An investigation into Grade 7 learners' knowledge of ratios

by

Siduduzile Bango

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Supervisor: Dr R. D. Sekao

Co-supervisor: Prof. J. C. Engelbrecht

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Declaration

I declare that the dissertation, which I hereby submit for the degree Magister Educationis at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

Siduduzile Bango

30 March 2020

Ethics Statement

I, Siduduzile Bango, have obtained, for the research described in this work, the applicable ethics approval. I declare that I have observed the ethical standards required in terms of the **University of Pretoria's** "Code of ethics for researchers and the policy guidelines for responsible research".

30/03/2020

SIGNATURE OF STUDENT DATE

Dedication

I dedicate this research to Asah Sibanda, my late grandmother, who always motivated me in my academic work and instilled a strong work ethic in me. I also dedicate it to my children Tamilika and Ntombikayise, to show them that education does not end. I hope this research will motivate them to work even harder in their studies.

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Abstract

Ratio is one of the key mathematics concepts included in the South African Mathematics curriculum. It is applied in other topics of the Grade 7 curriculum, including geometry, functions and relationships, algebra, similarity and congruency.

The aim of this qualitative research study was to explore the difficulties that learners experience in learning *ratio*. The primary research question for the study was: *What is Grade 7 learners' knowledge of ratio?* This research question was answered through the following secondary research questions: *How do learners solve problems involving ratio? What is learners' conceptual knowledge of ratio? And what learning difficulties do learners experience when learning about ratio?*

The study was informed by Kilpatrick, Swafford and Findell's (2001) five strands of mathematical proficiency; however, the focus was on *conceptual* and *procedural knowledge* of *ratio.* The interpretivist paradigm and the single exploratory case study design were used to gain insight into the learning of *ratio*. Data was collected from Grade 7 learners (23 of the 35 learners originally sampled) through a self-developed test that followed the prescripts of the Grade 7 Mathematics curriculum in South Africa and through semi-structured interviews. The test scripts were analysed using the Atlas.ti™ windows coding system and the results were used to construct questions for the semi-structured interviews. The interviews were used to corroborate data emerging from the test.

The results of the study indicated that Grade 7 learners can do simple and routine manipulations of ratio as well as non-proportional ratio problems but struggle to solve problems that require multiplicative thinking and proportional reasoning skills. Although there could be other factors contributing to learners' struggle to tackle proportional ratio problems requiring multiplication and proportional reasoning, a lack of *conceptual knowledge* seemed to contribute significantly.

Key Terms

Ratio, conceptual knowledge, procedural knowledge, proportional reasoning, mathematical errors multiplicative thinking, misconceptions,

Language Editor

WORDPLAY EDITING
Copy Editor and Proofreader
Email: karien.hurter@gmail.com
Tel: 071 104 9484
Website: http://wordplayediting.net/
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To Whom It May Concern:

This letter is to confirm that the Master's dissertation *An investigation into Grade 7 learners' knowledge of ratios* by Siduduzile Bango was edited by a professional language practitioner.

Regards,

Karien Hurter

List of Abbreviations

ANA:	Annual National Assessment
CAPS:	Curriculum and Assessment Policy Statement
DBE:	Department of Basic Education
TIMSS:	Trends in International Mathematics and Science Study

Description of Key Terms

Proportion

"Proportion is a relationship between numbers or quantities in which the ratio of the first pair equals the ratio of the second pair; written a: b = c: d" (Ekawati, Lin & Yang, 2015, p. 86).

Proportional reasoning

Proportional reasoning is "detecting, expressing, analysing, explaining and providing evidence in support of assertions about proportional relationships" (Lamon, 2014, p.10). In this study, proportional reasoning refers to the ability to think appropriately in situations involving simple direct proportions and inverse proportions and being able to justify the assertions made about the relationships.

Multiplicative thinking

Multiplicative thinking is the capacity to work flexibly with the concepts, strategies and representations of multiplication and division as they occur in a wide range of contexts (Lamon, 2014). In this study, it refers to the ability of learners to conceptualise *ratio* as a measure of the strength of the invariant relationship.

Learner performance

This study uses *learner achievement* as the amount of academic contents (in this case *ratio*) a learner learns in a determined amount of time (Kilpatrick, Swafford & Findell, 2001).

Conceptual knowledge

Conceptual knowledge is the "comprehension of mathematical concepts, operations and relations" (Kilpatrick et al., 2001, p. 18). In this study, it refers to the ability of learners to identify proportional and non-proportional problems, demonstrate an understanding of *ratio* in solving problems.

Procedural knowledge

Procedural knowledge is the skill of "carrying out procedures flexibly, accurately, efficiently and appropriately" (Kilpatrick et al., 2001, p. 18). In this study it refers to the ability of learners to correctly solve problems involving *ratio*, accurately following mathematically-sound procedures.

Learners

According to the South African School Act (1996) "*Learner* means any person receiving education or obliged to receive education in terms of this act" (p. 4). In the South African context this refers to children who attend formal or non-formal education at any age group. Universally the commonly used term is pupils instead of learners.

Misconceptions

Ojose (2015) defined misconceptions as "misunderstandings and misinterpretations based on incorrect meanings; they are due to naïve theories that impede rational reasoning of learners" (p. 30). In this study misconception refers to the entrenched incorrectly conceived mathematical facts about *ratio*. These include conceptual understanding and procedural skills that learners apply wrongly when solving *ratio* problems.

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CHAPTER 1: OVERVIEW OF THE STUDY

1.1 The context of this study

Mathematics is perceived as one of the most difficult subjects to learn and teach in South African schools, however, much of this perceived difficulty stems from the differing Mathematics content taught in the various grades and the numerous changes to the Mathematics curriculum since 1994 (McCarthy & Oliphant, 2013). Since 1994, when South Africa gained its democratic freedom, the government introduced different policies to address the imbalances created by the apartheid era. Most of these policies were introduced to improve the Mathematics and Science performance of learners in schools. For instance, in 1995 the White Paper on Education and Training Policy was introduced specifically to improve Mathematics, Science and Technology education; in 2001 the National Strategy for Mathematics, Science and Technology Education was introduced and gave rise to the Dinaledi schools project; and in 2012 the government introduced the National Development Plan 2030 policy to increase the number of students achieving above 50% in Mathematics (McCarthy & Oliphant, 2013).

Despite the numerous changes in the education system of South Africa, learners, especially in the Senior Phase (Grades 7 – 9), continue to perform poorly in Mathematics as evidenced in international assessments of educational achievements such as the Trends in International Mathematics and Science Study (TIMSS, 2015) as well as national assessments such as the Annual National Assessments (ANA) results (McCarthy & Oliphant, 2013). An analysis of results of these assessments revealed “a variety of frameworks and methodologies, but across these there was an unanimous agreement on the very low and highly unequal performance of South African learners” (Venkat & Spaull, 2015, p.123). Venkat and Spaull (2015) illustrated that “these studies, and particularly those that focus on mathematics, have identified that learners acquire learning deficits early in their schooling careers and that these backlogs are the root cause of underperformance in later years” (p. 125).

Ratio is known to be an area of mathematics that learners find difficult (Spaull & Kotze, 2015). This difficulty is mostly a result of a lack of *conceptual understanding* as evidenced by other research studies. Calisici (2018) claimed that "the acquisition of abstract concepts such as *ratio* is more difficult in mathematics because mathematics is a field where the abstract and prerequisite relationship is in its nature intense" (p. 98). The concept *ratio* is included in the South African Mathematics curriculum, the Curriculum and Assessment Policy Statement (CAPS). When I compared the content knowledge that is covered on *ratio* in Grade 7 and the *conceptual* and *procedural knowledge* that is required for learners to gain mathematical proficiency in *ratio,* there is a discrepancy because the curriculum limits the content knowledge and time for learners to deeply understand the concept of *ratio* (Department of Basic Education [DBE], 2013 p. 42). Consequently, this results in learners performing poorly in Grade 7 Mathematics where *ratio* is an underlying concept for most topics. Calisici (2018) claimed that "when a topic is not completely comprehended in mathematics, learning difficulties progress hampering understanding of other mathematics topics" (p. 164).

1.2 Problem statement

Learning of *ratio* and proportion concepts is the base for learning important concepts such as linear functions, scale drawing, similarity, geometry probability and algebra in the Senior Phase. Although research has documented children's difficulties in learning about *ratio*, not much has been done to try and investigate learners' *conceptual* and *procedural knowledge* of *ratio*. The results of the previous research tended to identify a list of things that learners can and cannot do. I, therefore, need to move beyond identifying variables that affect problem difficulty towards identifying components that explain children's performance in the domain. In fact, "developing an understanding of and applying proportional relationships in Grade 7 are deemed to be one of the critical areas of focused instructional time" (Jitendra, Star & Dupuis, 2013, p. 56).

1.3 Rationale and significance of the study

As a Grade 7 Mathematics educator, I have noted that most learners in Grade 7 perform poorly in Mathematics. Their performance declines as they move up in grades. It has been noted through the Annual National Examination and international examinations that South African learners, especially in the Senior Phase, struggle to solve problems in *ratio*, and as a result, they perform poorly in these examinations as *ratio* is a concept that cuts across all topics in the curriculum. Therefore, it appears frequently in the assessment tests given to the learners. This has motivated me to try and find out learners' *conceptual* and *procedural knowledge* of *ratio*. I hope that the results of this study will help me and other teachers to find strategies to help learners acquire knowledge on *ratio*. Although the results of this study cannot be generalised, it will give us a starting point from which to help learners, especially Grade 7 learners. This will be done through assessing Grade 7 learners' *conceptual* and *procedural knowledge* of *ratio*.

1.4 Purpose of the study

The purpose of this study was to determine the learning difficulties that Grade 7 learners experience while learning *ratio.* I focused on learners' *conceptual* and *procedural knowledge* of *ratio*. To achieve this, I formulated the following research questions.

1.5 Primary research question

The primary research question for the study is: What is Grade 7 learners' knowledge of *ratio*?

The primary research question will be answered through the following secondary research questions.

1.6 Secondary research questions

a) **How do learners solve problems involving *ratio*?**
b) **What is learners' conceptual knowledge of ratio?**
c) **What learning difficulties do learners experience while solving *ratio*?**

1.7 Theoretical framework and literature overview

1.7.1 Theoretical framework

This study was guided by strands of mathematical proficiency advocated by Kilpatrick, Swafford & Findell (2001). They identified five strands of mathematical proficiency, namely, *conceptual understanding, procedural fluency,* strategic competence, adaptive reasoning and productive disposition. According to Kilpatrick et al. (2001), the five strands are interwoven and interdependent, since they are orchestrated to help learners acquire mathematical proficiency. In this study, I chose to use two of the strands of mathematical proficiency, *conceptual understanding* and *procedural fluency*, to frame my study. I refer to these two constructs as two different types of knowledge, namely, *conceptual knowledge* and *procedural knowledge*, needed to gain insight into learners' knowledge of ratio. This will be discussed in detail in Section 2.1.

1.7.2 Literature overview

In this study, I reviewed literature gleaned from local and international studies to gain insight into the concept *ratio*. This helped me support my findings and discussions thereof, as well as make recommendations for future studies. In this section I will give an overview of the literature review as a precursor of the comprehensive literature review in Section 2.2.

Numerous studies have shown that early adolescents and many adults have difficulty with the basic concepts of fractions, *ratio* and proportion and with problems involving these concepts (Singh, 2001). One of the reasons learners experience difficulties with *ratio* is that they intuitively apply additive strategies rather than using multiplicative thinking (Lamon, 2007). Therefore, this issue of reliance on additive thinking is considered a significant challenge to teaching and learning *ratio*. Hart (as cited in Lamon, 2007) reported that less than 42% of students in Grade 7 succeeded in solving simple problems of enlargement, and the most common source of error was additive reasoning where learners focused on the difference between the given quantities rather than the proportionality illustrated in the context. The National Assessment of Educational Progress (NAEP, 2015) assessment results showed that less than half of Grade 8 learners correctly answered a basic proportional reasoning question when they were given a ratio and asked to find the missing value in a second ratio. Livy and Vale (2012) claimed that the language and range of types of ratio situations may be reasons for confusion when working with ratio situations. For over three decades researchers have stressed the general importance of proportional reasoning (Che, 2009). They have identified mastery of proportional reasoning as a signpost to signal an understanding of Senior Phase

Mathematics and as a foundation for future learning of mathematics in high school. Livy and Vale (2012) defined proportional reasoning as an understanding of the multiplicative relationship between variables in proportion situations. When solving mathematical problems, learners must be able to solve proportional problems (multiplicative ratio problems) and non-proportional (additive ratio problems) problems to display their *conceptual understanding* of *ratio.* In a research study done by Misailidou and William (2003) on proportional reasoning, the results showed that non-proportional problems were solved with the lowest success rate, while proportional problems were solved with the highest success rate. These results, therefore, indicate the importance of *conceptual knowledge* as well *procedural knowledge* in the learning of *ratio*.

1.8 Research methodology

This study employed a qualitative approach guided by an interpretative paradigm. This research approach enabled me to closely interact, observe and interpret learners' methods and skills used to solve *ratio* problems. I was able to explore in-depth the Grade 7 learners' knowledge of *ratio.* The case study design enabled me to achieve a comprehensive understanding of the learners' *conceptual* and *procedural knowledge* of *ratio.* I used two data collection instruments making it possible for me to focus on different aspects of learners' knowledge of *ratio.* These methods are covered in more depth in Chapter 3.

1.8.1 Sampling

I chose to use purposive sampling to select a sample of 35 Grade 7 learners. This type of sampling was relevant for the study because it allowed me to collect data from the learners in Grade 7 (the entry grade of the Senior Phase in South Africa). This is against the backdrop that learners in the Senior Phase (Grades 7 – 9) have been performing unacceptably low in mathematics in the national and international studies.

1.8.2 Data collection and analyses

The data collection instruments used in this study included a self-developed test on *ratio* and semi-structured interviews. The assessment test was used to collect data on how learners solved *ratio* problems (see Appendix A). The written assessment test guided me in understanding learners' procedural skills when solving *ratio* as well as their conceptual understanding of *ratio.* The test was used to answer research question 1 and 3.

The assessment test was analysed using the Atlas.ti™ windows 8 coding system. Codes were created and grouped to form themes that I analysed and interpreted. Selected scripts which fell under different themes were recorded for further investigation.

Semi-structured interviews were used in this study to corroborate data emerging from the assessment test and to further explore learners' conceptual knowledge of *ratio.* The interviews answered the research question 2. The interview enabled the participants to provide detailed information on their conceptual and procedural knowledge of *ratio*.

Analysis of data collected through semi-structured interview started with the transcriptions of data from the recordings. I used Atlas.ti™ windows 8 coding software for coding. The created codes were grouped together to form themes which informed the findings of the study.

1.9 Trustworthiness

In any study, researchers should establish the protocols and procedures necessary for a study to be considered credible (Connelly, 2016). To ensure the quality of this study, I observed the following criteria: credibility, transferability, dependability as well as confirmability. Each of these criteria are discussed in detail in Chapter 3.

1.10 Ethical considerations

The basic principles of ethics include the principles of justice, mutual respect and the avoidance of doing harm to children subject to any research (Ferdousi, 2015). Due to the potential vulnerabilities and mental immaturity, the interest and rights of children as partcipants in a research study need to be protected. This research focused on the following ethical principles: informed consent, voluntary participation, confidentiality, anonymity, confidence and no harm or risk to participants. These principles are dealt with in detail in Chapter 3.

1.11 Outline of study

Chapter 1

This chapter gives the context for the study, the problem statement, the research questions and the rationale for the study.

Chapter 2

This chapter gives a literature review for this research that encompasses evidence, views and opinions from prior studies conducted on the learning of *ratio*. The theoretical framework on which the study is based is discussed in depth.

Chapter 3

I present the research methodology and discuss the research design. The research paradigm, including the ontological perspective, the epistemological perspective and the methodological perspective of the study are included. The sampling methods and the tools to be used for data collection follows.

Chapter 4

This chapter presents the analysis of the collected data and the findings of the study in an integrated manner, presenting the evidence from the participant test responses as well as evidence from the interview sessions.

Chapter 5

I present the interpretation of the research findings. The discussions include evidence from learners' test responses that were presented in Chapter 4. I also respond to the research questions of the study and discuss how the theoretical framework guided the study. Recommendations and conclusions are also included.

1.12 Conclusion

Chapter one presented an overview and general outline of the study. The next chapter is the literature review of related studies previously conducted in South Africa and internationally on similar issues.

CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Theoretical framework

This chapter will first explain in detail the theoretical framework for this study. This will help guide the literature review of the study because theory helps researchers explain social facts explicitly (Thomas, 2017). Popper (2002) claimed that theories are like nets cast to catch what we call 'the world' to analyse, to explain things and to act in a consistent fashion. Therefore, in a research study, a theoretical framework is a guide for the research that is often 'borrowed' by the researcher to build their own research inquiry (Adom, Joe & Hussein, 2018). It gave me a foundation on which to construct the research and prevented me from deviating from the confines of the accepted theories to make my final contribution scholarly and academic (Adom et al., 2018).

This study used the strands of mathematical proficiency by Kilpatrick, Swafford & Findell (2001) as a theoretical framework. Although mathematical proficiency, according to Kilpatrick et al. (2001), is characterised by five strands of mathematical proficiency, this study focused on only two strands, namely *procedural fluency* and *conceptual understanding*. Figure 1 illustrates the strands of mathematical proficiency according to Kilpatrick et al. (2001).

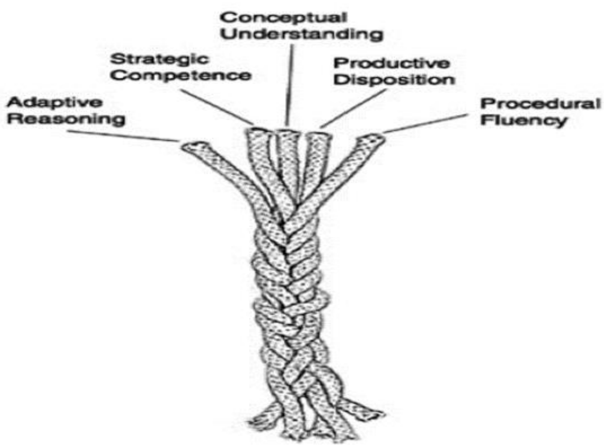


Figure 1: The strands of mathematical proficiency (Groves & Susie, 2012, p. 117)

In their theory of mathematical proficiency, Kilpatrick et al. (2001) identified five components or strands that are needed by people, especially learners, to promote mathematical knowledge, understanding, skills and beliefs that constitute mathematical proficiency. These strands are interwoven and interdependent in the development of proficiency in mathematics (Kilpatrick et al., 2001) as shown in Figure 1. The five strands are:

- *"Conceptual understanding:* Comprehension of mathematical concepts, operations, and relations".
- *"Procedural fluency*: Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately".
- *"Strategic competence*: Ability to formulate, represent, and solve mathematical problems".
- *"Adaptive reasoning*: Capacity for logical thought, reflection, explanation, and justification".
- *"Productive disposition*: Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy". (Kilpatrick et al., 2001, p. 116)

As I mentioned earlier, the study focuses on only two of the strands, namely *conceptual understanding* and *procedural fluency.* I chose to use these two strands because in learning *ratio* it is vital that learners grasp the *conceptual knowledge* or *conceptual understanding* of the concept. The words *conceptual understanding* and *conceptual knowledge*, *procedural fluency* and *procedural knowledge* will respectively be used interchangeably in this study. Although the focus is on *conceptual understanding* and *procedural fluency,* I have also shed light on the other three strands in this chapter.

Kilpatrick et al. (2001) defined *conceptual understanding* as "an integrated and functional grasp of mathematical ideas" (p. 118). Learners display their *conceptual understanding* in *ratio* by knowing all the facts and methods related to the concept. They must be able to use their proportional reasoning skills to tackle ratio problems and be able to differentiate additive and multiplicative ratio problems.

When learning *ratio*, learners show their *conceptual understanding* by being able to connect pieces of knowledge they learnt on fractions, division, percentages and multiplication to use it to construct their own knowledge of *ratio*. According to Kilpatrick et al. (2001), "competence in an area of inquiry depends upon knowledge that is not merely stored but represented mentally and organised (connected and structured) in ways that facilitate appropriate retrieval and application" (p. 118).

Kilpatrick et al. (2001) claimed that "learning with understanding is more powerful than simply memorising because the organisation improves retention, promotes fluency, and facilitates learning related material" (p. 118). In the Senior Phase, the concept *ratio* cuts across the Mathematics curriculum from Grade 7 up to Grade 9. This means that most of the concepts covered in the curriculum in Senior Phase, especially in Grade 7, involve the concept *ratio.* Therefore, a *conceptual understanding* of *ratio* in Grade 7 can result in good mathematical performance by learners. This is supported by Kilpatrick et al. (2001), who stated:

> When learners have acquired *conceptual understanding* in an area of mathematics, they see the connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence, which then provides a base from which they can move to another level of understanding (p. 118).

Although *conceptual knowledge* seems to round up all the mathematical skills that lead to mathematical competency, mathematical proficiency will not be complete if it does not include *procedural knowledge*. According to Kilpatrick et al. (2001), "*procedural knowledge* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 118). *Procedural knowledge* is of great importance in solving ratio problems because it helps learners with mental calculating skills, such as estimating. In this way learners can detect their own errors when solving problems. *Procedural fluency* makes it possible for learners to develop different calculating strategies that are essential in solving problems that involve addition, multiplication and division. McCarthy and Oliphant (2013) reported that most learners who are not proficient in multiplicative skills become frustrated by this realisation when their thinking process has to stop and they have to work out a simple multiplication fact. In their theory of mathematical proficiency, Kilpatrick et al. (2001) illustrated that *procedural fluency* and *conceptual understanding* work hand in hand to attain mathematical competence of a concept. It is, therefore, important that Grade 7 learners attain both types of knowledge to be able to solve ratio problems. Kilpatrick et al. (2001) claimed that if learners know only rules or learn computational skills without understanding, they work with symbol rules and procedures in a routine way and practice them as isolated bits of knowledge that do not connect, making the learning of new concepts difficult. The interdependency between these two types of knowledge is vital in the learning of *ratio* because it helps learners to be able to apply their *conceptual knowledge* in *ratio* to solve everyday problems.

Adaptive reasoning refers to the capacity for logical thought, reflection, explanation and justification (Kilpatrick et al., 2001). For learners to develop adaptive reasoning, it is important that they understand the concept taught very well. Conceptual understanding provides representations that can serve as a source of adaptive reasoning (Mathematics Assessment Resource Service [MARS], 2017).

Strategic competence refers to the ability to formulate mathematical problems, represent and solve them (MARS, 2017). Learners who have developed strategic competence in mathematics are able to come up with several approaches to solve non-routine problems. MARS (2017) explains that the development of strategies for solving problems depends on understanding the quantities involved and their relationships and fluency in solving problems.

Lastly, productive disposition refers to the tendency to see sense in mathematics, perceive it as both useful and worthwhile (MARS, 2017). For learners to see school mathematics as a powerful tool that can be used to solve problems in everyday life, they must develop conceptual understanding, procedural fluency, strategic competence and adaptive reasoning abilities (Kilpatrick et al., 2001).

Literature review

The review of literature is essential as it provides a link between existing knowledge and the research problem being investigated. This enhances the significance of the study (MacMillan & Schumacher, 2014). Additionally, Creswell (2014) stated that literature reviews should, "… build on existing knowledge and add to the accumulation of findings on the topic" (p. 178). In this literature review, I first discuss the learning of mathematics in South Africa, highlighting the challenges that learners experience in learning *ratio.* Then, I discuss the learning of *ratio* from an international perspective. After that I will explain proportional reasoning and its significance in the learning of *ratio*. Furthermore, learners' *conceptual* and *procedural knowledge* of *ratio* will be highlighted, and lastly, I will discuss the errors and misconceptions that learners encounter when solving ratio problems.

2.2.1 Learning of mathematics in the South African context

"Mathematics is considered a key requirement for not only entry into higher education, but also for most modern, knowledge intensive jobs" (Jojo, 2019, p. 89). "South Africa's development as a knowledge economy depends partly on improving the teaching of mathematics and numeracy" (McCarthy & Oliphant, 2013, p.264). Research commissioned by the President's Education Initiative in South Africa concluded that at all levels investigated "the *conceptual knowledge* of learners is well below than expected at the respective grades and the development of higher order skills is stunted" (Spaull & Kotze, 2015, p. 18).

The "poor Mathematics results at the primary and secondary level in South Africa severely limit the youth's capacity to exploit further training opportunities and intervening early to prevent, diagnose and correct these learning deficits is the only appropriate response" (Spaull & Kotze, 2015, p.19). Recently in 2018, the Minister of the Department of Basic Education (DBE), introduced a document called the Mathematics Teaching and Learning Framework for South Africa to try and support the current curriculum (CAPS) in improving learner performance in Mathematics (DBE, 2018). This document was compiled by a ministerial task team and other Mathematics stakeholders in South Africa. The framework is expected to be used by South African Mathematics educators to teach and guide learners in learning mathematics with *conceptual understanding*.

The Mathematics framework provides guidance to the Mathematics education community in two ways. Firstly, the framework provides a theoretical background to the teaching and learning of Mathematics. The framework adopted the five strands of mathematical proficiency by Kilpatrick et al. (2001) as its theoretical framework. The framework only concentrated on the four key dimensions namely *conceptual understanding*, *mathematics procedures*, *strategic competence* and *reasoning.* The framework added its own fifth dimension referred to as *a learning-centred classroom*. It is anticipated that using this framework correctly will improve learner performance in Mathematics. Performance targets for Grade 8 have respectively been set at 500 and 600 in TIMSS (DBE, 2014).

Secondly, the framework includes worked examples for all the different phases in the South African education system to guide teachers on how to use the framework during teaching and learning Mathematics in classrooms. The framework must result in *conceptual understanding* of the concepts taught. It emphasises *conceptual understanding* as the key dimension that will address the challenges that learners experience in the classroom during the teaching and learning of Mathematics.

2.2.2 Performance of South African learners in local and international assessments

The intention of this section is to paint a vivid picture of the performance of South African learners in local and international assessments in Mathematics. The study will only present observations on learner performance from the ANA (2014) and the TIMSS (2015).

2.2.2.1 Learner performance in the Annual National Assessment

The ANA were introduced by the DBE in 2009. Although suspended from 2015, ANA was a standardised diagnostic test administered in Grade 1–9 to test the numeracy and literacy skills of South African learners. These tests were put in place to try and improve learner performance in Mathematics, especially focusing on international assessments such as TIMSS. In 2014, the overall results for ANA in Grade 1–6 showed a slight improvement in learner performance. While there was no improvement in Grade 9 Mathematics, there was a decline in learner performance in the Senior Phase.

Table 1 shows the average national percentages that learners achieved in Mathematics in the ANA.

Table 1: Summary of ANA results (DBE, 2014)

Grade	Mathematics Average Percentage Mark		
	2012	**2013**	**2014**
1	68	60	68
2	57	59	62
3	41	53	56
4	37	37	37
5	30	33	37
6	27	39	43
9	13	14	11

Van Der Berg (2012) claimed that the results of the ANA cannot be used as evidence of improvement in education. Spaull and Kotze (2015) argued that “there is absolutely no statistical or methodological foundation to make any comparison of ANA results over time or across grades” (p. 28). Although these results show an improvement of learner performance across the years, it can also be seen that as learners progress from Grade 1 to Grade 9, the level of performance deteriorates with steeper declines in the senior level. This correlates with the research done by Spaull & Kotze (2015) on learner deficits. The results of their research showed that “learners continue falling further and further behind while they proceed to higher grades, eventually leading to a situation of silent exclusion” (Spaull & Kotze, 2015, p. 29). Figure 2 shows the percentage of learners who obtained at least 50% in Mathematics over 3 years (DBE, 2014).

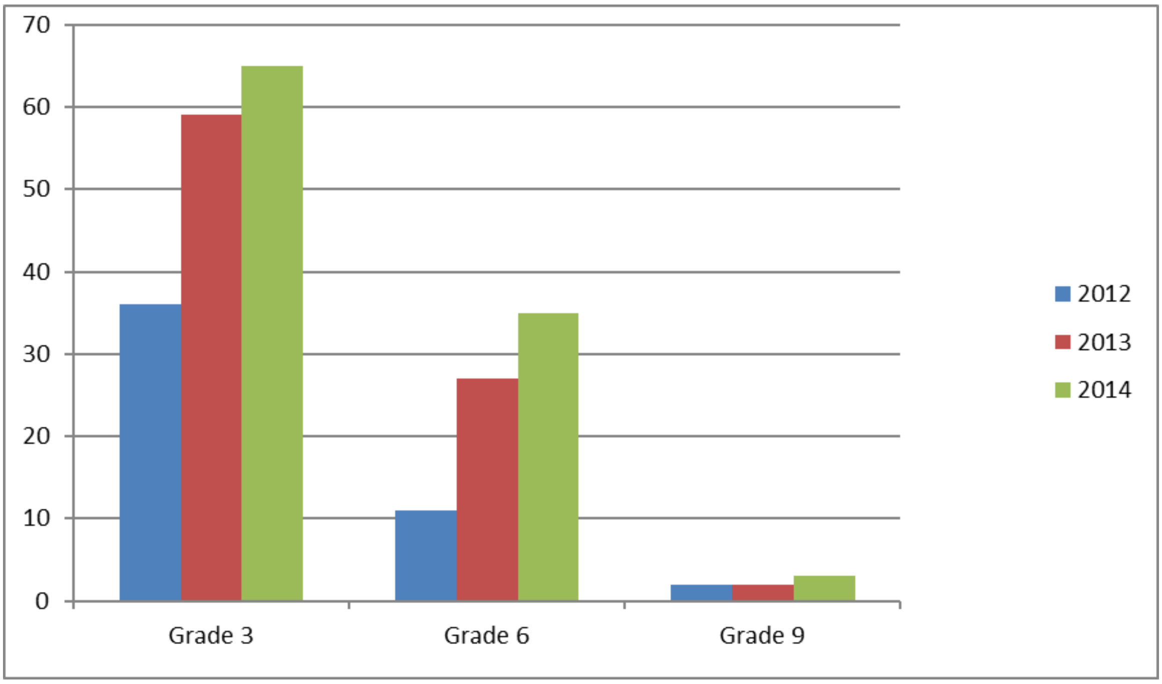


Figure 2: Percentage of learners who achieved 50% or more in ANA (DBE, 2014)

From these results, it is evident that learners in the Senior Phase performed poorly in Mathematics, with less than 5% of learners achieving 50%. This is a cause for concern for educators and all the stakeholders. Intervention strategies are needed to improve the situation. More research is needed to find a solution to this crisis. In the Senior Phase, it is vital that learners grasp concepts such as *ratio* at the beginning of the phase, as these concepts are structured in a hierarchical order and intertwine with each other as the grades progress, making it almost impossible for learners to perform well, especially in Grade 9, if they lack foundational knowledge of concepts such as *ratio*. The ANA 2014 results analysis of learner responses identified the following as challenges for Grade 9 learners:

a) **Learners were unfamiliar with mathematical terminology and properties and often used them incorrectly;**
b) **Basic algebra skills had not been mastered; and**
c) **Learners did not know how to solve applications in geometry and problems involving *ratio* (DBE, 2014).**

These concepts are part of the topic *ratio.* For learners to be proficient in these concepts, learners must first grasp the concept of *ratio*, which includes proportion and entails proportional reasoning. Failure to grasp these will result in learners being unable to construct their knowledge on algebra and geometry, and therefore, hamper their mathematical learning.

2.2.2.2 The Trends in International Mathematics and Science Study 2015 results analysis

“The TIMSS is an assessment of the Mathematics and Science knowledge of fourth and eighth grade learners around the world” (Alex & Juan, 2016, p. 8). In South Africa TIMSS was conducted in 1995, 1999, 2003, 2011 and 2015. “TIMSS is a widely recognised international testing programme aimed largely at assessing whether countries are making progress in education over time” (DBE, 2014, p. 264). South Africa takes part in TIMSS by testing its Grade 9 learners, despite this being a Grade 8 test internationally. The TIMSS 2015 South African sample consisted of 292 schools, 12 500 learners and 330 Mathematics and Science teachers (Alex & Juan, 2016).

According to TIMSS (2015), the South African average for Grade 9 was 372 points for Mathematics (38th out of 39 countries). TIMSS (2015) has the following categories:

> Four categories of benchmarks namely: Scores between 400–475 points are classified as achievement at a low level, scores between 475–550 points as achievement at an intermediate level, scores from 550–625 points as achievement at a high level and scores above 625 points as achievement at an advanced level. (p. 153)

The South African TIMSS results for 2015 reflected a very poor level of competence in Mathematics among learners, especially in the Senior Phase.

According to Alex and Juan (2015), “34% of Grade 9 Mathematics learners in South Africa achieved a score of over 400 points” (p. 10). This means that only one third of South African Grade 9 learners demonstrated achievement of the minimal level in Mathematics and only 3,2% of learners can be categorised at the high levels of achievement, scoring over 550 points (Alex & Juan, 2016).

In the item analysis of TIMSS (2015) Grade 8 Mathematics assessment, it was found that most South African learners could not solve the problem of finding the ratio of “shaded and unshaded parts and the item on finding the ratio of rectangle width and its perimeter" (Kristian & Pepin, 2013, p. 148). This resulted in them performing poorly, falling in the category of lower than the international average, because about 70% of problems in the assessment were on *ratio* and proportion.

2.2.3 International perspectives on the learning of ratio

Son (2013) defined *ratio* as "a comparison of two things with respect to size that can be represented by a fractional expression $\frac{a}{b}$ " (p. 58). Van de Walle, Karp and Williams (2015) stated that a *ratio* is "a number that relates two quantities or measures within a given situation in a multiplicative relationship (in contrast to a difference or additive relationship)" (p. 454). In their definition, Van de Walle et al. (2015) stressed the multiplicative relationship in *ratio* as opposed to the additive relationship. This suggests that for learners to gain access to *ratio*, they must have a solid understanding of multiplicative reasoning. Pelen and Artut (2016) claimed that during the years of primary, secondary and high school, learners make several major transitions in their mathematical thinking, and a central change in the thinking required is a shift from natural numbers to rational numbers and from additive concepts to multiplicative concepts.

According to Singh (2001) "numerous studies have shown that early adolescents and many adults have difficulty with the basic concepts of fractions, *ratio* and proportion and with problems involving these concepts" (p. 284). One of the reasons learners experience difficulties with *ratio* is that many learners intuitively apply additive strategies rather than using multiplicative thinking (Lamon, 2007). In other words, the issue of reliance on additive thinking is considered a significant challenge to teaching and learning *ratio*.

In a study done by Damon and Hart (as cited in Lamon, 2007) found that "two thirds of 13 to15-year olds answered the following incorrectly"

> When measured with paper clips, Mr Short is 6 paper clips tall. Mr Short has a friend Mr Tall. When you measure their heights with match sticks, Mr Short's height is 4 match sticks and Mr Tall's height is 6 match sticks. What would be Mr Tall's height if you measured it in paper clips?

In this study, most learners who answered incorrectly said 8. They reasoned additively, adding 2 to 6. They did not see the multiplicative relationship in the situation: Mr Tall is one and a half times as tall as Mr Short (Lamon, 2007).

According to Lo & Watanabe (1997), "there is a growing theoretical consensus that the concepts of *ratio* and proportion do not develop in isolation, rather they are part of the individual's multiplicative conceptual field, which includes other concepts such as multiplication, division and rational numbers" (p. 134). For learners to fully grasp the concept of *ratio*, it is critical that they fully attain their *conceptual* and *procedural knowledge* of multiplication, division and fractions. Their proficiency in these concepts must be at the level expected in their grades. This will help them as they construct their own knowledge on *ratio.* Spaull (2013) explained that "all more complex mathematics depends, in the first instance, on an instinctive understanding of prior knowledge combined with an ability to readily perform basic calculations and see numeric relationships" (p. 142).

In the teaching and learning of *ratio*, a wide variety of physical empirical situations and representations can be used in the classroom to help learners connect the physical representations with prior knowledge and be able to construct their own knowledge with the guidance of the teacher. Teachers are encouraged to be resourceful and creative in the Mathematics classroom to create an environment for learning. When learning mathematics, it is very important that learners feel relaxed and take an active role during the learning process to reduce anxiety levels that normally come with the subject (Spaull, 2013).

Simon and Placa (2012) claimed the following:

> In order for students to gain access to *ratio* and proportion they must have a solid understanding of multiplicative reasoning … Multiplicative reasoning involves new quantities that are integral to multiplication (intensive quantities) in contrast to extensive quantities, such as length, mass, area or volume which can be measured directly" or counted. (p. 35)

Concepts such as *ratio* involve intensive quantities that cannot be measured directly, and as such, are more conceptually demanding than those that are evaluated by counting or measuring. According to Singh (2001), multiplicative reasoning is an entry point to the world of *ratio.* If learners can reason about intensive quantities such as speed or concentration, they will be developing a functional concept of *ratio* and will be able to conceptualise *ratio* as a measure.

Van de Walle et al. (2015) identified four types of ratios:

a) **Part-part ratios (for example one part apple juice to 4 parts water 1:4);**
b) **Part-whole ratios (for example one of two parts are girls or $\frac{1}{2}$);**
c) **Ratios as quotients (for example the ratio of 3:5 can be written as $\frac{3}{5}$, as a fraction); and**
d) **Ratios as rates (for example, comparing distance to time (speed) that will be km/hr).**

Livy and Vale (2012) claimed that the language and range of types of ratio situations may be the reasons for confusion when working with ratio situations. In addition, the situation is compounded by the potential confusion arising between ratios and fractions. For instance, if 3 boys and 5 girls sit around the table, the fraction representation of this scenario is $\frac{3}{8}$ and $\frac{5}{8}$ respectively, while the ratio of boys to girls is 3:5 (often equated as $\frac{3}{5}$).

In most cases when learners are given the ratio 2:3 to represent the boys and girls, respectively, and are asked to calculate as an example the number of girls in the choir if there are 25 learners in the choir, learners normally get confused with part-part ratios and part-whole ratios. Instead of having the fractions as $\frac{2}{5}$ and $\frac{3}{5}$ to calculate they have their fractions as $\frac{2}{3}$ and $\frac{3}{2}$. Watson, Beswick and Geist (2007) explained that when a ratio connects two parts of the same whole relationship, learners may not adequately differentiate the part-part from the part-whole relationship. This indicates that learners have not fully grasped the concept. Lamon (2014) asserted that "part of understanding a concept is knowing what it is not and when it does not apply" (p. 5).

Lamon (2014) stated that "covariance in *ratio* refers to when two quantities are linked to each other in such a way that when one quantity changes, the other one also changes in a precise way with the first quantity" (p. 62). As an example, a loaf of bread costs R10, 50. Therefore 5 loaves of bread will cost R52, 50. Learners must be aware that these two quantities are in a multiplicative relationship. As the number of loaves of bread varies, so does the cost, and as the cost changes, so does the number of loaves of bread. Lobato, Ellis and Zbiek (2010) stated that “forming a ratio is a cognitive task” (p.174). In other words for learners to understand the concept of *ratio* they must be able to think proportionally and apply their proportional reasoning ability when faced with ratio problems.

2.2.4 Proportional reasoning in Mathematics learning (ratio)

Dole and Wright (2015) stated that “as *ratio* and proportion permeate so many topics in Mathematics and Science; the importance of study of the two concepts in the school curriculum is highlighted” (p. 56). According to Lamon (2014) “a proportion expresses an equality between two ratios (written as $\frac{3}{4} = \frac{6}{8}$ or in the form 3:4 = 6:8) (p.63)”, and in most cases, learners are asked to solve a proportional equation when they are given three numbers in the problem and have to find the missing number. An example is $\frac{3}{4} = \frac{12}{k}$. Finding that k = 16 involves solving a proportion. In introducing the concept *ratio* to learners, it is important that teachers do not teach the concept in a single chapter in which symbols and mathematical procedures are introduced before sufficient *conceptual knowledge* has been laid down for learners to understand (Dole & Wright, 2015).

I have noted in many Senior Phase (Grade 7–9) classes in South Africa that learners are normally taught a cross-multiplication algorithm such as $\frac{a}{b} = \frac{c}{d}$. Learners are taught cross-multiplication as a strategy to solve ratio and proportion problems, and in most cases, learners do not understand the need for it and will forget the formula or apply it wrongly, resulting in conceptual gaps. Muttaqin, Putri and Somakim (2017) explained that the cross-multiplication algorithm is efficient but less meaningful for learners. In fact, it is impossible to explain why one would want to find that product of contrasting elements from two different rate pairs. It has no physical referent and therefore lacks meaning for learners and for the rest of us as well. Therefore, proportion equations must be introduced to learners in a meaningful manner, with students provided with problems to solve that are in a familiar context to enable them to develop their own solution strategies. Lamon (2014) argued that “reasoning is not associated with rule driven or mechanised procedures, but rather with mental, free flowing processes that require conscious analysis of the relationship among quantities" (p. 66).

For learners to be able to solve ratio and proportion mathematical problems, it is important that they can reason proportionally. "Proportional reasoning enables us to make comparisons between entities in multiplicative terms" (Dole & Wright, 2015, p. 68). As an example, the ratio of girls to boys in a class can be 2:3, which can be written as $\frac{2}{3}$. If, in a class there are 15 boys, it means that class will have 10 girls, using the multiplicative relationship and not 14 girls. In most cases, learners will get the answer for the number of girls as 14 because they usually perceive the difference between the ratio as additive and not multiplicatively. Dole and Wright (2015) explained that proportional reasoning is being able to explain, to interpret, to recognise proportionality and to be able to represent proportional relationships in a number of ways that include graphs, tables and equations.

Proportional reasoning is essential for learners to succeed in many mathematical areas, including *ratio* and proportion (Hilton & Hilton, 2018). Lamon (2014) explained that "one of the most compelling tasks for researchers has been to discover how instruction can facilitate the joint development of rational number understanding and proportional reasoning" (p. 8). She continues and states that "by deeply analysing mathematical content, children's thinking, and adult thinking we have begun to understand some of the knowledge that contributes to the development of these critical concepts and ways of thinking" (p. 8). In other words, in teaching mathematical concepts such as *ratio*, it is important for the teacher to arrange learning materials in such a way that learners make connections of concepts learnt previously to help them grasp the new concept. In the learning process of *ratio*, we find that concepts such as measurement, sharing and comparing, fractions and quantities as well as covariations intertwine together to help learners grasp the concept. In addition, these concepts help the learner to develop relative thinking, which leads to the development of proportional reasoning. Figure 3 shows how these highly interrelated concepts, contexts, representations, operations, and ways of thinking are linked to help learners to learn proportional concepts and develop proportional reasoning.

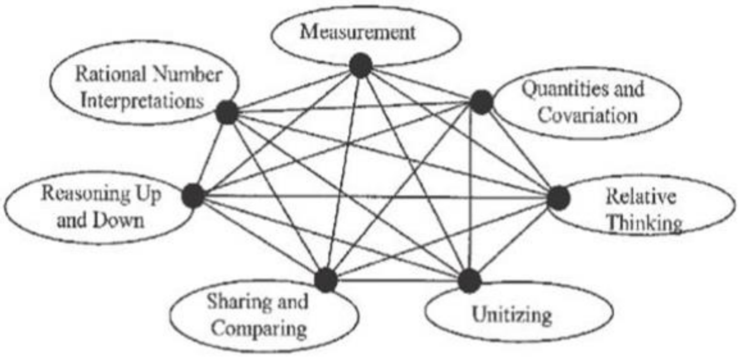


Figure 3: Interconnected concepts of proportional reasoning (Lamon, 2014)

This web diagram of interrelated proportional concepts helps in the teaching and learning of the concept *ratio*. It illustrates, especially to the teachers, how to link these proportional concepts during instructional time in the mathematics learning process to help learners to be able to connect these related topics together in a web like manner. During the learning process, learners acquire their *conceptual knowledge* through well planned activities that foster relative thinking and enable the learners to understand the sharing and comparing of quantities as well as the measurement of these concepts. Spaull and Kotze (2015) stated that "*conceptual knowledge* is characterised most clearly as knowledge that is rich in relationship, it can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (p. 15).

Proportional reasoning and *ratio* have a dependent relationship, and the absence of one, affects the other concept negatively, making it difficult for either of the concepts to function independently. Lamon (2014) stated that "proportional reasoning encompasses not only reasoning about the holistic relationship between two rational expressions but wider and more complex spectra of cognitive abilities which includes distinguishing proportional and non-proportional situations" (p. 31).

The following example is a non-proportional problem that can help teachers to identify learners who have developed proportional reasoning and gained *conceptual knowledge* on *ratio*.

> Two cyclists, Peter and Sam, are cycling at the same speed around a cycling track. Peter starts cycling before Sam arrived at the track and had completed **9** laps when Sam had completed **3.** When Peter had completed **15** laps. How many laps will Sam have completed? (Singh, 2001)

Learners who use proportion to solve this problem will get 5 as their answer instead of 9, because they are accustomed to learnt procedures. Instead of using the additive strategy to solve the problem, learners will use cross-multiplication. Lamon (2014) explained that proportional reasoning refers “to detecting, expressing, analysing, explaining and providing evidence in support of assertions about, proportional relationships” (p. 4). He also stated that “the word *reasoning* further suggests that we use common sense, good judgement and a thoughtful approach to problem solving, rather than plucking numbers from word problems and blindly applying rules and operations” (Lamon, 2014, p. 78).

For decades researchers have emphasised the general importance of proportional reasoning in mathematics (Dole & Wright, 2015). The mastery of proportional reasoning among learners is perceived as an indicator for understanding mathematics and as a foundation for learning mathematics as they progress to high school (Che, 2009). In a research study done by Misailidou and William (2003) on proportional reasoning, the results showed that “non-proportional problems were solved with the lowest success rate and direct proportional problems with the highest success rate” (p. 352). The study showed that learners have difficulty distinguishing proportional and non-proportional problem statements. According Dole and Wright (2015), “one of the key aspects of proportional reasoning is being able to consider situations of change in both additive and multiplicative terms, adjusting appropriately according to context” (p. 9).

According to Lamon (2014), a “large portion of the adult population does not reason proportionally” (p. 81). This suggests that certain kinds of thinking do not occur spontaneously, and that instruction must take an active role to facilitate thinking that will lead to proportional reasoning. In a classroom situation, it is therefore the role of the teacher to make sure that learners are provided with the necessary instructions for them to be able to acquire the necessary skills of a given concept. Shulman (1986) stated that teachers must not only be capable of defining for learners the accepted truths in a domain but also be able to explain why a particular concept is important and how it relates to other concepts both in theory and practice.

2.2.5 Conceptual and procedural knowledge of ratio

Lamon (2014) claimed that instruction plays a very important role in the learning situation, especially in Mathematics. In research done by Lamon in 2014, children were given time to develop their reasoning for 4 years without being taught the standard algorithms for operations with *ratio*. The results of the study produced "a dramatic increase in learners' reasoning abilities, including their proportional reasoning" (Lamon, 2014, p. 87). Even for learners who will never pursue work in mathematics related fields, it is important to have a sound knowledge of *ratio* as this will help them with reasoning in many everyday contexts. Dole and Wright (2015) stated that previous research has shown that most learners' understanding of *ratio* and proportion is not achieved during their time at school because of teaching methods or because the concepts are too difficult.

Previous research has identified that mathematical competence rests on developing knowledge of concepts and procedures (*conceptual* and *procedural knowledge*) by learners during the teaching and learning process of Mathematics (Rittle-Johnson & Schneider, 2014). It explains that *conceptual knowledge* encompasses not only what is known (knowledge of concepts) but also a way that concepts can be known deeply and with rich connections. "*Procedural knowledge* involves knowing the various steps required that will lead to the correct answer of a mathematical problem" (Rittle-Johnson & Schneider, 2014, p.24). "It is possible for learners to rely on procedural proportion knowledge to obtain correct answers to proportion problems even when they lack conceptual proportion knowledge" (Ekawati et al, 2015, p. 523). Jitendra et al. (2013) argued that the "ability to give correct answers is no guarantee that proportional reasoning is taking place:

> *Conceptual proportion knowledge* means providing reasons in support of claims made about the structural relationships among four quantities, say abcd in a context simultaneously involving covariance of quantities and invariance of ratios, this would consist of the ability to discern a multiplicative relationship between 2 quantities as well as the ability to extend the same relationship to other pairs of quantities". (p. 56)

Kilpatrick et al. (2001) defined a concept as "an abstract or generic idea generalised from particular instances" (p. 25). They continued to explain that knowledge of concepts is often referred to as *conceptual knowledge* and is not tied to any particular problem types. In their theory of mathematical proficiency, they identified five strands of mathematical proficiency. The intertwining of the strands indicates how learners acquire mathematical proficiency. It therefore implies that for learners to acquire mathematical concepts both learners' *conceptual* and *procedural knowledge* must be developed.

Learners' *conceptual* and *procedural knowledge* of a concept forms a very important part of mathematical thinking. Stacey (2014) defined mathematical thinking as "a process that involves deep mathematical knowledge, general reasoning abilities and knowledge of heuristic strategies" (p. 42). Mathematical thinking is very important in the learning of mathematics as it helps learners to be able to think mathematically and to be able to use mathematics in their everyday lives. In the learning of *ratio*, learners who can think mathematically are able to solve ratio problems using heuristic strategies. Learners are able to monitor their procedural steps in a way that they can easily identify their mistakes or errors during problem solving. Like the stages of development, mathematical thinking in learners also develops as children grow. It is therefore very important that in learning and teaching Mathematics, learners are given mathematical problems that will help them develop their mathematical thinking. This process will improve their mathematical problem solving skills and their *conceptual knowledge* of mathematics.

2.2.6 Misconceptions and errors in ratio

Ojose (2015) defined misconceptions as "misunderstandings and misinterpretations based on incorrect meanings; they are due to naïve theories that impede rational reasoning of learners" (p. 30). They also stated that "some elementary and even middle school learners believe that $\frac{1}{4}$ is larger than $\frac{1}{2}$ because 4 is greater than 2, additionally, a common misunderstanding is that the operation of multiplication will always increase a number" (Ojose, 2015, p. 31).

On the topic *ratio*, "one of the most difficult tasks for children to understand the multiplicative nature of the change in proportional situations. Children who cannot yet tell the difference indiscriminately employ additive transformations" (Lamon, 2014, p. 57). Here is an example of such incorrect reasoning:

> It takes 2 hours for 4 men to dig a foundation of a house. How many hours will it take 6 men to dig the foundation of the same house?

When approaching this problem, learners will write down 2hrs = 4men, so 4hrs = 6 men. The 4 hours will be from adding 2 + 2 = 4. Therefore, to get the missing number in the ratio, the learners will note that there is a difference of two between 2 and 4 and also between 4 and 6. Therefore to find the missing number they need to add 2. In a situation like this, learners would have failed to recognise the multiplicative nature of *ratio* and have applied the additive error. They tend to do that to avoid situations where they have to multiply. McCarthy and Oliphant (2013) reported that most learners are not proficient in their multiplication facts and they become frustrated with this realisation when their thinking process has to stop and work out, for example, 3 × 8.

According to Ojose (2015), learners generally make two types of errors:

> Conceptual errors and execution errors. Conceptual errors are related to lack of understanding while executive errors arise not from failure to understand how the problem should be tackled, but in some failure to actually carrying out the manipulations required.

Oliver (1992) distinguished between slips, errors and misconceptions as follows:

> Slips are wrong answers due to processing; they are not systematic but are sporadically carelessly made by both experts and novices, they are easily detected and are spontaneously corrected. Errors are wrong answers due to planning; they are systematic in that they are applied regularly in the same circumstance. Errors are symptoms of the underlying conceptual structures that are the cause of errors. It is these underlying beliefs and principles in the cognitive structure that are the cause of systematic conceptual errors that we call misconceptions.

During the teaching and learning process of mathematical concepts such as *ratio*, it is important that teachers identify errors they make when solving problems and use correct strategies to correct the errors so that the process leads to a *conceptual understanding* of the concept. Mathematics teachers are encouraged to guide learners to identify their own errors and to use the opportunity to closely interact with the learner, thereby guiding the learner step by step in solving the problems. In doing so, learners will be able to solve the problems and correct errors in a way that will lead to *conceptual understanding* of *ratio*.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

This study explored learners' *conceptual* and *procedural knowledge* of *ratio.* This chapter outlines the research methodology that was used to conduct the study. According to Cohen, Manion and Morrison (2005), methods refer to the techniques and procedures used in the process of data gathering; therefore, the aim of methodology is to describe and analyse these methods, showing their presuppositions and consequences and relating their potentialities to the frontiers of knowledge. This chapter presents the research paradigm, research methodology, data collection techniques and data analysis for the study. It also focuses on the study's ethical issues.

3.2 Research paradigm

This study used a qualitative research design approached from an interpretivist perspective. It focused on interpreting the learning of *ratio* in the Mathematics classroom. Interpretivists believe that reality is constructed by social actors and people's perceptions of it (Dina, 2012). The interpretivist paradigm emphasises "understanding the individual and their interpretation of the world around them" (Kivunja & Kuyini, 2017, p. 28). In this study, the focus was on learners' *conceptual understanding* as well as *procedural knowledge* of the concept *ratio*. My intention was to gain a deep understanding of learners' interpretation of *ratio* and the knowledge and skills they have acquired in solving problems related to *ratio*. The interpretivist paradigm gave me the opportunity to use my own knowledge, understanding and perspective to interpret learners' *conceptual* and *procedural knowledge* of *ratio*.

Leitch, Hill and Harrison (2010) described the interpretivist paradigm as "a non-positivist research concerned with the investigation of social reality" (p. 68). I used this paradigm to investigate *ratio*, one of the most important subjects in the Mathematics curriculum in South African and one that learners struggle with. I interacted with learners in their Mathematics classrooms and immersed myself into their world. This paradigm helped me to find the challenges that Grade 7 learners experience in learning *ratio*.

3.2.1 Ontological perspective

Kivunja & Kuyini (2017) stated that “relativist ontology means that you believe that the situation studied has multiple realities, and that those realities can be explored and meaning made of them” (p. 19). This research showed different ways to demonstrate learners’ *conceptual* and *procedural knowledge* of *ratio.* Through the use of interview questions, I was able to dig deeper into learners’ minds and reveal the different truths of learners’ knowledge of *ratio.* Learners demonstrated different procedural skills as they solved ratio problems. This revealed that as individuals, learners perceived and interpreted concepts differently. Consequently, different factors, such as cultural background, affected the learning of mathematics; therefore, I used interviews to explore and discover the multiple realities of the phenomenon.

3.2.2 Epistemological perspective

Subjective epistemology means that I “make meaning of the data through the participants’ own thinking and cognitive processing of data informed by their interactions with the data” (Kivunja & Kuyini, 2017, p. 19).

In this research, I analysed and interpreted learners’ test scripts using my knowledge of the concept *ratio.* The interpreted transcripts were cross checked with the learners to make sure it was the learners’ truth. The analysed scripts provided me with a guide on how to deepen my understanding of the phenomenon. The research was subjective as the focus was only Grade 7 learners at one particular school in Soweto. The results are based on my own view of a particular case; they cannot be generalised

3.2.3 Methodological perspective

Sefotho (2015) explained that “methodology is the strategy or plan of action which lies behind the choice and the use of particular methods” (p. 31). Learners wrote an assessment test on *ratio*; this was followed by semi-structured interviews. I used a qualitative case study research methodology to give the research the required credibility and consistency. A case study design helped me to understand the perceptions of participants in the study. Data was collected using a self-developed test on *ratio* and semi-structured interviews. A pilot study was done to test the instruments’ dependability and credibility. The test was moderated by Mathematics subject specialists for the Senior Phase from the Gauteng Department of Education and North West Department of Education.

3.3 Research design

This research followed the qualitative research method. MacMillan and Schumacher (2014) claimed that "a distinguishing characteristic of qualitative research is that behaviour is studied as it occurs naturally, there is no manipulation or control of behaviour or settings, nor are there any externally imposed constraints" (p. 345).

The qualitative research method allowed me to collect rich descriptive data on learners' *conceptual* and *procedural knowledge* of *ratio.* Creswell et al.'s (2010) statement supported this: "In qualitative research; the emphasis is on the quality and depth of information and not on the scope or breadth of the information provided as in quantitative research" (p. 36).

In this study, I intended to understand how learners solved problems involving *ratio* and to investigate the learning difficulties that learners experienced while solving ratio problems. Graebner, Martin and Roundy (2012) stated that "qualitative data can be concrete and vivid; the concreteness and vividness activate cognitive processes that foster the development and communication of ideas" (p. 278). The qualitative methodology allowed me to closely capture learners' subjective experiences and interpretations of the concept *ratio.* Widdowson (2011) explained that the "purpose of qualitative research is to understand the perspectives or experiences of individuals or groups and the contexts in which these perspectives or experiences are situated" (p. 98).

I chose to use qualitative methodology because it allowed me to collect data directly from the source that is Grade 7 learners, thereby giving the data authenticity. MacMillan and Schumacher (2014) claimed that "data obtained directly from participants is valid even though they may represent particular views or have been influenced by the researcher's presence" (p. 345). Henning, Gravett and Rensburg (2012) said that in a qualitative study, "the variables are usually not controlled because it is this freedom and natural development of action and representation that must be captured" (p. 53).

This study used a single exploratory case study design. Baxter and Jack (2008) explained that a qualitative case study as a research approach facilitates exploration of a phenomenon within its context using a variety of data sources. In this study, I used two research instruments to collect data from the participants. The data collection instruments allowed me to collect a rich, thick description of the phenomenon under study. Case study research designs are "particularistic, meaning it focuses on a particular situation or phenomenon; descriptive, meaning it yields a rich, thick description of the phenomenon under study; and heuristic, meaning it illuminates the reader's understanding of the phenomenon under study" (Yazan, 2015, p. 148). Therefore, I could conduct a feasible research study with minimal hindrances.

The disadvantages of a case study research design meant I needed strategic measures to help me overcome these hindrances. Widdowson (2011) argued that "it is not possible to generate inferential statistics from a single case; however, it is possible to use simple descriptive statistics to enable the reader to draw logical conclusions regarding the outcome(s) of the research" (p.101). I was able to analyse data with the Atlas.ti™ Windows 8 coding system. This helped me to draw rich descriptive conclusions to interpret as findings of the research study. The Atlas.ti™ coding system made the results of the study more easily understandable for a wide audience, including non-academics, as they were written in simple, non-professional language.

Another limitation of the case study is that findings cannot be generalised (Yin, 2012). This might be true, but it does not erase the fact that case studies are strong on reality and allow readers to judge the implications of the study for themselves. Therefore, generalisation is not of paramount importance in this study, and the study had no intentions of making generalisations beyond the boundaries of this case study.

One of the advantages of a case study is that it allows me and other researchers to continue with further research of the phenomenon by focusing on the gaps in this research study. This is possible because "case studies form an archive of descriptive material that is sufficiently rich to admit subsequent reinterpretation" (Cohen et al., 2005, p. 28). In this way, the research becomes a valuable, rich source of information. Widdowson (2011) illustrated that "case study records outcomes of different but similar cases that can be compared, and that the specific variables that might have impacted on the difference in outcome can be investigated separately" (p. 105).

Another advantage of a case study is that it allowed me to carry out the study on my own without a full research team. In this way, I was able to maintain confidentiality between me and the participants.

3.4 Selection and description of the sample

The participants in this study were Grade 7 learners from a township school in Soweto. The school is a government combined school with 450 learners, serving Grade R–9 learners. The suburb in which the school is located is characterised by small houses, informal settlements and hostels. The school's catchment area includes learners from Zone 2 and 3 from the suburb and a few from other zones. The school fell into Quintile 1 and 2 during apartheid, which resulted in the school lacking infrastructural development and teaching and learning resources, especially for Mathematics and Science. However, the school has greatly benefited from the government programmes introduced to redress the imbalances of the apartheid era.

The school is one of the first Soweto schools to be technologically resourced with teaching and learning resources such as computers and smart boards to enhance the teaching and learning of subjects such as Mathematics and Science. The school has only one class of Grade 7 learners with approximately 35 learners. The suburb where the school is located has a larger percentage of people speaking IsiZulu, and as a result, IsiZulu is the home language in the school and English is the first additional language, normally used as the medium of instruction for learners from Grade 4–9.

Previous research done on mathematics performance in South Africa has shown that Senior Phase learners perform poorly in mathematics and the performance declines as they move up the grades. This has motivated me to conduct a study on Grade 7 learners' knowledge of ratio since the Senior Phase in South Africa starts at Grade 7. In conducting this research, I chose to use purposeful sampling, also referred to as purposive sampling to collect data for this study. According to Etikan, Musa and Alkassim (2016), the purposive sampling technique is a deliberate choice of participants because of the qualities the participants possess. According to Cohen et al. (2005) purposive sampling involves choosing the participants who demonstrate distinct characteristics relevant for the study thereby building up a sample that suits specific needs of the researcher. A sample of 35 Grade 7 learners was chosen from a population of 450 learners in the school. The participants were chosen to obtain a comprehensive understanding of learners' *conceptual* and *procedural knowledge* of the concept *ratio*, which, in the Senior Phase, starts to be taught in Grade 7 and progresses to the next grades.

Data collection procedures

Data were collected after I received permission to conduct this study from the University of Pretoria Research Ethics Committee (see Ethics Certificate at the beginning of this dissertation) and the Gauteng Department of Education (Appendix C). I conducted the pilot study at one of the neighbouring schools to test the data collection instrument for dependability and credibility. The time frame for the test was set at 60 minutes after the pilot study. This study was done to assess the feasibility of my main study especially the data collection instruments as well as the estimated time frame for the main study. The pilot study was aimed at assessing any troubleshooting unforeseen issues in the study. Van Teijlingen and Vanora (2013) explain that the advantages of conducting a pilot study is that it can give advance warning about where the main research project could fail, or where research protocols might not be followed, or whether proposed methods or instruments are inappropriate or too complicated. Data collected in the pilot study was not meant to produce results for the study but to guide me in my main study.

After the pilot study, I visited the selected school and conducted an information session with the participants to give them the details about the study. Consent letters were given to the principal, participants, counsellor as well as the parents, and permission was granted through the consent forms by the principal, participants, parents and counsellor (Appendix D–G). The consent letters stated clearly that participation in the study was voluntary and that participants had the right to withdraw from the study at any stage during data collection. I set the date for data collection. The assessment test on *ratio* was written by 23 participants after school hours for an hour. This was followed by semi-structured interviews that were ` scheduled for 10–15 minutes per session after school hours for about 2 weeks.

I kept the learners' test scripts safely in a locked cupboard, and I also kept the raw data from interviews (audio recordings) and made it available to my supervisor and co-supervisor. I protected all documents related to the study on my computer with a password. When I finished the research, all the materials were stored at the University of Pretoria according to the University's storage policy regulations.

Data were collected using a self-developed assessment test on *ratio* and semi-structured interviews.

3.5.1 Test

A self-developed assessment test on *ratio* was written by participants to enable me to collect data on how learners solve ratio problems and to see the different strategies they use to solve ratio problems (Appendix A). The test also gave me some insight into learners' *conceptual knowledge* of *ratio*. The test lasted 60 minutes and was conducted after school hours. The questions in the test were varied in order to measure all four different types of *ratio* as identified by Van de Walle et al. (2015). These included questions that involved part-part ratios, part-whole ratios, ratios as quotients as well as ratios as rates. The questions were also classified according to multiplicative ratio problems (proportional problems) and additive ratio problems (non-proportional problems).

The data collected in this research was analysed using Atlas.ti™ Windows 8 software. I created codes emanating from learners' test responses. These codes were put into group codes (themes) that I interpreted and used to create themes. These themes were interpreted and used to formulate semi-structured interview questions. The interviews helped me gain insight into learners' knowledge of *ratio*. Table 2 summarises the questions in the assessment test that was written by the participants. Although all questions focused on *conceptual* and *procedural knowledge* of *ratio*, the table below presents the specific mathematical knowledge and skills that each question addressed.

Table 2: Summary of questions in assessment test

Question	Focus/ purpose
The length and width of the rectangle is shown in the diagram below. Write the ratio of the length to width.	Non-proportional problem
Write each of the following as a ratio in its simplest form: 15:25 3 hours:20 minutes	Proportional reasoning skills Multiplicative reasoning
A bag of beads contains 7 white beads and 9 red beads. What is: The ratio of white beads to red beads? The ratio of red beads to the total number of beads?	Proportional reasoning skills Additive reasoning Non-proportional problem
Diepkloof has a population of 100 000. One tenth of the population has red hair. What is the ratio of the number of red hair to the rest of the population?	Proportional reasoning skills Multiplicative reasoning
A man divides his estate of R360 000 in the ratio 4:3:3 among his daughter and his two sons. How much does each child receive?	Proportional reasoning skills Multiplicative reasoning
Jack and his sister Thandi have the same birthday. Jack was 15 years old when Thandi turned 5 years. How old will Thandi be when Jack turns 75 years?	Non-proportional problem Proportional reasoning skills
If 5 chocolates cost R75. How much will 13 chocolates cost?	Proportional reasoning skills Multiplicative reasoning
A rectangle is drawn in the ratio 3:2. Determine the length of the longer side if the shorter side is 13, 2 cm long.	Proportional reasoning skills Multiplicative reasoning
If today Zodwa ran fewer laps in more time than she did yesterday, would her running speed be: (a) Faster (b) slower (c) exactly the same Explain your answer.	Conceptual knowledge Proportional reasoning skills Non-proportional problem Additive reasoning
A wedding ring contains 2 parts of gold and 3 parts of silver by mass. What fraction of the ring is gold? What fraction of the ring is silver? If the total mass of the ring is 25 g, what fraction of the total mass does each metal contain?	Proportional reasoning skills Proportional problem Multiplicative reasoning
Musa travelled at a constant speed of 60 km and it took 4 hours to complete the journey. Calculate her average speed.	Conceptual knowledge Proportional reasoning skills Proportional problem Multiplicative reasoning
Two cyclists, Joseph and Kagiso, are cycling at the same speed around a cycling track. Kagiso started cycling before Joseph arrived at the track and had completed 9 laps when Joseph had completed 3. When Kagiso had completed 15 laps, how many laps will Joseph have completed?	Additive reasoning (conceptual knowledge) Non-proportional problem Proportional reasoning
Water is flowing into a dam at a constant rate of 600 litres per	Multiplicative reasoning

hour. How much water will flow into the dam in 2 hours? How long, in minutes, will it take for 10 000 litres of water to flow into the dam?	Proportional problem Proportional reasoning skills Conceptual knowledge.

3.5.2 Semi-structured interviews

Semi-structured interviews were used in this study to corroborate data that emerged from the assessment test and to gain further insight into the learners' *conceptual* and *procedural knowledge* of *ratio* (Appendix B). The interviews gave more room for the participants to provide detailed information on the steps they took to solve the given *ratio* problems and to gain more insight on their *conceptual knowledge* of *ratio*.

Gill, Stewart, Treasure and Chadwick (2008) claimed that "semi-structured interviews consists of several key questions that help to define the areas to be explored, but also allows the interviewer or interviewee to diverge in order to pursue an idea or response in more detail" (p. 291). The interviews were audio recorded and participants gave consent before the interviews. The participants who took part in the interviews were selected based on the results of the test analysis. The test was analysed using the Atlas.ti™ coding system. I created codes that led to themes, which I interpreted and used to select learners who could provide more information on the themes. These themes also helped me develop the interview questions. Fifteen participants were interviewed. The interviews were audio recorded. The recordings were analysed after each session using Atlasi.ti™ coding system. Codes were created which led to themes which I interpreted as research findings. From the interview sessions that were done, only seven interviews provided valuable data. Some of the sessions, the participants were unable to express themselves verbally. They were not to explain their procedural steps. Most of them said they did not remember (they were guessing answers).

3.5 Data analysis and interpretation

Data collected in this research study was analysed using the Atlas.ti™ Windows 8 coding system. The codes were interpreted and grouped together to constitute themes. The following themes emerged from the codes: *conceptual knowledge of ratio, procedural knowledge of ratio and challenges experienced by learners in solving ratio problems.* The themes were informed by the research questions.

Categories were used to construct questions for the semi-structured interviews, which were audio recorded. The codes were grouped together to create code groups/themes or categories (see Figure 5 and Figure 12 in Chapter 4).

3.6 Quality measures

I took the following measures to ensure the quality of the research study:

- Credibility: To guarantee credibility, I piloted the study to make sure the data collection instrument could measure according to expectations. Furthermore, transcripts were cross examined to check if the data collected was truthful. Lastly, to make sure the research was credible, I spent more time with the learners in the field and immersed myself in the participants' world to gain insight into the context of the study.
- Transferability: The findings of this study can be used by other researchers to build more cases around the concept *ratio* or be used as a source to find ways to improve learners' performance in Mathematics Senior Phase in South Africa.
- Confirmability: In this study, I was not known to the participants, making me a neutral person in the field; therefore, no form of bias was experienced during the data collection phase. Before finalising my data analysis (interview transcripts), I cross checked the transcripts with the learners to make sure that what I interpreted was the truth from the participants and not my own ideas.
- Dependability: The results for this research are based on the studied case and cannot be generalised. Prolonged field study was done to gain more information on learners' cultural background.
- Reliability: In this study, a pilot study was done to test the reliability of the self-developed test on *ratio*.
- Validity: The self-developed test was given to Mathematics Senior Phase specialists who specialise in the curriculum and teaching of Senior Phase learners to moderate the test and make sure the content of the test follows the grade level requirement according to the South African curriculum.

3.7 Ethical considerations

I followed the following ethical considerations when conducting the research:

- Informed consent: I obtained consent from the participants before conducting the study. The school principal, parents, school counsellor and participants signed informed consent letters for the study to take place. I informed them about the purpose of the study, highlighting all the benefits and possible risks they might encounter during the study. The role of the participants was clearly stated in the consent letters. Participants were also familiarised with the wording of the informed consent letters and given the opportunity to ask questions and raise any concerns they had about the study. Before data collection, a fair explanation of the whole process was given to the participants and informed consent letters were given to the participants who took part in the interviews.
- Voluntary participation: Participants were not compelled to take part in the study, and I explained to them that they had the right to withdraw at any time from the study with no harm or penalty.
- Confidentiality: I assured the participants that all data collected would be kept safe and confidential to protect their privacy and that their real names would not be used when analysing the data to avoid linking the results with individual participants. During interviews, I reassured participants that the process was kept confidential and all recordings were kept safe by me and made available only to my supervisors. No names or any personal identity were given in the recordings and the recordings cannot be traced back to them.
- Anonymity: Participants remained anonymous throughout the study and no information provided by the participants revealed their identity. All research data was kept confidential.
- Confidence: I created an atmosphere of trust and openness during the study. Participants were assured of the confidentiality during data collection. Although this study was conducted in English, learners were free to use IsiZulu language during the study to express themselves freely. Learners were told about the school counsellor who was available during data collection to give services to them.

- Privacy: Participants were assured that all data collected during the study was for research purpose only and only I and my supervisors had access to it. The assessment test and interviews were done in class after school hours and I was the only one who took part in the process.
- No harm or risk to participants: Participants and parents were assured that the study did not anticipate any physical or mental discomfort to the participants. If the participants felt discomfort during data collection, the school counsellor was available to assist them. Any physical harm during the study would be dealt with accordingly by me.

CHAPTER 4: DATA ANALYSIS AND PRESENTATION OF FINDINGS

4.1 Introduction

In this section I will present the findings of the analysed data collected during the study. The self-developed test and semi-structured interviews were used to collect data. The results of the test were used as guidelines to structure the questions for the interviews. First, I will present the findings of the test moderation that was done by Mathematics specialists. The analysis will be organised to show the integrated evidence from the test and from the interviews to deepen the understanding of learners' *conceptual* and *procedural knowledge* of *ratio.* Data will be organised into codes and themes. The themes in this research will then be analysed and interpreted by me to get findings for the study.

4.2 Findings of the test moderation by Mathematics experts

The test was moderated by Mathematics subject specialists from the Gauteng Department of Education and North West Department of Education who were knowledgeable about the Mathematics curriculum for the Senior Phase. The test was moderated to measure its dependability.

I was asked to change Question 2: *Write each of the following as a ratio in its simplest form. (a) 15:25 (b) 30:80.* This question only consisted of simplifying straight forward numbers and I added a problem with two different units of measurements. It was noted that in the test analysis, learners could not solve part (b) of the question where they had to first change the units of measurement, except Learner 2 who managed to change the units of measurements but could not simplify the problem completely. During the interview, Learner 2 corrected his error.

In Question 4: *In a population of 46 000. One tenth of the population has red hair. What is the ratio of the number of red hair to the rest of the population?*, I was advised to add a more contextual number to the population, to change the question to be more contextual by adding a place that the learners know, and to include fractions in the problem to check learners' *conceptual understanding* and *procedural knowledge* of fractions as *ratio*. Only one participant managed to solve the problem (Learner 2); he identified the fraction and calculated the fraction as part of a whole.

I was also asked to change Question 7: *If 5 chocolates cost 70. How much will 13 chocolates cost?* I was advised to change the digits so that the quotient will be a decimal number quotient so as to check learners' knowledge of working with decimal numbers.

In Question 10 I was advised to change the context of the problem from alloys to something that learners will understand, like a wedding ring.

Lastly I was advised to increase spaces for learners' answers and to specify in the instructions that learners are not allowed to use calculators during the test.

4.3 Summary of the findings on learners performance

The self-developed test was the first instrument that I used and it provided me with information on the different procedural skills that learners used to solve ratio problems and the difficulties they experienced while solving ratio problems. The interviews were used to collect data on learners' *conceptual knowledge* of *ratio* and to deepen my understanding of learners' *procedural knowledge* of *ratio* as well as the challenges that they face when solving *ratio* problems.

Question 1 in the test was a non-proportional ratio problem that assessed learners' *conceptual* and *procedural knowledge* of *ratio*. The question did not require learners' multiplicative or additive reasoning skills and had a diagram (visual) to help learners recall the concept. As a result, most learners managed to solve the problem very well. It was a procedural recall question that required learners to recall the basic concept of *ratio* they have learnt. It focused more on learners' *procedural knowledge* of *ratio* than their *conceptual knowledge*. Question 3, 6, 9 and 12 were also answered fairly well by most learners, especially Question 6. These questions were non-proportional or additive problems. Most participants could solve these problems that required additive reasoning skills (non-proportional) better than the problems that required multiplicative reasoning skills (proportional).

Question 2, 4, 5, 7, 8, 10, 11 and 13 were proportional problems and most learners struggled to solve these problems. Most learners answered Question 5 and 8 wrong. None of the learners tried to solve the problems correctly. The problems required a good *conceptual understanding* and *procedural knowledge* of *ratio.* Only Question 2 that was a proportional problem was fairly answered by the learners. A quarter of the learners solved the problem in Question 2, and approximately three quarters could solve only Part A of the question, which required learners to simplify 15:25, but struggled to simplify Part B, which required them to simplify *3 hours:20 minutes*. The results showed that only two learners managed to partly solve Part B of this problem. This indicated that most learners could not apply their proportional reasoning skills and their multiplicative reasoning skills to solve the problem. This also showed that the learners' *conceptual knowledge* of *ratio* was limited to only the simple, basic ratio problems that are less challenging and has procedural steps that are easy to recall. They could not solve those questions that required a deep *conceptual understanding* of *ratio* and proportional reasoning skills.

In Question 13, only a quarter of the participants managed to answer Part A of the question and struggled to solve Part B. The question was as follows: *Water is flowing into a dam at a constant rate of 600 litres per hour. (a) How much water will flow into the dam in 2 hours? (b) How long, in minutes, will it take for 10 000 litres of water to flow into the dam?* Part A was a simple recall procedural type question that required multiplicative reasoning, and only six learners managed to solve it. Part B required learners to apply their proportional reasoning skills as well as their multiplicative reasoning skills, and most learners struggled with it. Although the study was not quantitative, Figure 4 summarises the percentage analysis per question of the test (n = 23)

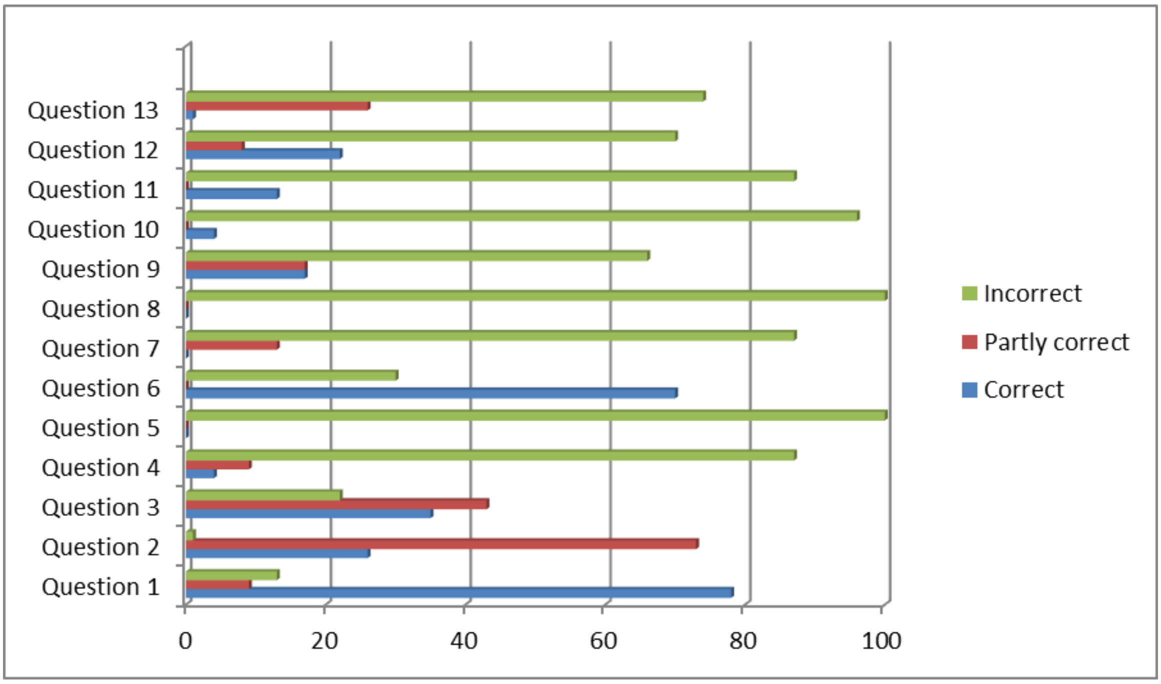


Figure 4: Summary of learner responses to the test questions

From the analysis of the data, three themes emerged that I will interpret as findings. These themes are the following:

1) **Conceptual knowledge of ratio**
2) **Procedural knowledge of ratio**
3) **Challenges experienced by learners in solving ratio problems**

These themes were formed using the coding system Atlas.ti^{TM}. Different codes were grouped together to form these themes. The first theme I will discuss is *conceptual knowledge of ratio*.

4.4 Conceptual knowledge of ratio

This theme was formed by five different sub-themes, including *correct answers* from solving the given problems, which were either *non-proportional problems* or *proportional problems.* This was also evidence that *learners could reason proportionally.* The code *non-proportional problems done well* were also evidence of *learners' conceptual* and *procedural knowledge* of *ratio,* which is associated with *correct answers*. Figure 5 shows the theme *conceptual knowledge of ratio* and the codes that formed the theme.

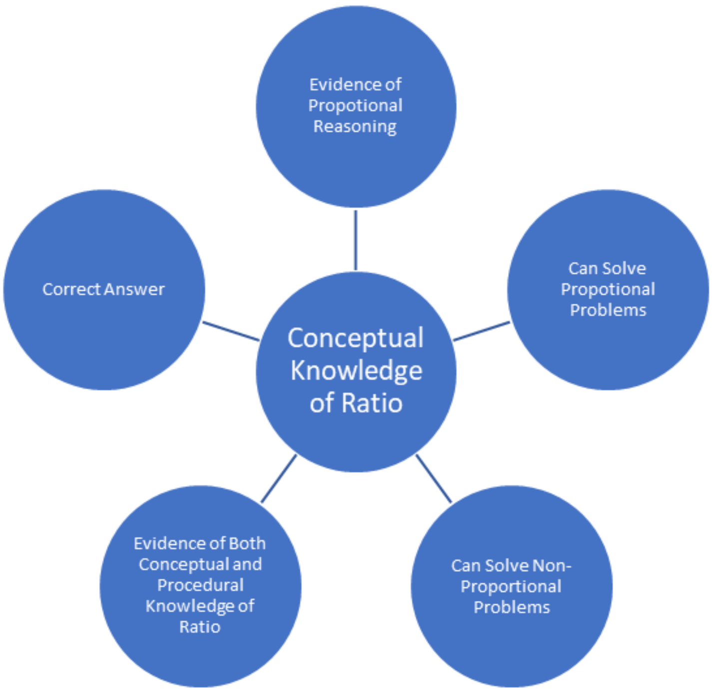


Figure 5: Theme: Conceptual knowledge of ratio

The analysis indicated that most learners completed Question 1 of the test correctly. Below is the interview transcript of Learner 2, who was one of the participants who got Question 1 right and was the only learner who got Question 4 right.

Researcher: Can the answer 8:5 also be written as 5:8

Learner 2: No.

Researcher: Explain why?

Learner 2: Because they asked the ratio of the length to the width, therefore the ratio 5:8 is wrong, because it is now the ratio of the width to the length not the length to width.

Researcher: Will I be correct if I write the answer in Question 1 as 16:10?

Learner 2: Yes.

Researcher: Ok. Explain how these two are the same.

Learner 2: 8:5 is equal to 16:10. You get 16:10 by multiplying the ratio 8:5 by 2. I can also write the ratio 7:9 as 14:18.

Researcher: That's good. Can I write the ratio 8:5 as $\frac{8}{5}$? That is, as a fraction?

Learner 2: Yes. Ratios can also be written as fractions

Researcher: In Question 4 you have your answer as 10 000:100 000. Where did you get the 10 000 from?

Learner 2: I calculated one tenth of 100 000 and I got 10 000. Therefore I wrote the ratio as 10 000:100 000.

Researcher: Show me how you worked it out.

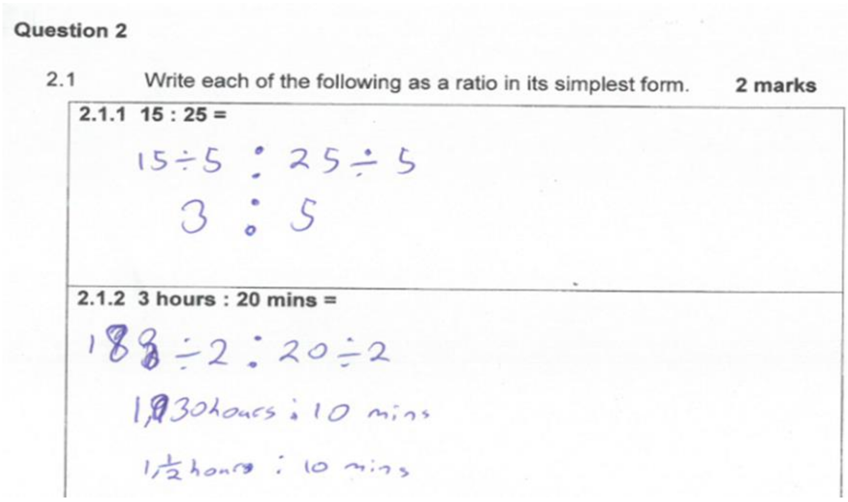
Learner 2: I divided 100 000 by 10 and I got 10 000 for the people with red hair, and the population is 100 000, making the ratio to be 10 000:100 000.

Researcher: This is good, but where did you get the 10 that you divided with?

Learner 2: The statement says one tenth of the population has red hair, therefore I can write one tenth as a fraction ($\frac{1}{10}$) and when I am calculating $\frac{1}{10}$ of 100 000, I divide by 10.

Researcher: This is brilliant.

Learners 2 and 12 were the only participants who could partly solve the problem in Question 2.1.2, which required learners to simplify 3 hours: 20 minutes. Although they could not carry out their procedural steps correctly to the end, they managed to take note of the two different units of measurement of time. They changed the hours to minutes and got 180:20, then divided by 2, and got 90mins:10mins, which they changed back to hours, making their final answer $1\frac{1}{2}$:10mins. During the interview, the learners could explain their procedural steps to me and rectify their mistakes by simplifying the ratio, although it took time for them to recognise their mistakes. Figure 6 shows Learner 2's answer to the two problems of simplifying the ratio.


Question 2

2.1 Write each of the following as a ratio in its simplest form. **2 marks**

2.1.1 15 : 25 =

15÷5 : 25÷5

3 : 5

2.1.2 3 hours : 20 mins =

1̶8̶0̶ ÷2 : 20÷2

1,30 hours : 10 mins

$1\frac{1}{2}$ hours : 10 mins

Figure 6: Learner 2's response to Question 2

During the interview, I could see that the cancelled digits was 180 minutes, as the learner explained how he changed the 3 hours to 180 minutes, further divided the results by 2, and got 90 minutes, which he changed to $1\frac{1}{2}$ hours:10 minutes. This was correct, but he could not simplify the problem further.

Learner 4 was also one of the participants who solved Question 6 correctly, showing evidence of *conceptual understanding* of the problem as well as evidence of proportional reasoning. The learner could also identify the non-proportional problem and solve it using a correct procedural skill. Figure 7 shows how Learner 4 used the ratio-unit/build-up strategy to solve Question 6.

Learner 7 also showed evidence of *conceptual understanding* of *ratio* by solving Question 10 correctly. She was the only learner who solved the problem correctly showing all the procedural steps. The learner could also explain her procedural steps during the interview. Figure 8 shows Learner 7's response to Question 10.

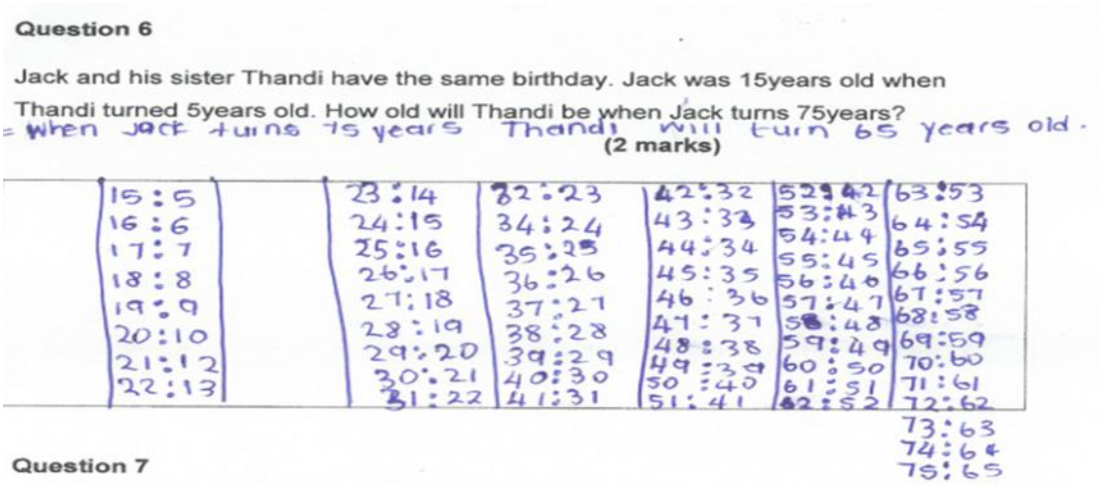


Figure 7: Learner 4’s ratio-unit/build-up strategy for Question 6

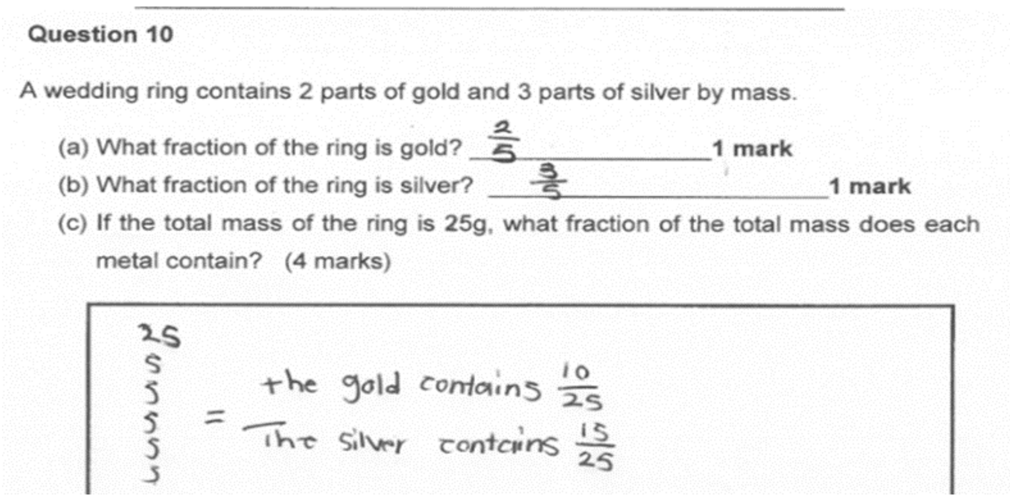


Figure 8: Learner 7’s response to Question 10

The analysis indicated that most participants solved problems that had an additive relationship (non-proportional) better than problems that had a multiplicative relationship. Although the test had five different problems that were non-proportional problems, most participants solved the problem in Question1 and 6 very well compared to Question 3, 9 and 12 that were also additive problems. The learners solved these additive problems by either subtracting or adding a constant number.

Figure 9 and 10 show how learners used sticks to solve multiplicative problems. Both these learners used sticks to calculate ratio multiplicative problems. Figure 9 shows how Learner 16 drew the sticks to help her solve the problem in Question 7: *If 5 chocolates cost R62. How much will 13 chocolates cost?* The first sets of sticks were drawn to divide 62 by 5, and the learner got 12 remainder 2, which was partly correct. During the interview, when I asked her to explain her procedural steps she explained that the "… far left groups of sticks were calculating 62 ÷ 5 = 12 rem 2". When asked why she had to divide, she said "… so as to get the cost of 5 chocolates". On the far right she made 5 groups of 13 sticks, and she was not sure why she made the groups and what she wanted to solve using those sticks but continued to say that she was calculating the cost of 13 chocolates. At the end, the learner had a final answer of R806, but there was no evidence of how she arrived at the answer and she could not explain how she arrived at the answer.

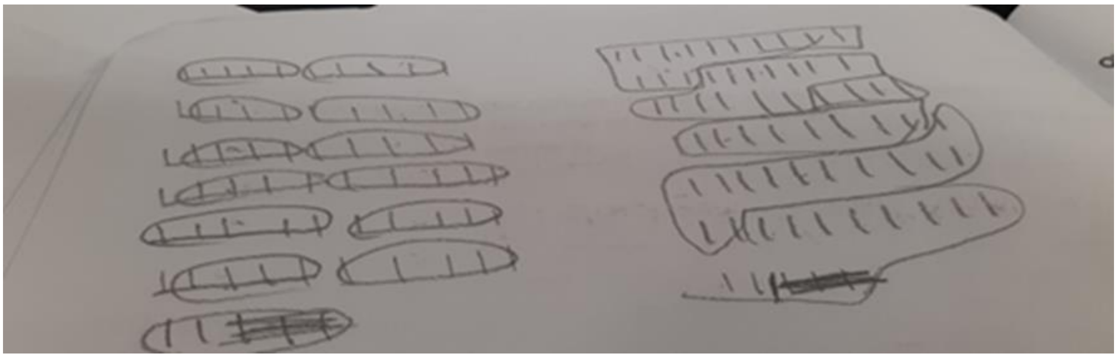
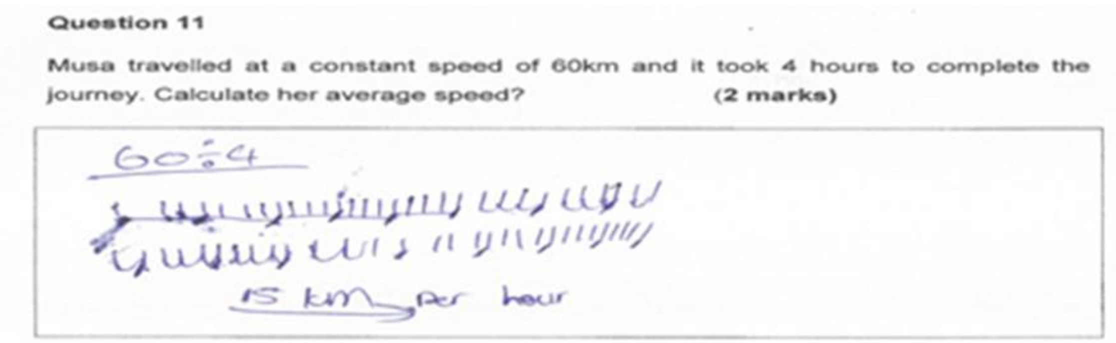


Figure 9: Learner 16's response to Question 7



Question 11

Musa travelled at a constant speed of 60km and it took 4 hours to complete the journey. Calculate her average speed? (2 marks)

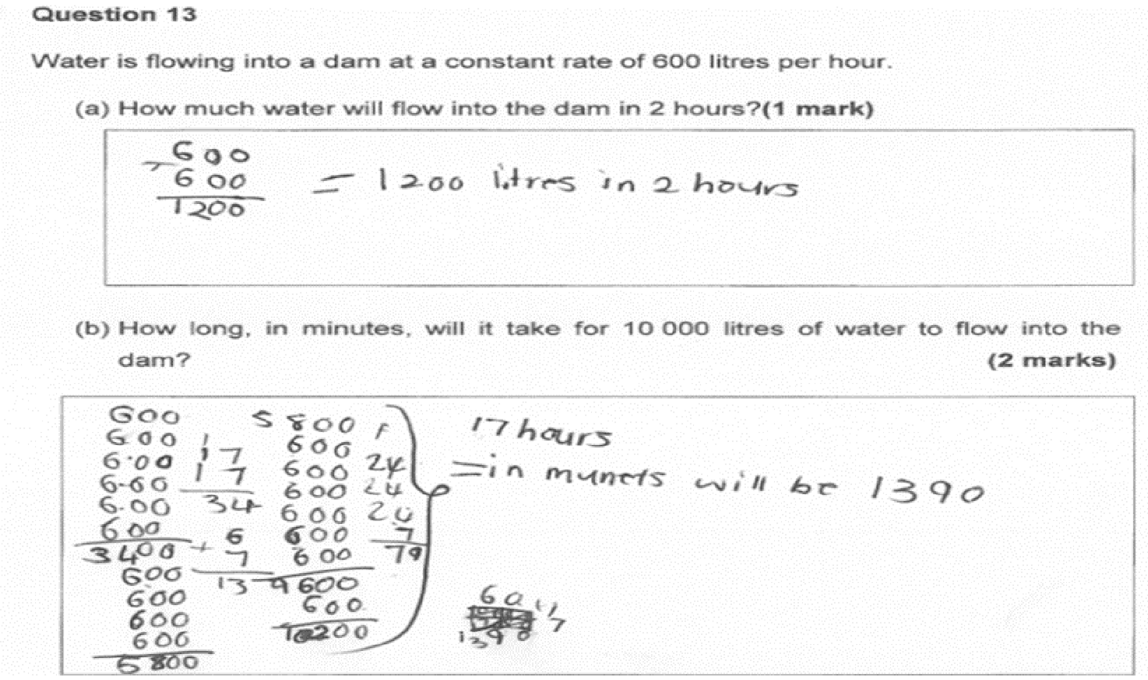
60÷4

15 km per hour

Figure 10: Learner 5's response to Question 11

Learner 5 managed to use sticks effectively to solve the problem in Question 11, which read: *Musa travelled at a constant speed of 60 km and it took 4 hours to complete the journey. Calculate her average speed?* When he was asked why he chose to use sticks to calculate, he said "… I have not yet mastered my multiplication tables; that are why I use sticks to help me count when multiplying or dividing". He indicated that the process is tiring and prone to mistakes, but at the moment he depends on it to solve his multiplicative problems, including division. However, the learner solved the problem correctly.

Most participants could not solve Question 13. Only one participant solved the problem using the repeated addition method of multiplication. Figure 11 shows Learner 7's response to Question 13. The response to the question shows that the participant had a *conceptual understanding* of the problem as well as *procedural knowledge* and could reason proportionally. During the interview, the learner explained her procedural steps and showed understanding of the concept.



Question 13

Water is flowing into a dam at a constant rate of 600 litres per hour.

(a) How much water will flow into the dam in 2 hours?**(1 mark)**

600 + 600 = 1200 = 1200 litres in 2 hours

(b) How long, in minutes, will it take for 10 000 litres of water to flow into the dam? **(2 marks)**

17 hours = in munets will be 1390

Figure 11: Learner 7's answer to Question 13

4.5 Procedural knowledge of ratio

The second theme formed from the data analysis was *procedural knowledge of ratio*. The theme was formed from five sub-themes. These included learners who had *correct procedural steps that were incomplete*; this is evidence of *learners' procedural knowledge of ratio.* Some learners could also *simplify straight forward ratio problems*, and this showed learners' *conceptual* and *procedural knowledge* of *ratio,* which resulted in *correct answers*. Some participants could *solve non-proportional ratio problems* as evidence of *both procedural knowledge* and *conceptual knowledge* of *ratio.* Figure 12 shows the theme *procedural knowledge of ratio* and the sub-themes that forms it.

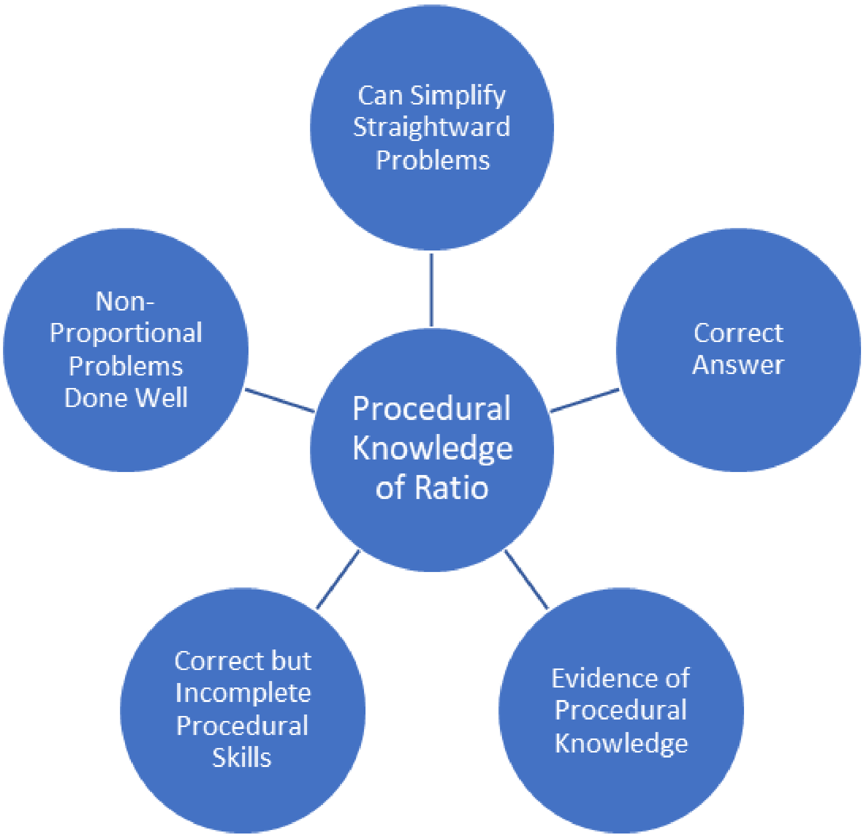
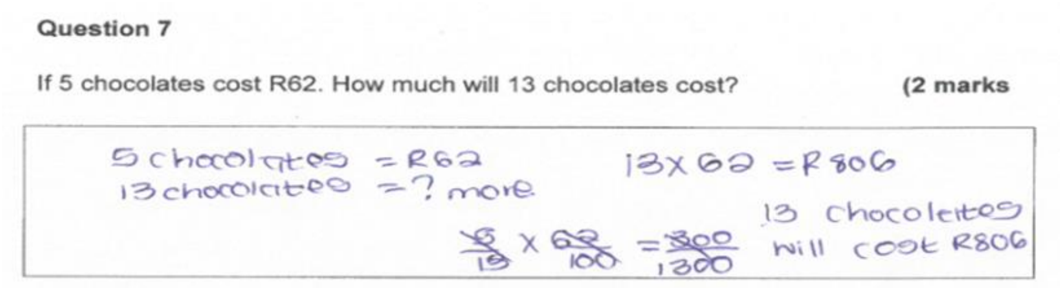


Figure 12: Theme: Procedural knowledge of ratio

According to the analysis, most learners could not solve ratio problems because their knowledge of *ratio* was mainly *procedural knowledge* and not *conceptual knowledge.* In trying to solve the given problems, learners ended up using related procedural steps that showed a lack of *conceptual understanding* of the problem. In some cases, learners could not complete their procedural steps because of a lack of understanding of the problem and the procedures they were doing to solve the problems.

Learner 6 was the only participant who partly solved the problem in Question 7, which read *If 5 chocolates cost R62. How much will 13 chocolates cost?* Learner 6 could write the correct number sentence as the first step to solve the given problem, but could not continue to carry out the steps correctly to the end. Figure 13 shows Learner 6's response to Question 7.


Question 7

If 5 chocolates cost R62. How much will 13 chocolates cost? (2 marks

5 chocolqtes = R62
13 chocolates = ? more

13 × 62 = R806

$\frac{5}{13} \times \frac{62}{100} = \frac{300}{1300}$

13 chocoletes will cost R806

Figure 13: Learner 6's cross-multiplication in response to Question 7

During the interview session I wanted to gain insight into the learner's understanding of the problem and the procedural steps used. The following is the interview transcript between me and Learner 6.

Researcher: I see here in trying to solve this problem you started by writing the number sentence: *5 chocolates = R62*

13 chocolates =? more

Tell me the meaning of this number sentence. Explain how you arrived at it.

Learner 6: Mam. (Silent for a minute and looking confused) I cannot remember what I wanted to calculate with the number sentence, and I cannot explain what it means.

Researcher: I see here at the bottom you have your next step as $\frac{5}{13} \times \frac{62}{100} = \frac{300}{1300}$

Can you try and remember how you arrived at this stage and what you wanted to find with this number sentence?

Learner 6: (Silent for a moment) Mmm, mam, I cannot remember anything, and I do not know where and why I wrote the number sentence. (Sighing) I have forgotten mam.

Researcher: Ok. Tell me now after reading this problem; do you understand what it requires you to do?

Learner 6: Yes, I am calculating the cost of 13 chocolates?

Researcher: Ok. From this problem, what is it that you are given or told?

Learner 6: I am told that the cost of 5 chocolates is R62.

Researcher: So how would you use this information to help you find the cost of 13 chocolates?

Learner 6: Mmm. Mam. (silent) The cost of 13 chocolates will be 13 × 62.

Researcher: Where are you getting this number sentence?

Learner 6: Mmm. Mam. They told us that the cost of 5 chocolates is R62; therefore, for me to get the cost of 13, I will multiply 13 by 62.

Researcher: Ok. (Not convincing)

Learner 6: Is it correct mam? (Curious about wrong and correct answers)

Researcher: We need to cross check and prove our answer. Ok?

Learner 6: Ok mam.

Learner 15 used a different procedural skill to try and solve the same problem in Question 7. In his procedural steps, the learner wanted to calculate the cost of 13 chocolates. Therefore, his number sentence was 62 × 13. The learner was convinced that his number sentence was correct and his procedural steps were good, because during the interview session he was eager to explain all his steps and did it with lots of confidence, as seen in his facial expression. Although the learner could not explain to me how the cost of 5 chocolates would help him find the cost of 13 chocolates, he was convinced that his method was correct and did not give me the chance to change his thinking. In his method, the learner had to draw sticks to help him calculate accurately. He drew 13 sticks to represent the 13 chocolates, and each stick was to be multiplied by 62. Therefore, underneath each stick he wrote the number 62. These numbers were then grouped in twos and added together to make the multiplication simpler. The addition continued until the final answer. The method was good and interesting, although it was prone to mistakes. In the end, his final answer was R654. When I tried to look back at the procedural steps to check the answer, he had committed some errors in addition. Figure 14 shows how Learner 15 used his repeated addition method of multiplication to try and solve the problem in Question 7. This is evidence that most learners lacked *conceptual understanding* of ratio problems and ended up using *procedural skills* they remembered as associated with the problems.

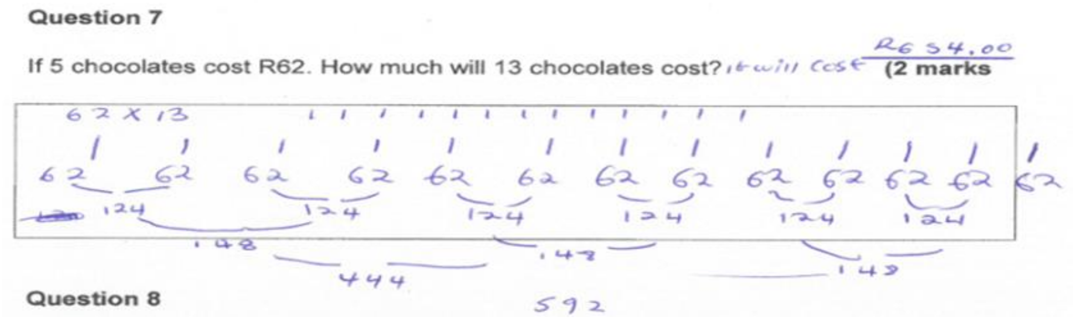
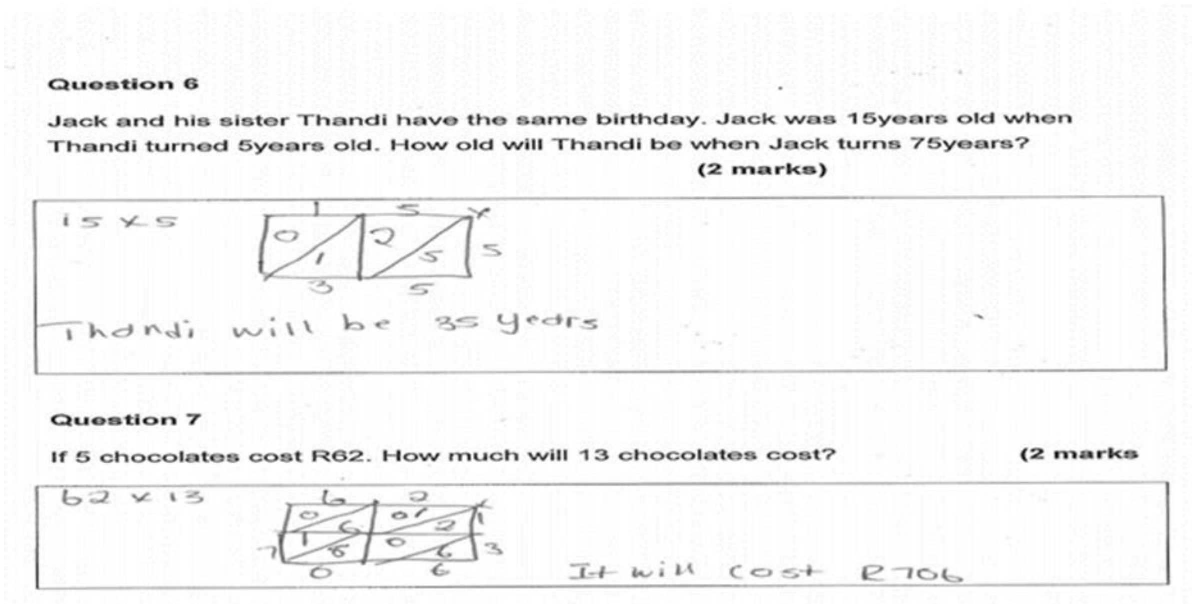


Figure 14: Learner 15's repeated addition method as response to Question 7

Another group of participants used the lattice method to help them solve proportional ratio problems (Figure 15). Most learners who used the lattice method could not explain how it works and why they have to divide and add using the boxes they drew. I am not familiar with this method; therefore, the learners' explanations were inconclusive and did not help me understand the method.

I asked the teacher, who explained how the method works. The analysis showed that most of them did not use the method effectively and did not understand why they used it; it was just a procedural skill that they used to help them solve proportional problems in *ratio.* The lattice method allowed learners to break down numbers into single numbers that were easy and manageable to multiply, but unfortunately, the participants could not use it properly. Figure 15 shows how Learner 17 used the lattice method solve the problems in Question 6 and 7.

Most learners who used the lattice multiplication method to solve proportional ratio problems did not apply the *procedural skill* appropriately. They did not understand why they had to draw the boxes and all the calculations that were to be done. As a result most learners got the calculations with errors and they were not able to reflect on their steps to correct the errors.


Question 6

Jack and his sister Thandi have the same birthday. Jack was 15years old when Thandi turned 5years old. How old will Thandi be when Jack turns 75years?

(2 marks)

15 x 5

Thandi will be 35 years

Question 7

If 5 chocolates cost R62. How much will 13 chocolates cost? (2 marks

62 x 13

It will cost R706

Figure 15: Learner 17's lattice multiplication method

4.6 Challenges experienced by learners in solving ratio problems

The last theme is the *challenges experienced by learners in solving ratio problems.* The theme was made up of 11 different codes, which were linked together to form the theme. These included, *learners who experienced challenges in simplifying ratio problems*, which resulted in learners getting *wrong answers, learners who were not sure of the concept and as a result gave two options* for me to choose. This was maybe due to a lack of *conceptual knowledge* of *ratio*, which is also associated with *partly incorrect answers* or *no answers* at all. The other codes that formed the theme included *misconceptions*, which were a result of *conceptual errors* and were also caused by *wrong procedural skills that lead to correct answers.* Furthermore, codes such as a *lack of procedural fluency* resulted in learners *not able to complete the steps of solving a problem*, which was associated with *partly incorrect answers.* Lastly, a *lack of proportional reasoning resulted in learners using incorrect procedural skills in solving problems,* which was maybe the result of a *lack of conceptual knowledge* of *ratio.* Figure 16 shows the sub-themes that built this theme.

Figure 16: Challenges experienced by learners in solving ratio problems

During the data analysis, I noticed that Learner 5 was one of the participants who got Question 1 correct, which required them to write the ratio of the length to the width of the given triangle. The learner had written the answer as 16:10, which was correct; therefore, I had to interview the learner to gain insight into the learner's *conceptual understanding* of the problem. Below is the interview transcript between me and Learner 5.

Researcher: How did you get the answer 16:10?

Learner 5: I added the two sides of the rectangle which are (5 + 5 = 10; 8 + 8 = 16) and I got 16:10.

Researcher: Why did you add the sides of the rectangle?

Learner 5: I wanted to find the ratio of the rectangle.

Researcher: Do we find the ratio by adding the sides?

Learner 5: Yes.

Researcher: Ok. What if I have 3 boys and 5 girls? What will be the ratio of girls to boys?

Learner 5: It will be 6:10.

Researcher: How did you get 6:10?

Learner 5: I added 3 + 3 = 6 and 5 + 5 = 10

Researcher: Where did you get the other 3 and 5 that you are adding?

Learner 5: (silent)

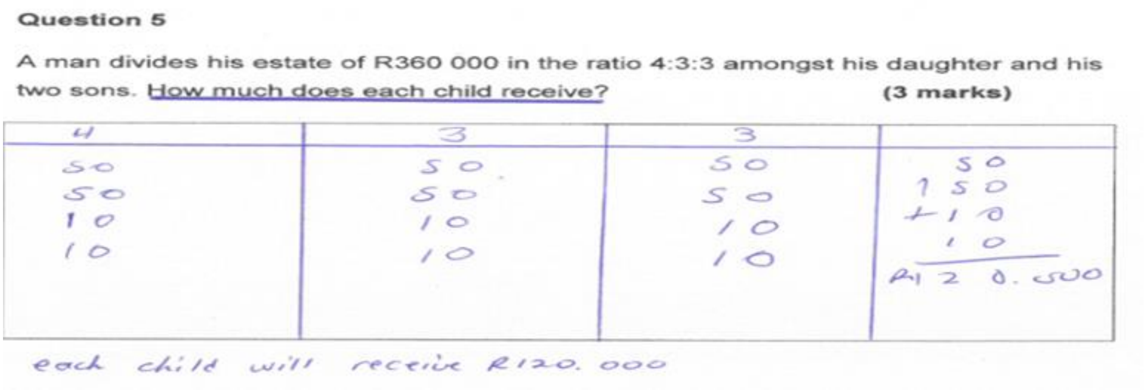
Researcher: When finding the ratio of girls to boys, do we use the same method as finding the ratio of the rectangle?

Learner 5: Yes, because we are finding ratio.

Researcher: Ok.

From the interview session, it was evident that the learner had a misconception. Instead of finding the ratio of the rectangle, the participant had calculated the perimeter of the rectangle. This was evidence that the learner had not yet grasped the concept of *ratio.* Hansen et al., (2017) explained that a misconception may be due to learners' inability to comprehend what the task is asking and a lack of relevant experience or knowledge related to the topic or concept. During the learning process, it is of utmost importance that concepts such as *ratio* that are conceptually demanding are taught using a wide variety of physical empirical situations and representations to help learners construct their own knowledge. Prior knowledge of natural numbers, fractions, decimals, percentages, factors and multiples is very important in the process of learning *ratio*. These concepts must be fully grasped by learners in order for them to be able to successfully acquire new concepts such as *ratio*. A lack of acquisition of prior knowledge can lead to misconceptions, such as the one shown above. It is therefore the teacher's duty to identify misconceptions and correct them so that learners gain a *conceptual understanding* of *ratio*.

In solving the given ratio problems, most learners experienced challenges, and some of these challenges were misconceptions. This was the case when solving Question 5: *A man divided his estate of R360 000 into the ratio 4:3:3 among his daughter and his two sons. How much does each child receive?* When solving this problem, some learners did not recognise that they were sharing using the *ratio.* As a result, they solved the problem as if they were sharing using whole numbers. That was a misconception that was exposed by most learners. Figure 17 shows an example of how Learner 15 solved the problem.



Question 5

A man divides his estate of R360 000 in the ratio 4:3:3 amongst his daughter and his two sons. How much does each child receive? **(3 marks)**

4	3	3	
50	50	50	50
50	50	50	150
10	10	10	+10
10	10	10	10
			R120.000

each child will receive R120.000

Figure 17: Learner 15's misconception in response to Question 5

During the interview session I asked the learner about his procedural steps so as to understand his *conceptual understanding* as well as his *procedural understanding* of the problem. The following is the interview transcript between me and Learner 15.

Researcher: Explain the steps you took to arrive at the final answer.

Learner 15: I divided R360 000 by 4 by 3 by 3 and I got 120 000.

Researcher: Ok. Why did you divide by 4, 3 and 3?

Learner 15: Mmm. (silent for a minute) I wanted to find out how much each child was going to get.

Researcher: Did you understand what the question requires you to do?

Learner 15: Yes, mam. I have to share R360 000 in the ratio 4:3:3. (learner reads the ratio as 4 is to 3, 3 are to 3)

Researcher: Ok. Do you remember how we share using ratio?

Learner 15: Mmm. Yes, mam. Here we divide by 3 because we are sharing between 3 children.

Researcher: Does the statement say we are sharing between 3 children?

Learner 15: (looking confused) Mmm. Mam. (reads the statement again) Yes! There are 2 boys and 1 girl, making them 3.

Researcher: Ok. Are you saying sharing in the ratio 4:3:3 (I read the ratio correctly) is the same as sharing between 3 children, because the statement said we are sharing in the ratio 4:3:3, and you are saying we must share between 3 children?

Learner 15: (looking confused; silent for a minute) Mmm … Yes, mam! Here I shared in the ratio 4:3:3 and each child got R120 000.

Researcher: Ok. So does that mean when we are sharing in the ratio 4:3:3 each person will get an equal share.

Learner 15: Yes.

Researcher: Ok.

Not all learners did well in Question 5. Most could not recall how to share using *ratio*, and what made it more difficult was using a ratio with 3 digits. Learner 6 had an interesting answer. In solving Question 5, the learner changed the ratio 4:3:3 to the whole number 433 (misconception). She then divided $\frac{360\ 000}{433}$ = 1 168. Figure 18 shows all her procedural steps for trying to solve the problem.

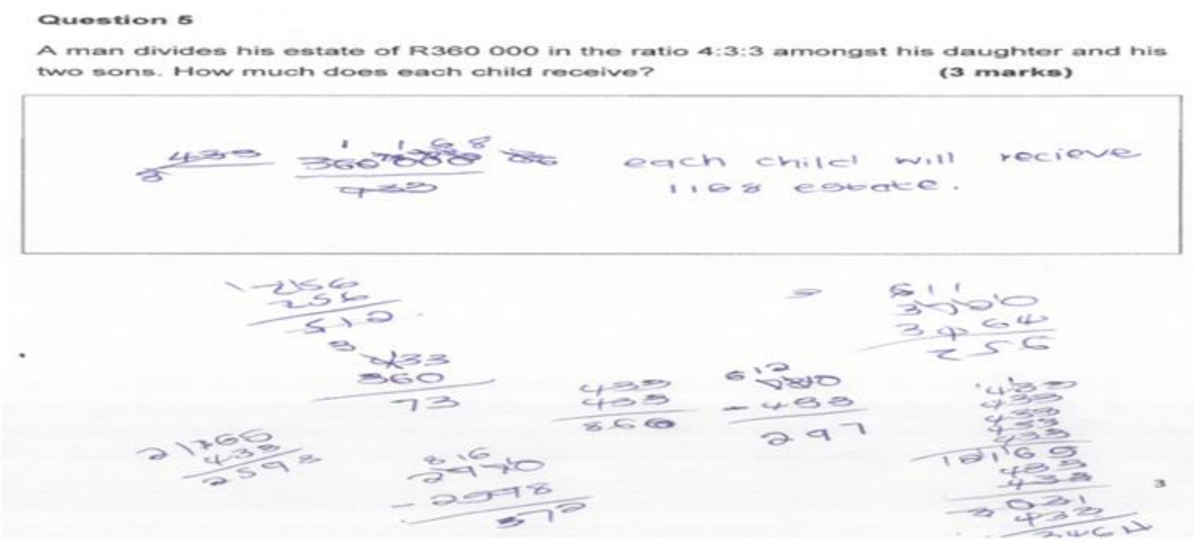


Figure 18: Learner 6's misconception in responding to Question 5

During the interview session, I asked her how she got 433, and why she divided by 433. She said that she was dividing by the ratio 4:3:3, which she could not read properly. When asked if 433 are the same as the ratio 4:3:3, the learner said no.

This same question was solved in a different way by Learner 17. The learner showed a glimpse of *procedural knowledge* of sharing using *ratio*, although the procedural steps were incomplete and ended up being another misconception. Figure 19 shows how Learner 17 solved the problem in Question 5.

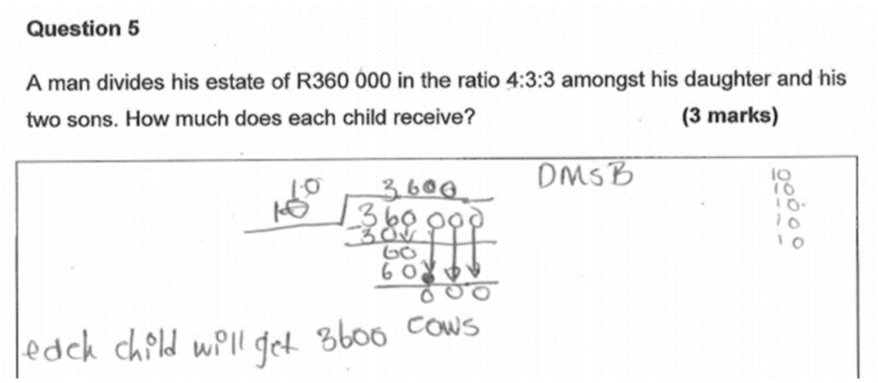


Figure 19: Learner 17's misconception in response to Question 5

During the interview session, I asked Learner 17 to explain her procedural steps. Below is the interview transcript between me and Learner 17.

> **Researcher:** Where did you get the number 10 that you divided with?
>
> **Learner 17:** I added the ratio 4:3:3 (4 + 3 + 3 = 10) and I got 10.
>
> **Researcher:** Why did you add the ratio 4:3:3?
>
> **Learner:** (silent)
>
> **Researcher:** Is dividing by the ratio 4:3:3 the same as dividing by 10.
>
> **Learner:** Mmm. Yes, because if I add 4:3:3, I get 10.
>
> **Researcher:** Ok

When closely analysing the learner's procedural steps to solve the problem, I could guess that the learner could recall the first steps of sharing using *ratio*, but unfortunately her knowledge was not good enough to recall all the procedural steps, and this resulted in a misconception.

4.7 Conclusion

In this chapter, I analysed all the findings of this study using examples from the learners' test scripts and the interview sessions. This analysis shed light on learners' *conceptual* and *procedural knowledge* of *ratio.* I identified the different strategies that learners used to solve *ratio* problems as well as their *conceptual understanding* of *ratio.* Furthermore, I identified learning difficulties such as misconceptions and errors that learners experienced while solving ratio problems. The difficulties identified were mostly the result of a lack of *conceptual knowledge* of *ratio.* I will further discuss these results in the next chapter, and I will draw conclusions and discuss the limitations as well as recommendations for the study.

CHAPTER 5: DISCUSSION OF RESEARCH FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The purpose of this study was to gain an in-depth understanding of learners' knowledge of *ratio*. This was done through research questions that I developed to achieve the objectives of the study. The research findings will be discussed according to the themes developed during the data analysis. The three secondary research questions will also be addressed. In addition, I will reflect on how the theoretical framework practically helped to guide the entire study. I will use of the learners' test responses that were used in the findings to support the discussion. The chapter ends with recommendations and conclusions based on the findings of this study.

5.2 Conceptual knowledge of ratio

This theme was created based on the participants who could solve the given ratio problems correctly and could clearly explain their procedural steps, demonstrating a deep understanding of the problem that was visible in the way they solved the given problem and in the interview.

Most participants could solve the ratio problem that was presented visually using a diagram. The physical representation helped learners' cognitive processes, thereby stimulating their reasoning abilities. According to Schwedt and Gates (2018), "visual representation can reduce the cognitive load during engagement. Specifically, when used in a supportive way with text, visuals can represent the text and provide additional nonverbal memory prompts" (p. 55). It could therefore be argued that diagrammatic representation should be used during the teaching of *ratio* to enhance *conceptual understanding*.

Problems that involved simplifying ratios and part-part ratios were not done well by most participants. Only a few participants could solve these problems fairly well, demonstrating their proficiency in *ratio*. Kilpatrick et al., (2001) explain that learning with understanding is more powerful than simply memorising because the organisation improves retention, promotes fluency, and facilitates learning related material. These learners demonstrated their proportional reasoning skills and showed all their procedural steps in their solutions. This was evidence of the learners' procedural fluency in *ratio,* which results in learners' computational fluency. These participants could also further explain their understanding of the problems, clarifying all the possible connections between *ratios* and the different representations of *ratio*. Lamon (2014) asserted that "part of understanding a concept is knowing what it is not and when it does not apply" (p. 5). The few learners, who could solve the problems that involved simplifying ratios and part-part ratios, demonstrated their proportional reasoning skills and *conceptual knowledge* of the different types of *ratios* by identifying and articulating equivalent ratios. By giving learners the opportunity to explain their understanding, I could understand their thinking as well as their *conceptual knowledge* of *ratio.* This process is very important in the teaching and learning of concepts such as *ratio*, because it gives teachers the opportunity to know the learners' strengths and weaknesses in particular concepts so as to plan the lessons in a way that will help the learners to improve their proficiency in mathematics. Kilpatrick et al. (2001) stated that "in order to understand the success or failure of a problem solving attempt, one needs to know about the individual's knowledge base, problem solving strategies, metacognitive actions, beliefs and practices through discussion" (p. 119).

Looking at the analysis, only few learners could solve the *ratio-rate* problems in the test. These learners used sticks to help them solve the problems. At this stage of the Senior Phase, learners are expected to have mastered the basic multiplication facts and to be able to solve abstract multiplicative problems using methods that are less concrete; however, a few learners were still using sticks to solve multiplicative problems. This is evidence that the learners' multiplication, division and fraction concepts are still at a lower level than the required standard in the Senior Phase. This was evidenced by (Spaull & Kotze, 2015) in a research commissioned by the President's Education initiative in South Africa, in this research it was concluded that the conceptual knowledge of learners in mathematics is well below than expected at the respective grades and the development of higher order skills is stunted.

Grade 7, learners are expected to be able to solve problems with whole numbers up to at least 9 digits, and it will become difficult for learners still relying on sticks to calculate multiplicative problems that involve more numbers. It is important that learners master their multiplicative concepts, which include fractions, multiplication and division, at an early stage to help them acquire their knowledge of *ratio* easier. Simon and Placa (2012) argued that “for learners to gain access to *ratio* and proportion, they must have a solid understanding of multiplicative reasoning” (p. 36).

If learners at this stage are still using sticks to help them solve *ratio* problems, it shows that further grades will be more mathematically challenging for them as they will be expected to work with much more challenging multiplicative problems that require them to have basic multiplicative skills. These challenges eventually lead to a situation of “silent exclusion” as learners continue falling further behind in their conceptual understanding even when they proceed in higher Grades (Spaull & Kotze, 2015). In Section 2.2.2 and 2.2.3, I discussed that most learners perform very poorly in Grade 9 national and international Mathematics examinations and this might be the cause. Using sticks and repeated addition to solve proportional problems make it difficult for them to acquire concepts easily and solve problems fluently.

It was evidenced that most learners in the study could solve problems that were non-proportional (additive). Some could also explain their procedural steps to solve the problem and give alternative methods that can be used to solve the problems. This was evidence that these learners had a *conceptual understanding* of *ratio*, as they could represent the given problems in different ways and explain how the different representations were useful for different purposes. This reflected that the learners' performance in *ratio* might be because of the different intellectual abilities that appear in a classroom situation. In most classroom situations we find learners of different abilities mixed together to form a class. In any class there will be those who are outstanding in their academic performance, and in most cases, they constitute the smallest percentage of the class. These are the learners who can grasp concepts taught regardless of how it is presented. They take a very active part in their learning, can think relatively and apply their proportional reasoning skills when solving problems. These are the learners who can link related learnt concepts to help them acquire new concepts such as *ratio.* They show a great *conceptual understanding* of *ratio.*

It is the teachers' duty to take note of the different abilities in a Mathematics classroom and use this knowledge to help them plan their lessons so that all learners in class benefit equally and acquire their own knowledge of a particular concept.

Kilpatrick et al. (2001) claimed that “competence in an area of inquiry depends upon knowledge that is not merely stored but represented mentally and organised (connected and structured) in ways that facilitate appropriate retrieval and application” (p. 118).

As noted earlier, most learners could solve non-proportional problems better than proportional problems. This shows that these learners have not yet moved from additive thinking to multiplicative thinking, as required at this level. Another reason why they could solve the non-proportional problems might be the natural instinct they have when solving ratio problems. Dole and Wright (2015) argued that learners “naturally tend to consider change situations in additive terms when solving ratio problems” (p. 73). They usually see change or react to change, especially in *ratio*, as additive rather than multiplicative. This is also evidence that the learners’ cognitive development is still at a lower level. Their multiplication reasoning skills are not yet fully developed and they are still reliant on additive methods, which is still good but become difficult to do at this stage. It shows that their mathematical thinking has not yet developed as required at this stage when they are doing concepts that require them to think multiplicatively instead of to additively

In the study, some participants committed slips during their procedural steps, which were a result of carelessness on their part. Some of these slips were committed when learners were transferring their work from the scrap books they used to the spaces provided for them in the test. This was done because learners felt that the spaces provided in the test to express their procedural skills were too small. According to Ojose (2015), “slips are wrong answers due to processing. They are not systematic but are sporadically carelessly made and are easily detected and spontaneously corrected” (p. 8). Some of these errors were not due to a lack of *conceptual understanding* of the problem. Learners committed these errors because of carelessness, which is also my fault as I did not provide the participants with enough space to express themselves freely. The moderators noted the problem earlier during the pilot study as I had first put only a few lines for learners to show their calculations. The moderators corrected me and I changed my space allocation to satisfaction, and the pilot study went well without detecting this problem. It is therefore advisable to the teachers to always provide enough space to cater for all the procedural steps those learners might need to do so as to avoid learners committing slips as shown in Figure 20.

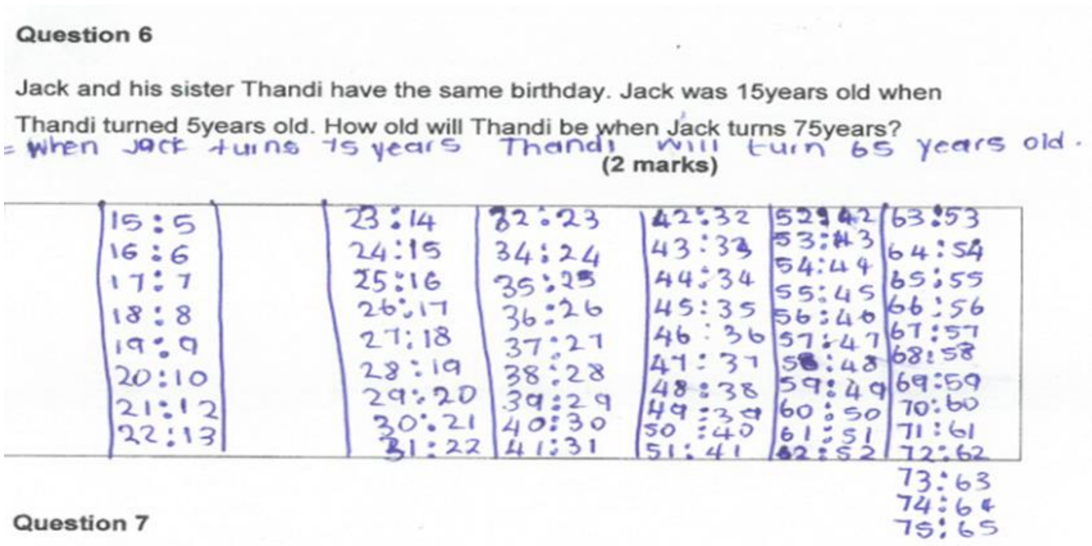
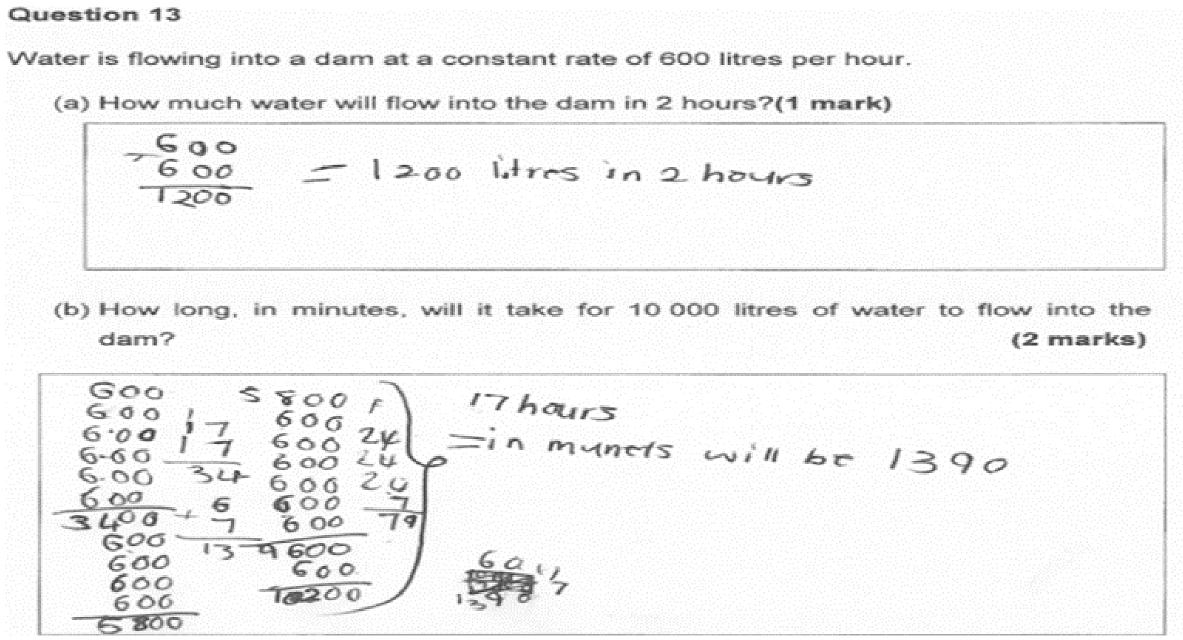


Figure 20: Errors in learner's response when copying their procedural steps from the scrap paper

When solving Question 6, the learner recognised the significance of numbers in the ratio problem and kept the balance by adding equivalent amounts of each value up to the end, but in the process he missed 11 years for Thandi's age and 33 years for Jack's age. This was a slip caused by carelessness.

Most participants in the study solved Question 6 in the same way as shown in Figure 20. Sparrow, Kissane and Hurst (2010) explained that learners who "recognise the need to preserve an equivalent relationship between two values are starting to think relatively, and use a multiplication strategy to build up to a new value" (p. 112). The learners who did not use the ratio-unit/built-up strategy to solve the same problem, solved the problem by subtracting the constant value as follows: 75 – 10 = 65 years. This was evidence that those learners could think proportionally and used their proportional skills to solve the problem. They also showed evidence of *conceptual knowledge* of *ratio.* Dole and Wright (2015) claimed that "one of the key aspects of proportional reasoning is being able to consider situations of change in both additive and multiplicative terms, adjusting appropriately according to context" (p. 9).

Besides using sticks to help them solve proportional problems, it was evidenced that those few learners who could solve proportional problems used repeated addition as their strategy to calculate multiplicative problems. This might be because learners' multiplicative thinking is still at a lower level than their grade level, and they are still thinking additively rather than multiplicatively when solving proportional problems. Jacob and Willis (2001) argue that repeated addition may be an appropriate beginning to maintain that interpretation of multiplication, but it is ultimately disabling because it does not provide children with important multiplicative structures. This will be a barrier to their learning, especially when they have to solve problems with 5-digit to 9-digit numbers. Figure 21 shows how learners used the repeated addition strategy to solve proportional problems.


Question 13

Water is flowing into a dam at a constant rate of 600 litres per hour.

(a) How much water will flow into the dam in 2 hours? **(1 mark)**

= 1200 litres in 2 hours

(b) How long, in minutes, will it take for 10 000 litres of water to flow into the dam? **(2 marks)**

17 hours

=in munets will be 1390

Figure 21: Learner's response using repeated addition

Most learners in the study could not solve problems that involved part-whole ratio. This reflected a lack of knowledge of fractions as ratios. Learners were unable to differentiate the part-part relationship of fractions to the part-whole relationship. Only a few learners could solve these types of problems. Watson et al. (2007) explained that when a ratio connects two parts of the same whole relationship, learners may not adequately differentiate the part-part and the part-whole relationship. This indicates that learners have not fully grasped the concept, "as part of understanding a concept is knowing what it is not and when it does not apply" (Lamon, 2014, p. 5).

5.3 Addressing the first research question: Conceptual knowledge

The theme *conceptual knowledge* of *ratio* goes hand in hand with the research question that asks *What is learners' conceptual knowledge of ratio?* It was noted earlier that most Grade 7 learners could solve ratio problems with visual representations. The use of visuals in problems seemed to stimulate learners' cognitive processes, thereby helping them retrieve previous knowledge learnt on a related concept. Arcavi (2009) writes that visual display of information enables learners to "see" the story, to envision some cause-effect relationships, and possibly to remember it vividly. It was also evidenced that most Grade 7 learners could solve non-proportional, additive problems better than proportional, multiplicative problems because their poor multiplicative skills normally interrupt their thinking. In most cases, learners still relied on using sticks and/or the repeated addition method to solve multiplicative ratio problems. This is evidence that learners have not yet developed their multiplicative reasoning skills and have not yet made the major transition in their mathematical thinking and still depend on additional reasoning skills to solve ratio problems. Pelen and Artut (2016) stated that learners make several major transitions during their years of primary, secondary and high school, and one of these changes is a shift from additive concepts to multiplicative concepts. Learners could also solve problems with part-part ratios much better that part-whole ratios, and they could interpret the 2-dimensional shape given in Question 1 to find the ratio.

It can be deduced that Grade 7 learners could identify the ratio of given items more easily, especially if the problem has a diagram to help them recall the concept. Learners could solve part-part ratio problems very well. They could also understand additive (non-proportional problems) ratio problems. Only a few could solve ratio problems involving rate.

5.4 Procedural knowledge of ratio

From the analysis, this theme was formed by sub-themes that included cases where the participants used correct procedural steps to solve the problem, but could not complete the steps correctly to find the solution of the problem, as well as those learners who used the correct procedural steps to solve the problems correctly. The study evidenced a few learners who used the correct procedural steps to solve the problems but could not complete the steps. This might be due to a lack of *conceptual understanding* of *ratio*, which hindered the learners from completing the procedural steps. Kilpatrick et al. (2001) claimed that if learners know only rules and procedures in a routine way and practice them as isolated bits of knowledge that do not connect, they find learning new concepts difficult. *Procedural knowledge* of *ratio* is very important in the learning of *ratio*, but it is very important that learners' knowledge of *ratio* is not only acquired procedurally, resulting in the learner only mastering the different procedural skills of solving *ratio* routinely but lacking the *conceptual knowledge* of *ratio*. This results in learners being unable to apply their procedural skills correctly to solve problems because they easily forget concepts or procedures that they practice without a deep *conceptual understanding* of the concept. Figure 22 is an example of how some learners could not complete their procedural steps correctly.

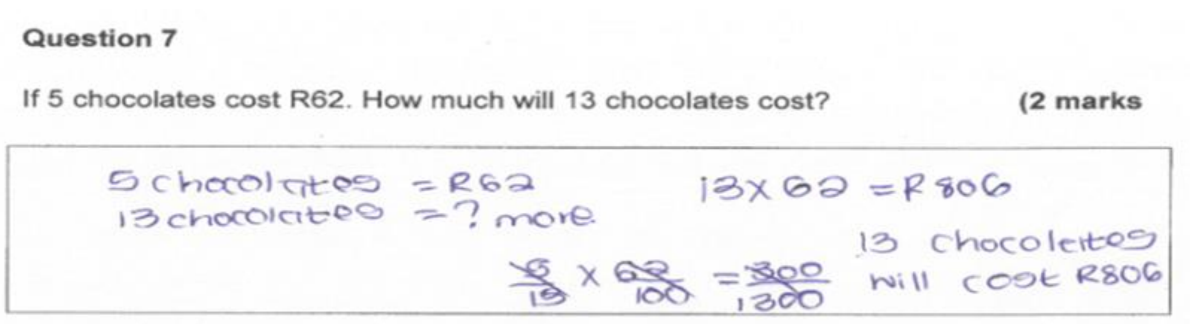


Figure 22: Example of learner being unable to correctly complete their procedural steps

Looking closely at the problem, it is evident that the learner was attempting to solve the problem using the cross-multiplication method, which was used by most learners to solve proportional ratio problems. Unfortunately, the learner could not remember the steps correctly, got confused, and ended up not remembering how to link her first steps correctly with the problem. Muttaqin et al. (2017) claimed that using the cross-multiplication algorithm to solve ratio problems is efficient but less meaningful for learners. This suggests that the cross-multiplication method is just a procedural skill that learners grasp to solve ratio problems without any relevant reasoning or understanding of why numbers have to be cross-multiplied when linking with the proportional problems to be solved. The conceptual gap that is created between the procedural and the problem results in learners making errors when solving problems using the cross-multiplication method, as shown in Figure 22.

It was also evidenced that there were some learners in the study who used the lattice method to solve proportional problems. Most of the learners who used the lattice method were unable to use the method correctly. This is evidence that learners did not understand how to use the lattice method or understand the given problems. They used the method as a procedural skill that was learnt to help them solve proportional problems.

5.5 Addressing the second research question: Procedural knowledge

The theme *procedural knowledge of ratio* helped me to answer the research question that asked *how do learners solve problems involving ratio?* It is clear that most learners preferred using the repeated addition strategy to solve proportional problems that require multiplicative skills. This might be because most have not yet mastered multiplication facts and multiplicative thinking. In order to try and bridge this gap, learners resorted to repeated addition and even drawing sticks that they physically counted and made into groups to help them find products or quotients of the given problems. This will become a serious problem that causes retardation in their acquisition of concepts such as *ratio*, which require multiplicative skills. Lamon (2014) stated that one of the reasons learners experience difficulties with *ratio* is that many learners intuitively apply additive strategies rather than using multiplicative thinking. It can be concluded that most learners in this study have not yet reached a level of proficiency in the concept *ratio* as most of them have not yet grasped basic multiplication skills.

Some learners used the ratio-unit/build-up strategy to solve the problem in Question 6: *Jack and his sister Thandi have the same birthday. Jack was 15 years old when Thandi turned 5 years old. How old will Thandi be when Jack turns 75 years?* As mentioned earlier, this was a non-proportional problem that required an additive strategy, which most of them did well. The ratio-unit/build-up strategy they used to solve this problem can be evidence that the participants can reason proportionally. Lamon (2014) explained that "proportional reasoning refers to detecting, expressing, analysing, explaining and providing evidence in support of assertions about proportional and non-proportional relationships" (p. 12). On the other hand, Sparrow et al (2010) argued that when they start recognising the importance of the numbers in problems involving ratio , learners will, at the beginning, try to maintain the balance by adding equivalent amounts to each value. By so doing, they are applying an additive strategy that does not ensure relativeness. It is a sign of pre-proportional reasoning because the learners achieved correct answers without recognising the structural similarities on both sides of the equation. Therefore, the ability to use the ratio-unit/build-up strategy to solve the problem cannot be used as evidence of learners' multiplicative reasoning, but it can be safely said that learners can reason proportionally.

The analysis also showed that most learners used the lattice method to try and solve proportional problems. It was evidenced that most learners who used this method were using it as a procedural skill to help them solve multiplicative problems. It was also noted that most learners who used the method could complete all the steps correctly and did not understand the problems stated as they were just plucking out numbers from the given problems and making number sentences; some were reasonable, but others were completely off the mark. Only one learner used this method successfully. It can therefore be said that learners in this study used the lattice method and the cross-multiplication method as rule driven or mechanised procedures, as they could apply them even in problems where they lacked understanding (Lamon, 2014).

In this study, it could be seen in the way learners solved the given problems that most were just applying learnt procedural skills that lacked procedural understanding as well as *conceptual knowledge* of *ratio*. In this study most learners used repeated addition to solve multiplication problems and some used semi-concrete objects such as sticks to help them divide and multiply. Only one learner attempted to use the cross-multiplication method, most used the lattice method to multiply, and a few used the ratio-unit/build-up strategy to solve non-proportional problems. In most cases learners could not complete their procedural steps correctly, showing gaps in their *procedural knowledge* of *ratio* and using some of the procedures incorrectly.

5.6 Challenges experienced by learners in solving ratio problems

The theme *challenges experienced by learners in solving ratio problems* was formed by 11 sub-themes that focus on those areas where learners did not write answers for the problems, wrote incorrect answers or partly incorrect answers. It also looked at those learners who struggled to simplify ratio problems; who were unsure of their answers; who gave two options as answers; whose steps for solving the problem were incomplete; and who made errors, especially in trying to solve proportional (multiplicative) problems. These errors mostly happened when they used sticks to help them to multiply or divide and when they used the lattice method. Most learners could not correctly draw and count their sticks to solve problems, resulting in them making errors that they were unable to detect during the process, as most of them did not understand the problem and were unsure of the procedural steps they were using. The same applied to those who used the lattice method. Kilpatrick et al. (2001) claimed that "learning with understanding is more powerful than simply memorising, because the organisation improves retention, promotes fluency, and facilitates learning related concepts" (p.119).

This theme is the most important theme formed because it has the most sub-themes. Some of the challenges experienced by the learners were a result of a lack of proportional reasoning as well as a lack of *conceptual* and *procedural knowledge* of *ratio.* Some participants had misconceptions about certain ratio problems. Most learners in the study showed evidence of a lack of proportional reasoning by the way they solved the given problems. Some of them were using any procedural skill they remembered to try and solve the problems. Lamon (2014) explained that the word *reasoning*, taken from proportional reasoning, "suggests that we use common sense, good judgement and a thoughtful approach to solve a problem, rather than plucking numbers from word problems and blindly applying rules and operations" (p. 12).

The misconceptions that were noted during the study were due to a lack of *conceptual understanding* by learners of the problems given. Learners were unable to comprehend what the task required them to do. Hansen et al. (2017) argued that misconceptions may be due to relevant experience or knowledge related to the concept. Figure 23 is an example of one of the misconceptions that was experienced by the participants.

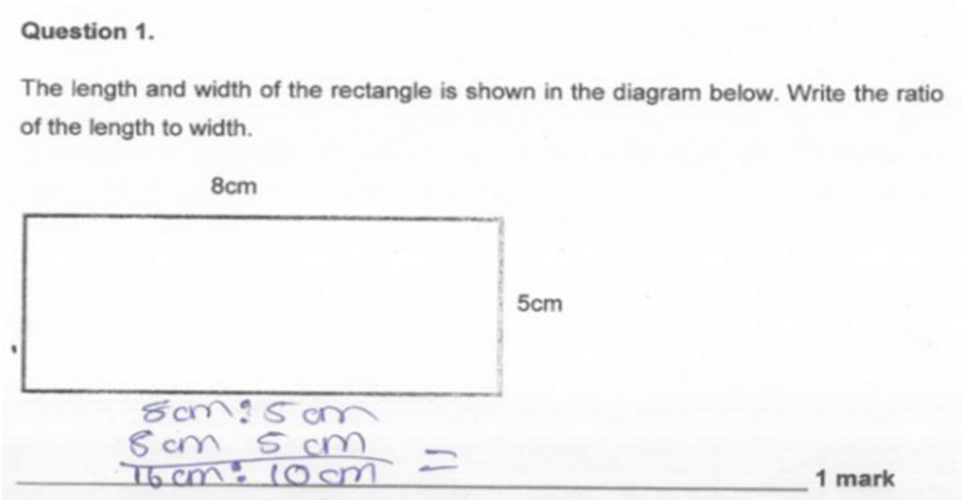


Figure 23: Example of a misconception by a learner

There were other misconceptions that were noted during the analysis, especially when the participants tried to solve a problem that involved simplifying. Some learners misunderstood the word *simplify* and also could not properly interpret the ratio symbol. They thought the question was focused on the commutative laws of addition, even though there were no addition sign; therefore, they wrote their answers as follows:

15:25 = 25:15 and 3hours:20mins = 20mins:3hours.

This is a sign that the learners had no *conceptual knowledge* of how to simplify *ratios*, and they had to use their acquired knowledge that they could easily remember and associate with the given problem to solve the problems. During the interviews, these learners were unable to explain their procedural steps. I came to the conclusion that all these misconceptions were due to a lack of *conceptual* and *procedural knowledge* of *ratio.* Kilpatrick et al. (2001) stated the following:

> When learners have acquired conceptual understanding in an area of mathematics, they see connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence, which then provides a base from which they can move to another level of understanding (p. 118).

I noted that another misconception arose when learners were trying to divide using *ratio*. Most could not solve the problem that involved ratios as quotients. This was evidence of a lack of *conceptual understanding*. In most cases when learners are given a problem to solve, they will usually use familiar problem solving strategies to try and understand the problem. Ojose (2015) claimed that misconceptions exist in part because of learners' overriding need to make sense of the instruction that they receive. Some participants had misconceptions about part-part ratios and part-whole ratios. The confusion between the two ratios made it impossible to solve problems with these types of *ratio*. Watson et al. (2007) argued that learners become confused when dealing with *ratios* and fractions, especially when the *ratio* connects two parts of the same whole relationship. Learners may not adequately differentiate the part-part from the part-whole relationship, leading to a misconception.

In this study, I also noted that there were some learners that did not provide me with adequate data in their test responses or in the interviews. Most of them were just guessing answers: Some were guessing reasonable answers, but most were completely off the mark. Therefore, I did not discuss them much. The theme *challenges experienced by learners in learning ratio* goes hand in hand with the research question that asks *What learning difficulties do learners experience while solving ratio problems?* I will answer this research question in the next section.

5.7 Addressing the third research question: Learning difficulties

The last research question asked *What learning difficulties do learners experience while solving ratio problems?* As discussed before, learners experience numerous challenges in solving ratio problems. These include a lack of *conceptual* and *procedural knowledge* of *ratio*, which resulted in some learners leaving questions unanswered (i.e. without attempting to solve them), others having incorrect answers, and some writing two different answers for me to choose from. Some learners were unable to complete their procedural steps correctly, leading to partly correct answers (Figure 13), and others were using the wrong procedural skills, leading to correct answers (Figure 23).

Learners encounter these problems mainly because they have not yet acquired a deep understanding of *ratio* that includes understanding multiplicative concepts, which involve multiplication and division of whole numbers and fractions. Simon and Placa (2012) claimed that "in order for learners to gain access to *ratio* and proportion, they must have a solid understanding of multiplicative reasoning, which involves new quantities that are integral to multiplication (intensive quantities), in contrast to extensive quantities such as length, mass, area and volume, which can be measured directly or counted" (p. 39). This lack of understanding of multiplicative concepts also affects learners' proportional reasoning skills. The inability of learners to master the multiplicative facts results in most learners making errors when solving proportional problems because of the methods that they use to solve multiplicative ratio problems, such as the lattice method that most learners could not apply correctly. Besides the lattice method, learners used sticks and the repeated addition method to help them solve proportional ratio problems.

Lastly learners also encountered different misconceptions when trying to solve ratio problems in this study. These misconceptions were a result of a lack of understanding of the ratio problems given.

In solving ratio problems, learners experienced difficulties such as a lack of *conceptual* and *procedural knowledge* of *ratio*, which made it difficult for them to understand what the question required them to do. Learners could not express themselves multiplicatively and were still reliant on addition, which they used to try and solve multiplicative ratio problems, while some resorted to using sticks to try and solve proportional problems. This difficulty stems from their lack of understanding of multiplication facts. Errors/slips were also noted as the participants were solving ratio problems.

Next I will focus on how the chosen theoretical framework guided me in the study.

5.8 The theoretical framework's rapport with the research

I chose the strands of mathematical proficiency by Kilpatrick et al. (2001) as a theoretical framework to guide the study. This theory was useful to me as it helped to create parameters for the research to avoid broadening the focus and making the objectives of the research difficult to attain. I only focused on two selected strands of mathematics proficiency, *conceptual understanding* and *procedural fluency,* in learning of the concept *ratio* (Kilpatrick et al., 2001). In this study I looked at these strands as different types of knowledge that facilitate the learning of mathematical concepts, especially the learning of *ratio*. I named the two strands *conceptual knowledge* and *procedural knowledge.* According to Kilpatrick et al. (2001), these two strands of mathematical proficiency must be intertwined to attain mathematical competence in a concept.

The theoretical framework guide d me to reach the secondary research questions. I was able to formulate the three research questions based on the theoretical framework of this study that focused on the *conceptual* and *procedural knowledge* of *ratio. Conceptual knowledge* included learners' understanding of the concept *ratio,* which is their *knowledge* of *ratio*. This looked at what the participants knew about *ratio* and the challenges they have with the concept*. Procedural knowledge* included how the participants solved ratio problems; that is, what were their procedural steps to solve the given problems?

The theoretical framework also helped me to find previous research studies pertaining to the study. This made it easier for me to articulate my literature review as I could search for relevant literature from previous studies to support the findings of this study. Furthermore, I developed the data collection instruments guided by the theory. The self-developed test questions were focused mainly on the two strands of knowledge, namely *conceptual knowledge* and *procedural knowledge* of *ratio*, which was informed by the theoretical framework.

Lastly, the theoretical framework guided me in the data analysis and discussions as I was able to develop codes that were grouped together to form themes/categories, which helped with the articulation of the findings of the study. In conclusion, I view these two strands of mathematical proficiency as the pillars of mathematical proficiency, especially in relation to this study. These two types of knowledge contribute to the basic foundation of mathematical learning of and thinking about *ratio*.

5.9 Limitations of the study

In this study, the first limitation that I experienced was that I did not conduct a lesson observation to see how the learning and teaching of the concept unfolded. This would have helped me understand learners' knowledge of *ratio* better. It also would have helped me understand why the learners are at this stage still reliant on using semi-concrete methods such as sticks to solve proportional problems and why they prefer the additive method when solving multiplicative problems.

While learners attempted almost all questions in the test, there were few learners who did not answer some questions, and when I tried to find out why, they all said that they did not understand the problems. Although, this was a limitation to this study, it did not impact it negatively as I was able to gain knowledge about the learners' *conceptual* and *procedural knowledge* of *ratio* through those learners that were able to express themselves in writing and verbally. However, I would have preferred getting information from all the learners so as to understand their knowledge of *ratio*.

Another limitation was the inability of most learners to explain their procedural steps during the interviews. Some said that they could not remember, while others said they just guessed answers. This made it difficult for me to get a meaningful understanding of learners' knowledge of *ratio*. As participating in the study was not compulsory, this could not be overlooked as a limitation to the study since the learners who did not volunteer to participate might have been the participants who could have provided me with crucial information.

Another limitation to the study was that the study was carried out in a school in Soweto with only one Grade 7 class. Questions are bound to arise from this, such as: Would a bigger sample have produced more informative findings? What findings could have been generated had the study been spread wider to other schools? However, because an in-depth, extensive study was carried out, it led to findings that would not have been generated had the study been done on a larger scale. In light of the above, it should be stated that the study has made a concerted effort to better understand the *conceptual* and *procedural knowledge* of *ratio* of Grade 7 learners.

5.10 Recommendations

Although I do not claim that the results are applicable to all Grade 7 learners in South Africa, a possibility exists that there will be commonalities between this group and others from similar backgrounds. It is therefore recommended that further research should be considered to investigate teaching and learning methods that can facilitate better acquisition of the concept *ratio* in learners and promote proportional reasoning skills at the same time to help learners acquire *conceptual* and *procedural knowledge* of *ratio.*

5.11 Conclusions

In proposing this research, I had set my goal on gaining a deeper understanding of Grade 7 learners' *conceptual* and *procedural knowledge* of *ratio.* My aim was to investigate *knowledge* of *ratio,* because this research showed that *ratio* is a concept that is included in almost all the topics that learners do in the Senior Phase in the South African curriculum. Therefore, the poor performance of Senior Phase learners in Mathematics as evidenced in this research might be caused by, inter alia, the lack of *conceptual* and *procedural knowledge* of *ratio* in Grade 7 learners.

In Chapter 2, an outline of similar studies carried out in other parts of the world was presented. The findings of this study and other previous studies have some similarities in that they conclude that most learners lack *conceptual* and *procedural knowledge* of *ratio.* Most of the participants' proportional reasoning skills were at a lower level and or have not yet been developed. Learners struggled to solve proportional ratio problems that are multiplicative and required them to use their multiplicative reasoning skills. This study also found that most learners' multiplicative facts were poor and they solved multiplicative problems using the repeated addition method and sticks. In most cases their procedural steps were not correctly completed, mostly due to a lack of *conceptual understanding* of the problem and errors. The use of sticks showed that learners were still reliant on semi-concrete objects.

The only difference that this study evidenced is that it was able to investigate deeply into learners' *conceptual* and *procedural knowledge* of *ratio*. It was able to further investigate all the procedural skills displayed by learners in a way that clarified and explained their knowledge of *ratio*, including their ability to solve non-proportional problems using the ratio-unit/build-up strategy and the challenges they experienced when solving *ratio* problems.

Teachers should also be reminded that errors and misconceptions result from knowledge construction by the learners using prior knowledge. It is therefore very important that teachers use teaching methods that are child centred and cater for individual differences to allow each learner in the classroom to be able to construct their knowledge in a way that allows them to make mistakes and create misconceptions that facilitates the construction of new knowledge. In this way learners can continue learning in a way that will facilitate good acquisition of new knowledge, which will lead to mathematical proficiency as described by Kilpatrick et al. (2001). In the teaching and learning process of *ratio*, it is vital that learners are given an opportunity to reflect on their procedural steps in solving problems by explaining their thoughts verbally; this way the teacher can detect learners' *conceptual* and *procedural knowledge* of *ratio* as well as their errors and misconceptions. By giving learners this opportunity, teachers can help learners acquire new knowledge by using the zone of proximal development in this learning situation and learners will be given an opportunity to correct their errors and misconceptions in a learner-centred way that promotes growth.

The chosen theoretical framework helped me to construct the primary and secondary research questions. I particularly focused on two strands of mathematical proficiency: The *conceptual understanding* of *ratio* and the *procedural fluency* of *ratio*, which I termed *conceptual* and *procedural knowledge* of *ratio.* This study was a qualitative research design approached from an interpretivist perspective. The methodology and the case study research design helped me to be actively involved with the learners and to deepen my understanding of the Grade 7 learners' knowledge of *ratio*. I could thoroughly investigate more learners' knowledge of *ratio* because of the selected sampling method and interviews that shed more light on the topic.

It can be summarised that the Grade 7 learners in the study know how to solve simple ratio problems that require them to recall basic ratio facts when given a diagram to help them recall the concept. They can also solve non-proportional, additive ratio problems much better than proportional, multiplicative ones. It was also found that learners struggle with their multiplication facts and use semi-concrete objects such as sticks to help them solve division and multiplication problems. In most cases they relied on repeated addition to solve multiplicative problems, showing that their mathematical thinking is still below their grade level; therefore, they find it difficult to solve proportional ratio problems. They need to grow mathematically and start thinking multiplicatively instead of additively; in this way they will be able to acquire concepts such as *ratio* that are multiplicatively easier.

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ANNEXURES

ANNEXURE A: ASSESSMENT TEST

Concept: Ratio Grade 7

Term 3 Date: ____________________

Assessor: Mrs S. Bango Moderator: __________________

Time: 60 minutes

Learner's name: ____

Class: ____

Instructions:

1. Read and understand the given questions carefully.
2. Answer all questions in the spaces provided.
3. The use of calculators is not allowed.
4. Show all your working.
5. Write neatly.

Question 1.

The length and width of the rectangle is shown in the diagram below. Write the ratio of the length to width.

____

Question 2

2.1 Write each of the following as a ratio in its simplest form.

2.1.1) 15:25 =
2.1.2) 3 hours:20 mins =

Question 3

A bag of beads contains 7 white beads and 9 red beads.

What is:

(a) The ratio of white beads to red beads?

(b) The ratio of red beads to the total number of beads?

Question 4

Diepkloof has a population of 100 000. One tenth of the population has red hair. What is the ratio of the number of people with red hair to the rest of the population?

Question 5

A man divides his estate of R360 000 in the ratio 4:3:3 among his daughter and his two sons. How much does each child receive?

Question 6

Jack and his sister Thandi have the same birthday. Jack was 15years old when Thandi turned 5years old. How old will Thandi be when Jack turns 75years?

Question 7

If 5 chocolates cost R62. How much will 13 chocolates cost?

Question 8

A rectangle is drawn in the ratio 3:2. Determine the length of the longer side if the shorter side is 13,2 cm long

Question 9:

Circle the correct answer below

If today Zodwa ran fewer laps in more time than she did yesterday, would her running speed be

(a) faster (b) slower (c) exactly the same

Explain your answer

Question 10

A wedding ring contains 2 parts of gold and 3 parts of silver by mass.

(a) What fraction of the ring is gold? ________________

(b) What fraction of the ring is silver? _______________________

(c) If the total mass of the ring is 25 g, what fraction of the total mass does each metal contain? (4 marks)

Question 11

Musa travelled at a constant speed of 60 km and it took 4 hours to complete the journey. Calculate her average speed?

Question 12

Two cyclists, Joseph and Kagiso, are cycling at the same speed around a cycling track. Kagiso started cycling before Joseph arrived at the track and had completed **9** laps when Joseph had completed **3**. When Kagiso had completed **15** laps, how many laps will Joseph have completed?

Question 13

Water is flowing into a dam at a constant rate of 600 litres per hour.

(a) How much water will flow into the dam in 2 hours?

(b) How long, in minutes, will it take for 10 000 litres of water to flow into the dam?

End of the question paper

ANNEXURE B: SEMI-STRUCTURED INTERVIEWS

These interview questions will be a follow-up on the written assessment. They will help the researcher gain a deep understanding on how learners solve problems involving ratio. (*Procedural knowledge* and *conceptual knowledge*)

a) I see here you wrote your answer as 8:5, and it is correct. Well done! Will I be correct if I write my answer as 5:8? (Yes/No) Why?

Do you understand what is meant by the word simplest form? When we are simplifying numbers we usually divide both numbers. Which number can we use to divide 25 and 15 without leaving a remainder? Can you simplify $\frac{6}{8}$? Is $\frac{6}{8}$ the same as 6:8?

Is the ratio 7:9 the same as 9:7

Explain how you got this answer? Do you understand what the question requires you to find? How can you use the given information to find the unknown?

Explain in your own words the given problem? Do you understand what it means to divide the estate in the ratio 4:3:3 among the 3 children? Is the money going to be equally shared? Is the ratio 4:3:3 the same as 433?

Explain your method to me.

- I see in your answer you divided 62 by 5. Explain what you wanted to find and why? You also used the square block diagram to help you multiply. Explain how this multiplication method works?

1.2 I see in your answer you have a statement (5 = R62 therefore 13 =? more) this is correct. Well done! Explain how you will use this information to calculate your next step and to arrive at your final answer?

Why did you multiply 13, 2 × 3?

Do you understand what you are asked to find?

I see you got the answer for question 10 correct. This is good. Explain to me your steps that led to these correct answers.

1.3 Why did you write $\frac{2}{3}$ as a fraction to represent the gold part of the ring and $\frac{3}{2}$ to represent the silver part of the ring? Write down the total of these fractions that make a whole (ring).

Explain to me why you are adding 60 + 60 + 60 + 60 = 240. Do you understand what you looking for (the unknown) speed? You have heard friends and family at home talk about speed of cars. Tell me what do they say?

Explain in your own words the given problem. I see in your answer you have 9 – 6 = 3 therefore to find Joseph's laps when Kagiso had ran 15 laps you calculated 15 – 6 = 9. Excellent. I love it. Can you use the same method to calculate this problem?

Susan can walk 6 km in 60 minutes. How many kilometres can she walk in 75 minutes?

1.4 Can you use the same method to calculate this problem? If (yes/no) show me how you will solve the given problem.

Do you understand what you are asked to find? How can you use the given information to help you find the unknown?

-End-

ANNEXURE C: DEPARTMENT OF BASIC EDUCATION ETHICS CLEARANCE

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**06 September 2019**
Validity of Research Approval:	**04 February 2019 – 30 September 2019** **2019/265**
Name of Researcher:	**Bango S**
Address of Researcher:	**1232 Mum Street , Fleurhof**
	Extension 5, Fleurhof
	Florida, 1709
Telephone Number:	**071 100 3334**
Email address:	**siduduzile.bango@gmail.com**
Research Topic:	**An investigation into Grade 7 learners' knowledge of ratio.**
Type of qualification	**MEd: Mathematics Education**
Number and type of schools:	**One Primary School**
District/s/HO	**Johannesburg North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

10/9/2019

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

1. Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.
2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.
4. A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
5. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
6. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
7. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
8. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
9. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
10. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
11. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
12. On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
13. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
14. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 10|09|2019.............................

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

ANNEXURE D: INVITATION LETTER: PARTICIPANT

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

Department of Science, Mathematics and technology Education

1232 Mum Street
Fruehauf extension 5
Florida
1709
4 March 2019

Dear participant:

(Grade 7 Mathematics learner)

Invitation to participate in a study

I am a Master's student at the University of Pretoria in the Faculty of Education. I wish to invite you to participate in a study titled: **"An investigation into learners' knowledge of ratio"**. The purpose of the study is to investigate the *conceptual* and *procedural knowledge* of Grade 7 learners in the learning of *ratio*. The intent is to explore the difficulties that the learners experience with reference to the learning of *ratio*, and also investigate the possible sources of these difficulties.

This letter intends to inform you of what will happen if you agree to participate in the study. You can decide if you want to participate or not. If you agree, you will be asked to sign the attached consent form as acceptance of the invitation and agreement to participate in this study. You may refuse to participate in the study or withdraw your participation at any stage during the study without forwarding any reason(s).

The research details are explained below:

- The process will take place at the school.

- You will be requested to spend about 30 minutes after lessons with the researcher so that the process may be explained to you. You will be afforded the opportunity to ask any questions relating to the study.

If you agree to participate in the study, the following events will take place.

- I, the researcher, will administer you an assessment test on ratio. Mark it and analyse your responses. The test will shed light on the types of errors you make when solving ratio problems.
- I will then select a few learners to interview based on the assessment test results in order to gain more understanding of the areas of difficulty that you experience with regards to solving mathematics problems on ratio.
- The test scripts will be stored in safe place during and after the study.
- To protect your privacy as a participant, I will keep your name and that of the school anonymous and confidential at all times.
- I do not foresee any harm or risk against you during participation in the research.
- You will not receive any incentives for agreeing to participate in the study. However, we hope that your participation in this study will contribute reasonably towards learning ratio more effectively.

Should you have any questions or concerns pertaining to this study, you can contact the researcher, Siduduzile Bango or siduduzile.bango@gmail.com and the supervisor Dr R. D. Sekao at 012 420 4640 or david.sekao@up.ac.za.

____________________ ____________________

Researcher-Signature Date

____________________ ____________________

Supervisor-Signature Date

ANNEXURE E: INVITATION LETTER: PARENTS

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

1232 Mum Street
Fleurhof extension 5
Florida
1709
4 March 2019

Dear Parent/guardian

Invitation to participate in a study

I am a Master's student at the University of Pretoria in the Faculty of Education. I wish to invite your child to participate in a study titled: **"An investigation into Grade 7 learners' knowledge of ratios"**. The purpose of the study is to investigate the *conceptual* and *procedural knowledge* of Grade 7 learners on ratio. The aim is to explore the learning difficulties that the Grade 7 learners experience in learning *ratio.*

You can decide if you want your child to participate or not. If you agree, you will be asked to sign the attached consent form as an indication that you grant permission for your child to participate in this study. You may refuse to allow your child to participate in the study or withdraw his/her participation at any stage during the study without giving any reason(s).

The research details are explained below:

- The process will take place at the school premises after school times.
- I, the researcher, will request permission from the school Principal to administer an assessment test on ratio after school times.

- The test scripts of the learners will be marked and analysed and stored in a safe locked cupboard during and after the study will be stored at the University of Pretoria for minimum 15 years.
- A pilot study will be done before your child is administered the test to check its reliability and specialist Mathematics Senior Phase facilitators from the Department of Basic Education will moderate the test to check its validity.
- I will thereafter conduct semi-structured interviews with some learners as part of the data collection process for the research. The purpose of the interviews is to gain a deeper understanding of how learners solve ratio problems.
- I will make audio recordings of the interviews and take field notes throughout.
- To ensure privacy of the learners as participants, I will keep your name, the name of your child and that of the school anonymous and confidential.
- I do not foresee any harm or risk against any of the learners who will participate in the research.
- In case your child experience emotional trauma or anxiety during data collection, a school counsellor will be available to assist them in this regard.
- No teachers will be available during data collection.
- If you agree that your child takes part in the research, your child must be available during data collection process.
- You will not receive any benefits or incentives for allowing your child to participate in the study. However, we hope that your child's participation in this study will contribute significantly towards effective learning of ratio.

Should you have any questions or concerns pertaining to this study, you can contact the researcher, Siduduzile Bango on 071 100 3334 or siduduzile.bango@gmail.com, my supervisor Dr R. D. Sekao on david.sekao@up.ac.za and Co-supervisor

______________________	______________________
Researcher-Signature	Date
______________________	______________________
Supervisor-Signature	Date

ANNEXURE F: INVITATION LETTER: SCHOOL

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Faculty of Education

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

1232 Mum Street
Fleurhof extension 5
Florida
1709
22 February 2019

The Principal

Vulamazibuko Combined School

Diepkloof, Soweto

Johannesburg

Request for permission to conduct research at your school.

I am a Master's student at the University of Pretoria in the Faculty of Education. I wish to apply for permission to conduct a research on the study titled **"An investigation into Grade 7 learners' knowledge of ratios".** The purpose of the study is to investigate the *conceptual* and *procedural knowledge* of Grade 7 learners in learning ratio. The study intends to explore the difficulties that learners experience in learning *ratio* and find the possible sources of these difficulties as this will assist with coming up with interventions that will minimise the learning difficulties in question and possibly lead to more effective teaching and learning of *ratio*.

Please see more details below and note that should you grant me permission you will be requested to release a signed letter permitting the study to take place.

The process of field work is detailed below:

- The process will take place at school after school times.
- Data will be collected through an assessment Test and semi-structured interviews.
- I, the researcher, will administer the test after school for minimum 60 minutes.
- No teachers will be involved in the data collection process or in any part of the research.
- Participants will not be given their answer scripts.
- This process will be followed by interviews with the learners. The purpose of the interviews is to gain understanding of the areas of difficulty that learners experience with regards to solving mathematics problems on ratio.
- The test scripts will be stored in safe locked cupboard during and after the study at the University of Pretoria for minimum 15 years.
- The name of the school will be kept private at all times in order to ensure anonymity and confidentiality.
- A pilot study will be done to test the reliability of the assessment test and specialist Mathematics facilitators from the Department of Basic Education will moderate the assessment to assess its validity.
- Interviews with the learners will also be scheduled to take after school times.
- Participation in the study is fully voluntary and all participants will be given the option of withdrawing from the study at any time should they wish to do so.
- I do not foresee any harm or risk against the participants.
- A school counsellor will be available to assist learners who may feel traumatised or anxiety during data collection. No extra fee will be needed to pay the counsellor.
- The school will not receive any incentives or direct benefit for granting permission for the study to take place. However, I hope that the findings and recommendations reported upon completion of the study, which a copy will be forwarded to the school, will reasonably improve teaching and learning of ratio.
- Should you have any questions or concerns pertaining to this study, you can contact the researcher, Siduduzile Bango orsiduduzile.bango@gmail.com and the supervisor Dr R. D. Sekao at david.sekao@up.ac.za.

Researcher-Signature Date

Supervisor-Signature Date

ANNEXURE G: INVITATION LETTER: COUNSELLOR

DEPARTMENT OF SCIENCE, MATHEMATICS AND TECHNOLOGY EDUCATION

1232 Mum Street
Fleurhof extension 5
Florida
1709
4 March 2019

Dear School counsellor

Invitation to participate in a study

I am a Master's student at the University of Pretoria in the Faculty of Education. I wish to invite you to participate in a study titled: **"An investigation into Grade 7 learners' knowledge of ratios"**. The purpose of the study is to investigate the *conceptual* and *procedural knowledge* of Grade 7 learners on ratio. The aim is to explore the learning difficulties that the Grade 7 learners experience in learning *ratio.*

I would like you to be available at school during the data collection process to assist me in case there are any learners who might require your assistance during this time. You can decide if you want to participate or not.

The research details are explained below:

- The process will take place at the school premises after school times.
- I, the researcher, will request permission from the school Principal to administer an assessment test on ratio after school times.
- I will thereafter conduct semi-structured interviews with some learners as part of the data collection process for the research. The purpose of the interviews is to gain a deeper understanding of how learners solve ratio problems.
- I will make audio recordings of the interviews and take field notes throughout.
- To ensure privacy of the participants, I will not mention your name in any document.
- I do not foresee any harm or risk against any of the learners who will participate in the research.
- In case a learner experience emotional trauma or anxiety during data collection, I will ask you to assist them in this regard.

- No teachers will be available during data collection.
- Be available at the school during data collection times.
- You will not receive any benefits or incentives for taking part in the study.

Should you have any questions or concerns pertaining to this study, you can contact the researcher, Siduduzile Bango on 071 100 3334 or siduduzile.bango@gmail.com, my supervisor Dr R. D. Sekao on david.sekao@up.ac.za and Co-supervisor

____________________	____________________
Researcher-Signature	Date
____________________	____________________
Supervisor-Signature	Date