 (7.21%) companies had contributions higher than 2.5% (half the maximum). Furthermore, 35 of the companies (16.83%) had contributions of more than 1%. It is interesting to note that although all but two of the companies were selected, 139 companies, more than two thirds (66.83%) of all the companies, had contributions of less than 0.1%. Of the 69 companies that had contributions of more than 0.1%, only eight (11.59%) do not appear in at least 50% of the selected portfolios. Furthermore, it was found that the companies with the smallest contributions, were selected the least. This implies that, with a few exceptions, the more a company was selected, the higher its contribution was.

From these results, it is clear that there are companies that were selected more than others, and there were some companies that had more significant contributions than others. Thus, it is clear that specific companies can be considered to be more important to invest in than others. Furthermore, since 69 companies had contributions greater than 0.1%, it is also clear that there was a collection of important companies, rather than only one or two companies that dominated all the selected portfolios.

Given that all the companies except two were selected, it was decided to analyse how companies were selected based on the ROR, liquidity, risk and ESG rating values, henceforth collectively known as the metric values, achieved by each company in the training period. These results are presented in the next section.

4.1.2 Companies selected based on the metric values

In the training period, 139 companies had ROR values greater or equal to zero and 112 of these companies (80.78%) were selected as part of at least 50% of the selected portfolios. It was found that only 52 companies that had positive ROR values had contributions higher than 0.1%. As seen in Figure 4.2, generally, it seemed that the companies with higher ROR values had smaller contributions than the low ROR companies. Furthermore, as seen in Figure 4.3, generally, companies with higher ROR values were selected as part of more portfolios than companies that had smaller ROR values. These result are interesting as it was expected that the it was expected that the companies with greater ROR values would have higher contributions and be selected more often. Yet, this was not the case.

When considering the risk objective, it was found that 171 companies (82.21%) carried a risk of less than 10% in the training period. Of these 171 companies, 141 were selected as part of more than 50% of the selected portfolios. As seen in Figure 4.4, generally, it seemed that the higher risk companies had smaller contributions than the lower risk companies. This result is intuitive because it was expected that the models, especially the risk-adjusted and social models, would prioritise selecting companies with lower risk. This is especially true for the portfolios that were selected in the low to moderate risk categories. Thus, it is shown that the risk objective influences the proportions invested in the selected companies. From Figure 4.5 it is seen that companies that had lower Conditional Value-at-Risk (CVaR) values were selected as part of fewer portfolios than the higher risk companies. Thus, it is shown that the risk objective influences the number of times that a company was selected.

It was found that all the sample companies achieved an exchange flow ratio of at least 0.97, indicating that all the sample companies had high liquidity. Furthermore, it was found that no correlation exists between the number of times a company was selected or the contribution of that company and the exchange flow ratio. Companies with higher

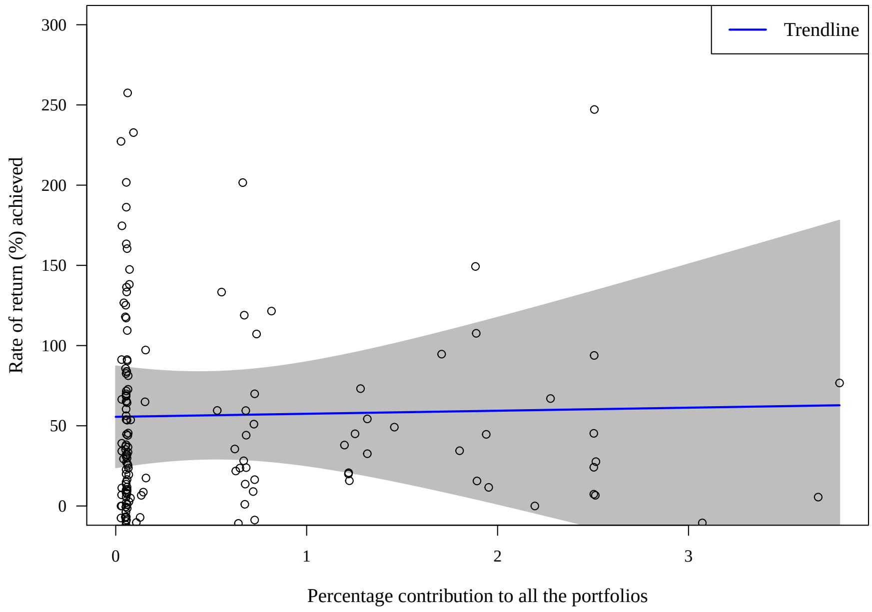

Figure 4.2: The correlation between the percentage that the companies contributed to all the portfolios and the ROR achieved by the companies

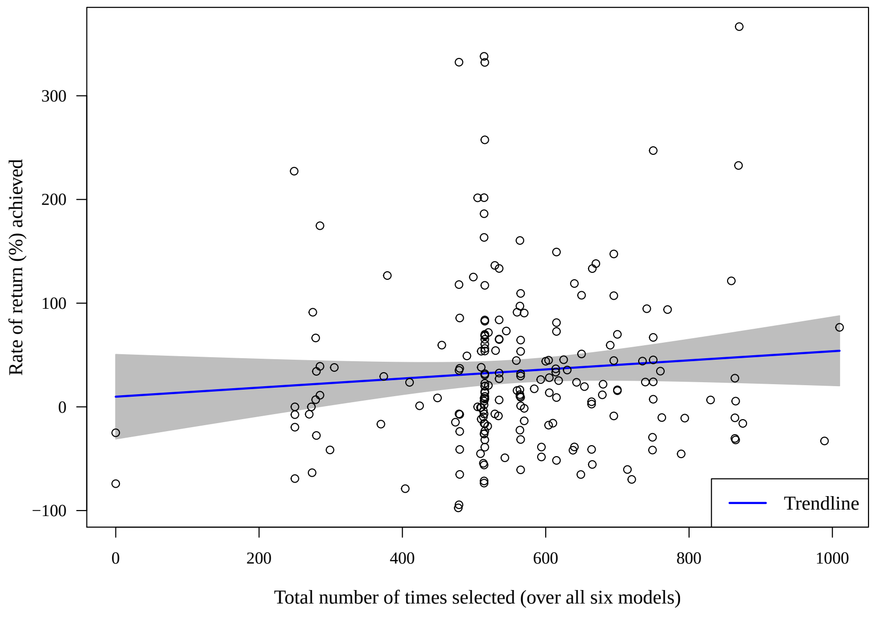

Figure 4.3: The correlation between the total number of times the companies were selected and the ROR achieved by the companies

liquidity were not selected more, or less, than those with slightly lower exchange flow ratio values. This phenomenon can be attributed to the fact that all the companies achieved a near-ideal exchange flow ratio. Thus, it can be seen that the liquidity objective had a negligible impact on the companies that were selected.

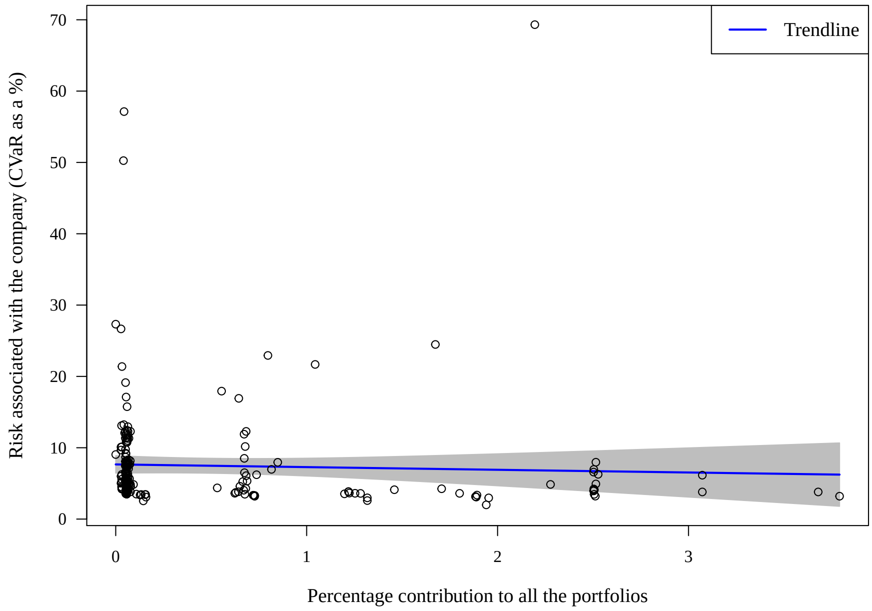

Figure 4.4: The correlation between the percentage that the companies contributed to all the portfolios and the CVaR achieved by the companies

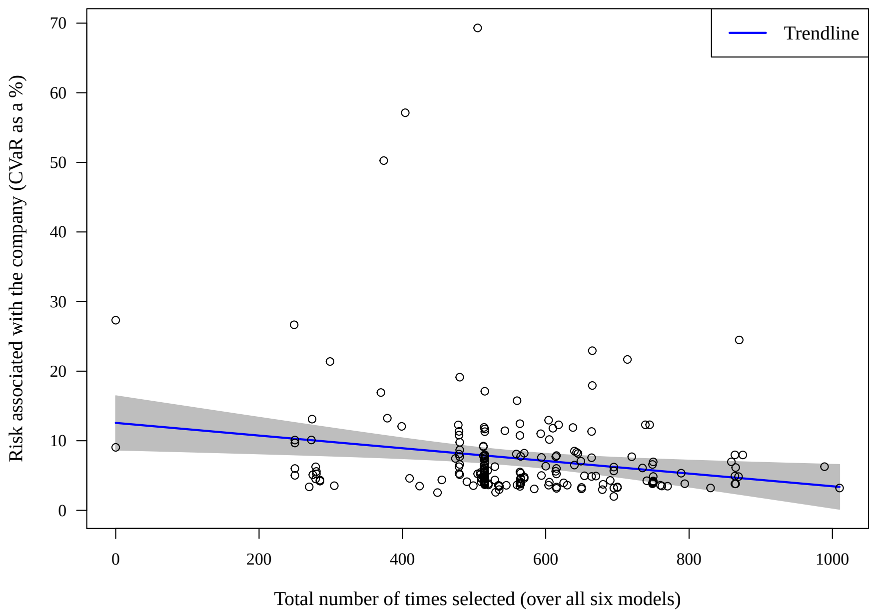

Figure 4.5: The correlation between the total number of times the companies were selected and the CVaR achieved by the companies

The companies with the highest ESG ratings were always selected, even by the Markowitz and risk-adjusted models, which did not specifically aim to select Socially Responsible (SR) companies. It was found that of the 208 companies, only 48 (23.08%) were SR companies (ESG $\geqslant 2.5$). Of these 48 companies, 45 were selected as part of at least 50% of the selected portfolios. Yet, only 21 of the SR companies had contributions that were higher than 0.1%. Generally, with a few exceptions, it appeared that the companies with lower ESG ratings had higher contributions. Given that four of the six models did not aim to select SR companies, it was expected that the contributions of the SR companies would not be high. The results also show that all the SR companies carried a risk of less than 10% and were thus low-risk companies. Thus it can be said that generally SR companies are also low-risk companies. It has already been established that the models prioritised selecting low-risk companies.

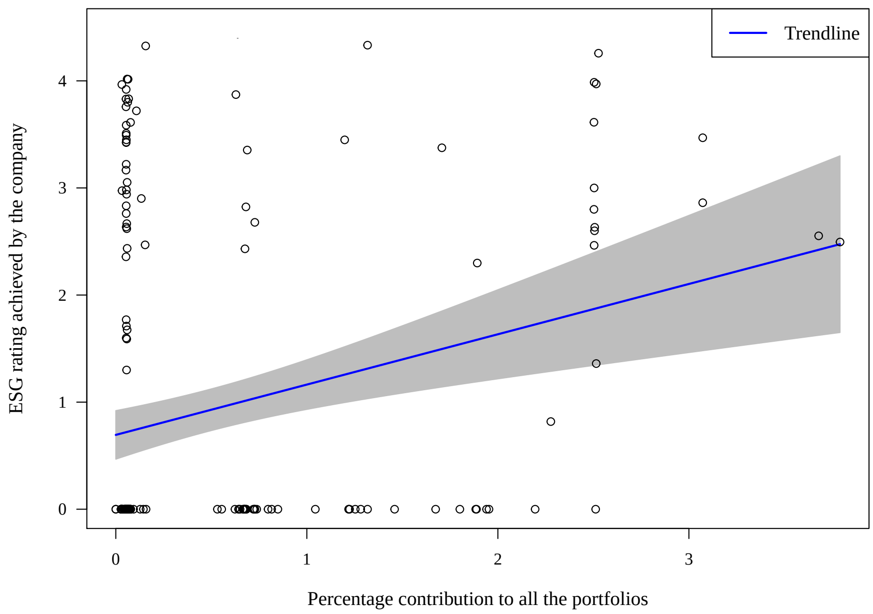

Figure 4.6: The correlation between the percentage that the companies contributed to all the portfolios and the ESG rating achieved by the companies

Another factor that influences the selection of the companies is the minimum number of companies that a portfolio must contain, as constrained by Equation (3.13). It was found that the Expected Value (EV) Markowitz model always selected 201 companies, while the Chance-Constrained Programming (CCP) Markowitz model selected portfolios containing 144, 176 or 204 companies. All the portfolios selected by the CCP risk-adjusted model as well as ten EV risk-adjusted portfolios contained the minimum of 30 companies. All the other portfolios selected by the EV risk-adjusted model contained 240 companies. The EV social model selected portfolios containing 148, 149 and 179 companies, with 230 of 250 (92%) selected portfolios containing 179 companies. The CCP social model had the most variation in the number of companies selected, with portfolios containing anything from 30 to 151 companies, and no more than 50 portfolios ever contained the same number of companies.

It should be noted that the results obtained in this dissertation may not be efficient. This is because goal programming was not implemented in such a way to guarantee an

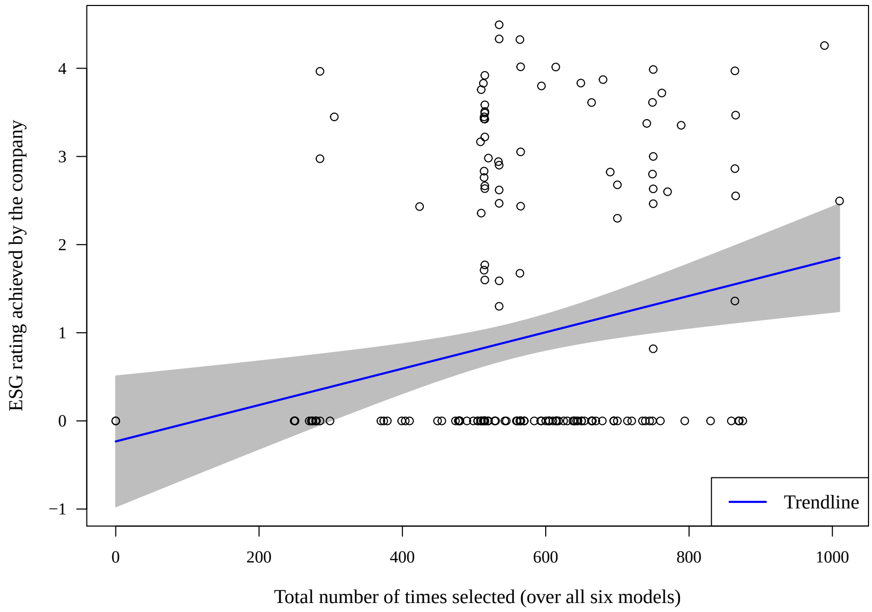

Figure 4.7: The correlation between the total number of times the companies were selected and the ESG rating achieved by the companies

efficient solution. An efficient solution is one where the value of one of the objectives can not be improved without deteriorating the value of any of the other objectives (Masmoudi and Abdelaziz, 2018). When perusing the literature, it was found that efficiency was not emphasised. Thus, goal programming was implemented in such a way that efficient solutions are not guaranteed. Rardin (1998) shows that simple multi-objective problems may have multiple efficient points. Conversely, the bigger and more complex a multi-objective problem is, the greater the likelihood that the solution obtained is efficient. Given that the models considered in this dissertation are highly complex, with three to four objectives and hundreds of variables, it is highly likely that the solutions obtained are in fact efficient. However, it is not guaranteed that these solutions are efficient.

From these results, it can be seen that there is a significant overlap between the companies selected by the various models. Given that many of the selected portfolios contain the same companies, the question is raised as to whether or not the portfolios selected by the different models are similar, and whether or not the six models considered each produced a distinctive set of results. If two models produce identical results, it is unnecessary to consider the results of both models. Rather, only the results of one of the models needs to be considered. Given these questions, the next section investigates this similarity.

4.1.3 Similarity between the selected portfolios

The aim of this dissertation is to *compare* a generic and individualised portfolio selection approach. If any two models produced identical results, comparing them would be less impactful and would influence the aim of the dissertation. Thus, it was necessary to determine whether or not the six models produced six distinct sets of results or not. To achieve this, it was necessary to measure the degree to which the selected portfolios are dissimilar. For this purpose a new metric, called the Absolute Dissimilarity Percentage (ADP), was developed and used. The ADP measures how dissimilar two portfolios are based on the proportion invested in the selected companies (w_j) and is calculated as follows:

Let:

$\boldsymbol{J}$ be the set of the 208 sample companies such that $\boldsymbol{J} = \{1, ..., 208\}$

$\boldsymbol{T}$ be the set of models produced such that

$$\boldsymbol{T} = \begin{cases} 1 \text{ is the Markowitz model solved using the EV method} \\ 2 \text{ is the Markowitz model solved using CCP} \\ 3 \text{ is the risk-adjusted model solved using the EV method} \\ 4 \text{ is the risk-adjusted model using CCP} \\ 5 \text{ is the social model solved using the EV method} \\ 6 \text{ is the social model solved using CCP} \end{cases}$$

$\boldsymbol{A}$ be the set of portfolios selected by model $t \in \boldsymbol{T}$ such that $\boldsymbol{A} = \{1, ..., M\}$

$\boldsymbol{B}$ be the set of portfolios selected by model $t \in \boldsymbol{T}$ such that $\boldsymbol{B} = \{1, ..., M\}$

where

$$M = \begin{cases} 5 \text{ in the case of the Markowitz model} \\ 250 \text{ in the case of the risk-adjusted and social models} \end{cases}$$

$w_{ja} =$ the proportion of capital invested in company $j \in \boldsymbol{J}$ in portfolio $a \in \boldsymbol{A}$

$w_{jb} =$ the proportion of capital invested in company $j \in \boldsymbol{J}$ in portfolio $b \in \boldsymbol{B}$

Then

$$\text{ADP}_{a,b} = \frac{\sum_{j \in \boldsymbol{J}} |w_{ja} - w_{jb}|}{2} \times 100 \qquad \forall a \in \boldsymbol{A}, \forall b \in \boldsymbol{B}$$

If two portfolios selected identical companies and invest identical proportions in each of the selected companies, then ADP $= 0\%$. However, if the two portfolios do not contain any identical companies, then ADP $= 100\%$.

It was decided that the EV and CCP versions of each of the three models would be compared first to determine whether or not there was merit in considering both the EV and CCP versions of the models. If the EV and CCP versions of a model produced the same results, only one of these models needed to be considered in any further analysis. Thus, the first comparison that took place was between the EV and CCP versions of the three models.

All the portfolios produced by the EV version of the model were compared to all the portfolios produced by the CCP version, and the ADP values of these comparisons calculated accordingly. Table 4.1 shows these ADP values achieved at each of the ROR goals. Furthermore, since 50 portfolios were selected for each ROR goal in the risk-adjusted

Table 4.1: ADP achieved at each ROR goal when comparing the EV and CCP versions of the three models

	Markowitz	**Risk-adjusted**			**Social**		
		Min	Median	Max	Min	Median	Max
R = 0%	1.50	0	75.55	90.70	2.00	16.40	24.90
R = 5%	1.50	0	75.55	90.70	2.00	16.40	24.90
R = 10%	48.03	0	75.55	90.70	2.00	16.40	24.90
R = 15%	45.30	0	75.55	90.70	2.00	16.40	24.90
R = 20%	1.53	0	75.55	90.70	2.00	16.40	24.90

and social models, the minimum, median and maximum ADP values for these models are shown.

A notable observation from the results was that for all three models, different portfolios were selected for each of the simulated investor goals. This indicates that the variability of the three-year ROR distributions were skew enough that it was necessary to account for this variability. Thus because of the different results obtained, there was merit in comparing both methods.

From Table 4.1 it is clear that for the Markowitz model, not a single portfolio selected by the EV version was exactly the same as any portfolio selected by the CCP version (ADP $\neq 0$). When $R = \{0\%, 5\%, 20\%\}$, the selected portfolio were almost identical. When $R = \{10\%, 15\%\}$, the selected portfolios were noticeably different. These results can be attributed to the fact that when $R = \{0\%, 5\%, 20\%\}$, for both the EV and CCP models, `LINGO` used an exact approach to find the solution to the model. Yet, when $R = \{10\%, 15\%\}$, `LINGO` used an exact approach when solving the EV version of the model, but used an approximate approach when solving the CCP version of the model. Thus, it was expected that the two versions of the model would produce different results at these two R values.

For both the risk-adjusted and the social models, the ADP distributions are identical for all values of R. The risk-adjusted models selected portfolios that were identical (ADP = 0). Despite this, given that the median ADP value is 75.55%, it is clear that more than half of the selected portfolios were significantly different. These results show that the variation in the distribution of the ROR variable has a significant impact on the selection of investment portfolios and investment proportions within the risk-adjusted model.

The social models had the smallest variation in ADP values achieved of all three models, with all the selected portfolios being similar. This indicates that the addition of the ESG constraint and deficiency variable results in the two versions of the social model selecting many of the same companies and investing similar proportions into those selected companies. This shows that the ESG consideration significantly constrains the solution space. Furthermore, this small variation indicates that the incorporation of an ROR distribution as opposed to a mean ROR value into the model has a lesser impact on the selection of companies and investment proportions than in the risk-adjusted models.

An interesting observation was that the ADP distributions were identical at each R value for the risk-adjusted and social models. Thus, it was decided to investigate why this was the case. It was found that the value of R does not have an influence on differentiating the portfolios that were selected. However, these models have changing R values as well as changing T values. Thus since the R value has no significant effect on the selection of the portfolio, it is necessary to investigate the effect that the T values have on the selection of the portfolios.

For the risk-adjusted and social models, the EV and CCP versions were compared and the ADP value was calculated at each T. These results can be seen in Table B.1 in Appendix B. It was found that the ADP distributions within each risk category (T 20–38, 39–42, 43–48, 49–52, 53–69) were highly similar or identical. Thus, it was discovered that no additional value could be obtained by examining the results at each individual T value. As such, it was decided to group the results into their respective risk categories and present the results accordingly. Table 4.2 shows the ADP results obtained when comparing the portfolios selected by the EV and CCP versions of the models within each risk category.

Table 4.2: ADP values observed across risk categories when comparing the portfolios produced by the EV and CCP approach

	Risk category	**Min**	**Median**	**Max**
Risk-adjusted	conservative	0	61.20	61.20
	moderately conservative	61.20	61.20	61.20
	moderate	61.20	90.70	90.70
	moderately aggressive	90.70	90.70	90.70
	aggressive	90.70	90.70	90.70
Social	conservative	6.70	16.40	16.40
	moderately conservative	16.40	16.50	16.50
	moderate	16.50	16.50	16.50
	moderately aggressive	16.50	20.10	20.10
	aggressive	16.40	16.40	20.10

From this table, it is seen that although some of the ADP distributions are similar, they are not identical for all the risk categories. These results confirm that with the exception of some portfolios in the conservative risk category, the EV and CCP versions of the models did select different portfolios. From these results, it appears as though the value of T does differentiate the portfolios. It was decided to investigate the extent to which the changing values of T may differentiate the portfolios. For this purpose, the ADP values obtained when comparing the portfolios selected by a specific model to each other within each risk category were calculated. This was done for EV and CCP versions of both the risk-adjusted and social models. These results can be seen in Table 4.3.

From Table 4.3 it can be seen that for all four of these models, for the majority of the risk categories, highly similar and identical portfolios were selected. It can be seen that for all four models, the portfolios in the conservative risk category have the highest variation in the ADP values obtained. This can be attributed to the fact that 205 of 208 the sample companies had conservative CVaR values in the training period. When starting at $T = 20$ and increasing up to $T = 38$, more and more companies became eligible for selection. Thus, the range of companies that the models could select from increased considerably in this risk category. Thanks to this phenomenon, there was variation in the portfolios that were selected in this risk category.

Despite the preceding results, the median ADP value in the conservative risk category for all four models was zero. This indicates that the majority of the portfolios selected by these models in this risk category were identical or highly similar. A notable result is that in the moderate risk category, the risk-adjusted model had a maximum ADP value of 85.5. This shows that some portfolios selected by this model in this risk category were highly dissimilar. This can be attributed to the fact that the three remaining sample companies

Table 4.3: ADP values observed across risk categories when comparing the portfolios produced separately by the EV and CCP versions of risk-adjusted and social models

	Risk category	Min	Median	Max
	conservative	0	0	57.90
	moderately conservative	0	0	0
Risk-adjusted EV	moderate	0	0	0
	moderately aggressive	0	0	0
	aggressive	0	0	0
	conservative	0	0	40.00
	moderately conservative	0	0	0
Risk-adjusted CCP	moderate	0	0	85.50
	moderately aggressive	0	0	0
	aggressive	0	0	0
	conservative	0	0	8.20
	moderately conservative	0	0	0
Social EV	moderate	0	0	0
	moderately aggressive	0	0	0
	aggressive	0	0	0
	conservative	0	17.44	22.80
	moderately conservative	0	0	7.00
Social CCP	moderate	0	0	0
	moderately aggressive	0	0	5.30
	aggressive	0	0	5.20

that did not fall into the conservative risk category fall into this risk category. However, given that the median ADP value for this model for this risk category was zero, it is clear that the majority of portfolios selected were highly similar. From this table, it is clear that each model selected highly similar results for each risk category. Thus it appeared that the value of T was in fact, not a differentiating factor within each individual model.

Upon even further investigation it was found that at each T value, the EV risk-adjusted model selected exactly the same portfolio. For example, when examining the portfolios selected by the EV risk-adjusted model at $T = 20$, it was found that the minimum, median and maximum ADP values were are zero. For this model, it was found that this was the case for all the T values. Furthermore, it was found that this was the case for the CCP risk-adjusted, EV social and CCP social models. From these results, it is clear that for each individual T value, the models select exactly the same portfolio. Yet the results in Table 4.2 indicate that the EV and CCP versions of the models selected different portfolios. Thus, it is shown that although each individual model selects exactly the same portfolio at a specific T values, the portfolio selected at a specific T value is not identical between the four models. In other words, the portfolio selected by the EV risk-adjusted model for $T = 20$ may not be identical to the portfolio selected by the CCP risk-adjusted model for $T = 20$. Thus it is clear that the consideration of an ROR distribution as opposed to an average ROR value has an impact on the portfolios that were selected.

These results demonstrate that there is value is considering the EV and CCP versions of the models. Yet, they still do not confirm whether or not the six models each produce a distinctive set of results. It is still possible that two or more of the three models produce the same results. Thus, it was then decided to determine the ADP values produced when comparing the three models to each other to determine whether or not they produce identical, similar or different results. These results are explored in the next section.

4.1.4 Similarity between the three models

The Markowitz model was compared to the risk-adjusted and social models, and the risk-adjusted model was compared to the social model. These comparisons were conducted for both the EV and CCP versions of the models, and the results are presented in Table 4.4.

Table 4.4: ADP values when comparing the Markowitz, risk-adjusted and social models

	EV			**CCP**		
	Min	Median	Max	Min	Median	Max
Markowitz and risk-adjusted	74.04	74.04	89.16	76.90	89.21	93.25
Markowitz and social	76.14	76.14	78.84	78.18	86.95	92.04
Risk-adjusted and social	74.00	77.70	92.90	76.86	89.00	94.80

From Table 4.4 it can be seen that none of the models selected identical portfolios. From these results, it can also be seen that the range of ADP values is always greater when considering the CCP versions of the models. This once again confirms that there is significant variation in the ROR variable, and this influences the results achieved. From these results, it can be seen that all of the selected portfolios differ from the other selected portfolios by more than 74%, indicating that the different models selected highly dissimilar portfolios. These results indicate that the six models being considered in this dissertation do in fact each produce a distinctive set of results. Thus, it is worthwhile to consider all six of these models.

Although it is known that it is worthwhile to consider the results of all six models presented in this dissertation, it is unknown whether the portfolios selected by these models are viable investment options. A portfolio is a viable investment option when it meets or exceeds the investment goals of an investor, or has negligibly small deficiency variables, during the training period. Non-viable portfolios hold no value for an investor and can be disregarded from any further evaluation. To determine if the selected portfolios are viable investment options, it was necessary to determine how these portfolios performed, in terms of achieving the simulated investor goals, in the training period.

4.2 Training period goal performance

When solving the models in `LINGO`, it was found that the selected portfolios do not always satisfy all the goals in the training period. This indicates that given the sample of companies, there may not exist a combination of companies and their respective investment proportions that are fully capable of satisfying all the investor investment goals. To this end, it was decided to evaluate the deficiency variables generated by `LINGO`. Furthermore, the deficiency variables were evaluated to determine whether the models produced viable investment portfolios. The deficiency variable results for each of the three models are presented in the following sections.

4.2.1 Markowitz model goal performance

For the Markowitz model, the deficiency variables for all the portfolios selected are found in Table 4.5.

Table 4.5: Deficiency variables produced by the EV and CCP Markowitz models

	EV			CCP		
	δ_1	δ_2	δ_3	δ_1	δ_2	δ_3
Portfolio 1 (ROR = 0%)	0	1×10^{-5}	0	0	3×10^{-5}	0
Portfolio 2 (ROR = 5%)	0	0	0	0	3×10^{-5}	0
Portfolio 3 (ROR = 10%)	0	3×10^{-5}	0	0	3×10^{-5}	0
Portfolio 4 (ROR = 15%)	0	0	0	0	0	0
Portfolio 5 (ROR = 20%)	0	0	0	0	3×10^{-5}	0

From Table 4.5 it can be seen that three of the five (60%) portfolios selected using the EV Markowitz model met all three the goals ($\delta_1 = \delta_2 = \delta_3 = 0$). Furthermore, when using the CCP version of this model, only one (20%) of the selected portfolios met all the goals. This result can be attributed to the conservative nature of CCP models. CCP models are inherently conservative. Due to this conservative nature, these models tend to select the more conservative companies, in terms of ROR, liquidity and risk, within the sample. Thus, it makes sense that the CCP model have weaker performance than the EV model.

The above results indicates that when considering only goal adherence, the EV version of this model is better suited to selecting investment portfolios that satisfy the goals of the investor at the moment of investment (the present). However, this could be attributed to the current configuration and assumptions of the models. If certain conditions within the models were changed, such as the weighting of the goals, it may be found that the CCP Markowitz model is better suited for this purpose. Both the EV and CCP Markowitz models fall short on the liquidity goal, implying that these models select companies with higher ROR values and lower risk values, but compromise on the portfolio liquidity value. Furthermore, it can be stated that due to the conservative nature of the CCP model, the companies that had lower liquidity values were selected more often than the companies that achieved the goal liquidity value of 1.

Although the liquidity goal is not always satisfied, it should be noted that the deficiency values are minimal ($\leqslant 3 \times 10^{-5}$), indicating that this goal is not met by a tiny margin. Thus, the selected portfolio still have very high liquidity and are still viable investment portfolios.

It has been shown that all the portfolios produced by the Markowitz model are viable investment options. Yet, these portfolios only account for 0.99% ($\frac{10}{1\,010}$) of all the portfolios that were selected by the models. It is unknown whether or not the portfolios selected by the risk-adjusted and social models are viable investment options. Thus, the next section examines the training period goal performance of the risk-adjusted model portfolios.

4.2.2 Risk-adjusted model goal performance

When evaluating the `LINGO` outputs for the risk-adjusted models it was found that for both the EV and CCP versions, 240 of the 250 selected portfolios (96%) met all the goals in the training period ($\delta_1 = \delta_2 = \delta_3 = 0$). If a model portfolio met all the goals, that portfolio is globally optimal. This is because the objective function is a minimisation function and is constrained so that its minimum bound is zero. Thus, if a selected portfolio met all the goals, the objective function is zero and can not be improved. Given that multiple portfolios selected by the risk-adjusted models had objective function values of zero, these results indicate that there were multiple globally optimal portfolios. The remaining 10 portfolios, for both the EV and CCP versions, met the ROR and liquidity goals, but did not meet the risk goal ($\delta_1 = \delta_2 = 0,\ \delta_3 > 0$). The δ_3 deficiency variables generated by `LINGO` for the risk-adjusted models are given in Table 4.6.

Table 4.6: δ_3 values of the risk-adjusted portfolios that did not meet the risk goal

	Min (δ_3)	**Median** (δ_3)	**Max** (δ_3)
Risk-adjusted EV	1.07×10^{-2}	2.09×10^{-2}	3.12×10^{-2}
Risk-adjusted CCP	1.07×10^{-2}	2.09×10^{-2}	3.12×10^{-2}

As seen from Table 4.6, the percentages by which these portfolios did not meet the risk goals are very small ($< 4\%$). Upon investigation, it was found that, for both the EV and CCP versions of the risk-adjusted model, the risk goal was not met when the simulated investor had a very small Risk Tolerance Score (RTS) value of 20 and 21. For investors who are so risk-averse that their RTS value is 20 or 21, there was no portfolio of companies in the sample that could meet their risk tolerance goals. These results align with typical investment advice within the South African market. Investment practitioners recommend that extremely risk-averse investors should rather invest in lower-risk investments, such as government bonds or money-market accounts, as opposed to investing in shares. These 4% of portfolios may not be suitable for the simulated investors with RTS values of 20 or 21, but they are still viable investment options for the simulated investors with higher RTS values. Thus, they are still considered in the rest of this evaluation. From these results, it is clear that all the portfolios selected by the risk-adjusted model are viable investment options.

It is interesting to note that for this model, the deficiency results are identical for the EV and CCP versions. This is interesting because as seen in Section 4.1.3 the EV and CCP versions of the models did not select identical portfolios. However, the goal values were identical between the two versions of the model. As such, it can be seen that several different combinations of companies and investment proportions can yield the same ROR, liquidity and risk results.

All the portfolios produced by the Markowitz and risk-adjusted models are viable investment options. Yet, it is unknown how the social model portfolios performed in the training period. This model performance is investigated in the next section.

4.2.3 Social model goal performance

For the social models it was found that no portfolios were selected that met all the goals ($\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$). The ROR goal was always achieved ($\delta_1 = 0$), but portfolios were selected that did not meet the liquidity, risk or ESG goals. Some portfolios selected by these models only met two goals, while others met three. Table 4.7 lists the goals that were met, the percentage of social model portfolios that met them and the associated deficiency variables.

From Table 4.7 it can be seen that for both versions of the social model, 230 of the 250 selected portfolios (92%), met the ROR, liquidity and risk goals ($\delta_1 = \delta_2 = \delta_3 = 0,\ \delta_4 > 0$). Furthermore, the EV version of this model did not select any portfolio that met the ESG goal. On the contrary, the CCP version of this model did select portfolios that met the ESG goal.

Table 4.7: Goals met and the deficiency variables produced by the social models

Goals met	% of portfolios	Min (δ_2)	Median (δ_2)	Max (δ_2)	Min (δ_3)	Median (δ_3)	Max (δ_3)	Min (δ_4)	Median (δ_4)	Max (δ_4)
Social EV										
ROR and liquidity	6	0	0	0	9.78×10^{-3}	3.17×10^{-2}	5.01×10^{-2}	4.20×10^{-2}	4.44×10^{-2}	4.48×10^{-2}
ROR and risk	2	3×10^{-5}	3×10^{-5}	3×10^{-5}	0	0	0	4.91×10^{-2}	4.90×10^{-2}	4.90×10^{-2}
ROR, liquidity and risk	92	0	0	0	0	0	0	5.90×10^{-2}	5.90×10^{-2}	5.90×10^{-2}
Social CCP										
ROR and liquidity	2	0	0	0	2.07×10^{-5}	2.07×10^{-5}	2.07×10^{-5}	4.20×10^{-2}	4.20×10^{-2}	4.20×10^{-2}
ROR, liquidity and risk	92	0	0	0	0	0	0	1.71×10^{-1}	2.85×10^{-1}	3.54×10^{-1}
ROR, liquidity and ESG	6	0	0	0	1.20×10^{-2}	2.63×10^{-2}	5.14×10^{-2}	0	0	0

However, these portfolios only account for 6% of all the selected portfolios, indicating this model has a low success rate for achieving this goal. The CCP social model's portfolios had smaller δ_4 deficiencies than the EV model portfolios. This can once again be attributed to the conservative nature of the CCP model. Furthermore, this indicates that consideration of a distribution of ROR values, as opposed to a mean ROR value, results in higher ESG companies being prioritised in the model.

The unfavourable ESG performance of the social model portfolios can be attributed to the fact that only 48 of the 208 sample companies (23.08%) were SR. Given that there were very few SR companies for the models to choose from, more none-SR companies were selected, resulting in lower portfolio ESG ratings. This was especially true in the EV version of this model, where all the portfolios contained at least 148 companies. This was also true for the CCP model, but to a lesser extent. For this model, 80% of the selected portfolios contained more than 48 companies, resulting in the none-SR companies decreasing the overall ESG rating of the portfolio. The remaining 20% of portfolios contained 30 companies, the majority of which were not SR, but had low risk. These results, combined with the fact that this model always met the ROR goal, confirms that there is a trade-off between the ROR and risk, and ESG goals in this model. This unsatisfactory ESG goal performance does not exclude the social model portfolios from consideration in the rest of this evaluation. This is because the percentages by which the selected portfolios did not meet the ESG goal are so minuscule that they can be considered to be negligible. Thus, these portfolios were still considered in the rest of this evaluation.

As seen in Table 4.7, both versions of the social model selected portfolios that did not meet the risk goal. In the EV version, 15 of the selected portfolios did not meet the risk goal, and these portfolios were selected using simulated investor RTS values of 20, 21 and 22. The CCP version selected 20 portfolios that did not meet the risk goal, and these portfolios were selected using simulated investor RTS values of 20, 21, 22 and 23. All of these RTS values (20–23), are low RTS values and once again indicates that extremely risk-averse investors should not invest in shares. Interestingly, the EV social model had three simulated investor RTS values at which portfolios did not meet the risk goal, as opposed to the risk-adjusted models, which had only two. Thus, it is shown that the EV social model selected marginally higher risk companies than the risk-adjusted models. Similarly, the CCP social model had four simulated investor RTS values at which the risk goal was not met, whereas the EV version of this model had only three. Thus, it is clear that the CCP version of this model selected marginally higher risk companies than the EV version of this model. These results indicate that the addition of the SR objective and constraint in the models constrains the solution space, resulting in higher-risk companies being selected. Nevertheless, these portfolios are still viable investment options for investors with RTS values of 23 and above and are thus still considered in this evaluation.

The last objective to consider for the social model portfolios is liquidity. All the CCP social model portfolios met the liquidity goal. As shown in Table 4.7, five portfolios selected by the EV social model did not meet the liquidity goal. However, just like with the Markowitz model portfolios, the percentages by which these portfolios did not meet this goal are so minuscule (0.003%), that they can be considered to be negligible. These portfolios are also considered to be viable investments. Thus, all the portfolios produced by the social model are viable investment options.

From the results given in the preceding sections, it is evident that given the model configurations and assumptions, all six models work and select portfolios that are viable investment options. Yet, it is essential to consider these results within the context of the aim of this dissertation, which is to compare the real-world performance of the generic Markowitz models to individualised risk-adjusted and social models.

4.2.4 Generic versus individualised portfolio selection approach

From the deficiency variable results in the preceding sections, it can be seen that although all the portfolios selected by the models were viable investment options, they did not all meet all the goals. When considering the goal adherence performance of the three models, it was seen that 96% of the portfolios selected by the EV and CCP risk-adjusted models met all the goals. In contrast, in the EV Markowitz model, only 60% of the portfolios met all the goals, and only 20% of the CCP Markowitz model portfolios met all the goals. This difference between the EV and CCP model performance can be attributed to the fact that the CCP model followed a more conservative selection approach, resulting in fewer portfolios that met the goals. Furthermore, none of the portfolios selected by the social models, EV or CCP versions, met all the goals. Thus, it is evident that the risk-adjusted models had the best performance in the training period with regard to goal adherence.

When considering the goal adherence results, it is clear that an individualised portfolio selection approach that includes the investment goals of the investor and an individualised risk objective outperforms the generic Markowitz model. In addition to outperforming the Markowitz models, the individualised risk-adjusted model also outperformed the individualised social models. Yet, it is interesting to note that the generic Markowitz model outperformed the individualised approach that incorporates the investment goals of the investor, an individualised risk objective and SRI. Thus, it is clear that the addition of the SRI objective and constraint significantly decreases the goal adherence of the selected portfolios. Based purely on these results, it can be stated that it is not beneficial for an investor to be SR in their investments. Thus, existing portfolio selection models should not be updated to incorporate SRI.

In terms of the aim of this dissertation, it is clear that when selecting investment portfolios that meet the goals of the investor in the training period, an individualised portfolio selection approach that does not incorporate SRI is superior to a generic approach. Furthermore, an individualised portfolio selection approach that does not incorporate SRI is superior to an individualised portfolio selection approach that does incorporate SRI. Thus, the risk-adjusted models, and not the Markowitz or social models, should be used to select investment portfolios that meet the investor goals in the training period.

This section shows that the risk-adjusted models are the most suited to select investment portfolios that meet the investor goals in the training period. Nevertheless, an investor is not interested in how their investment portfolio performs in the present, but rather what ROR and ESG rating they will achieve in the future. If past financial behaviours and trends repeated themselves in the future, these portfolios would always achieve the investor goals. Unfortunately, this is not the case, and financial trends change over time and are unpredictable due to the uncertain nature of the financial markets. Furthermore, for a portfolio selection model to be suitable for its intended purpose, it must at least achieve the ROR and SRI goals of the investor in the future. Thus, to determine the suitability of the models considered in this dissertation, it is necessary to determine the ROR and ESG performance of the selected portfolios in the testing period.

4.3 Portfolio performance in the testing period

As explained in Section 3.4.4, an investor is only interested in what ROR and ESG rating they will achieve in the future. Thus, in this dissertation, a portfolio selection model is only suitable for its intended purpose if it meets or exceeds the ROR and SRI goals of the investor in the testing period. In this section, the ROR and ESG rating values achieved by the selected portfolios in the testing period were calculated, and it was evaluated whether or not these values met the investor goals. These results were also used to compare the individualised risk-adjusted and social models to the generic Markowitz model, which is the aim of this dissertation.

4.3.1 Testing period rate of return performance

The ROR values achieved by the selected portfolios in the testing period were calculated using Equation (3.39). These ROR values were then compared to the simulated investor ROR goals to determine whether or not the models are capable of achieving these ROR goals in the future. This process was executed for the Markowitz, risk-adjusted and social models, and the results are presented in the following sections.

Markowitz model performance

The ROR values obtained by the portfolios selected using the EV and CCP Markowitz models are found in Table 4.8.

Table 4.8: The testing period ROR values (%) of the portfolios selected by the EV and CCP Markowitz models at each of ROR goal

	EV	**CCP**
Portfolio 1 (ROR = 0%)	0.86	0.89
Portfolio 2 (ROR = 5%)	0.85	0.89
Portfolio 3 (ROR = 10%)	0.85	−4.08
Portfolio 4 (ROR = 15%)	0.94	−16.65
Portfolio 5 (ROR = 20%)	0.83	0.89

From this table, it can be seen that all the Markowitz portfolios produced dismal ROR performance. This is because not a single portfolio's value increased by more than 1% and two portfolios selected by the CCP model had negative returns. CCP models are inherently conservative, thus this performance can be attributed to the conservative nature of CCP. Although the CCP portfolios 1, 2 and 5 had higher ROR values than four of the five portfolios produced by the EV model, it is still better to invest in the EV model portfolios. This is because all the portfolios produced by the EV model have positive ROR values, while some of the portfolios produced by the CCP model have negative ROR values. Furthermore, the portfolio that had the highest ROR value is produced by using the EV model.

The most significant result that can be seen from this table is that regardless of which version of this model was used, only one of the five ROR goals, namely ROR $= 0$, was ever achieved. This is interesting because the ROR goals were always achieved in the training period. This indicates that, in this scenario, the Markowitz models are only useful to select investment portfolios that will not make a loss and do not select portfolios that can achieve high ROR values. These results show that these models do not consistently select portfolios that achieve the ROR goals of the investor. Therefore, in this scenario, these models are not suitable for the purpose for which they are created. If the scenario of this study was changed, it may be found that the Markowitz models could be suitable for their intended purpose.

It seems strange that the Markowitz model would perform well in the training period, but then have dismal performance in the testing period. To explain this phenomenon, it is necessary to consider these results within the context of the South African market in the testing period. The South African economy was in the doldrums during the testing period. As a result, most companies listed on the JSE experienced poor performance in this period. This is evidenced by the fact that 146 of the 208 (70.19%) sample companies in this study experiences a decrease in ROR from the training period to the testing period. Thus, it was expected that the Markowitz model portfolios would not perform well in this period.

The poor performance of the South African market can be qualified by considering the ROR performance of the JSE all share index. In the testing period, the JSE all share index achieved an ROR of 9.04%. Given that a reasonable three-year ROR is 17.09%, and this ROR of 9.04% is just slightly more than half of this value, it is evident that the market did not perform well in this time (Bernstein, 1997). When comparing this ROR value to the ROR values achieved by the Markowitz model portfolios, it was seen that the market significantly outperformed the model portfolios. Thus, it would have been better for an investor to invest in the JSE all share index than to invest in a Markowitz model portfolio. These results also show that in addition to not selecting portfolios that meet the ROR goals of the investor, the Markowitz models do not select market-competitive investments. Thus, in this scenario, these models can not be considered to be useful models and should not be used to select investment portfolios for investors. If the scenario of this study was changed, it may be found that these models could be useful.

From this section it is clear that it would not be useful for an investor to invest in a Markowitz model portfolio. Yet, this dissertation aims to compare the performance of a generic and individualised approach. Thus, it is still necessary to evaluate the testing period performance of the individualised risk-adjusted and social models.

Risk-adjusted model performance

For the risk-adjusted models, the ROR values achieved by the selected portfolios were calculated and sorted according to the risk category within which they fall. When analysing these results, it was seen that the differences between the minimum, median and maximum ROR values for each risk category were so small, that considering the boxplots of the values would not be meaningful. This is because these ROR distributions provide no additional information than a single value, such as the average. Thus, the average of the ROR values achieved for each risk category was determined and plotted, as seen in Figure 4.8.

From Figure 4.8 it can be seen that EV risk-adjusted model portfolios performed even worse than the portfolios selected by the Markowitz models as all the portfolios selected by this model had negative ROR values in the testing period. It is also seen that the average ROR achieved by the selected portfolios are virtually inseparable for four of the five risk categories. The portfolios produced for the conservative risk category (RTS values of 20–38) had slightly higher ROR values than the portfolios produced for the other four risk categories. Furthermore, it is interesting to note that the average ROR value for each risk category is identical at all five of the ROR values. This shows that regardless of the RTS value of the simulated investor, the ROR achieved remains constant. This is because

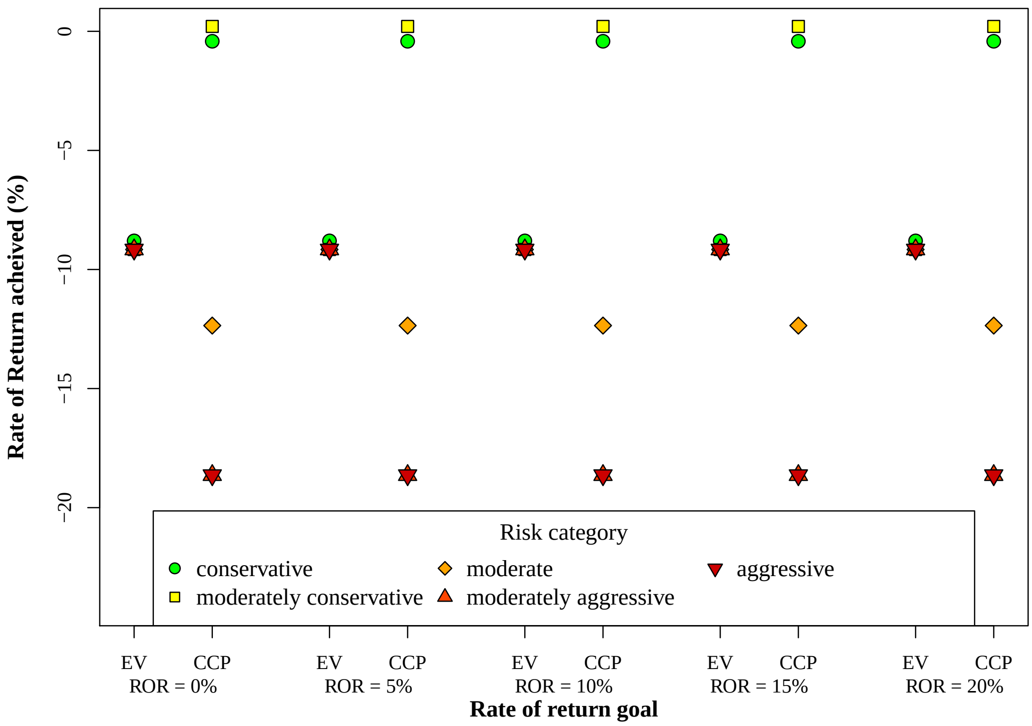

Figure 4.8: The testing period ROR values achieved by the EV and CCP risk-adjusted model portfolios for each risk category at each ROR goal

the underlying portfolios are identical. When portfolios are selected using this model, none of the ROR goals are ever achieved. As with the Markowitz models, this result is interesting because all the ROR goals were met in the training period. This shows that, in this scenario, this model is not suitable for its intended purpose and should not be used to select investment portfolios for investors.

Unfortunately, the CCP risk-adjusted model did not perform much better. This is because, in this model, some positive ROR values were achieved, but for the most part, the selected portfolios had negative ROR values. This can once again be attributed to the conservative nature of the CCP model, which results in more conservative portfolios being selected, resulting in worse financial performance. In the moderately conservative risk category, the average ROR is only 0.21% and highest ROR achieved by any portfolio selected by this model is 0.21%. This indicates that as with both the Markowitz models, only one of the five ROR goals was ever achieved. Thus, the portfolios selected by this model experienced poor performance in the testing period. This shows that this model can only be used to select investment portfolios that will achieve an ROR slightly greater than zero, but less than 5%, given that the investor has a moderately conservative risk tolerance. Despite that fact that the average ROR value for each risk category is easily differentiable, the values for the two highest risk categories, moderately aggressive and aggressive, are identical. It is also seen that as with the EV model, the average ROR values for each risk category are identical at each ROR goal. From these results, it is seen that portfolios selected at lower RTS values, had higher average ROR values. This is an interesting result as it is contrary to the widespread perception that the higher the risk of an investment is, the higher the ROR will be (Kaiser et al., 2014).

Given these results, it can be concluded that, in this scenario, both the EV and CCP risk-adjusted models are not suitable for their intended purpose. As with the Markowitz models, this is because they do not always select portfolios that achieve the ROR goals of the investor and thus do not achieve the purpose for which they are created. Should the scenario of this study be altered, such as assigning different weights to the different goals, or a different dataset be used, the results may be different, and it may be found that the models may be suitable.

As with the Markowitz models, it is important to consider these results within the context of the South African market. As discussed in Section 4.3.1, the market performed very poorly during the testing period, so it was expected that the risk-adjusted models would perform poorly during this period. It was found that this was the case as no risk-adjusted model portfolio achieved an ROR of more than 0.21%. It is known that the JSE all share index achieved an ROR of 9.04% in the testing period. Thus, it is clear that the market outperformed all the risk-adjusted model portfolios. Furthermore, the JSE all share index had a positive ROR, whereas the majority of the risk-adjusted model portfolios experienced a loss. Therefore, it would have been more beneficial for an investor to invest in the JSE all share index than to invest in a risk-adjusted model portfolio. Thus, in addition to not selecting investment portfolios that meet the investment goals of the investor, the risk-adjusted models do not select market-competitive investments. Thus, in this scenario, these models can not be considered to be useful models and should not be used to select investment portfolios for investors.

It has been shown that the generic Markowitz model and a model that incorporates the investment goals of the investor and an individualised risk objective should not be used to select investments for an investor. Yet, in this dissertation, a third model that incorporate the investment goals of the investor, an individualised risk objective and SRI is also considered. Thus, it is necessary to determine how this model performed in the testing period and whether or not it is suitable for its intended purpose.

Social model performance

For the social models, the ROR values achieved by the selected portfolios were calculated and sorted according to the risk category within which they fall. As with the risk-adjusted models, it was found that no additional information could be obtained from considering the ROR distributions as opposed to considering only the average ROR value. Thus, the average of the ROR values achieved for each risk category was determined and plotted, as seen in Figure 4.9.

The social models achieved some exciting results, significantly outperforming the Markowitz and risk-adjusted models. As seen in Figure 4.9, as with the risk-adjusted models, the ROR values achieved are identical, regardless of the ROR goal for which portfolios were selected. Nevertheless, unlike the Markowitz and risk-adjusted models, all the portfolios selected by the EV and CCP models achieved positive returns. Even more impressive is that all the portfolios selected by these models not only met all five ROR goals, but greatly exceeded them. According to Bernstein (1997), a reasonable three-year ROR is 17.09%. It was found that all the model portfolios selected by the social models had ROR values higher than 17.09%. Furthermore, with the exception of 5 portfolios selected by the CCP version of this model (2%), all the social model portfolios had ROR values that are more than double this reasonable ROR. The fact that the CCP version of this model selected fewer high ROR portfolios than the EV version is once again evidence of the conservatism of the CCP approach. Due to these results, unlike the Markowitz and

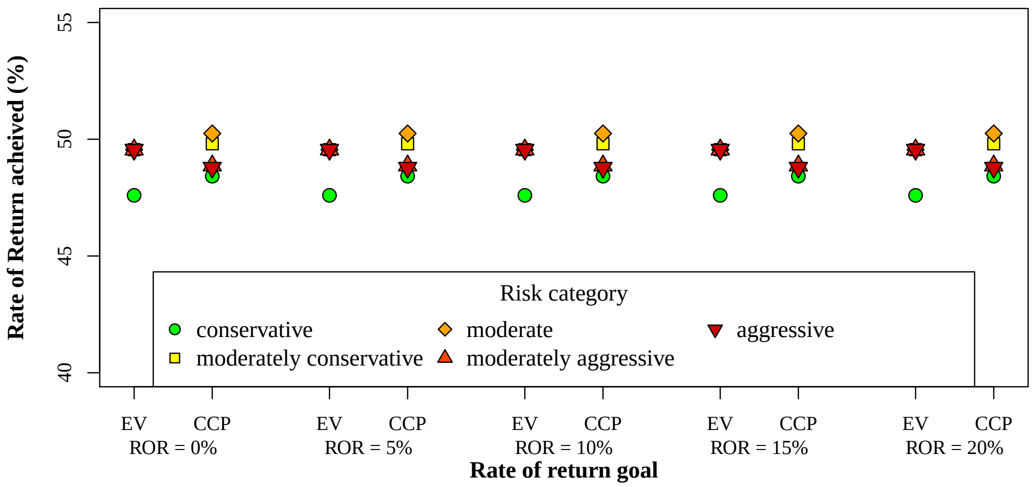

Figure 4.9: The testing period ROR values achieved the EV and CCP social model portfolios for each risk category at each ROR goal

risk-adjusted models, in this scenario the social models can be considered to be suitable for their intended purpose, if ROR is the only consideration. This assertion is supported by the work of Revelli and Viviani (2015) who find that the level of performance of a SR investment portfolio depends on the methodological choices that were made by the researcher. Given that the only difference between the risk-adjusted and social models is the inclusion of the SRI consideration, these results show that this addition does have an impact on the companies that were selected and the proportions that were invested into the selected companies.

The social models always selected the companies with the highest ESG rating values (in the training period). This suggests that companies that are SR in the present will have high returns in the future. This result makes sense given the recent shift towards SRI. Investors want to invest in SR companies. Thus, companies that are classified as being SR, or are more SR than others, gain interest and more investors invest in them. This increase in investment results in the value of these companies increasing. As the value of a company increases, its share price increases, leading to an increase in its investment appeal. This once again leads to an increase in investments, leading to an increase in value. Also, ESG ratings usually increase over time, as explained in Section 2.1.4. This is attributed to the fact that as companies become aware of how SR they are, they begin to make conscious efforts to incorporate and increase ESG factors within the company, leading to increased ESG performance. Thus, generally, over time, companies become more SR, which again increases their investment appeal. Moreover, this cycle continues every time the value of the companies or its ESG rating increases.

These results appear very promising. Yet, it still appears strange that the social models achieved such high ROR values and so drastically outperformed the Markowitz and risk-adjusted models, especially considering that the same dataset was used. When investigating this phenomenon, it was found that the significant difference between the social model portfolios and the portfolios selected by the Markowitz and risk-adjusted models was the allocation of the total investment to the selected companies. The social models allocated much higher proportions (w_j) to the SR companies than the Markowitz and risk-adjusted models. Furthermore, the social models invested the highest possible contribution of 5% into 14 of the SR companies. The results show that 80% of these 14 companies experienced an increase in ROR from the training period to the testing period. It was found that 128 companies achieved a negative ROR in the testing period. For 122 (96.88%) of these companies, the EV social model invested less than 0.3% of the total investment into any of these companies. The CCP social model invested proportions of less than 0.3% into 121 (94.53) of these companies. Both the Markowitz and risk-adjusted models allocated much greater proportions to these companies than the social models did. It was also was found that the companies that the social models invested the highest proportions into, with minor exceptions, achieved at least a reasonable ROR of 17.09% in the testing period. Thus, generally, the social models invested high proportions into companies that became more profitable and low proportions into companies that because less profitable.

Upon further investigation it was found that the high ROR values achieved by the social models align with the findings of De and Clayman (2015) and Kempf and Osthoff (2007). De and Clayman (2015) found that the shares that achieved the highest returns were the shares that had better ESG profiles than the shares that achieved lower returns. Furthermore, they state that investors could increase their profitability by removing lower-tail ESG shares from their portfolios. In their study, Kempf and Osthoff (2007) created portfolios, bought shares that had high SR ratings and sold shares that had low SR ratings. They found that adopting this strategy led to their portfolios achieving high abnormal returns. Furthermore, the research of Revelli and Viviani (2015) shows that the well-established theory within the financial world of "doing well while doing good" can be applied to SR investments, and thus SR investments achieve higher returns than traditional investment portfolios. These studies demonstrate that being SR is also profitable and SR investment portfolios outperform non-SR portfolios. In light of these studies, it makes sense that the social model portfolios outperformed the Markowitz and risk-adjusted portfolios.

When comparing the EV and CCP versions of this model, it can be seen that for the moderately conservative and moderate risk categories, the CCP version selected portfolios that had the highest returns. Thus, the CCP social model is possibly more suitable for selecting portfolios for investors who fall into these two risk categories. For the remaining three risk categories, the EV social model is possibly more suitable for selecting portfolios for the investor as these model portfolios achieved higher returns than the portfolios selected by the CCP version for these risk categories. The fact that the EV version of the model outperformed the CCP version for the majority of the risk categories is attributed to the fact that the CCP model generally selected more conservative portfolios, resulting in poorer performance.

When considering the ROR results of the social models within the context of the South African market, it is clear that all the social models significantly outperformed the JSE all share index. This is as all the social model portfolios achieved an ROR that is at least three times as much as the 9.04% return achieved by the JSE all share index. Thus, it is clear that in addition to selecting portfolios that meet the ROR goals of the investor, the social models select market-competitive investment portfolios. Given that the social models select market-competitive portfolios, it can be stated that the social models are useful models.

From the results in this section, it is clear that the social models are far superior to the Markowitz and risk-adjusted models when it comes to selecting investment portfolios that meet or exceed the ROR goal of the investor in the future. Yet, ROR is not the only consideration for determining whether or not a model is suitable for its intended purpose. For this dissertation, a model can only be regarded as suited for its intended purpose if it selects investment portfolios that achieve the ROR and SRI goals of the investor in the future. Thus, it is necessary to investigate the ESG performance of the selected portfolios in the testing period.

4.3.2 Testing period environmental, social and governance performance

Although only the social models specifically aimed to select SR portfolios, it was decided to investigate what ESG rating was achieved by all the selected portfolios in the testing period. This was decided so that it can be determined whether adding the SRI constraint and deficiency variable does result in portfolios being selected with a higher ESG ratings in the testing period than the portfolios selected without incorporating SRI. The ESG ratings of the selected portfolios in the testing period were calculated using Equation (3.40). Table 4.9 presents the ESG rating achieved by the EV and CCP Markowitz portfolios.

Table 4.9: Testing period ESG rating achieved by the EV and CCP Markowitz portfolios

	EV	**CCP**
Portfolio 1 (ROR = 0%)	1.09	1.09
Portfolio 2 (ROR = 5%)	1.09	1.09
Portfolio 3 (ROR = 10%)	1.09	1.03
Portfolio 4 (ROR = 15%)	1.10	0.69
Portfolio 5 (ROR = 20%)	1.09	1.09

From this table, it can be seen that none of the portfolios selected by the Markowitz models can be considered to be SR (ESG < 2.5). These results, coupled with the results given in Section 4.3.1, show that the Markowitz models are not capable of selecting portfolios that will achieve an investor's ROR and SRI goals in the future. Thus, in this scenario, these models are not suitable for their intended purpose. If the scenario of this study was changed, it may be found that the Markowitz models could be suitable for their intended purpose.

For the risk-adjusted and social models, the ESG ratings achieved by the selected portfolios were calculated and sorted according to the risk category within which they fall. The average of the ESG ratings achieved for each model for each risk category was determined and plotted, as seen in Figure 4.10. As with the Markowitz models, none of the risk-adjusted portfolios were SR. These results, combined with the dismal ROR performance of these portfolios leads to the conclusion that, in this scenario, the risk-adjusted models are not suitable for their intended purpose. If the scenario of this study was changed, it may be found that the risk-adjusted models could be suitable for their intended purpose.

Both the Markowitz and risk-adjusted models had unsatisfactory performance in the testing period and are not suitable for their intended purpose. Thus, it is clear, when looking at only these two models, that there is not a significant difference between using a generic and individualised approach to select investment portfolios. Given that all the portfolios selected by these models are not SR, the only real differentiator between these models is the ROR achieved. When looking at only the ROR achieved by these portfolios, the Markowitz models had better performance, and thus it is shown when looking at these two models, a generic portfolio selection approach produces better performance than an individualised approach.

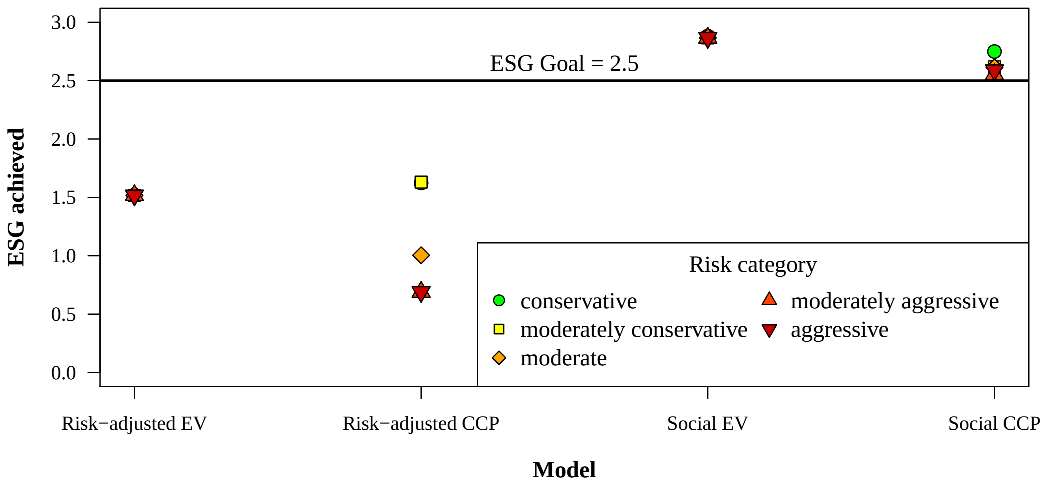

Figure 4.10: The testing period ESG ratings achieved by the portfolios selected by the EV and CCP risk-adjusted models and the EV and CCP social models

From Figure 4.10, it can be seen that the average ESG rating achieved for both social models for each risk category is always higher than the ESG goal of 2.5. It was found that all the portfolios selected by the social models had ESG ratings higher than 2.5 in the testing period. This indicates that the social models select portfolios that will likely be SR in the future. Furthermore, from this figure, it can be seen that the portfolios selected by the EV social model had higher ESG ratings than the portfolios selected by the CCP social model. These ESG results, combined with the ROR results, show that both the social models are suitable for their intended purpose. This is because all the social portfolios achieve the ROR and ESG goals of the investor in the testing period. Thus, the EV social model can be used to select investment portfolios for investors who fall into the conservative, moderately aggressive and aggressive risk categories. The CCP social model can be used to select investment portfolios for investors who fall into the remaining two risk categories. Selecting portfolios for these investors this way will ensure that a portfolio is selected which is SR and results in the highest possible ROR for the investor, given their risk category.

From these results, it can also be conclusively stated that the social models are superior to the Markowitz and risk-adjusted models. This is because these models significantly outperform the other models in both the ROR achieved and SRI. These results also show that an individualised approach is superior to a generic one, but that SRI and not the risk is the differentiating factor.

Although the social portfolios did have satisfactory performance in the testing period, it may not be worthwhile to invest in them if the investor can achieve a higher ROR by investing in existing investment options. Thus, it is necessary to compare the ROR achieved by all the model portfolios in the testing period to the ROR achieved by existing investment options, such as unit trusts.

4.4 Portfolio rate of return performance compared with unit trusts

The investment portfolios selected by the models can only be classified as worthwhile if they have an ROR that is equal to or greater than the ROR achieved by existing investment options, such as unit trusts. The 25 unit trusts listed in Table 3.1, reproduced here in Table 4.10, were used for this comparison. This comparison was conducted for each of the three models, and the results and discussion of these analyses are presented in the following sections.

Table 4.10: The unit trusts, with their ROR values, used in the comparison

Risk category	Unit trust name	Three-year ROR achieved (%)
Conservative	Allan Gray Optimal Fund (Allan Gray, 2019c)	−5.59
	Coronation Money Market Fund (Coronation, 2019d)	25.27
	Momentum Money Market Fund (Momentum, 2019d)	24.23
	Prudential High Yield Bond Fund (Prudential, 2019c)	29.50
	Stanlib Enhanced Yield Fund (Stanlib, 2019c)	25.55
Moderately conservative	Allan Gray Stable Fund (Allan Gray, 2019e)	20.12
	Coronation Balanced Defensive Fund (Coronation, 2019a)	20.46
	Momentum Diversified Income Fund (Momentum, 2019b)	27.02
	Prudential Enhanced Income Fund (Prudential, 2019d)	23.54
	Stanlib Balanced Cautious Fund (Stanlib, 2019a)	17.76
Moderate	Allan Gray Balanced Fund (Allan Gray, 2019a)	14.77
	Coronation Balanced Plus Fund (Coronation, 2019b)	19.10
	Momentum Odyssey Moderate Aggressive (Momentum, 2019e)	16.76
	Prudential Balanced Fund (Prudential, 2019e)	20.12
	Stanlib Balanced Fund (Stanlib, 2019b)	17.49
Moderately aggressive	Allan Gray-Orbis Global Fund of Funds (Allan Gray, 2019d)	12.81
	Coronation Market Plus Fund (Coronation, 2019c)	16.43
	Momentum Aggressive Growth (Momentum, 2019a)	13.46
	Prudential Enhanced SA Property Tracker Fund (Prudential, 2019a)	−14.26
	Stanlib Global Balanced Feeder Fund (Stanlib, 2019e)	39.67
Aggressive	Allan Gray Equity Fund (Allan Gray, 2019b)	11.19
	Coronation Top 20 Fund (Coronation, 2019e)	20.12
	Momentum Equity Fund (Momentum, 2019c)	13.20
	Prudential Equity Fund (Prudential, 2019b)	15.76
	Stanlib Equity Fund (Stanlib, 2019d)	16.96

4.4.1 Markowitz models

For the Markowitz models, the ROR achieved by the selected portfolios in the testing period were compared to the ROR achieved by the 25 unit trusts given in Table 4.10. All of these unit trusts, except the Allan Gray Optimal Fund and the Prudential Enhanced SA Property Tracker Fund, had positive ROR values in the testing period. From Table 4.8 it can be seen that the highest ROR achieved by any Markowitz portfolio is 0.94%.

This ROR value is significantly less than the ROR achieved by all the unit trusts that had positive returns in the testing period. All of the EV Markowitz portfolios and four of the five CCP Markowitz portfolios (80%) did outperform the Allan Gray Optimal Fund and the Prudential Enhanced SA Property Tracker Fund. The fact that the EV version of this model had better performance than the CCP version when compared to the unit trusts is attributed to the conservative nature of the CCP model. Yet, the portfolios that did outperform these two unit trusts were outperformed by all the other unit trusts and are thus still not more desirable investments than the unit trusts. Given these results, it can be concluded that the unit trusts unequivocally outperformed the Markowitz models' portfolios. Thus, it is not worthwhile for an investor to invest in a Markowitz model portfolio. Given that the Markowitz models do not select portfolios that are market-competitive or worthwhile, it can be stated that, in this scenario, these models are not useful and they should not be used to select investment portfolios for investors.

The Markowitz models are only two of the six models being considered in this dissertation. Thus, even though these models do not produced worthwhile investment portfolios, the four remaining models may produce worthwhile portfolios. Thus, it is necessary to evaluate the portfolios selected by the risk-adjusted and social models.

4.4.2 Risk-adjusted models

The portfolios selected by the risk-adjusted models can be divided into five risk categories, just like the unit trusts. Thus, for each risk category, the model portfolios ROR values were compared to the ROR values achieved by the five unit trusts in the same risk category, over the testing period. To aid in this comparison, the range of ROR values achieved by the risk-adjusted model portfolios in the testing period was determined and tabulated in Table 4.11.

Table 4.11: The minimum, median and maximum testing period ROR values achieved by the risk-adjusted model portfolios in each of the five risk categories

	Risk-adjusted EV			Risk-adjusted CCP		
Risk category	**Min**	**Median**	**Max**	**Min**	**Median**	**Max**
conservative	−9.16	−9.16	−5.70	−5.70	0.21	0.21
moderately conservative	−9.16	−9.16	−9.16	0.21	0.21	0.21
moderate	−9.16	−9.16	−9.16	−18.64	−18.64	0.21
moderately aggressive	−9.16	−9.16	−9.16	−18.64	−18.64	−18.64
aggressive	−9.16	−9.16	−9.16	−18.64	−18.64	−18.64

The highest ROR achieved by any risk-adjusted portfolio is 0.21%. As with the Markowitz model portfolios, this ROR is substantially less than the ROR achieved by 23 of the 25 unit trusts. For the moderately conservative, moderate and aggressive risk categories, the unit trusts outperformed all the portfolios produced by the EV and CCP risk-adjusted portfolios. Thus, unit trusts are more desirable investments than the risk-adjusted model portfolios for investors who fall into the moderately conservative, moderate and aggressive risk category.

For the EV risk-adjusted model, it was found that for the conservative risk category, even though one unit trust made a loss, all the model portfolios were outperformed by all five unit trusts. For the moderately aggressive risk category, it was found that the four positive unit trusts outperformed all the model portfolios. Yet, the Prudential Enhanced SA Property Tracker Fund was outperformed by all the model portfolios. Although portfolios do exist that perform better than the moderately aggressive Prudential unit trust, these portfolios are outperformed by all the other unit trusts in this risk category. They are thus still not more desirable investments than the unit trusts. For the CCP risk-adjusted model, it was found that 89.47% of the conservative model portfolios outperformed the Allan Gray unit trust. Despite this, these portfolios, and all the other CCP risk-adjusted conservative portfolios, were outperformed by the other four unit trusts in this risk category. Thus, unit trusts are preferred investment options over the CCP risk-adjusted conservative portfolios. In the moderately aggressive risk category, all the CCP risk-adjusted portfolios had lower ROR values than all of the unit trusts in this category. Thus, the risk-adjusted moderately aggressive portfolios are also not desirable investment options.

Given that all the EV and CCP risk-adjusted portfolios were significantly outperformed by the unit trusts, in all five risk categories, it is evident that risk-adjusted models do not produce worthwhile investments. It would be more beneficial for an investor to invest in a unit trust than to invest in a risk-adjusted model portfolio. Given that the risk-adjusted models do not selected portfolios that are market-competitive or worthwhile, in this scenario, these models are not useful models.

The risk-adjusted models did not selected market-competitive or worthwhile investments and are thus not useful models. Yet, it has already been shown that the social models are useful models because the produce market-competitive portfolios. It may be found that the social model select portfolios that are market-competitive and worthwhile. Thus, an investigation to this end is required.

4.4.3 Social models

As with the risk-adjusted models, the portfolios selected by the social models can be divided into five risk categories. Thus the ROR values achieved by the social models' portfolios in the testing period, summarised in Table 4.12, were compared to the ROR values achieved by the five unit trusts in the same risk category.

Table 4.12: The minimum, median and maximum testing period ROR values achieved by the social model portfolios in each of the five risk categories

	Social EV			**Social CCP**		
Risk category	**Min**	**Median**	**Max**	**Min**	**Median**	**Max**
conservative	38.48	49.55	49.55	33.71	48.47	54.81
moderately conservative	49.55	49.55	49.55	48.47	50.25	50.25
moderate	49.55	49.55	49.55	50.25	50.25	50.25
moderately aggressive	49.55	49.55	49.55	48.41	48.41	50.25
aggressive	49.55	49.55	49.55	48.41	49.04	49.04

The highest ROR achieved by any portfolio selected by the social models in the testing period was found to be 54.80%. This result is the opposite of the results achieved by Markowitz and risk-adjusted models. This is because this ROR value is higher than the ROR achieved by any of the unit trusts. Furthermore, it was found that all the portfolios selected by the social models achieved higher ROR values than any of the unit trusts that fall within the same risk category. For the moderately conservative, moderate and aggressive risk categories, the portfolios selected by both the EV and CCP models outperformed the unit trusts by at least 20%. Given that a reasonable three-year ROR is 17.09%, a difference of at least this percentage between two investments can be considered to be a substantial difference (Bernstein, 1997). For the other two risk categories, this difference was found to be smaller, with most of the differences being less than 10%. However, the model portfolios still outperformed all the unit trusts in those risk categories.

From these results, it is clear that it is far more advantageous for an investor to invest in a social model portfolio, according to their risk category, than it is to invest in a unit trust. Thus, it can be concluded that not only are the social models useful models and select market-competitive investment portfolio; they also select worthwhile portfolios. In addition to being useful models, it is known that these models produce SR portfolios. From the published data, it can not be determined whether or not the unit trusts that were considered are SR. Thus, in addition to achieving high ROR values, investing in a social model portfolio will ensure that an investor is practising SRI.

The unit trust ROR results indicate that the industry fund managers were able to overcome the difficulties experienced by the South African market during the testing period and achieve, for the most part, favourable ROR values. Given that these unit trusts significantly outperformed two of the three models, and were outperformed by the third, it was decided to investigate the differences that exist between the unit trusts and the models.

4.4.4 Differences between the unit trusts and the models

The significant difference between the unit trust ROR values and the ROR values achieved by the Markowitz and risk-adjusted models can be credited to three main factors. Firstly, unit trusts are not constrained to the 208 sample companies considered in this dissertation but can include any company listed on the JSE, companies listed on international exchange markets, and other assets, such as bonds. The inclusion of the additional companies and assets is another way in which risk and diversification can be incorporated into a portfolio selection model. This inclusion means that the unit trusts incorporate, and thus hedge, the covariance of the different economies. Furthermore, the included international assets and companies can counterbalance unfavourable market conditions within the South African market. The second difference is that unit trusts are rebalanced regularly, sometimes even daily, as opposed to only being rebalanced after three years. Third and finally, the selection and rebalancing of unit trusts are not solely dependent on market data. The managers of these funds also consider qualitative market data as well as their experience when compiling and rebalancing unit trusts. Given these differences, it makes sense that the majority of the unit trusts achieved favourable ROR values.

The same differences exist between the unit trusts and the social models, so it was expected that the social models would also be outperformed by the unit trusts. Yet, these models selected portfolios that thrived in the testing period. Thus, it is clear that the inclusion of SRI into the portfolio selection model ensures that the selected portfolios overcome the challenges experienced by the Markowitz and risk-adjusted models. Given all the results produced in this chapter, it is evident that the social models are superior to the Markowitz and risk-adjusted models and can and should be used to select investment portfolios for investors.

By determining how the selected portfolios performed in the unknown future, the models have been validated according to the definition of model validation used in this dissertation. However, from the result it is still not clear whether or not the risk-adjusted and social models do in fact select *individualised* investment portfolios. Thus, to some extent it can still be argued that these two models are unvalidated. Thus, it is necessary to perform further model validation and determine if the models succeed in this aim.

4.5 Model validation

In order to show that risk-adjusted and social models select *individualised* investment portfolios it is necessary to demonstrate that the portfolios selected for investors with low RTS values have different risk distributions than the portfolios selected for investors with greater RTS values. For this purpose it is necessary to determine what CVaR (risk value) the selected portfolios achieved at each of the T values. For the risk-adjusted EV model, these results can be seen in Figure 4.11.

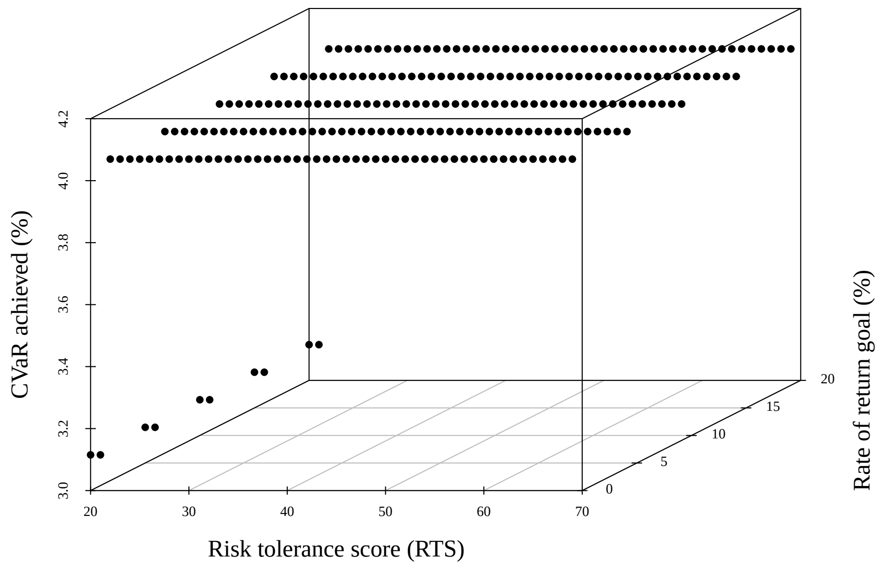

Figure 4.11: The CVaR values achieved at each RTS value and ROR goal value for the Risk-adjusted EV model

From Figure 4.11 it can be seen that at each T value, the model selected exactly the same portfolio, as already shown by the ADP results. Furthermore, it is also seen that, as shown by the ADP results, at each ROR goals value, the model selected identical portfolios. An interesting result that can be seen from the Figure is that this model, essentially, only selected two portfolios. The ten the portfolios selected at RTS values of 20 and 21 achieved an ROR of 41.70% and a CVaR of 3.12%. All the other portfolios selected by this model, 240 portfolios, achieved an ROR of 45.62% and a CVaR of 4.07%.

It is interesting that all the portfolios selected by this model achieved extremely low risk values (< 5%). This can be attributed to fact that 205 of the 208 (98.56%) of the sample companies have a conservative CVaR value in the training period (CVaR < 36.73%). From the way in which the model was formulated it was expected that as the RTS increases, the model would start to select more risky companies, and thus the portfolios selected would have high risk values. However, this simply was not the case in this dissertation. Although the portfolios selected at the extremely small RTS values of 20 and 21 did have smaller CVaR values than the portfolios selected at the other 48 higher RTS values, there is no clear and increasing differentiation between the portfolios selected at the different RTS values. This can be attributed to the selection of the sample for this study.

Given that almost all of the sample companies had extremely low risk values it is unsurprising that the portfolios selected from this sample achieved such low risk values. Yet, this means that it is not possible to evaluate how the model would have selected if there were higher risk companies in the sample. This also means that given this dataset it is not possible to determine if the portfolios selected for investors with higher RTS values would contain higher risk companies than the portfolios selected for investors with lower RTS values. As such, this dataset does not allow for the proper validation of larger RTS values. From this explanation it is clear that this evaluation is constrained by the dataset that was used. Thus, it can not be said that the risk-adjusted EV model achieves its aim of selecting *individualised* portfolios and as such this model is considered to be unvalidated. If a different dataset was used, this result may be different.

For the portfolios selected by the risk-adjusted CCP model, the CVaR values achieved were calculated and plotted. These results can be seen in Figure 4.12. From this figure it can be seen that as with the risk-adjusted EV portfolios, all the portfolios achieved very small CVaR values ($< 8\%$). Given the conservative nature of CCP models it was expected that the risk-adjusted CCP portfolios would have lower CVaR values than the risk-adjusted EV portfolios. However, the majority of the risk-adjusted CCP portfolios actually have higher risk than the risk-adjusted EV portfolios. As with the risk-adjusted EV model it can be seen that the model selected identical portfolios at each RTS and ROR goals value. This has already been shown and discussed in the ADP results. An interesting result that can be seen from the Figure is that this model, essentially, only selected three portfolios, at RTS values of $20-21$, $22-44$ and $45-69$.

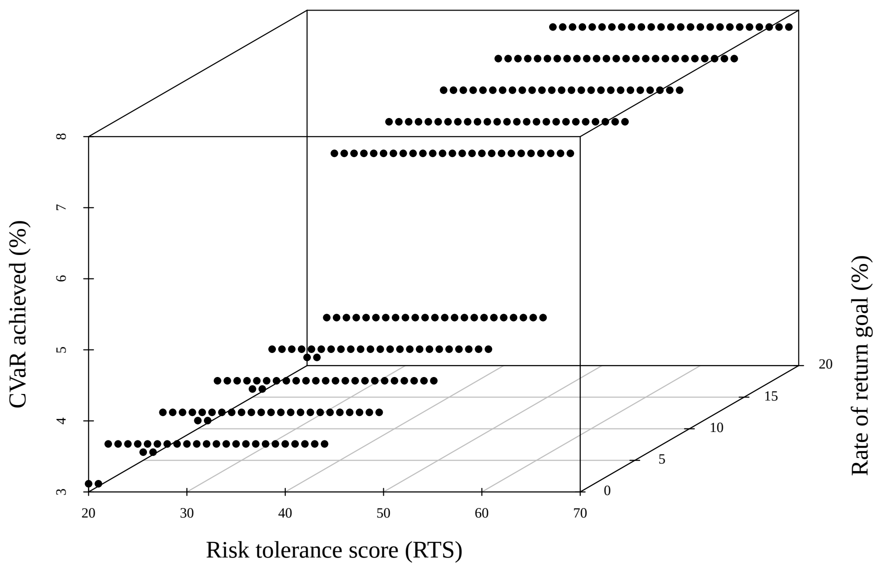

Figure 4.12: The CVaR values achieved at each RTS value and ROR goal value for the Risk-adjusted CCP model

From this figure it can be seen that, to a small degree, the CVaR of the selected portfolios do increase as the RTS values increases. That is, for the three aforementioned groups of RTS values, as the RTS value of the group increased, so did the CVaR achieved. Yet, these groupings of RTS values encompass a wide range of RTS values and there is no variation in the CVaR values achieved within these groupings. Furthermore, all the portfolios achieved CVaR values that fall into the conservative risk category, which once again can be attributed to the lack of high risk companies in the selected sample. Thus, as with the risk-adjusted EV model, it is not possible to evaluate how the model would have selected if there were higher risk companies in the sample. As such, once again, it can be stated that the model is constrained by the dataset and it is not possible to properly validated the larger RTS values. Thus, it can not be said that the risk-adjusted CCP model achieves its aim of selecting *individualised* portfolios and as such this model is considered to be unvalidated. If a different dataset was used, it may be found that this outcome is different.

For the social EV model, for each portfolio that was selected the CVaR achieved was calculated and plotted. These results can be seen in Figure 4.13. From this figure it can be seen that in essence only five unique portfolios were selected by this model. At RTS values of 20, 21, 22 and 23, the model selected a distinct portfolio, but from an RTS value of 24 upwards, all the portfolios selected were identical. This high degree of similarity has already been addressed in the ADP results. It is seen that once again, all the portfolios achieved conservative CVaR values. Given the aforementioned constraints of the dataset, this outcome is unexpected. In line with the aim to select individualised portfolios, from the figure it is clear that for very small RTS values, the model does tend to select higher risk portfolios for higher RTS values. With the exception of at an RTS value of 23, the CVaR values achieved do tend to increase as the RTS values increase. Nevertheless, all the portfolios selected at RTS values of 24−69 are identical and a range of four RTS values is too small of a sample to make a definitive judgement. Thus, it can not definitively be determined whether this model does in fact select portfolios that carry higher risk values as the RTS values increase. As with both the risk-adjusted models, this can be attributed to the limiting nature of the sample that was used. From these results it can not be said that the social EV model achieves its aim of selecting *individualised* portfolios and as such this model is considered to be unvalidated. Given a different dataset, this result may be different.

When considering the social CCP model, it was found that there was a broader range of CVaR values achieved by the selected portfolios that with the other three models that have been considered. These CVaR results can be seen in Figure 4.14. It can be seen from this Figure that this model essentially selected nine unique portfolios, with the most individualised portfolios being selected at smaller RTS values. Yet, the portfolios selected by this model achieved higher CVaR values than the portfolios selected by any of the other models. This indicates that the inclusion of the ESG constraint, in combination with the consideration of an ROR distribution, results in higher risk companies being selected as part of the portfolios.

From Figure 4.14 it can be seen that generally, as the RTS values increase, so do the CVaR values that were achieved by the portfolios. There are essentially nine groupings of RTS values that can be seen in this figure, these are: RTS values of 20, 21, 22, 23, 24−29, 30−39, 40−49, 50−59 and 60−69. With the exception of the CVaR values achieved at RTS values of 21, 40−49 and 60−69, the CVaR values increase as the RTS values increase. This is an interesting results, as with all three the other models, there was no clear increase in CVaR values as the RTS values increased expect at very small RTS values. This indicates that the inclusion of the ESG constraint, as well as the conservative nature of CCP results

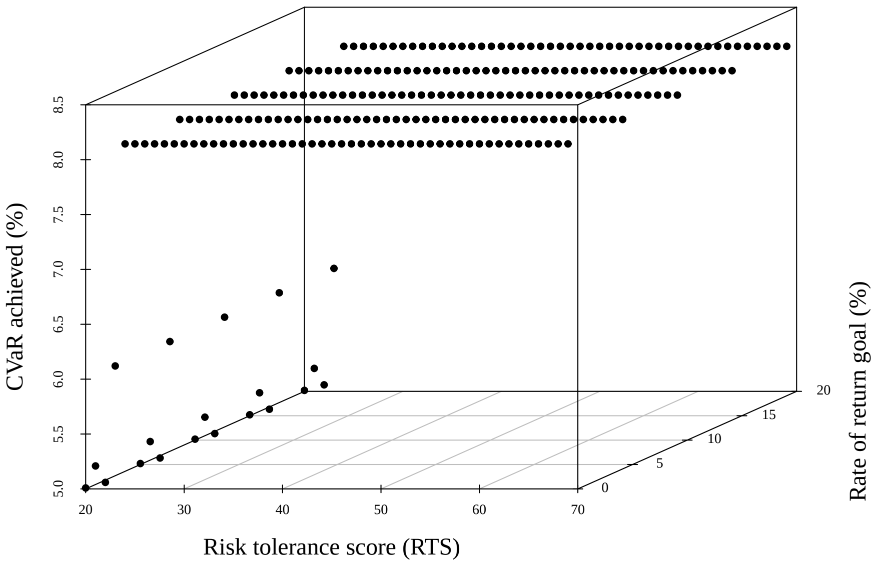

Figure 4.13: The CVaR values achieved at each RTS value and ROR goal value for the Social EV model

in a more differentiated set of portfolios. It also means that this model is better at selecting portfolios that are in line with an investor's risk tolerance than the other three models. Yet, once again all the portfolios that were selected by this model achieved conservative CVaR values. This is once again attributed to the fact that the sample used in this study is very limiting in terms of risk.

Although there seems to be a general trend that the CVaR values do increase as the RTS values increase, there are three clear exceptions and many of the RTS values are grouped together. Thus, it can not definitively be determined whether this model does in fact select portfolios that carry higher risk values as the RTS values increase. As with both the risk-adjusted models, this can be attributed to the limiting nature of the sample that was used. From these results it can not be said that the social EV model achieves its aim of selecting *individualised* portfolios and as such this model is considered to be unvalidated. If a different dataset was used, this outcome may be different.

Although none of these four models can be considered to be able to select higher risk portfolios for investors with higher risk tolerances, it is interesting to note that for all four models, for the extremely small risk values, the CVaR of the selected portfolios do tend to increase as the RTS values increase. It has already been established that almost all of the sample companies achieved conservative CVaR values during the testing period. Yet, not all of these companies were viable options for selection as part of a portfolio at all the conservative RTS values. For example, only 54 of 208 sample companies achieved CVaR values of at most 4.08%, which corresponds to an RTS value of 22. Then at an RTS of 23, 103 of the 208 sample companies achieved CVaR values that are less than the risk tolerance goal risk percentage. Thus, the increasing CVaR values as the RTS values increase at the very small RTS values can be attributed to the fact that as the RTS values increase, more and more companies become viable investment options. Thus, the sample that the models have to choose from keeps increasing. Yet, when an RTS value of 23 is

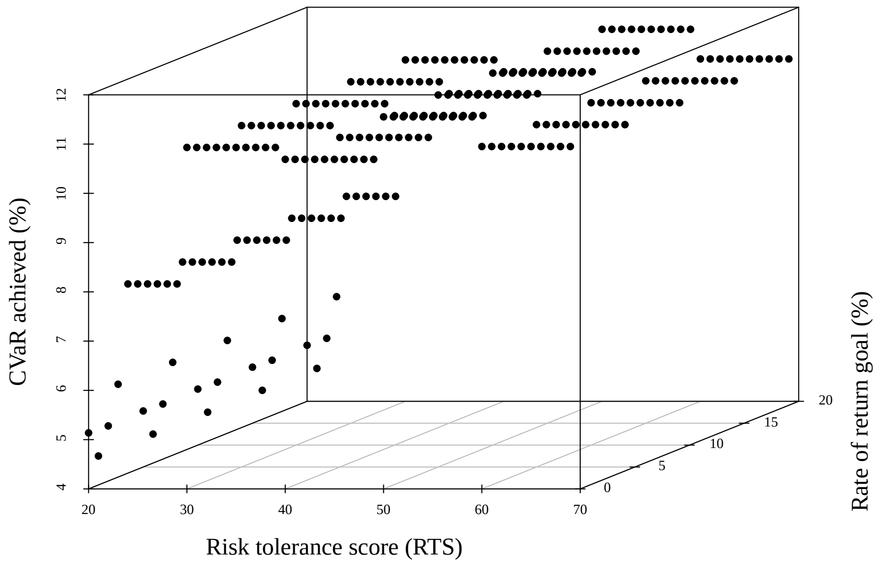

Figure 4.14: The CVaR values achieved at each RTS value and ROR goal value for the Social CCP model

reached, there are more than enough companies for the models to select from to create portfolios of 30 companies. Also, the models aim to minimise risk, so it makes sense that as soon as there is a large enough sample of relatively low risk companies to composes 30-stock portfolios from, the models would continue to favour these 30 or more companies, regardless of what the RTS value is. Because of the very conservative risk nature of the sample, for most of the models, this "turning point", where the models start to select the same low risk companies, is at an RTS value of 23. From this explanation it makes sense that it is can only be seen that the CVaR values of the selected portfolios increase and the RTS values increase for extremely small RTS values.

It was expected that as the RTS value increases, the portfolios that were selected would consist of higher risk companies and thus have higher risk. But due to the limiting nature of the sample that was used, no definitive conclusions can be drawn about the models' ability to select *individualised* investment portfolios that are in line with the risk tolerance of the investor. It was argued that existing portfolio selection models are insufficient in the current investment market because they are not individualised. However, given the results in this section it is clear that adding a risk objective that aims to select investment portfolios for investors based on their risk tolerance does not in fact guarantee that individualised portfolios are produced. Thus, in the absence of any other evidence, it can only be concluded that the models do not produce individualised investment portfolios.

The results in this section shown that one of the principle aims of the portfolio selection models that were developed in this dissertation is not achieved. Yet, to a large extent, this can be attributed to the sample that was selected. Thus, the question was raised, as to how the models would have performed if the sample that was used consisted of companies that had a range of CVaR values. For this reason it was decided to generate such a dataset. The next section discusses how this was achieved and the results that were found.

4.5.1 Theoretical risk values

To determine how the models would have performed if the companies in the sample had a range of CVaR values, it was decided that the same sample of companies would be used. However, a random range of CVaR values, between zero and 100, would be generated. These CVaR values were generated using the `runif()` function in `R` (R Core Team, 2020). The resulting list of CVaR values consisted of 73 conservative risk values, 23 moderately conservative risk values, 25 moderate risk values, 19 moderately aggressive risk values and 68 aggressive risk values. This list of CVaR values represents a much wider range of CVaR values than the original CVaR values. Once a list of CVaR values was simulate, each of the 208 sample companies was randomly assigned a new CVaR value. Note, that the ROR, liquidity and ESG values achieved by the companies remained unchanged. This was done so that it could specifically be investigated what effect the risk had on the CVaR values that the portfolios achieved.

Two of the four models considered in the previous section were run. The CVaR values achieved by the portfolios selected by the risk-adjusted EV model can be seen in Figure 4.15. From this figure it can be seen that despite the increase range of CVaR values, all the portfolios that were selected achieved extremely conservative risk values ($< 5\%$). This could be explained by the fact that the model does aim to minimise the CVaR value that the selected portfolios achieve. Furthermore, it can be seen that as was the case with this model with the original dataset, the model essentially only selected two portfolios. Given that a selected portfolio only needs to consist of 30 companies and there were at 73 companies that had conservative risk, it can be derived that the model prioritised selecting these low risk companies. As with the original results, the CVaR achieved only increases as the RTS values increase once, at an RTS value of 22. Thus, from these results it can be see that for the risk-adjusted EV model, the inclusion of a risk objective that incorporates the financial risk tolerance of the investor does not produce individualised portfolios. Thus, for the risk-adjusted EV model, one of the principle hypotheses of this dissertation is disproven.

When considering the social EV model, it was found that the portfolios that were selected from this "new" dataset achieve higher CVaR values. These results can be seen in Figure 4.16. Yet, all the CVaR values achieved are conservative or moderately conservative, and are thus still relatively low. This is attributed to the fact that model aims to minimise risk, so this result indicates that the model is performing as expected. The fact that this model selects higher risk portfolios than the risk-adjusted EV model indicates that the inclusion of the ESG objective in the model formulation, creates a trade-off where risk is traded-off in favour of another objective, most likely ESG.

When considering the results in this figure it can be seen that generally the CVaR values of the selected portfolios that do increase as the RTS values increase. This is especially true at RTS values of $32-38$, where at each of these RTS values as the RTS value increases, the CVaR value achieved increases. Yet, there were still groupings of RTS values, and it was expected that for the aggressive RTS values, the model would select aggressive risk portfolios. Thus, although this model shows promising results, it is still not enough to determine with confidence whether or not the model can select highly individualised portfolios. Thus, it is concluded that this model does not produce investment portfolios that are aligned with the financial risk tolerance of the investor.

Given the results achieved for the risk-adjusted EV and the social EV model it can be surmised that the results for the risk-adjusted CCP and social CCP models are similar. Thus, from all these results it is concluded that the models formulated in this dissertation do not achieve one of their principle aims, which is to select *individualised* investment

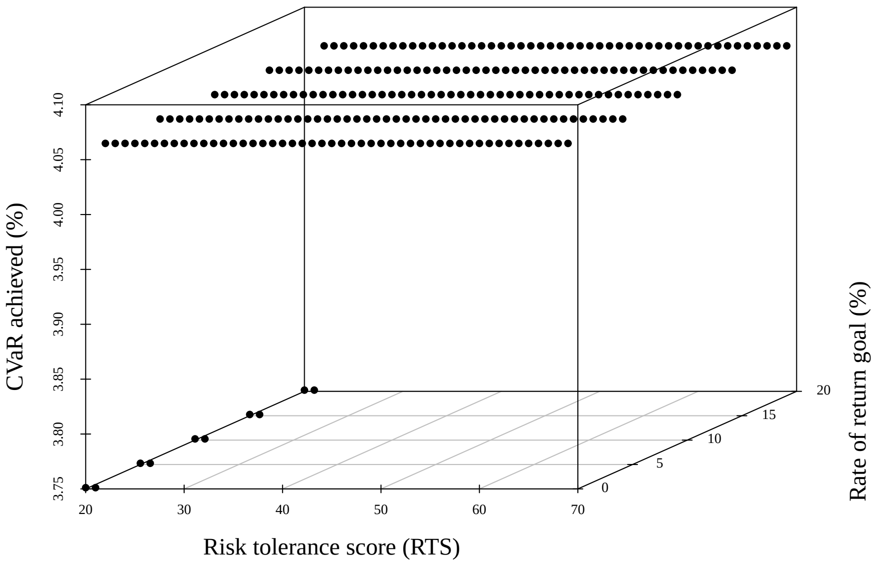

Figure 4.15: The CVaR values achieved at each RTS value and ROR goal value for the Risk-adjusted EV model with adjusted risk values

portfolios for investors.

This chapter explores all the results that were achieved when solving the generic and individualised portfolio selection models. The last chapter gives the conclusions of this dissertation and explores opportunities for future research stemming from this study.

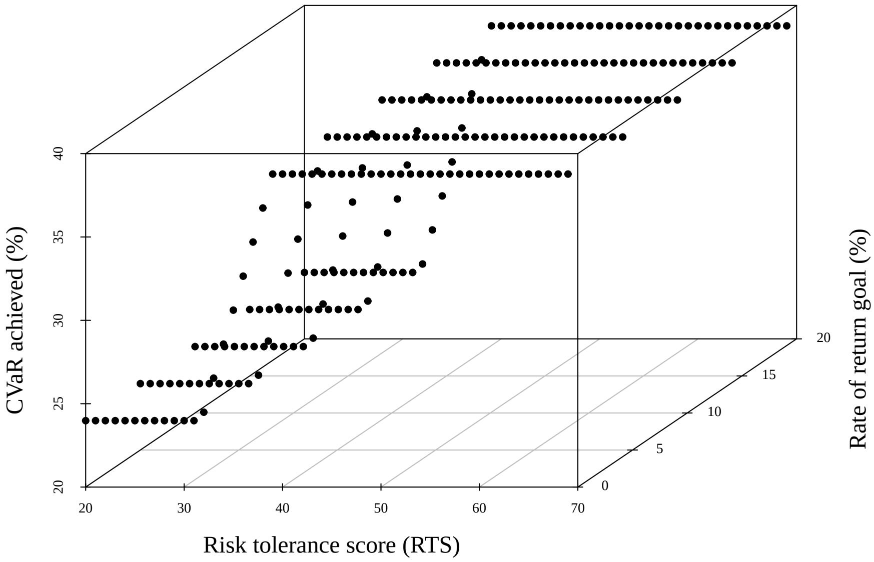

Figure 4.16: The CVaR values achieved at each RTS value and ROR goal value for the Social EV model with adjusted risk values

Chapter 5

Conclusion

Since 1952, portfolio selection models have continued to evolve with more objectives being considered in the formulation of these models. The general consensus is that three objectives should be considered. These objectives are to maximise the uncertain Rate Of Return (ROR), to maximise liquidity and to minimise risk.

Despite being a well-establish and researched field, existing portfolio selection models could be inadequate to select investment portfolios for investors in the current investment climate because of three identified gaps in the existing portfolio selection models. Firstly, generally these models are generic and do not account for the *individual* investment goals of the investor. This is especially evident in the risk objective, which does not account for the financial risk tolerance of the investor. Secondly, the majority of portfolio selection models do not account for Socially Responsible Investing (SRI) principles. Thus an investment portfolio that specifically aligns with the moral and ethical beliefs of the investor can not be selected. Finally, the majority of existing portfolio selection models are considered to be unvalidated. This is because the existing models, with minor exceptions, are formulated and solved using historical data, but these models are not tested on an unknown future. For a portfolio selection model to be relevant in the current investment climate, it must account for the investment goals of the investor, especially in terms of risk, incorporate SRI and be validated.

In this dissertation, an individualised risk objective was developed that consolidates the "riskiness" of the selected portfolio and the financial risk tolerance of the investor. Then two individualised portfolio selection models were developed, both of which incorporate the individual investment goals of the investor as well as the individualised risk objective. Furthermore, one of these models incorporated an SRI objective. These two models were called the risk-adjusted and social models, respectively. These individualised models were compared to an existing Markowitz model with the same ROR and liquidity objectives as the individualised models and a generic risk objective.

Given the multi-objective nature of portfolio selection models, it is necessary to employ an appropriate Multi-Objective Optimisation (MOO) technique so that the models can be formulated in such a way so that they can be solved. For this purpose, goal programming was used in this dissertation. The investment market is plagued with uncertainty, and thus the portfolio selection models contain uncertainty. To account for the uncertainty within these models, two stochastic programming methods, namely the Expected Value (EV) method and Chance-Constrained Programming (CCP) were used.

A sample of 208 companies was selected from JSE Limited. The historical market data for these companies was extracted from `Yahoo Finance` for a period of ten years (2010/01/01 – 2019/12/31). The Environmental, Social and Governance (ESG) rating data for these companies were obtained directly from the JSE. This data was divided into two independent datasets, a training set (2010/01/01 – 2016/12/31) and a testing set (2017/01/01 – 2019/12/31). Based on research, it was determined what ROR, liquidity, risk and ESG goal values an investor may have, and a set of investor goals was simulated. The three models were solved in LINGO using goal programming with the simulated investor goals as the ideal values and the training dataset. In total six models were solved — an EV and CCP version of each of the three models — producing a total of 1 010 portfolios.

5.1 Summary of the results

It was found that all of the sample companies except two were selected as part of at least one portfolio. Furthermore, it was found that more than two-thirds of the sample companies had little to no impact on the goal performance of the selected portfolios in the training period.

For all three models, it was found that the EV version selected different portfolios to the CCP version. This indicates that the variability of the three-year ROR distributions was large enough that it is necessary to account for this variability. Thus, both the EV and CCP versions of the models were considered. Another observation was that the Absolute Dissimilarity Percentage (ADP) distributions for the social models were significantly smaller than those for the risk-adjusted models. From this, it is clear that the inclusion of the SRI objective constrains the solution space and results in highly similar portfolios being selected by the EV and CCP versions.

When comparing the models to one another, the ADP values obtained from these comparisons clearly indicated that the three models selected highly dissimilar portfolios. This was true for both the EV and CCP versions. Thus, it is clear that both the individualised risk objective and the SRI objective had a significant impact on the portfolios that were selected.

Next, it was investigated whether or not the selected portfolios met the simulated investor goals in the training period. All the portfolios selected by all the models always met the ROR goal. For the Markowitz models, it was found that there were multiple globally optimal portfolios. The portfolios that did not meet all the goals had negligibly small deficiency variable values. Thus all the portfolios selected by these models were viable investment options.

The risk-adjusted models also selected multiple globally optimal portfolio, with 96% of the selected portfolios meeting all three goals. Only the portfolios that were selected for the extremely risk-averse simulated investors (Risk Tolerance Score (RTS) = 20, 21) did not meet the risk goal. However, these portfolios were still viable investment options for investors with higher RTS values. Thus, all the portfolios selected by the risk-adjusted models were considered to be viable investment options.

None of the portfolios selected by the social models were globally optimal. This indicates that the addition of the SRI objective significantly constrains the solution space. With this model, no combination of companies existed that could satisfy all the goals. This poor goal performance was attributed to the SRI goal, which was rarely met. However, it was found that the margins by which this goal was not met were so small, that they were negligible. Thus, the portfolios that did not meet this goal were not excluded from consideration in the remainder of the study.

Some of the social model portfolios also did not meet the risk or liquidity goals. As with the risk-adjusted models, only the portfolios that were selected for the extremely risk-averse simulated investors did not meet the risk goal. Interestingly, it was found that the CCP social model selected higher-risk companies than the EV social model, which, in turn, selected higher-risk companies than the risk-adjusted models. As with the Markowitz models, the percentage by which these portfolios did not meet the liquidity goal was minuscule, and these portfolios were still viable investment options. Thus, it was shown that all the social model portfolios were viable investment options.

The results for the training period show that the three models do select different portfolios and that the EV and CCP versions of the models do select different portfolios. Both the Markowitz portfolios and the risk-adjusted portfolios achieved the simulated investor goals and selected multiple globally optimal portfolios. However, the inclusion of the SRI objective in the social models meant that the simulated investor goals were not met, and no globally optimal portfolios were selected. It should be noted that these results are highly dependent on the configuration and assumptions of the models, the selected holding period, and the dataset that was used. Should certain changes be made within the models, such as using a different dataset (such as the data of companies listed on the New York Stock Exchange) or using different weightings for the goals, these results and the resulting assertions may no longer hold true. Thus, the findings of this dissertation are restricted to this individual study.

If this dissertation ended after this analysis, as most portfolio selection studies do, the conclusion would have been that there is a significant difference between a generic and individualised portfolio selection approach. This is because based on the goal adherence results, the individualised risk-adjusted model significantly outperformed the other two models, and the generic Markowitz model outperformed the individualised social model. Thus, the risk-adjusted models, and not the Markowitz or social models, should be used to select investment portfolios that meet the investor goals in the training period. Given the dismal goal adherence performance of the social model, it could be concluded that it would not be beneficial for an investor to be Socially Responsible (SR) in their investments. Thus existing portfolio selection models should not be updated to incorporate SRI.

Nevertheless, investors invest to achieve their ROR and SRI goals in the future, not in the present. Thus, this dissertation determined and evaluated how the portfolios selected by all three models would have performed in an unknown future. A summary of these results is given in the next section.

5.1.1 Portfolio performance in the testing period

It was found that portfolios selected by both the Markowitz and the risk-adjusted models achieved dismal ROR values in the testing period. For both of these models, only the lowest ROR goal, namely ROR $\geqslant 0\%$, was ever achieved. Furthermore, none of the portfolios selected by these models achieved an ROR of more than 1%. All the portfolios selected by the Markowitz model, except two, achieved positive ROR values. On the contrary, except for a few portfolios in the moderately conservative risk category, all the portfolios selected by the risk-adjusted models had negative ROR values. Thus, it is evident that if only three objectives are considered, a generic portfolio selection approach outperforms an individualised approach. When comparing these two models' portfolio ROR values to the ROR of a known market index, the JSE all share index, it was found that all the model portfolios underperformed the market. The JSE all share index achieved an ROR of 9.04% in the testing period, which is far greater than the maximum 0.94% ROR achieved by any of the model portfolios. Thus, it was concluded that neither the Markowitz nor the risk-adjusted models selected market-competitive investments.

Given the dismal ROR performance of Markowitz and risk-adjusted models, it was expected that the social models would have unsatisfactory performance as well. The opposite was found to be true. Not only did all the portfolios selected by the social models achieve their ROR goals, they exceeded the highest goal of 20%. The lowest ROR achieved by any of the social model portfolios was 38.48%. Furthermore, all the social model portfolios achieved greater ROR values than the JSE all share index, and thus outperformed the market. Given that these models do select portfolios that consistently meet the ROR goals of the investor, in the future, it was concluded that these models are suitable for their intended purpose. Furthermore, it was concluded that the social models selected market-competitive portfolios. Thus, these models can, and should, be used to select investment portfolios for investors within the South African market. It was found that the EV social model should be used to select investment portfolios for investors that fall into the conservative, moderately aggressive and aggressive risk categories. The CCP social model should be used to select investment portfolios for investors that fall into the remaining two risk categories.

Similar results were achieved when looking at the ESG ratings obtained by the model portfolios in the testing period. As expected, the portfolios selected by the Markowitz and risk-adjusted models did not achieve an ESG rating of at least 2.5 and were thus not SR. All the social model portfolios achieved an ESG rating of at least 2.5 and were thus SR. These results confirmed that the social models were capable of selecting portfolios that meet both the ROR and SRI goals of the investor in the future.

The results also showed that the social models are far superior to both the Markowitz and risk-adjusted models because it was the only one of the three models that achieved the ROR and SRI goals of the simulated investor in the future. Upon investigation, it was found that generally, the social models invested high proportions into companies that became more profitable and low proportions into companies that became less profitable. Thus, the social models selected portfolios that were highly profitable in the testing period. Furthermore, it was found that the ROR results achieved by the social models align with the findings of other SRI researchers.

In this dissertation, a model was considered to be suitable for its intended purpose if it selected investment portfolios that met or exceeded the ROR and SRI goals of the investor in the testing period. Given that neither the Markowitz nor risk-adjusted models could select portfolios that would consistently meet the ROR goals of the investor it was concluded that, for this study, these models are not suitable for their intended purpose. This does not mean that these models can never be suitable for their intended purpose. If any of the conditions of this study were changed, such as using a different dataset, holding period or weightings for the goals, it may be found that these models may be suitable for their intended purpose. Given that neither the Markowitz nor the risk-adjusted models selected market-competitive portfolios, it was concluded that, according to the definitions of this dissertation, that they are not useful models. On the contrary, the social model portfolios always met the ROR and SRI goals of the simulated investors in the testing period and selected market-competitive investments. Thus, these models were classified as being useful and suitable for their intended purpose.

Although a portfolio selection model may not select investment portfolios that achieve the investment goals of the investor, it may still be worthwhile for an investor to invest in these portfolios. This is the case if the model portfolios achieves an ROR that is greater than or equal to the ROR achieved by existing investment options, such as unit trusts. Thus, all the model portfolio testing period ROR values were compared to the ROR achieved by 25 unit trusts in the same period. Once again the Markowitz and risk-adjusted models performed dismally, being significantly outperformed by the unit trusts. Thus, it was concluded that it would have been more beneficial to invest in a unit trust than to invest in a Markowitz or risk-adjusted portfolio. This strengthened the argument that the Markowitz and risk-adjusted models are not useful models and should not be used to select portfolios for investors.

On the contrary, the social model portfolios significantly outperformed all the unit trusts. Thus, it was found that would have been more beneficial for an investor to invest in a social model portfolio than to invest in a unit trust. Given that the social models selected portfolios that met the investor goals in the future, outperformed the market, and outperformed existing investment options, it was evident that they are useful models and should be used to select portfolios for investors.

This section shows that of the three models that were considered, only the social model was considered to be useful and suitable for its intended purpose. The next section discusses how these results reflect on the aim of this dissertation.

5.1.2 Reflection on research question

From the results, it was clear that there was a considerable difference between a generic and individualised portfolio selection approach. When only changing the risk objective, it was found that the portfolios selected by the Markowitz model had better performance than the risk-adjusted model portfolios. These results are contrary to a fundamental assumption of this dissertation; that it is necessary to account for the individual risk tolerance of investors when selecting investment portfolios for investors. Furthermore, these results substantiate why the Markowitz risk objective is the standard within the investment world.

When an SRI investing objective was added to the model formulation, the social model portfolios significantly outperformed the Markowitz model portfolios. Furthermore, the social model portfolios also significantly outperformed the risk-adjusted model portfolios. These results showed that investors can invest ethically and still achieve their financial goals, strengthening the argument that existing models should be updated to align with the shift towards SRI.

These results also confirm that portfolio selection models should be validated by being tested on an unknown future. The Markowitz and risk-adjusted models had satisfactory results in the training period, while the social models had unsatisfactory results in this period. If only these results were considered, as is the norm in portfolio selection literature, it would have been concluded that the risk-adjusted model is the best model to use for portfolio selection. Furthermore, it would have been concluded that an individualised portfolio selection approach that does not incorporate SRI outperforms a generic approach. However, the testing period results indicated that these conclusions are inaccurate. Thus, all portfolio selection models should be validated by being tested on an unknown future.

5.1.3 Model validation

Although the models were considered to be validated because they were test on an unknown future, it was still unclear whether or not the risk-adjusted and social models achieved the aim of selecting *individualised* investment portfolios. For this reason it was argued that these models were still unvalidated. The Conditional Value-at-Risk (CVaR) values achieved by the portfolios selected by these four models were plotted against the RTS values at which they were achieved as well as the ROR goal values. It was expected that as the RTS values increase, the models would select riskier portfolios and thus the CVaR values would increase. However, this simply was not the case in this dissertation. It was found that for all four models, although there was some degree to which the CVaR values increased as the RTS values increased, especially at very low RTS values, there was no clear and increasing differentiation between the portfolios selected at the different RTS values. After investigation it was found that this lack of higher CVaR values can be attributed to the sample that was used in the study.

It was found that 205 of the 208 sample companies achieved CVaR conservative risk values in the training period. Thus, it was unsurprising that the portfolios selected from this sample achieved conservative CVaR values. Yet, this meant that it was not possible to evaluate how the models would have selected portfolios if a sample that had a larger variation of CVaR values was used. This also meant that given the dataset that was used, it was not possible to determine if the portfolios selected for investors with higher RTS values would contain higher risk companies than the portfolios selected for investors with lower RTS values. As such, the dataset did not allow for the proper validation of larger RTS values. Thus, it became clear that the evaluation was significantly constrained by the dataset that was used. Owing to this lack of variation within the dataset, it could not be said that the models achieved the aim of selecting *individualised* portfolios. Thus, it was concluded that one of the principle hypotheses of this dissertation was disproven. The addition of an individualised risk objective does not ensure that individualised portfolios are selected. Furthermore, there is no clear differentiation between the risk distribution achieved by different investors who have different RTS values. Thus, these models were classified as being unvalidated. If a different dataset was used, this result may be different.

Given the constraints of the dataset that was used, it was decided to investigate whether or not using a sample of companies that have a broad range of CVaR values would yield a different result. For this purpose a random list of 208 CVaR values between zero and 100 was generated and each of the sample companies was assigned one of the CVaR values. The risk-adjusted EV and social EV models were rerun using this new adjusted dataset. It was found that the results did not differ much. Once again, there was some degree to which the portfolio CVaR values increased as the RTS values increased, but this was not substantial enough to make a definitive judgement about the degree to which this means that the models achieve the aim of selecting individualised portfolios. Thus, from all the results it was concluded that having an risk objective that incorporates the financial risk tolerance does not ensure that portfolios are selected that align with this risk tolerance. Furthermore, it was concluded that the model formulated in this dissertation do not achieve one of their principle aims, which is to select *individualised* investment portfolios for investors.

From the results in this section, it was clear that this dissertation produced some novel findings within the field of portfolio selection. However, this study has many limitations which creates opportunities for further study and research. The next section gives a summary of the contributions, limitations and future work opportunities of this dissertation.

5.2 Research contributions, limitations and future work

Given the recent shift towards SRI, this dissertation formulated a model that included an objective aimed at maximising SRI. When analysing the results produced by the social model, it was found that its performance was unparalleled.

Given this unparalleled performance, as well as a novel model formulation, this model makes a contribution to the field of portfolio selection.

In addition, this dissertation highlights the importance of testing portfolio selection models on an unknown future and demonstrates the potentially horrific consequences of neglecting this analysis. Thus it should become standard practice for portfolio selection models to be tested on an unknown future. This assertion, and its evidence, is another contribution of this dissertation.

Despite making contributions to the field of portfolio selection, the research conducted for this dissertation has many limitations. An explicit limitation of this dissertation is the sample size. The sample is limited to the 208 JSE Limited companies for which ten years of data could be extracted from `Yahoo Finance`. This sample should be increased to include all the companies listed on the JSE (344). Furthermore, industry practitioners and researchers are not constrained to the companies listed on the JSE. They often consider internationally listed companies, as well as other investments such as bonds, options, and others, when performing portfolio selection. This research could be expanded by using a dataset representative of the datasets used in practice. This research is also limited by the generalisability of the results. This limitation could be addressed by repeating the experiments conducted in this study on many other exchange markets as well.

Another limitation is the period for which the models are developed. The models could be developed for long time periods to evaluate the long-term impacts of the results of the portfolios. An alternative to developing the models for more extended time periods is to develop models that rebalance dynamically. Investment portfolios in industry are continuously rebalanced, being rebalanced monthly, weekly or even daily. Yet, the portfolios selected in this dissertation are static for a period of three years. Thus, portfolio selection models could be developed that rebalance dynamically.

When the same objectives were considered (ROR, liquidity and risk), the generic Markowitz model outperformed the individualised risk-adjusted model. However, when a fourth objective was added, the Markowitz model was significantly outperformed. Thus, it is clear that the SRI objective is the differentiating factor in the success or failure of the selected portfolios. Thus, this research can be expanded by adding an SRI objective to the Markowitz model. This SRI-Markowitz model could then be compared to the generic Markowitz model and the social model. This will allow for a more in-depth analysis of the impact of considering SRI within portfolio selection models.

The models developed in this dissertation are based on the assumption that share trading occurs under ideal circumstances. This means that these model assume that any shares can be bought as part of the portfolio, there are no bounds on the number of shares that one can buy from a specific company, there are no transaction costs, and there are no holding constraints imposed by the JSE. In reality, this is not the case. Thus, this research can be expanded by developing models that account for these additional considerations.

Throughout this dissertation, it has been stated that the results that were obtained are highly depended on the assumptions and configurations of the models. One such assumption is the weightings of the goals within the model formulation. In this dissertation, the goals were considered to be of equal importance. In reality, this may not be the case. This research could be expanded by using weighted goal programming and setting the goals to have differing levels of importance. Furthermore, different combinations of weightings could be used to investigate the relative importance and impact of each of the goals. Another possible avenue for future research is that the weights of the objective functions could be derived from information gathered on the investor, as done by Kaiser et al. (2014). Alternatively the objectives could be weighted according to the inputs of the investors (Kaiser et al., 2014).

The solutions found in the dissertation may be efficient, but goal programming was not implemented in such a way to ensure that the solutions obtained are efficient. Thus, this research could be expanded in the future by implementing goal programming in such a way that an efficient solution is assured.

The body of portfolio selection knowledge is ever-growing and expanding. This dissertation made contributions to the field of portfolio selection. Nevertheless, there are still many opportunities to explore.

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Appendix A

Distribution graphs

Appendix A.1 ABSA Group Limited

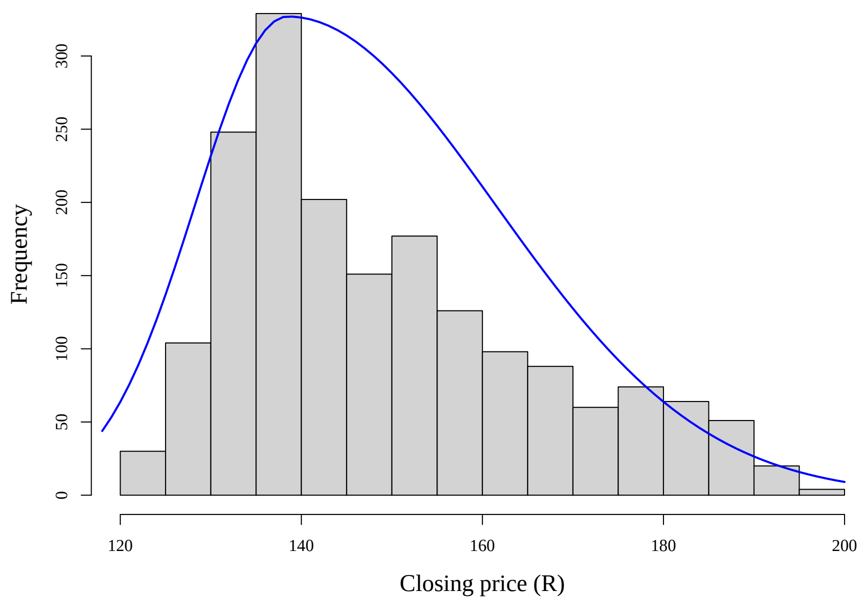

Figure A.1: The closing prices distribution of ABSA Group Limited during the training period

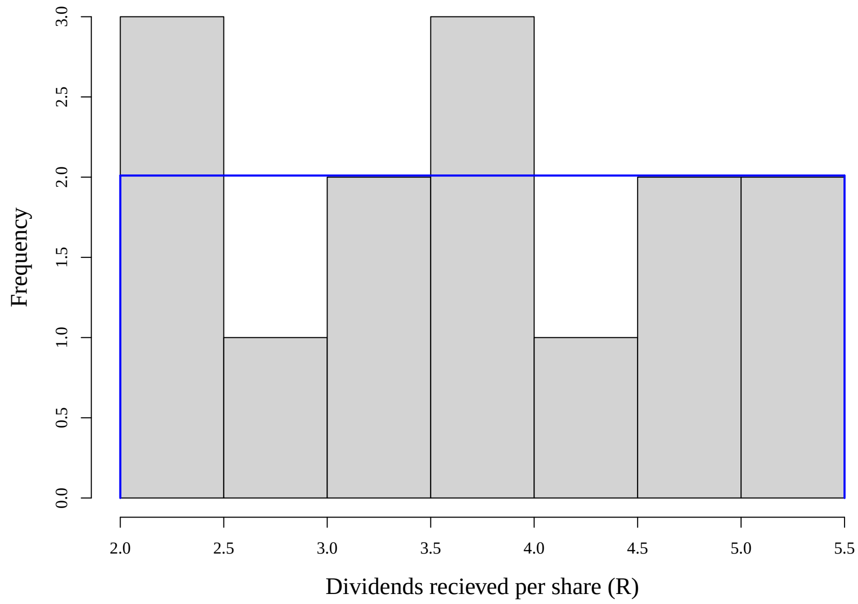

Figure A.2: The dividends distribution of ABSA Group Limited during the training period

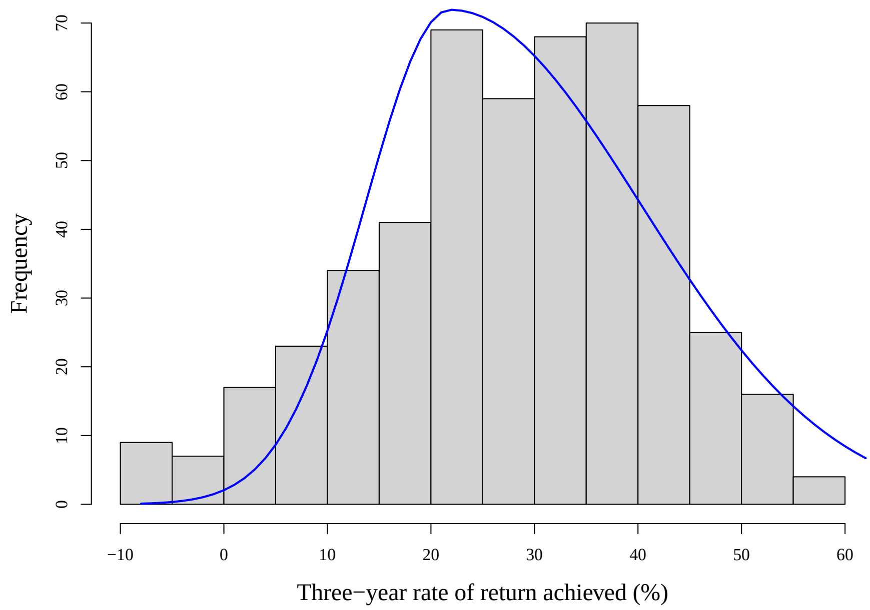

Figure A.3: The three-year rate or return distribution of ABSA Group Limited during the training period

Appendix A.2 Combined Motor Holdings Limited

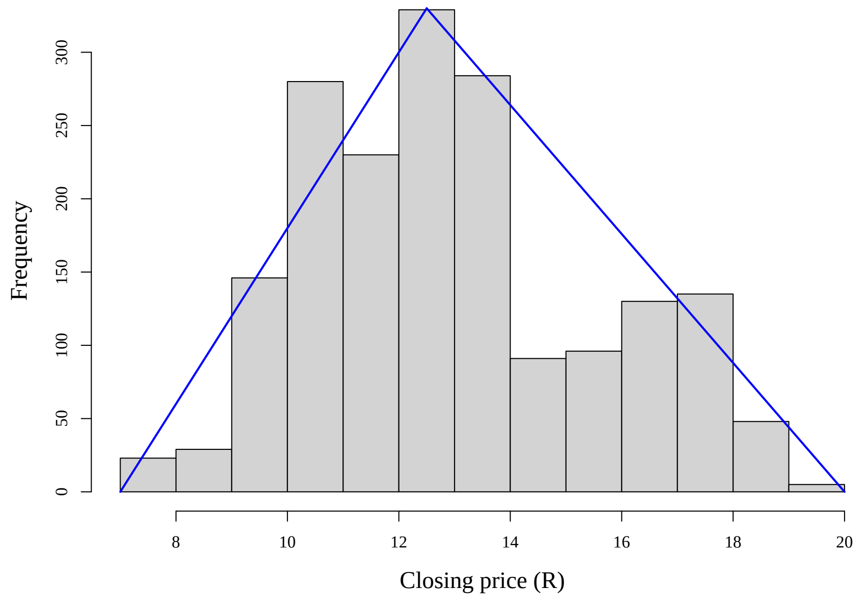

Figure A.4: The closing prices distribution of Combined Motor Holdings Limited Limited during the training period

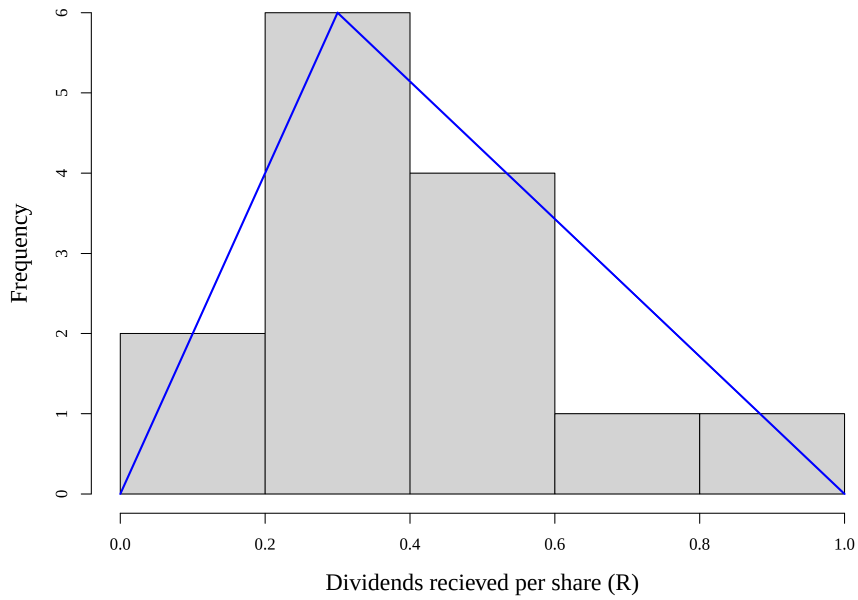

Figure A.5: The dividends distribution of Combined Motor Holding Limited during the training period

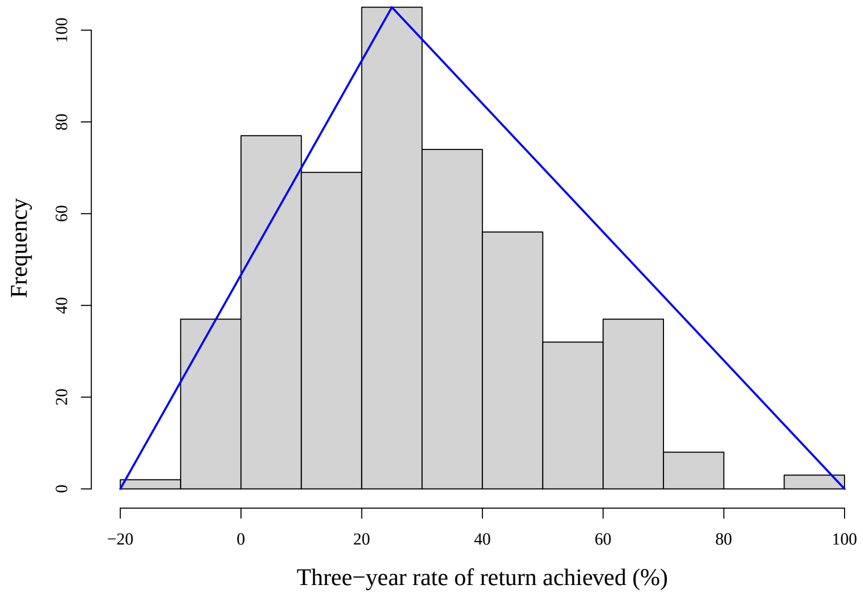

Figure A.6: The three-year rate of return distribution of Combined Motor Holding Limited during the training period

Appendix A.3 Invicta Holdings Limited

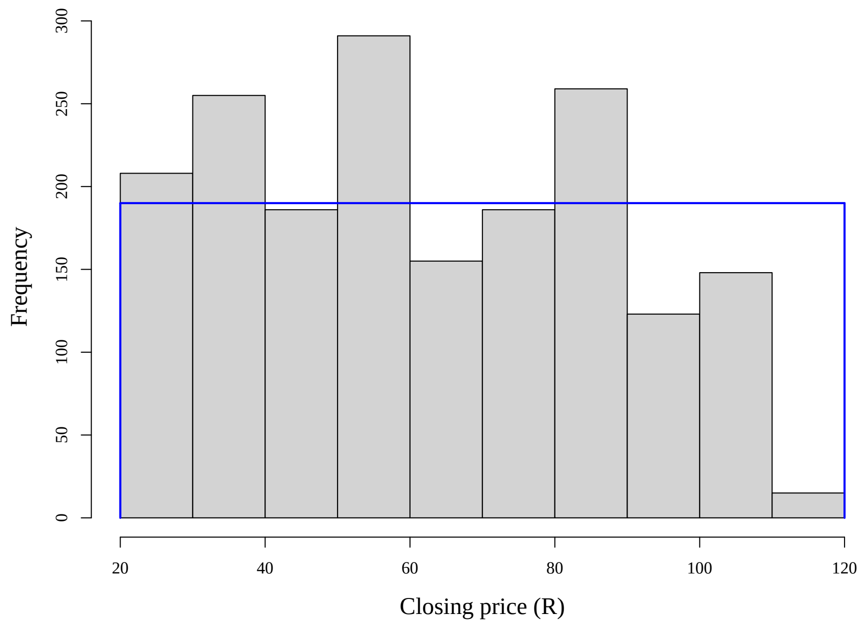

Figure A.7: The closing prices distribution of Invicta Holdings Limited Limited during the training period

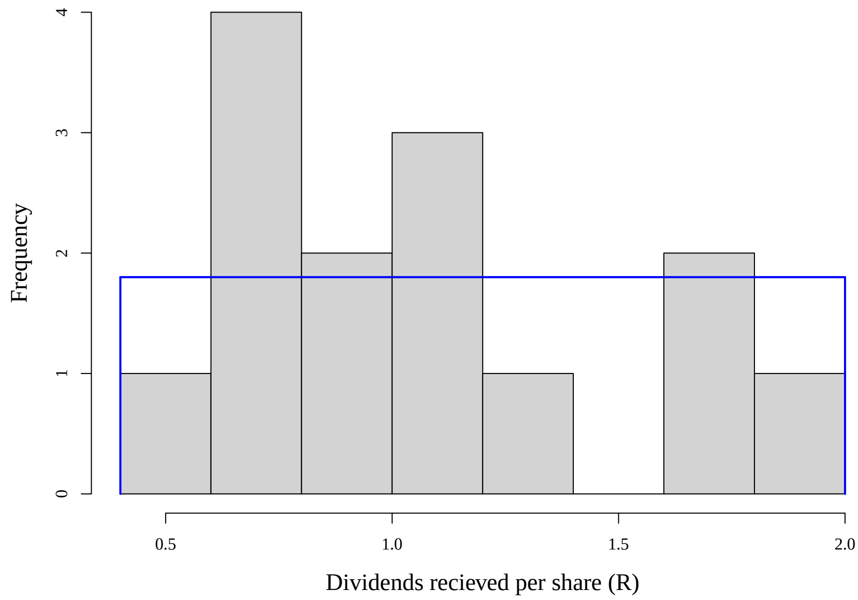

Figure A.8: The dividends distribution of Invicta Holdings Limited during the training period

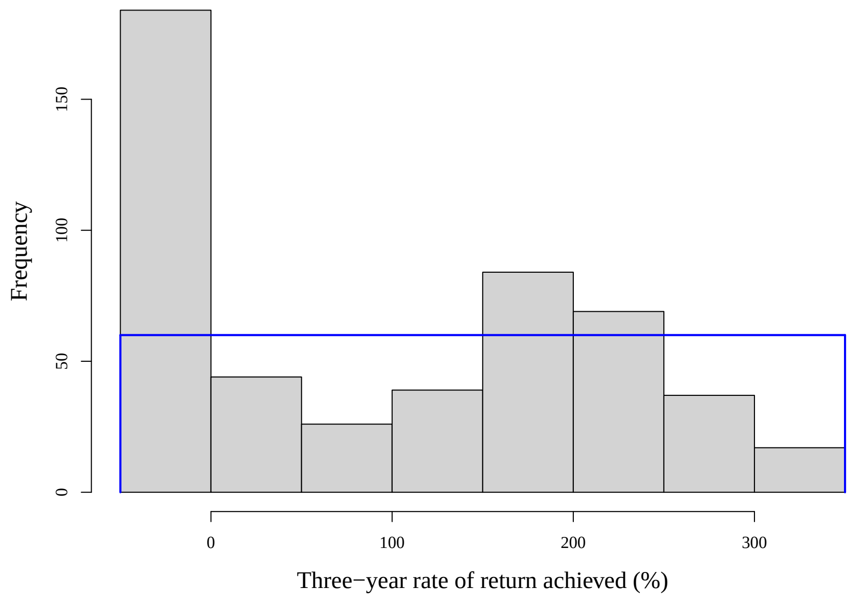

Figure A.9: The three-year rate of return distribution of Invicta Holdings Limited during the training period

Appendix B

Additional ADP tables

Table B.1: ADP achieved at each RTS value when comparing the EV and CCP versions of the Risk-adjusted and Social Models

	Risk-adjusted (EV vs CCP)			**Exclusion (EV vs CCP)**		
RTS	Min	Median	Max	Min	Median	Max
20	0	0	0	6.7	6.7	6.7
21	0	0	0	16.4	16.4	16.4
22	61.2	61.2	61.2	6.8	6.8	6.8
23	61.2	61.2	61.2	5.223	5.223	5.223
24	61.2	61.2	61.2	13.64	13.64	13.64
25	61.2	61.2	61.2	13.64	13.64	13.64
26	61.2	61.2	61.2	13.64	13.64	13.64
27	61.2	61.2	61.2	13.64	13.64	13.64
28	61.2	61.2	61.2	13.64	13.64	13.64
29	61.2	61.2	61.2	13.64	13.64	13.64
30	61.2	61.2	61.2	16.4	16.4	16.4
31	61.2	61.2	61.2	16.4	16.4	16.4
32	61.2	61.2	61.2	16.4	16.4	16.4
33	61.2	61.2	61.2	16.4	16.4	16.4
34	61.2	61.2	61.2	16.4	16.4	16.4
35	61.2	61.2	61.2	16.4	16.4	16.4
36	61.2	61.2	61.2	16.4	16.4	16.4
37	61.2	61.2	61.2	16.4	16.4	16.4
38	61.2	61.2	61.2	16.4	16.4	16.4
39	61.2	61.2	61.2	16.4	16.4	16.4
40	61.2	61.2	61.2	16.5	16.5	16.5
41	61.2	61.2	61.2	16.5	16.5	16.5
42	61.2	61.2	61.2	16.5	16.5	16.5
43	61.2	61.2	61.2	16.5	16.5	16.5
44	61.2	61.2	61.2	16.5	16.5	16.5

	Risk-adjusted (EV vs CCP)			Exclusion (EV vs CCP)		
RTS	Min	Median	Max	Min	Median	Max
45	90.7	90.7	90.7	16.5	16.5	16.5
46	90.7	90.7	90.7	16.5	16.5	16.5
47	90.7	90.7	90.7	16.5	16.5	16.5
48	90.7	90.7	90.7	16.5	16.5	16.5
49	90.7	90.7	90.7	16.5	16.5	16.5
50	90.7	90.7	90.7	20.1	20.1	20.1
51	90.7	90.7	90.7	20.1	20.1	20.1
52	90.7	90.7	90.7	20.1	20.1	20.1
53	90.7	90.7	90.7	20.1	20.1	20.1
54	90.7	90.7	90.7	20.1	20.1	20.1
55	90.7	90.7	90.7	20.1	20.1	20.1
56	90.7	90.7	90.7	20.1	20.1	20.1
57	90.7	90.7	90.7	20.1	20.1	20.1
58	90.7	90.7	90.7	20.1	20.1	20.1
59	90.7	90.7	90.7	20.1	20.1	20.1
60	90.7	90.7	90.7	16.4	16.4	16.4
61	90.7	90.7	90.7	16.4	16.4	16.4
62	90.7	90.7	90.7	16.4	16.4	16.4
63	90.7	90.7	90.7	16.4	16.4	16.4
64	90.7	90.7	90.7	16.4	16.4	16.4
65	90.7	90.7	90.7	16.4	16.4	16.4
66	90.7	90.7	90.7	16.4	16.4	16.4
67	90.7	90.7	90.7	16.4	16.4	16.4
68	90.7	90.7	90.7	16.4	16.4	16.4
69	90.7	90.7	90.7	16.4	16.4	16.4